The role of $\sigma_{\rm hadronic}$ for the future of the precision determinations of the muon $g-2$ and the running $\alpha_{\rm em}$

F. JEGERLEHNER

DESY Zeuthen/Humboldt-Universität zu Berlin

Colloqium , 13th Meeting of the Radio MonteCarLOW Collaboration

ETC* Trento, Italy, April 10-12, 2013

supported by DFG – SFB 1044 at Mainz University

Abstract

Precision measurements of hadronic cross-sections in electron-positron annihilation play a key role in high precision predictions of the muon anomalous magnetic moment a_μ and for a precise determination of the effective fine structure constant $\alpha_{\rm em}(M_Z)$. The role of the effective fine structure constant for the future of high precision electroweak fits and possible progress is outlined. I present a brief summary of the status of the anomalous magnetic moment of the muon and then discuss the requirements for the next generation experiments. The hope is to be able to substantially enhance the significance of known "deviations" between SM predictions and experiments, and eventually establish new physics not yet included in the Standard Model.

Outline of Talk:

① **Hadronic Effects in Electroweak Observables**

② **The role of $\alpha(M_Z)$ in precision physics**

③ **Evaluation of $\alpha(M_Z)$**

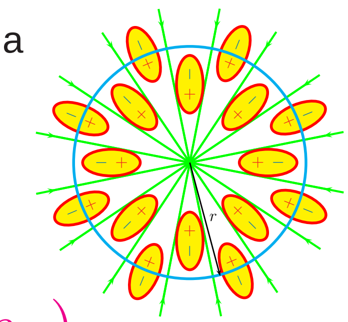
④ **The role of VP effects in muon $g-2$**

④ **Hadronic light-by-light in $g-2$**

⑤ **Muon $g-2$: where we are, what do we learn**

① Hadronic Effects in Electroweak Observables

Non-perturbative hadronic effects in electroweak precision observables, main effect via



effective fine-structure "constant" $\alpha(E)$

(charge screening by vacuum polarization)

Of particular interest:

$$\alpha(M_Z) \text{ and } a_\mu \equiv (g-2)_\mu/2 \Leftrightarrow \alpha(m_\mu)$$

- **electroweak effects (leptons etc.) calculable in perturbation theory**
- **strong interaction effects (hadrons/quarks etc.) perturbation theory fails**

$\Longrightarrow$ **Dispersion integrals over e^+e^-–data**

encoded in $$R_\gamma(s) \equiv \frac{\sigma(e^+e^-\to\gamma^*\to\text{hadrons})}{\sigma(e^+e^-\to\gamma^*\to\mu^+\mu^-)}$$

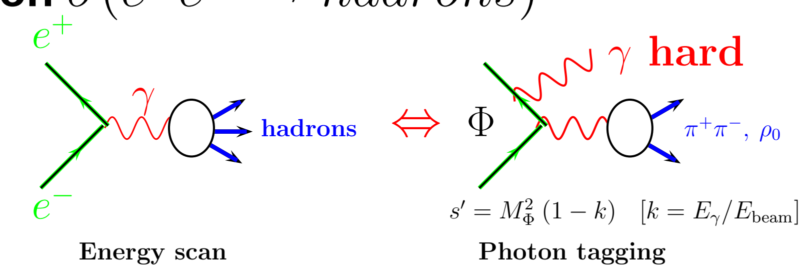
Errors of data $\Longrightarrow$ theoretical uncertainties !!!

The art of getting precise results from non-precision measurements !

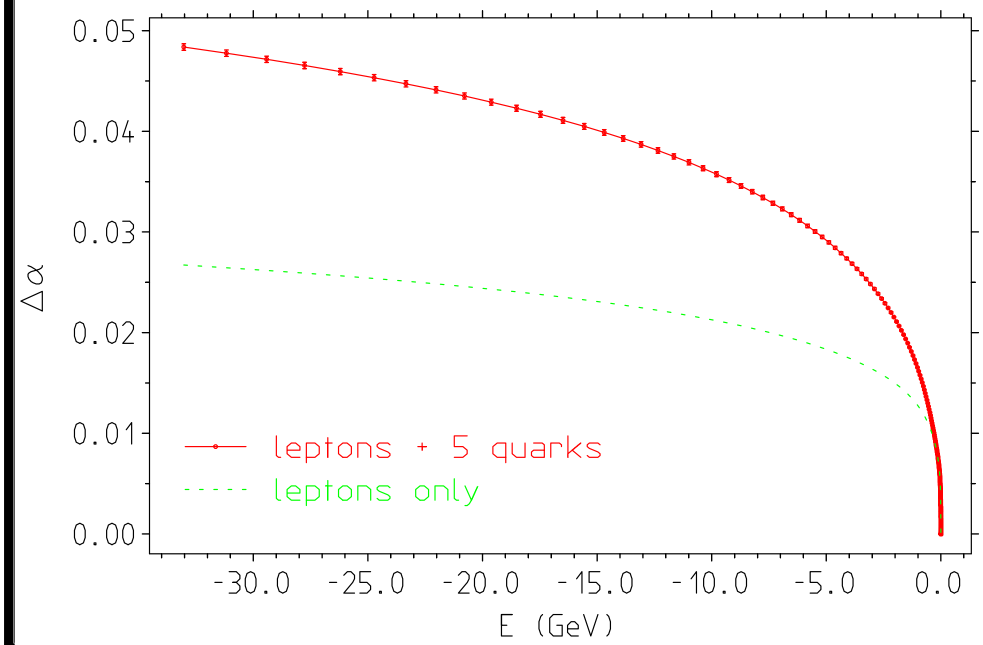
New challenge for precision experiments on $\sigma(e^+e^- \to hadrons)$

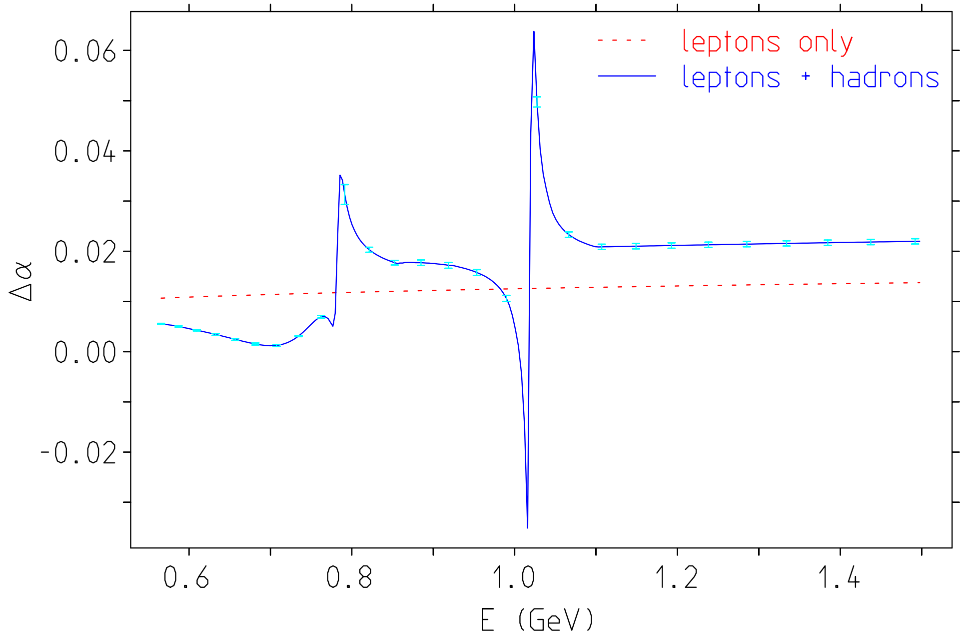
KLOE, BABAR, Belle via radiative return:

CMD, SND, KEDR, BES via scan:



- **Need to know running of $\alpha_{\rm QED}$ very precisely.**
  Large corrections, steeply increasing at low E





The running of α. The “negative” E axis is chosen to indicate space-like momentum transfer. The vertical bars at selected points indicate the uncertainty. In the time-like region the resonances lead to pronounced variations of the effective charge (shown in the $\rho-\omega$ and ϕ region).

$\boxed{\alpha}$ $\alpha^{-1}(a_e) = 137.0359991657(331)(68)(46)(24)[0.25\ \mathrm{ppb}]$

$\boxed{\alpha(E)}$:

$$\begin{array}{rcll} \Delta\alpha^{(5)}_{\rm hadrons}(M_Z^2) & = & 0.027510 \pm 0.000218 & \\ & & 0.027498 \pm 0.000135 & \textbf{Adler} \\ \alpha^{-1}(M_Z^2) & = & 128.961 \pm 0.030 & \\ & & 128.962 \pm 0.018 & \textbf{Adler} \end{array}$$

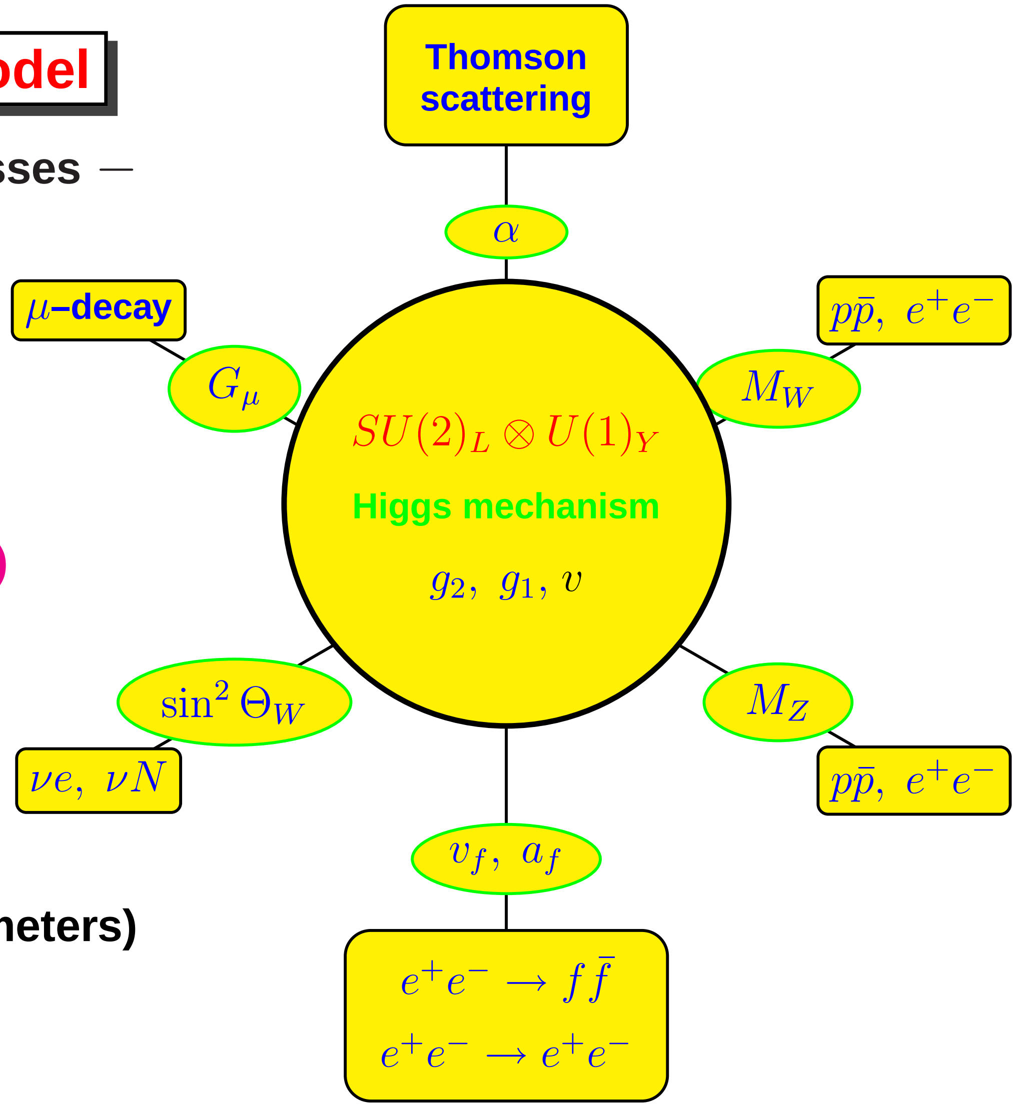
- **0.25 ppb ⇔ 139.58 ppm loose** $\boxed{5.3 \cdot 10^5}$ **in precision**
- **effective fine structure constant least well known SM parameter for W and Z boson physics**

muon $g-2$ **:** $\alpha^{-1}(m_\mu) = 136.067675(978)$

- **0.25 ppb ⇔ 0.72 ppm loose** $\boxed{1.4 \cdot 10^3}$ **in precision**

The Parameters of the Standard Model

— in four fermion and vector boson processes —



unlike in QED and QCD in SM (SBGT)

parameter interdependence

☞

only 3 independent quantities

(besides fermion masses and mixing parameters)

α, G_μ, M_Z

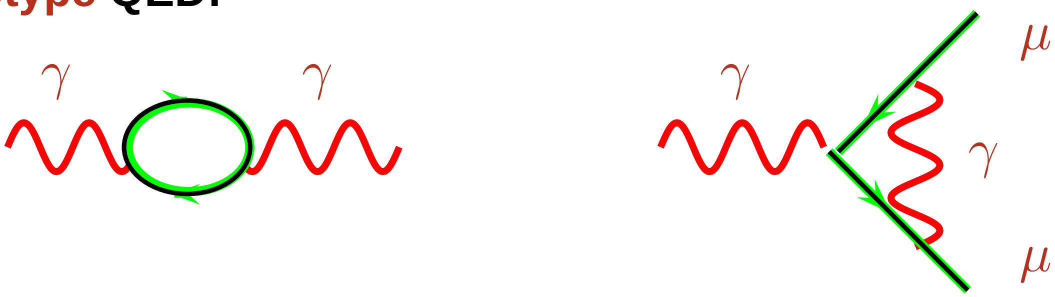
$\Downarrow$

parameter relationships between very precisely measurable quantities

precision tests, possible sign of new physics

Test of quantum effects

Prototype QED:



vacuum polarization
$\longrightarrow$ Lamb shift

form factors
$\longrightarrow$ anomalous magnetic moment

LEP/SLC version of $g-2$:

$$\sqrt{2}G_\mu M_Z^2 \sin^2\Theta_i \cos^2\Theta_1 = \pi\alpha\,(1+\delta_i)$$

$\sin^2\Theta_i = (1 - v_\ell/a_\ell)/4, 1 - M_W^2/M_Z^2, e^2/g^2, \cdots$ differ by quantum corrections only

SM: renormalizable theory ('t Hooft 1971)

➤ δ_i uniquely calculable

$$\delta_i \equiv \Delta r_i = \Delta r_i(\underbrace{\alpha, G_\mu, M_Z}_{\text{very precisely known}}, M_H, m_t, m_b, \cdots)$$

- News from LHC: Higgs found, last essential free parameter fixed

$$M_H = 125 \pm 5 \text{ GeV}$$

- all SM parameters rather precisely known now

$$\begin{aligned} M_Z &= 91.1876(21) \text{ GeV}, \quad M_W = 80.385(15) \text{ GeV}, \quad M_t = 173.5(1.0) \text{ GeV},^{a} \\ G_F &= 1.16637 \times 10^{-5} \text{ GeV}^{-2}, \quad \alpha^{-1} = 137.035999, \quad \alpha_s^{(5)}(M_Z^2) = 0.1184(7). \end{aligned} \quad (1)$$

Precision predictions:

$$\sin^2\Theta_f, v_f, a_f, M_W, \Gamma_Z, \Gamma_W, g_2 \cdots$$

all depend on α effective!

