QUARK MASSES

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SFB TR9 Computational Theoretical Particle Physics

I. GENERALITIES

- The Concept of Quark Masses
- Why

II. CHARM and BOTTOM MASSES

- m_Q from Sum Rules
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- Experimental Analysis: m_b

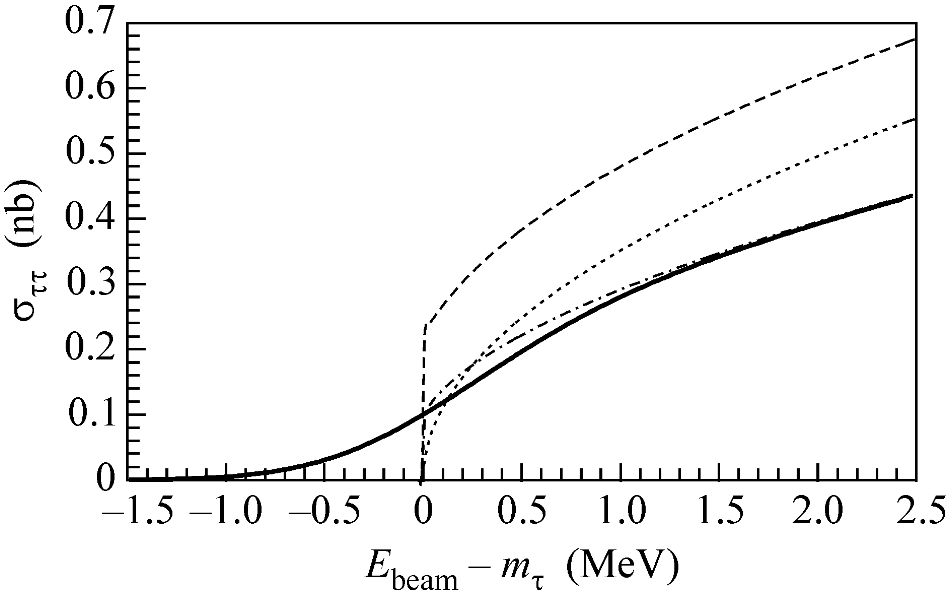
III. SUMMARY

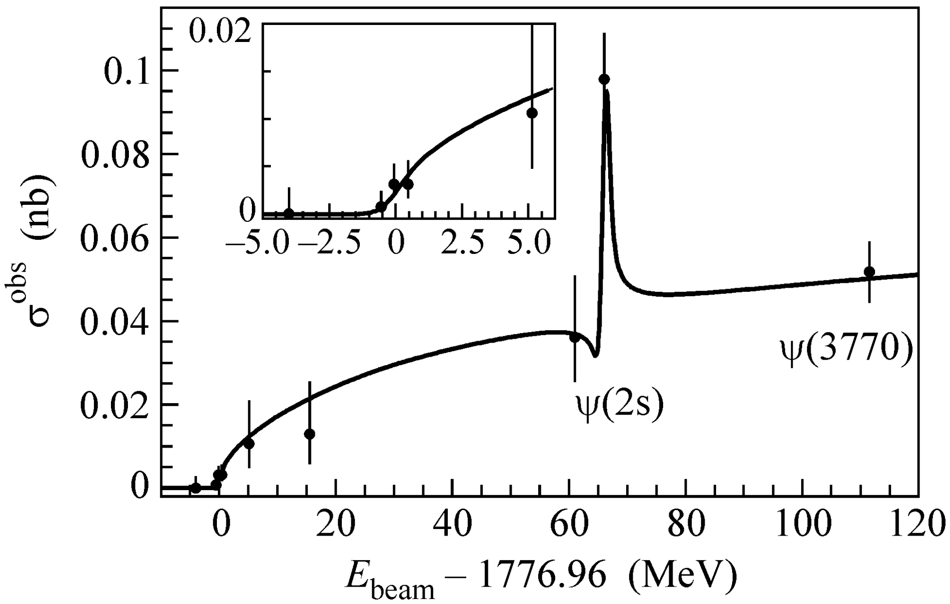
I. GENERALITIES

I. 1. The Concept of Quark Masses

Masses of elementary particles

τ-lepton: $m_\tau = 1776.82 \pm 0.16$ MeV





kinematics: location of threshold

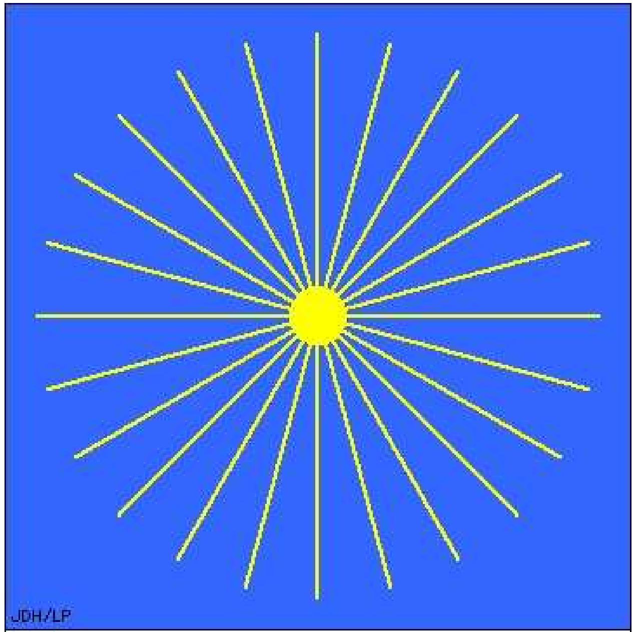
(Novosibirsk,

"pole mass" $M^2_{pole} = E^2 - \vec{p}^{\,2}$

JETP Letts 85, 347)

intuitively clear

Naive question: Can we separate core and cloud (field energy)?

