Analysis of η & η′ transition form factors with rational approximants

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(hopefully appearing soon in arXiv)

JG|U

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THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

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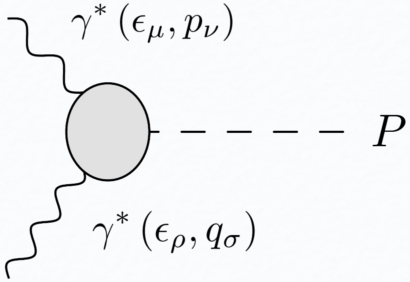
Introduction & Motivations

Pseudoscalar TFF
Padé approximants

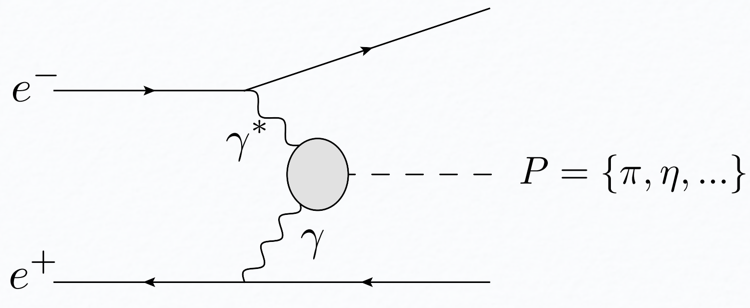
PSEUDOSCALAR TFF

Pseudoscalar TFF

- Pseudoscalar meson Transition Form Factors (TFF) describe interaction of pseudoscalar mesons with 2 photons



$$i\mathcal{M} = ie^2 \varepsilon^{\mu\rho\nu\sigma} \epsilon^*_\mu \epsilon^*_\rho p_\nu q_\sigma \times F_{P\gamma^*\gamma^*}(p^2, q^2)$$



$$F_{P\gamma\gamma^*}(Q^2, 0)$$

- This form factor is well known from perturbative QCD (pQCD) (Brodsky & Lepage, PRD **22**, 1980) and the ABJ anomaly, respectively (π^0 case)

$$F_{\pi\gamma^*\gamma}(-q^2, 0) \equiv F_{\pi\gamma}(Q^2) = \frac{2f_\pi}{3Q^2} \int \frac{dx}{x} \phi_\pi(x, Q^2) \left(1 + \mathcal{O}(\alpha_s(Q^2))\right); \qquad F_{\pi\gamma\gamma}(0,0) = \frac{1}{4\pi^2 f_\pi}$$

Where ϕ_π, the pion (pseudoscalar) distribution amplitude (DA), encodes all the non-perturbative information.

- pQCD predicts asymptotic behavior $\lim_{Q^2 \to \infty} Q^2 F_{\pi\gamma}(Q^2) = 2f_\pi$

Pseudoscalar TFF

- However, the DA is not known and we don't have a good description at low energies: χPT does not give a precise description and only Q^2=0 value is well known (decay through ABJ anomaly)

- Finally, we must invoke some model either modeling the TFF itself (i.e. Vector Meson Dominance (VMD), Sum Rules) or the DA instead (i.e. Light Cone pQCD, Holographic)

All of them lead to different predictions ...
•Which one is the most accurate?
•Could we estimate the systematic error from model dependency?

- We could take a systematic and model independent approach able to estimate systematics.

Padé Approximants

Padé approximants

Padé Approximants

- Padé Approximants (PA) are rational functions, given by the quotient of 2 polynomials of order (N,M)

$$P_M^N(Q^2) = \frac{T(x;N)}{R(x;M)} = \frac{\sum_i^N a_i x^i}{\sum_j^M b_j x^j}$$

- Where a_i and b_i are such to match the Taylor expansion of the function to be approximated, details in G.A. Baker and P. Grave-Morris, Encyclopedia of Mathematics and its applications, Cambridge University Press 1996

- We can obtain them from fitting data: using systematically higher sequences (increasing N, M) we improve the result

- Nevertheless at some (N,M) the new parameters are compatible with zero and we should stop fitting, which introduces systematic error.

- The different coefficients may be related to the Taylor Series of the function

$$F_{P\gamma}(Q^2) = a_0 \left(1 - a_P \frac{Q^2}{m_P^2} + b_P \frac{Q^4}{m_P^4} + \mathcal{O}(\frac{Q}{m_P})^6\right)$$

Padé Approximants

Use well-motivated Padé Approximants sequences to improve convergence

• Include poles (zeros of the denominator)	$P_1^N(Q^2) = \dfrac{T(Q^2;N)}{Q^2+M^2}$
• Asymptotic pQCD behavior encoded. Fixing asymptotic $\Rightarrow$ Brodsky-Lepage interpolation formula	$P_{N+1}^N(Q^2) = \dfrac{T(Q^2;N)}{R(Q^2;N+1)}$ $P_1'^0 = \dfrac{a_0}{1+\frac{a_0}{2f_\pi}Q^2}$
• Well known models such as VMD or LMD are nothing, but constrained PA's	$VMD = \dfrac{1}{4\pi^2 f_\pi}\dfrac{1}{1+\frac{Q^2}{M_\rho^2}}$

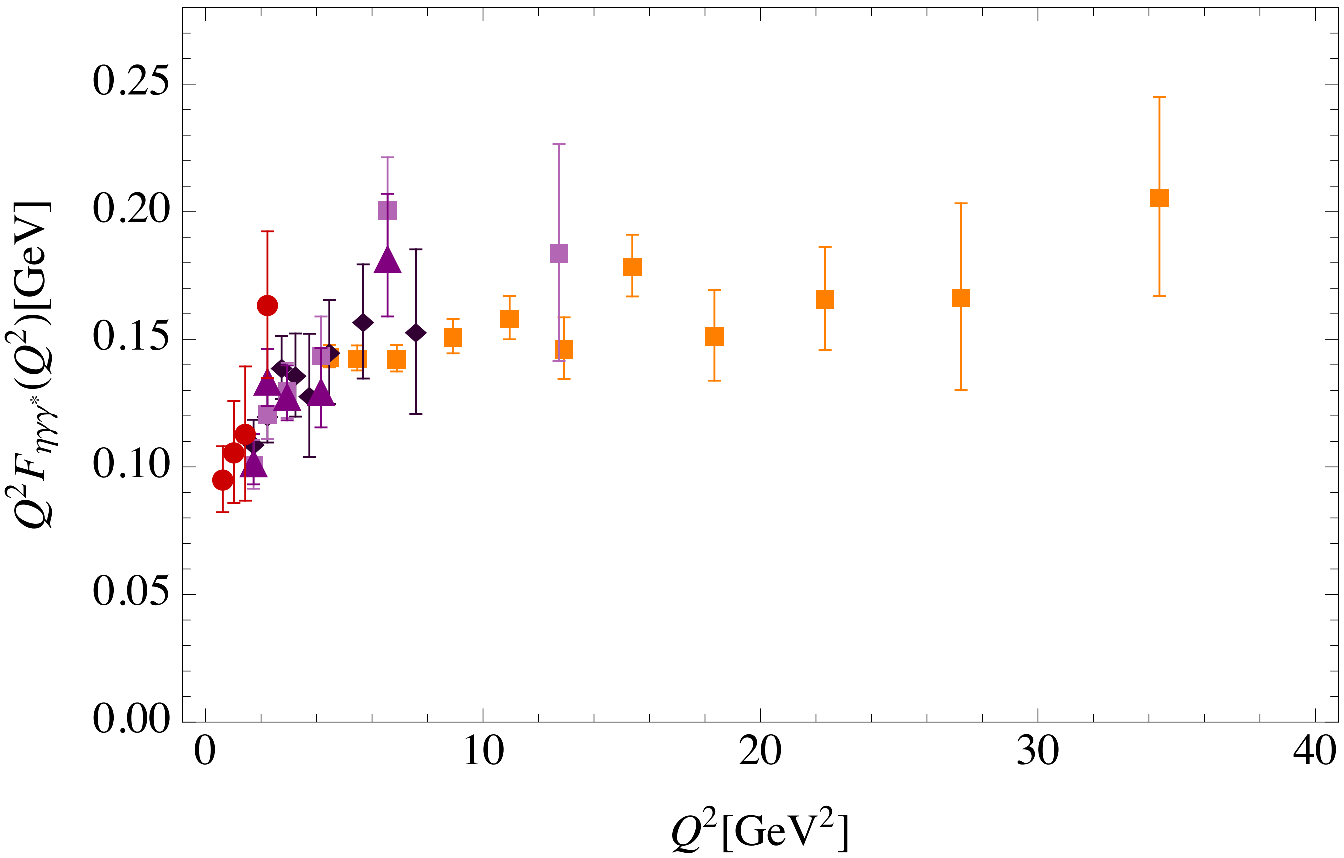
Increasing N leads to stable convergent coefficients which should approach the "real" value. Different sequences lead to similar results checking robustness