Impact of $\delta\Delta\alpha$:

specifically M_W , $\sin^2\theta$, etc

$$\frac{\delta\sin^2\theta}{\sin^2\theta} \sim \frac{\cos^2\theta}{\cos^2\theta - \sin^2\theta}\,\delta\Delta\alpha \sim 1.54\,\delta\Delta\alpha$$

$$\frac{\delta M_W}{M_W} \sim \frac{1}{2}\frac{\sin^2\theta}{\cos^2\theta - \sin^2\theta}\,\delta\Delta\alpha \sim 0.23\,\delta\Delta\alpha$$

Pre LHC: Indirect

Higgs boson mass “measurement”

$m_H = 87^{+35}_{-26}$ **GeV**

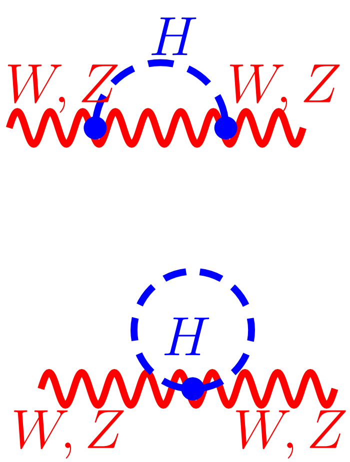
CDF/D0 exclude 160-170 GeV 95% C.L.

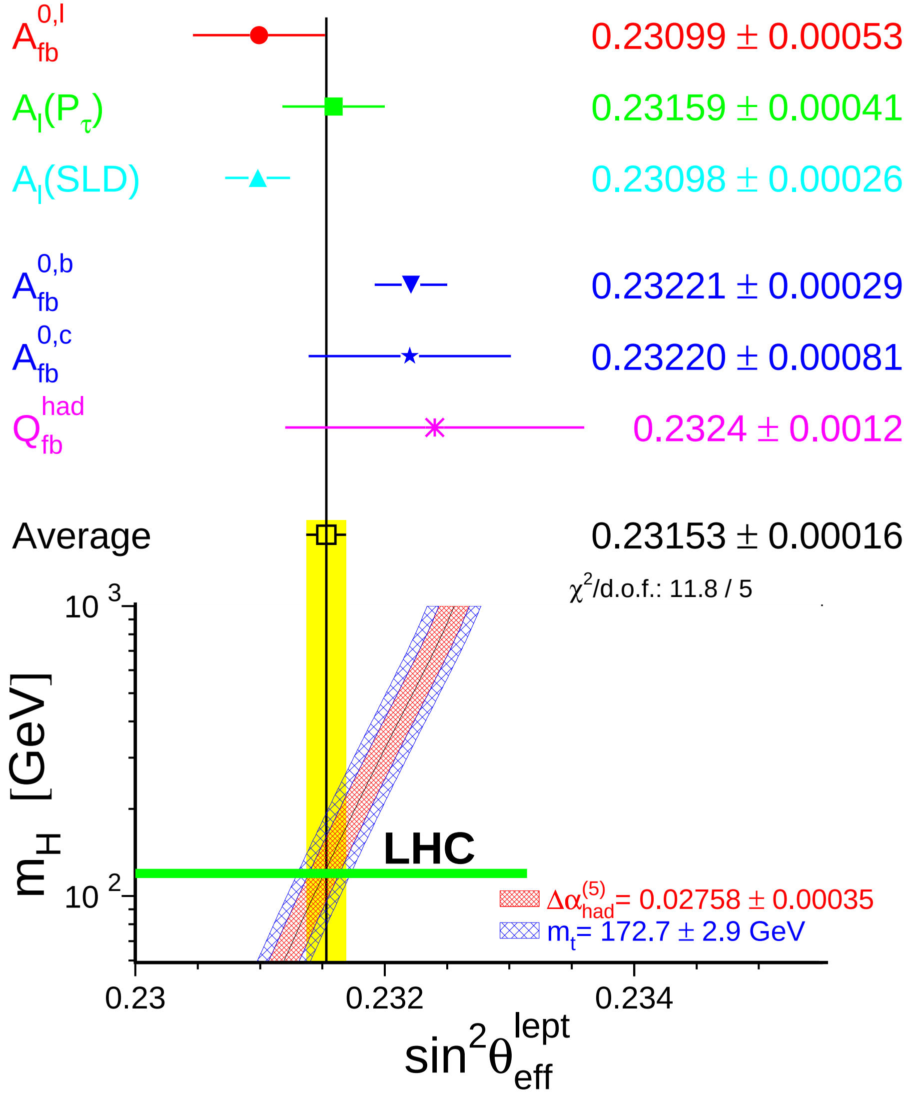
Direct lower bound:

$m_H > 114$ **GeV at 95% CL**

Indirect upper bound:

$m_H < 186$ **GeV at 95% CL**



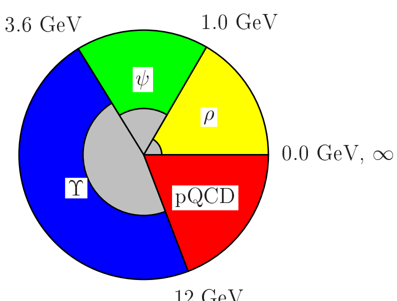


(LEP Electroweak Working Group: D. Abbaneo et al. 05)

③ Evaluation of $\alpha(M_Z)$

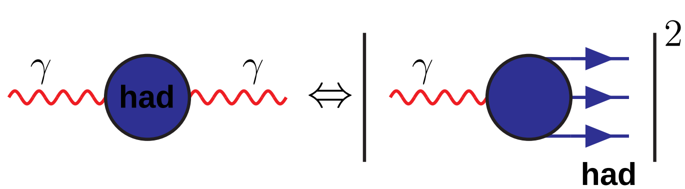
Non-perturbative hadronic contributions $\Delta\alpha^{(5)}_{\rm had}(s)$ can be evaluated in terms of $\sigma(e^+e^- \to \text{hadrons})$ data via dispersion integral:

$$\Delta\alpha^{(5)}_{\rm had}(s) = -\frac{\alpha s}{3\pi}\left(\mathrm{P}\!\!\int_{4m_\pi^2}^{E^2_{\rm cut}} ds' \frac{R^{\rm data}_\gamma(s')}{s'(s'-s)} + \mathrm{P}\!\!\int_{E^2_{\rm cut}}^{\infty} ds' \frac{R^{\rm pQCD}_\gamma(s')}{s'(s'-s)} \right)$$



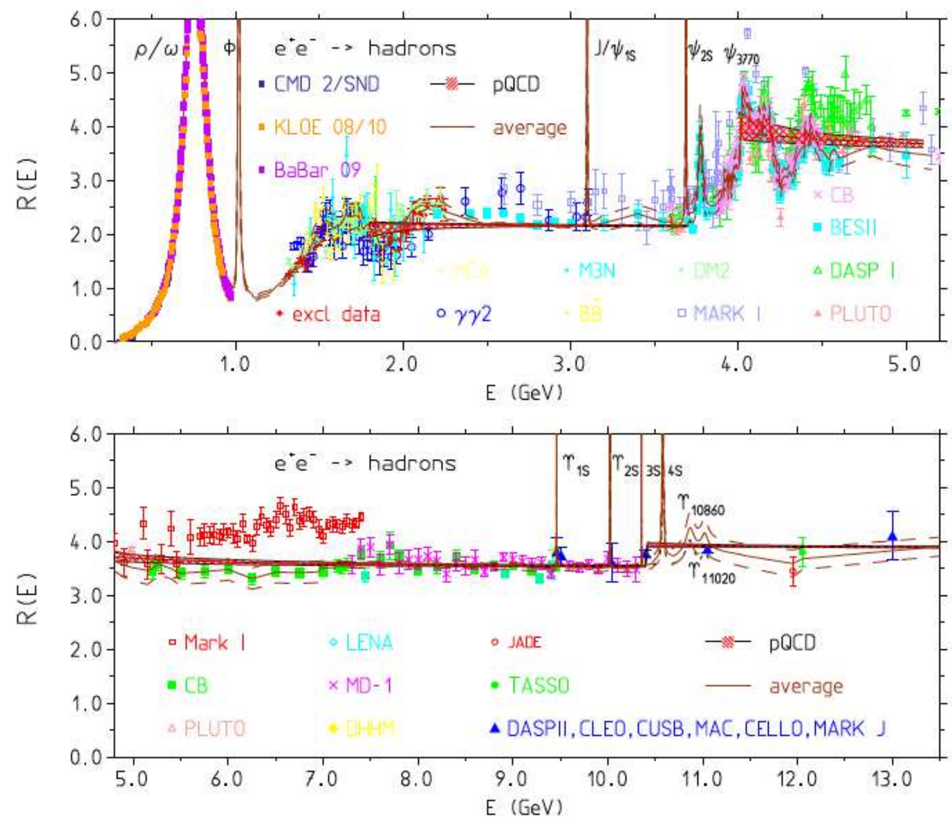
where

$$R_\gamma(s) \equiv \frac{\sigma^{(0)}(e^+e^- \to \gamma^* \to \text{hadrons})}{\frac{4\pi\alpha^2}{3s}}$$



$\Pi'_\gamma(q^2)$ $\sim \sigma^{\rm had}_{\rm tot}(q^2)$

Compilation: F.J. 2012
Theory = pQCD: Gorishny et al. 91, Chetyrkin et al. 97



Evaluation FJ 2012 update: **at $M_Z = $ 91.1876 GeV**

- $R(s)$ **data up to $\sqrt{s} = E_{cut} = 5$ GeV and for Υ resonances region between 9.6 and 11.5 GeV**
- **perturbative QCD from 5.2 to 9.6 GeV and for the high energy tail above 11.5 GeV**

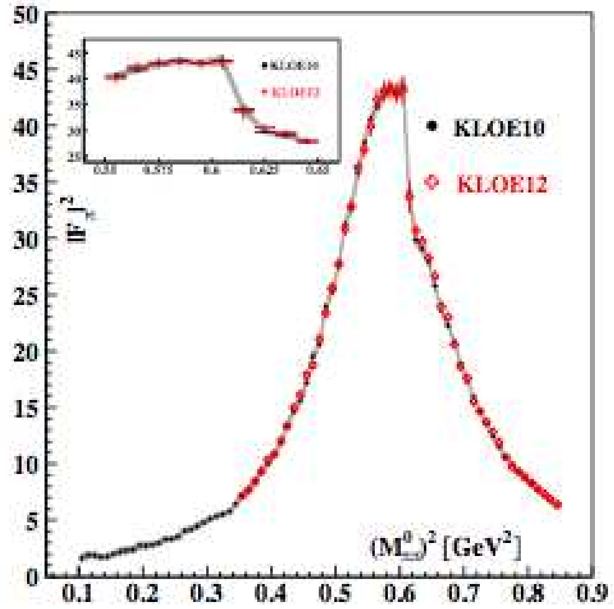
$$\Delta\alpha^{(5)}_{\rm hadrons}(M_Z^2) = 0.027510 \pm 0.000218$$

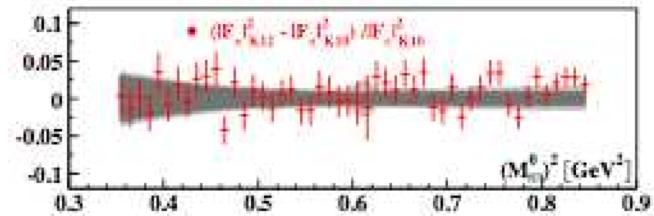
$$0.027498 \pm 0.000135 \quad \textbf{Adler}$$

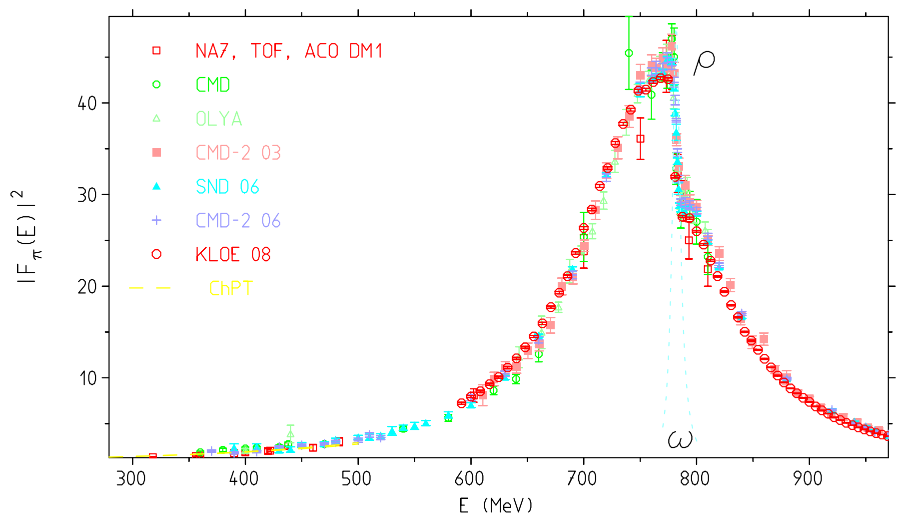
$$\alpha^{-1}(M_Z^2) = 128.961 \pm 0.030$$

