R measurements, and hadronic contributions to muon $g - 2$ and $\alpha_{\rm em}(M_Z)$

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Constraining the hadronic contributions to
the muon's anomalous magnetic moment.
Mini-Workshop: R-Measurements at BES-III
and 13th Meeting of the Radio MonteCarLOW Collaboration

ETC* Trento, Italy, April 10-12, 2013

Tanks for support by the DFG – SFB 1044 at Mainz University

Outline of Talk:

- ❖Introduction
- ❖Motivations
- ❖R data what can we do, what do we need
- ❖Comment on VP subtraction
- ❖Adler function controlled QCD
- ❖Remark on $\gamma\gamma \to \pi\pi$
- ❖Conclusion

Introduction

Real and virtual photons are everywhere! couple to charged matter with strength

α one of our most fundamental parameters.

$$\alpha^{-1}(a_e) = 137.0359991657(331)(68)(46)(24)[0.25 \text{ ppb}]$$

How do virtual photons propagate? quantum fluctuations produce and absorb virtual charged particle pairs ⇒ vacuum polarization and charge screening

$\alpha(E)$

- leading hadronic effects in electroweak precision observables:

muon anomaly $a_\mu = (g_\mu - 2)/2 \Leftrightarrow \alpha(m_\mu)$ and $\alpha(M_Z)$

Present situation: (after KLOE & BaBar)

$$\Delta\alpha^{(5)}_{\rm hadrons}(M_Z^2) = 0.027510 \pm 0.000218$$

$$0.027498 \pm 0.000135 \quad \text{Adler}$$

$$\alpha^{-1}(M_Z^2) = 128.961 \pm 0.030$$

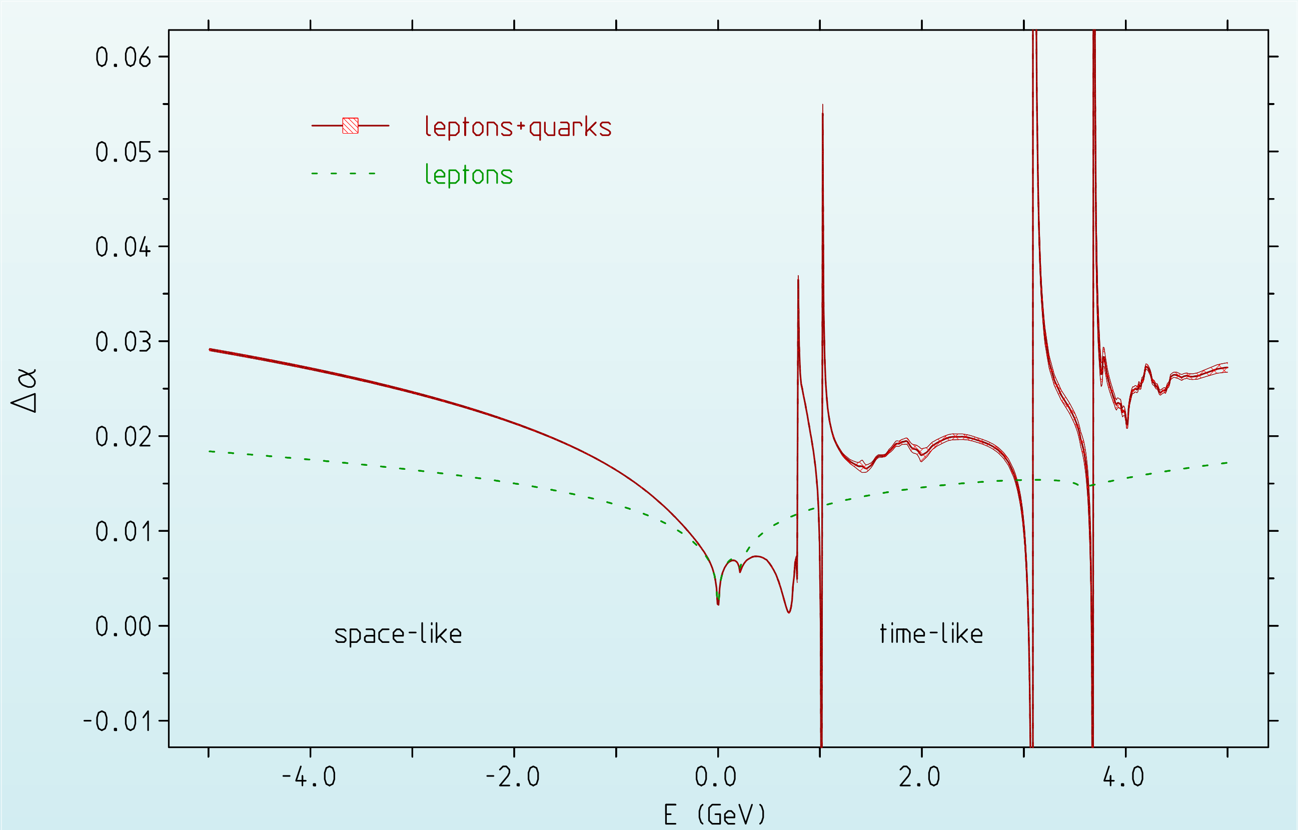
$$128.962 \pm 0.018 \quad \text{Adler}$$

❖0.25 ppb ⇔ 139.58 ppm loose 5.3×10^5 in precision

❖effective fine structure constant least well known SM parameter for W and Z boson physics

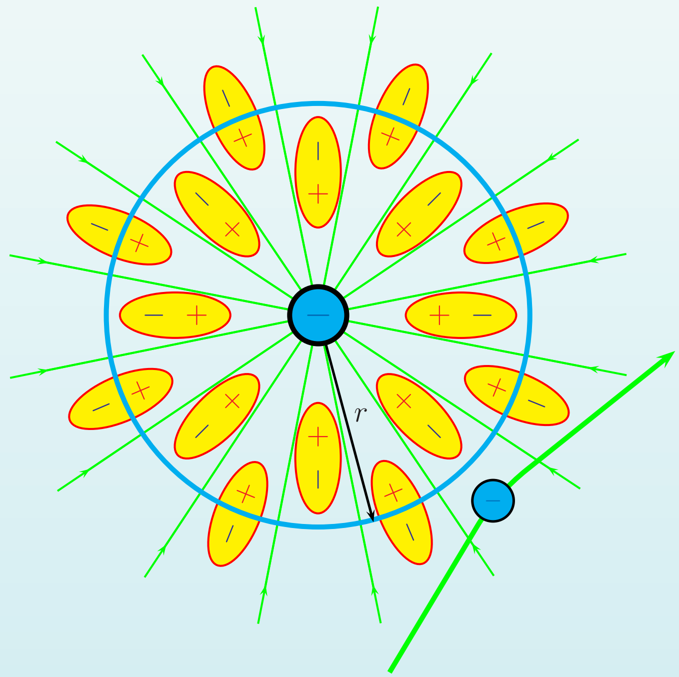
muon $g-2$: $\alpha^{-1}(m_\mu) = 136.067675(978)$

❖0.25 ppb ⇔ 0.72 ppm loose 1.4×10^3 in precision



Shift of the effective fine structure constant $\Delta\alpha$ as a function of the energy scale in the time–like region $s > 0$ ($E = \sqrt{s}$) vs the space–like region $-s > 0$ ($E = -\sqrt{-s}$). The band indicates the uncertainties

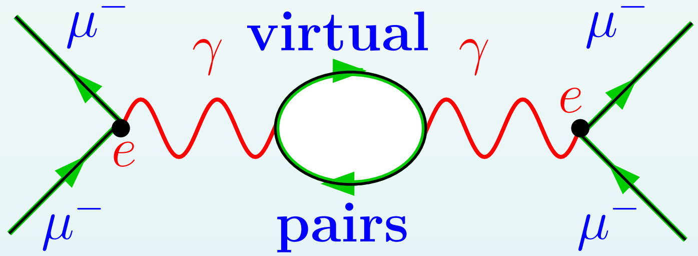
- large effects several % , complicated energy dependence !



In a collision of impact energy E the effective charge is the charge contained in the sphere of radius $r \simeq 1/E$, which due to vacuum polarization is larger than the classical charge seen in a large sphere ($r \to \infty$)

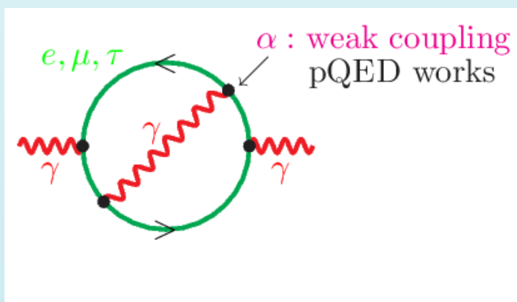
$\Downarrow$

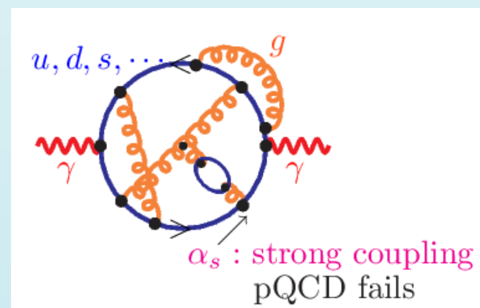
charge screening (charge renormalization).



$$\gamma^* \to e^+e^-, \mu^+\mu^-, \tau^+\tau^-, u\bar{u}, d\bar{d}, \cdots \to \gamma^*$$

❒ lepton contribution reliably calculated in perturbation theory, $\alpha(0) = 1/137.036..$ small.





