CRC 1044

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Gamma-gamma sum rules and their implication on the hadronic LbL contribution to $(g-2)_\mu$

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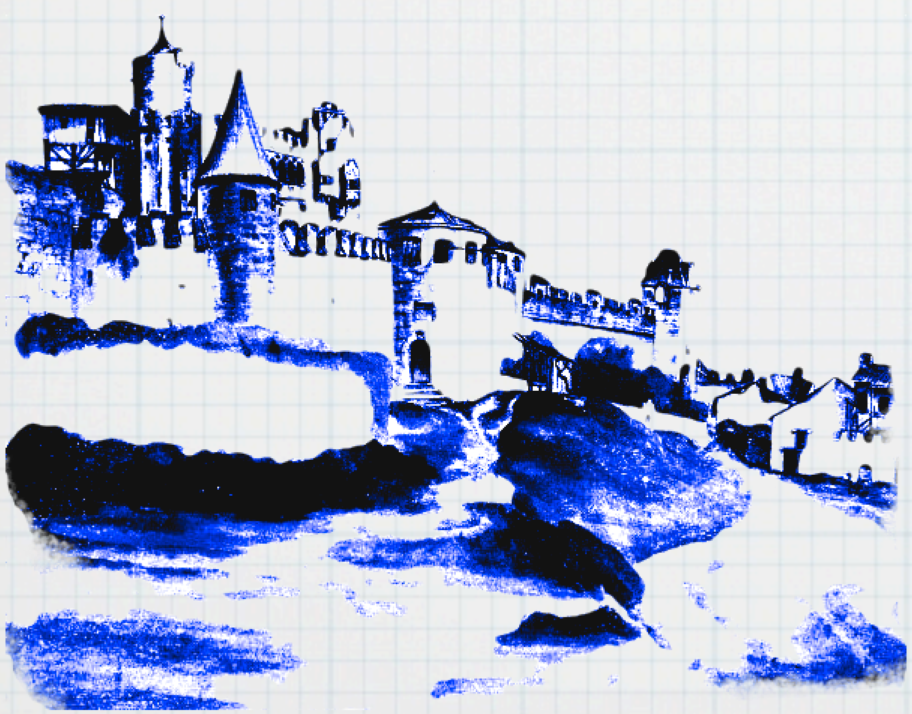


RMCWG meeting at ETC,
Trento, April 11, 2013

Outline

- $(g-2)_\mu$: motivation
- sum rules and low-energy light-by-light scattering
- hadronic contribution to $(g-2)_\mu$
  - pseudoscalar meson contribution
  - axial-vector contribution
  - perspectives

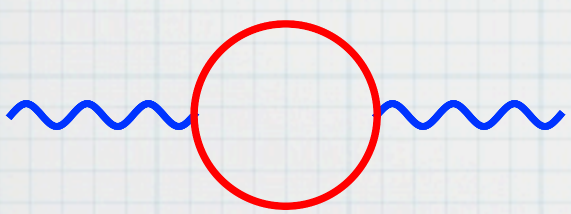
$(g-2)_\mu$ - the crystal ball of particle physics

predictability of the g-2 for testing theory:
in a renormalizable local relativistic QFT
g-2 vanishes at tree level
→ a calculable quantity!

test of the relativistic QFT with
unprecedented accuracy:

- strict limits on deviations from the SM
- a window to new physics

mediates helicity-flip amplitude $\sim m_{lepton}^2$
enhanced short distance sensitivity



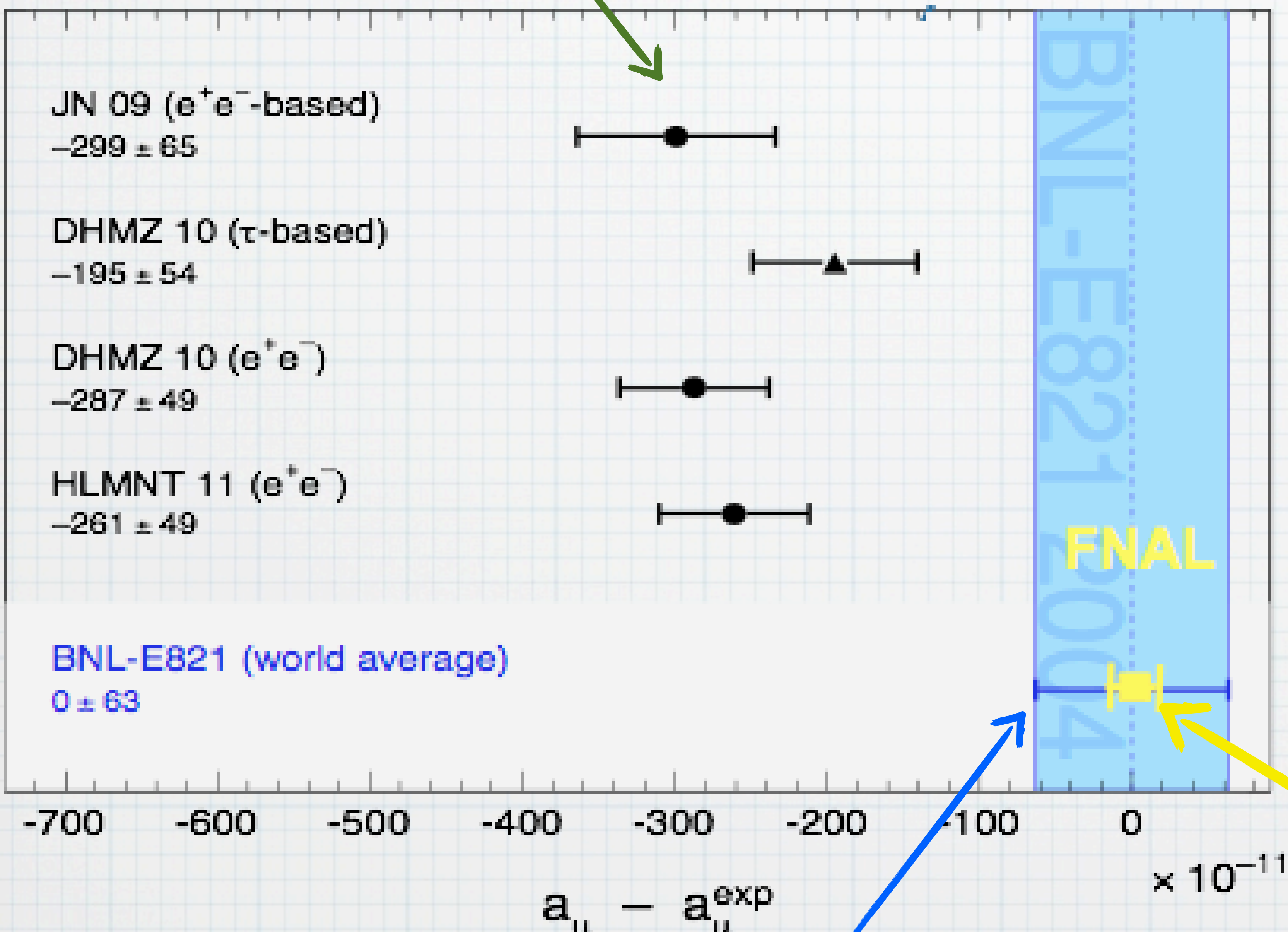
$$\frac{\delta a_l}{a_l} \propto \frac{m_l^2}{M^2}$$

$(g-2)_\mu$: theory vs experiment

SM predictions for $(g-2)_\mu$

$a_\mu^{exp} - a_\mu^{th} = (24.9 \pm 8.7) \times 10^{-10}$

(2.9σ)



Error(s) or New Physics?

→ Clarify situation!

New FNAL $(g-2)_\mu$ measurement (2015):

factor 4 improvement in experimental error

→ improve theory!

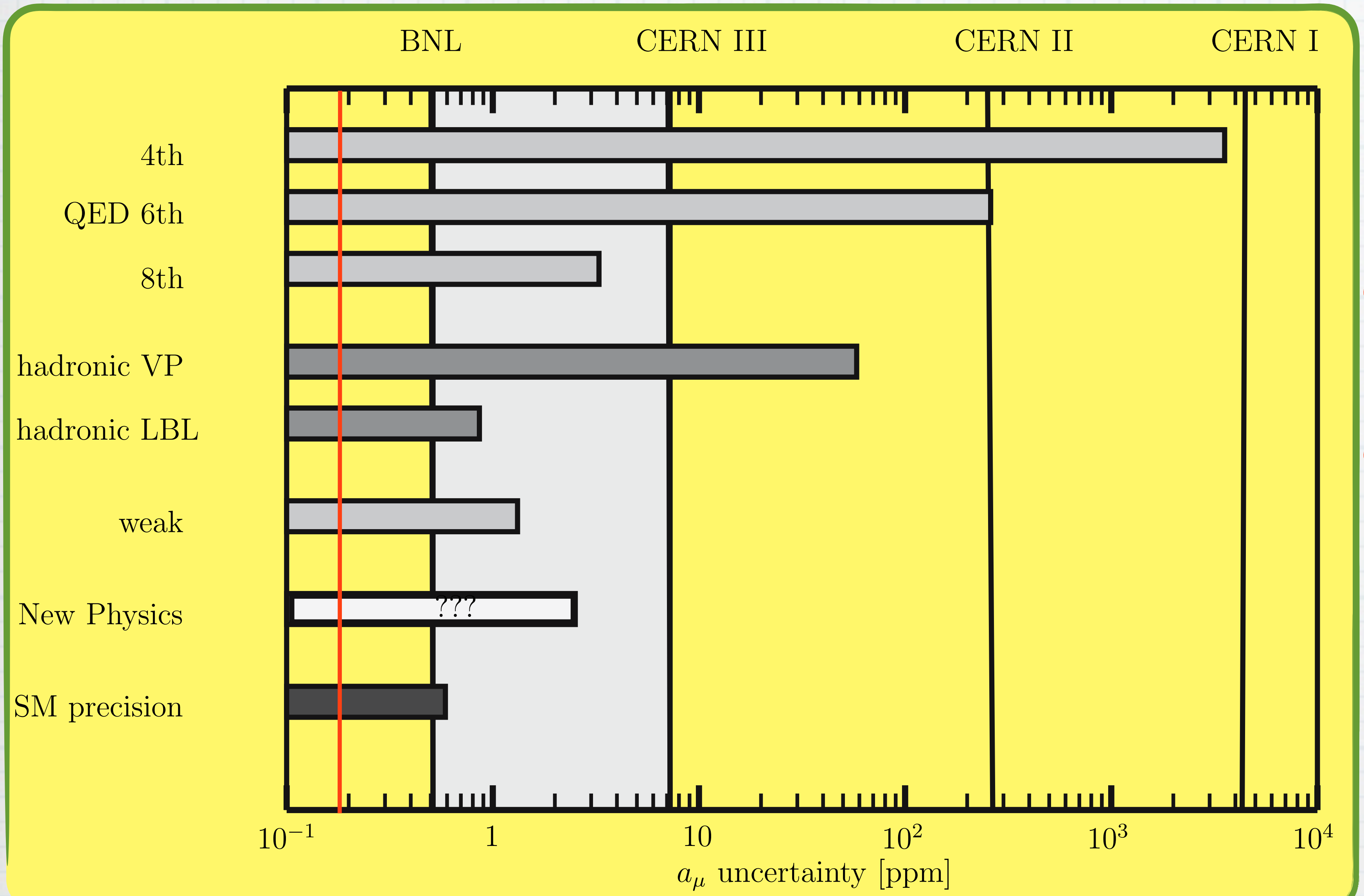
$\pm 1.6 \cdot 10^{-10}$

E821 measurement of $(g-2)_\mu$

$a_\mu^{exp} = (11\ 659\ 208.9 \pm 6.3) \cdot 10^{-10}$

(g-2)$_\mu$: SM predictions

sensitivity of g-2 experiments to various contributions

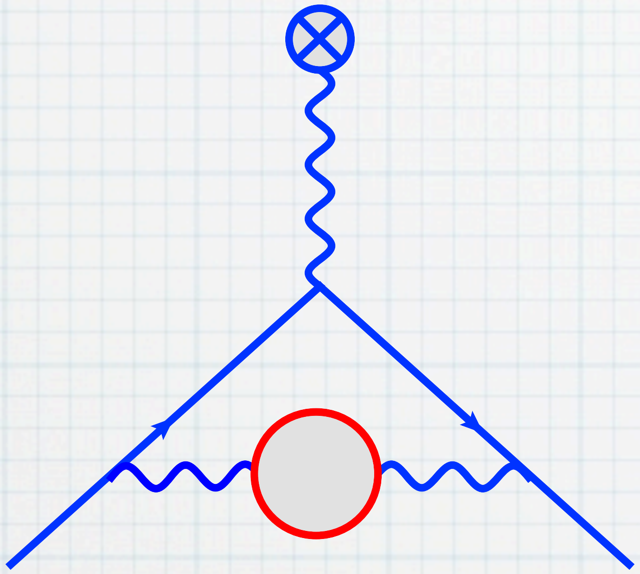


the various types of contributions to (g-2)$_\mu$ in units 10^{-6}

L.O. universal	1161.409 73	(0)
e–loops	6.194 57	(0)
H.O. universal	−1.757 55	(0)
L.O. hadronic	0.069 21	(56)
L.O. weak	0.001 95	(0)
H.O. hadronic	−0.001 00	(2)
LbL. hadronic	0.000 93	(34)
τ–loops	0.000 43	(0)
H.O. weak	−0.000 41	(2)
$e+\tau$–loops	0.000 01	(0)
theory	1165.917 86	(66)
experiment	1165.920 80	(63)