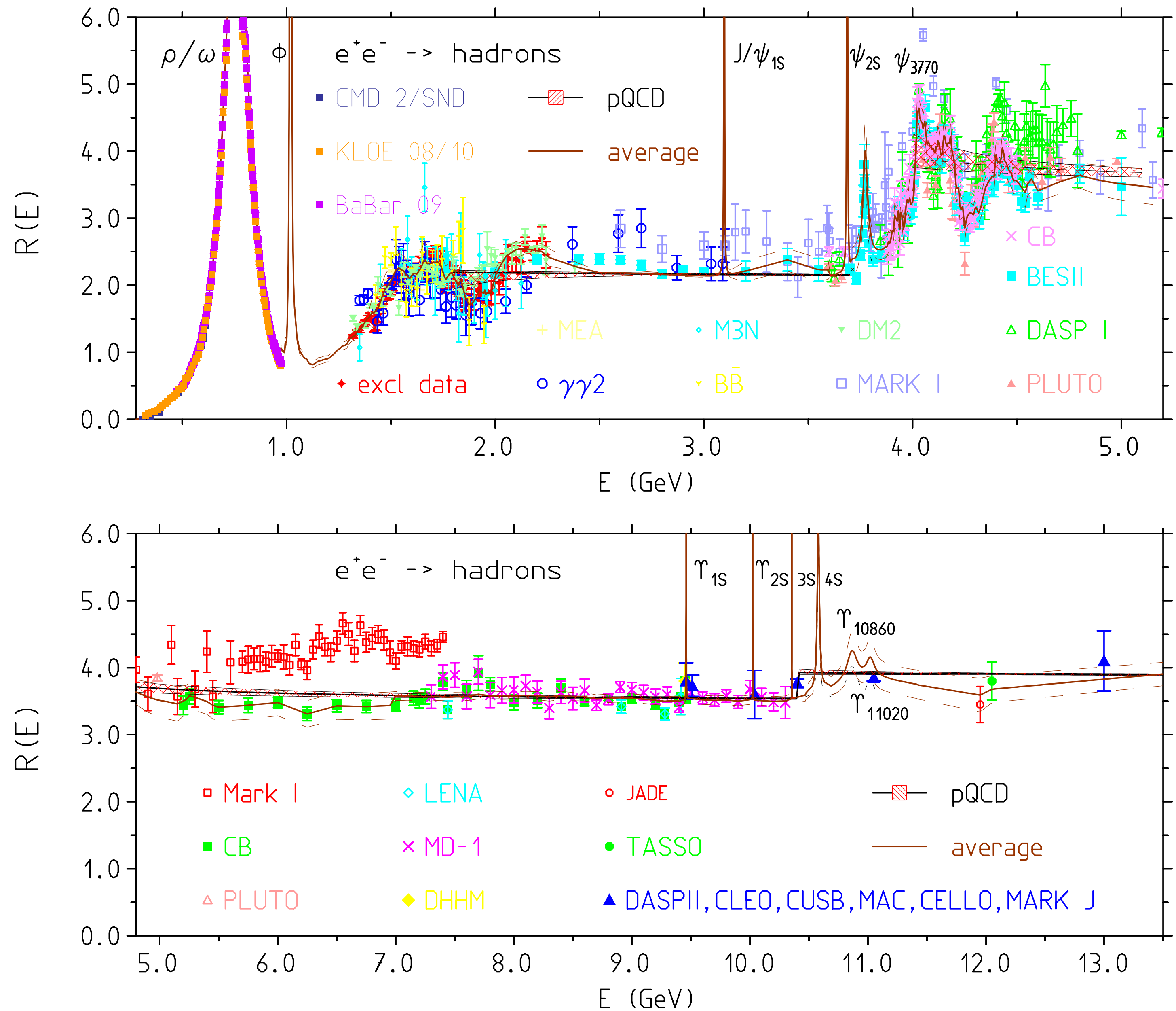
$$128.962 \pm 0.018 \quad \textbf{Adler}$$

A closer look at the e^+e^-–data



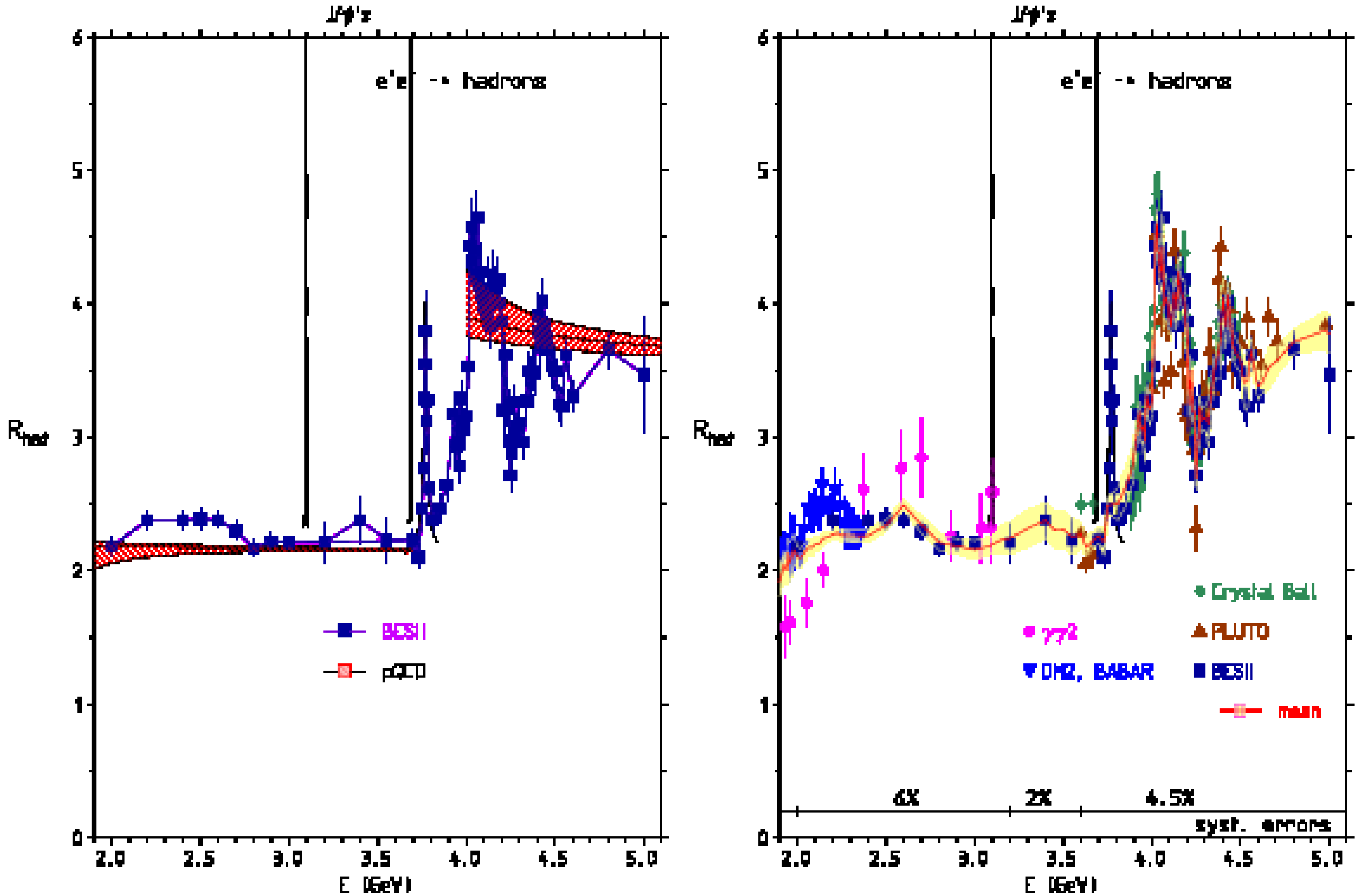






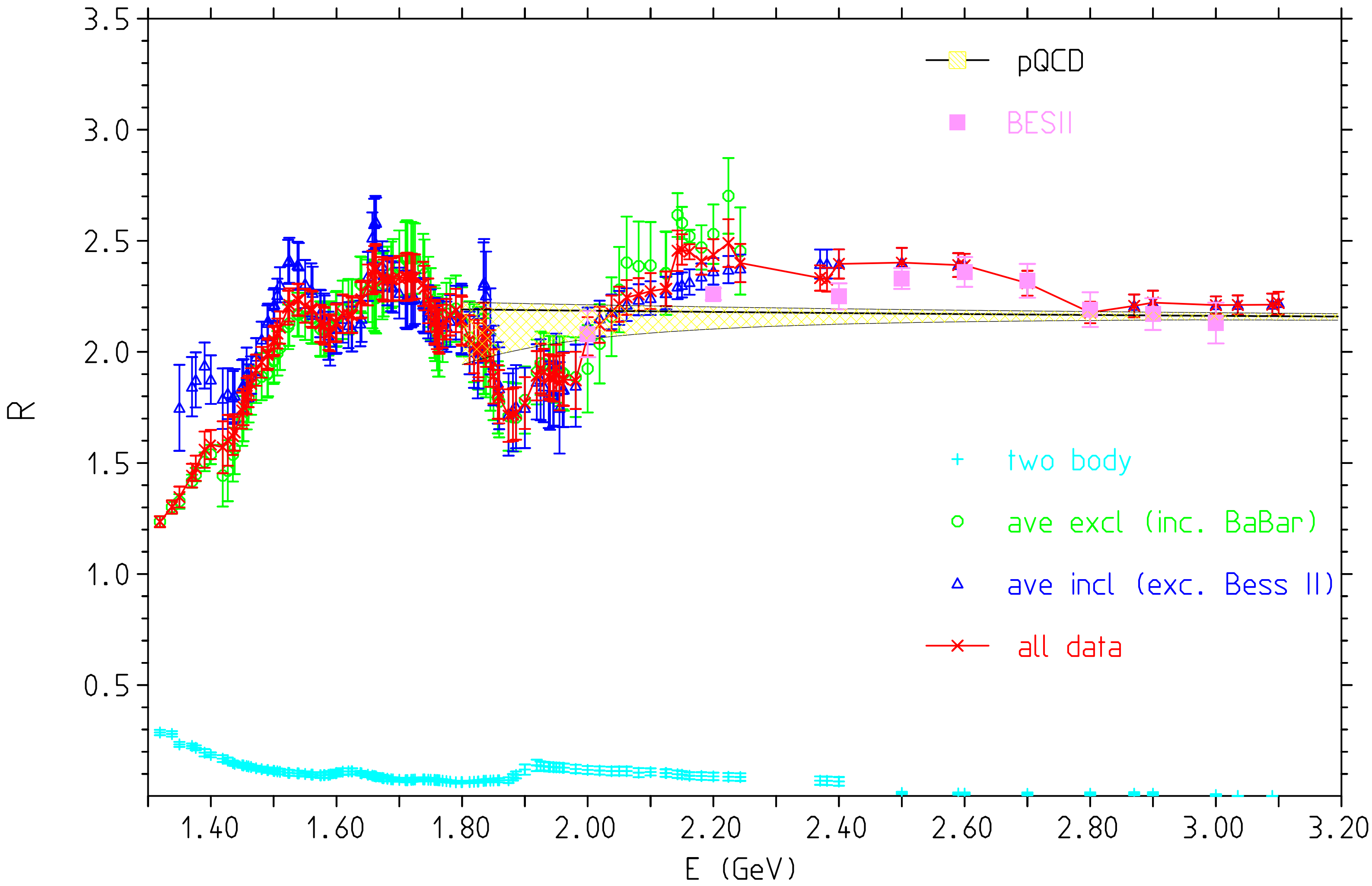
e+e- -> hadrons
ρ/ω
φ
J/ψ1S
ψ2S ψ3770
CMD 2/SND
KLOE 08/10
BaBar 09
pQCD
average
CB
BESII
MEA
M3N
DM2
DASP I
excl data
γγ2
BB̄
MARK I
PLUTO
R(E)
E (GeV)
Υ1S
Υ2S 3S 4S
Υ10860
Υ11020
Mark I
LENA
JADE
CB
MD-1
TASSO
PLUTO
DHHM
DASPII,CLEO,CUSB,MAC,CELLO,MARK J

R data BES region



❑ **primary goal:** R **as precise as possible** $\Rightarrow a_\mu\ , \alpha(M_Z)$

Note for muon $g-2$ region 1 - 2 GeV very important (VEPP 2000)

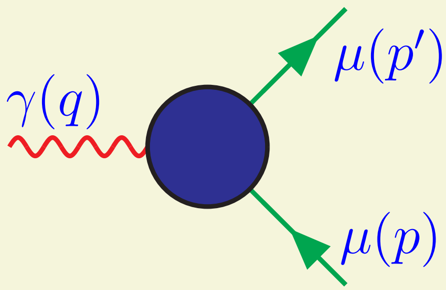


③ The Anomalous Magnetic Moment of the Muon

$$\vec{\mu} = g_\mu \frac{e\hbar}{2m_\mu c}\,\vec{s}\ ;\quad g_\mu = 2\,(1+a_\mu)$$

Dirac: $g_\mu = 2$, a_μ muon anomaly

Stern, Gerlach 22: $g_e = 2$; Kusch, Foley 48: $g_e = 2\,(1.00119 \pm 0.00005)$



$$= (-ie)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + i\frac{\sigma^{\mu\nu}q_\nu}{2m_\mu}F_2(q^2)\right]u(p)$$

$$F_1(0) = 1\ ;\quad F_2(0) = a_\mu$$

a_μ responsible for the Larmor precession

directly proportional at magic energy $\sim$ 3.1 GeV

CERN, BNL g-2 experiments

$$\vec{\omega}_a = \frac{e}{m}\left[a_\mu\vec{B} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\vec{\beta}\times\vec{E}\right]_{\text{at "magic }\gamma\text{"}}^{E\sim 3.1\,\text{GeV}} \simeq \frac{e}{m}\left[a_\mu\vec{B}\right]$$

The role of a_μ in precision physics

Precision measurement of a_μ provides most sensitive test of magnetic helicity flip transition

$$\bar{\psi}_L \, \sigma_{\mu\nu} \, F^{\mu\nu} \, \psi_R \qquad (\text{ dim 5 operator })$$

such a term must be absent for any fermion in any renormalizable theory at tree level

$$\Downarrow$$

a_μ is a pure "quantum correction" effect:

a finite model-specific prediction

in any renormalizable quantum field theory (QFT)