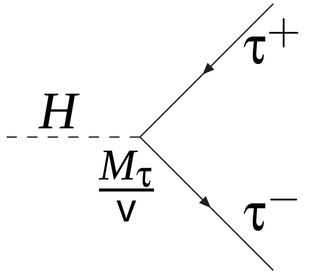


(a) Electric field lines around an electron.

kick the particle hard

$\Rightarrow$ perhaps only short distance part

consider Higgs boson decay: $H \to \tau^+ \tau^-$



$$\frac{M_\tau}{v}$$

$$v^2 = G_F\sqrt{2} = (246\,GeV)^2$$

$$\Gamma(H \to \tau^+\tau^-) \approx \frac{G_F M_\tau^2}{4\pi\sqrt{2}} M_H$$

Which mass M_τ?

Calculate quantum corrections:

QED: correction factor $\left(1 - \frac{\alpha}{\pi}\left(\frac{3}{2}\ln\frac{M_H^2}{M_\tau^2} - \frac{9}{4}\right)\right)$

large logarithm: negative;

can be absorbed in “running mass”

$$M_\tau(M_H) = M_\tau\left(1 - \frac{\alpha}{\pi}\left(\frac{3}{4}\ln\frac{M_H^2}{M_\tau^2} + 1\right)\right)$$

constant term proportional $\frac{\alpha}{\pi}$ depends on choice of mass definition

(concept of “running” mass (and coupling) was introduced in QCD around 1973-1975) convenient choice for QCD calculations

$\overline{MS}$ convention:

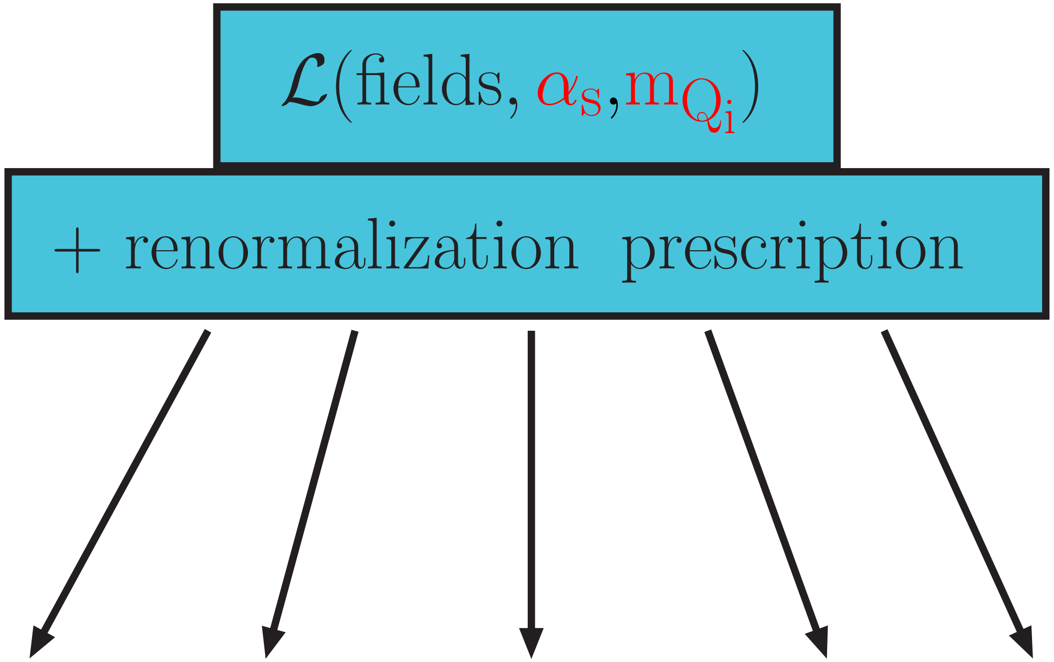
perform calculation in dimensional regularisation: $d = 4 - 2\varepsilon$

divergencies appear as poles in ε

$\Rightarrow$ subtract poles (and convention dependent constants)

m is a convention-dependent parameter in the Lagrangian and depends on the

renormalization scale μ.



Observables :

conversions: $M \Leftrightarrow \overline{m_{\rm b}}(\mu)$

$$\overline{m_{\rm b}}(\mu) = M\left\{1 - a_s\left[\frac{4}{3} + \ln\frac{\mu^2}{M^2}\right] - {a_s}^2\left[\# + \ln + \ln^2\right] + {a_s}^3\left[\# + \ldots\right]\right\}$$

${a_s}^3$: Chetyrkin+Steinhauser; Melnikov+Ritbergen

examples: $M_{\rm t} = 171\,{\rm GeV} \quad \Rightarrow \quad m_{\rm t}(m_{\rm t}) = 161\,{\rm GeV}$

$M_{\rm b} = 4796\,{\rm MeV} \quad \Rightarrow m_{\rm b}(m_{\rm b}) = 4165\,{\rm MeV}$