Strong contributions to $(g-2)_\mu$

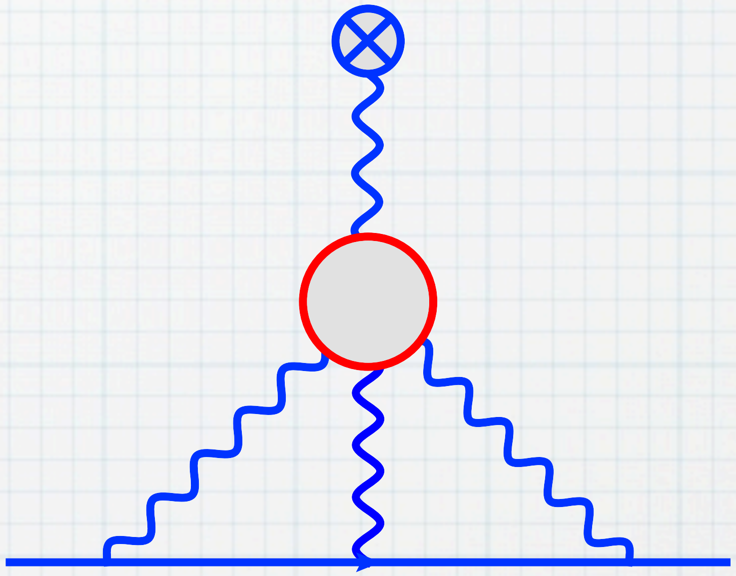
hadronic vacuum polarization



$a_\mu^{had, VP} = (692.3 \pm 4.2) \times 10^{-10}$

hadronic vacuum polarization determined by cross section measurements of e^+e^- -> hadrons

hadronic light-by-light scattering

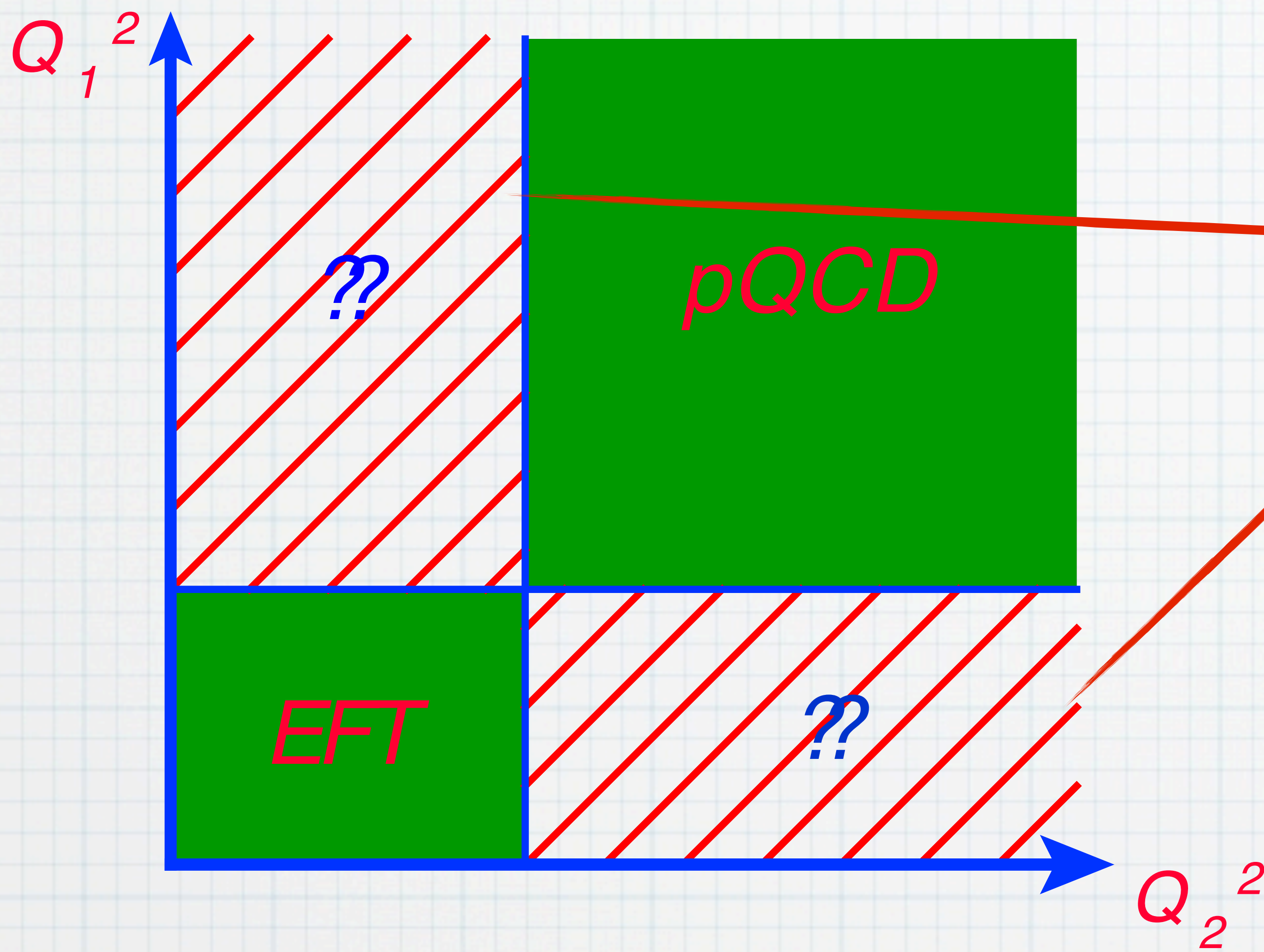


$a_\mu^{had, LbL} = (11.6 \pm 4.0) \times 10^{-10}$

measurements of meson transition form factors required as input to reduce uncertainty

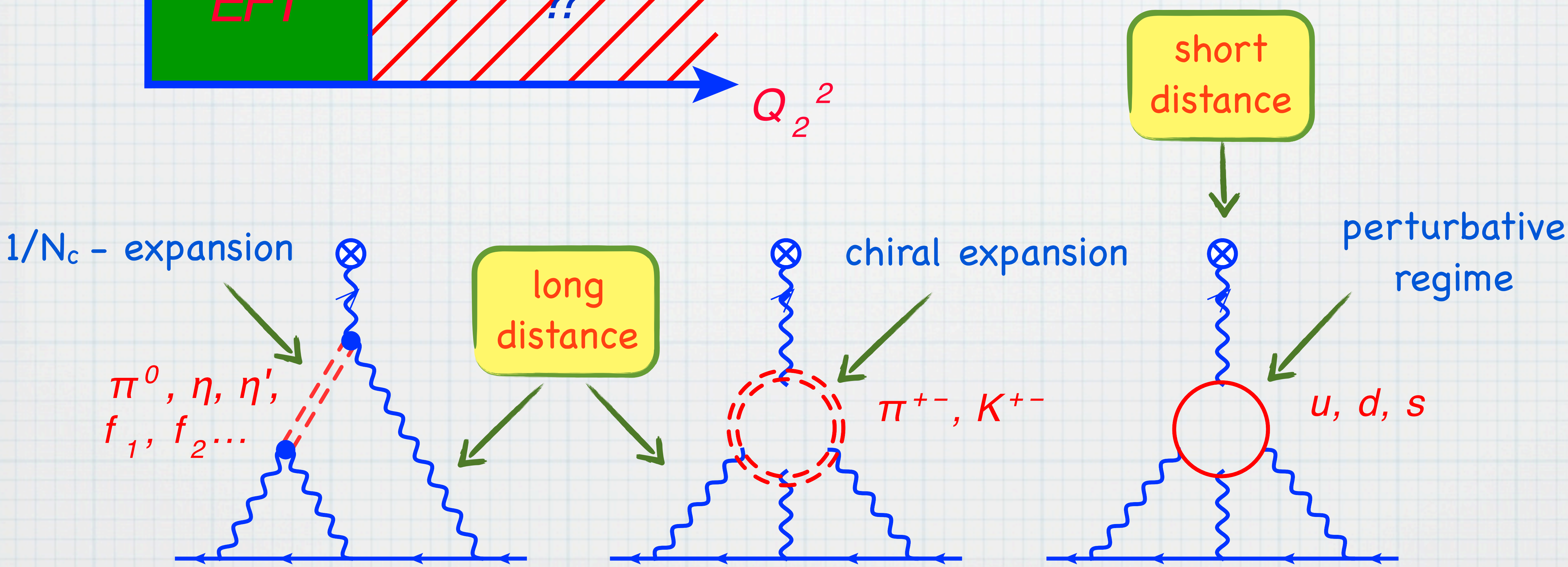
Jegerlehner, Nyffeler (2009)

Hadronic models of LbL scattering

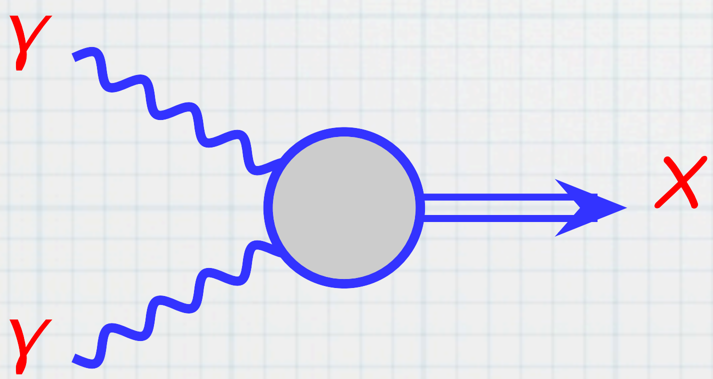


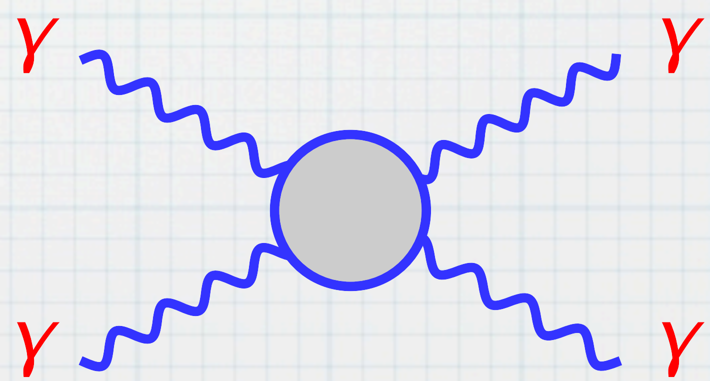
multi-scale problem -
mixed soft-hard regions

no answer is available in
general, unless we have data!

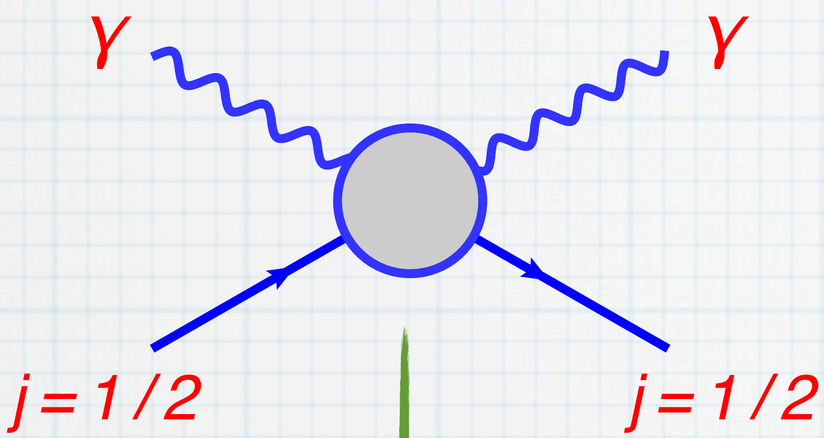


Sum rules
for light-by-light scattering





Sum rules

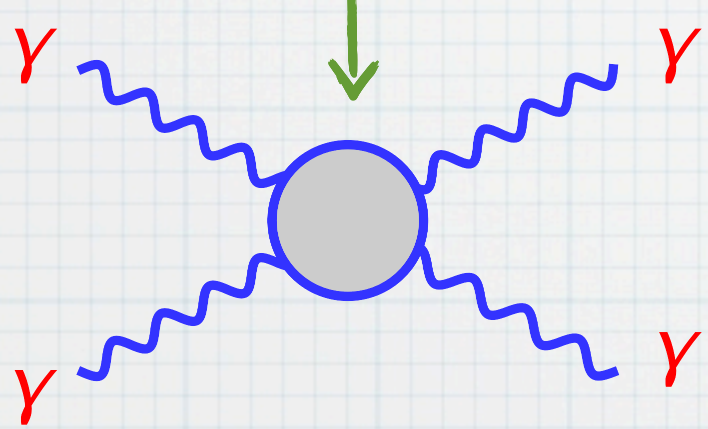


1966

Gerasimov-Drell-Hearn sum rule

$$\frac{e^2}{2M^2}\kappa^2 = \frac{1}{\pi}\int_0^\infty \frac{ds}{s}\left[\sigma_{3/2}(s) - \sigma_{1/2}(s)\right]$$

photon target → no anomalous moments!