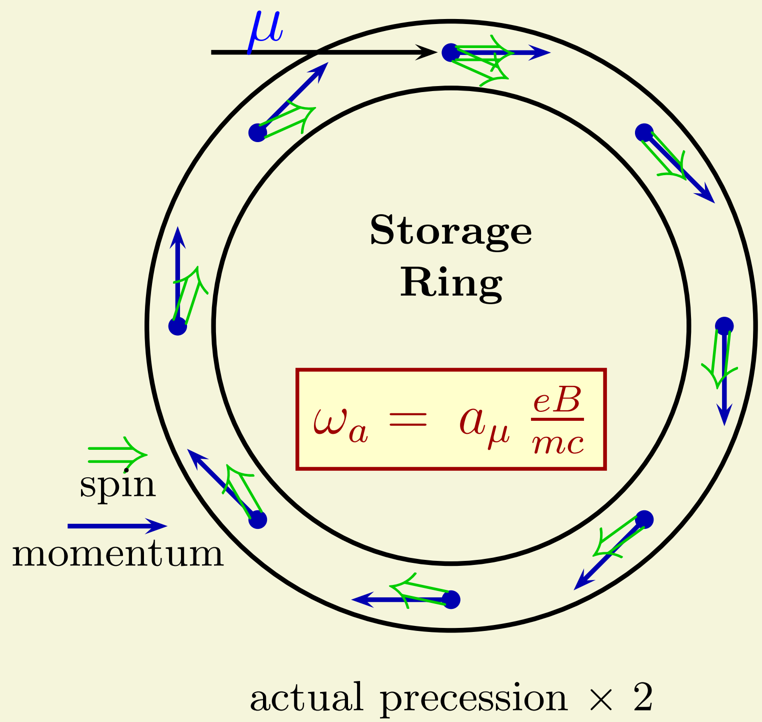
✗ – test of quantum structure

✗ – monitor for new physics

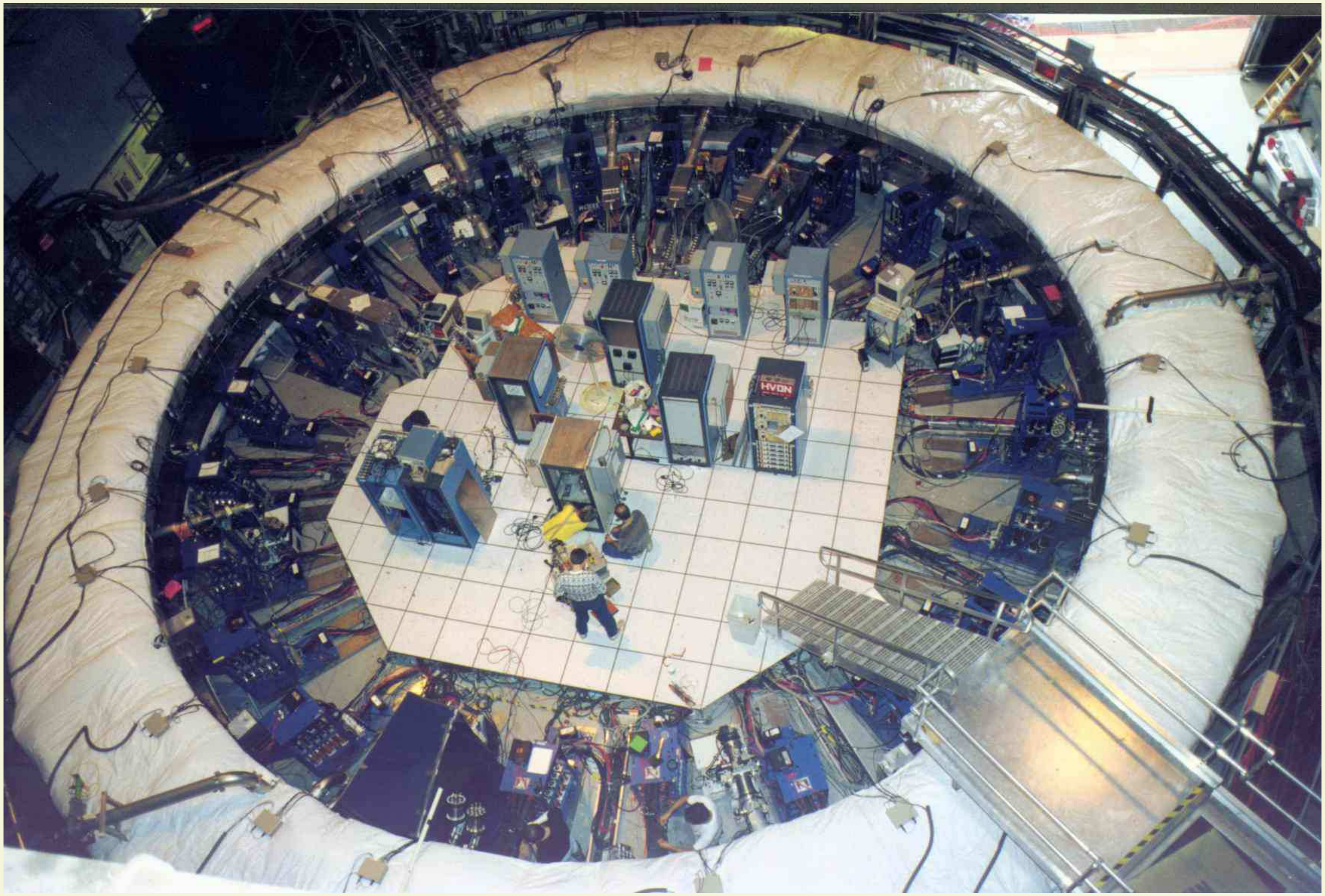
Most fascinating aspect highly complex mathematics meets reality !

Basic principle of experiment: measure Larmor precession of highly polarized muons circulating in a ring

$a_\mu = 0$ **would mean no rotation of spin relative to muon momentum!**



Spin precession in the $g-2$ ring ($\sim 12^\circ$/circle)

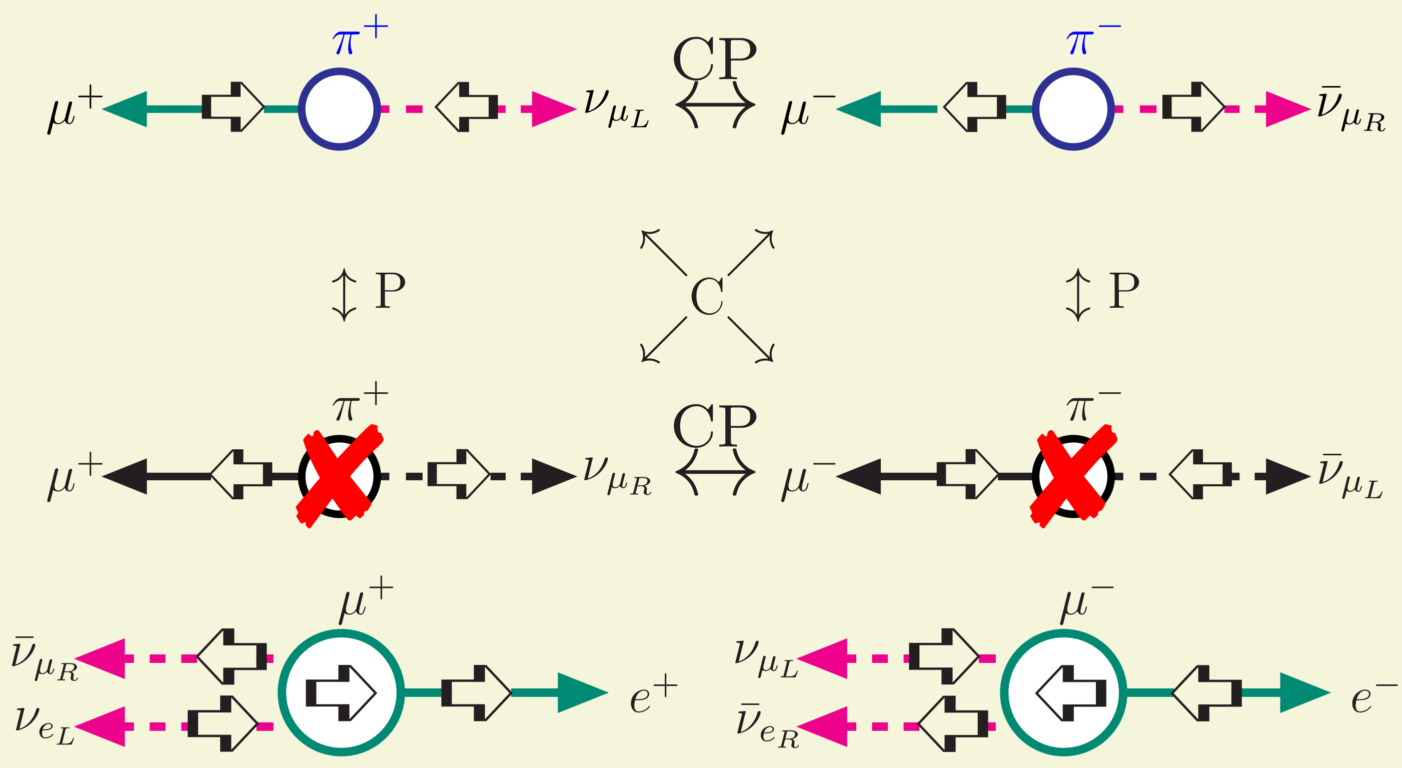


BNL muon storage ring: r= 7.112 meters, aperture of the beam pipe 90 mm, field 1.45 Tesla, momentum of the muon $p_\mu = 3.094$ GeV/c (see `http://www.g-2.bnl.gov/`)

Production and Decay of Muons

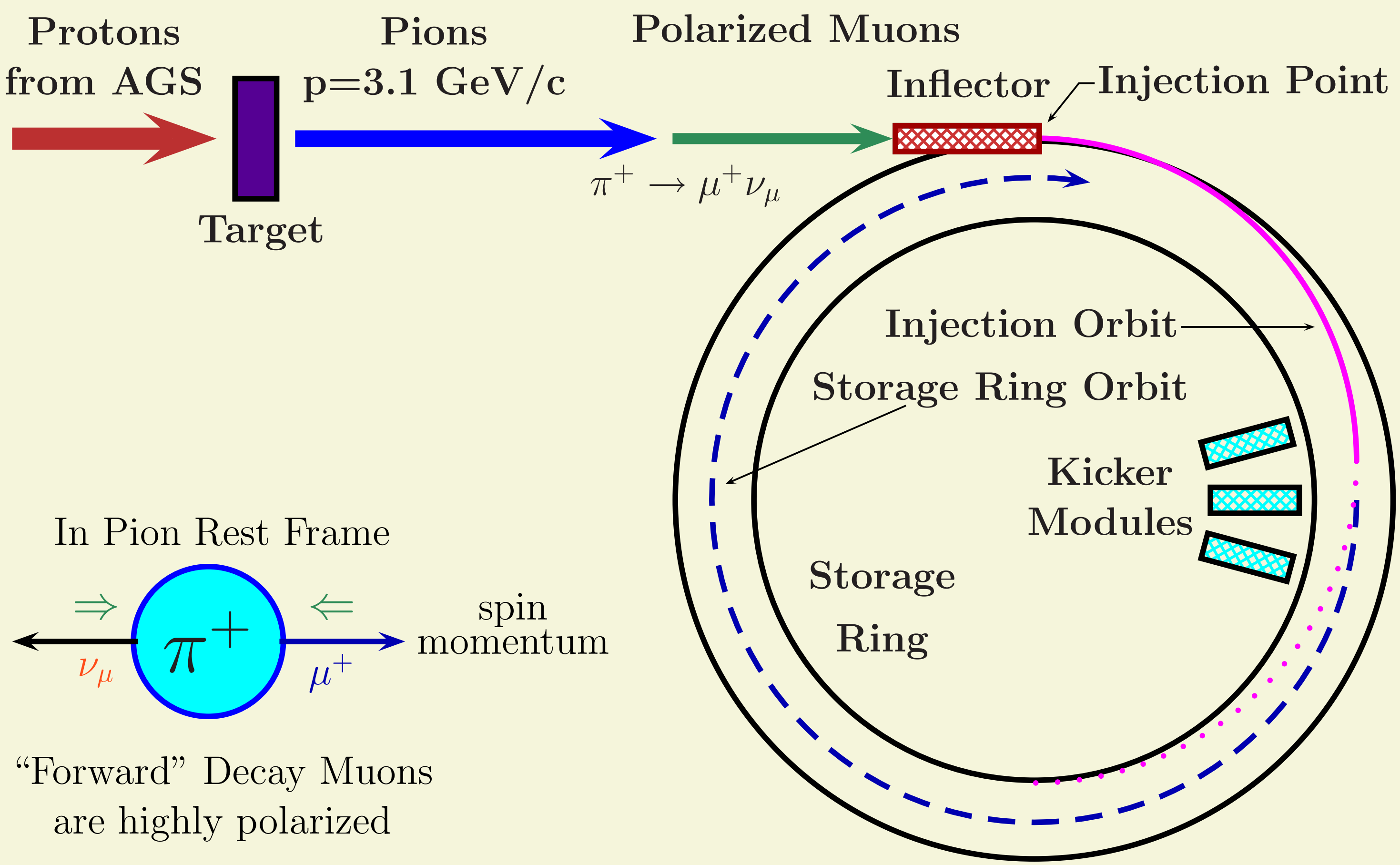
Relevant decay chain

$$\pi \;\longrightarrow\; \mu + \nu_\mu$$
$$\hookrightarrow\; e + \nu_e + \nu_\mu$$