large logarithms for $\mu^2 \gg M^2$ $\rightarrow$ renormalization group

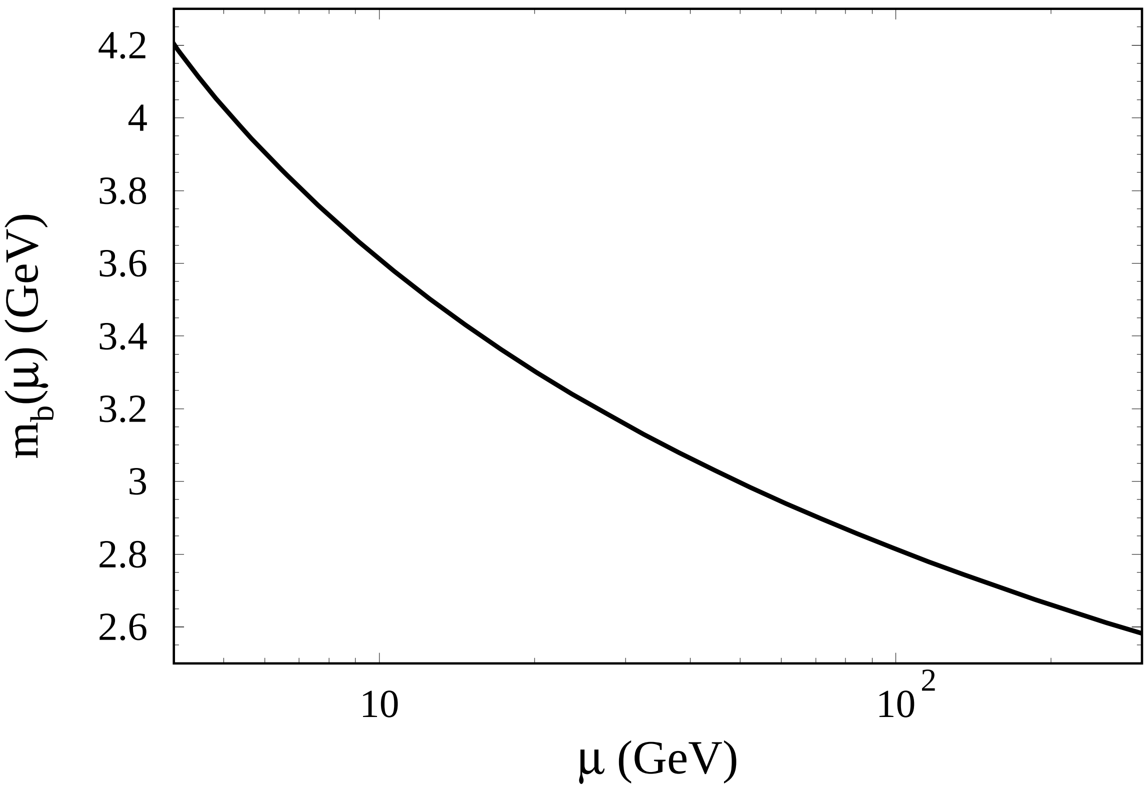
$$\mu^2 \frac{\rm d}{{\rm d}\mu^2}\overline{m}(\mu) = \overline{m}(\mu)\,\gamma(\alpha_s)$$

$\gamma(\alpha_s) = -\sum_{i\geq 0}\gamma_i {a_s}^{i+1}$, (known up to γ_3, Chetyrkin; Larin+...)

+matching at quark thresholds

(Preliminary results for γ_4: Baikov+Chetyrkin) solve RGE numerically or perturbatively

$$\overline{m}(\mu) = \overline{m}(\mu_0)\left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{\gamma_m^0/\beta_0}\left[1+\left(\frac{\gamma_m^1}{\beta_0}-\frac{\beta_1\gamma_m^0}{\beta_0^2}\right)\left(\frac{\alpha_s(\mu)}{\pi}-\frac{\alpha_s(\mu_0)}{\pi}\right)+\dots\right]$$



$m_{\rm b}(m_{\rm b}) = 4165\,\text{MeV}$

$m_{\rm b}(10\text{GeV}) = 3610\,\text{MeV}$

$m_{\rm b}(M_{\rm Z}) = 2836\,\text{MeV}$

$m_{\rm b}(161\text{GeV}) = 2706\,\text{MeV}$

$\overline{\text{MS}}$- vs. Pole-Mass

Pole-Mass (M_{pole}): close to intuition

- t $\rightarrow$ b W

  $M_{\text{pole}}(\text{b W}) = (173.2 \pm 0.9)\ \text{GeV} \pm \mathcal{O}(\Lambda?)$ (Tevatron +MonteCarlo)

- $e^+e^- \rightarrow \text{t}\,\bar{\text{t}}$

  ”peak” at $2M_{\text{pole}} + \mathcal{O}(\alpha_s^2)$

- $M_{\text{B}} \approx M_{\text{pole}} + \mathcal{O}(\Lambda)$

  $5280\ \text{MeV} \approx (4800 + 480)\text{MeV}$

But: large corrections for observables involving large momentum transfers

Implication for Higgs decay: (e.g. $M_H = 126\ GeV$)

$$\Gamma(H \to b\bar{b}) \approx 3\,\frac{G_F M_H}{4\pi\sqrt{2}}\overline{m}_b(M_H)^2\Big[1 + 5.667\Big(\frac{\alpha_s}{\pi}\Big) + 29.147\Big(\frac{\alpha_s}{\pi}\Big)^2 + 41.758\Big(\frac{\alpha_s}{\pi}\Big)^3 - 825.7\Big(\frac{\alpha_s}{\pi}\Big)^4\Big]$$

$\big(\overline{m}_b(M_H)/M_b\big)^2 \approx (2.8/4.8)^2 \approx 0.34$

$[1 + \ldots] \approx [1 + 0.207 + 0.039 + 0.002 - 0.001] \approx 1.247$

(with $\alpha_s = 0.115$)

$\Rightarrow$ dominant corrections from running mass!

Large corrections (often, not always!) absorbed by running mass.

other schemes:

pole mass contains unphysical long distance contributions

$\rightarrow$ subtract unphysical long distance terms

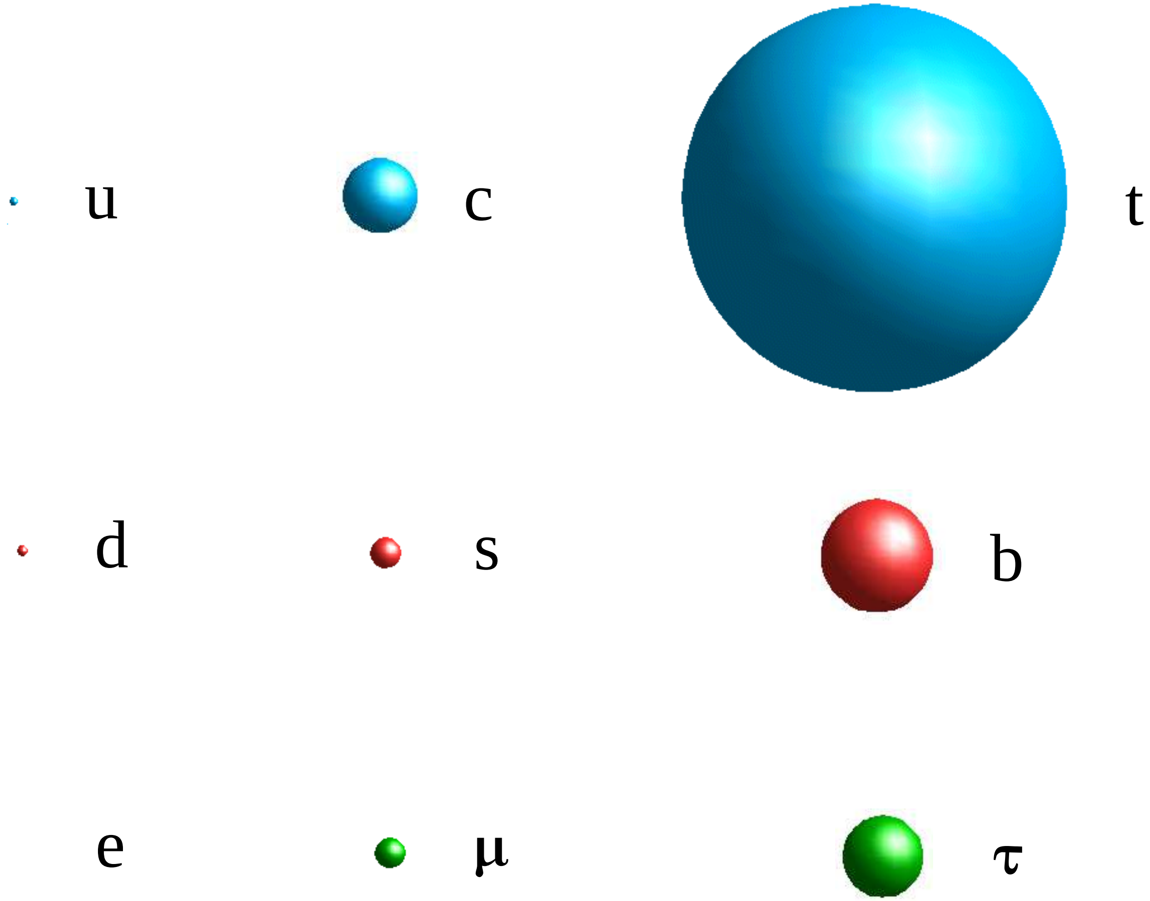
“potential subtracted (PS) mass” (Beneke)

“1 S-mass” (Hoang+Manohar)

often used for B-meson decays, Υ-spectroscopy, closer to pole mass definition

$\rightarrow$ residual uncertainty often larger than uncertainty of m_b in $\overline{MS}$-scheme.

The Puzzle



WHY precise masses?

B-decays:

$$\Gamma(B \to X_u l\bar{\nu}) \sim G_F^2\, m_b^5\, |V_{ub}|^2$$

$$\Gamma(B \to X_c l\bar{\nu}) \sim G_F^2\, m_b^5\, f(m_c^2/m_b^2)\, |V_{cb}|^2$$

moments of $\frac{dN}{dE_l}$, $\frac{dN}{dm(l\nu)}$,

$B \to X_s\gamma$: moments of m_{had}^2

Higgs decays

dominant decay mode for light Higgs

Υ-spectroscopy:

$$m\left(\Upsilon(1\mathrm{s})\right) = 2M_\mathrm{b} - \left(\frac{4}{3}\alpha_\mathrm{s}\right)^2 \frac{M_\mathrm{b}}{4} + \ldots$$

sum rules:

$$\int \frac{\mathrm{d}s}{s^{n+1}} R_\mathrm{Q}(s) \sim \frac{1}{m_\mathrm{Q}^{2n}}$$

perturbative vs. lattice:

recently (HPQC)

$\overline{m}_c(3 \text{ GeV}) = 986(6)$ MeV

$\overline{m}_b(10 \text{ GeV}) = 3617(25)$ MeV

Yukawa Unification

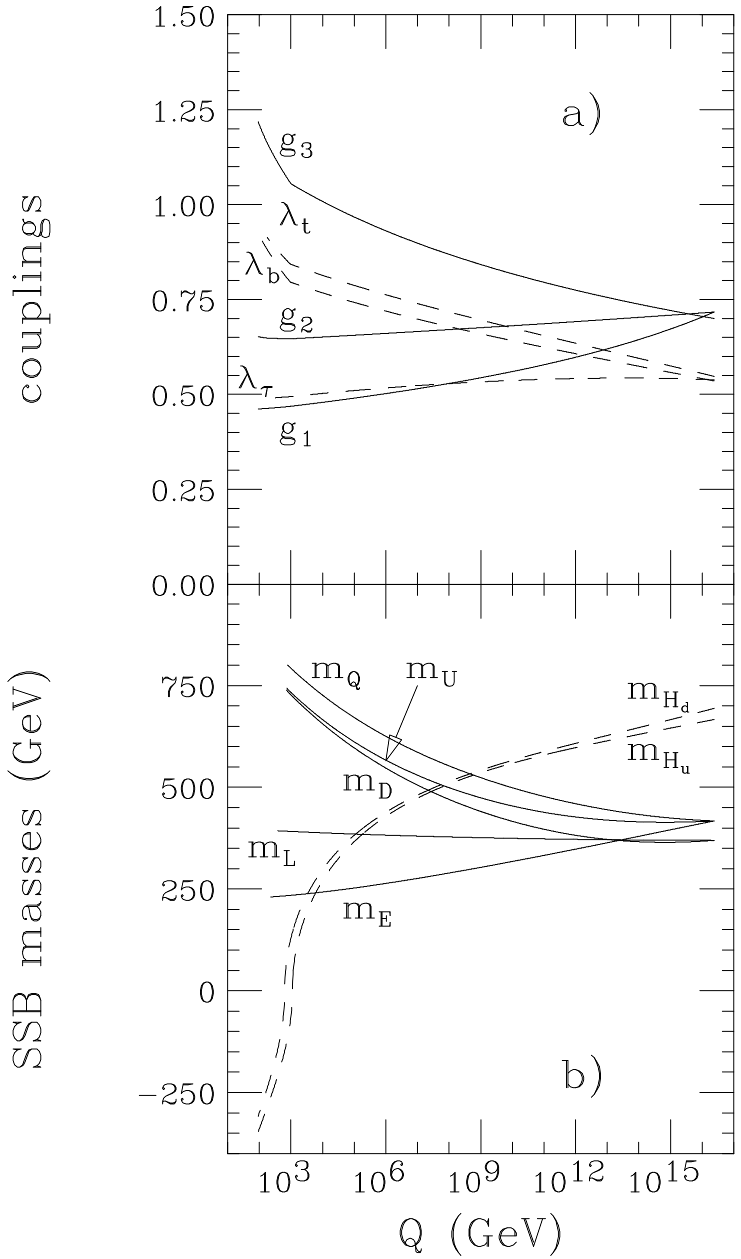
$\lambda_\tau = \lambda_b$ or $\lambda_\tau = \lambda_b = \lambda_t$

