Final state emission radiative corrections to the process $e^+e^- \to \pi^+\pi^-(\gamma)$. Contribution to muon anomalous magnetic moment

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Motivation

Analytic calculation of contribution to anomalous magnetic moment of muon from the channels of annihilation of electron-positron pair to the pair of charged pi-meson with radiative correction connected with final state, as well as corrections to the lowest order kernell are presented. The result with the point-like pi-meson assumptions is $a_\mu = a_\mu^{(1)} + \Delta a_\mu$, $a_\mu^{(1)} = 7.0866 \cdot 10^{-9}; \Delta a_\mu = -2.4 \cdot 10^{-10}$.

Introduction

It is known (M. Davier, et al., Eur.Phys.J. C 71(2011),1515; C72(2012),1874) that about seventy three percent of contribution of hadrons to the anomalous magnetic moment of muon $a_\mu = (g-2)/2$ (B.E. Lautrup, A. Peterman and E. de Rafael, Phys. Rep. 3 (1972),4. S. Brodsky, E. de Rafael, Phys.Rev. v 168, p 1620(1968))

$$a_\mu = \frac{1}{4\pi^3} \int\limits_{4m_\pi^3}^{\infty} ds \sigma^{e^+e^- \to \pi^+\pi^-}(s) K^{(1)}(\frac{s}{m_\mu^2}), \quad (1)$$

with

$$K^{(1)}(\frac{s}{m_\mu^2}) = \int\limits_0^1 dx \frac{x^2(1-x)}{x^2 + \frac{s}{m_\mu^2}(1-x)}, \quad (2)$$

with M is muon mass, arises from taking into account the simplest process $e^+e^- \to \gamma^* \to \pi^+\pi^-$, whereas about sixty percent of the error arises from the uncertainties associated with pion pair production from the mechanisms with the intermediate states of lightest vector meson ρ, ω (G.Venanzoni,private communication).

Introduction

It seems "natural" to use the result of experimental measuring of the cross section of process $e^+e^- \to \pi^+\pi^-$. But, unfortunately, the measured on experiment total cross section (omitting the effects of detection of the final particles), include the emission of both virtual and real photons by the initial electron and positron, (ISE) and final state emission (FSE) and possible, the interference of amplitudes of emission of initial and the final particles. Assuming that the contribution of this interference terms to the total cross section is zero (charge-blind set-up), we remain with the problem of icluding such an enhanced factors as form factor of the charged pion in the time-like region and the delicate procedure of extracting the effects of initial state emission (both photons and charged particles). Only part of radiative corrections FSE, connected with final $\pi^+\pi^-$ can be included in frames of one virtual photon polarization operator used above,since one imply $\sigma^{e^+e^-\to hadrons}(s) = ((4\pi\alpha)^2/s)Im\Pi(s)$.

Introduction

With polarization operator defined as a transverse part of virtual photon self-energy tensor $\Pi_{\mu\nu}(q) = (q_\mu q_\nu - q^2 g_{\mu\nu})\Pi(q^2)$ and applying the dispersion relation (B.E. Lautrup, A. Peterman and E. de Rafael, Phys. Rep. 3 (1972)4; S. Brodsky, E. de Rafael, Phys.Rev. 168 (1968)1620; B. Krause, arXiv: hep-ph/9607259)

$$\Pi(q^2) = -\frac{q^2}{\pi} \int\limits_{4m_\pi^2}^{\infty} \frac{ds}{s} \frac{Im\Pi(s)}{q^2 - s}. \tag{3}$$

Replacing the Green function of the virtual photon in the one-loop vertex function by the one containing the polarization operator

$$-i\frac{g_{\mu\nu}}{q^2} \to -\frac{1}{\pi} \int\limits_{4m_\pi^2}^{\infty} \frac{ds}{s} Im\Pi(s) \frac{-ig_{\mu\nu}}{q^2 - s}, \tag{4}$$

one arrives to the known result of lowest order contribution to a_μ from the hadronic intermediate state

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$$a_\mu^{(1)} = \frac{\alpha^2}{3\pi^2} \int\limits_{4m_\pi^2}^{\infty} ds \frac{R(s)K^{(1)}(s/M^2)}{s}, \quad R(s) = \frac{3s}{4\pi\alpha^2}\sigma^{e^+e^-\to had.}(s), \tag{5}$$

and the lowest order kernel is

$$K^{(1)}(s/M^2) = \int\limits_0^1 dx \frac{x^2(1-x)}{x^2+(1-x)(s/M^2)}. \tag{6}$$

Problem consist in removing from the experimentally measured cross section the radiative corrections associated with initial electron-positron state, including the emission of virtual and real photon emission.