❒ quark/hadron contributions not reliably calculated in perturbation theory; $\alpha_s(E)$ "strong", actually ill-defined at scale m_μ; pQCD fails

The way out:

What do we need for calculating $\alpha(s)$ and $a_\mu^{\rm had}$?

$$\alpha(s) = \frac{\alpha}{1-\Delta\alpha} \quad ; \quad \Delta\alpha = -e^2 {\rm Re}\,\left(\Pi'(s) - \Pi'(0)\right)$$

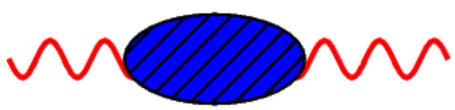
$$\Pi'_{\rm had}(q^2) \quad =$$

1pi blob

Dispersion relation:

$$\Pi'_\gamma(q^2) - \Pi'_\gamma(0) = \frac{q^2}{\pi} \int\limits_0^\infty ds \frac{{\rm Im}\Pi'_\gamma(s)}{s\,(s-q^2-i\varepsilon)}$$

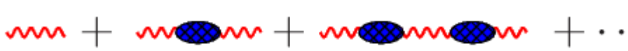
$$\frac{1}{1-\Delta\alpha} \quad \Leftrightarrow$$



full

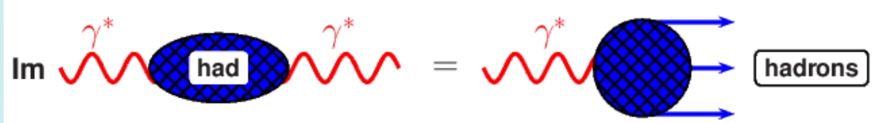
photon propagator

$\simeq$



Optical theorem:

$${\rm Im}\Pi'_\gamma(s) = \frac{1}{12\pi} R(s)$$



non–perturbative approach using experimental data and dispersion relation

What we need:

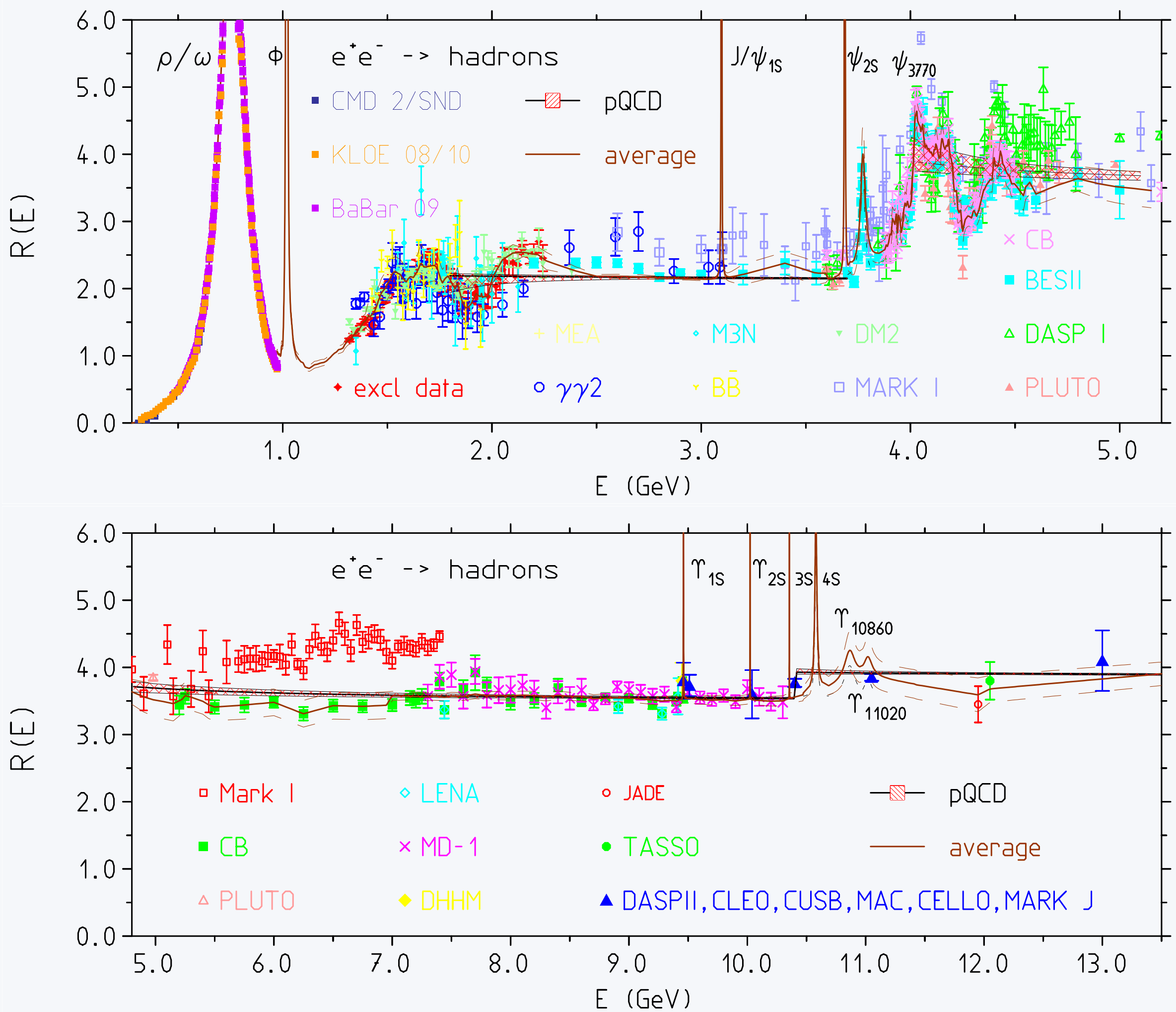
$$\Delta\alpha(s) \equiv -4\pi\alpha \mathrm{Re}\left[\Pi'_\gamma(s) - \Pi'_\gamma(0)\right]$$

What experimenters give us (after working hard):

$$R_\gamma(s) \equiv \frac{\sigma(e^+e^- \to \gamma^* \to \text{hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)} = 12\pi \mathrm{Im}\Pi'_\gamma(s) \quad \overset{DR}{\Rightarrow} \quad 12\pi \mathrm{Re}\Pi'_\gamma(s)$$

$$\Delta\alpha^{(5)}_{\mathrm{had}}(s) = -\frac{\alpha s}{3\pi}\left(\mathrm{P}\!\!\int_{4m_\pi^2}^{E^2_{\mathrm{cut}}} ds' \frac{R^{\mathrm{data}}_\gamma(s')}{s'(s'-s)} + \mathrm{P}\!\!\int_{E^2_{\mathrm{cut}}}^{\infty} ds' \frac{R^{\mathrm{pQCD}}_\gamma(s')}{s'(s'-s)} \right)$$

- Note only works thanks to **asymptotic freedom** of QCD; no data on tail
- Experimental **uncertainties** carry over to "prediction"



Data today vs. QCD prediction

Motivation I: the new muon $g-2$ experiments
Fermilab E989, J-PARC

❖ $\delta a_\mu = 16 \times 10^{-11}$ by 2015

❖ Magnetic field: $\frac{\delta \langle B \rangle_\mu}{\langle B \rangle_\mu} \leq 2 \times 10^{-8}$

❖ Requires 10% error on HLbL

❖ Improving HVP $\sigma(e^+e^- \to \text{hadrons})$ in progress

Present:

❒ $a_\mu^{\text{exp}} = 116\,592\,089(63) \times 10^{-11}$; $a_\mu^{\text{SM}} = 116\,591\,793 \pm 51 \times 10^{-11}$

E989: statistics $21\times$; total error factor 4 more precise

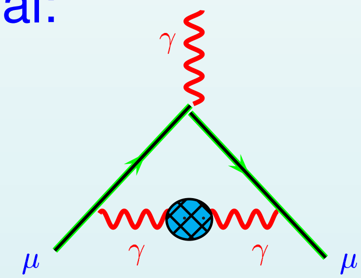
$$\left.\begin{array}{r} \sigma_{\text{stat}} = 0.1 \text{ ppm} \\ \sigma_{\text{syst}} = 0.1 \text{ ppm} \end{array}\right\} \sigma_{\text{tot}} = 0.14 \text{ ppm}$$