1995

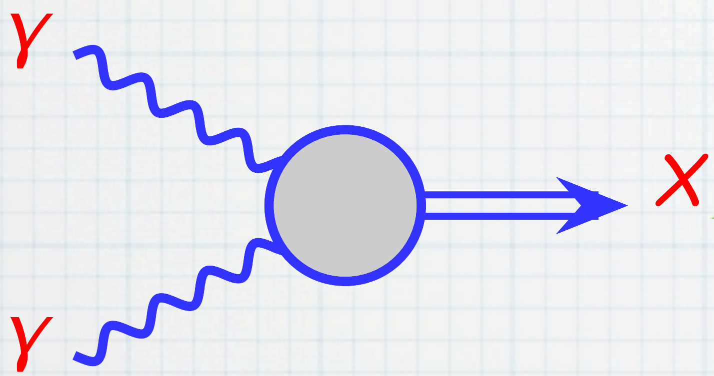
Light-by-light sum rule

$$0 = \int_0^\infty \frac{ds}{s}\left[\sigma_2(s) - \sigma_0(s)\right]$$

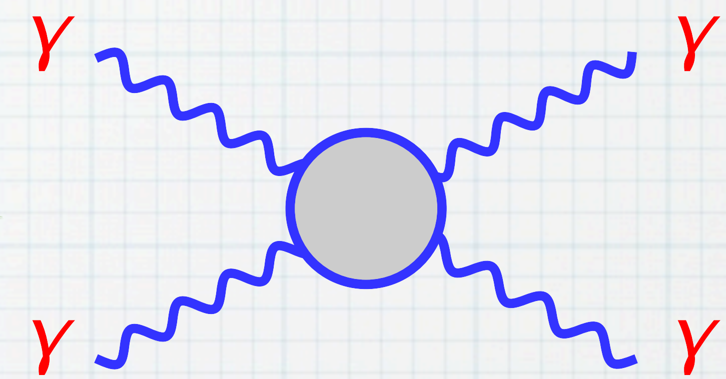
- hadron physics: leads to constraints on hadronic contribution to light-by-light scattering
- field theory: provide an explicit check of consistency with causality

Sum rules

micro-causality



unitarity → dispersion theory ← LETs



imaginary part of the amplitude – photon-photon fusion into leptons and hadrons:

$$\mathrm{Im} f^{(-)}(s) = -\tfrac{s}{8}\,[\,\Delta\sigma(s)\,]$$
$$\mathrm{Im} f^{(+)}(s) = -\tfrac{s}{8}\,[\,\sigma_{tot}(s)\,]$$

real part of the amplitude – low-energy structure of the elastic LbL scattering:

$$\mathcal{L}^{(8)} = c_1 (F_{\mu\nu}F^{\mu\nu})^2 + c_2 (F_{\mu\nu}\tilde{F}^{\mu\nu})^2$$

Sum rules

$$\int_{s_0}^{\infty} \frac{ds}{s}\,[\sigma_2(s) - \sigma_0(s)] = 0$$

$$c_1 \pm c_2 = \frac{1}{8\pi}\int_{s_0}^{\infty} \frac{ds}{s^2}\,[\sigma_\parallel(s) \pm \sigma_\perp(s)]$$

Sum rules

3 superconvergent relations:

helicity difference sum rule

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} \left[\sigma_0 - \sigma_2\right]_{Q_2^2=0}$$

sum rules involving longitudinal photons

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)^2} \left[\sigma_{\parallel} + \sigma_{LT} + \frac{(s+Q_1^2)}{Q_1 Q_2} \tau_{TL}^{a}\right]_{Q_2^2=0}$$

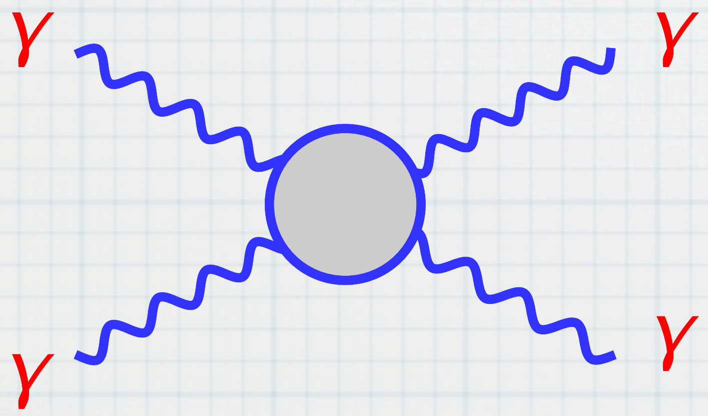
$$0 = \int_{s_0}^{\infty} ds \left[\frac{\tau_{TL}}{Q_1 Q_2}\right]_{Q_2^2=0}$$

SRs involving LbL low-energy constants:

$$c_1 \pm c_2 = \frac{1}{8\pi} \int_{s_0}^{\infty} \frac{ds}{s^2} \left[\sigma_{\parallel}(s) \pm \sigma_{\perp}(s)\right]$$

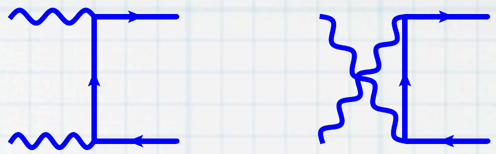
...

Low-energy LbL scattering using sum rules



$$\mathcal{L}^{(8)} = c_1 (F_{\mu\nu}F^{\mu\nu})^2 + c_2 (F_{\mu\nu}\tilde{F}^{\mu\nu})^2$$

Pair production in QED



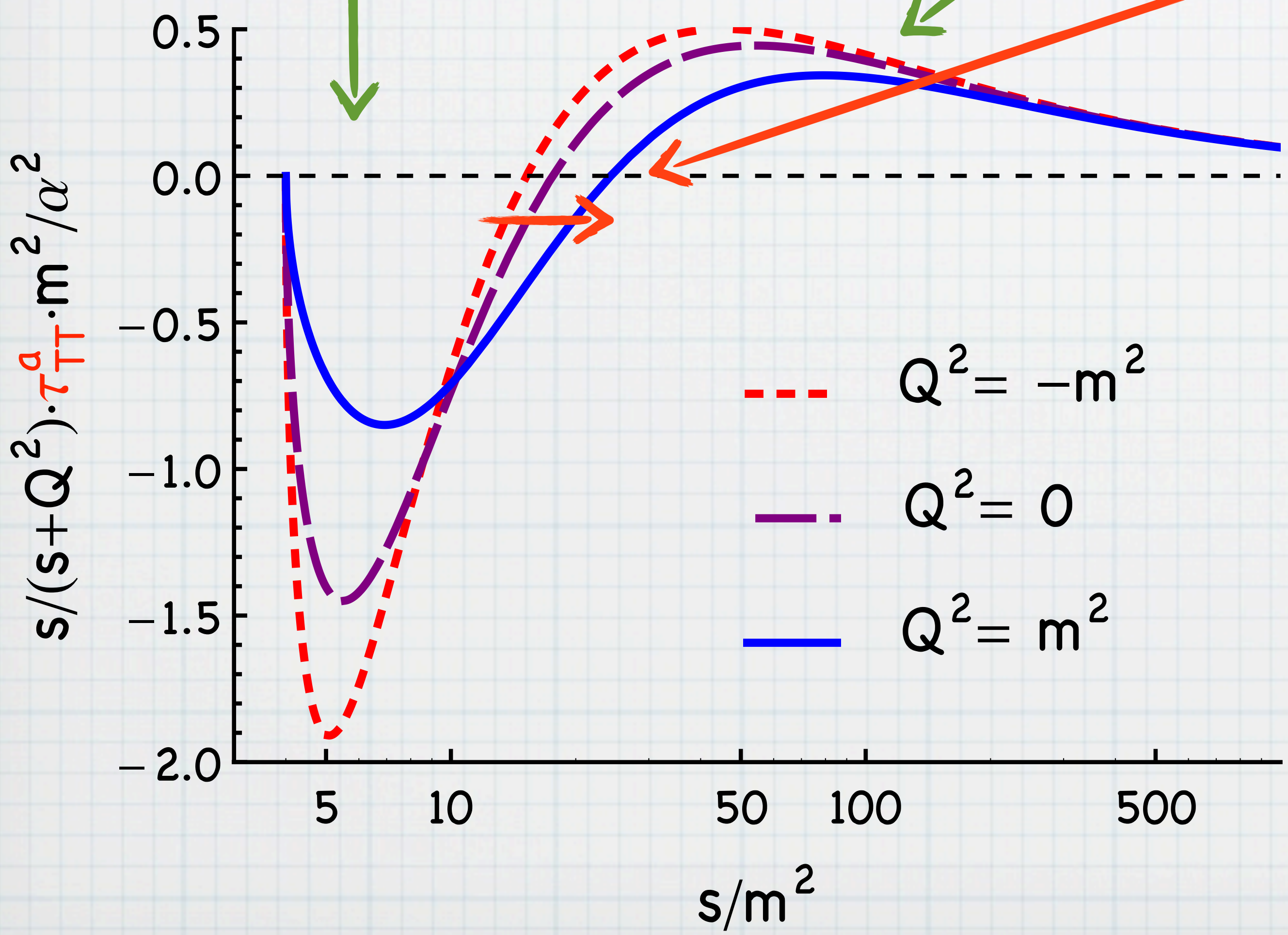
$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} \left[\sigma_2 - \sigma_0\right]_{Q_2^2=0}$$

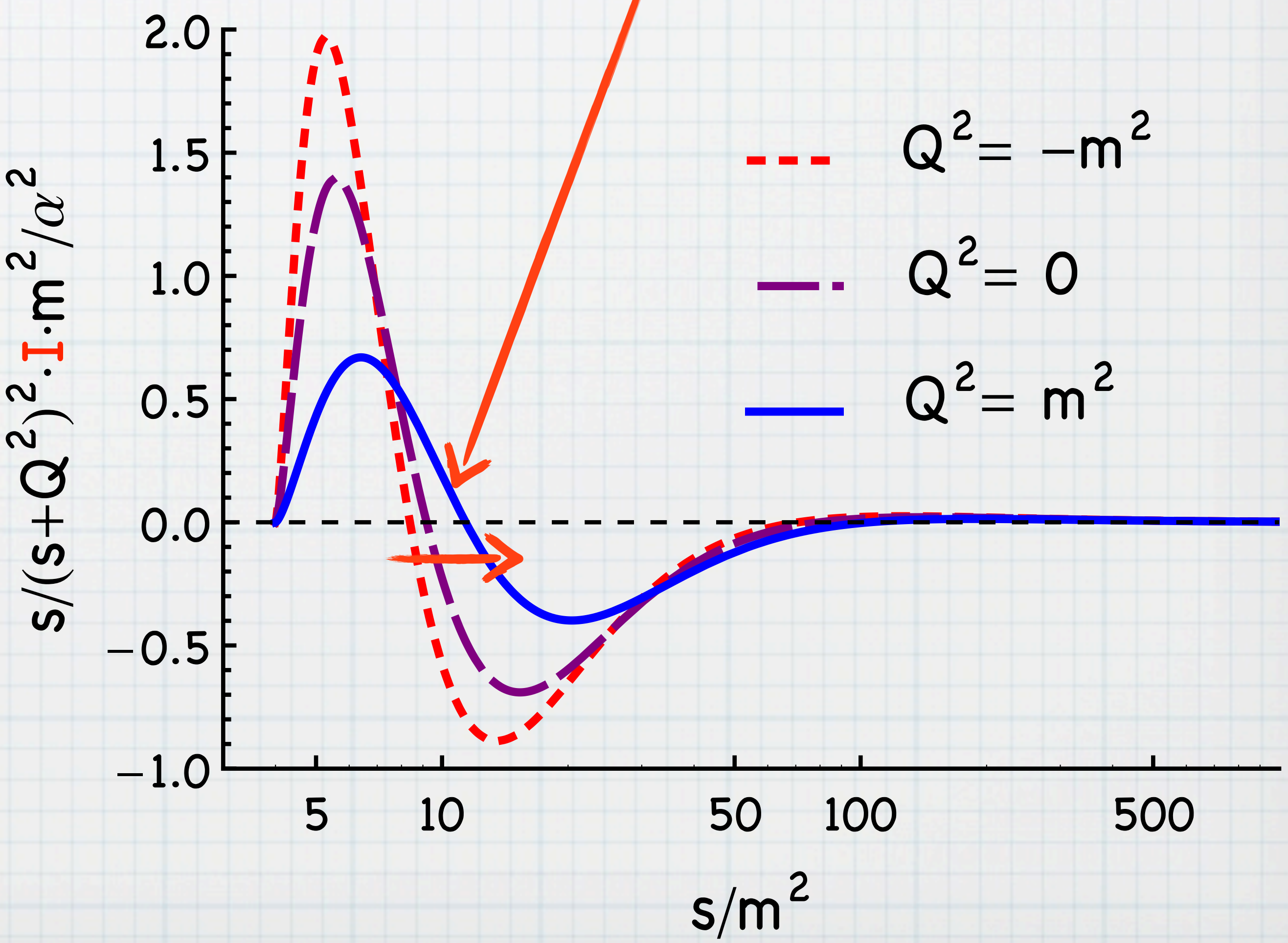
$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)^2} \left[\sigma_\parallel + \sigma_{LT} + \frac{(s+Q_1^2)}{Q_1 Q_2} \tau_{TL}^a \right]_{Q_2^2=0}$$