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This procedure can be the source of errors and uncertainties.
One can include the the pion form factor in form of the replacing

$$Im\Pi(s) \to F_\pi^2(s) Im\Pi(s). \quad (7)$$

Below we calculate the contribution to a_μ from the processes $e^+e^- \to \pi^+\pi^-$ and $e^+e^- \to \pi^+\pi^-(\gamma)$ assuming pion as a point-like particle, taking into account the emission of virtual and real photons by the charged pions only. To obtain the explicit formulae describing FSE is the motivation of our paper.

Lowest order contribution $a_\mu^{(1)}$

The differential (center of mass reference frame (cmf) is implied) and total cross sections of process

$$e_+(p_+) + e_-(p_-) \to \pi_+(q_+) + \pi_-(q_-) \tag{8}$$

are

$$\frac{d\sigma}{dc} = \frac{\pi\alpha^2\beta^3}{4s}(1-c^2); \sigma(s) = \frac{\pi\alpha^2\beta^3}{3s}, \tag{9}$$

with $s = (p_+ + p_-)^2 = 4E^2$ is the square of the total energy $c = \cos\theta$, and θ is the angle between the directions of initial electron and the negative charged pion in cmf. Inserting the explicit value of the total cross section we obtain

$$a_\mu^{(1)} = \frac{\alpha^2\rho^2}{6\pi^2}\int\limits_0^1 dx x^2(1-x)\int\limits_0^1 \frac{d\beta\beta^4}{4(1-x)+x^2\rho^2(1-\beta^2)},$$

$$\rho = \frac{m_\mu}{m_\pi}. \tag{10}$$

Lowest order contribution $a_\mu^{(1)}$

Performing the explicit integration on β we obtain

$$a_\mu^{(1)} = \frac{\alpha^2}{6\pi^2\rho^2}\int\limits_0^1 \frac{(1-x)dx}{x^2}\left[-4(1-x) - \frac{4}{3}\rho^2 x^2 + \frac{q}{2}[4(1-x)+\rho^2x^2] \times R\ln\frac{R+1}{R-1}\right],$$

$$R = \sqrt{1+\frac{4(1-x)}{\rho^2x^2}}. \quad (11)$$

Numeric estimation of both representation coincides leads to $a_\mu^{(1)} = 7.08665 \times 10^{-9}$.
In the next order of perturbation theory we must consider the contribution arising from the correction associated with emission of virtual and real photons (soft and hard) by the final $\pi_+\pi_-$ state. It result in replacement $\sigma(s) \to \sigma(s)(1+\delta(s))$.

Lowest order contribution $a_\mu^{(1)}$

Keeping in mind the correction to the kernel we obtain

$$a_\mu = \frac{1}{4\pi^3} \int\limits_{4m_\pi^2}^{\infty} ds\sigma(s)(1+\delta(s))[K^{(1)}(s/M^2) + \frac{\alpha}{\pi} K^{(2)}(s/M^2)]. \qquad (12)$$

The quantity $K^{(2)}(s/M^2)$ was computed in the paper (R.Barbieri , E. Remiddi, Nucl Phys. B 90,(1975),233) it it presented in Appendix. Radiative correction to the final state $\pi_+\pi_-$ will be considered below.