❒ $a_\mu^{\text{exp}} = 116\,59x\,xxx(16) \times 10^{-11}$

Evaluation of $a_\mu^{\rm had}$

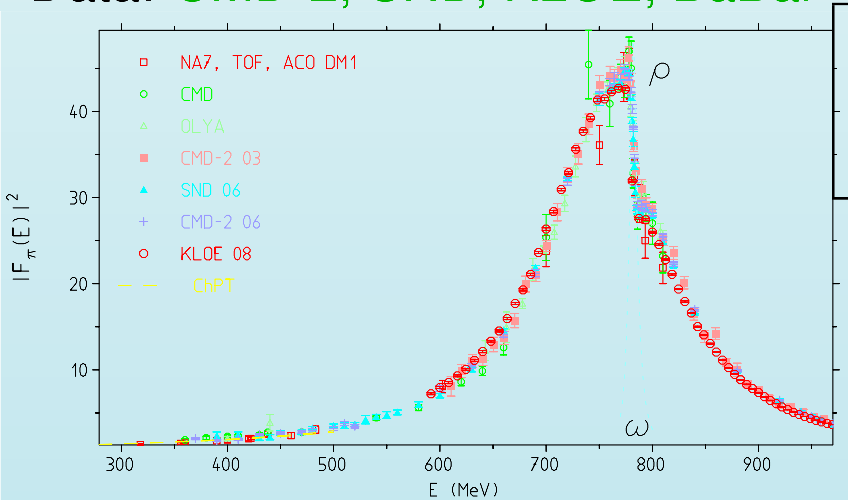
Leading non-perturbative hadronic contributions $a_\mu^{\rm had}$ can be obtained in terms of $R_\gamma(s) \equiv \sigma^{(0)}(e^+e^- \to \gamma^* \to \text{hadrons})/\frac{4\pi\alpha^2}{3s}$ data via dispersion integral:

$$a_\mu^{\rm had} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left(\int_{4m_\pi^2}^{E^2_{\rm cut}} ds \frac{R_\gamma^{\rm data}(s)\ \hat{K}(s)}{s^2} + \int_{E^2_{\rm cut}}^{\infty} ds \frac{R_\gamma^{\rm pQCD}(s)\ \hat{K}(s)}{s^2} \right)$$



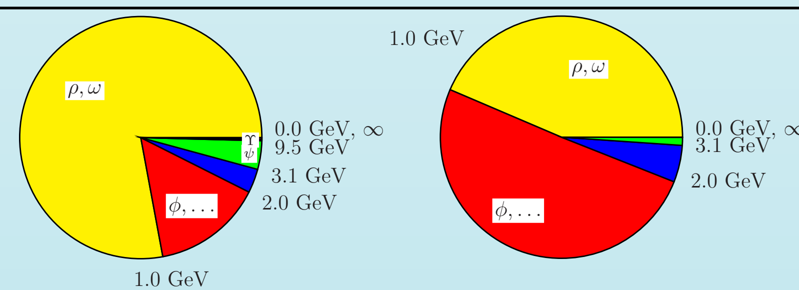
- Experimental error implies theoretical uncertainty!
- Low energy contributions enhanced: $\sim 75\%$ come from region $4m_\pi^2 < m_{\pi\pi}^2 < M_\Phi^2$

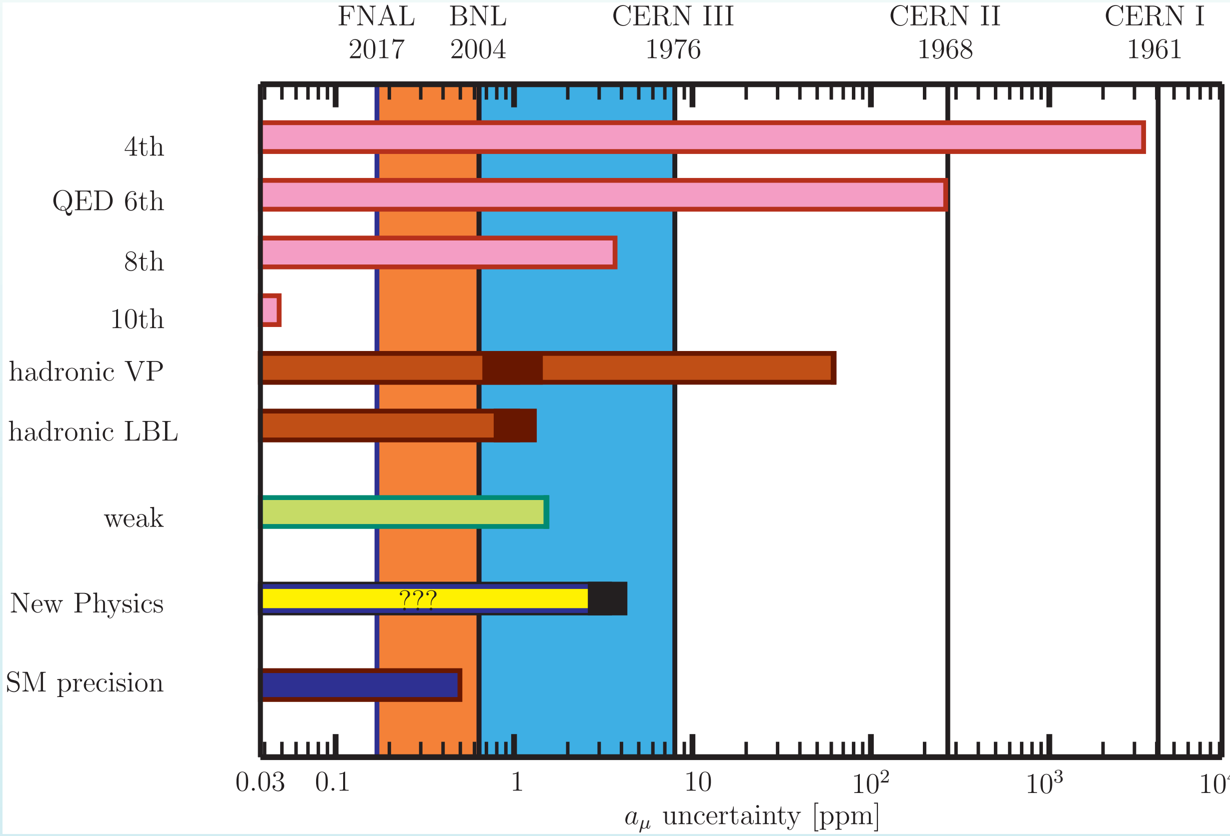
Data: CMD-2, SND, KLOE, BaBar



$$a_\mu^{\rm had(1)} = (690.7 \pm 4.7)[695.5 \pm 4.1]\ 10^{-10}$$

e^+e^-–data based [incl. BaBar MD09]





Sensitivity of $g-2$ experiments to various contributions. The increase in precision with the BNL $g-2$ experiment is shown as a cyan vertical band. New Physics is illustrated by the deviation $(a_\mu^{\rm exp}-a_\mu^{\rm the})/a_\mu^{\rm exp}$

The challenge:

$a_\mu^{\text{had,VP}}[LO]$	$(6923 \pm 42) \times 10^{-11}$	+58.82 ±0.36 ppm
$a_\mu^{\text{had,VP}}[NLO]$	$(-98 \pm 1) \times 10^{-11}$	
a_μ^{EW}	$(154 \pm 1) \times 10^{-11}$	
$a_\mu^{\text{had,LbL}}$	$[(105 \div 115) \pm (26 \div 40)] \times 10^{-11}$	+0.90 ±0.22 ppm
$\delta a_\mu^{\text{exp}}$ present	63×10^{-11}	±0.54 ppm
$\delta a_\mu^{\text{exp}}$ future	16×10^{-11}	±0.14 ppm

Next generation experiments require a factor 4 reduction of the uncertainty
optimistically feasible is factor 2 we hope

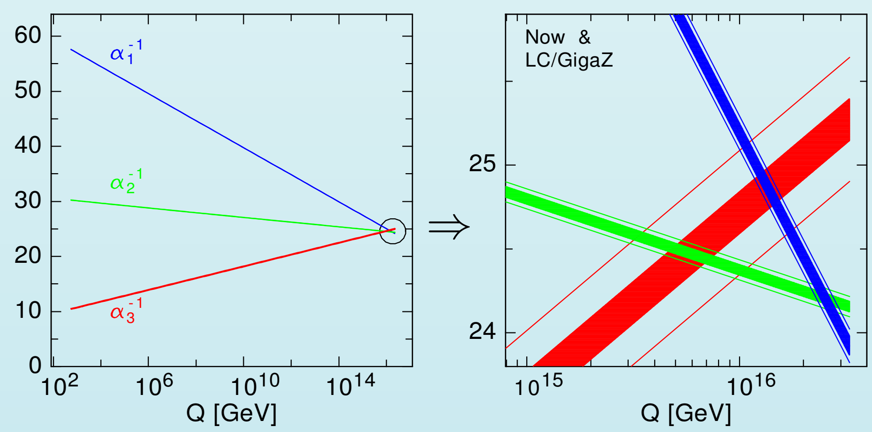
Most urgent R-Measurements above 1GeV, including VEPP 2000 and BES-III

