ARE ISOSPIN CORRECTIONS IN $\tau^- \longrightarrow \pi^-\pi^0\nu_\tau$ UNDERSTOOD?

Pablo Roig (IFAE, Barcelona)

Based on D. Gómez Dumm (U. La Plata), P. Roig (IFAE) arXiv:1301.6973 and work in progress

To be implemented in TAUOLA [PRD86 (2012) 113008] thanks to Olga, Tomasz and Zbigniew

XIIIth WG meeting on Radiative Corrections and MC Generators for Low Energies
Trento 11-12 April 2013

CONTENTS

INTRODUCTION

• The di-pion tau decay is the most frequent tau decay mode (**BR~25.5%**).

• It provides an ideal place to study the properties of the **ρ(770)** resonance [Also ρ(1450) and ρ(1700) can be studied]. The relevant form factor is very relevant to understand the hadronization of QCD currents at low energies (**chiral dynamics**)

• It allows to obtain an alternative evaluation of the $\pi\pi$ contribution to the anomalous magnetic moment of the muon, $\mathbf{a}_\mu$.

• These decays (together with $\tau^- \longrightarrow \pi^-\pi^-\pi^+\nu_\tau$, see Olga's talk) are fundamental in studies of **spin-parity** of the **Higgs** boson discovered at LHC.

INTRODUCTION

• The di-pion tau decay is the most frequent tau decay mode (**BR~25.5%**).

• It provides an ideal place to study the properties of the **ρ(770)** resonance [Also ρ(1450) and ρ(1700) can be studied]. The relevant form factor is very relevant to understand the hadronization of QCD currents at low energies (**chiral dynamics**)

• It allows to obtain an alternative evaluation of the ππ contribution to the anomalous magnetic moment of the muon, $\mathbf{a}_\mu$.

• These decays (together with $\tau^- \longrightarrow \pi^-\pi^-\pi^+\nu_\tau$, see Olga's talk) are fundamental in studies of **spin-parity** of the **Higgs** boson discovered at LHC.

⇒ The data (Belle '08) are so precise that **isospin breaking** corrections become important.

ARE ISOSPIN CORRECTIONS IN $\tau^- \longrightarrow \pi^-\pi^0\nu_\tau$ UNDERSTOOD?

DISPERSIVE REPRESENTATION OF THE RELEVANT FORM FACTOR

$$\frac{d\Gamma(\tau^- \to \pi^0\pi^-\nu_\tau)}{dt} = \frac{\Gamma_e^{(0)} S_{\rm EW}|V_{ud}|^2}{2m_\tau^2}\beta_{\pi^0\pi^-}(t)\,(1-\frac{t}{m_\tau^2})^2\left\{|f_+(t)|^2\left[(1+\frac{2t}{m_\tau^2})\beta^2_{\pi^0\pi^-}(t) + \frac{3\Delta_\pi^2}{t^2}\right] + 3|f_-(t)|^2 - 6{\rm Re}\,[f_+^*(t)f_-(t)]\frac{\Delta_\pi}{t}\right\} G_{\rm EM}(t) \qquad (4)$$

It **vanishes** even including isospin breaking corrections to first order (Cirigliano, Ecker, Neufeld '01)

$$\Gamma_e^{(0)} = \frac{G_F^2 m_\tau^5}{192\pi^3}\ ,\ \Delta_\pi = M_{\pi^+}^2 - M_{\pi^0}^2\ ,\ \beta_{\pi^0\pi^-}(t) = \lambda^{1/2}(1, M_{\pi^0}^2/t, M_{\pi^+}^2/t)$$

DISPERSIVE REPRESENTATION OF THE RELEVANT FORM FACTOR

$$\frac{d\Gamma(\tau^- \to \pi^0\pi^-\nu_\tau)}{dt} = \frac{\Gamma_e^{(0)} S_{\rm EW}|V_{ud}|^2}{2m_\tau^2}\beta_{\pi^0\pi^-}(t)\,(1-\frac{t}{m_\tau^2})^2\left\{|f_+(t)|^2\left[(1+\frac{2t}{m_\tau^2})\beta^2_{\pi^0\pi^-}(t) + \frac{3\Delta_\pi^2}{t^2}\right] + 3|f_-(t)|^2 - 6{\rm Re}\,[f_+^*(t)f_-(t)]\frac{\Delta_\pi}{t}\right\}G_{\rm EM}(t) \qquad (4)$$

It **vanishes** even including isospin breaking corrections to first order (Cirigliano, Ecker, Neufeld ‘01)

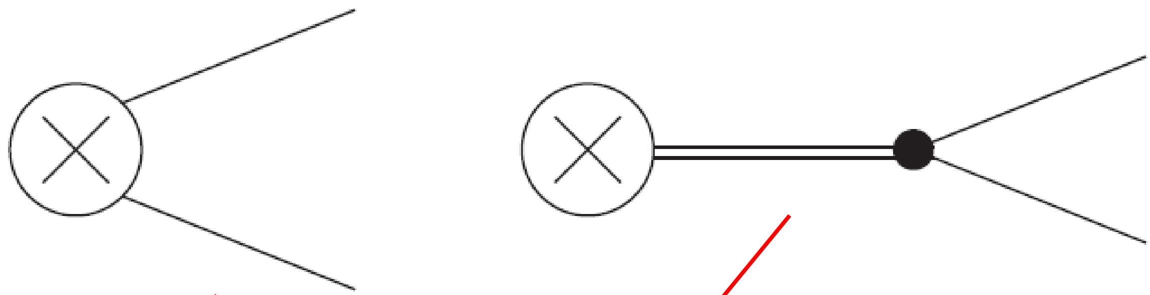
$$\frac{d\Gamma(\tau^- \to \pi^-\pi^0\nu_\tau)}{ds} = \frac{G_F^2\, m_\tau^3}{384\,\pi^3} S_{EW}\,|V_{ud}|^2 \left(1-\frac{s}{M_\tau^2}\right)^2\left(1+\frac{2s}{M_\tau^2}\right) \lambda^{3/2}\left(1,\frac{m_{\pi^0}^2}{s},\frac{m_{\pi^+}^2}{s}\right)|f_+(s)|^2\, G_{EM}(s)\ ,$$

Only one relevant form factor: Vector Form Factor

DISPERSIVE REPRESENTATION OF THE VFF

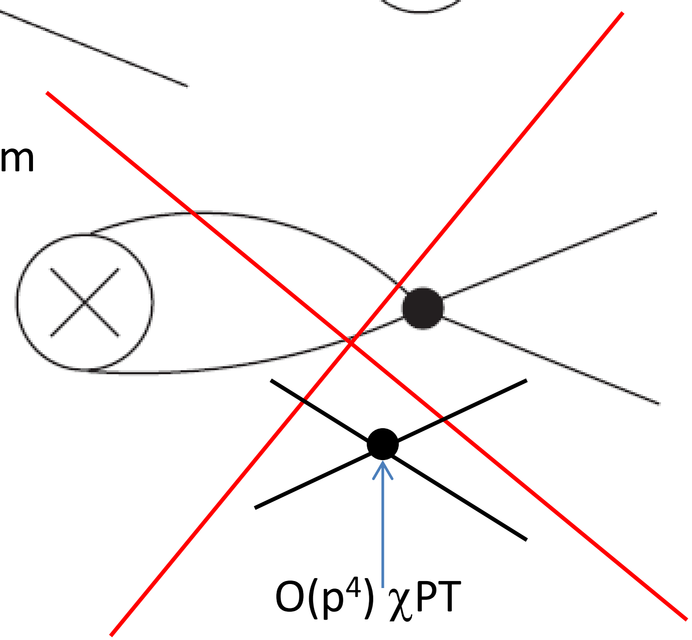
$$\langle \pi^-(p_{\pi^-})\pi^0(p_{\pi^0})|\bar{d}\gamma^\mu u|0\rangle = \sqrt{2}(p_{\pi^-} - p_{\pi^0})^\mu F_V^\pi(s)$$

Guerrero, Pich ’97



Antisymmetric tensor formalism for spin-one resonances

Ecker *et al.* ’89

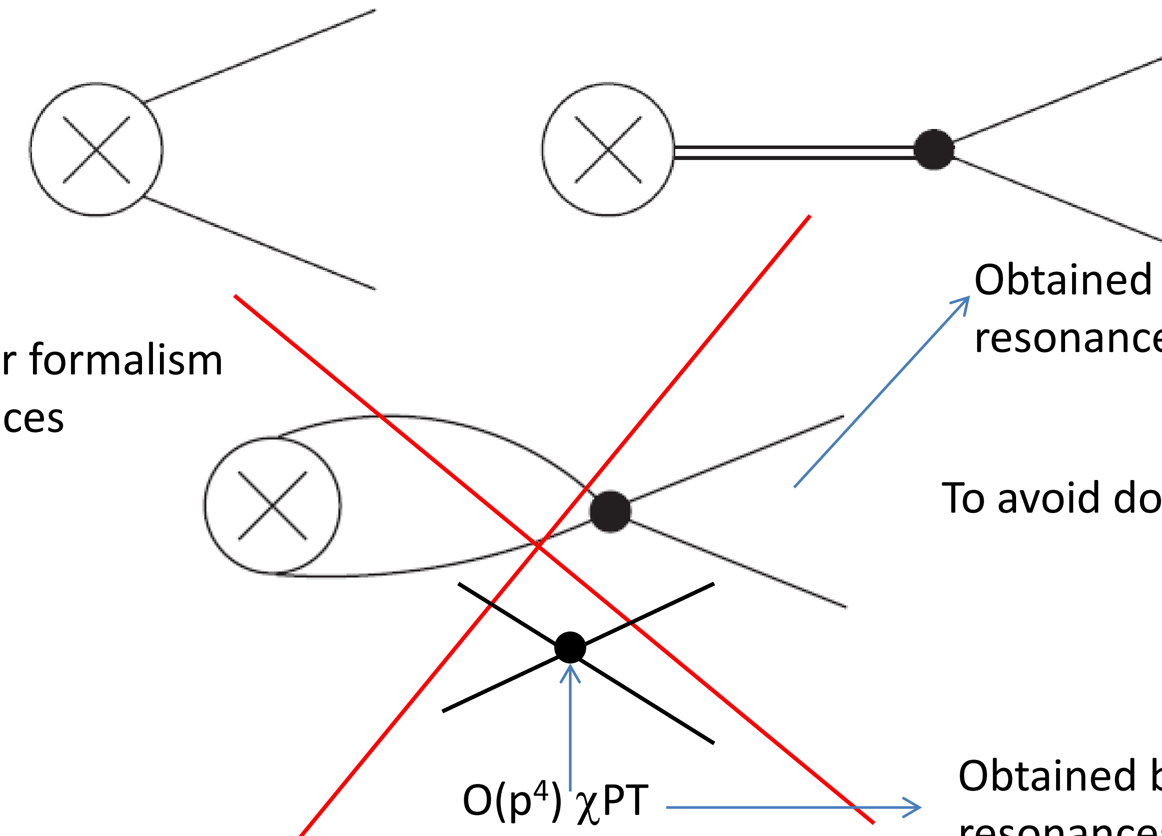


To avoid double counting

DISPERSIVE REPRESENTATION OF THE VFF

$$\langle \pi^-(p_{\pi^-})\pi^0(p_{\pi^0})|\bar{d}\gamma^\mu u|0\rangle = \sqrt{2}(p_{\pi^-} - p_{\pi^0})^\mu F_V^\pi(s)$$