Emission of virtual photons

To start with virtual correction we find first the vertex function for scattering of the charged pion in the external field. Than we write it down in the annihilation channel and use to calculate the relevant virtual correction to the cross section. Vertex function of process $\pi_-(p_1) + \gamma^*(q) \to \pi_-(p_2)$ have a form

$$\Gamma_\mu = \frac{\alpha}{4\pi} \int \frac{N_\mu dk}{(k)(1)(2)}, (k) = k^2 - \lambda^2; (1) = k^2 - 2p_1 k; (2) = k^2 - 2p_2 k,$$
$$dk = \frac{d^4k}{i\pi^2}, N_\mu = (p_1 + p_2 - 2k)_\mu (2p_1 - k)_\lambda (2p_2 - k)^\lambda. \quad (13)$$

Writing N_μ as $N_\mu = (p_1 + p_2 - 2k)_\mu [4p_1 p_2 + (1) + (2) - (k)]$ and performing the loop momenta integration, we obtain for the un-renormalized vertex function

$$\Gamma_\mu^{un} = \frac{\alpha}{4\pi} (p_1 + p_2)_\mu F^{un}(q^2),$$
$$F^{un}(q^2) = (2m^2 - q^2) \int_1^1 \frac{dx}{q_x^2} [\ln \frac{m^2}{\lambda^2} + \ln \frac{q_x^2}{m^2} - 1] + 3 + \ln \frac{\Lambda^2}{m2},$$
$$p_1^2 = p_2^2 = m^2, q_x = p_1 x + p_2(1-x), q_x^2 = m^2 - x(1-x)q^2, q = p_2 - p_1. \quad (14)$$

Emission of virtual photons

Here λ, λ are the ultraviolet cut-off parameter and the fictitious photon mass. The regularization consist in the construction $F(q^2) = F^{un}(q^2) - F^{un}(0)$. So we have

$$\Gamma_\mu = \frac{\alpha}{4\pi}(p_1 + p_2)_\mu F(q^2),$$

$$F(q^2) = (2m^2 - q^2)\int\limits_1^1 \frac{dx}{q_x^2}[\ln\frac{m^2}{\lambda^2} + \ln\frac{q_x^2}{m^2} - 2] + 4[1 - \ln\frac{m}{\lambda}]. \quad (15)$$

Introducing the new variable $(1-\theta)^2, \theta = -q^2, m^2$ and using

$$\int\limits_1^1 \frac{dx}{q_x^2} = \frac{2\theta}{m^2(1-\theta^2)}\ln\frac{1}{\theta},$$

$$\int\limits_1^1 \frac{dx}{q_x^2}\ln\frac{q_x^2}{m^2} = \frac{2\theta}{m^2(1-\theta^2)}[\frac{1}{2}\ln^2\theta - 2\ln\theta\ln(1+\theta) - 2Li_2(-\theta) - \frac{\pi^2}{6}], \quad (16)$$

Emission of virtual photons

we obtain

$$\Gamma_\mu(p_1,p_2) = \frac{\alpha}{\pi}(p_1+p_2)_\mu \Big[(\ln\frac{m}{\lambda}-1)(\frac{1+\theta^2}{1-\theta^2}-ln\frac{1}{\theta}-1)+ + \frac{1+\theta^2}{4(1-\theta^2)}[\ln^2\theta-4\ln\theta\ln(1+\theta)-4Li_2(-\theta)-\frac{\pi^2}{3}\Big]. \quad (17)$$

For the crossing channel $\gamma^*(q,\mu)\to\pi_-(q_-)+\pi_+(q_+)$ we use the substitutions (R. Barbieri, J. A. Mignaco, E. Remiddi, IL Nuovo Cimento, 11A (1972)824)

$$p_2\to q_-, p_1\to -q_+, \theta\to -x+i\epsilon, 0<\epsilon<<1,$$
$$x=\frac{1-\beta}{1+\beta}, \beta=\sqrt{1-(4m^2/s)}, s=(q_++q_-)^2=4E^2. \quad (18)$$

Emission of virtual photons

This quantity acquire the imaginary part for $s > 4m^2$:

$$\Gamma_\mu = \frac{\alpha}{\pi}(q_- - q_+)_\mu F(x),$$

$$F(x) = (\ln\frac{\lambda}{m} + 1)(\frac{1+x^2}{1-x^2}\ln x + 1 + i\pi) +$$

$$+\frac{1+x^2}{4(1-x^2)}\Big[\ln^2 x - \frac{4}{3}\pi^2 -$$

$$4Li_2(x) - 4\ln x\ln(1-x) + i\pi(2\ln x - 4\ln(1-x))\Big].$$

Emission of virtual photons

Writing the $ReF(x)$ as

$$ReF(x) = (-1 + \frac{1+\beta^2}{2\beta}L)\ln\frac{\lambda}{m} + F_V,$$
$$L = \ln\frac{1+\beta}{1-\beta}), \quad (19)$$