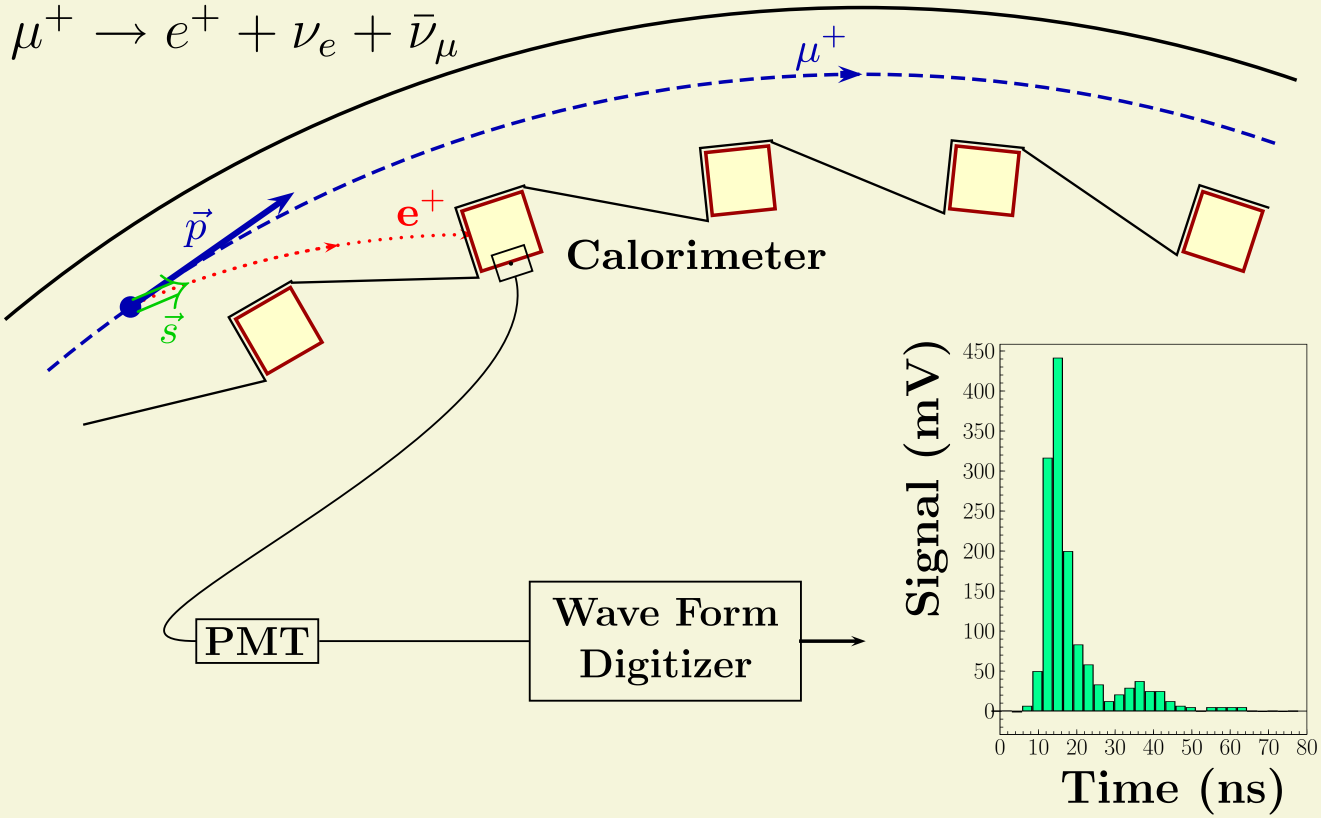


producing the polarized muons which decay into electrons which carry along in their direction of propagation the knowledge of the muon's polarization

How the muon $g-2$ experiment at Brookhaven works:



The schematics of muon injection and storage in the $g-2$ ring



Decay of μ^+ and detection of the emitted e^+ (PMT=Photomultiplier)

Decay spectrum: electrons of energy $> E$ yields very precise ω_a

$$N(t) = N_0(E)\ \exp\left(\frac{-t}{\gamma\tau_\mu}\right)\ [1 + A(E)\ \sin(\omega_a t + \phi(E))]\ ,$$

Distribution of counts

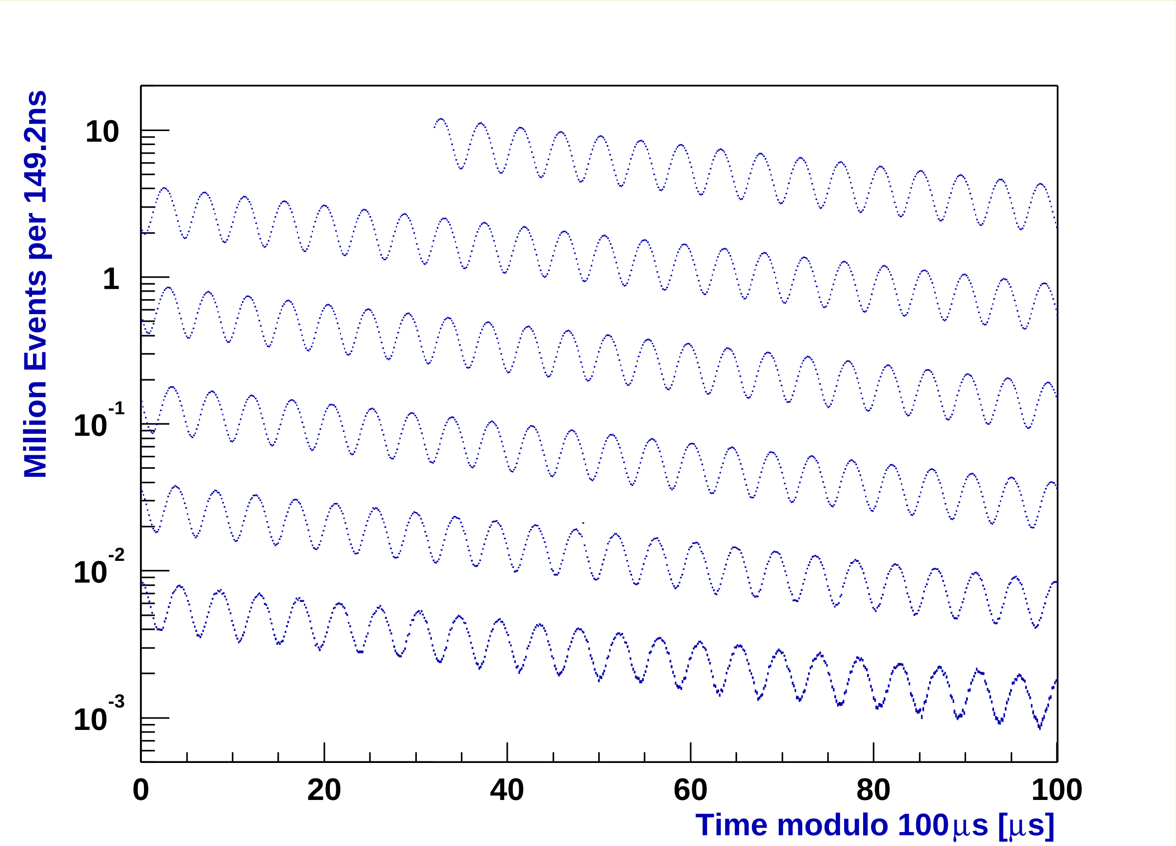
versus time

for the 3.6 billion decays

in the 2001

negative muon

data-taking period



BNL Result and Update

a_μ measured via a ratio of frequencies (measurement of a_μ and B)

$$B = \frac{\hbar\omega_p}{2\mu_p}\,,\; \omega_a = \frac{e a_\mu}{m_\mu c}\,B\,,\; \mu_\mu = (1+a_\mu)\,\frac{e\hbar}{2m_\mu c} \;\Leftrightarrow$$

$$\mu_\mu = (1+a_\mu)\,\frac{\hbar}{2}\,\frac{\omega_a}{a_\mu B} = \left(\frac{1}{a_\mu}+1\right)\,\frac{\omega_a}{\omega_p}\,\mu_p \Leftrightarrow$$

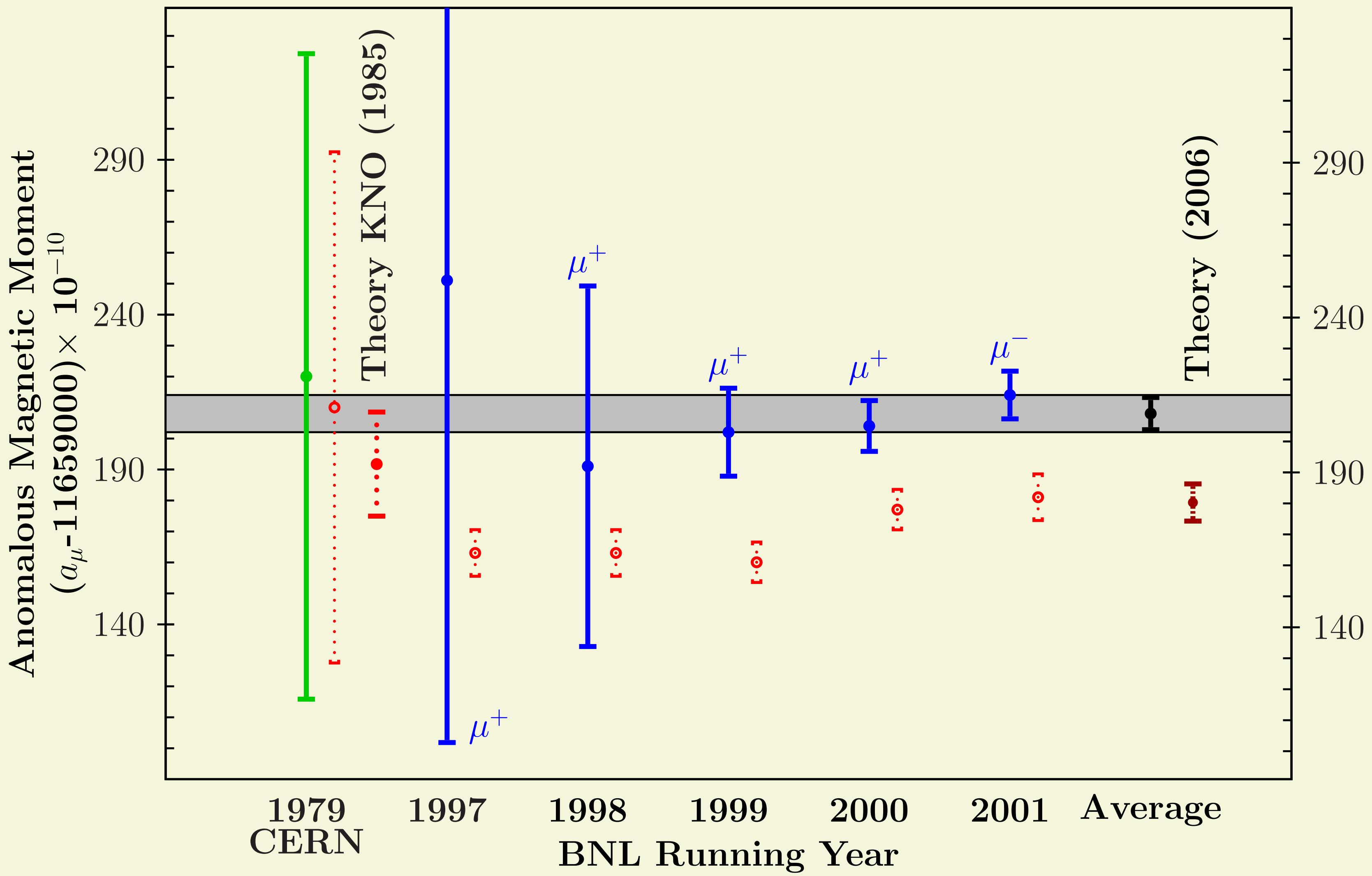
$$\boxed{a_\mu = \frac{\mathcal{R}}{\lambda-\mathcal{R}}}$$

- $\tilde{\omega}_p = (e/m_\mu)\langle B\rangle$ free proton NMR frequency
- $\mathcal{R} = \omega_a/\tilde{\omega}_p$ from E-821
- $\lambda = \omega_L/\tilde{\omega}_p = \mu_\mu/\mu_p$ from hyperfine splitting of muonium

value used by E-821	$3.18334539(10)$	
new value	$3.183345137(85)$	Mohr et al. RMP 80 (2008) 633

$\Rightarrow$ change in a_μ : $+0.92 \cdot 10^{-10}$ review in RPP2009

$$a_\mu^{\mathrm{exp}} = (11\,659\,208.9 \pm 5.4 \pm 3.3[6.3]) \cdot 10^{-10}$$ **updated**



Results of individual E821 measurements, together with last CERN result and theory values quoted by the experiments

② Standard Model Prediction for a_μ

❑ QED Contribution

The QED contribution to a_μ has been computed (or estimated) through 5 loops

Growing coefficients in the α/π expansion reflect the presence of large $\ln \frac{m_\mu}{m_e} \simeq 5.3$ terms coming from electron loops.

$a_e^{\rm exp} = 0.001\,159\,652\,180\,73(28)$ Gabrielse et al. 2008

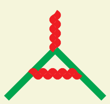
$\alpha^{-1}(a_e) = 137.0359991657(331)(68)(46)(24)[0.25 \text{ ppb}]$ Aoyama et al 2012

$$a_\mu^{\rm QED} = 116\,584\,718.851 \underbrace{(0.029)}_{\alpha_{\rm inp}} \underbrace{(0.009)}_{m_e/m_\mu} \underbrace{(0.018)}_{\alpha^4} \underbrace{(0.007)}_{\alpha^5} [0.36] \times 10^{-11}$$

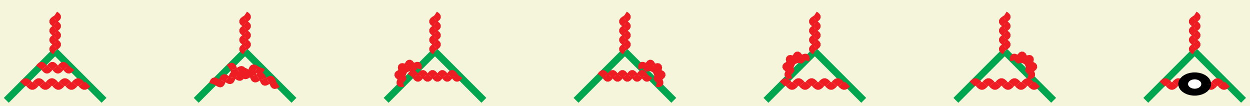
The current uncertainty is well below the $\pm 60 \times 10^{-11}$ experimental error from E821

# n of loops	$C_i\ [(\alpha/\pi)^n]$	$a_\mu^{\mathrm{QED}} \times 10^{11}$
1	+0.5	116140973.289 (43)
2	+0.765 857 426(16)	413217.628 (9)
3	+24.050 509 88(32)	30141.9023 (4)
4	+130.8796(63)	381.008 (18)
5	+753.290(1.04)	5.094 (7)
tot		116584718.851 (0.036)

❶ **1 diagram** **Schwinger 1948**



❷ **7 diagrams** **Peterman 1957, Sommerfield 1957**



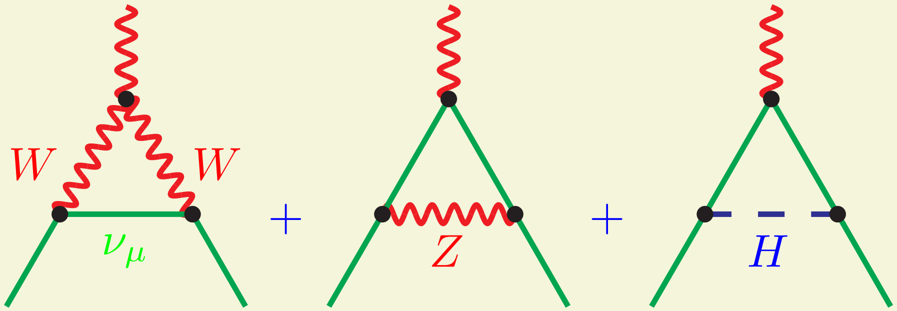
❸ **72 diagrams** **Lautrup, Peterman, de Rafael 1974, Laporta, Remiddi 1996**