Guerrero, Pich '97



Obtained via the resonances width

Antisymmetric tensor formalism for spin-one resonances

Ecker *et al.* '89

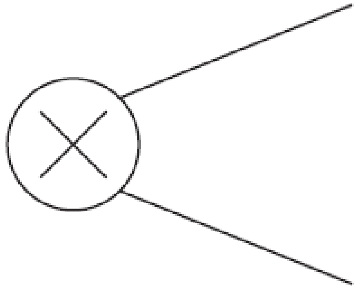
To avoid double counting

Obtained by integrating resonances out

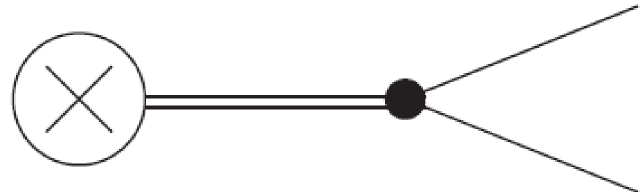
DISPERSIVE REPRESENTATION OF THE VFF

$$\langle \pi^-(p_{\pi^-})\pi^0(p_{\pi^0})|\bar{d}\gamma^\mu u|0\rangle = \sqrt{2}(p_{\pi^-} - p_{\pi^0})^\mu F_V^\pi(s)$$

Guerrero, Pich ’97



$$\mathcal{L}_\chi^{(2)} = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle$$



$$\mathcal{L}_2[V(1^{--})] = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{i\,G_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle$$

$$u_\mu = i\left\{u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu)u^\dagger\right\}$$

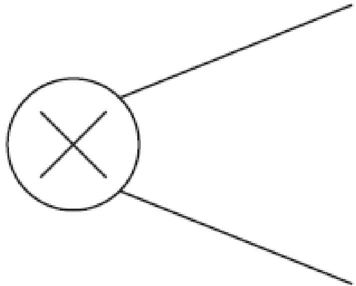
$$f_\pm^{\mu\nu} = u\,F_L^{\mu\nu}\,u^\dagger \pm u^\dagger\,F_R^{\mu\nu}\,u$$

$$u(\varphi) = \exp\left\{i\frac{\Phi}{\sqrt{2}\,F}\right\} \quad \Phi(x) \equiv \frac{1}{\sqrt{2}}\sum_{a=1}^{8}\lambda_a\,\varphi_a = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}$$

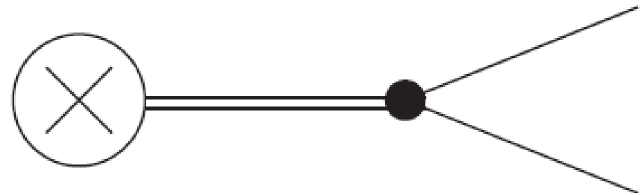
DISPERSIVE REPRESENTATION OF THE VFF

$$\langle \pi^-(p_{\pi^-})\pi^0(p_{\pi^0})|\bar{d}\gamma^\mu u|0\rangle = \sqrt{2}(p_{\pi^-}-p_{\pi^0})^\mu F_V^\pi(s)$$

Guerrero, Pich '97



$$\mathcal{L}_\chi^{(2)} = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle$$



$$\mathcal{L}_2[V(1^{--})] = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{i\,G_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle$$

$$u_\mu = i\left\{u^\dagger(\partial_\mu - i r_\mu)u - u(\partial_\mu - i l_\mu)u^\dagger\right\}$$

$$f_\pm^{\mu\nu} = u\,F_L^{\mu\nu}u^\dagger \pm u^\dagger F_R^{\mu\nu}u$$

$$u(\varphi) = \exp\left\{i\frac{\Phi}{\sqrt{2}F}\right\} \quad \Phi(x) \equiv \frac{1}{\sqrt{2}}\sum_{a=1}^{8}\lambda_a\varphi_a = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}$$

$$F(s)^V = 1 + \frac{F_V G_V}{f_\pi^2}\frac{s}{M_\rho^2 - s}$$

Short-distance constraints

F(s) →0, for s→∞ $\Rightarrow F_V\,G_V = F^2$

$$F(s)^{\mathrm{VMD}} = \frac{M_\rho^2}{M_\rho^2 - s}$$

DISPERSIVE REPRESENTATION OF THE VFF

$$\langle \pi^-(p_{\pi^-})\pi^0(p_{\pi^0})|\overline{d}\gamma^\mu u|0\rangle = \sqrt{2}(p_{\pi^-} - p_{\pi^0})^\mu F_V^\pi(s)$$

$$F(s)^{\mathrm{VMD}} = \frac{M_\rho^2}{M_\rho^2 - s}$$

Guerrero, Pich '97

$$F(s)^{\mathrm{ChPT}}_{O(p^4)} = 1 + \frac{2L_9^r(\mu)}{f_\pi^2} s - \frac{s}{96\pi^2 f_\pi^2}\left[A(m_\pi^2/s, m_\pi^2/\mu^2) + \frac{1}{2}A(m_K^2/s, m_K^2/\mu^2)\right]$$

$$A(m_P^2/s, m_P^2/\mu^2) = \ln\left(m_P^2/\mu^2\right) + \frac{8m_P^2}{s} - \frac{5}{3} + \sigma_P^3 \ln\left(\frac{\sigma_P + 1}{\sigma_P - 1}\right) \qquad \sigma_P \equiv \sqrt{1 - 4m_P^2/s}$$

DISPERSIVE REPRESENTATION OF THE VFF

$$\langle \pi^-(p_{\pi^-})\pi^0(p_{\pi^0})|\bar{d}\gamma^\mu u|0\rangle = \sqrt{2}(p_{\pi^-} - p_{\pi^0})^\mu F_V^\pi(s)$$

$$F(s)^{\mathrm{VMD}} = \frac{M_\rho^2}{M_\rho^2 - s}$$

Guerrero, Pich '97

$$F(s)^{\mathrm{ChPT}}_{O(p^4)} = 1 + \frac{2L_9^r(\mu)}{f_\pi^2}s - \frac{s}{96\pi^2 f_\pi^2}\left[A(m_\pi^2/s, m_\pi^2/\mu^2) + \frac{1}{2}A(m_K^2/s, m_K^2/\mu^2)\right]$$

$$A(m_P^2/s, m_P^2/\mu^2) = \ln\left(m_P^2/\mu^2\right) + \frac{8m_P^2}{s} - \frac{5}{3} + \sigma_P^3 \ln\left(\frac{\sigma_P + 1}{\sigma_P - 1}\right) \qquad \sigma_P \equiv \sqrt{1 - 4m_P^2/s}$$

ChPT+VMD

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s} - \frac{s}{96\pi^2 f_\pi^2}\left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}A(m_K^2/s, m_K^2/M_\rho^2)\right]$$

DISPERSIVE REPRESENTATION OF THE VFF

ChPT+VMD Guerrero, Pich '97

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s} - \frac{s}{96\pi^2 f_\pi^2}\left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}A(m_K^2/s, m_K^2/M_\rho^2)\right]$$

Unitarity+Analiticity Omnés, '58

DISPERSIVE REPRESENTATION OF THE VFF

ChPT+VMD Guerrero, Pich '97

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s} - \frac{s}{96\pi^2 f_\pi^2}\left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}A(m_K^2/s, m_K^2/M_\rho^2)\right]$$

Unitarity+Analiticity Omnés, '58

$O(p^2)$ result for $\delta^1{}_1$ (s)

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s} \exp\left\{\frac{-s}{96\pi^2 f^2}\left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}A(m_K^2/s, m_K^2/M_\rho^2)\right]\right\}$$

DISPERSIVE REPRESENTATION OF THE VFF

ChPT+VMD Guerrero, Pich '97

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s} - \frac{s}{96\pi^2 f_\pi^2}\left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}A(m_K^2/s, m_K^2/M_\rho^2)\right]$$

Unitarity+Analiticity Omnés, '58

$O(p^2)$ result for $\delta^1_1(s)$

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s} \exp\left\{\frac{-s}{96\pi^2 f^2}\left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}A(m_K^2/s, m_K^2/M_\rho^2)\right]\right\}$$

Guerrero, Pich '97

$$\Gamma_\rho(s) = \frac{M_\rho s}{96\pi f_\pi^2}\left\{\theta(s - 4m_\pi^2)\,\sigma_\pi^3 + \frac{1}{2}\theta(s - 4m_K^2)\,\sigma_K^3\right\}$$

Gómez-Dumm, Pich, Portolés '00

$$= -\frac{M_\rho s}{96\pi^2 f_\pi^2}\,\mathrm{Im}\left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}A(m_K^2/s, m_K^2/M_\rho^2)\right]$$

DISPERSIVE REPRESENTATION OF THE VFF

ChPT+VMD Guerrero, Pich '97

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s} - \frac{s}{96\pi^2 f_\pi^2}\left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}A(m_K^2/s, m_K^2/M_\rho^2)\right]$$

Unitarity+Analiticity Omnés, '58

O(p^2) result for δ^1_1 (s)

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s}\exp\left\{\frac{-s}{96\pi^2 f^2}\left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}A(m_K^2/s, m_K^2/M_\rho^2)\right]\right\}$$

Guerrero, Pich '97

Gómez-Dumm, Pich, Portolés '00

$$\begin{aligned}\Gamma_\rho(s) &= \frac{M_\rho\, s}{96\pi f_\pi^2}\left\{\theta(s-4m_\pi^2)\,\sigma_\pi^3 + \frac{1}{2}\theta(s-4m_K^2)\,\sigma_K^3\right\}\\ &= -\frac{M_\rho\, s}{96\pi^2 f_\pi^2}\,\mathrm{Im}\left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}A(m_K^2/s, m_K^2/M_\rho^2)\right]\end{aligned}$$