identical coupling to Higgs boson(s) at GUT scale

top-bottom $\rightarrow m_t / m_b \sim$ ratio of vacuum expectation values

request $\frac{\delta m_b}{m_b} \sim \frac{\delta m_t}{m_t}$

$\delta m_t \approx 1$ GeV $\Rightarrow \delta m_b \approx 25$ MeV



Baer *et al.*

Phys.Rev.D61,2000

II. CHARM and BOTTOM

MASSES

in collaboration with

K. Chetyrkin, Y. Kiyo, A. Maier, P. Maierhöfer, P. Marquard, A. Smirnov, M. Steinhauser, C. Sturm

and the HPQCD Collaboration

NPB 619 (2001) 588

EPJ C48 (2006) 107

NPB 778 (2008) 05413

PLB 669 (2008) 88

NPB 823 (2009) 269

PRD 80 (2009) 074010

NPB 824 (2010) 1

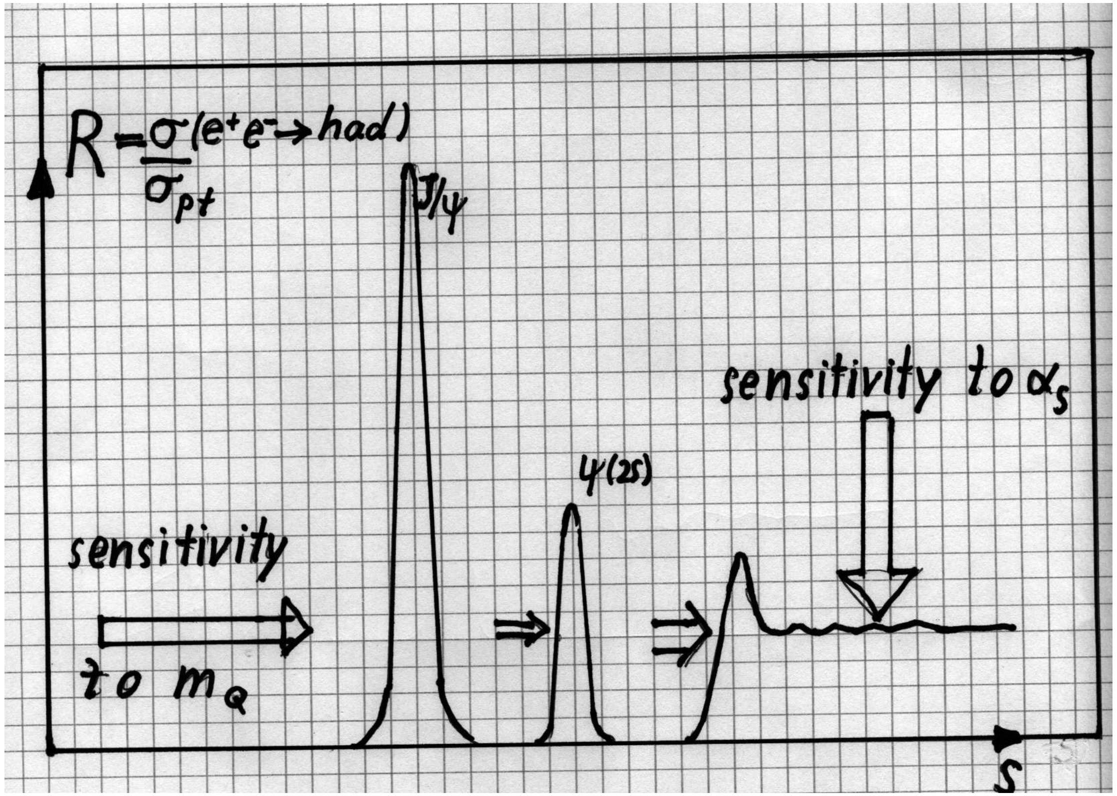
Proceedings of QUARKS 2010

(Kolomna, Russia)

arXiv:1010.6157

II. 1. m_Q from SVZ Sum Rules, Moments and Tadpoles

Main Idea (SVZ)



Sensitivity to m_Q from location of $Q\bar{Q}$ threshold.