❹ **about 1000 diagrams** **Kinoshita 1999, Kinoshita, Nio 2004, Aoyama et al. 2012**

❺ **estimates of leading terms** **Karshenboim 93, Czarnecki, Marciano 00, Kinoshita, Nio 05**

❏ **all 12672 diagrams (fully automated numerical) Aoyama et al. 2012**

❑ Weak Contributions

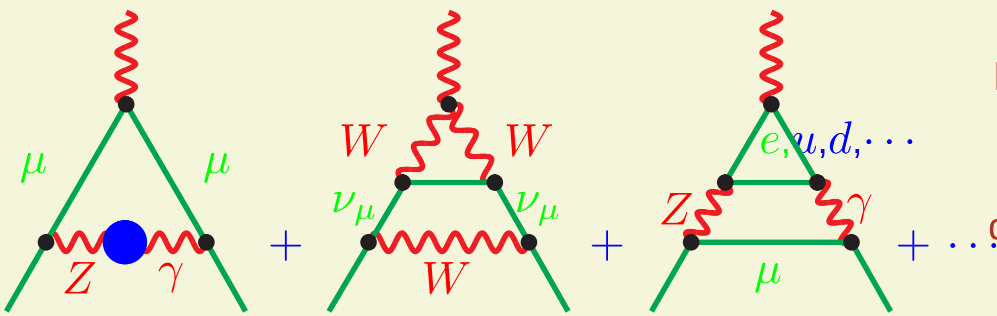


Brodsky, Sullivan 67, ...,

Bardeen, Gastmans, Lautrup 72

Higgs contribution tiny!

$$a_\mu^{\rm weak(1)} = (194.82 \pm 0.02) \times 10^{-11}$$



Kukhto et al 92

potentially large terms $\sim G_F m_\mu^2 \frac{\alpha}{\pi} \ln \frac{M_Z}{m_\mu}$

Peris, Perrottet, de Rafael 95

quark-lepton (triangle anomaly) cancellation

Czarnecki, Krause, Marciano 96

Heinemeyer, Stöckinger, Weiglein 04, Gribouk, Czarnecki 05 full 2–loop result

$$a_\mu^{\rm weak(2)} = (-42.08 \pm 1.5[m_H, m_t] \pm 1.0[\rm had]) \cdot 10^{-11}$$

Most recent evaluations: improved hadronic part (beyond QPM)

$$a_\mu^{\rm weak} = (154.0 \pm 1.0[\rm had] \pm 0.3[m_H, m_t, 3-loop]) \times 10^{-11}$$

New: M_H known!

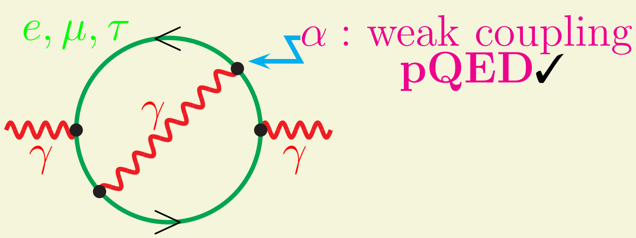
(Knecht, Peris, Perrottet, de Rafael 02, Czarnecki, Marciano, Vainshtein 02)

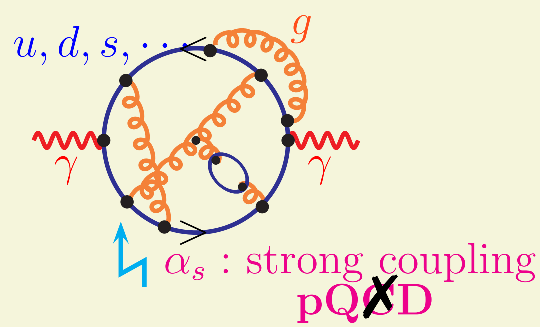
❑ Hadronic Contributions

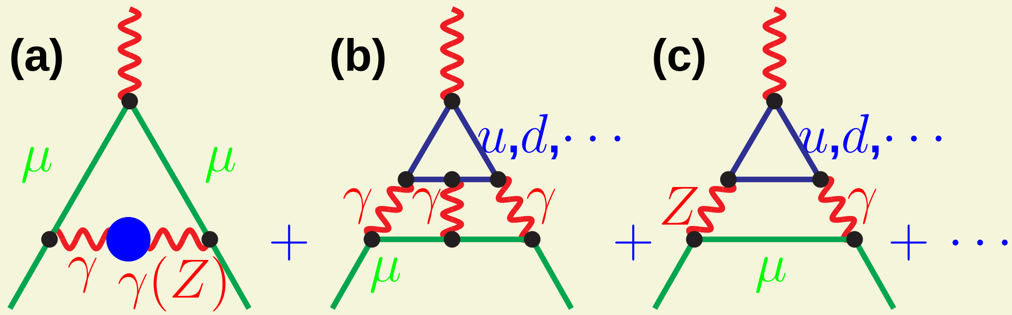
General problem in electroweak precision physics:

contributions from hadrons (quark loops) at low energy scales

Leptons **Quarks**







(a) Hadronic vacuum polarization $O(\alpha^2), O(\alpha^3)$

(b) Hadronic light-by-light scattering $O(\alpha^3)$

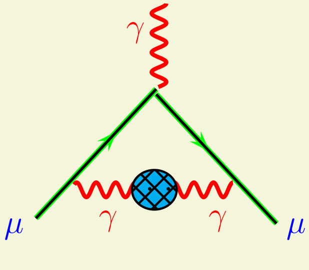
(c) Hadronic effects in 2-loop EWRC $O(\alpha G_F m_\mu^2)$

Light quark loops $\longrightarrow$ Hadronic "blob"

❑ Evaluation of $a_\mu^{\rm had}$

Leading non-perturbative hadronic contributions $a_\mu^{\rm had}$ can be obtained in terms of $R_\gamma(s) \equiv \sigma^{(0)}(e^+e^- \to \gamma^* \to \text{hadrons})/\frac{4\pi\alpha^2}{3s}$ data via dispersion integral:

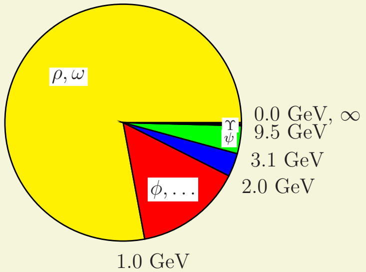
$$a_\mu^{\rm had} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left(\int\limits_{4m_\pi^2}^{E^2_{\rm cut}} ds \frac{R_\gamma^{\rm data}(s)\ \hat{K}(s)}{s^2} + \int\limits_{E^2_{\rm cut}}^{\infty} ds \frac{R_\gamma^{\rm pQCD}(s)\ \hat{K}(s)}{s^2} \right)$$

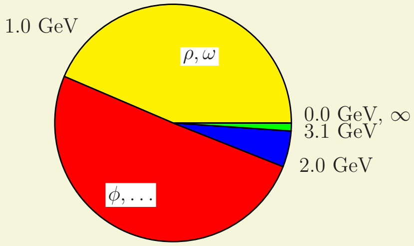


$$a_\mu^{\rm had(1)} = (690.7 \pm 4.7)[695.5 \pm 4.1]\ 10^{-10}$$

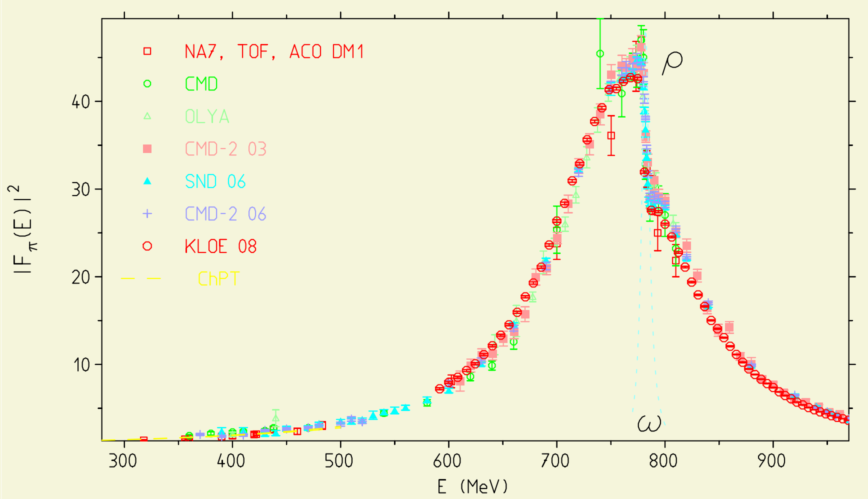
e^+e^-**–data based [incl. BaBar MD09]**

- **Experimental error implies theoretical uncertainty!**
- **Low energy contributions enhanced: $\sim 67\%$ of error on $a_\mu^{\rm had}$ comes from region $4m_\pi^2 < m_{\pi\pi}^2 < M_\Phi^2$**



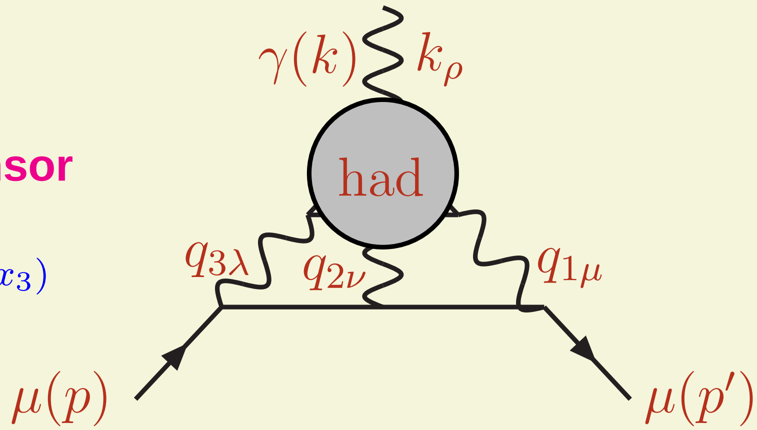


Data:CMD-2,SND,KLOE,BaBar



④ About the hadronic light-by-light scattering contribution

Hadrons in $\langle 0|T\{A^\mu(x_1)A^\nu(x_2)A^\rho(x_3)A^\sigma(x_4)\}|0\rangle$



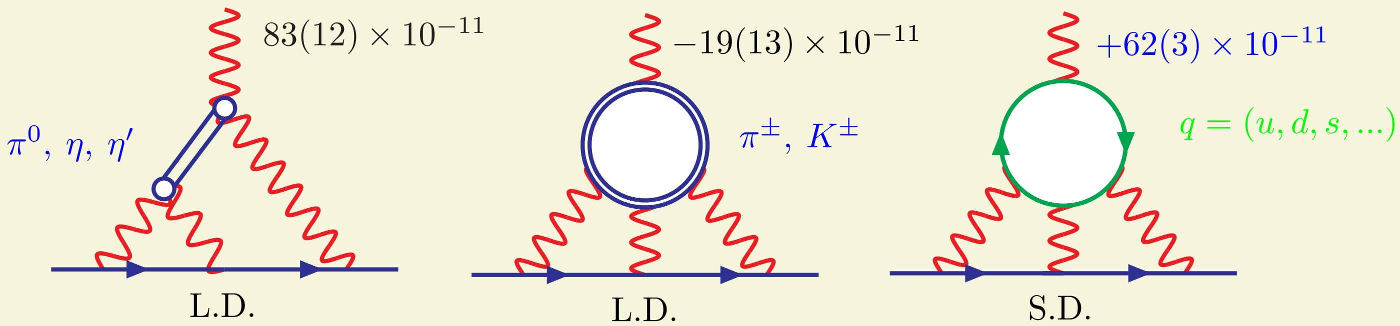
Key object full rank-four hadronic vacuum polarization tensor

$$\Pi_{\mu\nu\lambda\rho}(q_1,q_2,q_3) = \int \mathrm{d}^4x_1\,\mathrm{d}^4x_2\,\mathrm{d}^4x_3\,\mathrm{e}^{\mathrm{i}\,(q_1x_1+q_2x_2+q_3x_3)} \times\langle\,0\,|\,T\{j_\mu(x_1)j_\nu(x_2)j_\lambda(x_3)j_\rho(0)\}\,|\,0\,\rangle\ .$$

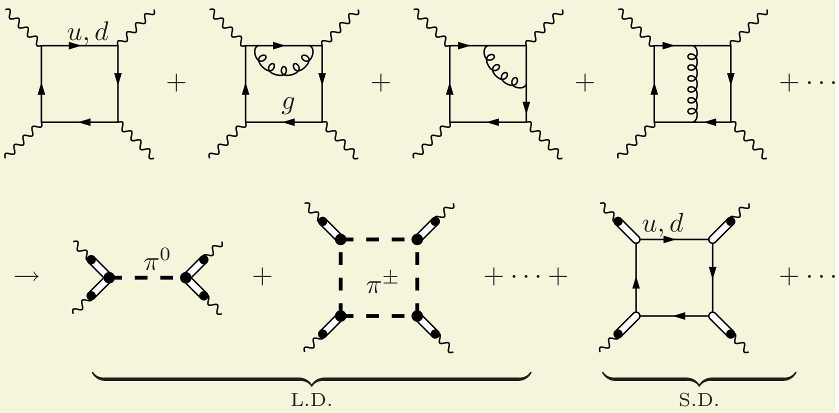
- **non-perturbative physics**
- **general covariant decomposition involves 138 Lorentz structures of which**
- **32 can contribute to $g-2$**
- **fortunately, dominated by the pseudoscalar exchanges $\pi^0,\eta,\eta',\ldots$ described by the effective Wess-Zumino Lagrangian**
- **generally, pQCD useful to evaluate the short distance (S.D.) tail**

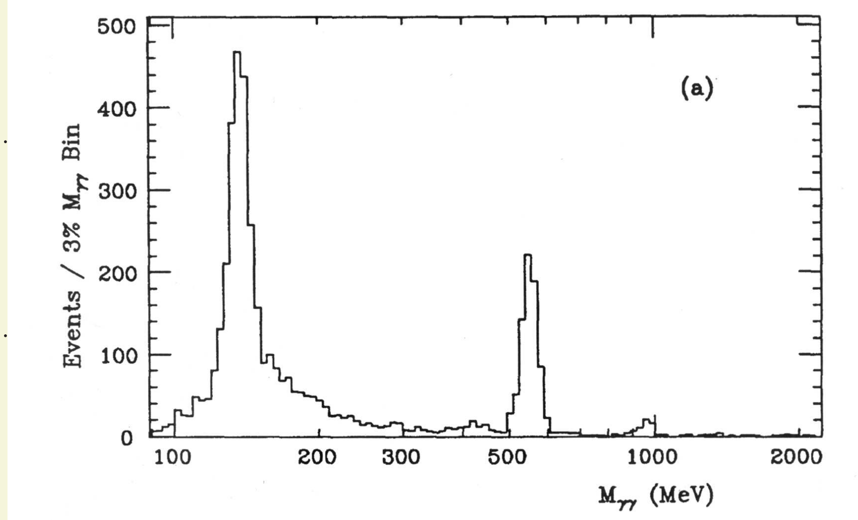
- the dominant long distance (L.D.) part must be evaluated using some low energy effective model which includes the pseudoscalar Goldstone bosons as well as the vector mesons which play a dominant role (vector meson dominance mechanism); HLS, ENJL, general RLA, large N_c inspired ansätze, and others