Motivation II: $\alpha_{\rm em}(E), \sin^2\theta_{\rm eff}(E)$ and all that

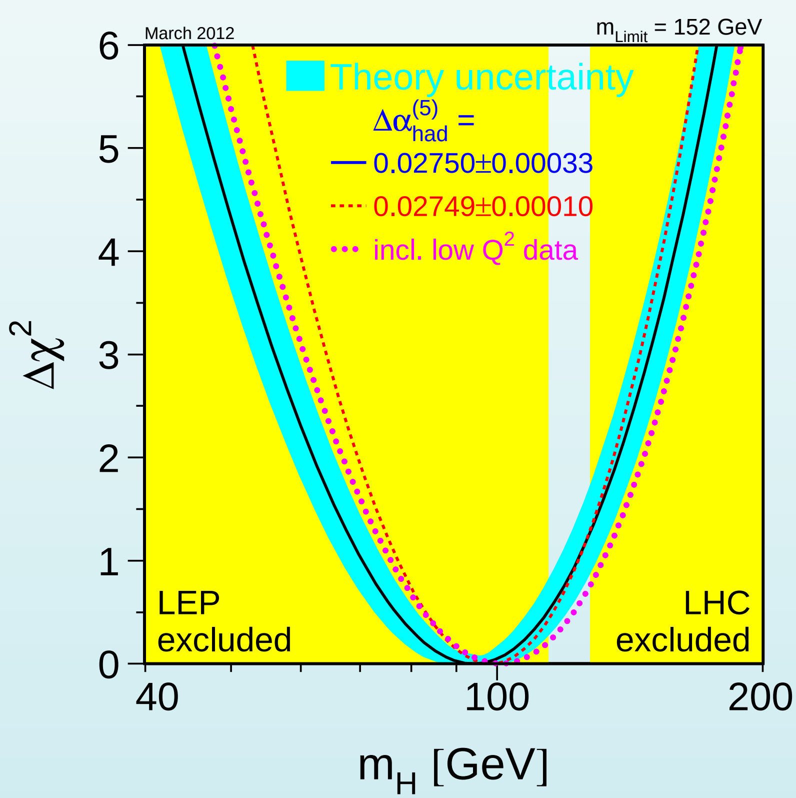
Precise SM predictions require to determine the $U(1)_Y \otimes SU(2)_L \otimes SU(3)_c$
SM gauge couplings $\alpha_{\rm em}$, α_2 and $\alpha_s \equiv \alpha_3$ (QCD) as accurately as possible
**** a theory can not be better than its input parameters ****
$\Rightarrow$ precision limitations due to non-perturbative hadronic contributions $\Leftarrow$

❖beyond SM physics gauge coupling unification?

from Zerwas



$\alpha_s = 0.1183 \pm 0.0027$ vs ± 0.0009



LEP best input α, G_mu, M_Z:

$$\sqrt{2} G_\mu M_Z^2 \sin^2\Theta_i \cos^2\Theta_i = \pi\alpha\,(1+\delta_i)$$

i process specific, M_W, $\sin^2\theta$, etc

$$\frac{\delta \sin^2\theta}{\sin^2\theta} \sim \frac{\cos^2\theta}{\cos^2\theta - \sin^2\theta}\,\delta\Delta\alpha \sim 1.54\;\delta\Delta\alpha$$

$$\frac{\delta M_W}{M_W} \sim \frac{1}{2}\frac{\sin^2\theta}{\cos^2\theta - \sin^2\theta}\,\delta\Delta\alpha \sim 0.23\;\delta\Delta\alpha$$

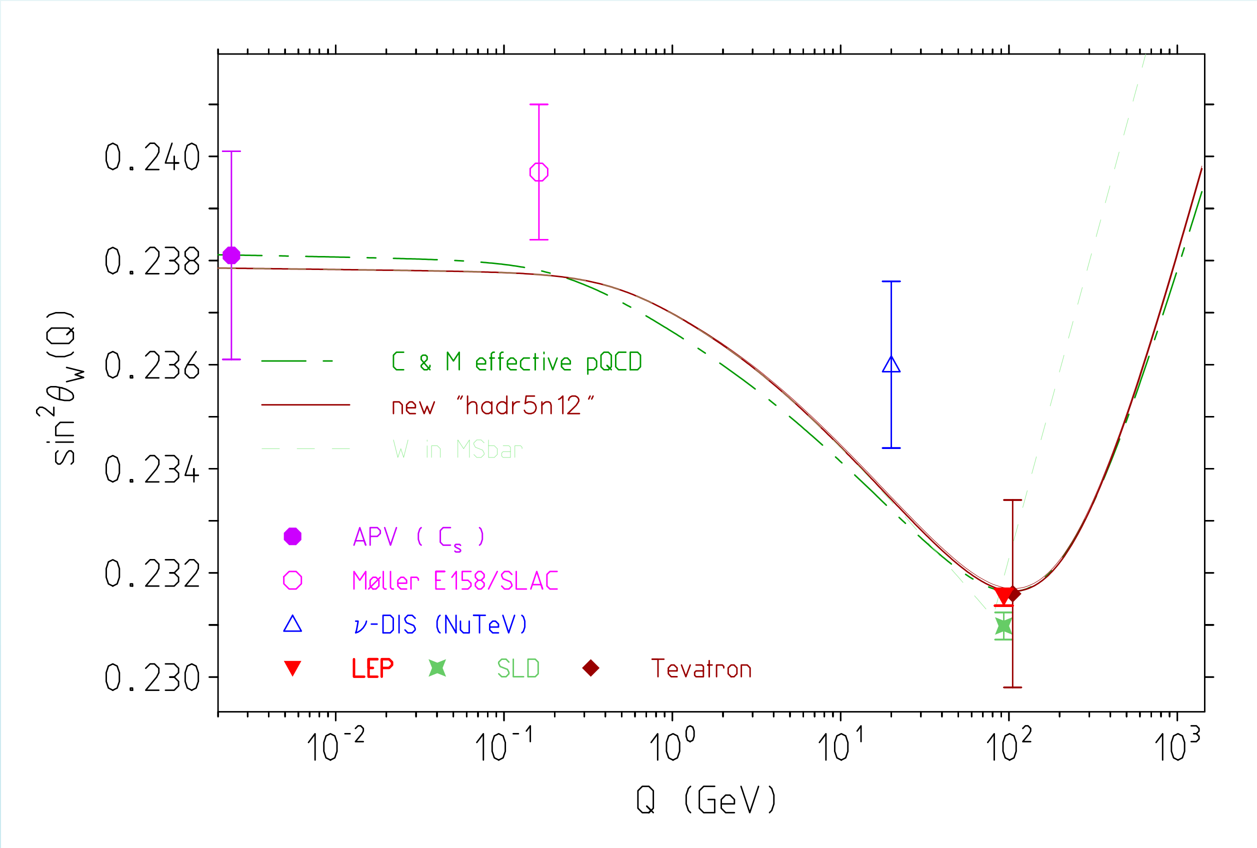
Precision predictions:

$$\sin^2\Theta_f, v_f, a_f, M_W, \Gamma_Z, \Gamma_W, \cdots$$

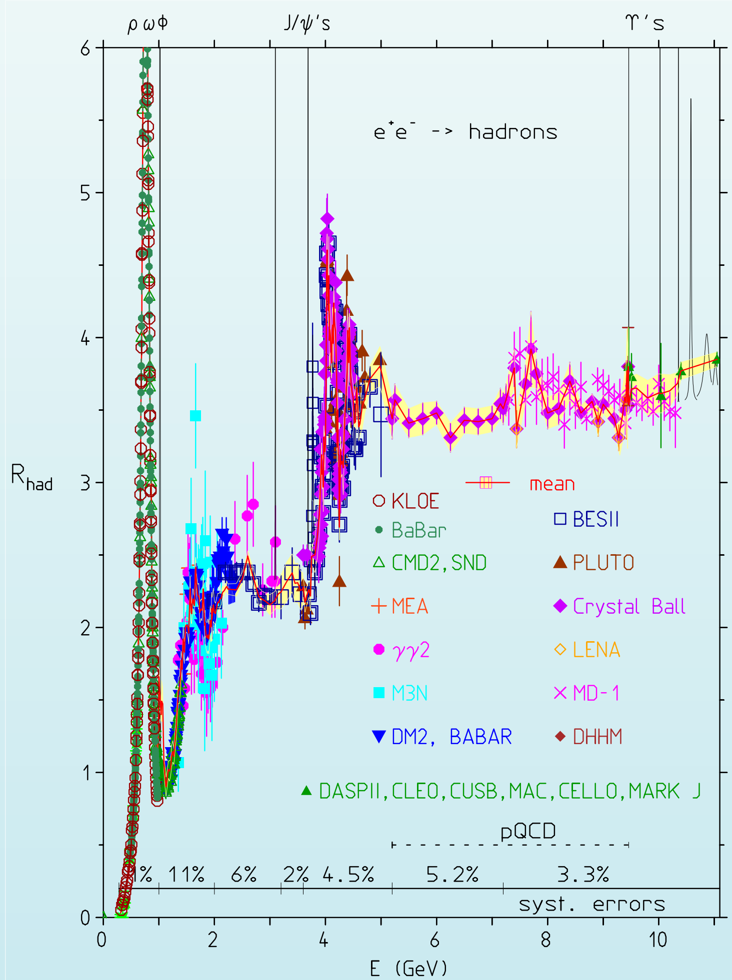
all depend on α effective!