Some definitions:

$$\left(-q^2 g_{\mu\nu} + q_\mu q_\nu\right) \Pi(q^2) \equiv i \int \mathrm{d}x\, e^{iqx} \langle T j_\mu(x) j_\nu(0) \rangle$$

with the electromagnetic current j_μ.

$$R(s) = 12\pi \, \mathrm{Im} \left[\Pi(q^2 = s + i\varepsilon) \right]$$

Taylor expansion: $\Pi_Q(q^2) = Q_Q^2 \frac{3}{16\pi^2} \sum_{n \geq 0} \bar{C}_n z^n$

with $z = q^2/(4m_Q^2)$ and $m_Q = m_Q(\mu)$ the $\overline{\mathrm{MS}}$ mass.

$$\bar{C}_n = \bar{C}_n^{(0)} + \frac{\alpha_s}{\pi} \bar{C}_n^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \bar{C}_n^{(2)} + \left(\frac{\alpha_s}{\pi}\right)^3 \bar{C}_n^{(3)} + \ldots$$

generic form

$$\bar{C}_n = \bar{C}_n^{(0)} + \frac{\alpha_s}{\pi}\left(\bar{C}_n^{(10)} + \bar{C}_n^{(11)} l_{m_c}\right) + \left(\frac{\alpha_s}{\pi}\right)^2\left(\bar{C}_n^{(20)} + \bar{C}_n^{(21)} l_{m_c} + \bar{C}_n^{(22)} l_{m_c}^2\right) + \left(\frac{\alpha_s}{\pi}\right)^3\left(\bar{C}_n^{(30)} + \bar{C}_n^{(31)} l_{m_c} + \bar{C}_n^{(32)} l_{m_c}^2 + \bar{C}_n^{(33)} l_{m_c}^3\right) + \ldots$$

with $\alpha_s = \alpha_s(\mu)$, $l_{m_c} = \ln\left(\frac{m_c^2}{\mu^2}\right)$

$\bar{C}_n^{(ij)}$ = pure numbers

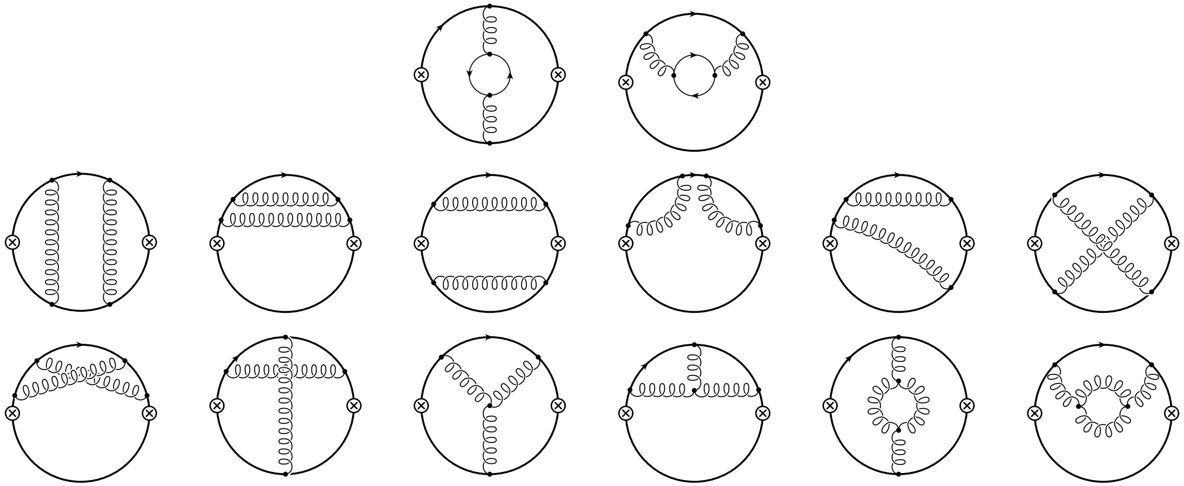
for $j \geq 1$ from RG

for $j = 0$: calculation

Analysis in NNLO

Coefficients $\overline{C}_n$ from three-loop one-scale tadpole amplitudes with

“arbitrary” power of propagators;



- FORM program MATAD

  Coefficients $\bar{C}_n$ up to $n = 8$

  (also for axial, scalar and pseudoscalar correlators)

  (Chetyrkin, JK, Steinhauser, 1996)

- up to $n = 30$ for vector correlator

  (Boughezal, Czakon, Schutzmeier 2007)

- up to $n = 30$ for vector, axial, scalar and pseudoscalar correlators

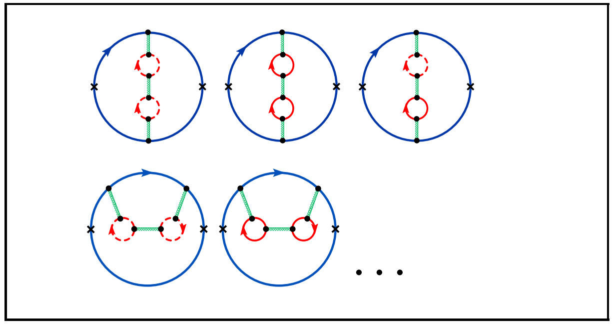
  (A. Maier, P. Maierhöfer, P. Marquard, 2007)