RESULTS FOR η, η' TFF

Fitting Results for η

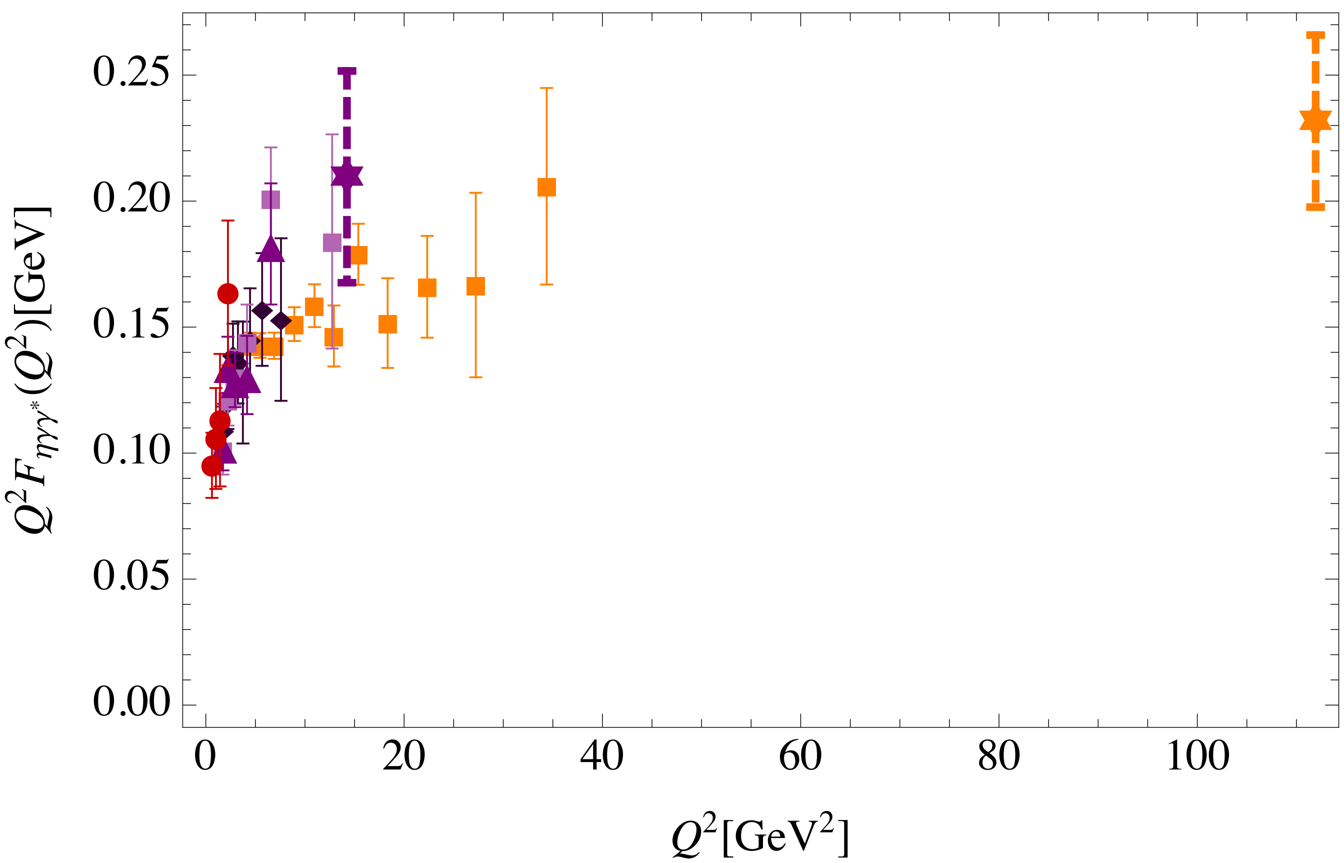
Take all the available data

CELLO (red), CLEO (purple), BABAR (orange), TFF(Q^2=0) from KLOE-2



Fitting Results for η

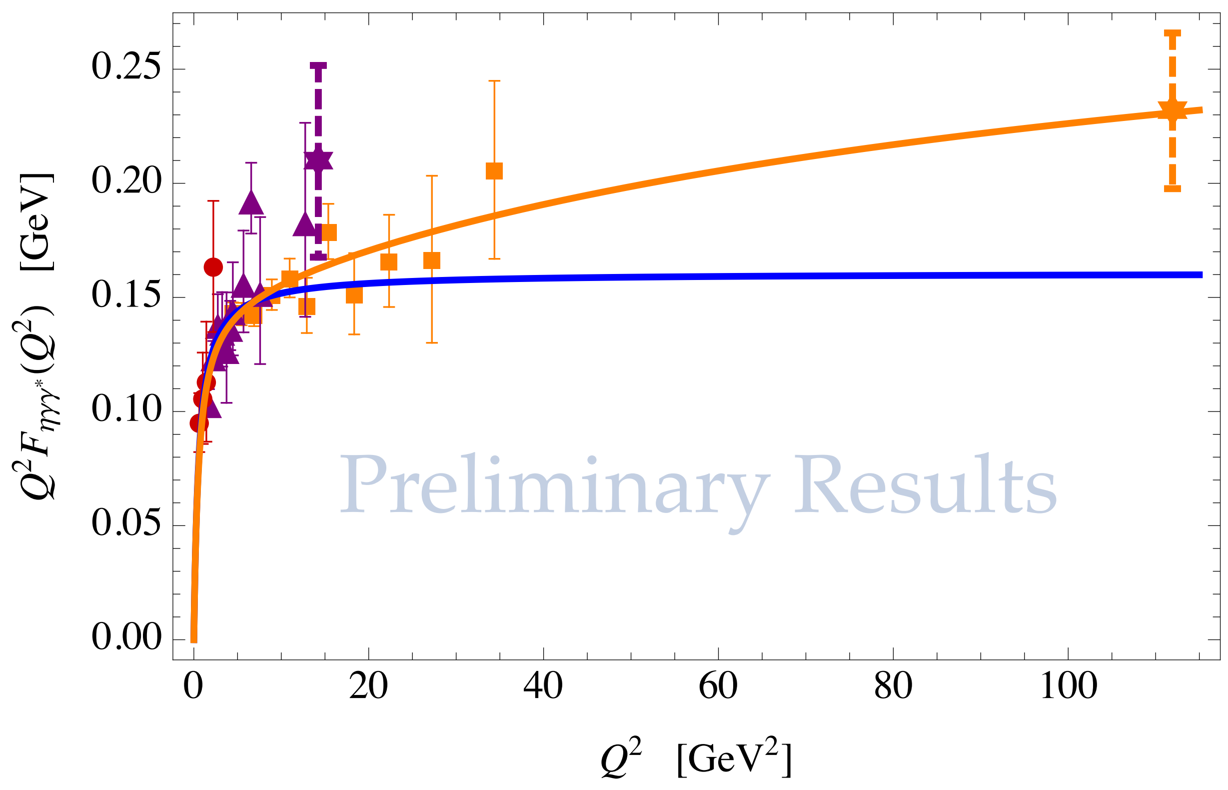
Additionally, we use the timelike data points from CLEO (14GeV^2) (purple dashed) and BaBar (112GeV^2) (orange dashed)
We combine some data points in next plots to avoid cluttering



Fitting results for η

We use the Padé sequence P(N,N+1)

$\Rightarrow$ We reach the P(1,2) which is shown orange, P(0,1) shown in blue



$$a_0 = 0.275(6)\ \text{(KLOE-2)};\quad a_\eta = 0.547(18)_{stat}(31)_{sys};\quad b_\eta = 0.304(25)_{stat}(64)_{sys}$$