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)}\exp\left\{\frac{-s}{96\pi^2 f_\pi^2}\left[\mathrm{Re}A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}\mathrm{Re}A(m_K^2/s, m_K^2/M_\rho^2)\right]\right\}$$

DISPERSIVE REPRESENTATION OF THE VFF

Starting point

Guerrero, Pich '97

Match χPT results to VMD using an Omnés solution for dispersion relation

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left\{ \frac{-s}{96\pi^2 f_\pi^2} \left[\mathrm{Re}A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}\mathrm{Re}A(m_K^2/s, m_K^2/M_\rho^2) \right] \right\}$$

DISPERSIVE REPRESENTATION OF THE VFF

Starting point

Guerrero, Pich '97

Match χPT results to VMD using an Omnés solution for dispersion relation

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left\{\frac{-s}{96\pi^2 f_\pi^2}\left[\mathrm{Re}A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}\mathrm{Re}A(m_K^2/s, m_K^2/M_\rho^2)\right]\right\}$$

- χPT up to $O(p^4)$ and leading $O(p^6)$ contributions Guerrero '98
- Right fall-off at high energies
- SU(2)
- Analiticity and unitarity constraints (NNLO)

Idea: Follow the approach of Boito, Escribano, Jamin '08 preserving analiticity and unitarity exactly using a dispersive representation of the VFF while retaining (some of) these nice properties

Also using a dispersive representation: Pich, Portolés '02

Hanhart '12

DISPERSIVE REPRESENTATION OF THE VFF

Starting point

Guerrero, Pich '97

Match χPT results to VMD using an Omnés solution for dispersion relation

Idea: Follow the approach of Boito, Escribano, Jamin '08 preserving analiticity and unitarity exactly using a dispersive representation of the VFF while retaining (some of) these nice properties

$$F_V(s) = \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(A_\pi(s)+A_K(s)/2\right)\right]-s}$$
$$= \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(\Re e A_\pi(s)+\Re e A_K(s)/2\right)\right]-s-iM_\rho\Gamma_\rho(s)}$$

DISPERSIVE REPRESENTATION OF THE VFF

Starting point

Guerrero, Pich '97

Match χPT results to VMD using an Omnés solution for dispersion relation

Idea: Follow the approach of Boito, Escribano, Jamin '08 preserving analiticity and unitarity exactly using a dispersive representation of the VFF while retaining (some of) these nice properties

$$F_V(s) = \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2 F_\pi^2}\left(A_\pi(s)+A_K(s)/2\right)\right]-s}$$

$$= \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2 F_\pi^2}\left(\Re e A_\pi(s)+\Re e A_K(s)/2\right)\right]-s-iM_\rho\Gamma_\rho(s)}$$

$$\tan\delta_1^1(s) = \frac{\Im m F_V(s)}{\Re e F_V(s)}$$

DISPERSIVE REPRESENTATION OF THE VFF

Starting point

Guerrero, Pich '97

Match χPT results to VMD using an Omnés solution for dispersion relation

Idea: Follow the approach of Boito, Escribano, Jamin '08 preserving analiticity and unitarity exactly using a dispersive representation of the VFF while retaining (some of) these nice properties

$$F_V(s) = \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(A_\pi(s)+A_K(s)/2\right)\right]-s}$$
$$= \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(\Re e A_\pi(s)+\Re e A_K(s)/2\right)\right]-s-iM_\rho\Gamma_\rho(s)}$$

$$\tan\delta_1^1(s) = \frac{\Im m F_V(s)}{\Re e F_V(s)}$$

$$F_V(s) = \exp\left\{\alpha_1 s + \frac{\alpha_2}{2}s^2 + \frac{s^3}{\pi}\int_{s_{thr}}^{\infty} ds' \frac{\delta_1^1(s')}{(s')^3(s'-s-i\epsilon)}\right\}$$

DISPERSIVE REPRESENTATION OF THE VFF

The dispersive representation is matched to

$$F_V^\pi(s) = \frac{M_\rho^2 + (\alpha' e^{i\phi'} + \alpha'' e^{i\phi''})\, s}{M_\rho^2 \left[1 + \frac{s}{96\pi^2 F_\pi^2}\left(A_\pi(s) + \frac{1}{2}A_K(s)\right)\right] - s} - \frac{\alpha' e^{i\phi'}\, s}{M_{\rho'}^2\, [1 + s\, C_{\rho'} A_\pi(s)] - s} - \frac{\alpha'' e^{i\phi''}\, s}{M_{\rho''}^2\, [1 + s\, C_{\rho''} A_\pi(s)] - s}$$

$$C_R = \frac{\Gamma_R}{\pi M_R^3\, \sigma_\pi^3(M_R^2)} \qquad \Gamma_R(s) = \Gamma_R \frac{s}{M_R^2} \frac{\sigma_\pi^3(s)}{\sigma_\pi^3(M_R^2)}\, \theta(s - 4m_\pi^2)$$

at higher energies

DISPERSIVE REPRESENTATION OF THE VFF

$$F_V(s) = \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(A_\pi(s)+A_K(s)/2\right)\right]-s}$$
$$= \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(\Re eA_\pi(s)+\Re eA_K(s)/2\right)\right]-s-iM_\rho\Gamma_\rho(s)}$$

⟷ ?

$$F_V(s) = \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(A_{\pi^-\pi^0}(s)+A_{K^-K^0}(s)/2\right)\right]-s}$$
$$\sim \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(\Re eA_{\pi^-\pi^0}(s)+\Re eA_{K^-K^0}(s)/2\right)\right]-s-iM_\rho\Gamma_\rho(s)}$$

But these are not the complete LO SU(2) corrections

Cirigliano, Ecker, Neufeld '01

$$\tan\delta_1^1(s) = \frac{\Im mF_V(s)}{\Re eF_V(s)}$$

$$F_V(s) = \exp\left\{\alpha_1 s + \frac{\alpha_2}{2}s^2 + \frac{s^3}{\pi}\int_{s_{thr}}^{\infty} ds'\, \frac{\delta_1^1(s')}{(s')^3(s'-s-i\epsilon)}\right\}$$

DISPERSIVE REPRESENTATION OF THE VFF

$$F_V(s) = \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(A_\pi(s)+A_K(s)/2\right)\right]-s}$$
$$= \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(\Re e A_\pi(s)+\Re e A_K(s)/2\right)\right]-s-iM_\rho\Gamma_\rho(s)}$$

?

$$F_V(s) = \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(A_{\pi^-\pi^0}(s)+A_{K^-K^0}(s)/2\right)\right]-s}$$
$$\sim \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(\Re e A_{\pi^-\pi^0}(s)+\Re e A_{K^-K^0}(s)/2\right)\right]-s-iM_\rho\Gamma_\rho(s)}$$

$$|F_V^-(s)|^2 \to |F_V^-(s)|^2 G_{\mathrm{EM}}(s)$$

But these are not the complete LO SU(2) corrections

Cirigliano, Ecker, Neufeld '01

Factor used by Belle to obtain the had LO contribution to a_μ, but not to fit the VFF

Belle '08 from Flores-Báez et. al. '08

$$\tan\delta_1^1(s) = \frac{\Im m F_V(s)}{\Re e F_V(s)}$$

$$F_V(s) = \exp\left\{\alpha_1 s + \frac{\alpha_2}{2}s^2 + \frac{s^3}{\pi}\int_{s_{thr}}^{\infty} ds' \frac{\delta_1^1(s')}{(s')^3(s'-s-i\epsilon)}\right\}$$

DISPERSIVE REPRESENTATION OF THE VFF

$$F_V(s) = \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(A_\pi(s)+A_K(s)/2\right)\right]-s}$$
$$= \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(\Re e A_\pi(s)+\Re e A_K(s)/2\right)\right]-s-iM_\rho\Gamma_\rho(s)}$$

Fit I SU(2)

?

$$F_V(s) = \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(A_{\pi^-\pi^0}(s)+A_{K^-K^0}(s)/2\right)\right]-s}$$
$$\sim \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(\Re e A_{\pi^-\pi^0}(s)+\Re e A_{K^-K^0}(s)/2\right)\right]-s-iM_\rho\Gamma_\rho(s)}$$

$$|F_V^-(s)|^2 \to |F_V^-(s)|^2 G_{EM}(s)$$

But these are not the complete LO SU(2) corrections

Cirigliano, Ecker, Neufeld '01

Factor used by Belle to obtain the had LO contribution to a_μ, but not to fit the VFF

Belle '08 from Flores-Báez et. al. '08

$$\tan\delta_1^1(s) = \frac{\Im m F_V(s)}{\Re e F_V(s)}$$

$$F_V(s) = \exp\left\{\alpha_1 s + \frac{\alpha_2}{2}s^2 + \frac{s^3}{\pi}\int_{s_{thr}}^{\infty} ds' \frac{\delta_1^1(s')}{(s')^3(s'-s-i\epsilon)}\right\}$$

DISPERSIVE REPRESENTATION OF THE VFF

$$F_V(s) = \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(A_\pi(s)+A_K(s)/2\right)\right]-s}$$
$$= \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(\Re eA_\pi(s)+\Re eA_K(s)/2\right)\right]-s-iM_\rho\Gamma_\rho(s)}$$

?

$$F_V(s) = \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(A_{\pi^-\pi^0}(s)+A_{K^-K^0}(s)/2\right)\right]-s}$$
$$\sim \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(\Re eA_{\pi^-\pi^0}(s)+\Re eA_{K^-K^0}(s)/2\right)\right]-s-iM_\rho\Gamma_\rho(s)}$$

Fit II **SU(2) kin**

$$|F_V^-(s)|^2 \to |F_V^-(s)|^2 G_{\rm EM}(s)$$

But these are not the complete LO SU(2) corrections

Cirigliano, Ecker, Neufeld '01

Factor used by Belle to obtain the had LO contribution to a_μ, but not to fit the VFF

Belle '08 from Flores-Báez et. al. '08

$$\tan\delta_1^1(s) = \frac{\Im m F_V(s)}{\Re e F_V(s)}$$

$$F_V(s) = \exp\left\{\alpha_1 s + \frac{\alpha_2}{2}s^2 + \frac{s^3}{\pi}\int_{s_{thr}}^{\infty} ds' \frac{\delta_1^1(s')}{(s')^3(s'-s-i\epsilon)}\right\}$$

DISPERSIVE REPRESENTATION OF THE VFF

$$F_V(s) = \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(A_\pi(s)+A_K(s)/2\right)\right]-s}$$
$$= \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(\Re e A_\pi(s)+\Re e A_K(s)/2\right)\right]-s-iM_\rho\Gamma_\rho(s)}$$