Need appropriate low energy effective theory $\Rightarrow$ amount to calculate the following type diagrams



LD contribution requires low energy effective hadronic models: simplest case $\pi^0\gamma\gamma$ vertex



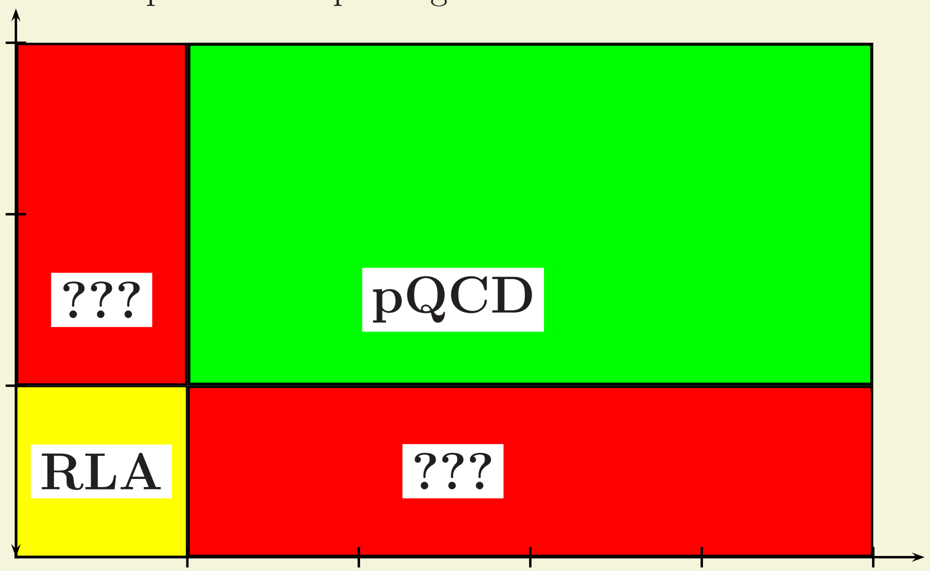


Data show almost background free spikes of the PS mesons! Substantial background form quark loop is absent. Clear message from data: fully non-perturbative, evidence for PS dominance. However, no information about axial mesons (Landau-Yang theorem). Illustrates how data can tell us where we are.

Low energy expansion in terms of hadronic components: theoretical models vs experimental data $\Rightarrow$ KLOE, KEDR, BES, BaBar, Belle, ?

Basic problem: (s, s_1, s_2)–domain of $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(s, s_1, s_2)$; here $(0, s_1, s_2)$–plane

Two scale problem: "open regions"



??? – Data, OPE,
– QCD factorization,
– Brodsky-Lepage approach

One scale problem: "no problem"

Novel approach: refer to quark–hadron duality of large-N_c QCD, hadron spectrum known, infinite series of narrow spin 1 resonances ('t Hooft 79) $\Rightarrow$ no matching problem (resonance representation has to match quark level representation)

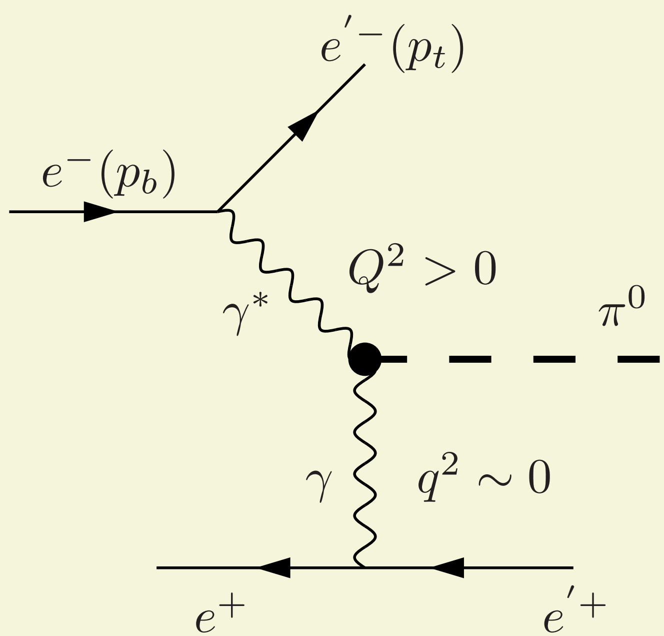
(De Rafael 94, Knecht, Nyffeler 02)

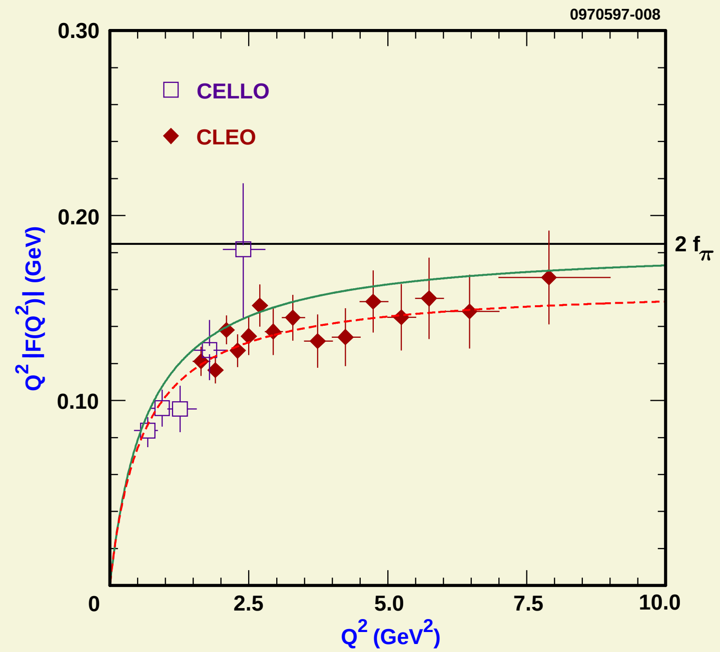
Constraints for on-shell pions (pion pole approximation)

❖ **General form–factor $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(s,s_1,s_2)$ is largely unknown**

- The constant $e^2\,\mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2,0,0)=\frac{e^2 N_c}{12\pi^2 f_\pi}=\frac{\alpha}{\pi f_\pi}\approx 0.025$ GeV^{-1} well determined by $\pi^0\to\gamma\gamma$ decay rate (from Wess-Zumino Lagrangian); experimental improvement needed!

- Information on $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0)$ from $e^+e^-\to e^+e^-\pi^0$ experiments





CELLO and CLEO measurement of the π^0 form factor $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0)$ at high space–like Q^2. outdated by BaBar? Belle conforms with theory expectations!

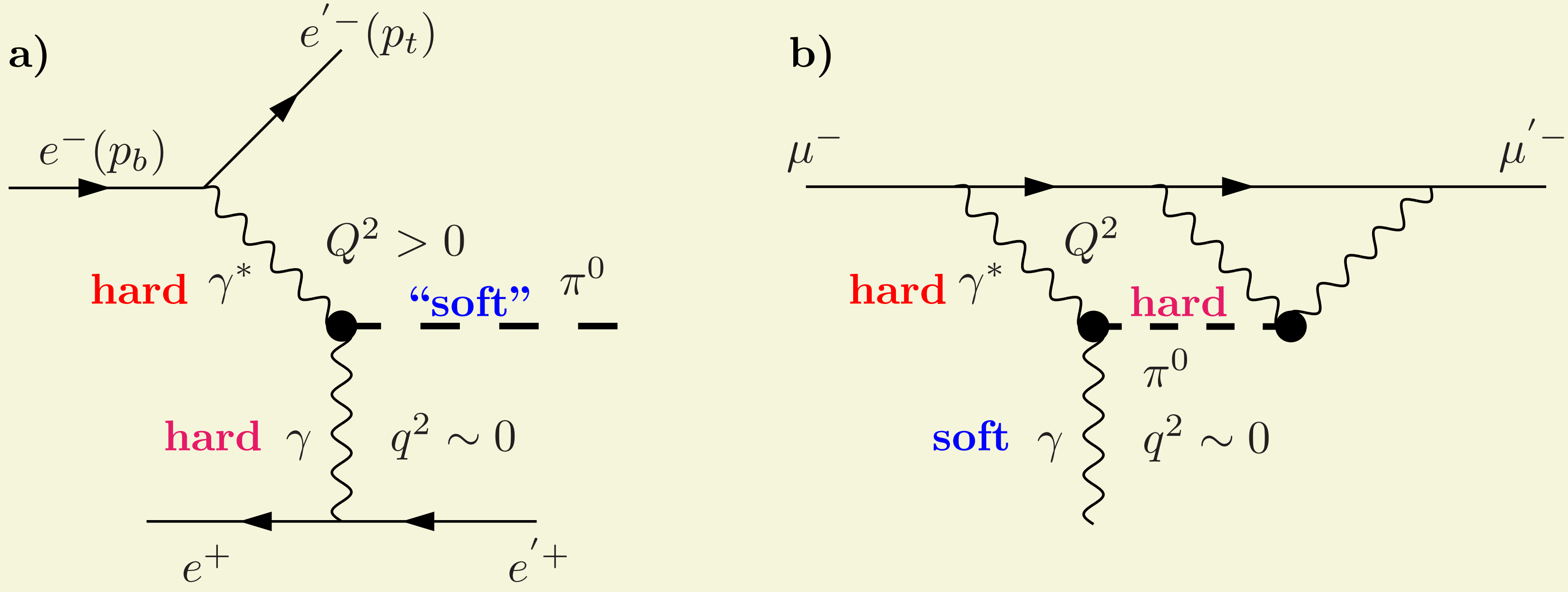
Brodsky–Lepage interpolating formula gives an acceptable fit.

$$\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0) \simeq \frac{1}{4\pi^2 f_\pi}\frac{1}{1+(Q^2/8\pi^2 f_\pi^2)} \sim \frac{2f_\pi}{Q^2}$$

Inspired by pion pole dominance idea this FF has been used mostly (HKS,BPP,KN) in the past, but has been criticized recently (MV and FJ07).

- Melnikov, Vainshtein: in chiral limit vertex with external photon must be non-dressed! i.e. use $\mathcal{F}_{\pi^0\gamma^*\gamma}(0,0,0)$, which avoids eventual kinematic inconsistency, thus no VMD damping $\Rightarrow$ result increases by **30%** !

- In $g-2$ external photon at zero momentum $\Rightarrow$ only $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(-Q^2,-Q^2,0)$ not $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0)$ is consistent with kinematics. Unfortunately, this off–shell form factor is not known and in fact not measurable and CELLO/CLEO constraint does not apply!. Obsolete far off-shell pion (in space-like region).



Measured is $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2, -Q^2, 0)$ at high space–like Q^2, needed at external vertex is $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(-Q^2, -Q^2, 0)$.

- I still claim using $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(0,0,0)$ in this case is not a reliable approximation!

Need realistic "model" for off–shell form–factor $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(-Q^2, -Q^2, 0)$!

Is it really to be identified with $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(0,0,0)$?

Can we check such questions experimentally or in lattice QCD?

Evaluation of $a_\mu^{\rm LbL}$ in the large-N_c framework

- Knecht & Nyffeler and Melnikov & Vainshtein were using pion-pole approximation together with large-N_c $\pi^0\gamma\gamma$–form-factor
- FJ & A. Nyffeler: relax from pole approximation, using KN off-shell LDM+V form-factor

$$
\begin{aligned}
\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(p_\pi^2, q_1^2, q_2^2) &= \frac{F_\pi}{3}\,\frac{\mathcal{P}(q_1^2, q_2^2, p_\pi^2)}{\mathcal{Q}(q_1^2, q_2^2)} \\
\mathcal{P}(q_1^2, q_2^2, p_\pi^2) &= h_7 + h_6\, p_\pi^2 + h_5\,(q_2^2 + q_1^2) + h_4\, p_\pi^4 + h_3\,(q_2^2 + q_1^2)\, p_\pi^2 \\
&\quad + h_2\, q_1^2\, q_2^2 + h_1\,(q_2^2 + q_1^2)^2 + q_1^2\, q_2^2\,(p_\pi^2 + q_2^2 + q_1^2)) \\
\mathcal{Q}(q_1^2, q_2^2) &= (q_1^2 - M_1^2)\,(q_1^2 - M_2^2)\,(q_2^2 - M_1^2)\,(q_2^2 - M_2^2)
\end{aligned}
$$

all constants are constraint by SD expansion (OPE). **Again, need data to fix parameters!**

- needed in HLBL amplitude in unphysical domain
- using hadronic model unavoidable
- mandatory: need experimental constraints in physical accessible region
  all possible variants of $\gamma\gamma \to {\rm hadrons}$

❖ looking for new ideas to get ride of model dependence

❑ Lattice QCD will provide an answer [far future ("yellow" region only)]!

My own calculation: $h_3 \in [-10, 10]$ **GeV**$^{-2}$

X	$a_\mu(\mathrm{LbL}; X) \times 10^{11}$				
π^0, η, η'	93.91 ± 12.40	a_1, f_1', f_1	28.13 ± 5.63	a_0, f_0', f_0	-5.98 ± 1.20

JN09 based on Nyffeler 09:

$$a_\mu^{\mathrm{LbL;had}} = (116 \pm 39) \cdot 10^{-11}$$

Summary of most recent results

Contribution	BPP	HKS	KN	MV	BP	PdRV	N/JN
π^0, η, η'	85 ± 13	82.7 ± 6.4	83 ± 12	114 ± 10	–	114 ± 13	99 ± 16
π, K **loops**	-19 ± 13	-4.5 ± 8.1	–	0 ± 10	–	-19 ± 19	-19 ± 13
axial vectors	2.5 ± 1.0	1.7 ± 1.7	–	22 ± 5	–	15 ± 10	22 ± 5
scalars	-6.8 ± 2.0	–	–	–	–	-7 ± 7	-7 ± 2
quark loops	21 ± 3	9.7 ± 11.1	–	–	–	2.3	21 ± 3
total	83 ± 32	89.6 ± 15.4	80 ± 40	136 ± 25	110 ± 40	105 ± 26	116 ± 39

❑ Theory vs Experiment; do we see New Physics?