σ_0 dominates at lower energies

σ_2 dominates at higher energies

at larger Q^2 higher energy contributions are required





Low-energy constants for scalar QED

evaluation of the low-energy constants
for LbL scattering using the sum rules:

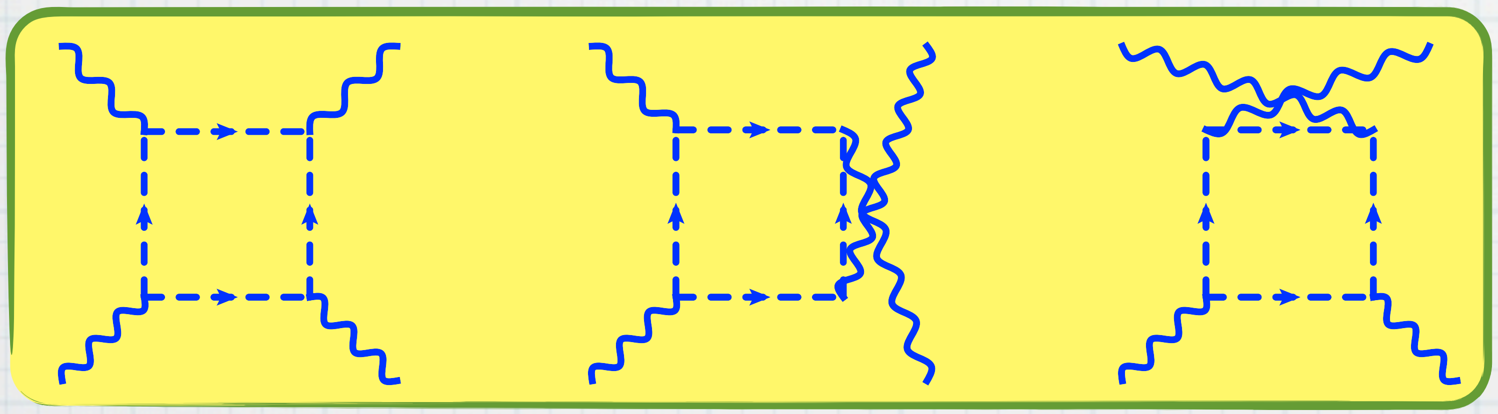
$$c_1 \pm c_2 = \frac{1}{8\pi} \int_{s_0}^{\infty} \frac{ds}{s^2} \left[\sigma_{\parallel}(s) \pm \sigma_{\perp}(s) \right]$$

LbL low-energy constants

$$c_1 = \frac{\alpha^2}{m^4} \frac{7}{1440}$$

$$c_2 = \frac{\alpha^2}{m^4} \frac{1}{1440}$$

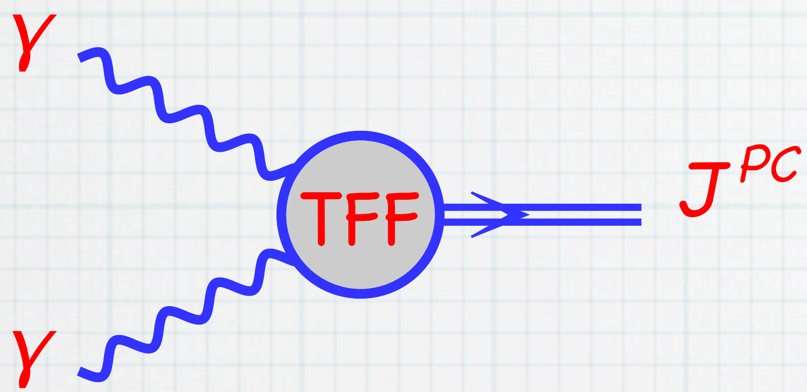
explicit one-loop calculation



one-loop result is defined by
tree-level amplitudes

Böhm, Schuster (1994)

Meson production in γγ collision



- two-photon state: produced meson has C=+1
- when both photons are real ~~J=1~~ final state is forbidden (Landau-Yang theorem);

the main contribution comes from J=0: 0^{-+} (pseudoscalar)

and 0^{++} (scalar)

and J=2: 2^{++} (tensor)

- the SRs hold separately for channels of given intrinsic quantum numbers: isoscalar and isovector mesons, $c\bar{c}$ states
- input for the absorptive part of the SRs: γγ-hadrons response functions, can be expressed in terms of γγ→M transition form factors

$$\sigma_{\Lambda}^{\gamma\gamma\to M}(s) \approx (2J+1)16\pi^2 \frac{\Gamma_{\gamma\gamma}}{m_M}\delta(s-m_M^2)$$

meson contribution to the cross-section in the narrow-resonance approximation

$$\Gamma_{\gamma\gamma}(\mathcal{P}) = \frac{\pi\alpha^2}{4} m^3 \, |F_{\mathcal{M}\gamma^*\gamma^*}(0,0)|^2$$

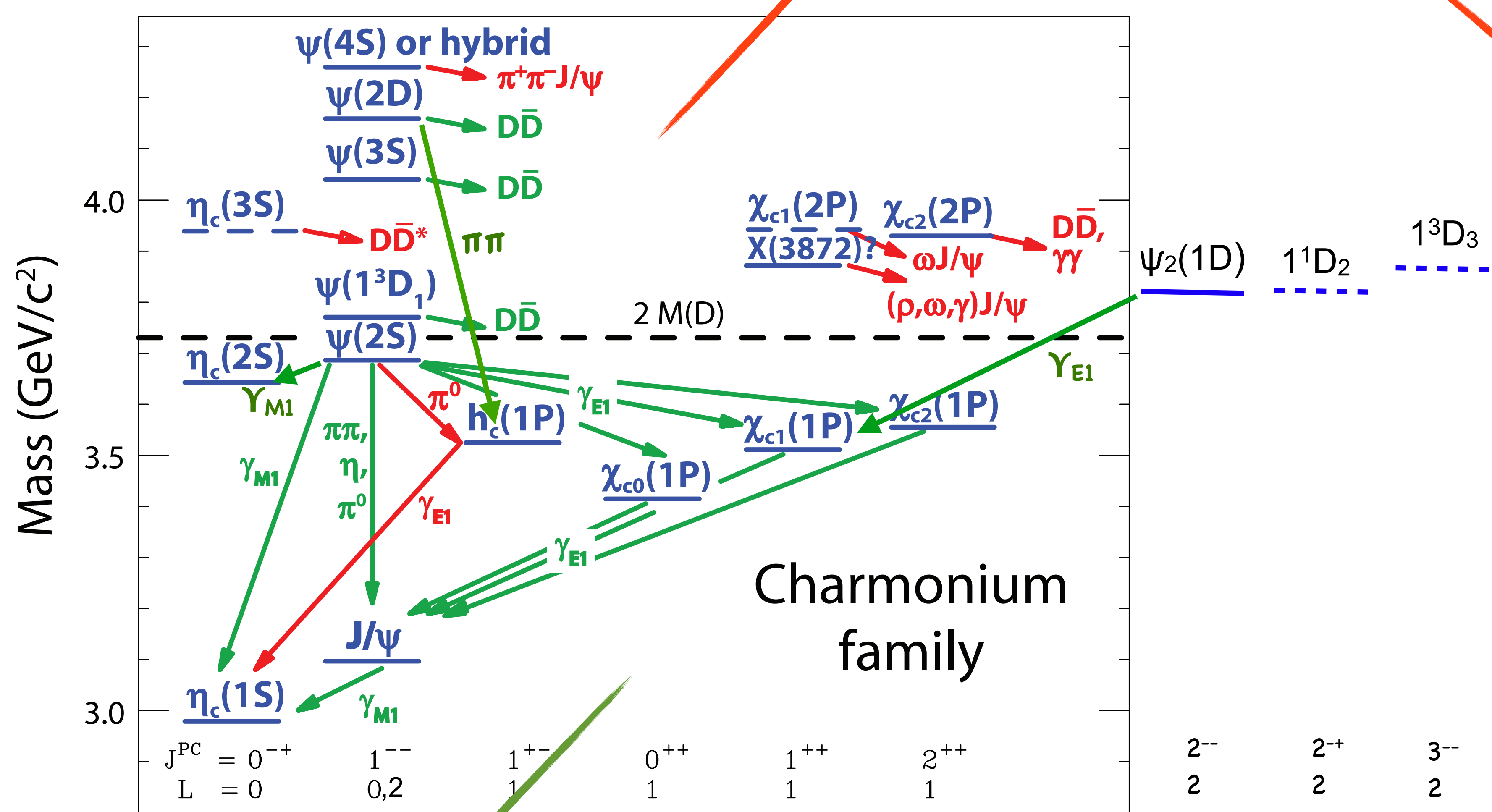
two-photons decay rate for the meson

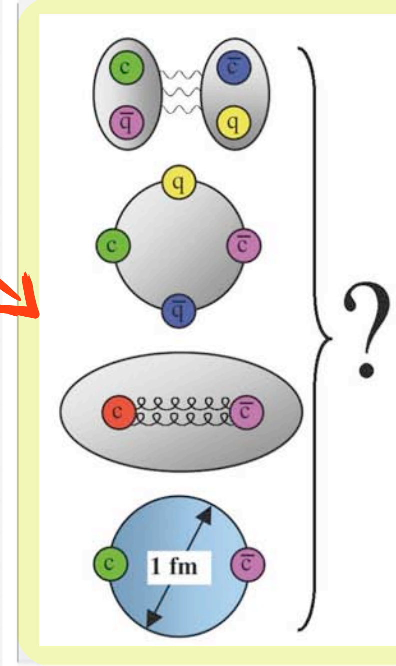
Charmonium states

charmonium spectrum

above DD̄ threshold:

- plethora of new states
- ? nature: molecules, tetra-quarks, hybrids,...





lower energies:

- well understood narrow cc̄ states
- only 2 remain to be observed

Meson production in γγ collision: $c\bar{c}$ mesons

the SRs evaluated for $c\bar{c}$ states

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} [\sigma_2 - \sigma_0]_{Q_2^2=0}$$

$$c_1 \pm c_2 = \frac{1}{8\pi} \int_{s_0}^{\infty} \frac{ds}{s^2} \sigma_{\parallel}(s) \pm \sigma_{\perp}(s)$$

		m_M [MeV]	$\Gamma_{\gamma\gamma}$ [keV]	$\int \frac{ds}{s}(\sigma_2 - \sigma_0)$ [nb]	c_1 $[10^{-7}\mathrm{GeV}^{-4}]$	c_2 $[10^{-7}\mathrm{GeV}^{-4}]$
0^{-+}	$\eta_c(1S)$	2980.3 ± 1.2	6.7 ± 0.9	-15.6 ± 2.1	0	1.79 ± 0.24
0^{++}	$\chi_{c0}(1P)$	3414.75 ± 0.31	2.32 ± 0.13	-3.6 ± 0.2	0.31 ± 0.02	0
2^{++}	$\chi_{c2}(1P)$	3556.2 ± 0.09	0.50 ± 0.06	3.4 ± 0.4	0.14 ± 0.02	0.14 ± 0.02
	Sum resonances			-15.8 ± 2.1	0.49 ± 0.03	1.97 ± 0.24
	duality estimate continuum ($\sqrt{s} \geq 2m_D$)			15.1		
	resonances + continuum			-0.7 ± 2.1		

unmeasured sizable contribution from states above the nearby $\bar{D}D$ threshold $s_D \simeq 14\,\mathrm{GeV}^2$

quark-hadron duality: replace the integral of the cross section for the γγ→X process (X - hadronic final state containing charm quarks) by the corresponding integral of the helicity-difference cross section for perturbative $\gamma\gamma \to c\bar{c}$ process

$$I_{cont} \equiv \int_{s_D}^{\infty} ds \frac{1}{s} [\sigma_2 - \sigma_0](\gamma\gamma \to X) \approx \int_{s_D}^{\infty} ds \frac{1}{s} [\sigma_2 - \sigma_0](\gamma\gamma \to c\bar{c})$$

interplay between production of $c\bar{c}$ states and charmed mesons

Meson production in γγ collision: I=1

angular distribution analysis of measurements at e^+e^- colliders of
$\gamma\gamma \to \pi^+\pi^-$ ($\pi^0\pi^0$, $\eta\pi^0$, K^+K^-):
tensor mesons are produced in predominantly (≃95% or more) helicity $\Lambda=2$ state

the SRs applied to the I=1 channel

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} [\sigma_0 - \sigma_2]_{Q_2^2=0}$$

$$c_1 \pm c_2 = \frac{1}{8\pi} \int_{s_0}^{\infty} \frac{ds}{s^2} \sigma_{\|}(s) \pm \sigma_{\perp}(s)$$

		m_M [MeV]	$\Gamma_{\gamma\gamma}$ [keV]	$\int \frac{ds}{s} (\sigma_2 - \sigma_0)$ [nb]	c_1 [10^{-4} GeV^{-4}]	c_2 [10^{-4} GeV^{-4}]
0^{-+}	π^0	134.9766 ± 0.0006	$(7.8 \pm 0.5) \times 10^{-3}$	-195 ± 13	0	10.94 ± 0.70
0^{++}	$a_0(980)$	980 ± 20	0.3 ± 0.1	-20 ± 8	0.021 ± 0.007	0
2^{++}	$a_2(1320)$	1318.3 ± 0.6	1.00 ± 0.06	134 ± 8	0.039 ± 0.002	0.039 ± 0.002
	$a_2(1700)$	1732 ± 16	0.30 ± 0.05	18 ± 3	0.003 ± 0.001	0.003 ± 0.001
	Sum			-63 ± 17	0.06 ± 0.01	10.98 ± 0.70