Analysis in N^{3}LO

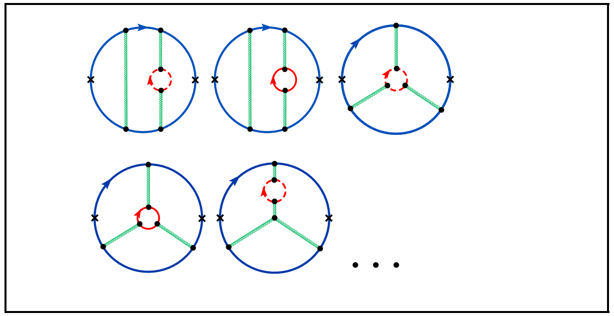
Algebraic reduction to 13 master integrals (Laporta algorithm);

numerical and analytical evaluation of master integrals

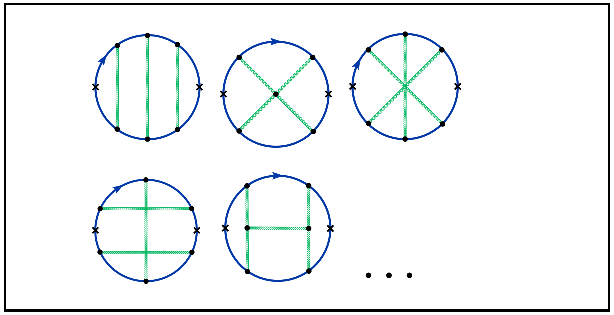
n_f^2-contributions



n_f^1-contributions



n_f^0-contributions



 : heavy quarks,  : light quarks,

n_f: number of active quarks

$\Rightarrow$ About 700 Feynman-diagrams

$\bar{C}_0$ and $\bar{C}_1$ in order α_s^3 (four loops!) (2006)

Reduction to master integrals

(Chetyrkin, JK, Sturm; Boughezal, Czakon, Schutzmeier)

$\bar{C}_2$ and $\bar{C}_3$ (2008)

(Maier, Maierhöfer, Marquard, A. Smirnov)

All master integrals known analytically and double checked.

(Schröder + Vuorinen, Chetyrkin et al., Schröder + Steinhauser,

Laporta, Broadhurst, Kniehl et al.)

$\bar{C}_4 - \bar{C}_{10}$: extension to higher moments by Padé method, using analytic information from low energy ($q^2 = 0$), threshold ($q^2 = 4m^2$), high energy ($q^2 = -\infty$) (Kiyo, Maier, Maierhöfer, Marquard, 2009)

Relation to measurements

$$\mathcal{M}_n^{\rm th} \equiv \frac{12\pi^2}{n!}\left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi_c(q^2)\Big|_{q^2=0} = \frac{9}{4}Q_c^2\left(\frac{1}{4m_c^2}\right)^n \bar{C}_n$$

Perturbation theory: $\bar{C}_n$ is function of α_s and $\ln\frac{m_c^2}{\mu^2}$

dispersion relation:

$$\Pi_c(q^2) = \frac{q^2}{12\pi^2}\int \mathrm{d}s\, \frac{R_c(s)}{s(s-q^2)} + \text{subtraction}$$

$$\Rightarrow \mathcal{M}_n^{\rm exp} = \int \frac{\mathrm{d}s}{s^{n+1}} R_c(s)$$

constraint: $\mathcal{M}_n^{\rm exp} = \mathcal{M}_n^{\rm th}$

$$\Rightarrow m_c = \frac{1}{2}\left(\frac{9}{4}Q_c^2\bar{C}_n/\mathcal{M}_n^{\rm exp}\right)^{1/(2n)}$$

qualitative considerations

$$\mathcal{M}_n = \int_{threshold} \frac{ds}{s^{n+1}} R_c(s) \sim \text{dimension } (m_c)^{-2n}$$

- depends moderately on α_s!

- $\Pi(q^2)$ is an analytic function with branch cut from $(2m_D)^2$ to ∞.

- averaging over resonances reduces influence of long distances

  (binding effects).

- $\Pi(q^2 = 0)$ and its derivatives at $q^2 = 0$ are short distance quantities.

  $\Rightarrow$ pQCD is applicable.

n	1	2	3	4
charm	-5.6404	-3.4937	-2.8395	$-3.349(11)$
lower $\|\|$ upper limits	—	$-6.0 \|\| 7.0$	$-6.0 \|\| 5.2$	$-6.0 \|\| 3.1$
bottom	-7.7624	-2.6438	-1.1745	$-1.386(10)$
lower $\|\|$ upper limits	—	$-8.0 \|\| 9.5$	$-8.0 \|\| 8.3$	$-8.0 \|\| 7.4$

Analytic results for the coefficients $\overline{C}_n^{(30)}$ in comparison with previous upper and lower limits.

II. 2. Experimental Analysis: m_c

$$\mathcal{M}_n^{\mathrm{exp}} \equiv \int \frac{ds}{s^{n+1}} R_{\mathrm{charm}}(s)$$

$$\Rightarrow m_c = \frac{1}{2}\left(\frac{9}{4} Q_c^2 \bar{C}_n / \mathcal{M}_n^{\mathrm{exp}}\right)^{1/(2n)}$$

Ingredients (charm)

experiment:

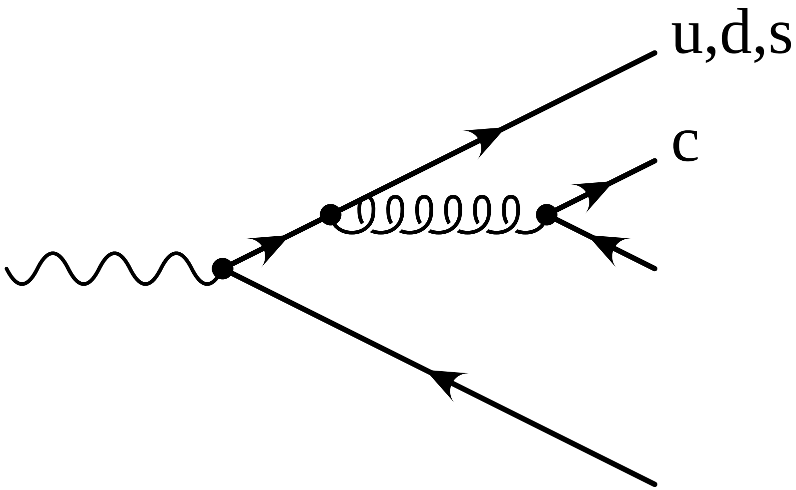
- $\Gamma_e(J/\psi, \psi')$ from BES & CLEO & BABAR (PDG)
- $\psi(3770)$ and $R(s)$ from BES
- $\alpha_s = 0.1187 \pm 0.0020$