we write down the relevant contribution to the total cross section as

$$\Delta_V\sigma(s) = \frac{2\alpha^3\beta^3}{3s}[(-1 + \frac{1+\beta^2}{2\beta}L)\ln\frac{\lambda}{m} + F_V(\beta)], \quad (20)$$

with

$$F_V(\beta) = -1 + \frac{1+\beta^2}{2\beta}[L - \frac{1}{4}L^2 + \frac{1}{3}\pi^2 + Li_2(\frac{1-\beta}{1+\beta}) - L\ln\frac{2\beta}{1+\beta}]. \quad (21)$$

Emission of soft photons

Consider now the contribution from the emission of soft real photon channel. It have a form

$$\Delta_S\sigma = -\frac{\alpha}{4\pi^2}\sigma_B(s)\int' \frac{d^3k}{\omega}(\frac{q_-}{q_-k} - \frac{q_+}{q_+k})^2, \tag{22}$$

where the sign prime means $\omega = \sqrt{\vec{k}^2 + \lambda^2} < \Delta E$ and it is implied $\Delta E << E$. Using the relations

$$\frac{\alpha}{4\pi^2}\int' \frac{d^3k}{\omega}\frac{m^2}{(q_-k)^2} = \frac{\alpha}{\pi}[\ln\frac{2\Delta E}{\lambda} - \frac{1}{2\beta}L], \qquad L = \ln\frac{1+\beta}{1-\beta}, \tag{23}$$

$$\frac{\alpha}{4\pi^2}\int' \frac{d^3k}{\omega}\frac{(q_+q_-)}{(q_+k)(q_-k)} = \frac{\alpha}{\pi}\frac{1+\beta^2}{2\beta}[\ln\frac{2\Delta E}{\lambda}L + J(\beta)], \tag{24}$$

Emission of soft photons

with

$$J(\beta) = Li_2(-\beta) - Li_2(\beta) + Li_2(\frac{1+\beta}{2}) - \frac{1}{4}L^2 + \frac{1}{2}\ln^2(\frac{1+\beta}{2}) - \frac{1}{12}\pi^2, \quad (25)$$

we obtain the contribution to the cross section

$$\Delta_S\sigma(s) = \frac{2\alpha^3\beta^3}{3s}[(-1 + \frac{1+\beta^2}{2\beta}L)\ln\frac{2\Delta E}{\lambda} + F_S(\beta)],$$

$$F_S(\beta) = -\frac{1}{2\beta L} + \frac{1+\beta^2}{2\beta}J(\beta). \quad (26)$$

Hard real photon emission

Consider at least the contribution from the hard photon emission channel $\omega > \Delta E$. Matrix element of this process have a form

$$M = \frac{(4\pi\alpha)^{3/2}}{s} J^\mu T_{\mu\nu} e(k)^\nu, \qquad (27)$$

with $e(k)$ is the polarization vector of photon, $J^\mu = \bar{v}(p_+)\gamma^\mu u(p_-)$-the current associated with the leptons, and

$$T_{\mu\nu} = \frac{1}{2(q_- k)}(2q_- + k)_\nu (Q+k)_\mu + \frac{1}{2(q_+ k)}(-2q_+ - k)_\nu (Q-k)_\mu - 2g_{\mu\nu}. \qquad (28)$$

It can be check that this expression obey the gauge invariance conditions $T_{\mu\nu}q^\mu = T_{\mu\nu}k^\nu = 0$.

Hard real photon emission

We use as well the relation (Akhiezer A. I., Berestetskij V. B., Quantum Electrodynamics, Moscow, Science, 1981; J. D. Bjorken, S. Drell "Relativistic Quantum Fields", McGraw-Hill (1965))

$$\sum_{spin}\int |M|^2 d\Gamma_3 = -\frac{1}{3} Tr \hat{p}_+ \gamma^\mu \hat{p}_- \gamma^\nu (g_{\mu\nu} - q_\mu q_\nu / q^2) \int I d\Gamma_3,$$

$$I = T_{\rho\sigma} T^{\rho\sigma}, \qquad (29)$$

with $d\Gamma_3$ is the element of the phase space of the final particles

$$d\Gamma_3 = \frac{d^3 q_-}{2E_-} \frac{d^3 q_+}{2E_+} \frac{d^3 k}{2\omega} \frac{1}{(2\pi)^5} \delta^4(q - q_- - q_+ - k). \qquad (30)$$