?

$$F_V(s) = \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(A_{\pi^-\pi^0}(s)+A_{K^-K^0}(s)/2\right)\right]-s}$$
$$\sim \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(\Re e A_{\pi^-\pi^0}(s)+\Re e A_{K^-K^0}(s)/2\right)\right]-s-iM_\rho\Gamma_\rho(s)}$$

Fit III

~~SU(2)~~ LO

$$|F_V^-(s)|^2 \rightarrow |F_V^-(s)|^2 G_{\rm EM}(s)$$

But these are not the complete LO SU(2) corrections

Cirigliano, Ecker, Neufeld ‘01

Factor used by Belle to obtain the had LO contribution to a_μ, but not to fit the VFF

Belle ’08 from Flores-Báez et. al. ‘08

$$\tan\delta_1^1(s) = \frac{\Im m F_V(s)}{\Re e F_V(s)}$$

$$F_V(s) = \exp\left\{\alpha_1 s + \frac{\alpha_2}{2}s^2 + \frac{s^3}{\pi}\int_{s_{thr}}^{\infty} ds' \frac{\delta_1^1(s')}{(s')^3(s'-s-i\epsilon)}\right\}$$

DISPERSIVE REPRESENTATION OF THE VFF

$$F_V(s) = \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(A_\pi(s)+A_K(s)/2\right)\right]-s}$$
$$= \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(\Re e A_\pi(s)+\Re e A_K(s)/2\right)\right]-s-iM_\rho\Gamma_\rho(s)}$$

Fit I SU(2)

?

$$F_V(s) = \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(A_{\pi^-\pi^0}(s)+A_{K^-K^0}(s)/2\right)\right]-s}$$
$$\sim \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(\Re e A_{\pi^-\pi^0}(s)+\Re e A_{K^-K^0}(s)/2\right)\right]-s-iM_\rho\Gamma_\rho(s)}$$

Fit II SU(2) kin

Fit III SU(2) LO

$$|F_V^-(s)|^2 \to |F_V^-(s)|^2 G_{EM}(s)$$

But these are not the complete LO SU(2) corrections

Cirigliano, Ecker, Neufeld '01

Factor used by Belle to obtain the had LO contribution to a_μ, but not to fit the VFF

Belle '08 from Flores-Báez et. al. '08

$$\tan\delta_1^1(s) = \frac{\Im m F_V(s)}{\Re e F_V(s)}$$

$$F_V(s) = \exp\left\{\alpha_1 s + \frac{\alpha_2}{2}s^2 + \frac{s^3}{\pi}\int_{s_{thr}}^{\infty} ds' \frac{\delta_1^1(s')}{(s')^3(s'-s-i\epsilon)}\right\}$$

DISPERSIVE REPRESENTATION OF THE VFF

Fit III ~~SU(2)~~ LO

$$\frac{d\Gamma(\tau^- \to \pi^-\pi^0\nu_\tau)}{ds} = \frac{G_F^2\, m_\tau^3}{384\,\pi^3} S_{EW}\, |V_{ud}|^2 \left(1-\frac{s}{M_\tau^2}\right)^2 \left(1+\frac{2s}{M_\tau^2}\right)$$

$$\lambda^{3/2}\left(1,\frac{m_{\pi^0}^2}{s},\frac{m_{\pi^+}^2}{s}\right) |f_+(s)|^2 G_{EM}(s)\,,$$

$S_{EW} \neq 1$; $m_{\pi^0}^2 \neq m_{\pi^+}^2$; $G_{EM}(s) \neq 1$

$$f_+(s) = F_V^{\pi^+}(s) + f_{\rm local}^{\rm elm}$$

Chiral LECs

$$f_{\rm local}^{\rm elm} = \frac{\alpha}{4\pi}\left(-\frac{3}{2}-\frac{1}{2}\log\frac{M_\tau^2}{\mu^2}-\log\frac{m_\pi^2}{\mu^2}+2\log\frac{M_\tau^2}{M_\rho^2}-X(\mu)\right)$$

Cirigliano, Ecker, Neufeld '01

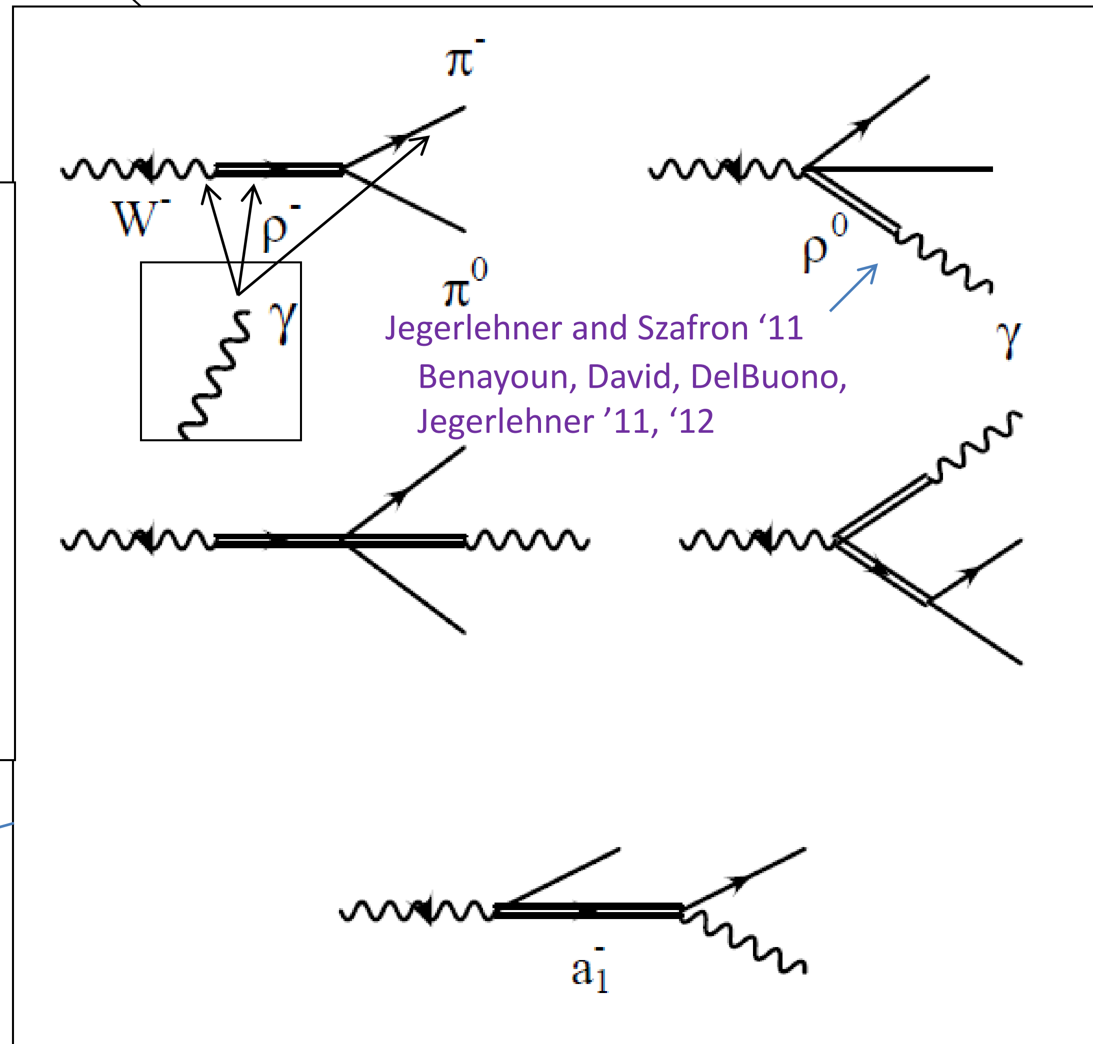
$$A_\pi(s) \text{ and } A_K(s) \longrightarrow A_{\pi^-\pi^0}(s) \text{ and } A_{K^-K^0}(s)$$