Contribution	Value	Error	Reference
QED incl. 4-loops+5-loops	11 658 471.8851	0.036	Remiddi, Kinoshita ...
Leading hadronic vac. pol.	691.0	4.7	2011 update
Subleading hadronic vac. pol.	-9.974	0.086	2011 update
Hadronic light–by–light	11.6	3.9	evaluation (J&N 09)
Weak incl. 2-loops	15.40	0.10	CMV06
Theory	11 659 179.7	6.1	–
Experiment	11 659 209.1	6.3	BNL Updated
Exp.- The. 3.3 standard deviations	29.2	8.8	–

Standard model theory and experiment comparison [in units 10^{-10}]. What represents the 3 σ deviation: ❑ new physics? ❑ a statistical fluctuation? ❑ underestimating uncertainties (experimental, theoretical)?

❖ do experiments measure what theoreticians calculate?

The new muon $g-2$ experiments

Fermilab E989, J-PARC

- $\delta a_\mu = 16 \cdot 10^{-11}$ **by 2017**
- **Magnetic field:** $\frac{\delta \langle B \rangle_\mu}{\langle B \rangle_\mu} \lesssim 2 \cdot 10^{-8}$
- **Requires 10% error on HLbL**
- **Improving HVP** $\sigma(e^+e^- \to \mathrm{hadrons})$ **in progress**

Present:

- $a_\mu^{\rm exp} = 116\,592\,089(63) \cdot 10^{-11}$; $a_\mu^{\rm SM} = 116\,591\,793 \pm 51 \cdot 10^{-11}$

E989: statistics $21\times$ **; total error factor** 4 **more precise**

$$\left.\begin{array}{l} \sigma_{\rm stat} = 0.1 \ \mathrm{ppm} \\ \sigma_{\rm syst} = 0.1 \ \mathrm{ppm} \end{array}\right\} \sigma_{\rm tot} = 0.14 \ \mathrm{ppm}$$

- $a_\mu^{\rm exp} = 116\,59x\,xxx(16) \cdot 10^{-11}$

The challenge:

$a_\mu^{\text{had,VP}}[LO]$	$(6923 \pm 42) \times 10^{-11}$	+58.82 $\pm$0.36 ppm
$a_\mu^{\text{had,VP}}[NLO]$	$(-98 \pm 1) \times 10^{-11}$	
a_μ^{EW}	$(154 \pm 1) \times 10^{-11}$	
$a_\mu^{\text{had,LbL}}$	$[(105 \div 115) \pm (26 \div 40)] \times 10^{-11}$	+0.90 $\pm$0.22 ppm
$\delta a_\mu^{\rm exp}$ present	63×10^{-11}	$\pm$0.54 ppm
$\delta a_\mu^{\rm exp}$ future	16×10^{-11}	$\pm$0.14 ppm

Next generation experiments require a factor 4 reduction of the uncertainty

optimistically feasible is factor 2 we hope

Most urgent R-Measurements above 1GeV, including VEPP 2000 and BES-III

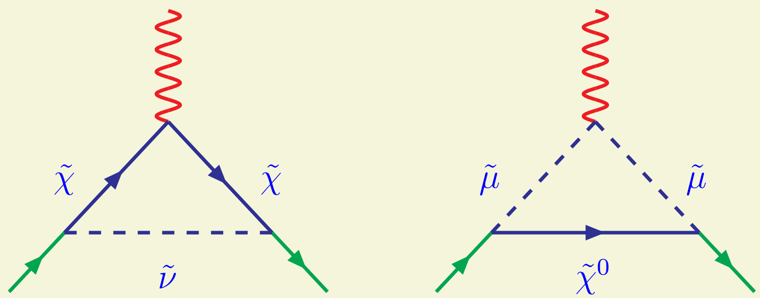
New physics sensitivity: **(example)**

$$\Delta a_\mu^{\rm SUSY}/a_\mu \simeq 1.25 {\rm ppm} \left(\frac{100\mathbf{GeV}}{\tilde{m}}\right)^2 \tan\beta$$

$\tilde{m}$ **lightest SUSY particle; SUSY requires two Higgs doublets**

$\tan\beta = \frac{v_1}{v_2}$, $v_i =< H_i > \;\; ; \;\; i = 1,2$

$\tan\beta \sim m_t/m_b \sim 40 \;\; [4-40]$



The supersymmetric contributions to a_μ stem from smuon–neutralino and sneutrino-chargino loops

- **muon $g-2$ in contrast requires moderately light SUSY masses and in the pre-LHC era fitted rather well with expectations from SUSY**

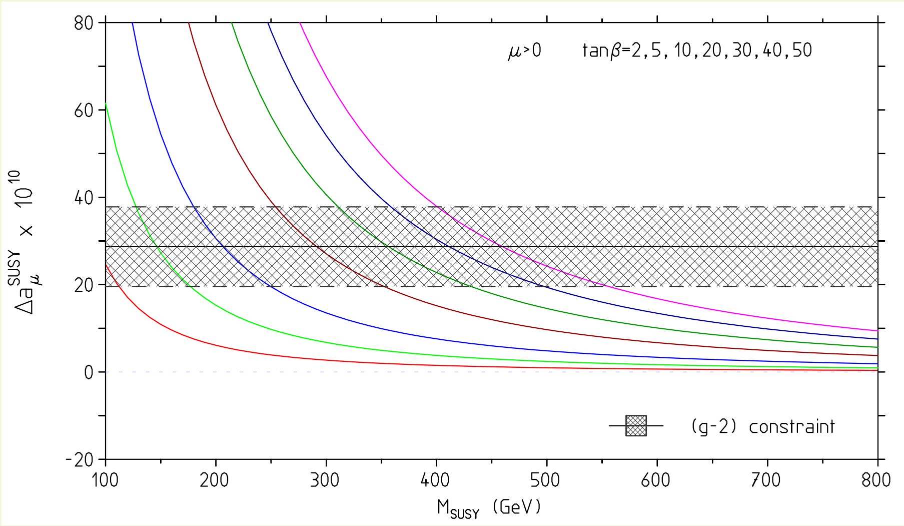
- **a particular role is played by the mass of the light Higgs**

At tree level in the MSSM $m_h \leq M_Z$. This bound receives large radiative corrections from the $t/\tilde{t}$ sector, which changes the upper bound to ((Haber & Hempfling 1990))

$$m_h^2 \sim M_Z^2 \cos^2 2\beta + \frac{3\sqrt{2} G_\mu m_t^4}{2\pi^2 \sin^2\beta} \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) + \cdots$$

which in any case is well below 200 GeV. A given value of m_h fixes the value of $m_{1/2}$ represented by $\{m_{\tilde{t}_1}, m_{\tilde{t}_2}\}$

- **if Higgs is established at 125 GeV (LHC/CERN) we must have $m_{1/2} > 800 GeV$ or higher!**



Constraint on large $\tan\beta$ SUSY contributions as a function of $M_{\rm SUSY}$. The horizontal band shows $\Delta a_\mu^{\rm NP} = \delta a_\mu$. The region left of $M_{\rm SUSY} \sim$ 500 GeV is excluded by LHC searches. If $m_h \sim 125 \pm 1.5$ GeV actually $M_{\rm SUSY} > 800$ GeV depending on details of the stop sector ($\{\tilde{t}_1, \tilde{t}_2\}$ mixing and mass splitting) and weakly on $\tan\beta$.

New physics typically:

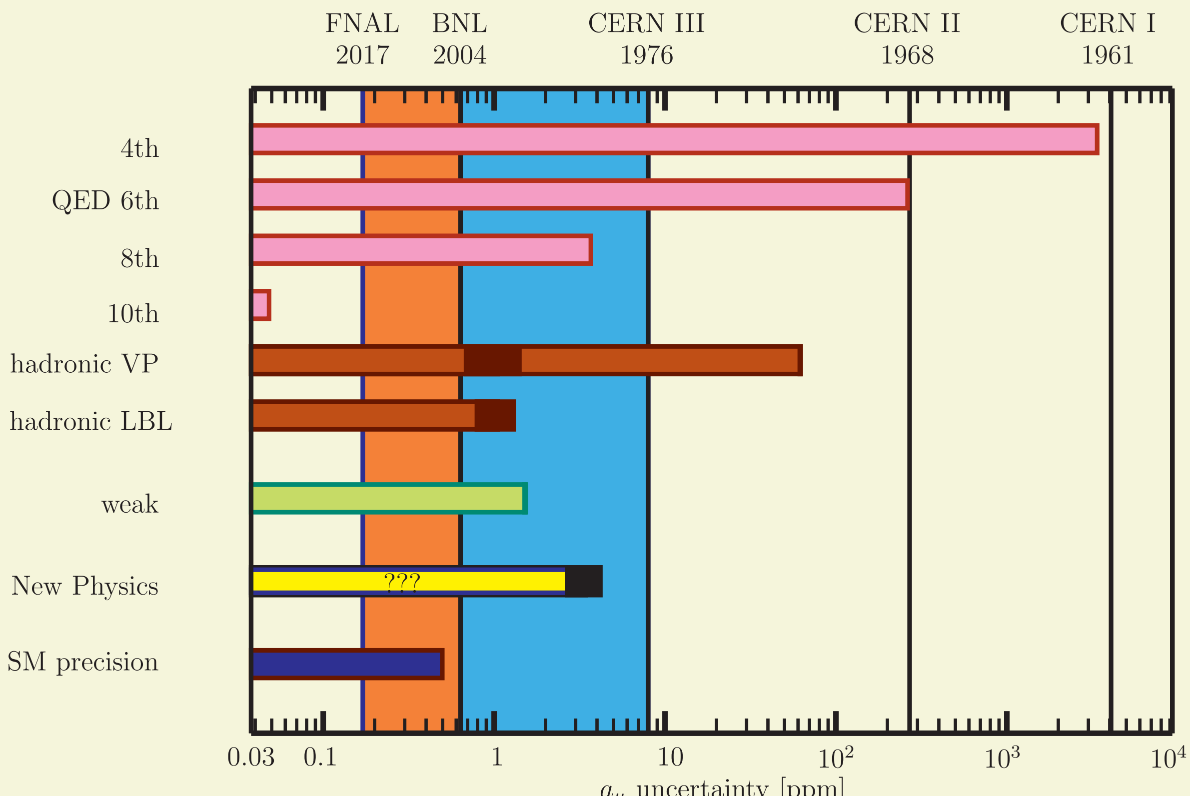
$$a_\mu^{\rm NP} = \mathcal{C}\,\frac{m_\mu^2}{M_{\rm NP}^2}$$

where naturally $\mathcal{C} = O(\alpha/\pi)$ **(**$\sim$ **lowest order** $a_\mu^{\rm SM}$**);**

Typical New Physics scales required to satisfy $\Delta a_\mu^{\rm NP} = \delta a_\mu$**:**

$\mathcal{C}$	**1**	α/π	$(\alpha/\pi)^2$
$M_{\rm NP}$	$2.0^{+0.4}_{-0.3}$ **TeV**	100^{+21}_{-13} **GeV**	5^{+1}_{-1} **GeV**

The Muon $g-2$
"the closer you look the more there is to see"



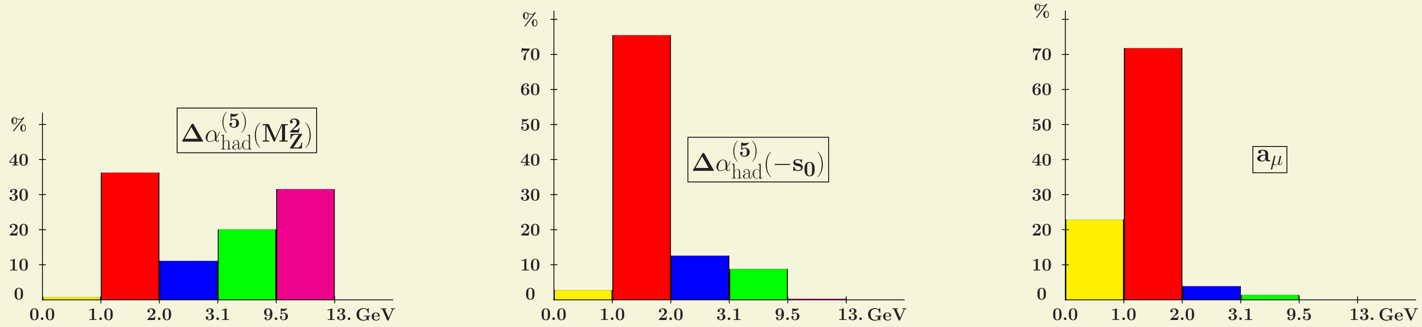
Sensitivity of $g-2$ experiments to various contributions. New Physics illustrated by deviation $(a_\mu^{\rm exp}-a_\mu^{\rm the})/a_\mu^{\rm exp}$

The anomalous magnetic moment of the muon by itself a tiny 0.116 % effect now measured at 5 $\cdot 10^{-7}$!

❑ The Future

- We are hoping for follow up experiment at Fermilab/USA or JPAC/Japan
- Improved hadronic VP needed (dominating present theory error)
  - Experimental program still urgently needed: VEPP-2000, BES III, Belle
  - Lattice QCD will provide results within a few years to cross check and hopefully improve hadronic VP calculations up to 2 GeV
- Hadronic LbL the touchstone for theory :
  - Progress possible: sort out “the” realistic resonance Lagrangian [as the true low energy effective version of QCD] by global fit strategies
  - Lattice QCD calculations can provide in steps important results to cross check model calculations [very long term project]

Comparison of error profiles between $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$, $\Delta\alpha^{(5)}_{\rm had}(-s_0)$ and a_μ:



Long term goal: 1% precision up to 12 GeV

Thanks you for your attention! Thanks to ETC* for the great hospitality

Announcement of MITP Workshop on muon g-2:

Hadronic contributions to the muon anomalous magnetic moment:

strategies for improvements of the accuracy of the theoretical prediction

Coordinators/Organizers: Tom Blum (University of Conneticut), Achim Denig (JGU Mainz), Simon Eidelman (INP Novosibirsk), Fred Jegerlehner (HU Berlin), Dominik Stöckinger (TU Dresden), Marc Vanderhaeghen (JGU Mainz)

March 31 - April 12, 2014 at Johannes Gutenberg University Mainz

in conjunction with SFB Collaboration Meeting (2nd week)