- helicity difference SR: $a_2(1320)$ compensates π^0 contribution to 70%: additional yet unmeasured two-photon strength at higher energies in the tensor channel contributing to the σ_2
- dominant contribution to low-energy LbL scattering constant c_2 comes from π^0

Meson production in γγ collision: I=0

the SRs applied to the I=0 channel

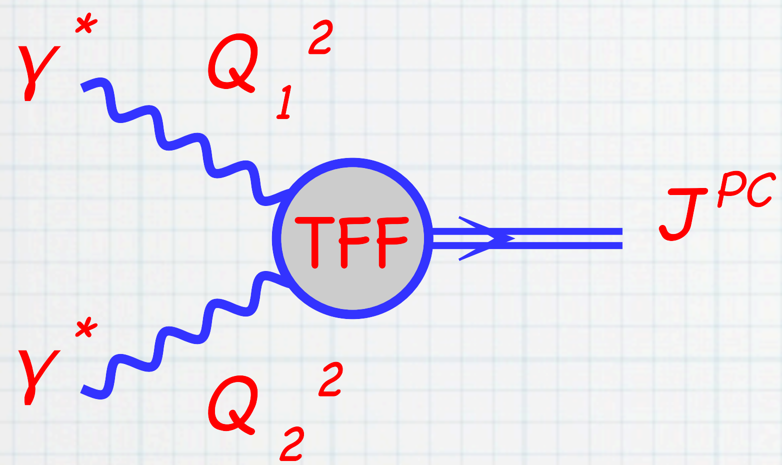
$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} [\sigma_2 - \sigma_0]_{Q_2^2=0}$$

$$c_1 \pm c_2 = \frac{1}{8\pi} \int_{s_0}^{\infty} \frac{ds}{s^2} \sigma_\parallel(s) \pm \sigma_\perp(s)$$

		m_M [MeV]	$\Gamma_{\gamma\gamma}$ [keV]	$\int \frac{ds}{s}(\sigma_2 - \sigma_0)$ [nb]	c_1 [10^{-4}GeV^{-4}]	c_2 [10^{-4}GeV^{-4}]
0^{-+}	η	547.853 ± 0.024	0.510 ± 0.026	-191 ± 10	0	0.65 ± 0.03
	η'	957.78 ± 0.06	4.29 ± 0.14	-300 ± 10	0	0.33 ± 0.01
0^{++}	$f_0(980)$	980 ± 10	0.29 ± 0.07	-19 ± 5	0.020 ± 0.005	0
	$f_0'(1370)$	$1200 - 1500$	3.8 ± 1.5	-91 ± 36	0.049 ± 0.019	0
2^{++}	$f_2(1270)$	1275.1 ± 1.2	3.03 ± 0.35	449 ± 52	0.141 ± 0.016	0.141 ± 0.016
	$f_2'(1525)$	1525 ± 5	0.081 ± 0.009	7 ± 1	0.002 ± 0.000	0.002 ± 0.000
	$f_2(1565)$	1562 ± 13	0.70 ± 0.14	56 ± 11	0.012 ± 0.002	0.012 ± 0.002
	Sum			-89 ± 66	0.22 ± 0.03	1.14 ± 0.04

- helicity difference SR: the contribution of η, η′ is entirely compensated by $f_2(1270)$, $f_2(1565)$ and $f_2'(1525)$
- dominant contribution to low-energy LbL scattering constant c_2 comes from η, η′ and $f_2(1270)$

Meson production in $\gamma^*\gamma$ collision



one photon is virtual Q_1^2, second photon is real or quasi-real $Q_2^2 \simeq 0$:

axial-vector mesons 1^{++} are also allowed if one of the photons is virtual $\gamma^*\gamma^* \to f_1(1285)$ / $f_1(1420)$ measured L3 Coll.

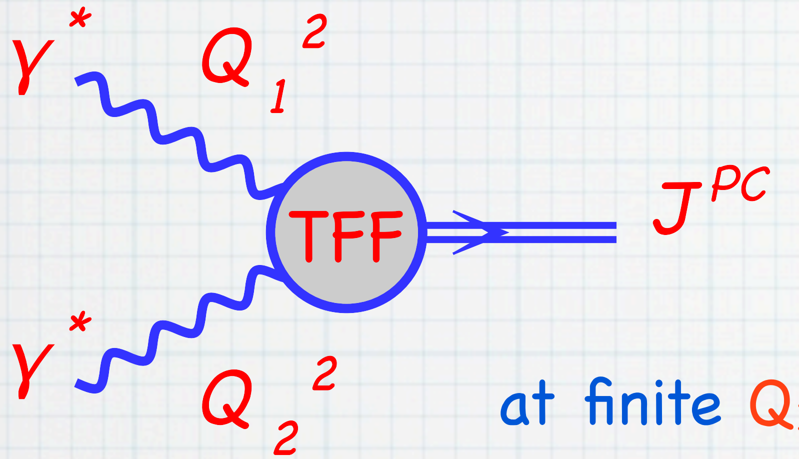
$$0 = \int_{s_0}^{\infty} ds \, \frac{1}{(s+Q_1^2)^2} \left[\sigma_{\parallel} + \sigma_{LT} + \frac{(s+Q_1^2)}{Q_1 Q_2} \tau_{TL}^a \right]_{Q_2^2=0}$$

sum rules involving longitudinally polarized cross-sections: cancelation mechanism between scalar, axial-vector and tensor mesons

	m_M [MeV]	$\Gamma_{\gamma\gamma}$ [keV]	$\int \frac{ds}{s^2}\sigma_{\parallel}(s)$ [nb / GeV^2]	$\int ds \left[\frac{1}{s}\frac{\tau_{TL}^a}{Q_1Q_2}\right]_{Q_i^2=0}$ [nb / GeV^2]	$\int ds \left[\frac{1}{s^2}\sigma_{\parallel} + \frac{1}{s}\frac{\tau_{TL}^a}{Q_1Q_2}\right]_{Q_i^2=0}$ [nb / GeV^2]
$f_1(1285)$	1281.8 ± 0.6	3.5 ± 0.8	0	-93 ± 21	-93 ± 21
$f_1(1420)$	1426.4 ± 0.9	3.2 ± 0.9	0	-50 ± 14	-50 ± 14
$f_0(980)$	980 ± 10	0.29 ± 0.07	20 ± 5	0	20 ± 5
$f_0'(1370)$	$1200 - 1500$	3.8 ± 1.5	48 ± 19	0	48 ± 19
$f_2(1270)$	1275.1 ± 1.2	3.03 ± 0.35	138 ± 16	$\gtrsim 0$	138 ± 16
$f_2'(1525)$	1525 ± 5	0.081 ± 0.009	1.5 ± 0.2	$\gtrsim 0$	1.5 ± 0.2
$f_2(1565)$	1562 ± 13	0.70 ± 0.14	12 ± 2	$\gtrsim 0$	12 ± 2
Sum					76 ± 36

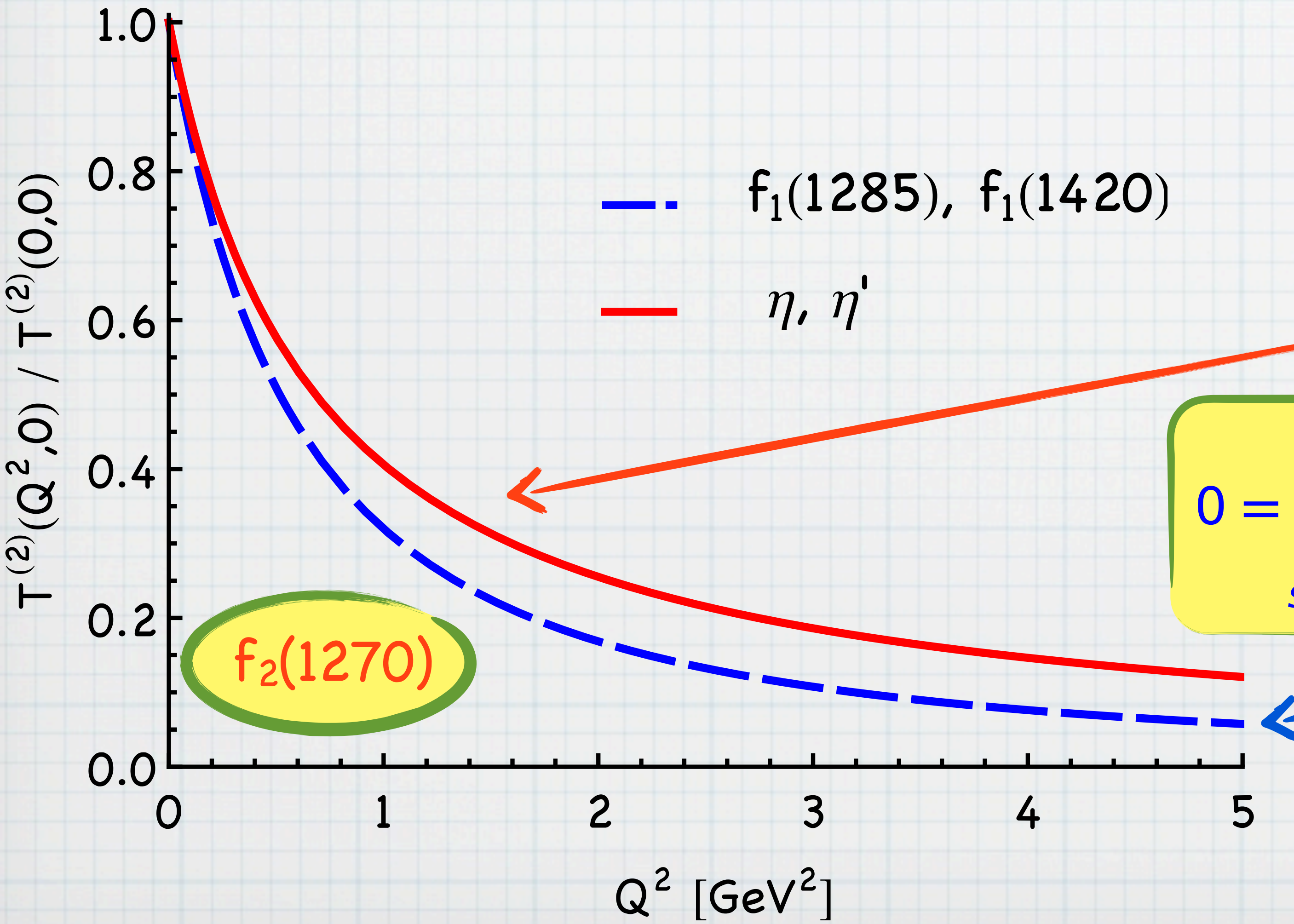
uncertainty: higher mass states or non-resonant contributions with axial-vector quantum numbers

Meson production in $\gamma^*\gamma$ collision: TFF



one photon is virtual Q_1^2, second photon is real or quasi-real $Q_2^2 \simeq 0$