Fitting Results for η

Compare our slope to other results

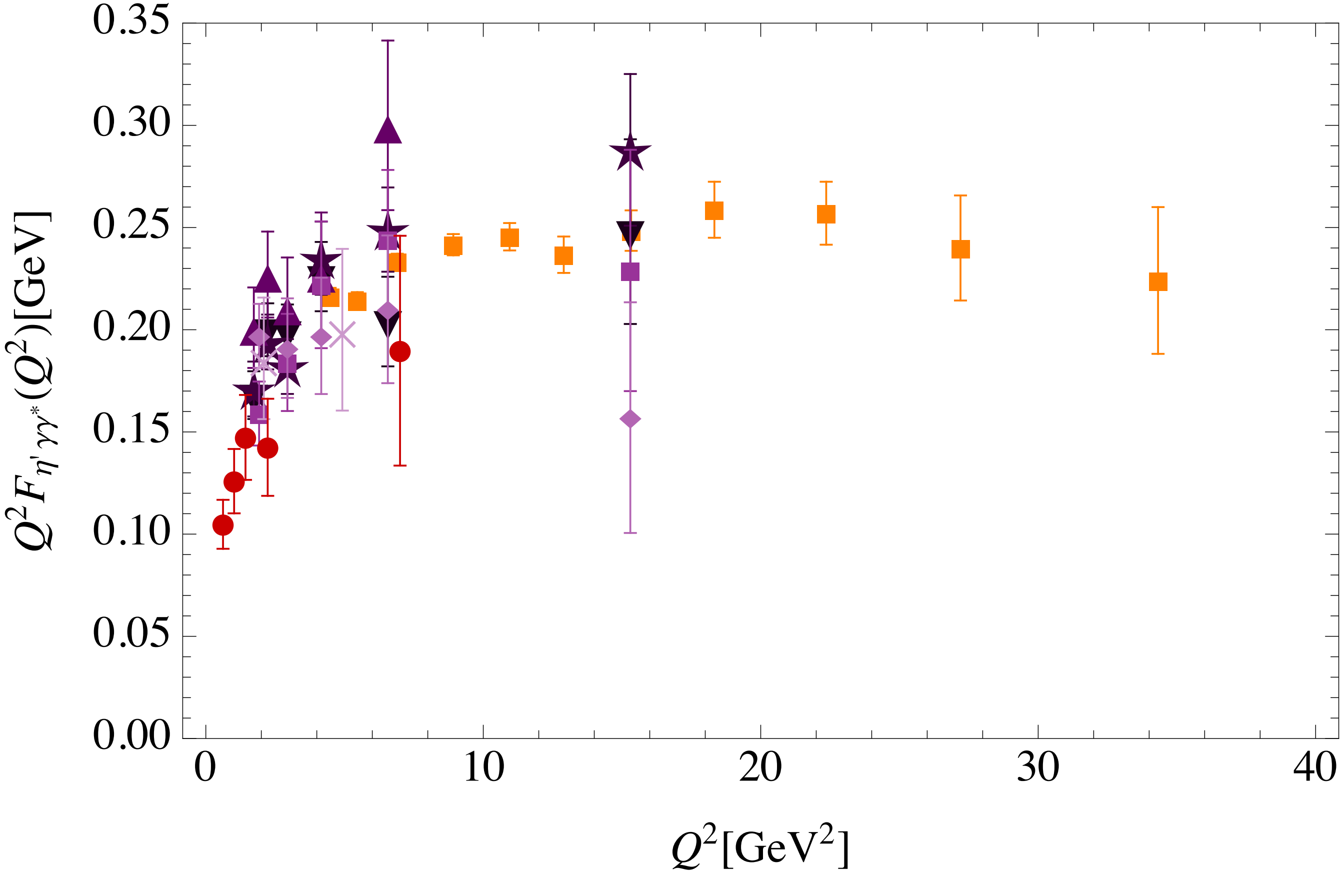
	Spacelike			Timelike				χPT, Ametller *et. al*	Czyż *et. al*
	Our	CELLO	CLEO	Lepton-G	NA60	MAMI	WASA	PRD **45**, (1992)	PRD **85**, (2012)
a_η	0.547(18)(31)	0.428(63)	0.501(38)	0.57(12)	0.585(51)	0.58(11)	0.68(26)	0.51	0.546(9)

It nicely agree with different results
All other experimental results use a VMD fit to obtain the result
We have an estimate for the systematic error in our final result

Fitting results for η'

Take all the available data

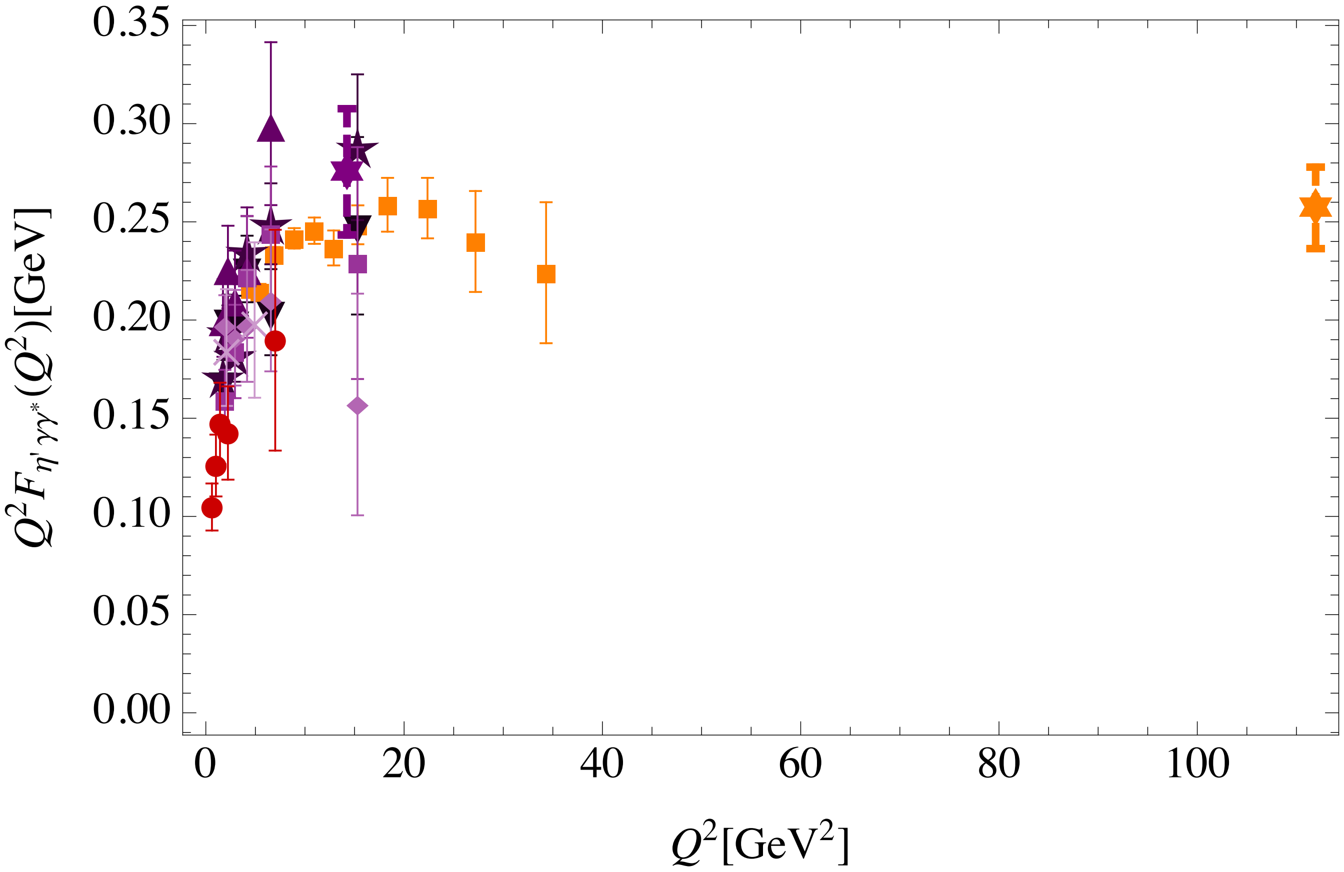
CELLO (red), CLEO (purple), BABAR (orange), TFF(Q^2=0) from PDG



Fitting results for η′

Additionally, we use the timelike data points from CLEO (14GeV²) (purple dashed) and BaBar (112GeV²) (orange dashed)