DISPERSIVE REPRESENTATION OF THE VFF

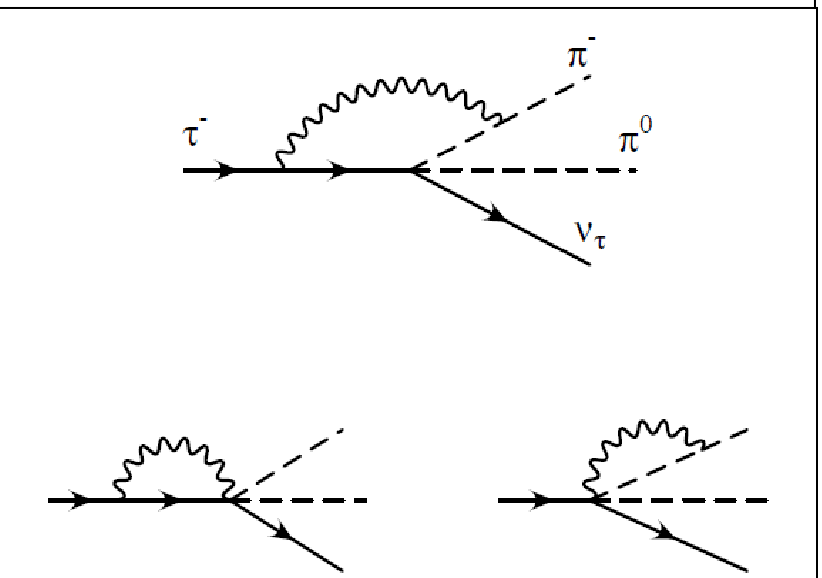
Fit III

SU(2) LO

Diagrams with external photon



Photon loop diagrams



Cirigliano, Ecker, Neufeld '01
Flores-Tlalpa et. al. '05

$G_{EM}(s)$

DISPERSIVE REPRESENTATION OF THE VFF

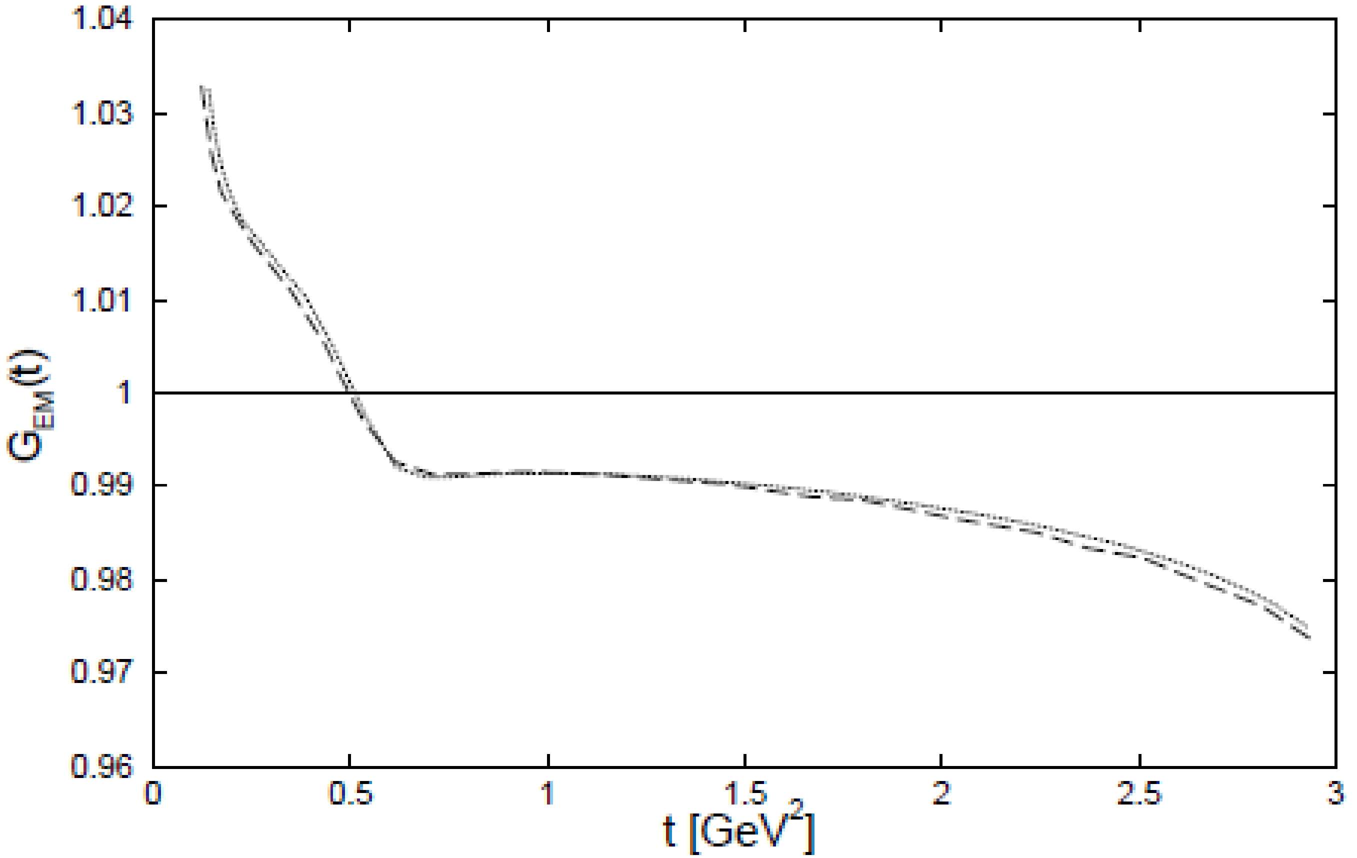
Fit III

~~SU(2) LO~~

$G_{EM}(s)$ has been calculated by Cirigliano, Ecker, Neufeld '01 (Chiral Perturbation Theory) '02 (including resonances) and Flores-Tlalpa et. al. '05.

Cirigliano, Ecker, Neufeld '01 → dashed-dotted

Flores-Tlalpa et. al. '05 → long-dashed



RESULTS: $\tau^- \longrightarrow \pi^- \pi^0 \nu_\tau$

	Fit value (I)	Fit value (II)	Fit value (III)
M_ρ [GeV]	0.8430(5)(17)	0.8427(5)(14)	0.8426(5)(20)
F_π [GeV]	0.0901(2)(5)	0.0902(2)(4)	0.0906(2)(4)
α_1 [GeV^{-2}]	1.87(1)(3)	1.87(1)(3)	1.81(1)(2)
α_2 [GeV^{-4}]	4.29(1)(7)	4.31(1)(7)	4.40(1)(6)
χ^2/dof	1.37	1.37	1.55
$\Gamma_\rho(M_\rho^2)$ [GeV]	0.206(1)(3)	0.206(1)(3)	0.204(1)(3)

Table 1: Results of our fits. The first and second numbers in brackets correspond to the statistic and theoretical systematic errors, respectively. $\Gamma_\rho(M_\rho^2)$ is obtained using the fitted values of M_ρ and F_π and is given only for reference.

$$\sqrt{s_{\rm pole}} = M_\rho^{\rm pole} - \frac{i}{2}\Gamma_\rho^{\rm pole} \longrightarrow M_\rho^{\rm pole} = (748.2 \pm 0.8)\,{\rm MeV}\,, \quad \Gamma_\rho^{\rm pole} = (153.0 \pm 0.7)\,{\rm MeV} \quad ({\rm Fit\ III})$$

The χ^2 is similar in all cases and the inclusion of isospin breaking corrections does not improve the quality of the fits.

RESULTS: $\tau^- \longrightarrow \pi^-\pi^0\nu_\tau$

	Fit value (I)	Fit value (II)	Fit value (III)
M_ρ [GeV]	0.8430(5)(17)	0.8427(5)(14)	0.8426(5)(20)
F_π [GeV]	0.0901(2)(5)	0.0902(2)(4)	0.0906(2)(4)
α_1 [GeV^{-2}]	1.87(1)(3)	1.87(1)(3)	1.81(1)(2)
α_2 [GeV^{-4}]	4.29(1)(7)	4.31(1)(7)	4.40(1)(6)
χ^2/dof	1.37	1.37	1.55
$\Gamma_\rho(M_\rho^2)$ [GeV]	0.206(1)(3)	0.206(1)(3)	0.204(1)(3)

Table 1: Results of our fits. The first and second numbers in brackets correspond to the statistic and theoretical systematic errors, respectively. $\Gamma_\rho(M_\rho^2)$ is obtained using the fitted values of M_ρ and F_π and is given only for reference.

The χ^2 is similar in all cases and the inclusion of isospin breaking corrections does not improve the quality of the fits.

Small contribution (→ EM)

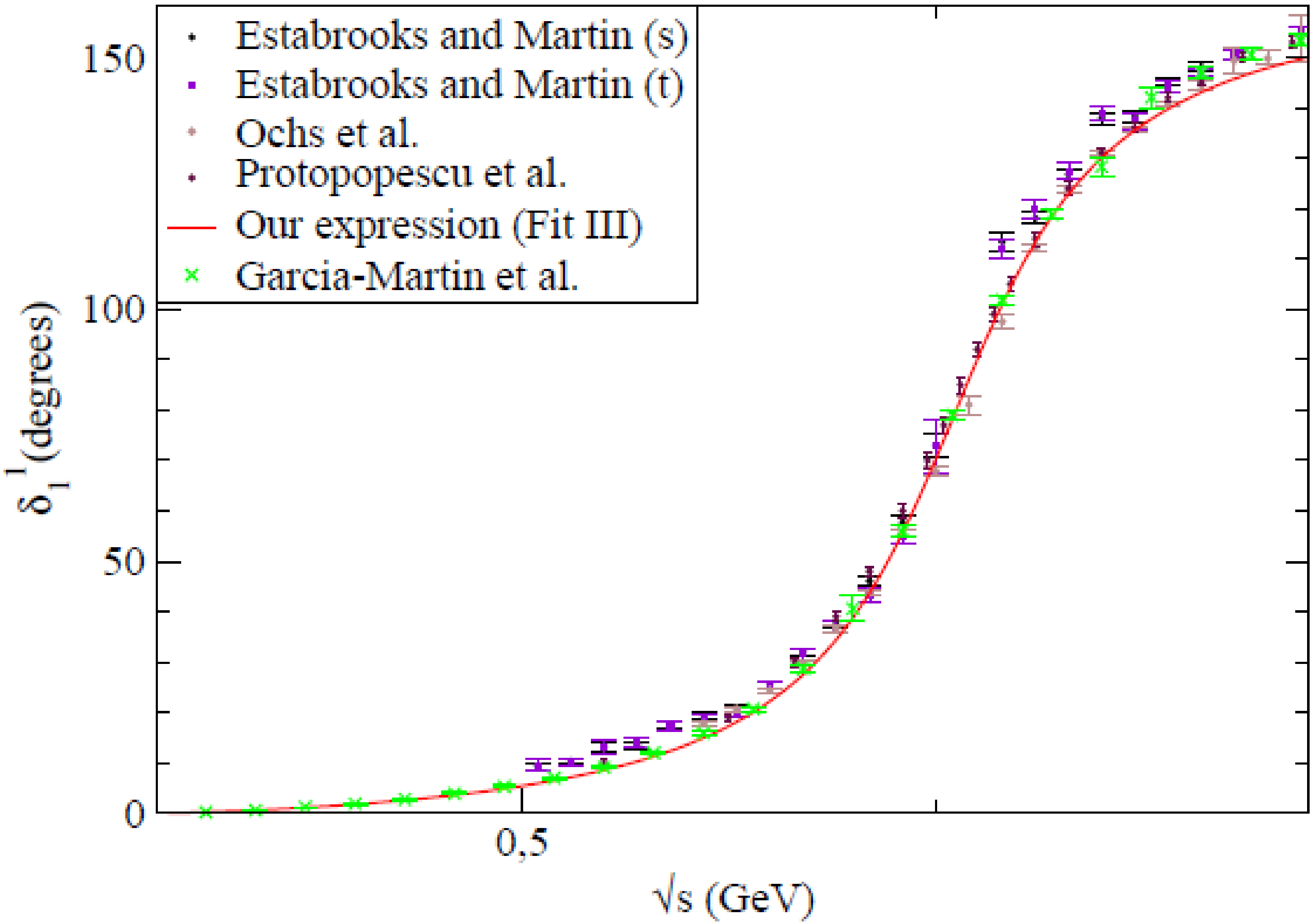
This is the key: However, the error is of the size of the discrepancy with the SM/10 (→ FF)

t>1 GeV^2 contributions are negligible (→ t_{max} = 1)