$\sin^2 \Theta_{\rm eff}(E)$

another challenging program: test running $\sin^2 \Theta_{\rm eff}(E)$

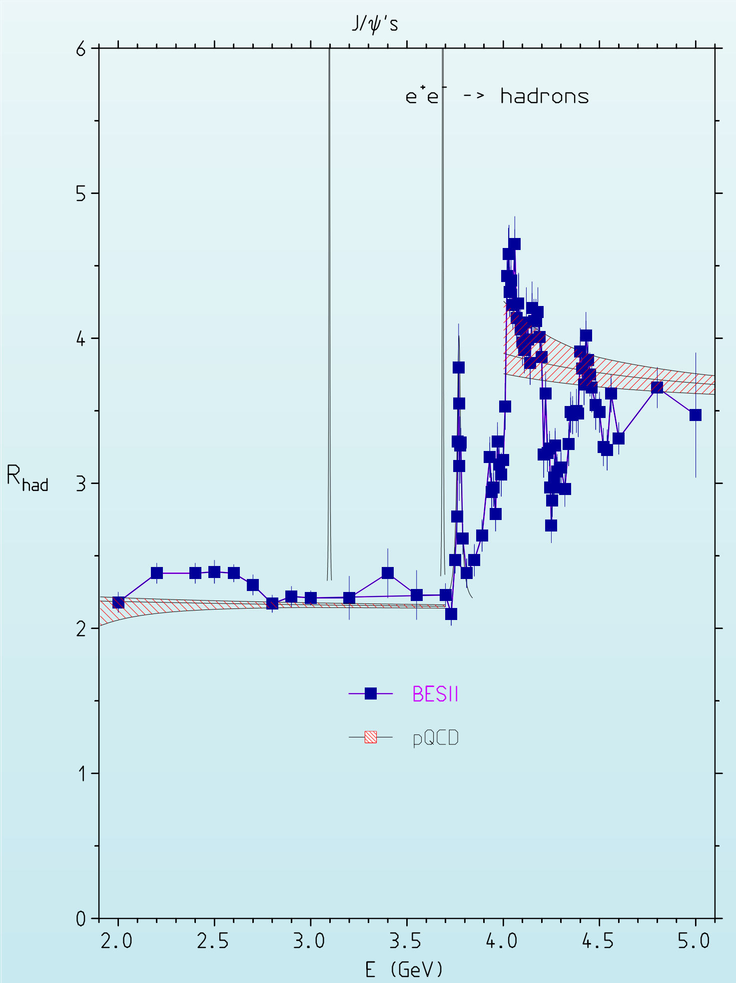


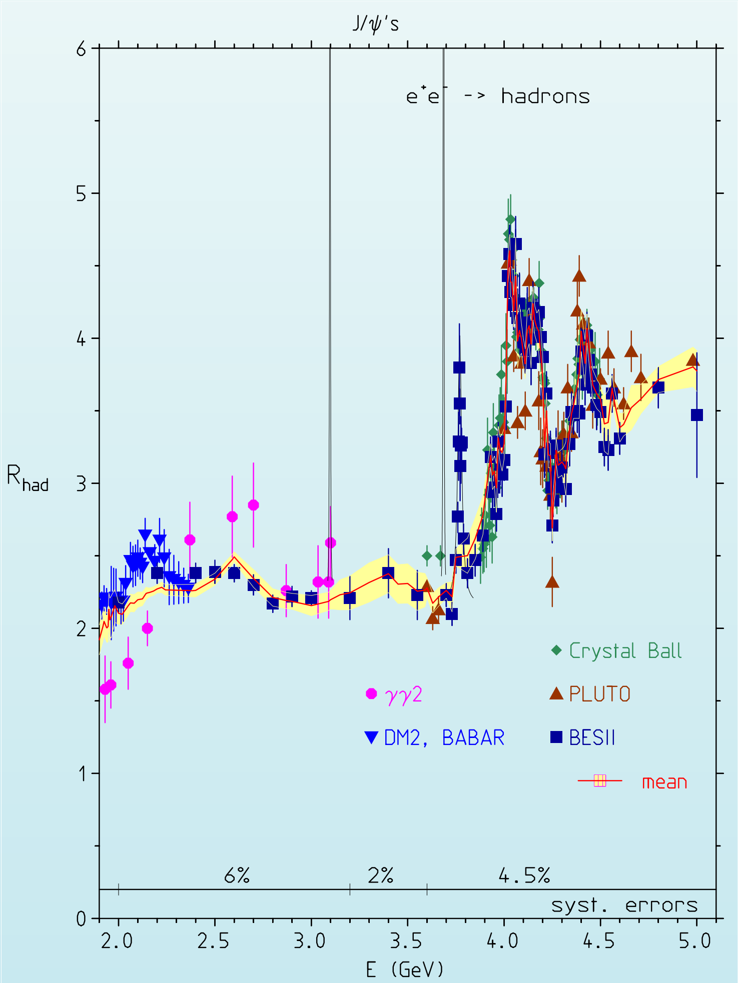
R data present and future



R 2012 compilation

R data BES region





- primary goal: R as precise as possible $\Rightarrow a_\mu$, $\alpha(M_Z)$
- testing pQCD below J/ψ resonances
- more precise resonances scan $\Rightarrow$test quark – hadron duality in time-like regime
- quark-hadron duality $\overline{\sigma(e^+e^- \to \text{hadrons})}(s) \simeq \sum_q \sigma(e^+e^- \to q\bar{q})(s)$, only is useful for scales where pQCD works, quark production cross-section not known and probably ill-defined below 2 GeV.
- quark-hadron duality prefect in space-like region for scales $\mu \gtrsim$ 2 GeV (Adler function see below)
- quark-hadron duality looks to work fine in charm and bottom resonance regions (non-perturbative windows in between perturbative regions)

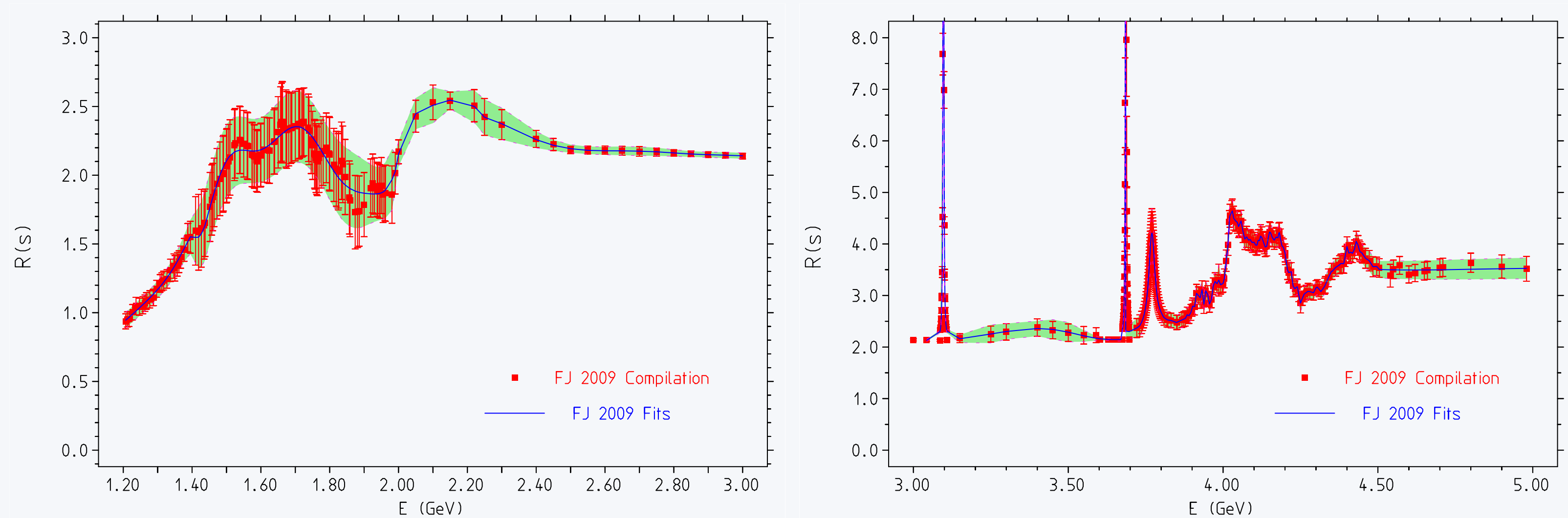
❖ more accurate tests very useful !

❖ can we test and confront exclusive channel method vs inclusive x-section measurements in transition region?

Technical issues: this Meeting is about

- Improving radiative corrections very important for precision x-section determination
- Luminosity monitoring and the Bhabha process: progess expected soon
- Tricks can help: like ISR cancellation by determining R from physical cross section ratios (KLOE, BaBar)
- likely not useful at BES as $\sigma(e^+e^- \to \mu^+\mu^-)$ is two small
- FSR at higher energies less an issue, likely well represented by QED of quarks