It can be written as

$$\frac{d^3 q_-}{2E_-} \frac{d^3 q_+}{2E_+} \frac{d^3 k}{2\omega} \frac{1}{2E_+ (2\pi)^5} \delta(q_0 - E_- - E_+ - \omega), \qquad (31)$$

with $E_+ = \sqrt{(\vec{q}_- + \vec{k})^2 + m^2} = \sqrt{\omega^2 + E_-^2 + 2\vec{q}_- \vec{k}}$.

Hard real photon emission

Performing the integration on $\cos\theta$, θ is the angle in cmf between 3-momenta of pion and photon we obtain

$$d\Gamma_3 = \frac{\pi^2 s}{4(2\pi)^5} d\nu d\nu_- d\nu_+ \delta(\nu + \nu_- + \nu_+), \nu = \frac{2\omega}{q_0}, \nu_- = \frac{2E_-}{q_0}, \nu = \frac{2E_+}{q_0}, q_0 = 2E. \quad (32)$$

In terms of energy fractions we obtain

$$I = 8 + \frac{2(1-\nu)}{1-\nu_+} + \frac{2(1-\nu)}{1-\nu_-} - \beta^2(1-\beta^2)[\frac{1}{(1-\nu_+)^2} + \frac{1}{(1-\nu_-)^2}] + + \frac{2}{\nu}(\nu-\beta^2)(\nu-1-\beta^2)[\frac{1}{1-\nu_+} + \frac{1}{1-\nu_-}]. \quad (33)$$

The domain of integration D is

$$\frac{\Delta E}{E} < \nu < \beta^2; \nu + \nu_- + \nu_+ = 2; (1-\nu_-)(1-\nu_+)(1-\nu) > \frac{m^2\nu^2}{s}, \frac{\nu}{2}(1-R) < 1-\nu_\pm < \frac{\nu}{2}(1+R), R = \sqrt{\frac{\beta^2-\nu}{1-\nu}}. \quad (34)$$

Hard real photon emission

Performing the integration on $\nu_\pm$ we obtain

$$\int I d\nu_- d\nu_+ \delta(2-\nu-\nu_- -\nu_+) =$$
$$4[2R(\nu - \frac{\beta^2(1-\nu)}{\nu}) + (\frac{\beta^2(1+\beta^2)}{\nu} - 2\beta^2)\ln\frac{1+R}{1-R}]. \quad (35)$$

Performing the further integration we use the substitution $t = R, 0 < t < t_m, t_m^2 = \beta^2 - (\Delta E/E)(1-\beta^2)$. The corresponding contribution to the cross section is

$$\Delta_H\sigma(s) = \frac{2\alpha^3\beta^3}{3s}[(\frac{1+\beta^2}{2\beta}L - 1)\ln\frac{E}{\Delta E} + F_H(\beta)], \quad (36)$$

with

$$F_H(\beta) = -\frac{1+\beta^2}{\beta}G(\beta) + \ln\frac{1-\beta^2}{4\beta^2} - \frac{1}{8\beta^3}(3+\beta^2)(1-\beta^2)L + \frac{3+7\beta^2}{4\beta^2}, \quad (37)$$

Hard real photon emission

and

$$G(\beta) = \int_0^\beta \frac{dt}{1-t^2} \ln \frac{1-t^2}{\beta^2 - t^2} =$$

$$Li_2(\frac{1-\beta}{2}) - Li_2(\frac{1+\beta}{2}) + Li_2(1+\beta) - Li_2(1-\beta). \quad (38)$$

The total contribution do not depend on " photon mass" λ as well as from the auxiliary parameter ΔE:

$$\Delta\sigma = \frac{2\alpha^3\beta^3}{3s}[\frac{1}{2}(\frac{1+\beta^2}{2\beta}L - 1)\ln\frac{1}{1-\beta^2} + F_V + F_S + F_H]. \quad (39)$$