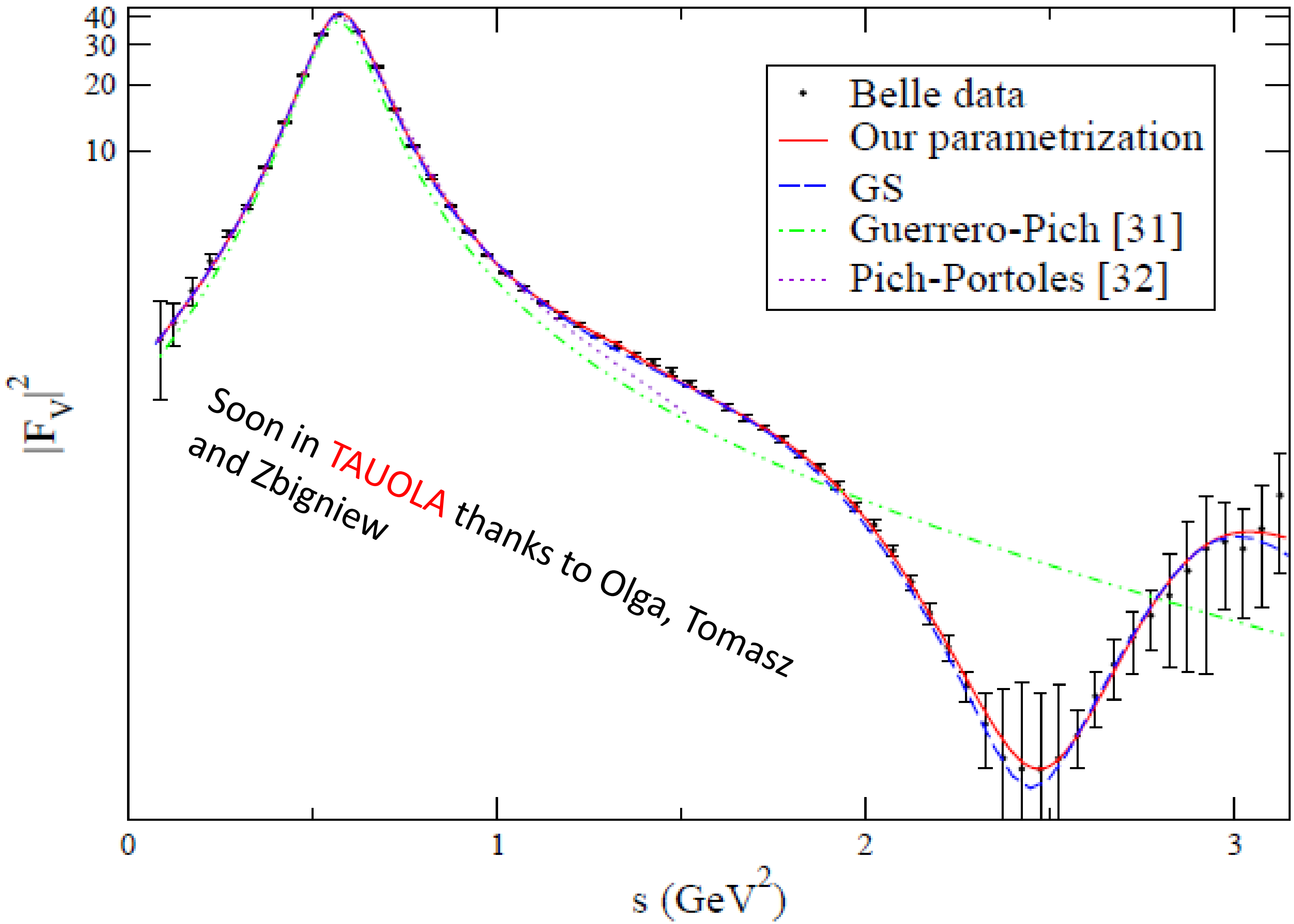
t_{max}	S_{EW}	KIN	EM	FF	Δa_μ^{vacpol} (total)
1	- 95	- 75	- 11	$61 \pm 26 \pm 3$	- 119
2	- 97	- 75	- 10	$61 \pm 26 \pm 3$	- 120
3	- 97	- 75	- 10	$61 \pm 26 \pm 3$	- 120

In units of 10^{-11}

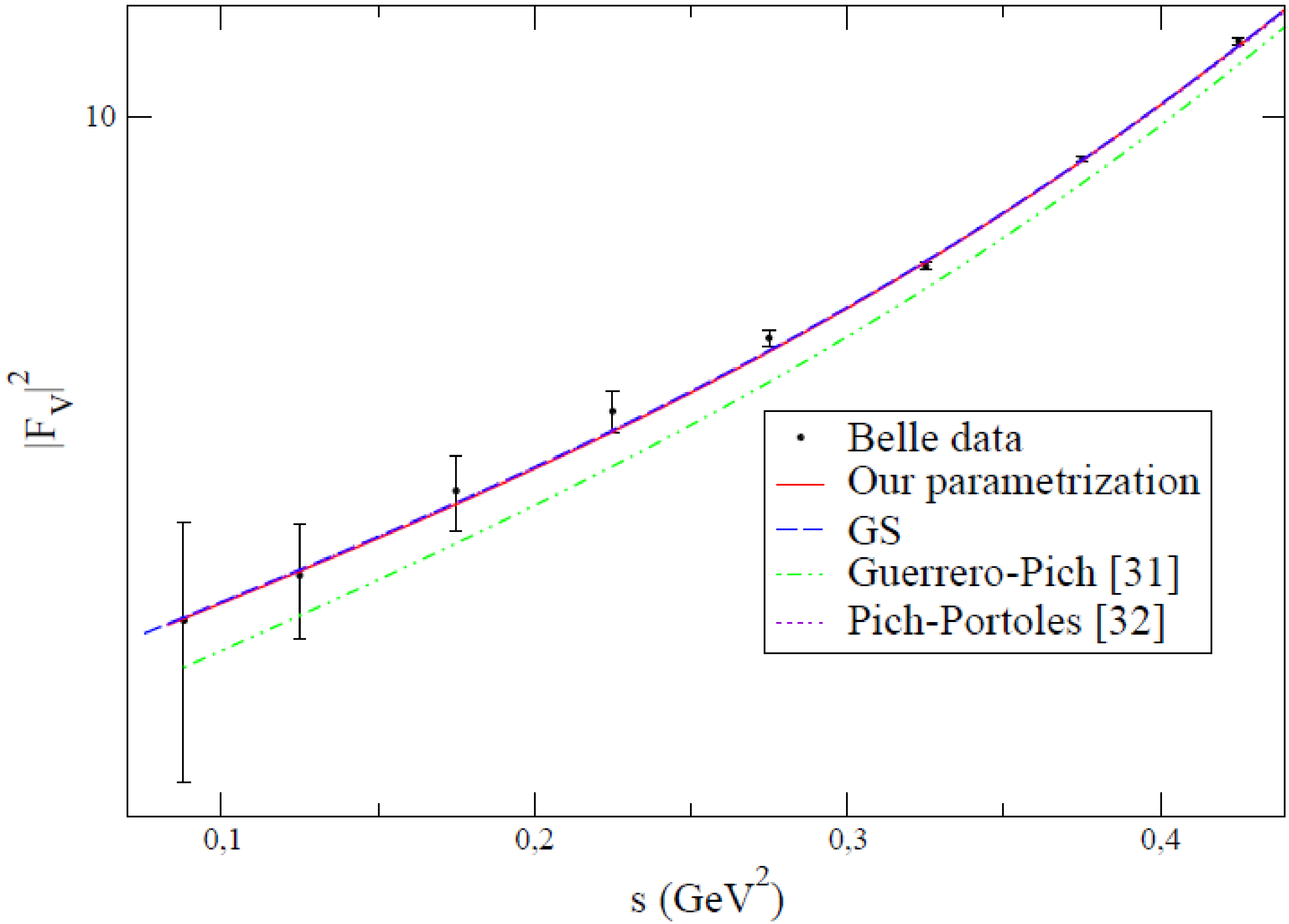
RESULTS: $\tau^- \longrightarrow \pi^-\pi^0\nu_\tau$



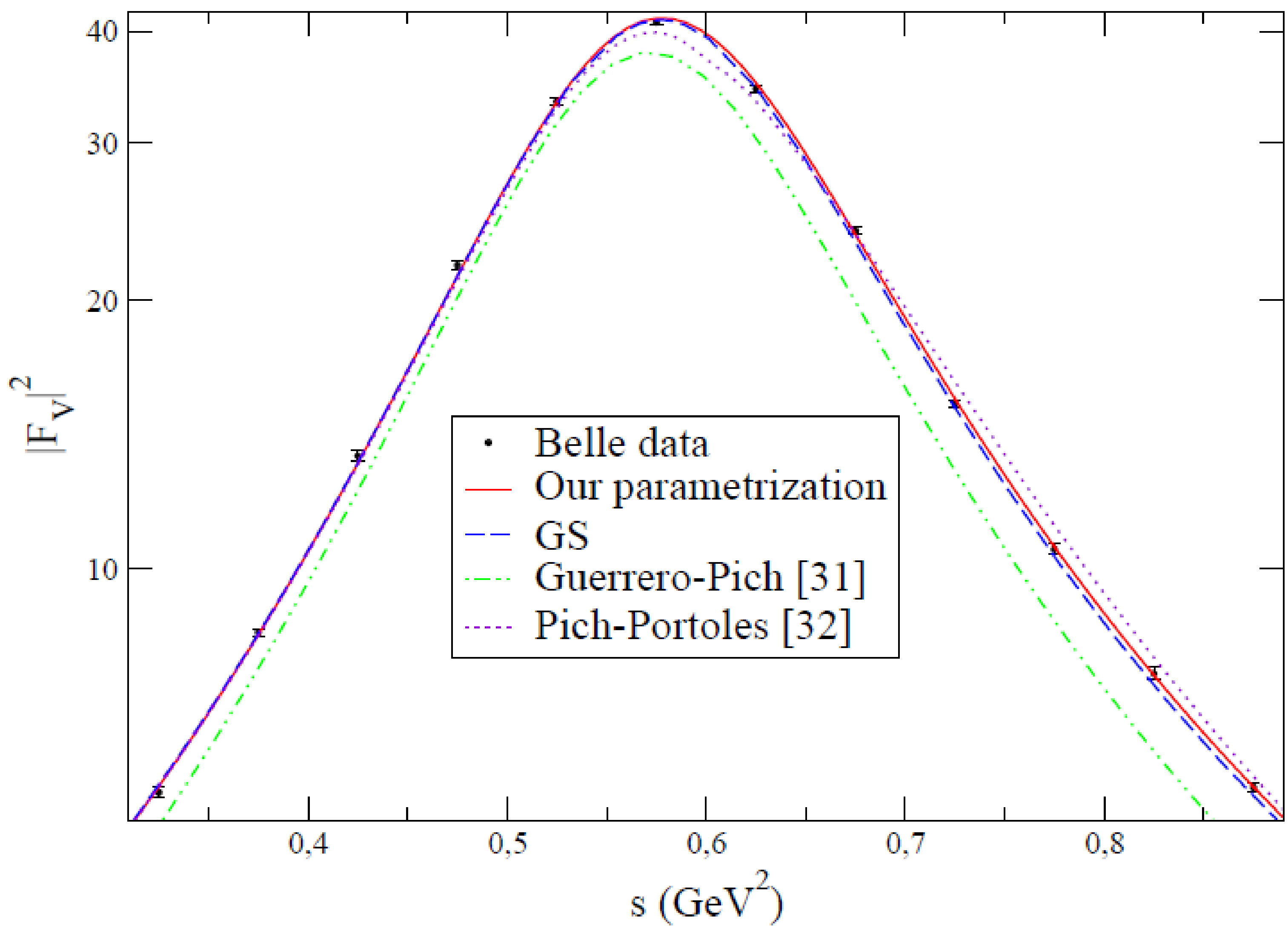
RESULTS: $\tau^- \longrightarrow \pi^-\pi^0\nu_\tau$