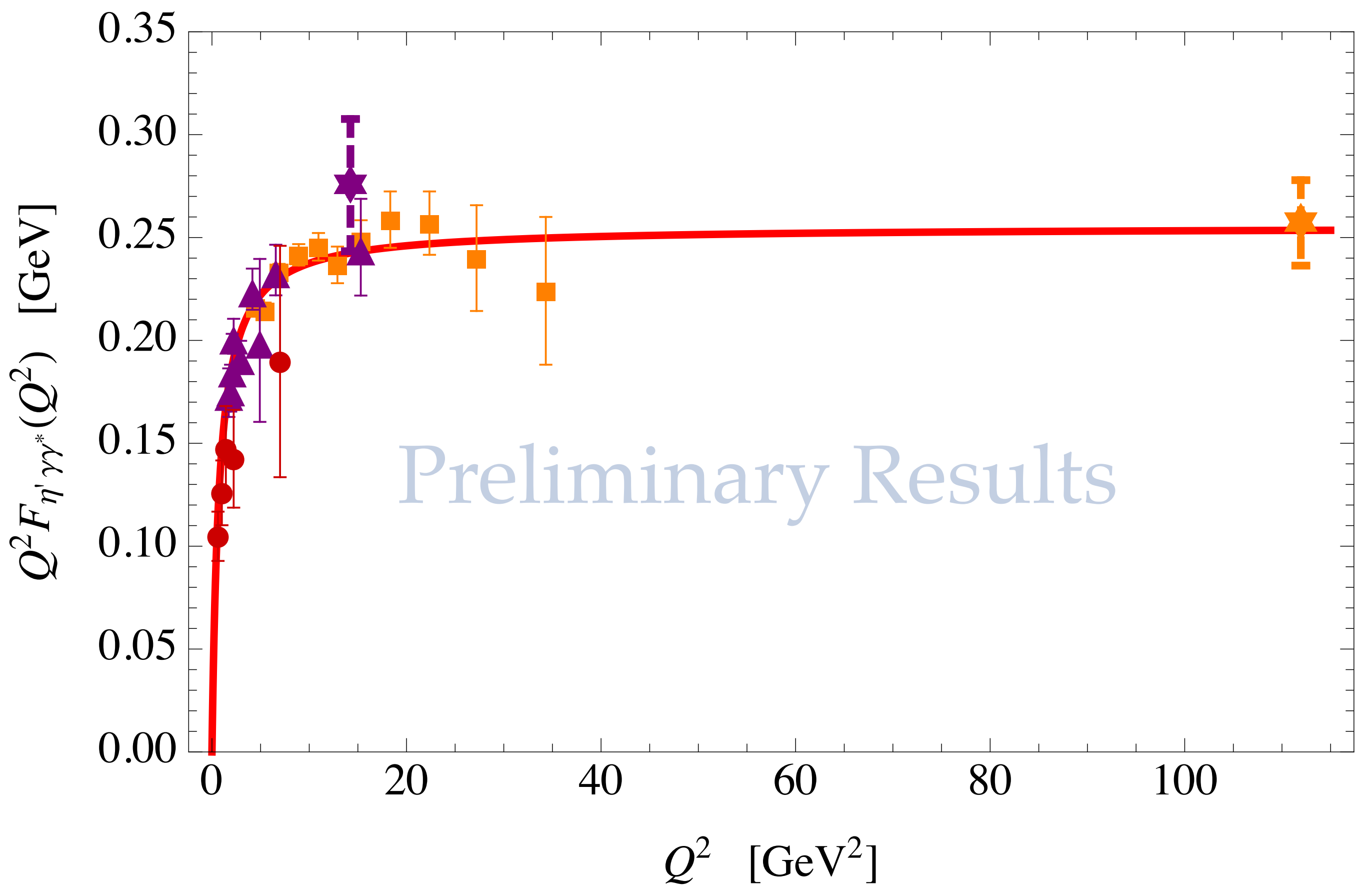
We combine some data points in next plots to avoid cluttering



FITTING RESULTS FOR η′

We use the Padé sequence P(N,N+1)

⇒ We are able to reach the P(0,1) approximant which is shown red



$$a_0 = 0.347(55)\ (PDG); \quad a_{\eta'} = 1.24(1)_{stat}(7)_{syst}; \quad b_{\eta'} = 1.54(4)_{stat}(32)_{sys}$$

Fitting Results for η'

Compare our slope to other results

	Spacelike			Timelike	χPT, Ametller *et. al*	Czyż *et. al*
	Our	CELLO	CLEO	Lepton-G	PRD **45**,(1992)	PRD **85**,(2012)
$a_{\eta'}$	1.24(1)(7)	1.46(16)	1.24(8)	1.6(4)	1.47	1.384(3)

It nicely agree with different results

All other experimental results use a VMD fit to obtain the result

We have an estimate for the systematic error in our final result

APPLICATIONS

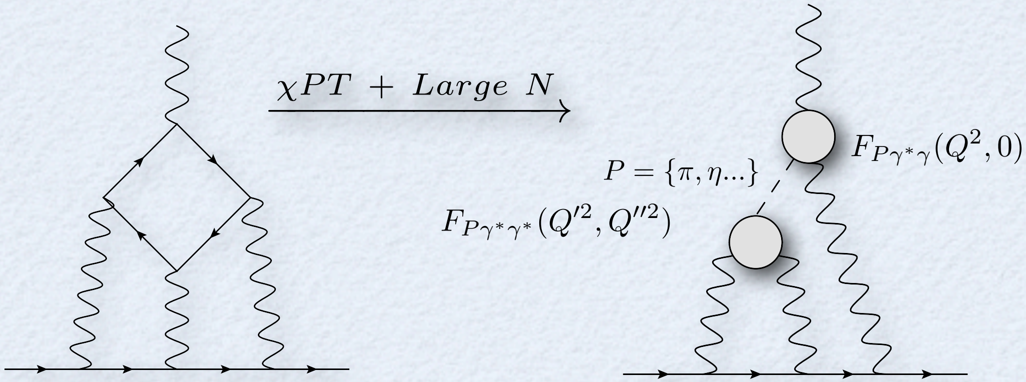
CALCULATION OF $(g\text{-}2)_\mu^{HLBL;PS}$

η-η' MIXING

CALCULATION OF $(g\text{-}2)_{\mu}^{HLBL;PS}$

ESTIMATION FOR $(g\text{-}2)_\mu^{HLBL;PS}$

- Our determination of the previous TFF may be used as an input to estimate the pseudoscalar Light by Light contribution to $(g\text{-}2)_\mu$ value (M. Knecht and A. Nyffeler, (2001)). We have the advantage of including systematic errors.



Latest measurement (E821 BNL)

$(g\text{-}2)_\mu = 116592089(54)_{st}(33)_{syst} \times 10^{-11}$

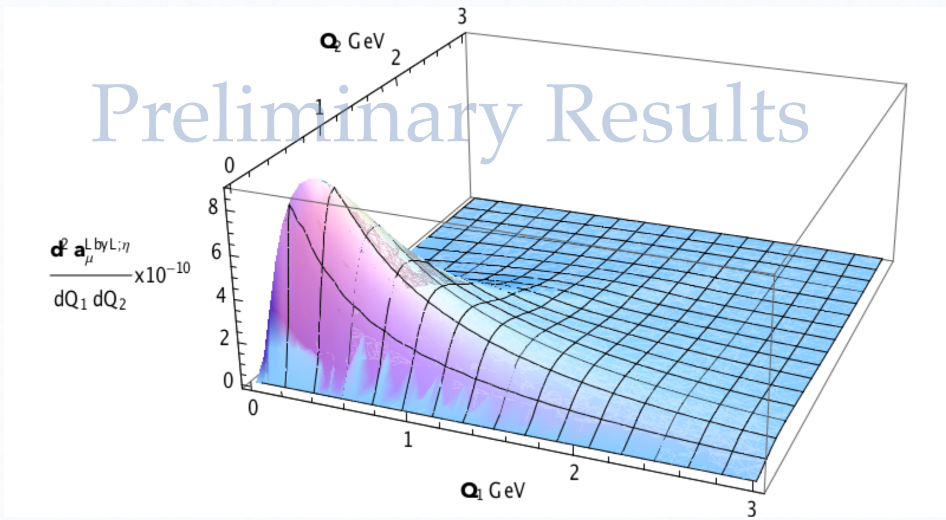
Theory

$(g\text{-}2)_\mu^{Th.} = 116591801(49) \times 10^{-11}$

QCD

EW

QED



- The integral over virtualitites is peaked at low energies (due to low muon mass)