Hard real photon emission

After some algebra one obtain

$$\Delta\sigma^{e\bar{e}\to\pi\bar{\pi}}(s) = 2\frac{\alpha}{\pi}\sigma_B(s)\Delta(\beta), \sigma_B(s) = \frac{\pi\alpha^2\beta^3}{3s}, \beta = \sqrt{1-\frac{4m_\pi^2}{s}}, \qquad (40)$$

and

$$\Delta(\beta) = -\frac{3}{2}\ln\frac{4}{1-\beta^2} - 2\ln\beta + \frac{1+\beta^2}{2\beta}[-\frac{1}{12}\pi^2 + \frac{5}{4}L + \frac{3}{2\beta}[1-\frac{1}{2\beta}L] - L\ln\beta + Li_2(\frac{1-\beta}{1+\beta}) + 3Li_2(-\beta) - Li_2(\beta) + 3Li_2(\frac{1+\beta}{2}) - 2Li_2(\frac{1-\beta}{2}) + 2\ln\beta\ln(1+\beta) - 2Li_2(1-\beta) + \frac{1}{2}\ln^2(\frac{1+\beta}{2})]. \qquad (41)$$

Results for FSE in point-like pion approximation

The total contribution to a_μ can be obtained from the general formulae (see(9)) by replacement

$$\beta^4 \to \beta^4[1 + \frac{2\alpha}{\pi}\Delta(\beta)] = \beta^4[1 + \delta(s)] \tag{42}$$

Numeric estimation leads to

$$\Delta^\pi a_\mu = -6.923 \times 10^{-11}. \tag{43}$$

A total set of the lowest order RC to $a_\mu^{(1)}$ takes into account as well the correction to the kernel

$$\Delta^{ker} a_\mu = \frac{\alpha^3}{3\pi^3} \int\limits_0^1 \frac{\beta_\pi^4 d\beta_\pi}{1-\beta_\pi^2} K^{(2)}(b), \quad b = \frac{s}{M^2} = \frac{4}{\rho^2(1-\beta_\pi^2)}. \tag{44}$$

Results for FSE in point-like pion approximation

Explicit form of the kernel $K^{(2)}(b)$ as well as it's expansion on powers b^{-1} are presented in Appendix. Using the explicit form of $K^{(2)}$ can be successfully applied for the region $1-\beta_\pi^2 \sim 1$ and is not convenient for the region $1-\beta_\pi^2 << 1$. In this region we apply the expansion of it in powers M^2/s, which was obtained in paper of Krause (B. Krause, arXiv: hep-ph/9607259). For this aim we choice an auxiliary parameter $\beta_0 \sim 1$

$$\Delta^{ker} a_\mu = \frac{1}{4\pi^3}[\int\limits_0^{\beta_0} \frac{\beta_\pi^4 d\beta_\pi}{1-\beta_\pi^2} K^{(2)}(b)_{BR} + \int\limits_{\beta_0}^{1} \frac{\beta_\pi^4 d\beta_\pi}{1-\beta_\pi^2} K^{(2)}(b)_{Kr}]. \quad (45)$$

The result do not depend on β_0 and is

$$\Delta^{ker} a_\mu = -1.73 \cdot 10^{-10}. \quad (46)$$

Results for FSE in point-like pion approximation

Total contribution of correction to a_μ both from RC to final $\pi_+\pi_-$ and to the kernel is

$$\Delta A_\mu = \Delta^\pi + \Delta^{ker} = -2.4 \cdot 10^{-10}. \quad (47)$$

Insertion of pion form factor. Discussion

The result, obtained in point-like approximation about an order of magnitude lower than one measured in experiment (M. Davier, et al., Eur.Phys.J. C 71(2011),1515; C72(2012),1874) $a_\mu \approx 7 \cdot 10^{-8}$. Conversion of a virtual photon to $\pi^+\pi^-(\gamma)$ state in time-like region is realiszed through the intermediate state with vector mesons $\rho(775), \omega(782), \phi(1020)$ with the following decay to the two pion state. Main contribution arise from $\rho(775)$ meson state. Keeping in mind the resonance nature of this transition, it can be taken into account by the replacement in (1)

$$\sigma_B(s) \to \sigma_B(s) Z(\frac{S}{M_\rho^2}),$$

$$Z(\frac{S}{M_\rho^2}) = \frac{M_\rho^4}{(s-M_\rho^2)^2 + M_\rho^2\Gamma_\rho^2}, \tag{48}$$

with $\Gamma_\rho \approx 149 MeV$ is the width of $\rho(775)$ meson.