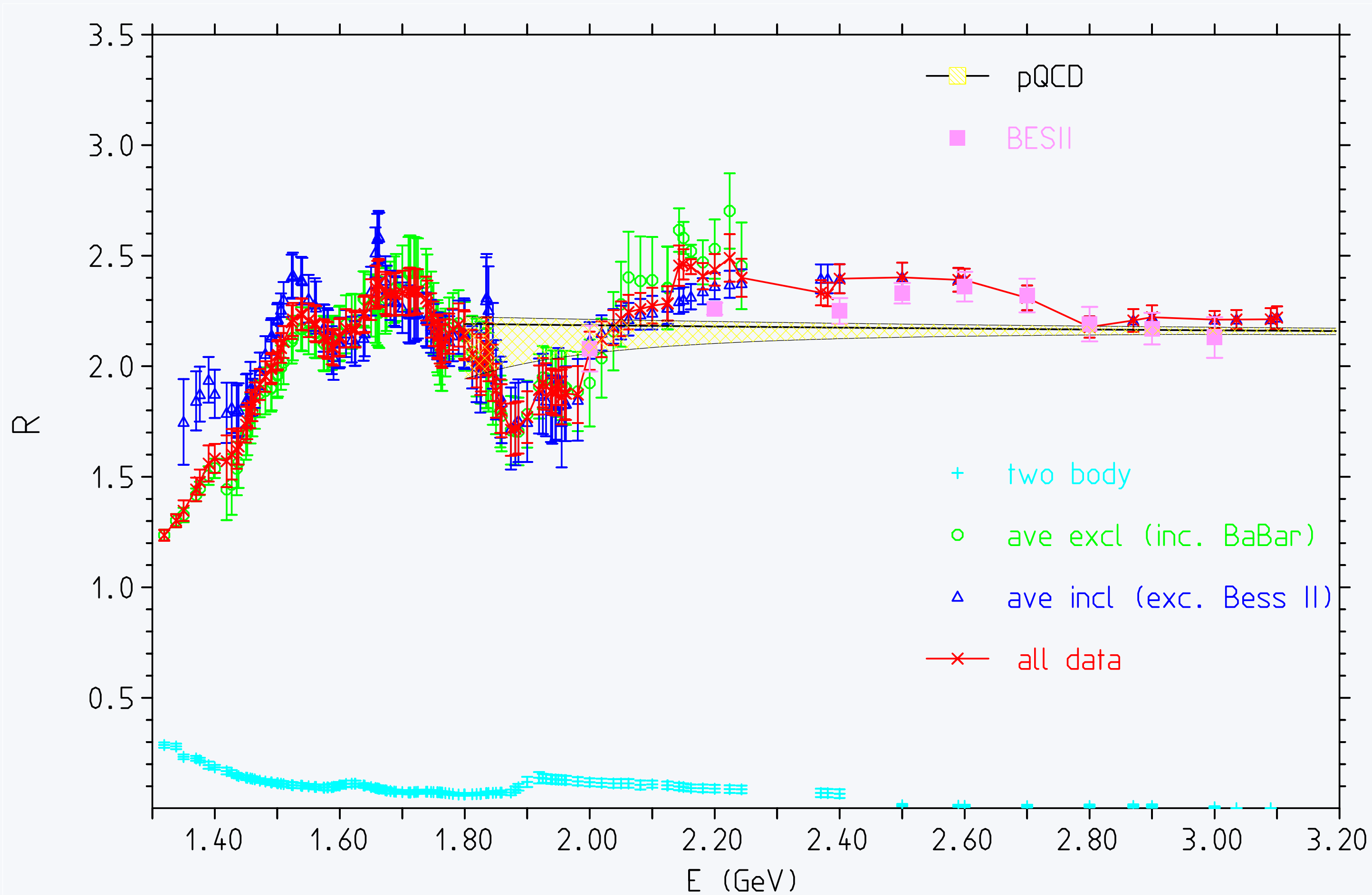
Present medium energy regime: results from my alphaQED and SMalpha

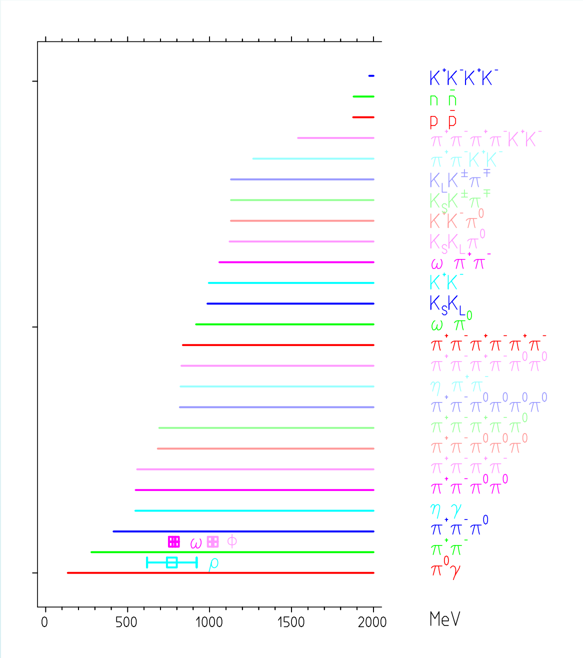


$R(s)\ e^+e^- \to$ hadrons data vs. Chebyshev polynomial fits

Hope for substantial progress at VEPP 2000 and BES III ! Very challenging very real physics (other than hunting for likely non-existent BSM phantoms)

Note for muon $g-2$ region 1 - 2 GeV very important (VEPP 2000)





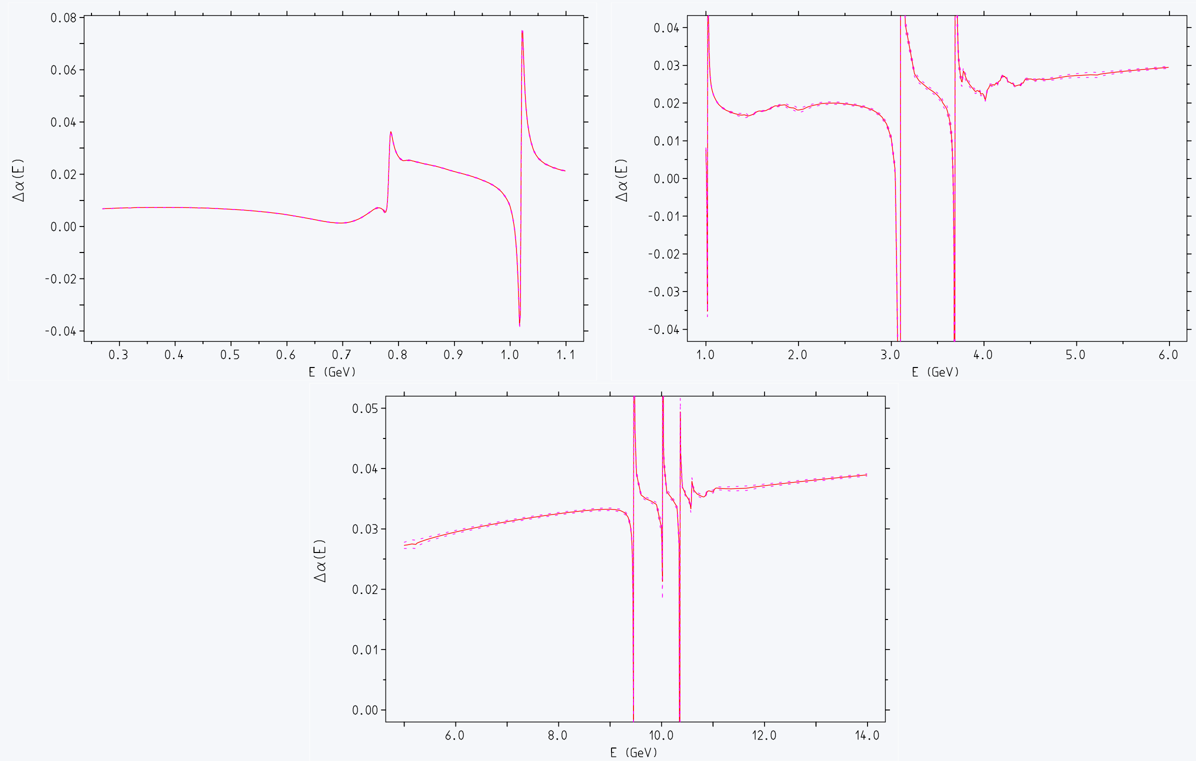
Thresholds for exclusive multi-particle channels below 2 GeV

> Good look to find them all!

Comments on VP subtraction

- VP subtraction in resonance regions very sensitive to self-consistency; huge effects, large imaginary parts
  need 1pi "blob" i.e. undressing physical data from effects of running α:

$$R^{(0}(s) = R^{\mathrm{phys}}(s) \cdot (\alpha/\alpha(s))^2$$



From alphaQED package

Complex vs. real α VP correction

- Usually adopted VP subtraction corrections: $\alpha(s) \to \alpha$

  $R(s)$ corrected by $(\alpha/\alpha(s))^2 = |1 - \mathrm{Re}\,\Pi'(s)|^2$ ($\Pi'(0)$ subtracted)

- more precisely, should subtract $|1 - \Pi'(s)|^2 = \alpha/|\alpha_c(s)|)^2$
  where $\alpha_c(s)$ complex version of running α