- Bigger for pseudoscalar closer to the muon mass

ESTIMATION FOR $(g\text{-}2)_\mu^{\mathrm{HLBL;PS}}$

- In order to obtain the double virtual form factor, we may use Bose symmetry to use invariance upon $Q^2{}_1 \Leftrightarrow Q^2{}_2$.

$$F_{P\gamma^*\gamma^*}(Q_1^2,Q_2^2) = a\frac{b}{(b+Q_1^2)}\frac{b}{(b+Q_2^2)} \qquad F_{P\gamma^*\gamma^*}(Q_1^2,Q_2^2) = \frac{a+bQ_1^2}{(c+Q_1^2)(d+Q_1^2)}\frac{a+bQ_2^2}{(c+Q_2^2)(d+Q_2^2)}$$

- This TFF are PA as well. Since there is no data, we take the above ansatz.
- The first one have the decay and slope parameters as inputs.
- The second one needs two more: we take the curvature and the ϱ mass.
- We determine a (conservative) 5% systematic error

ESTIMATION FOR $(g\text{-}2)_\mu^{HLBL;PS}$

- We obtain (Also π^0 contribution in P. Masjuan PRD**86**, (2012) for completeness)

$F_{\pi\gamma\gamma^*}(Q_1^2, Q_2^2)$	π^0	η	η'	total
$P_1^0(Q_1^2, Q_2^2)$	$5.53(27)$	$1.38(10)$	$1.26(8)$	$8.24(45)$
$P_2^1(Q_1^2, Q_2^2)$	$5.55(^{+0.24}_{-0.34})$	$1.41(^{+0.10}_{-0.13})$	$1.25(^{+0.17}_{-0.11})$	$8.21(55)$
$P_1^0(Q_1^2, Q_2^2) \times P'^N_{N+1}(Q^2, 0)$	$5.52(^{+0.23}_{-0.16})$	$1.33(7)$	$1.27(6)$	$8.12(32)$
$P_2^1(Q_1^2, Q_2^2) \times P'^N_{N+1}(Q^2, 0)$	$5.53(^{+0.24}_{-0.21})$	$1.34(8)$	$1.25(10)$	$8.12(39)$

Table 1: Collection of results for the $a_\mu^{HLBL;PS}$ for $PS = \pi^0$, η and η' contributions. Results in units of 10^{-10}. N = 2,1,0 respectively for π^0, η, η'

Preliminary Results

- We see that the 5% error is big enough. The result is

$$a_\mu^{HLBL;PS}(8.2(5)_{stat}(4)_{sys})10^{-10}$$

- Our result improves errors with respect to the Knecht-Nyffeler even when systematics are included 8.3(1.2). Other results: 8.3(0.6) HGS Hayakawa 8.5(1.3) ENJL Prades (10^{-10} units).
- Still, off-shellness is not included and sign is unclear.

η-η' MIXING

Applications: η-η' mixing

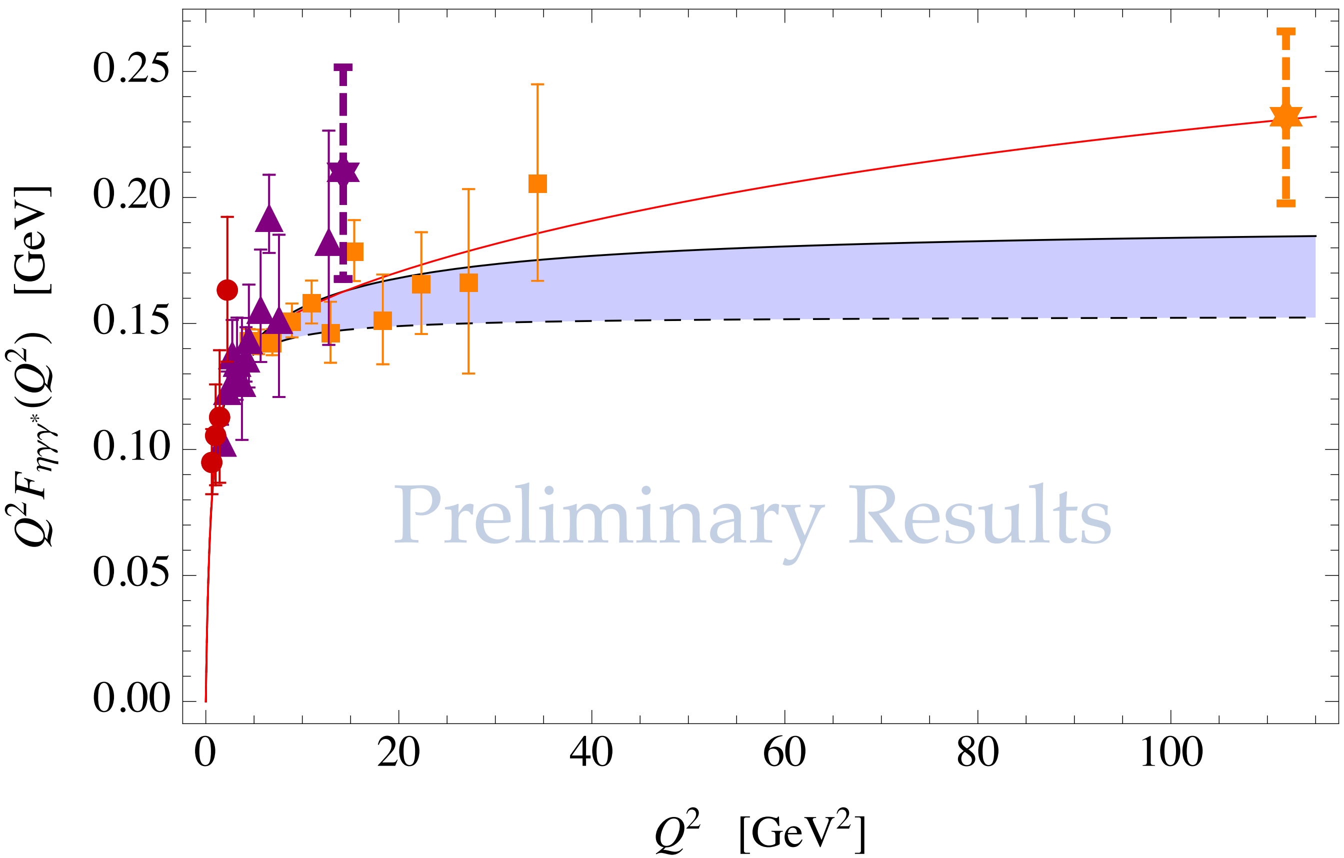
- The physical η, η' mesons are not those of the χPT SU(3) lagrangian: they mix each other
- However the mixing parameters are not known from first principles. Furthermore, their phenomenological values are not accurate enough

	SU(3)basis: $\{\eta_8 \sim \eta,\ \eta_0 \sim \eta'\}$	Flavor basis:$\{\eta_q,\ \eta_s\}$
Mixing Scheme	$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} cos\theta_8 & -sin\theta_0 \\ sin\theta_8 & cos\theta_0 \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix}$	$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} cos\phi_q & -sin\phi_s \\ sin\phi_q & cos\phi_s \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}$
Phenomenological results	$\frac{f_8}{f_\pi} = 1.26; \quad \frac{f_0}{f_\pi} = 1.17$ $\theta_8 = -21.2^\circ; \quad \theta_0 = -9.2^\circ$	$\frac{f_q}{f_\pi} = 1.07; \quad \frac{f_s}{f_\pi} = 1.34$ $\phi_q = \phi_s = 39.3^\circ$

- Constrained in the literature using $\eta(\eta') \rightarrow \gamma\gamma$ as well as hadron decays into $\eta(\eta')$
- η' gluonium content issues discussed later

η-TFF Asymptotics

Setting free the asymptotic limit (P(N,N+1)) we obtain 0.18(+0.15/-0.03)
Using results from the literature we obtain reasonable values ($\chi^2/_{\rm dof} < 1.3$) with asymptotic limit in the range (0.154-0.190)GeV