Insertion of pion form factor. Discussion

Rather rough approximation for a_μ is

$$a_\mu = \frac{\alpha^2}{12\pi^2}\int\limits_{4m_\pi^4}^{\infty}\frac{ds}{s}Z(\frac{s}{M_\rho^2})F(\beta_\pi) = \frac{\alpha^2}{12\pi^2}F(\beta_0)\frac{\pi M_\rho}{\Gamma_\rho}, \quad (49)$$

with

$$\beta_0 = \sqrt{1-\frac{4m_\pi^2}{M_\rho^2}} \approx 0.8;$$

$$F(\beta_0) = \beta_0^3\int\limits_0^1 dx\frac{x^2(1-x)}{x^2+(1-x)(M_\rho^2/M^2)} \approx 2.5\cdot 10^{-3}. \quad (50)$$

Insertion of pion form factor. Discussion

As a result we have $a_\mu^{(1)} \approx 2 \cdot 10^{-8}$. This quantity still $2-3$ times less than the experimental value. The contribution of $\omega(782)$ arise due to the rather small $\rho - \omega$ mixing. The contribution of $\phi(1020)$ as well can not expected to exceed the contribution of $\rho(775) meson$. The estimation of contributions of the next order of perturbation theory is

$$\Delta a_\mu = \frac{2\alpha}{\pi} a_\mu^{(1)} [\Delta(\beta_0) + K^{(2)}(\beta_0)] = \frac{2\alpha}{\pi} a_\mu^{(1)} [-0.8 - 3]. \quad (51)$$

Insertion of pion form factor. Discussion

Here we do not take into account the contribution of double vacuum polarizatioin-with two hadronic insertion and QED one with electron-positron intermediate state. Both of them was considered in the recent paper (D. Greynat and E. de !Rafael, JHEP 07(2012) 020).
In paper (A. Hoefer, J. Glusa, F. Jegerlehner, Eur. Phys..J. C 24, 59 (2002)) an attempt to take into account the initial state emission of an additional pair of charged particles was done. But it remain unclear the question of choice of a kernel used.
In paper (D. Greynat and E. de !Rafael, JHEP 07(2012) 020) the similar calculation was performed using the duality approximation (constituent quarks and gluons – hadrons) and applying the result of paper (G. Kallen and A. Sabry, Danske Videnskab, 29N 17 (1955),17) for the final state emission of fermion-anti-fermion pair.

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Appendix

The explicit form of the kernel $K^{(2)}(b)$ was obtained in paper of R. Barbieri and E. Remiddi (R.Barbieri , E. Remiddi, Nucl Phys. B 90 (1975)233). The contribution of 14 Feynman diagram was taken into account. It have a form

$$K^{(2)}(b)_{BR} = -\frac{139}{144} + \frac{115}{72}b + \left(\frac{19}{12} - \frac{7}{36}b + \frac{23}{144}b^2 + \frac{1}{b-4}\right)\log(b) + (-\frac{4}{3} + \frac{127}{36}b - \frac{115}{72}b^2 + \frac{23}{144}b^3)\frac{\log y}{\sqrt{b(b-4)}} + \left(\frac{9}{4} + \frac{5}{24}b - \frac{1}{2}b^2 - \frac{2}{b}\right)\xi(2) + \frac{5}{96}b^2\log^2 b + (-\frac{1}{2}b + \frac{17}{24}b^2 - \frac{7}{48}b^3)\frac{\log y}{\sqrt{b(b-4)}}\log b + \left(\frac{19}{24} + \frac{53}{48}b - \frac{29}{96}b^2 - \frac{1}{3b} + \frac{2}{b-4}\right)\log^2 y + (-2b + \frac{17}{6}b^2 - \frac{7}{12}b^3)\frac{1}{\sqrt{b(b-4)}}D_p(b) + \left(\frac{13}{3} - \frac{7}{6}b + \frac{1}{4}b^2 - \frac{1}{6}b^3 - \frac{4}{b-4}\right)\frac{D_m(b)}{\sqrt{b(b-4)}} + (\frac{1}{2} - \frac{7}{6}b + \frac{1}{2}b^2)T(b), \quad (52)$$