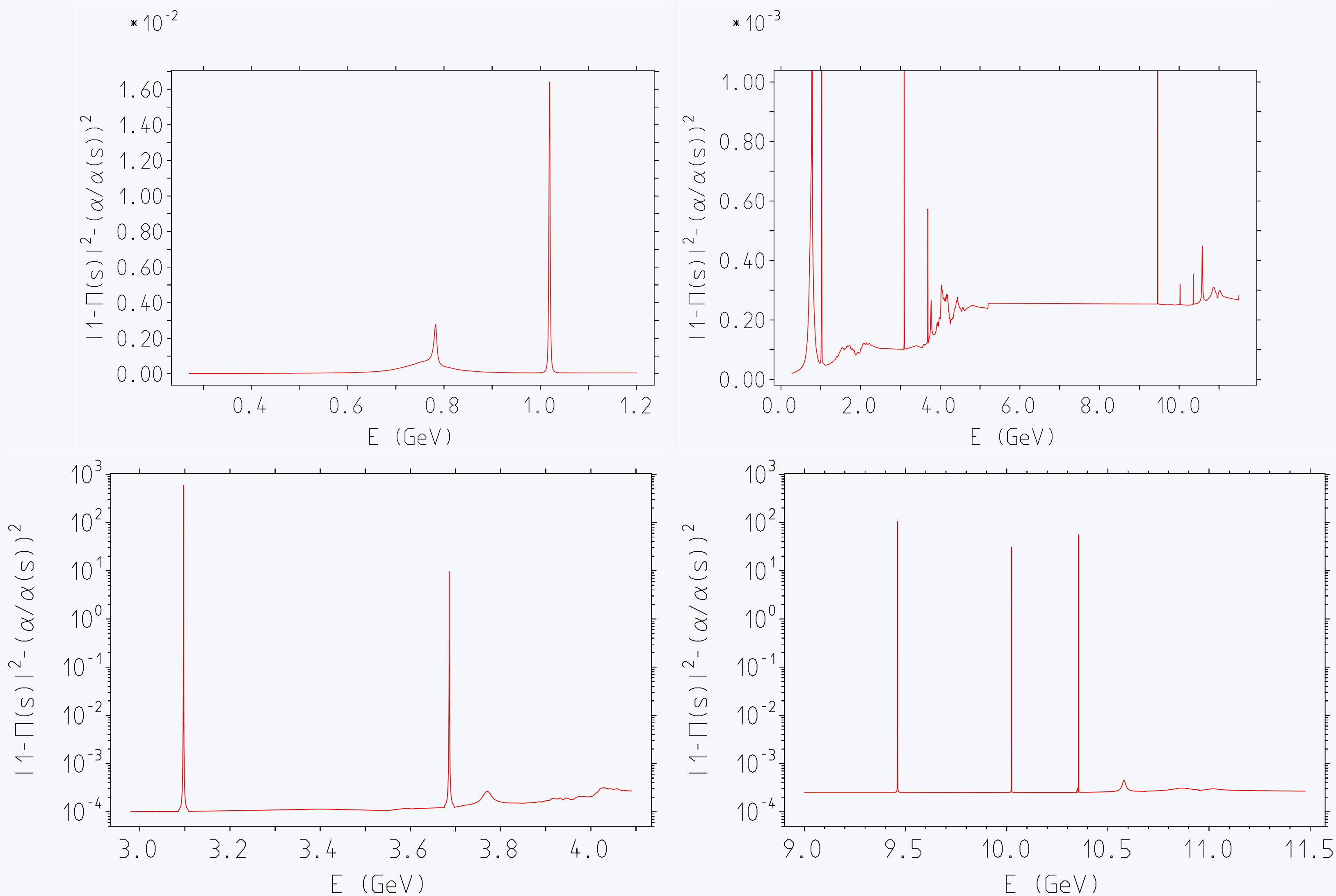
- complex version what the Novosibirsk CMD-2 Collaboration has been using in more recent analyzes [code available from Fedor Ignatov ⋙]

- Typically, corrections

$$1 - |1 - \Pi'(s)|^2/(\alpha/\alpha(s))^2$$

- non-resonance regions corrections $\lesssim 0.1$ %

- at resonances where corrections $\sim 1/\Gamma_R$



* 10^-2
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.00
0.4
0.6
0.8
1.0
1.2
E (GeV)
|1-Π(s)|^2-(α/α(s))^2
* 10^-3
1.00
0.80
0.60
0.40
0.20
0.00
0.0
2.0
4.0
6.0
8.0
10.0
E (GeV)
|1-Π(s)|^2-(α/α(s))^2
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
3.0
3.2
3.4
3.6
3.8
4.0
E (GeV)
|1-Π(s)|^2-(α/α(s))^2
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
9.0
9.5
10.0
10.5
11.0
11.5
E (GeV)
|1-Π(s)|^2-(α/α(s))^2

Note: imaginary parts from narrow resonances, $\text{Im}\,\Pi'(s)) = \frac{\alpha}{3}\,R(s) = \frac{3}{\alpha}\,\frac{\Gamma_{ee}}{\Gamma}$ at peak, are sharp spikes and are obtained correctly only by appropriately high resolution scans. For example,

$$|1 - \Pi'(s)|^2 - (\alpha/\alpha(s))^2 = (\text{Im}\,\Pi'(s))^2$$

at $\sqrt{s} = M_R$ is given by

1.23×10^{-3} $[\rho]$, 2.76×10^{-3} $[\omega]$, 1.56×10^{-2} $[\phi]$, 594.81 $[J/\psi]$, 9.58 $[\psi_2]$, 2.66×10^{-4} $[\psi_3]$, 104.26 $[\Upsilon_1]$, 30.51$[\Upsilon_2]$, 55.58 $[\Upsilon_3]$

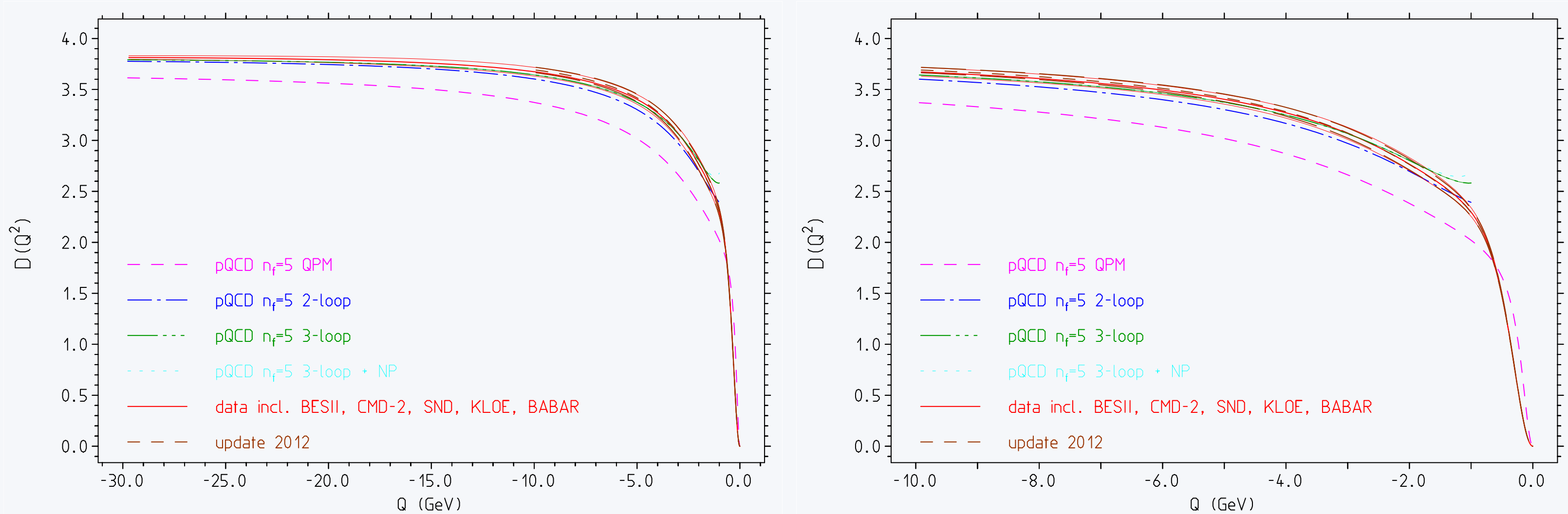
Adler function controlled QCD

Ideal monitor to control the applicability of pQCD is the Adler function

$$D(Q^2 = -s) = -(12\pi^2)\, s\, \frac{\mathrm{d}\Pi'_\gamma(s)}{\mathrm{d}s} = Q^2 \int\limits_{4m_\pi^2}^{\infty} \frac{R(s)}{(s+Q^2)^2}$$

which on the one hand for, Q^2 not too small, can be calculated in pQCD on the other hand it can be evaluated non-perturbatively in the standard manner using data at lower energies plus pQCD for perturbative regions and the perturbative tail.

Plots based on pQCD in BF-MOM scheme cut at 1 GeV:



"Experimental" Adler–function versus theory (pQCD + NP) Experimental data obtained by integrating $R^{\mathrm{had}}(s)$. Error includes statistical + systematic here (in contrast to most R-plots showing statistical errors only)!

Curvature (in perturbative region) almost entirely due to mass effects. Very sensitive to precise values of m_c and m_b!