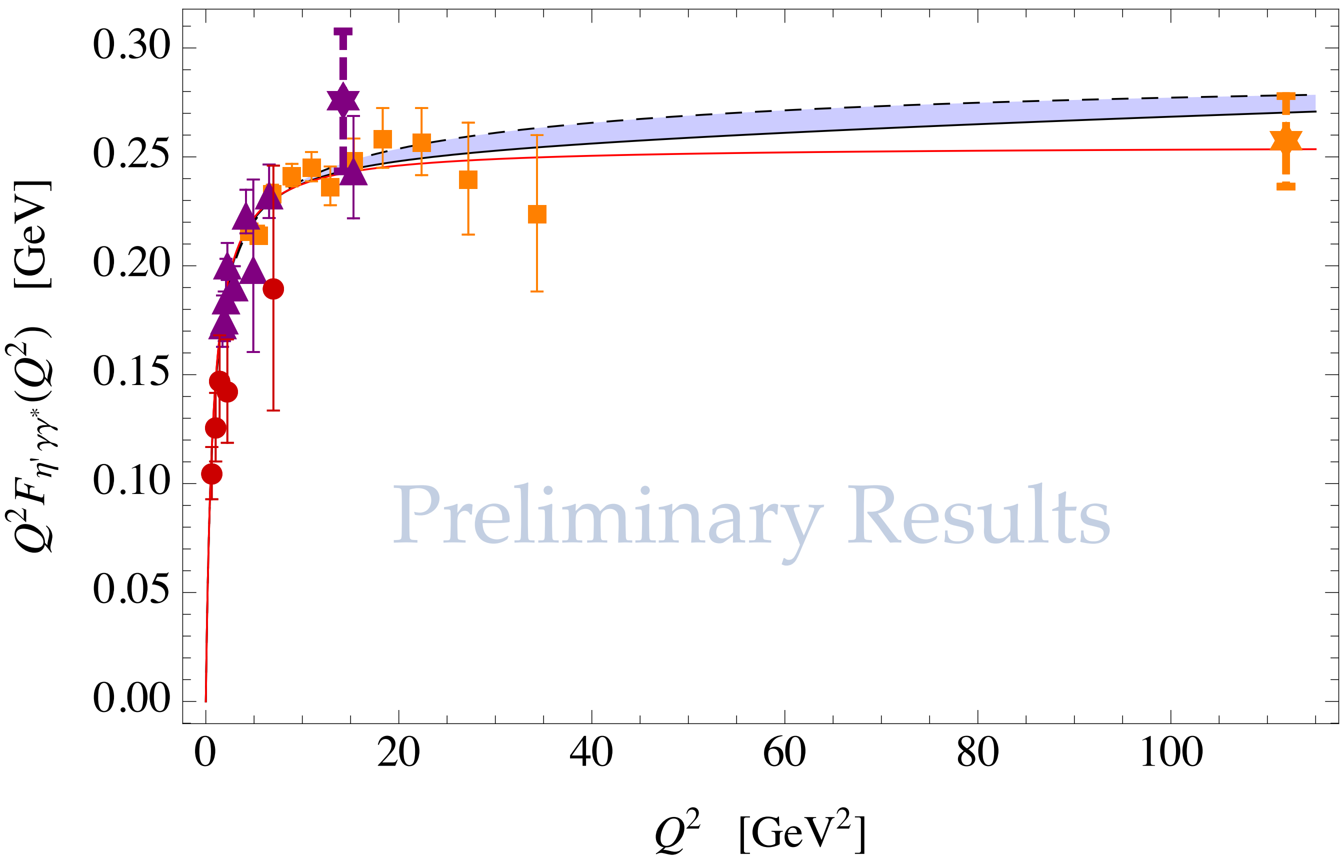


η′-TFF Asymptotics

Setting free the asymptotic limit (P(N,N+1)) we obtain 0.255(2) GeV

⇒ Compare with the literature (0.29-0.36) GeV using P′(N,N+1)

⇒ All descriptions similar and reach asymptotics quite far



η-η' Mixing Parameters

- Since η' TFF asymptotic limit was precisely determined using fitting procedures, fix it @ 0.255 GeV.
- For the η TFF, take the range we found: (0.154-0.190) GeV.
- Use decay amplitudes and asymptotics to find mixing pars. (need 4 pars.).

Preliminary Results

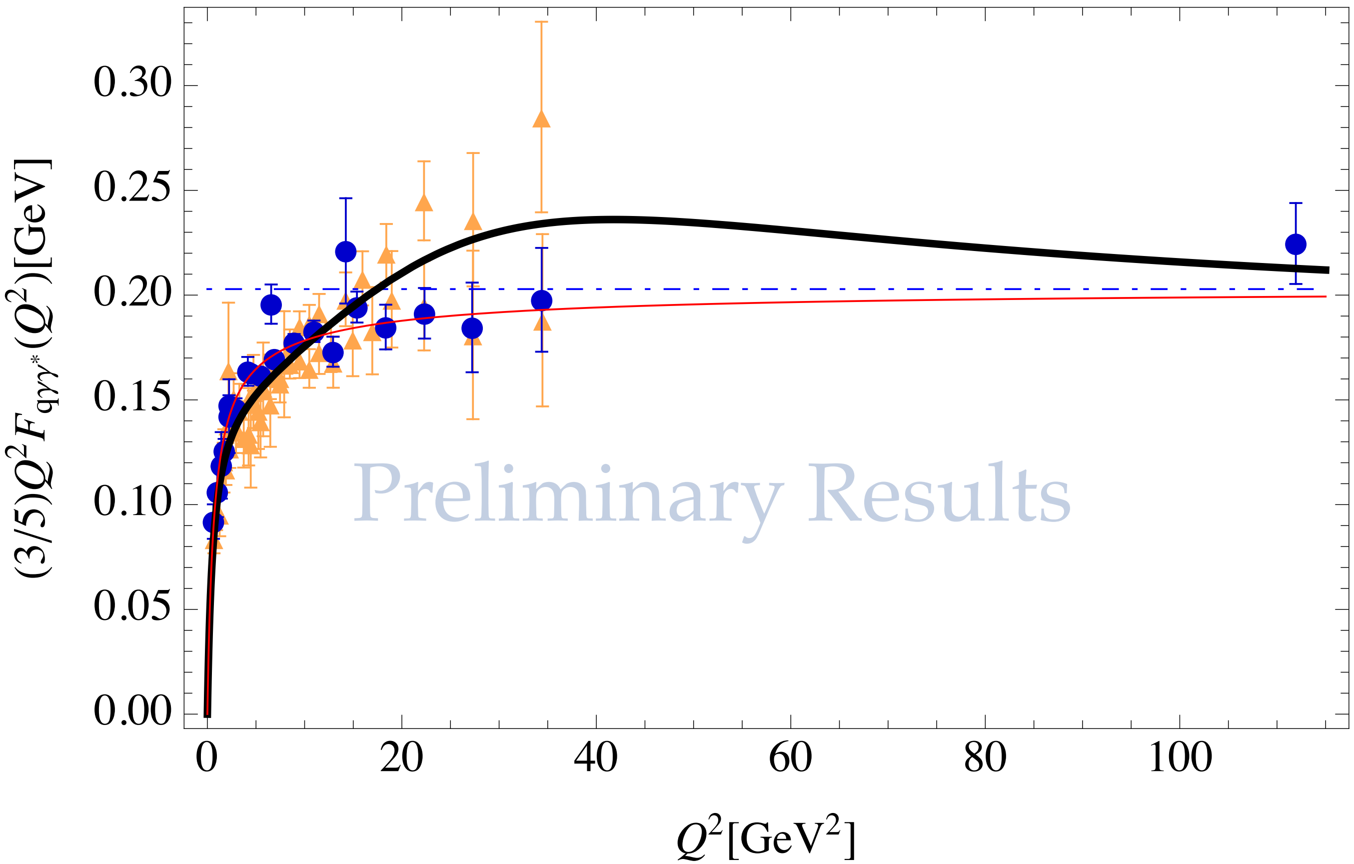
$\lim_{Q^2\to\infty} Q^2 F_{\eta\gamma\gamma^*}(Q^2)$	θ_8	θ_0	f_8	f_0
0.190	-33.8°	-4.3°	1.84	1.21
0.170	-36.3°	-3.6°	1.72	1.21
0.154	-35.6°	-7.2°	1.29	1.12