Appendix

where

$$y = \frac{\sqrt{b} - \sqrt{b-4}}{\sqrt{b} + \sqrt{b-4}},$$
$$D_p(b) = Li_2(y) + \log(y)\log(1-y) - \frac{1}{4}\log^2 y - \xi(2),$$
$$D_m(b) = Li_2(-y) + \frac{1}{4}\log^2 y + \frac{1}{2}\xi(2),$$
$$T(b) = -6Li_3(y) - 3Li_3(-y) + \log^2 y \log(1-y) +$$
$$\frac{1}{2}(\log^2 y + 6\xi(2))\log(1+y) + 2\log y(Li_2(-y) + 2Li_2(y)). \quad (53)$$

Appendix

The function $Li_2(y), Li_3(y)$ are the dilogarithm and trilogarithm defined through

$$Li_2(y) = -\int\limits_0^y \frac{dt}{t}\ln(1-t) = -\int\limits_0^1 \frac{dt}{t}\ln(1-ty),$$

$$Li_2(-y) = -\int\limits_0^y \frac{dt}{t}\ln(1+t) = -\int\limits_0^1 \frac{dt}{t}\ln(1+ty);$$

$$Li_3(y) = \int\limits_0^y \frac{dt}{t}[\ln t - \ln y]\ln(1-t) = \int\limits_0^1 \frac{dt}{t}\ln t\ln(1-ty),$$

$$Li_3(-y) = \int\limits_0^y \frac{dt}{t}[\ln t - \ln y]\ln(1+t) = \int\limits_0^1 \frac{dt}{t}\ln t\ln(1+ty). \tag{54}$$

Appendix

In paper of B.Krause (B. Krause, arXiv: hep-ph/9607259) the expansion on $M^2/s = 1/b$ was obtained

$$K^{(2)}(b)_{Kr} = \frac{1}{b}\Big\{[\frac{223}{54} - 2\xi_2 - \frac{23}{36}L] + \frac{1}{b}[\frac{8785}{1152} - \frac{37}{8}\xi_2 - \frac{367}{216}L + \frac{19}{144}L^2] + \frac{1}{b^2}[\frac{13072841}{432000} - \frac{883}{40}\xi_2 - \frac{10079}{3600}L + \frac{141}{80}L^2] + \frac{1}{b^3}[\frac{2034703}{16000} - \frac{3903}{40}\xi_2 - \frac{6517}{1800}L + \frac{961}{80}L^2]\Big\}. \quad (55)$$

with

$$\xi_2 = \frac{\pi^2}{6}, L = \ln b. \quad (56)$$

In Figure 1 the β_π dependence of the exact integrand $F_{BR}(\beta_\pi) = \beta_\pi^4 K^{(2)}_{BR}/(1-\beta_\pi^2)$ and it's expansion in powers M^2/s $F_{Kr}(\beta_\pi) = \beta_\pi^4 K^{(2)}_{Kr}/(1-\beta_\pi^2)$ is presented.

Figures

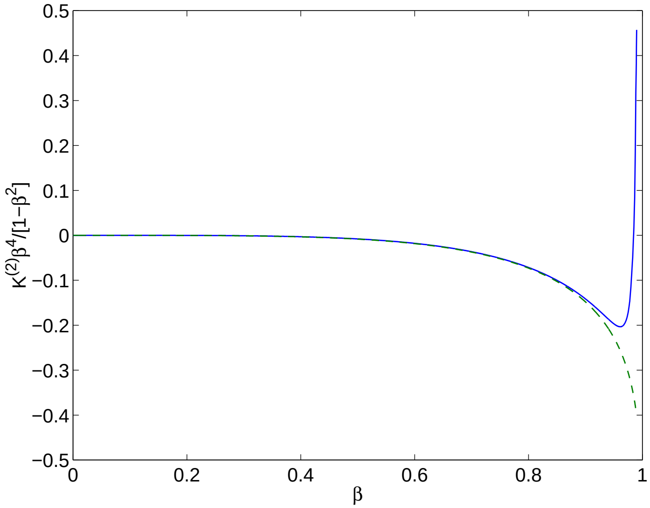

Figure : Dependence of $K^{(2)}(\beta)$ a) exact formulae, b) power M^2/s expansion.