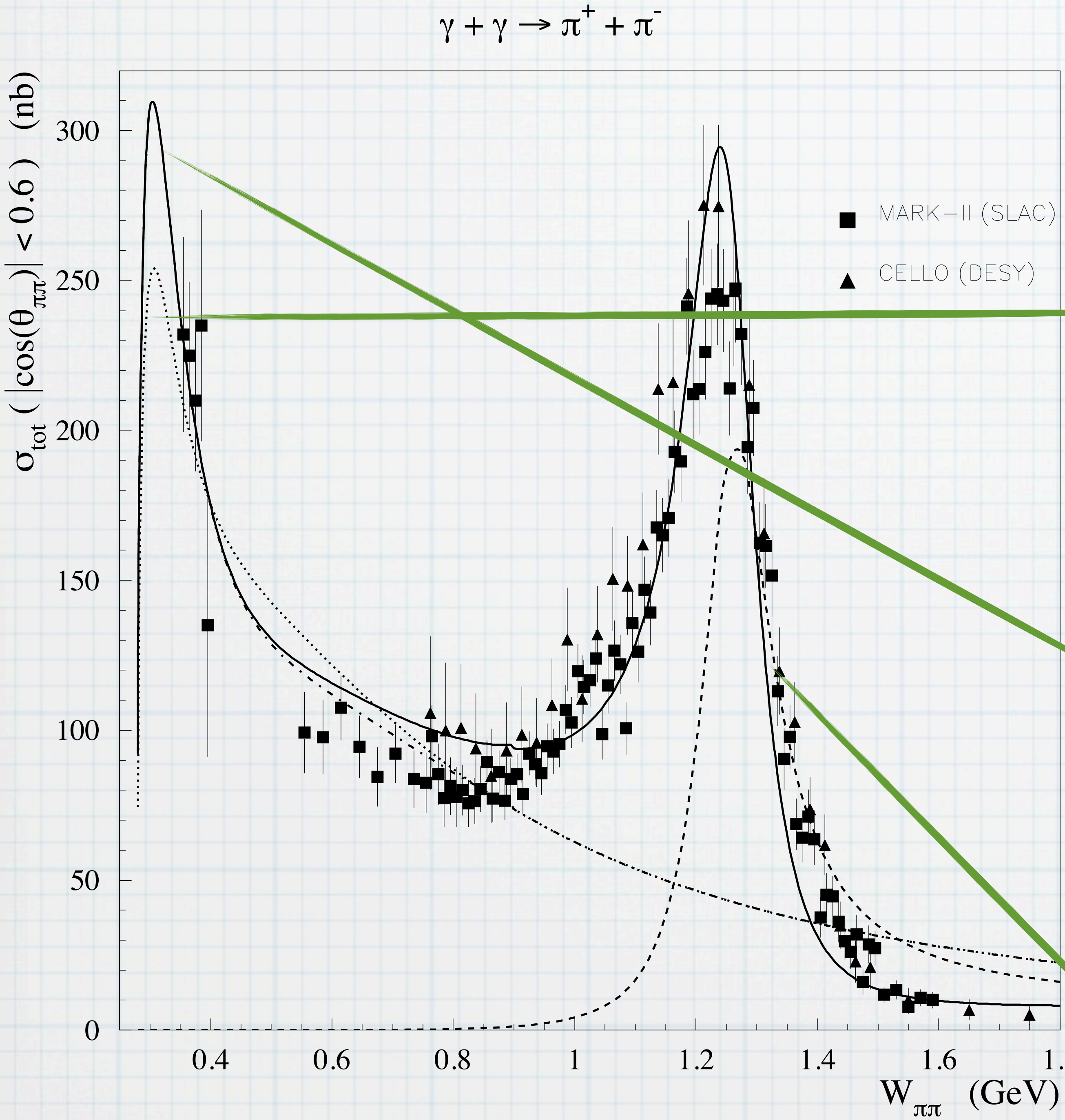
at finite Q_1^2 the SRs imply information on meson transition form-factors: estimate for the $f_2(1270)$ tensor FF in terms of the η, η' and f_1 FFs and for the $a_2(1320)$ tensor meson FF in terms of the π^0 FF.



$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} \left[\sigma_2 - \sigma_0\right]_{Q_2^2=0}$$

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)^2} \left[\sigma_\parallel + \sigma_{LT} + \frac{(s+Q_1^2)}{Q_1 Q_2} \tau_{TL}^{a}\right]_{Q_2^2=0}$$

$\gamma\gamma \to \pi^+\pi^-$: the LE constants



tree-level QED approximation:

$$[c_1 + c_2]^{Born} = \frac{\alpha^2}{m_\pi^4}\frac{1}{180} = 7.78 \cdot 10^{-4}\mathrm{GeV}^{-4}$$

unitarized QED approximation:

$$[c_1 + c_2]^{Born\ unitary} = 8.36 \cdot 10^{-4}\mathrm{GeV}^{-4}$$

unitarized QED approximation + f_2:

$$[c_1 + c_2]^{Born\ unitary + f_2} = 8.53 \cdot 10^{-4}\mathrm{GeV}^{-4}$$

low-energy LbL scattering: the same order of magnitude as for a pion-pole contribution

→ expect a sizable contribution to the $(g-2)_\mu$

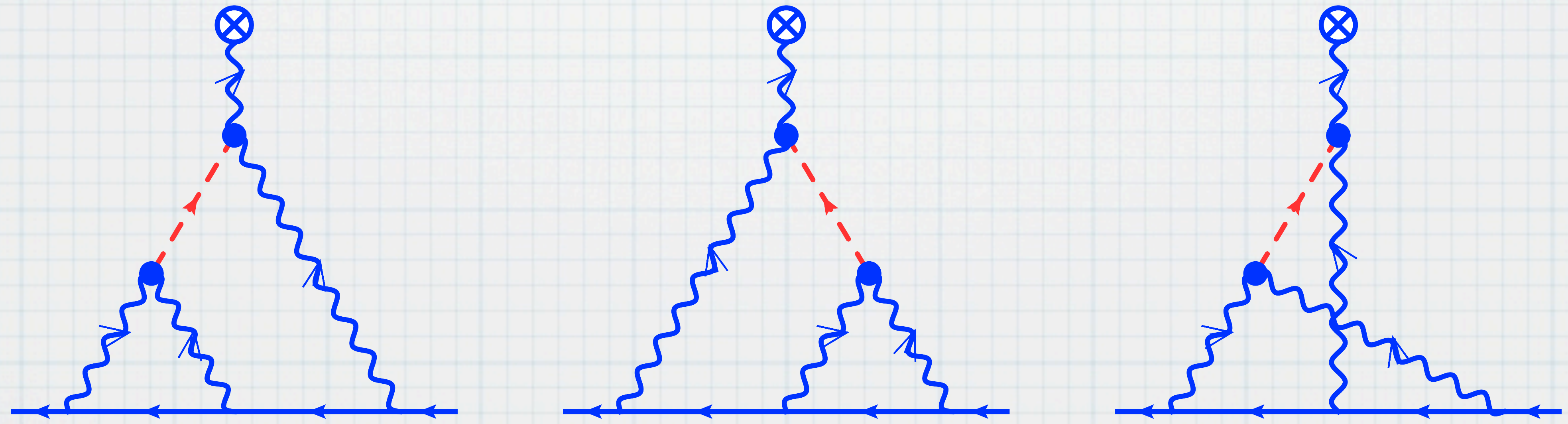
Summary H-contr. to LbL

sum rules allow to select relevant meson contributions:

- contributions of axial states: $f_1(1285)$ and $f_1(1420)$;
- contributions of tensor states: $f_2(1270)$, $f_2(1565)$, $a_2(1320)$;
- two-pion contribution beyond scalar QED ~ 10% on $c_1 + c_2$;

Hadronic contribution to the $(g-2)_\mu$.

Pole contributions

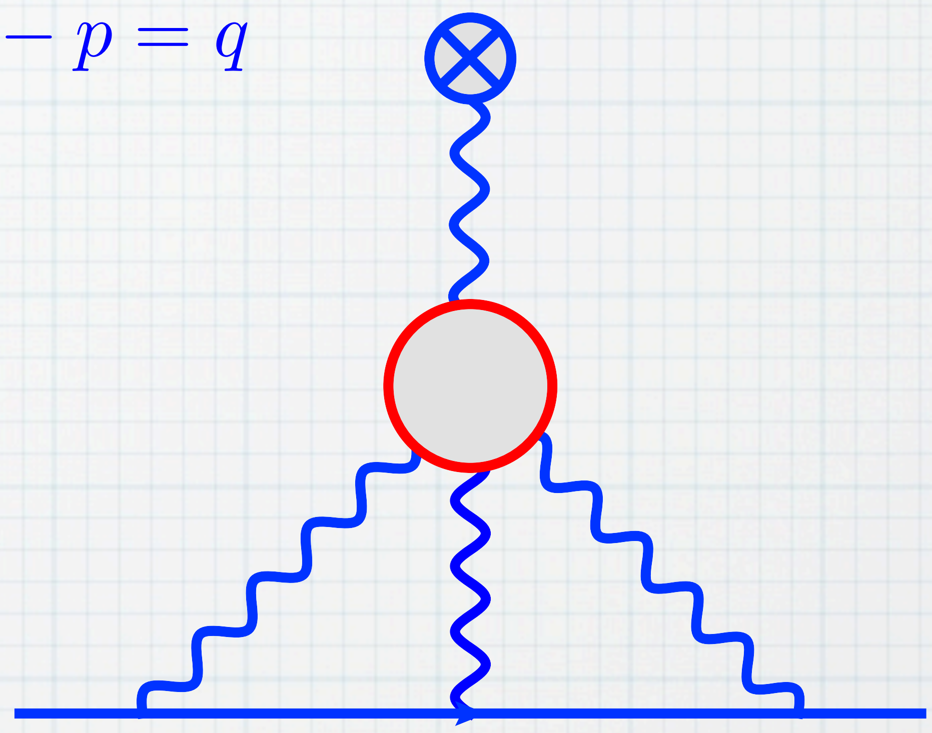


LbL contribution to the $(g-2)_\mu$

Electromagnetic current covariant decomposition:

$$\Gamma^\mu(p', p) = \gamma^\mu F_1(q^2) + i\frac{\sigma^{\mu\nu}q_\nu}{2m} F_2(q^2),$$

$$p' - p = q$$



$F_1(q^2)$ - Dirac form factor

$F_2(q^2)$ - Pauli form factor

$F_2(0) = a_\mu$ - anomalous magnetic moment

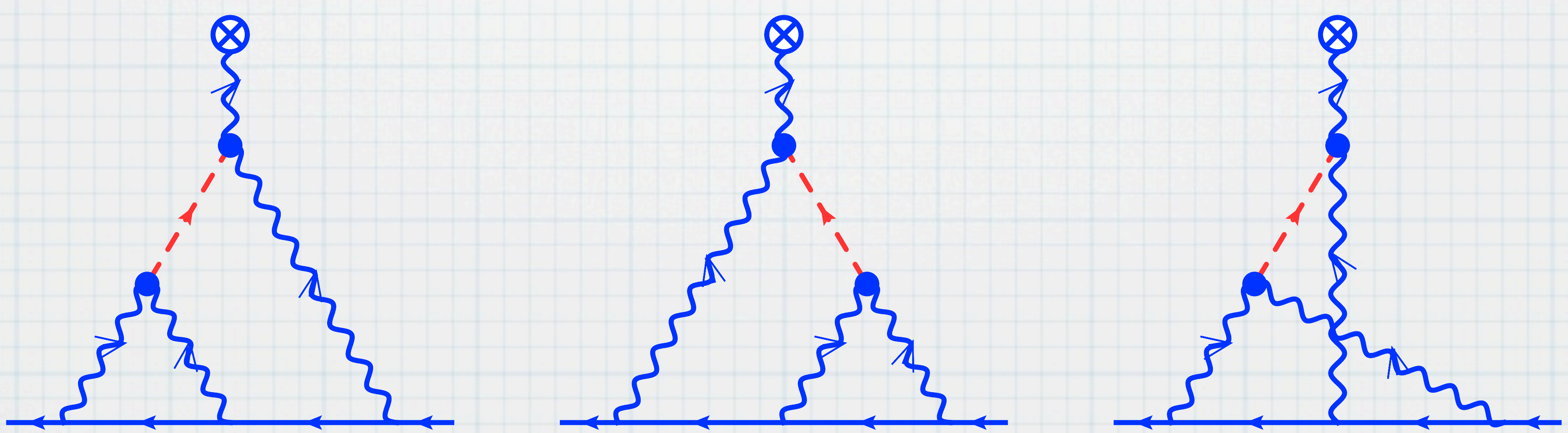
Light-by-light contribution to the $(g-2)_\mu$:

$$a_\mu^{LbL} = F_2(0) = \frac{-ie^6}{48m}\int\frac{d^4q_1}{(2\pi)^4}\int\frac{d^4q_2}{(2\pi)^4}\frac{1}{q_1^2q_2^2q_3^2}\frac{1}{(p+q_1)^2-m^2}\frac{1}{(p+q_1+q_2)^2-m^2}\times$$
$$\times\, \mathrm{Tr}\left[(\not p + m)[\gamma^\rho,\gamma^\sigma](\not p' + m)\gamma^\mu(\not p + \not q_1 + m)\gamma^\nu(\not p + \not q_1 + \not q_2 + m)\gamma^\lambda\right]$$
$$\times\,\frac{\partial}{\partial k^\rho}\Pi_{\mu\nu\lambda\sigma}(q_1, q_2, q_3)$$

elastic LbL scattering

Hadronic contribution to the $(g-2)_\mu$.