$\lim_{Q^2\to\infty} Q^2 F_{\eta\gamma\gamma^*}(Q^2)$	ϕ_q	ϕ_s	f_q	f_s
0.190	15.7°	36.9°	1.10	2.43
0.170	21.7°	37.9°	0.98	2.08
0.154	21.0°	34.7°	0.91	2.16

- ϕ_q and f_s specially sensitive to the asymptotics and decays ⇒ More input.
- We may be sensitive to gluonium content.
- Big differences respect to the literature ⇒ Use a more elaborated approach.
- Naive approach to be improved in a future work with R. Escribano and P. Masjuan

η, η‘ LIGHT QUARK CONTENT

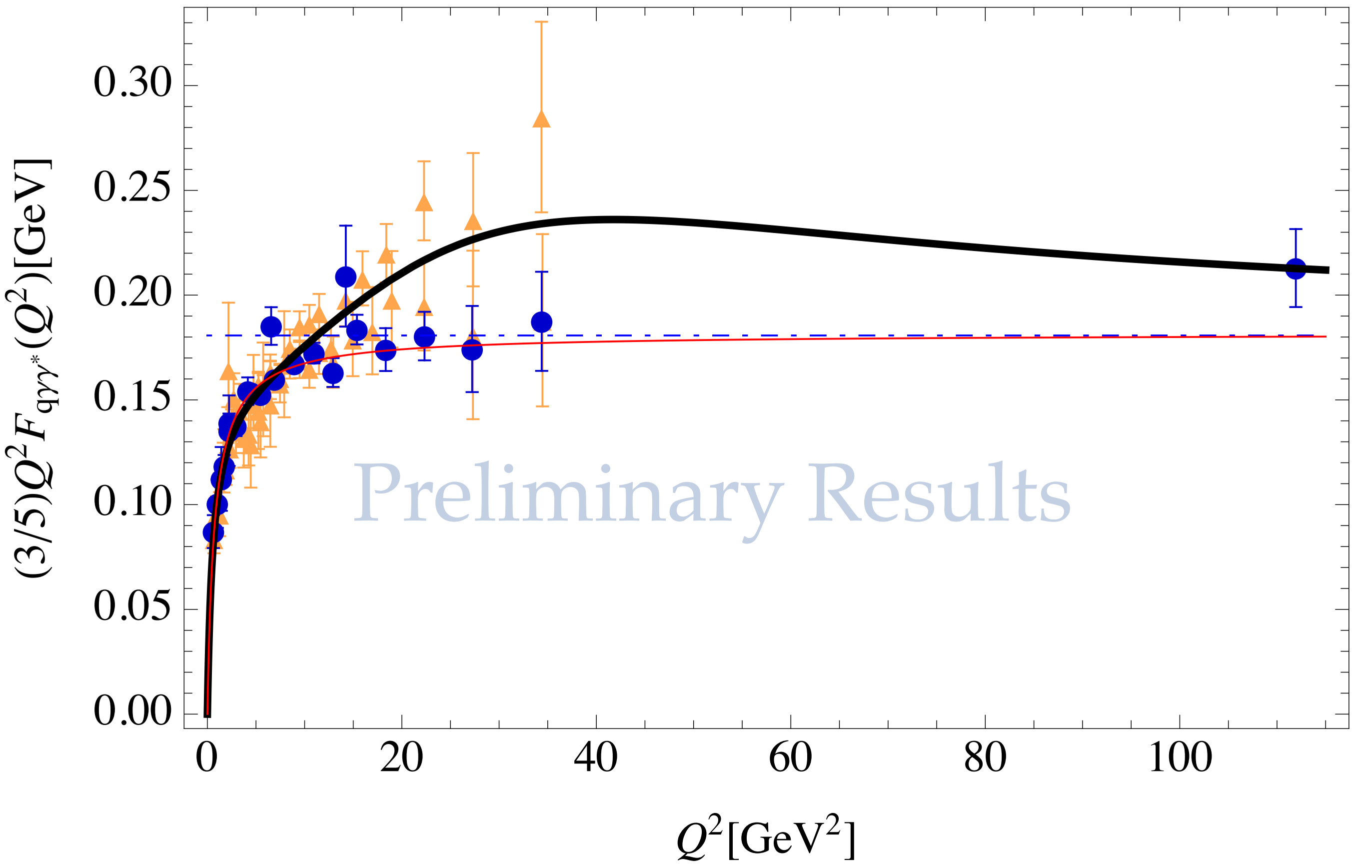
- Having the mixing angles we can find the light quark content of the η and η′
- We would expect it to be similar to the π TFF



Asymptotic η (η ‘) limit = 0.190 GeV (0.255 GeV)

η, η' Light Quark Content

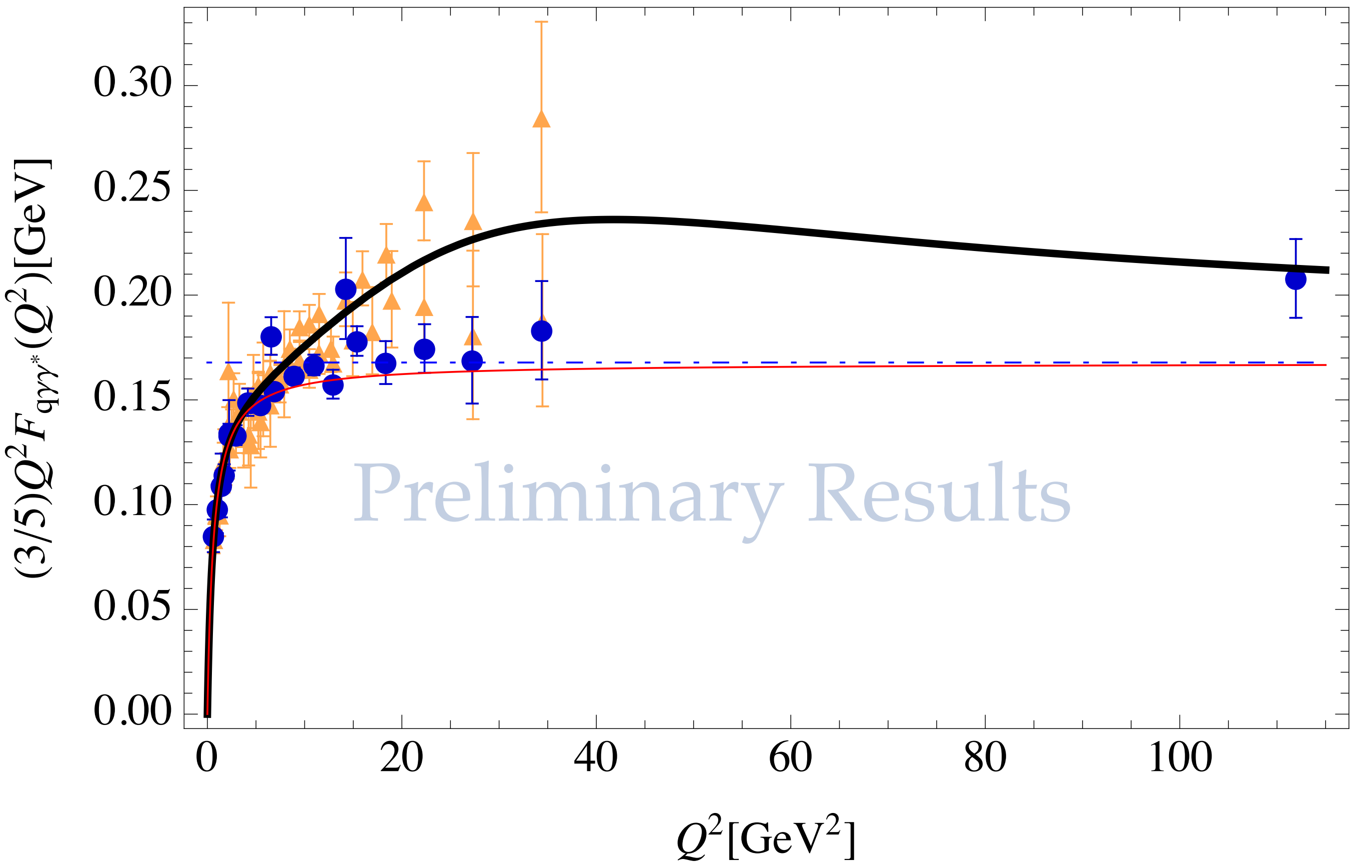
- Having the mixing angles we can find the light quark content of the η and η′
- We would expect it to be similar to the π TFF



Asymptotic η (η ') limit = 0.170 GeV (0.255 GeV)

η, η' LIGHT QUARK CONTENT

- Having the mixing angles we can find the light quark content of the η and η′
- We would expect it to be similar to the π TFF



Asymptotic η (η ') limit = 0.150 GeV (0.255 GeV)

CONCLUSIONS

Conclusions

- We have developed a method which is model independent, allows for systematics and may be a good tool for a variety of purposes, being user-friendly.
- We have found reasonable values (with systematics) for the low energy constants of the η, η' TFF.
- This methods allows us to compute $(g\text{-}2)_\mu^{HLBL;PS}$ contribution with improved accuracy and to include systematic error due to TFF parametrization.
- This method is useful to constrain the η-η' mixing.
- Forthcoming data from BES-III will improve the results shown here as well as the π^0 one. Some Belle analysis for the η and the η' would be very welcomed too.