Application I: leading hadronic contribution to muon anomaly

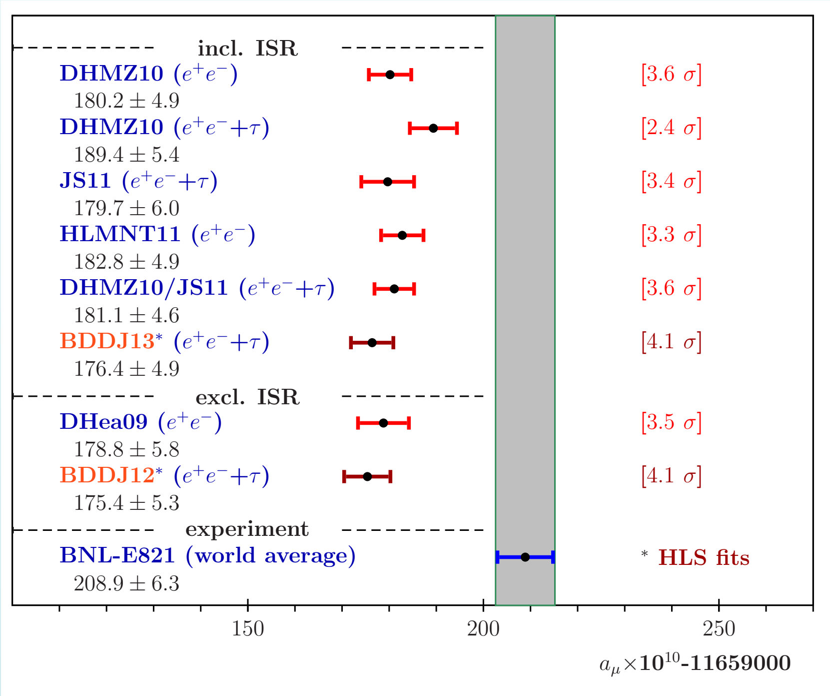
$$a_\mu^{\rm had} = \frac{\alpha}{\pi} m_\mu^2 \int\limits_0^1 dx\, x\,(2-x)\, \left(D(Q^2(x))/Q^2(x)\right)$$

where $Q^2(x) \equiv \frac{x^2}{1-x} m_\mu^2$ is the space–like square momentum–transfer.

year	$a_\mu^{\rm had} \times 10^{10}$	
2001	702.27 ± 11.23	
2002	682.97 ± 8.65	
2003	694.76 ± 8.64	
2006	694.44 ± 6.05	
2008	694.62 ± 6.05	
2013	694.42 ± 5.15	
2013	691.00 ± 4.70	standard approach

Alternative: other extreme

❖Global fit HLS resonance Lagrangian approach Benayoun et al



Data below $E_0 = 1.05$ GeV (just above the ϕ) constrain effective Lagrangian couplings, using 45 different data sets (6 annihilation channels and 10 partial width decays).

Application II: $\alpha_{\rm em}(M_Z^2)$ by the "Adler function controlled" approach

$$\begin{aligned}\alpha(M_Z^2) &= \alpha(-s_0) + \left[\alpha(-M_Z^2) - \alpha(-s_0)\right] + \left[\alpha(M_Z^2) - \alpha(-M_Z^2)\right] \\ &= \alpha^{\rm data}(-s_0) + \left[\alpha(-M_Z^2) - \alpha(-s_0)\right]^{\rm pQCD} + \left[\alpha(M_Z^2) - \alpha(-M_Z^2)\right]^{\rm pQCD}\end{aligned}$$

where the space-like $-s_0$ is chosen such that pQCD is well under controlled for $-s < -s_0$.

First calculate Euclidean $\Delta\alpha(-M_Z^2)$ via Adler function plus reminder

$$\delta\Delta\alpha(M_Z^2) \doteq \Delta\alpha(M_Z^2) - \Delta\alpha(-M_Z^2) = -\frac{2\alpha\, M_Z^2}{3\pi} \int_{4m_\pi^2}^{\infty} \frac{\mathrm{d}sR(s)}{(s-M_Z^2)(s+M_Z^2)}$$

- $\delta\Delta\alpha(M_Z^2) = 0.34(0.02) \times 10^{-4}$

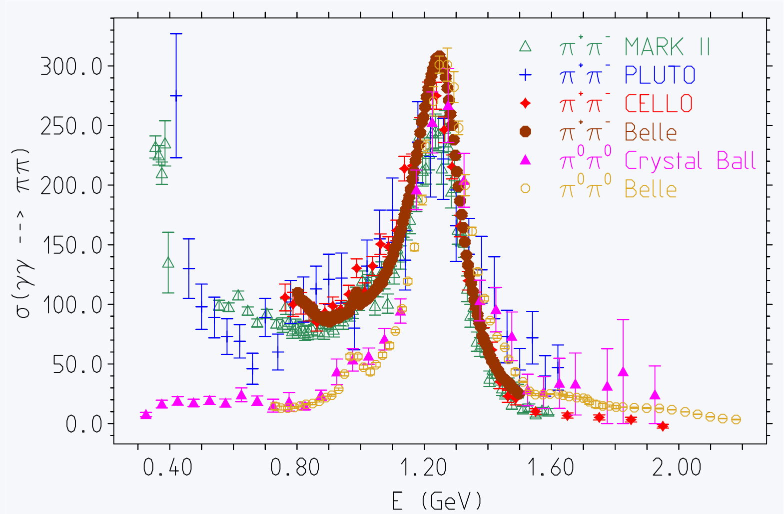
- low energy contribution up to s_0 suppressed by s_0/M_Z^2
  pQCD allows for reliable calculation
- for $M_Z^2 \to q^2$ and q^2 very large

Adler-function slope:

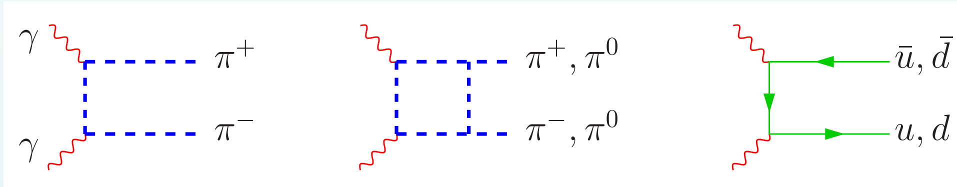
$$\lim_{Q\to 0} D(Q^2)/Q^2 = \int_{4m_\pi^2}^{\infty} \frac{\mathrm{d}s\, R(s)}{s^2} = 11.85(0.08)$$

consistent with the slope extracted from $D(Q^2)$ itself which is 11.885.

$\gamma\gamma \to \pi\pi$ sQED and isospin relations



How photons couple to pions? This is obviously probed in reactions like $\gamma\gamma \to \pi^+\pi^-, \pi^0\pi^0$. Data infer that below about 1 GeV photons couple to pions as point-like objects (i.e. to the charged ones overwhelmingly), at higher energies the photons see the quarks exclusively and form the prominent tensor resonance $f_2(1270)$. The $\pi^0\pi^0$ cross section in this figure is enhanced by the isospin symmetry factor 2, by which it is reduced in reality.



Di-pion production in $\gamma\gamma$ fusion. At low energy we have direct $\pi^+\pi^-$ production and by strong rescattering $\pi^+\pi^- \to \pi^0\pi^0$, however with very much suppressed rate. Above about 1 GeV, resolved $q\bar{q}$ couplings seen.

Strong tensor meson resonance in $\pi\pi$ channel $f_2(1270)$ with photons directly probe the quarks!

- Photons seem to see pions below 1 GeV
- Photons definitely look at the quarks in $f_2(1270)$ resonance region
- We apply the sQED model up to 0.975 GeV (relevant for a_μ). This should be pretty save (still we assume a 10% model uncertainty)
- Switching off the electromagnetic interaction of pions, is definitely not a realistic approximation in trying to describe what data we see in $e^+e^- \to \pi^+\pi^-$

- isospin relation completion of missing channels [1-2 GeV] in $e^+e^- \to \mathrm{hadrons}$ are not reliable [large IB]

Conclusion

- future muon $g-2$ experiments require substantial improvement on hadronic effects: VP most urgent but in principle doable, new R measurements mandatory, lattice QCD very good chance to compete at low energies

- Hadronic light-by-light scattering contribution to muon $g-2$, biggest challenge theory-wise, requires $\gamma\gamma \to \mathrm{hadrons}$ in all versions as constraints, for lattice QCD far future

- Future high precision physics at ILC requires substantial improvement in the determination of the effective fine structure constant

- The future of precision physice depends on you! Go ahead!

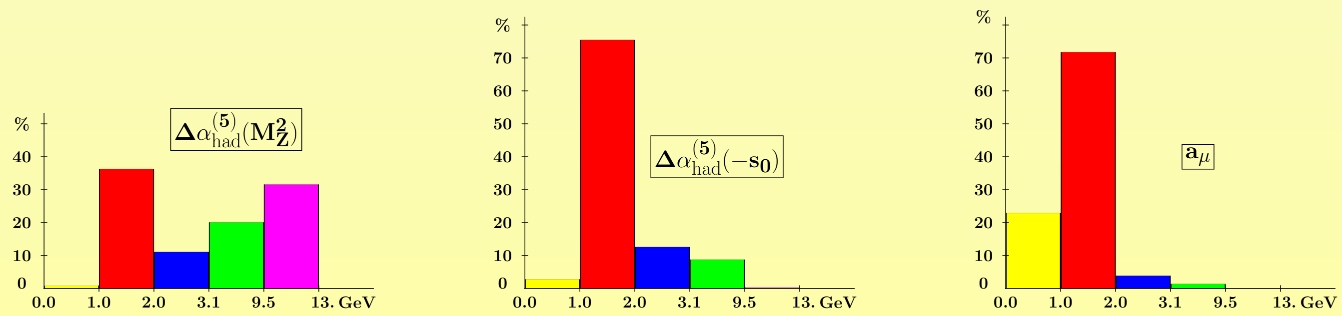
Thank you for your attention!

Theory: (QCD parameters) has to improve by factor 10 ! $\longrightarrow \pm 0.20$

Requirement may be realistic:

- **pin down experimental errors to 1% level in all non-perturbative regions up to 10 GeV**
- **switch to Adler function method**
- **improve on QCD parameters, mainly on m_c and m_b**

DAFNE-2 in conjunction with Adler function approach



Unique chance for DAFNE-2 to improve precision of $\alpha_{\rm eff}(E)$ substantially! In conjunction with improvement of QCD parameters (lattice QCD!). Mandatory for ILC project, but in many other places e.g. $g-2$ of the muon.