Pseudo-scalar mesons



The pion pole contribution

Knecht, Nyffeler (2001)

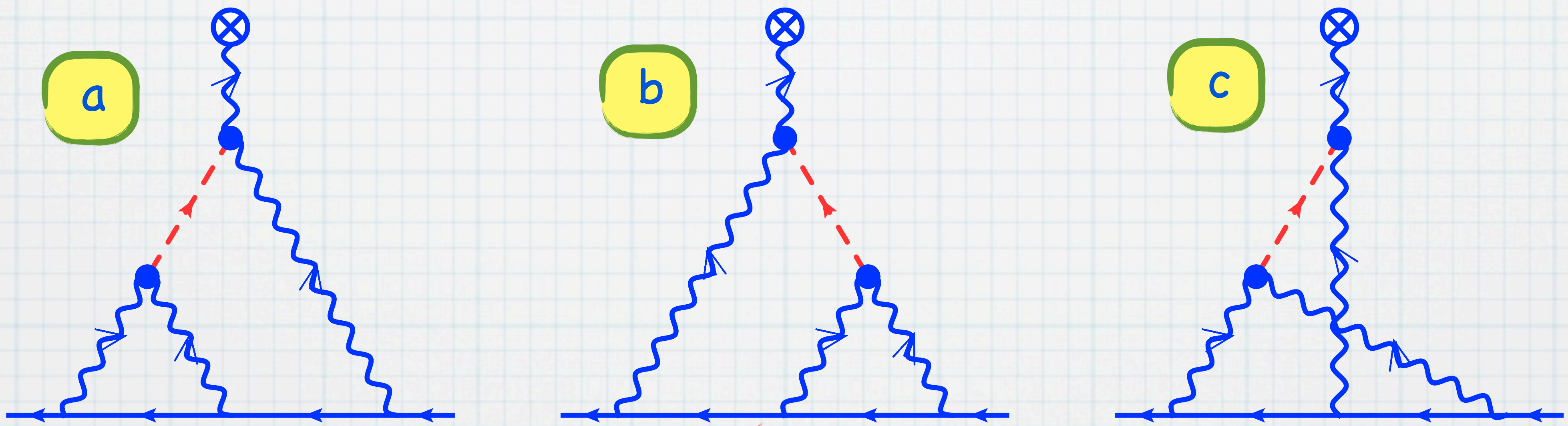
The pseudoscalar-meson pole contribution to the elastic LbL scattering:

$$\Pi^{P(a)}_{\mu\nu\lambda\sigma}(q_1,q_2,q_3) = (-i)\frac{1}{(q_1+q_2)^2-m_P^2}\mathcal{M}_{\mu\nu}(q_1,q_2)\mathcal{M}_{\lambda\sigma}(q_3,-q_1-q_2-q_3)$$

$\pi_0\gamma\gamma$ transition amplitude:

$$\mathcal{M}_{\mu\nu}(q_1,q_2) = -ie^2\varepsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta F(q_1^2,q_2^2)$$

pion is not necessarily near the pole, although pole-dominance might be expected to give a reasonable approximation



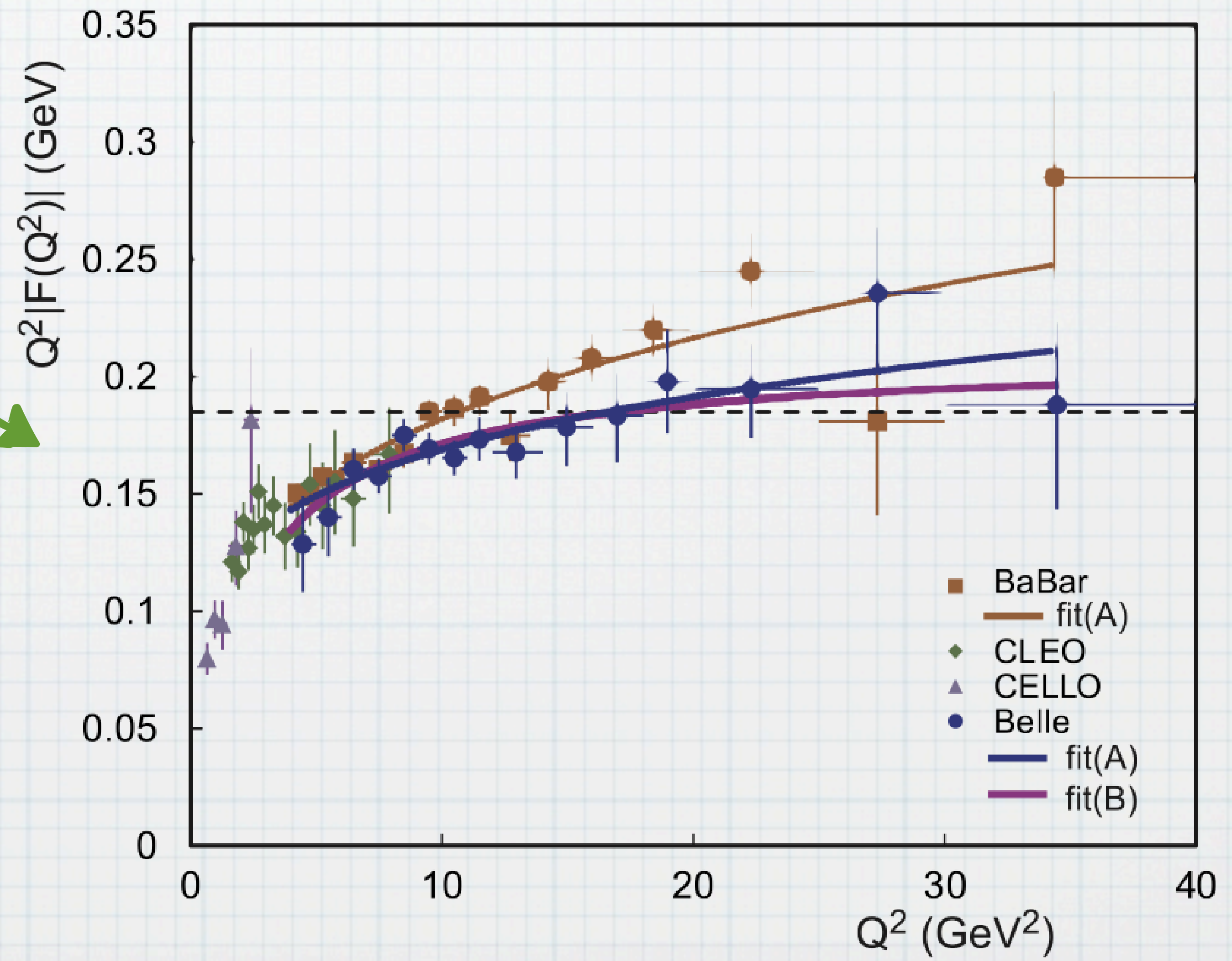
$$a_\mu^{LbL} = \frac{-e^6}{48m}\int\frac{d^4q_1}{(2\pi)^4}\int\frac{d^4q_2}{(2\pi)^4}\frac{1}{q_1^2q_2^2(q_1+q_2)^2}\frac{1}{(p+q_1)^2-m^2}\frac{1}{(p-q_2)^2-m^2}$$
$$\times\left[\frac{F\left(q_1^2,(q_1+q_2)^2\right)F\left(q_2^2,0\right)}{q_2^2-m_P^2}T_{ab}(q_1,q_2,p)+\frac{F\left(q_1^2,q_2^2\right)F\left((q_1+q_2)^2,0\right)}{(q_1+q_2)^2-m_P^2}T_c(q_1,q_2,p)\right]$$

The $\pi^0\gamma\gamma$ transition form factor

Universal parametrization for $\pi_0\gamma\gamma$ transition FF based on the large-N_C approximation

$$\mathcal{F}(q_1^2, q_2^2) = \frac{F_\pi}{3}\left[\sum_{M=M_\pi, M_{V_i}} f_M(q_2^2) - \sum_{M_{V_i}} f_{M_{V_i}}(q_2^2)\frac{M_\pi^2 - M_{V_i}^2}{q_1^2 - M_{V_i}^2}\right]$$

$\pi_0\gamma\gamma$ transition FF
in the space-like region



known experimentally in
space-like region
1.5 GeV^2 < Q_2^2 < 40 GeV^2
for Q_1^2=0

Angular integrations

Perform Wick's rotation, switch to the hyperspherical coordinates:

$$a_\mu^{LbL} = -\frac{e^6}{48m}\frac{F_\pi}{3}\sum_{M=M_\pi,M_{V_i}} f_M(0)\int \mathrm{d}Q_1^2\int \mathrm{d}Q_2^2\int\frac{\mathrm{d}\Omega(\hat{Q}_1)}{2\pi^2}\int\frac{\mathrm{d}\Omega(\hat{Q}_2)}{2\pi^2}\times$$
$$\times\frac{1}{(Q_1+Q_2)^2}\frac{1}{(P+Q_1)^2+m^2}\frac{1}{(P-Q_2)^2+m^2}\times$$
$$\times\left[\frac{F_\pi}{3}\frac{\mathcal{F}\left(-Q_2^2,0\right)}{Q_2^2+M_\pi^2}\left(\sum_{M=M_\pi,M_{V_i}} f_M(-Q_1^2)+\sum_{M_{V_i}} f_{M_{V_i}}(-Q_1^2)\frac{M_\pi^2-M_{V_i}^2}{(Q_1+Q_2)^2+M_{V_i}^2}\right)T_{ab}(Q_1,Q_2,P)\right.$$
$$\left.+\frac{F_\pi}{3}\mathcal{F}\left(-Q_1^2,-Q_2^2\right)\sum_{M=M_\pi,M_{V_i}}\frac{f_M(0)}{(Q_1+Q_2)^2+M^2}T_c(Q_1,Q_2,P)\right]$$

Use Gegenbauer polynomials technique:

$$\frac{1}{(K-L)^2+M^2} = \frac{Z_{KL}^M}{|K||L|}\sum_{n=0}^{\infty}\left(Z_{KL}^M\right)^n C_n(\hat{K}\cdot\hat{L})$$

generating functional

$$\int \mathrm{d}\Omega(\hat{K})C_n(\hat{Q}_1\cdot\hat{K})C_m(\hat{K}\cdot\hat{Q}_2) = 2\pi^2\frac{\delta_{nm}}{n+1}C_n(\hat{Q}_1\cdot\hat{Q}_2)$$

generalized orthogonality relation

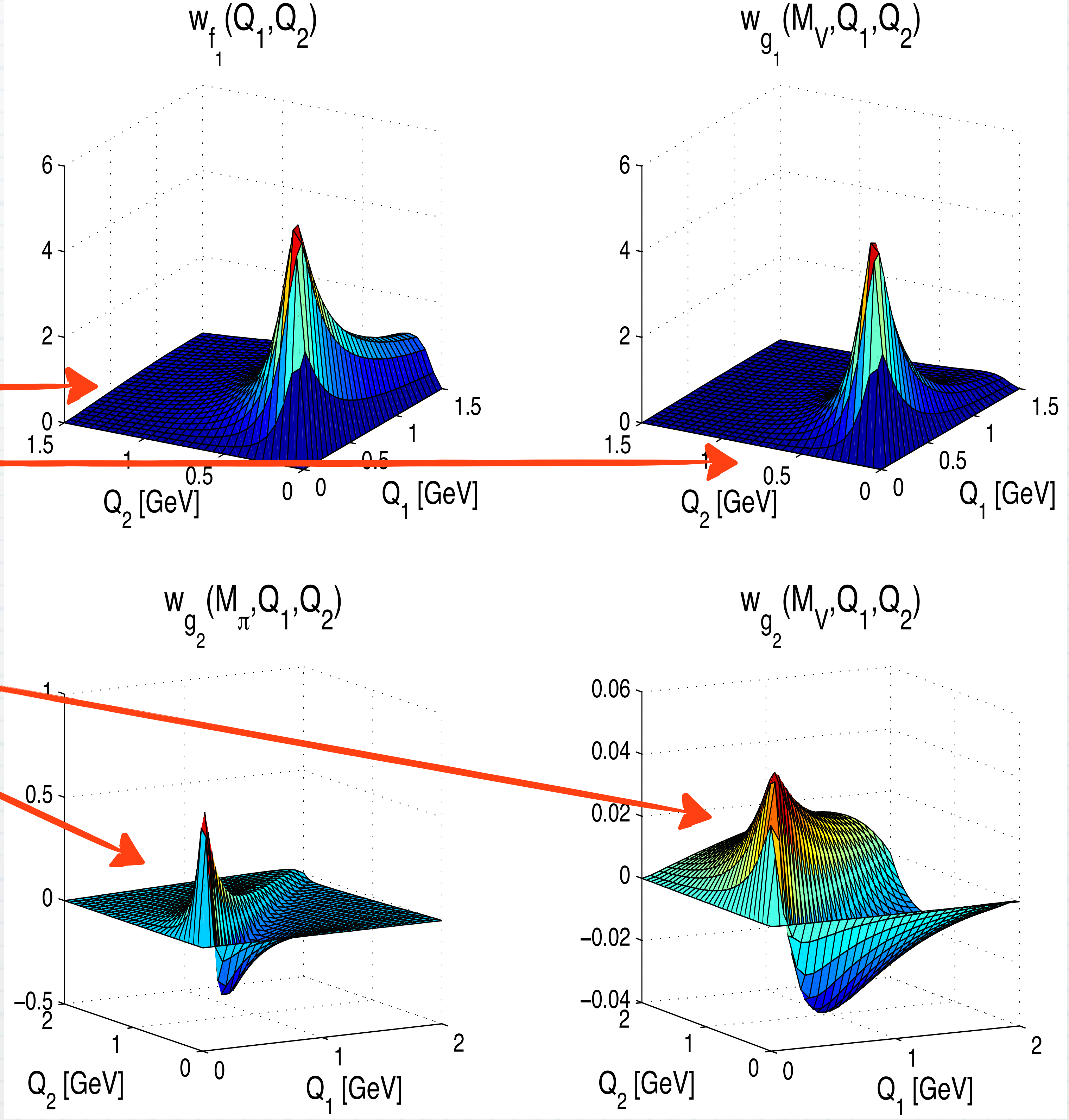
Two-dimensional representation

$(g-2)_\mu$ in two-dimensional integral representation

$$a_\mu^{LbL,\pi^0} = \left(\frac{\alpha}{\pi}\right)^3 \int_0^\infty dQ_1 \int_0^\infty dQ_2$$

$$\Big[(w_{f_1}(Q_1,Q_2) f^{(1)}(Q_1^2,Q_2^2) + \sum_{M_{V_i}} w_{g_1}(M_{V_i},Q_1,Q_2) g^{(1)}_{M_{V_i}}(Q_1^2,Q_2^2)) +$$

$$\sum_{M_{M_\pi,V_i}} w_{g_2}(M_{V_i},Q_1,Q_2) g^{(2)}_{M_{V_i}}(Q_1^2,Q_2^2)) \Big]$$