THANKS FOR YOUR ATTENTION!

Backup: Systematics

- Take well-motivated models describing transition from factors: Regge, Holographic & Log models

- Take values from this models at points similar to the available experimentally. Then fit and check convergence to the real PA for this model (not including noise, since we aim for systematics)

	P_1^0	P_1^1	P_1^2	P_1^3	P_1^4	P_1^5	$F_{\pi\gamma}$ (exact)
$a_0(GeV^{-1})$	0.2556	0.2694	0.2734	0.2746	0.2751	0.2752	0.2753
$a_1(GeV^{-1})$	0.1290	0.1716	0.1935	0.2051	0.2124	0.2166	0.22294
$a_2(GeV^{-1})$	0.0651	0.1147	0.1492	0.1725	0.1898	0.2013	0.2549

Table: Results for the $F_{\pi\gamma}(Q^2)$ in the log model appearing in Ref. [8]

Backup: PA sequences

- Values obtained with the different sequences are compatible within errors.
- We take the weighted average as the final value for slope and curvature, which will be ascribed a 5.6% and 21% respectively
- The value at the origin and its error is, essentially, the one from experiment

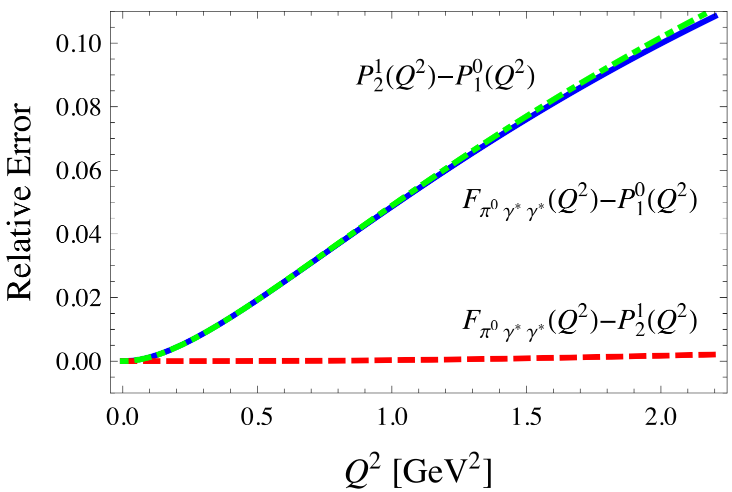
TABLE I. Pseudoscalar Transition Form Factor results from the fit to experimental data.

	πTFF				ηTFF				$\eta' TFF$			
	N	a_π	$b_\pi \times 10^{-3}$	χ^2/dof	N	a_η	b_η	χ^2/dof	N	$a_{\eta'}$	$b_{\eta'}$	χ^2/dof
$P[N,1]$	5	0.0340(35)	1.20(28)	0.79	5	0.569(60)	0.328(77)	0.92	5	1.29(10)	1.66(30)	0.81
$P[N,2]$	1	0.0324(20)	1.07(15)	0.76	1	0.545(24)	0.298(27)	0.85	0	1.24(3)	1.53(6)	0.81
$P[N,N+1]$	2	0.0331(45)	1.11(27)	0.76	1	0.545(24)	0.298(27)	0.85	0	1.23(2)	1.52(6)	0.83
$P'[N,N+1]$	2	0.0332(25)	1.13(19)	0.77	1	$0.582(^{+83}_{-70})$	$0.346(^{+120}_{-96})$	0.91	1	1.25(3)	1.56(9)	0.83
$PT[N,1]$	5	0.0302(28)	0.92(18)	0.82	6	0.545(30)	0.300(40)	0.95	6	1.29(5)	1.66(16)	0.83
Final		0.0324(12)	1.06(9)			0.547(18)	0.303(25)			1.24(1)	1.54(4)	

Estimation for $(g-2)_\mu^{HLBL}$

- In order to obtain the double virtual form factor, we may use Bose symmetry to use invariance upon $Q^2_1 \Leftrightarrow Q^2_2$.

$$F_{P\gamma^*\gamma^*}(Q_1^2,Q_2^2) = a\frac{b}{(b+Q_1^2)}\frac{b}{(b+Q_2^2)} \qquad F_{P\gamma^*\gamma^*}(Q_1^2,Q_2^2) = \frac{a+bQ_1^2}{(c+Q_1^2)(d+Q_1^2)}\frac{a+bQ_2^2}{(c+Q_2^2)(d+Q_2^2)}$$



- The errors may be estimated comparing our TFF(Q^2,0) PA convergence to a model (Regge).
- It is shown that difference between P(1,2) and P(0,1) is a good estimate to the real error.
- Still, since integral relevant until 1 GeV we take 5% systematics, though may be refined.
- Parametrizing this Q^2-dependent error we obtain 2% error, similar to the difference among approximants.

BACKUP: MIXING

Defining the decay constant using the mixing parameters in the following way

$$\begin{pmatrix} f_\eta^8 & f_\eta^0 \\ f_{\eta'}^8 & f_{\eta'}^0 \end{pmatrix} \equiv \begin{pmatrix} f_8 cos\theta_8 & -f_0 sin\theta_0 \\ f_8 sin\theta_8 & f_0 cos\theta_0 \end{pmatrix} \qquad \begin{pmatrix} f_\eta^q & f_\eta^s \\ f_{\eta'}^q & f_{\eta'}^s \end{pmatrix} = \begin{pmatrix} f_q cos\phi_q & -f_s sin\phi_s \\ f_q sin\phi_s & f_s cos\phi_s \end{pmatrix}$$

We obtain the following expression for the asymptotic limit

$$\lim_{Q^2\to\infty} Q^2 F_{\eta\gamma\gamma^*}(Q^2) = 6\sqrt{2}(f_\eta^q \frac{5}{9\sqrt{2}} + f_\eta^s \frac{1}{9})$$

$$\lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma\gamma^*}(Q^2) = 6\sqrt{2}(f_{\eta'}^q \frac{5}{9\sqrt{2}} + f_{\eta'}^s \frac{1}{9})$$

And the next one for the decay int photons

$$\Gamma_{\eta\gamma\gamma} = \frac{9\alpha^2}{32\pi^3} m_\eta^3 \left(\frac{f_{\eta'}^s \left(\frac{5}{9\sqrt{2}}\right) - f_{\eta'}^q \left(\frac{1}{9}\right)}{f_{\eta'}^s f_\eta^q - f_{\eta'}^q f_\eta^s} \right)^2$$

$$\Gamma_{\eta'\gamma\gamma} = \frac{9\alpha^2}{32\pi^3} m_\eta^3 \left(\frac{f_\eta^s \left(\frac{5}{9\sqrt{2}}\right) - f_\eta^q \left(\frac{1}{9}\right)}{f_{\eta'}^s f_\eta^q - f_{\eta'}^q f_\eta^s} \right)^2$$

In units where asymptotics for the $\pi = 2f_\pi$ and TFF(0,0) = $(4\pi^2 f_\pi)^{-1}$

η-TFF Asymptotics

Higher asymptotic (solid black) is preferred by data
Low energy data would help distinguishing different asymptotic values too
Hence forthcoming Belle as well as BES-III data may help in constraining asymptotic