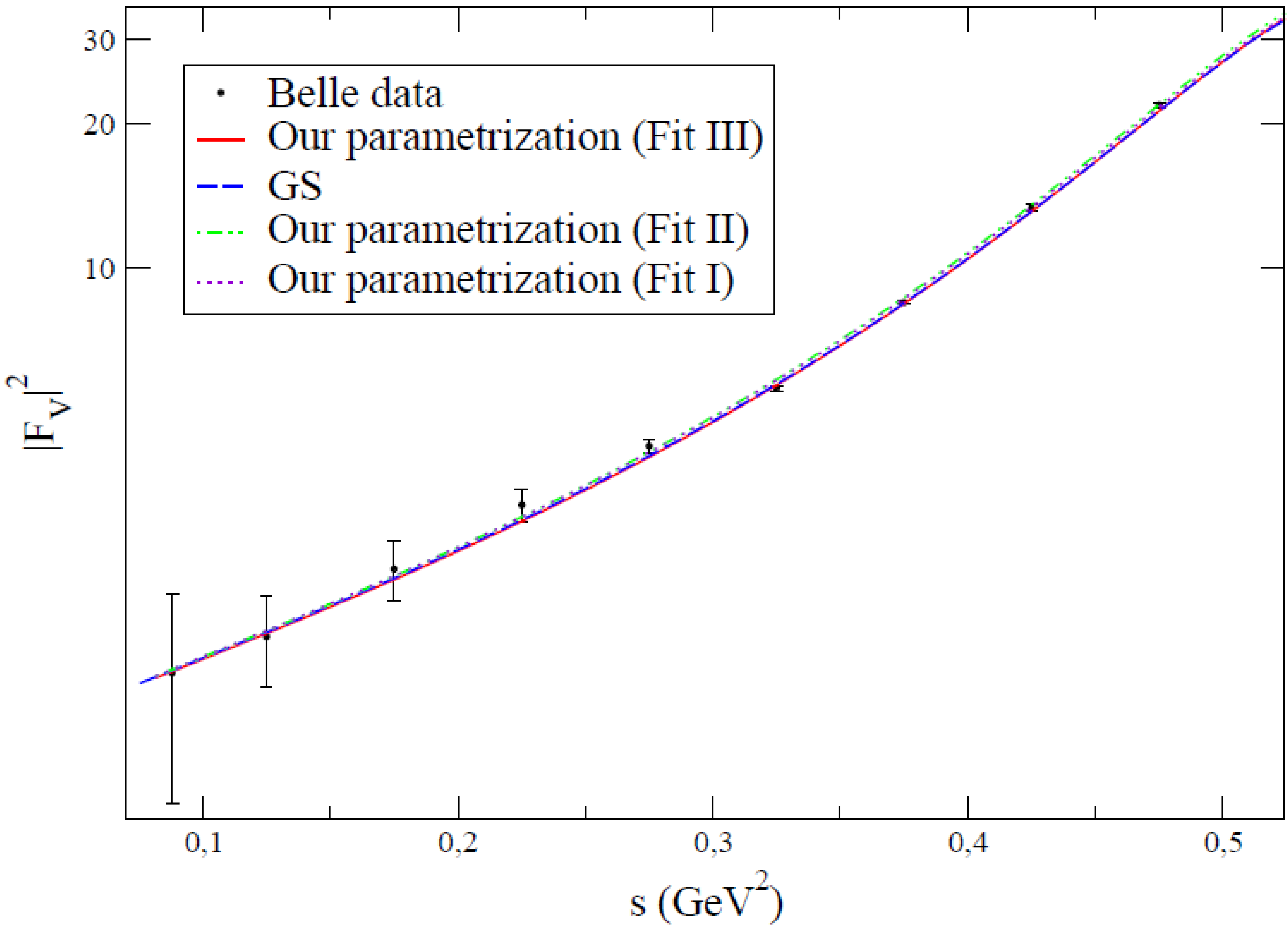
RESULTS: $\tau^- \longrightarrow \pi^- \pi^0 \nu_\tau$



RESULTS: $\tau^- \longrightarrow \pi^- \pi^0 \nu_\tau$



RESULTS: $\tau^- \longrightarrow \pi^-\pi^0\nu_\tau$



RESULTS: $\tau^- \longrightarrow \pi^- \pi^0 \nu_\tau$

Low-E expansion

$$F_V^\pi(s) = 1 + \frac{1}{6}\langle r^2\rangle_V^\pi\, s + c_V^\pi\, s^2 + d_V^\pi\, s^3 + \dots$$

$$\langle r^2\rangle_V^\pi = 6\,\alpha_1\ , \quad c_V^\pi = \frac{1}{2}\left(\alpha_2 + \alpha_1^2\right)$$

$$\alpha_k = \frac{k!}{\pi}\int_{4m_\pi^2}^\infty \mathrm{d}s'\ \frac{\delta_1^1(s')}{s'^{k+1}}$$

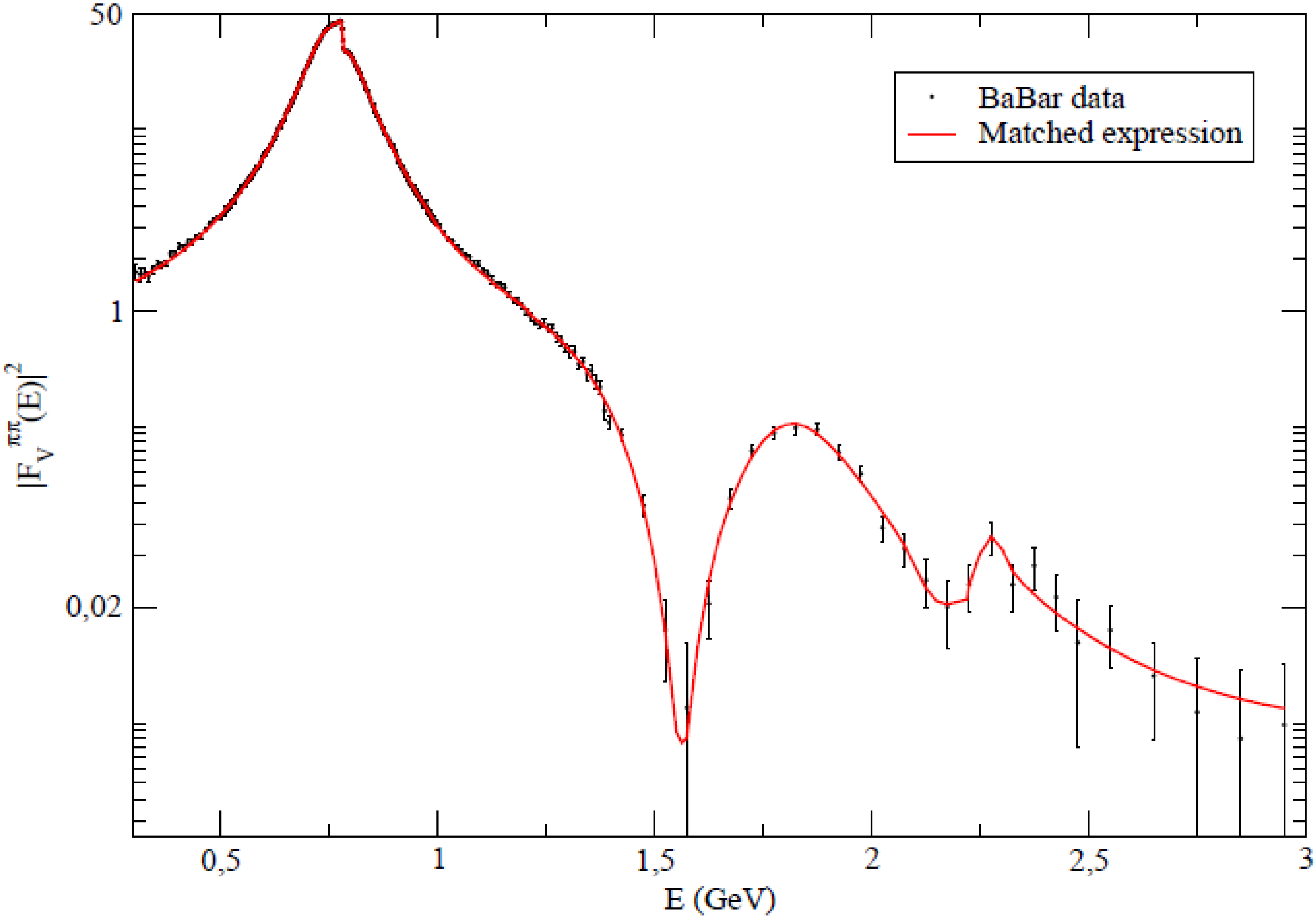
$$\langle r^2\rangle_V^\pi = 10.86 \pm 0.14\ \mathrm{GeV}^{-2}\ , \quad c_V^\pi = 3.84 \pm 0.03\ \mathrm{GeV}^{-4}$$

$$d_V^\pi = \frac{1}{6}\left(\alpha_3 + 3\alpha_1\alpha_2 + \alpha_1^3\right) = 9.84 \pm 0.05\ \mathrm{GeV}^{-6}$$