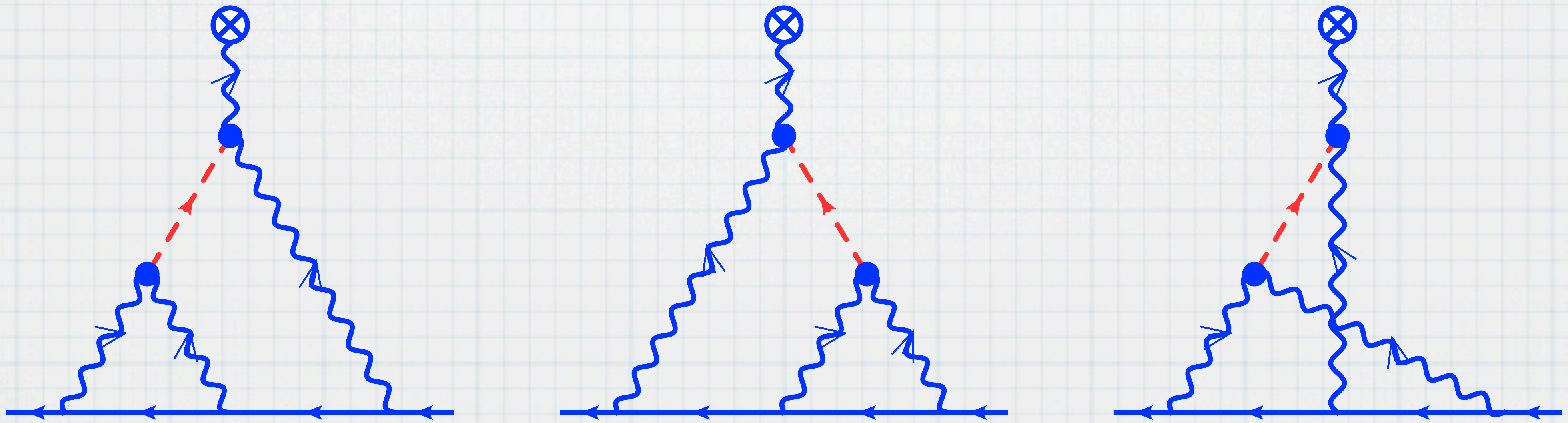
relevant contribution is defined by the region

$$Q_1 \sim Q_2 \sim 1\mathrm{GeV}$$

$a_\mu^{LbL,PS} = +8.3\ (1.2) \times 10^{-10}$

Hadronic contribution to the $(g-2)_\mu$.

Axial mesons



Aγγ transition amplitude

The axial-meson pole contribution to the elastic LbL scattering:

$$\mathcal{M}(\lambda_1, \lambda_2; \Lambda) = e^2\, \varepsilon^\mu(q_1, \lambda_1)\, \varepsilon^\nu(q_2, \lambda_2)\, \varepsilon^{\alpha *}(p_f, \Lambda)\; i\varepsilon_{\mu\nu\tau\alpha}\, \left(-Q_1^2\, q_2^\tau + Q_2^2\, q_1^\tau\right)\, A(Q_1^2, Q_2^2)$$

Aγγ transition FF:

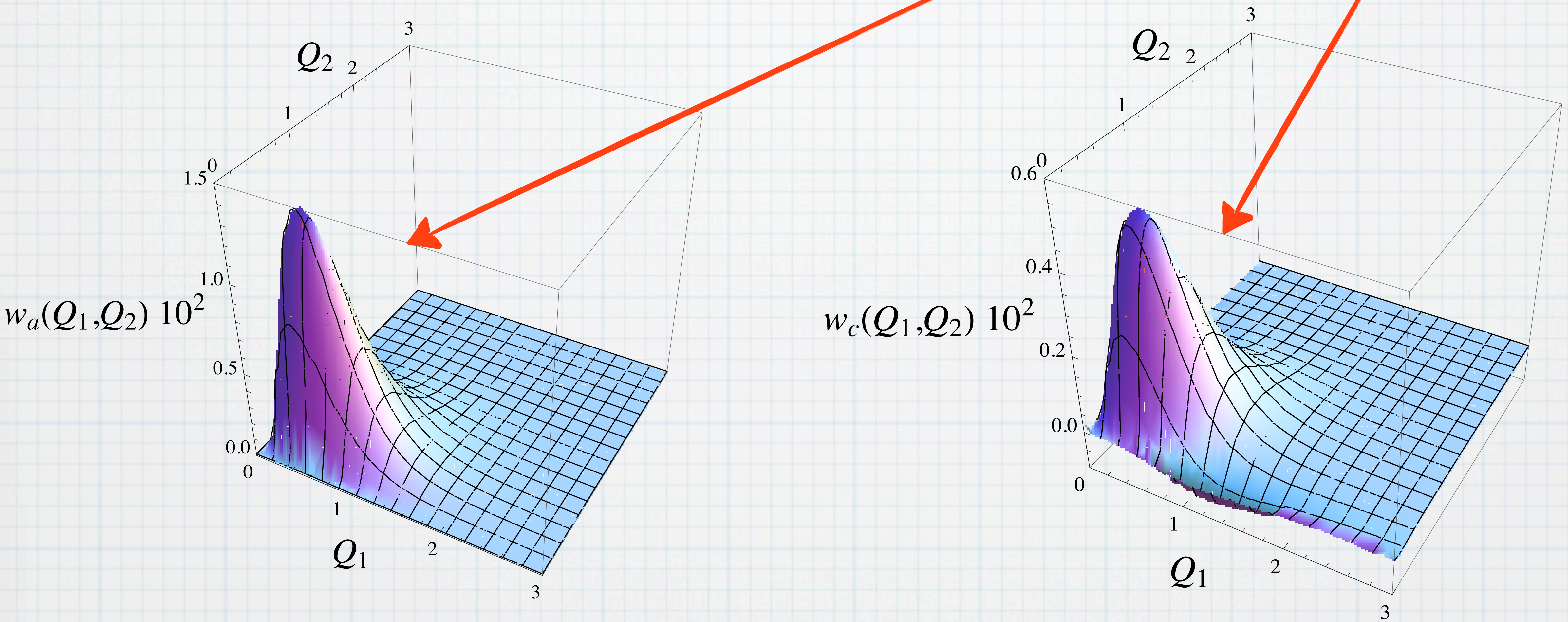
$$\frac{A(Q_1^2,0)}{A(0,0)} = \frac{1}{\left(1+Q_1^2/\Lambda_A^2\right)^2}$$

	m_A [MeV]	$\tilde{\Gamma}_{\gamma\gamma}$ [keV]	Λ_A [MeV]
$f_1(1285)$	1281.8 ± 0.6	3.5 ± 0.8	1040 ± 78
$f_1(1420)$	1426.4 ± 0.9	3.2 ± 0.9	926 ± 78

Present values of the f1(1285) meson and f1(1420) meson masses m_A, their equivalent 2γ decay widths $\Gamma_{\gamma\gamma}$, as well as their dipole masses Λ_A entering the FF and contributions to the anomalous magnetic moment of the muon $a^{LbL;A}$. For $\Gamma_{\gamma\gamma}$, we use the experimental results from the L3 Collaboration.

Two-dimensional representation

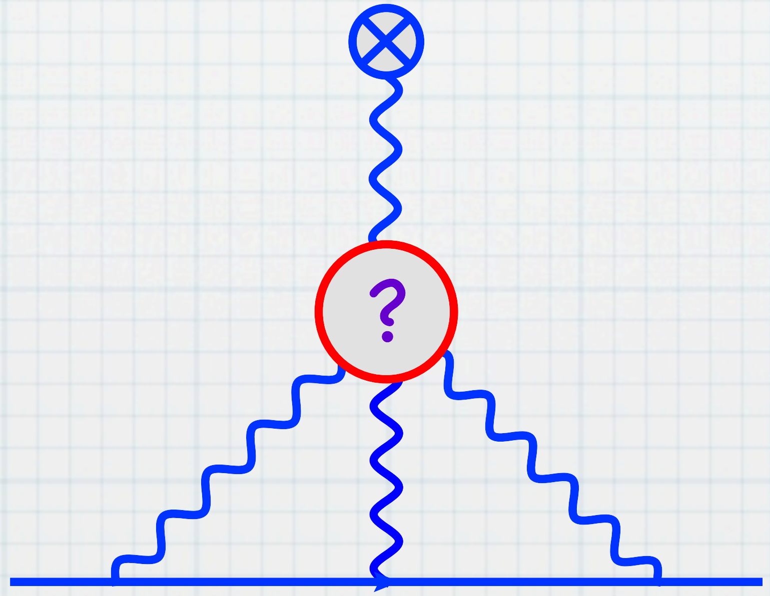
$$a_\mu^{LbL} = \frac{\alpha}{(2\pi)^2}\frac{\Lambda_{A1}^6\Lambda_{A2}^6\tilde{\Gamma}_{\gamma\gamma}(A)}{m\, m_A^5}\int dQ_1\int dQ_2\,[2w_a(Q_1,Q_2) + w_c(Q_1,Q_2)]$$



	m_A [MeV]	$\tilde{\Gamma}_{\gamma\gamma}$ [keV]	Λ_A [MeV]	$a_\mu^{LbL;A}\times 10^{10}$
$f_1(1285)$	1281.8 ± 0.6	3.5 ± 0.8	1040 ± 78	$0.50^{+0.20}_{-0.17}$
$f_1(1420)$	1426.4 ± 0.9	3.2 ± 0.9	926 ± 78	$0.14^{+0.07}_{-0.06}$

Hadronic contribution to the $(g-2)_\mu$.

Perspectives

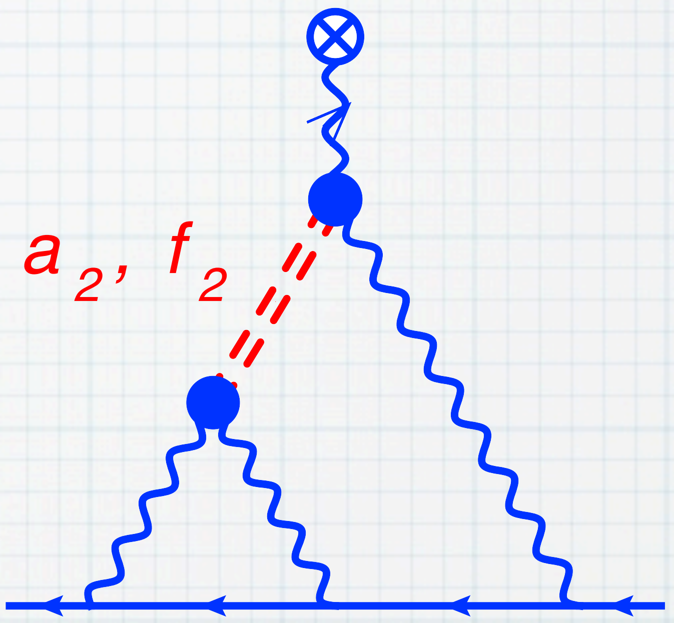


Perspectives

tensor meson contribution:
dominant at higher energies

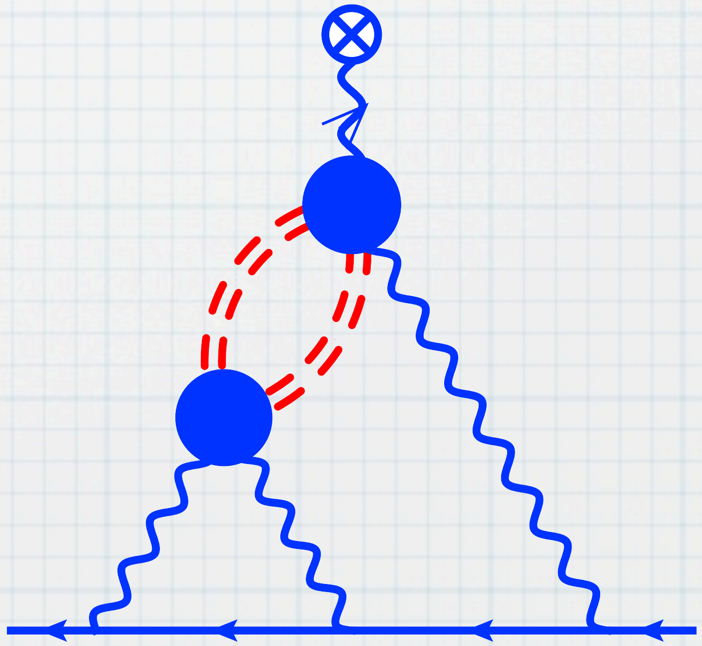
f2(1270), f2(1565), a2(1320)

not measured so far, experimental
input required



pion-pair:
dispersion framework

measurements for
the threshold region
required



Summary

- sum rules allow to select relevant meson contributions to the LbL elastic scattering amplitude (pseudoscalar, axial, tensor states and pion-pair production)

- axial-vector contribution has been evaluated:

$$a_\mu^{LbL,A} = 0.64^{+0.34}_{-0.23} \cdot 10^{-10}$$

- the calculation of the tensor-pole as well as the pion-pair production is under progress

- experimental input for axial, tensor and two-pion channels is needed

Thank You
for attention!