In good agreement with the literature with higher precision

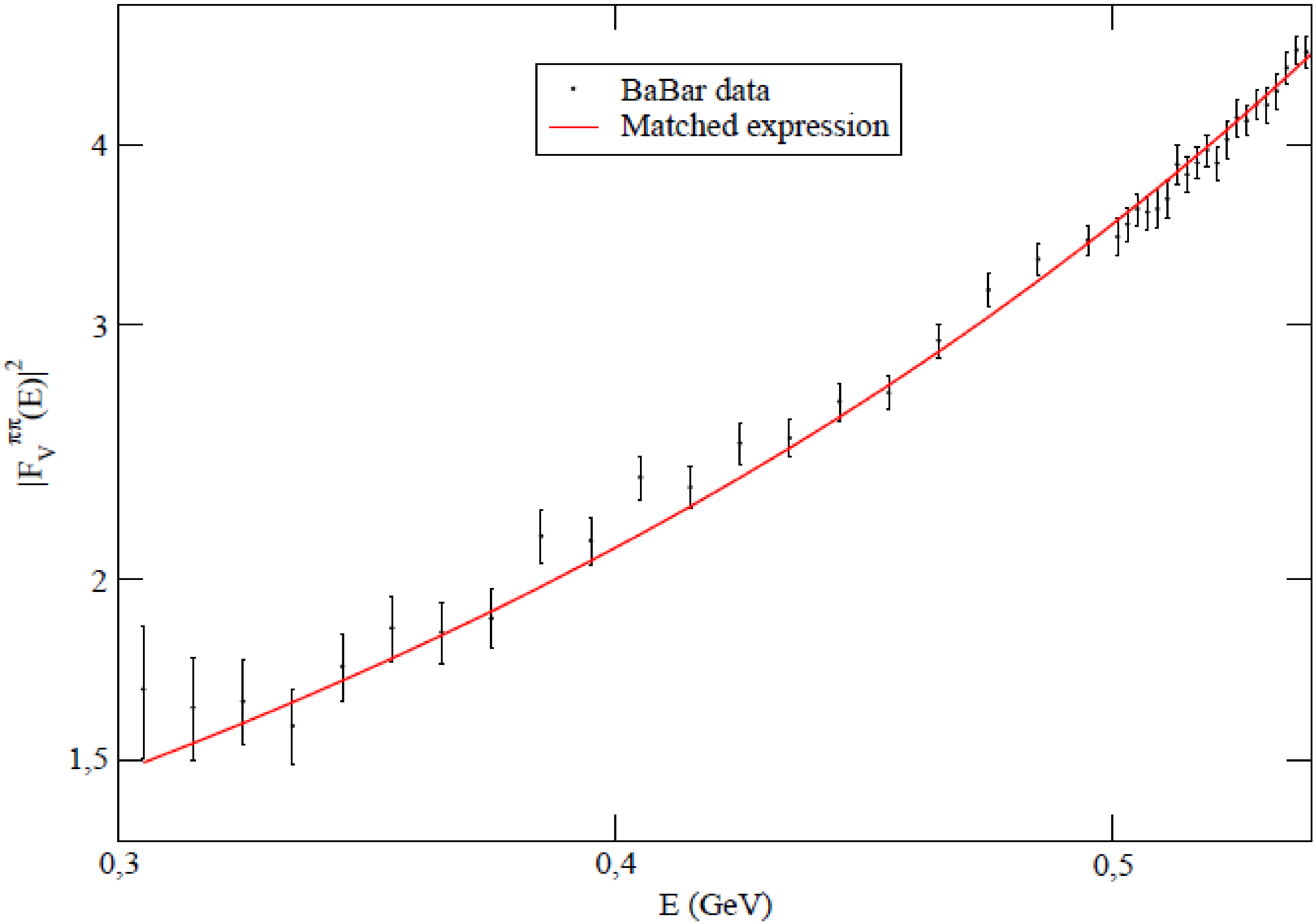
RESULTS: $e^+e^- \longrightarrow \pi^+\pi^-$

Analogous method is applied (Gómez Dumm and Roig, in progress)



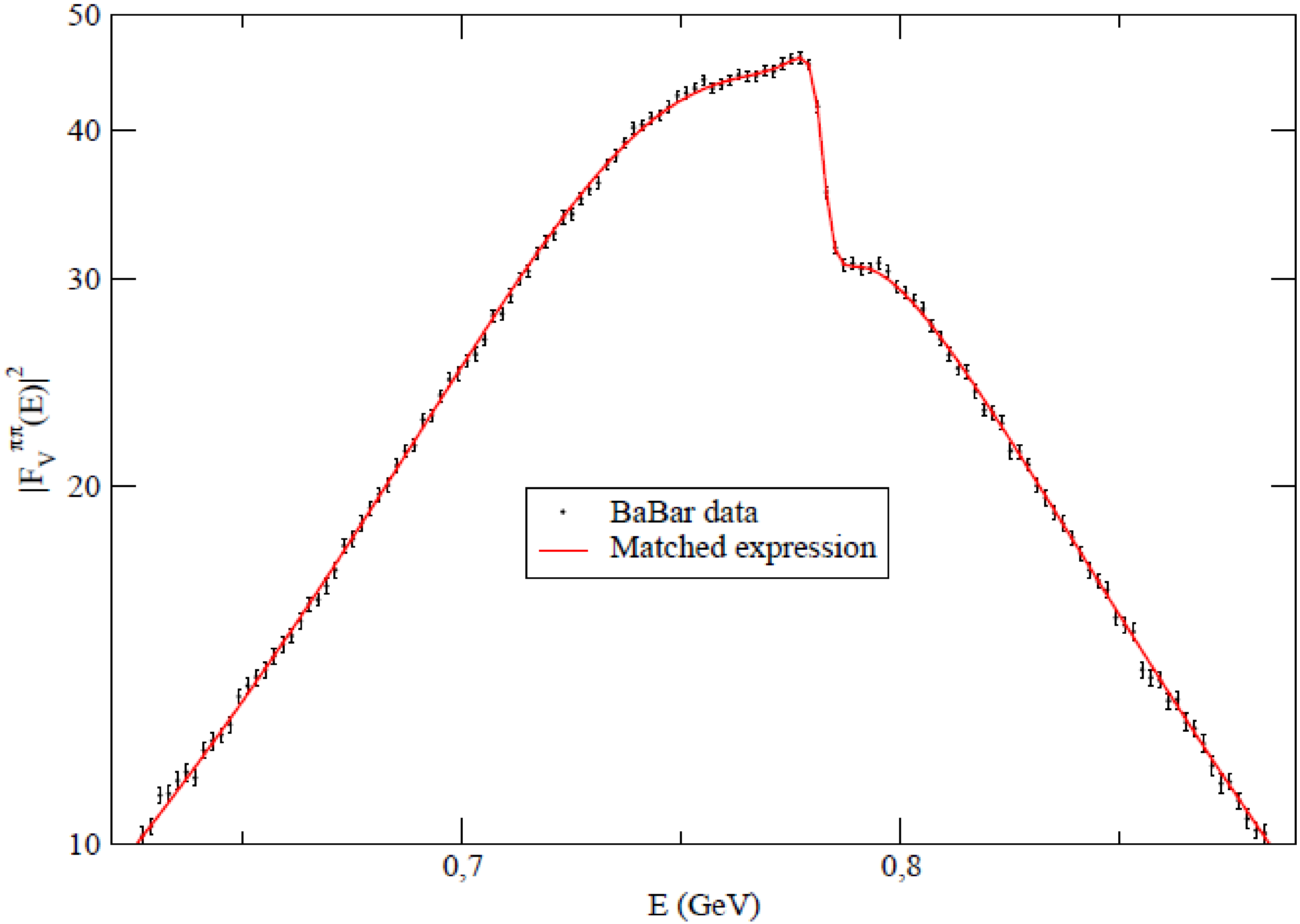
RESULTS: $e^+e^- \longrightarrow \pi+\pi-$

Gómez Dumm and Roig, in progress



RESULTS: $e^+e^- \longrightarrow \pi+\pi-$

Gómez Dumm and Roig, in progress



CONCLUSIONS

• We have elaborated a dispersive description of the $\pi^-\pi^0$ VFF which preserves **analiticity** and **unitarity** exactly and reproduces **χPT** up to $O(p^4)$ with leading $O(p^6)$ contributions (soon available in TAUOLA).

CONCLUSIONS

• We have elaborated a dispersive description of the $\pi^-\pi^0$ VFF which preserves **analiticity** and **unitarity** exactly and reproduces **χPT** up to $O(p^4)$ with leading $O(p^6)$ contributions (soon available in TAUOLA).

• We have studied the inclusion of **SU(2) breaking corrections** and concluded that they **do not improve** the **agreement with data**.

CONCLUSIONS

• We have elaborated a dispersive description of the $\pi^-\pi^0$ VFF which preserves **analiticity** and **unitarity** exactly and reproduces **χPT** up to $O(p^4)$ with leading $O(p^6)$ contributions (soon available in TAUOLA).

• We have studied the inclusion of **SU(2) breaking corrections** and concluded that they **do not improve** the **agreement with data**.

• Even in case we admit that SU(2) breaking corrections are not completely understood, **this does not affect the precision in the determination of $a_\mu^{\pi\pi}$**.

CONCLUSIONS

• We have elaborated a dispersive description of the $\pi^-\pi^0$ VFF which preserves **analiticity** and **unitarity** exactly and reproduces **χPT** up to O(p^4) with leading O(p^6) contributions (soon available in TAUOLA).

• We have studied the inclusion of **SU(2) breaking corrections** and concluded that they **do not improve** the **agreement with data**.

• Even in case we admit that SU(2) breaking corrections are not completely understood, **this does not affect the precision in the determination of $a_\mu^{\pi\pi}$**.

•We have evaluated several **LECs improving** the **precision** of previous determinations.

CONCLUSIONS

• We have elaborated a dispersive description of the $\pi^-\pi^0$ VFF which preserves **analiticity** and **unitarity** exactly and reproduces **χPT** up to $O(p^4)$ with leading $O(p^6)$ contributions (soon available in TAUOLA).

• We have studied the inclusion of **SU(2) breaking corrections** and concluded that they **do not improve** the **agreement with data**.

• Even in case we admit that SU(2) breaking corrections are not completely understood, **this does not affect the precision in the determination of $a_\mu^{\pi\pi}$**.

•We have evaluated several **LECs improving** the **precision** of previous determinations.

•Our framework is also able to provide **good quality fits** of **$\sigma(e^+e^- \longrightarrow \pi^+\pi^-)$** at low E. This will allow us to evaluate the **$a_\mu^{\pi\pi}$ both from e^+e^- and τ decays consistently**. The **WG** can collect evaluations of **$a_\mu^{\pi\pi}/a_\mu^{had}$ using different approaches** and obtain a **more robust error** determination from them (Discussion by Graziano, Simon and Thomas).