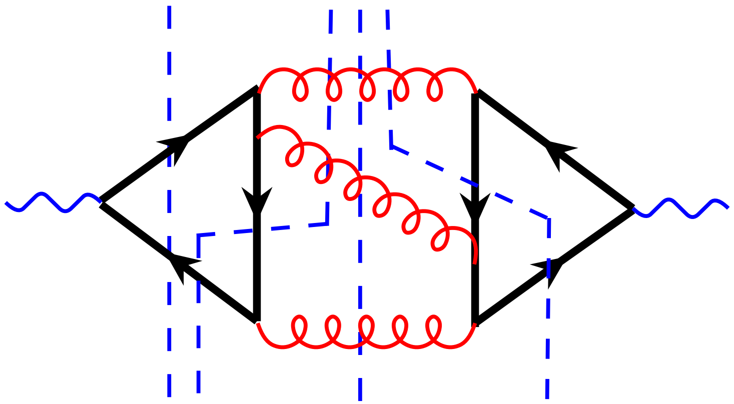
theory:

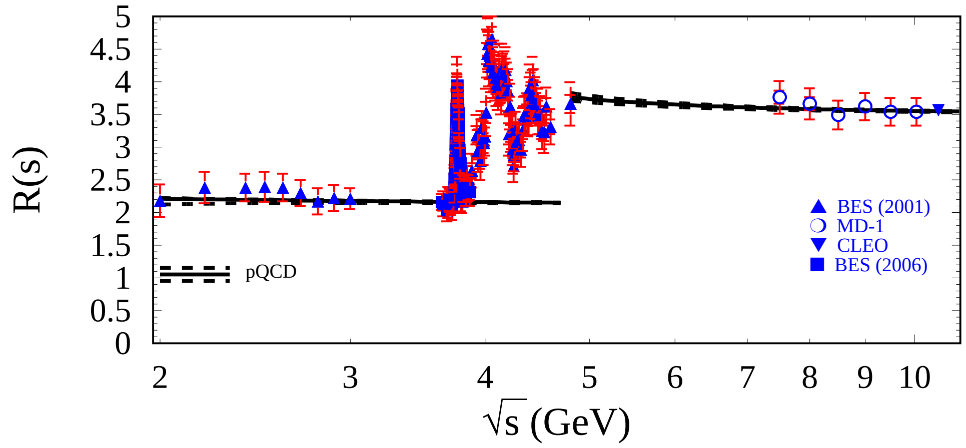
- N^3LO for $n = 1, 2, 3, 4$
- include condensates

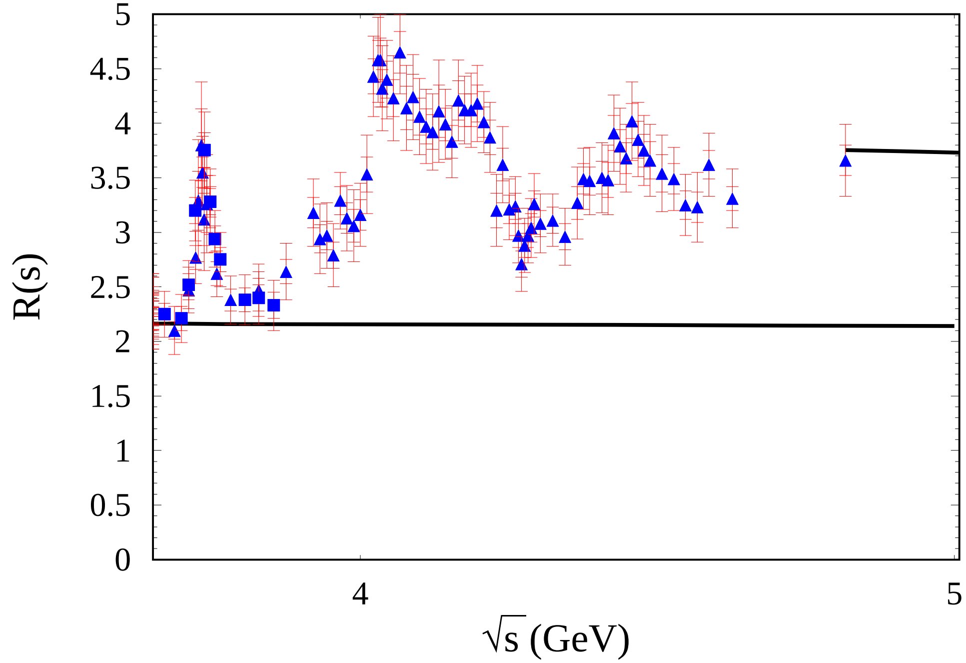
$$\delta\mathcal{M}_n^{\rm np} = \frac{12\pi^2 Q_c^2}{(4m_c^2)^{(n+2)}} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle a_n \left(1 + \frac{\alpha_s}{\pi} \bar{b}_n\right)$$

- estimate of non-perturbative terms (oscillations, based on Shifman)
- careful extrapolation of R_{uds}
- careful definition of R_c


u,d,s
c




R(s)
√s (GeV)
BES (2001)
MD-1
CLEO
BES (2006)
pQCD


R(s)
√s (GeV)

Contributions from

- narrow resonances: $R = \frac{9\Pi M_R \Gamma_e}{\alpha^2(s)} \delta(s - M_R^2)$ (PDG)
- threshold region (2 m_D – 4.8 GeV) (BESS)
- perturbative continuum ($E \geq$ 4.8 GeV) (Theory)

n	$\mathcal{M}_n^{\rm res}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\rm thresh}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\rm cont}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\rm exp}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\rm np}$ $\times 10^{(n-1)}$
1	0.1201(25)	0.0318(15)	0.0646(11)	0.2166(31)	−0.0001(2)
2	0.1176(25)	0.0178(8)	0.0144(3)	0.1497(27)	0.0000(0)
3	0.1169(26)	0.0101(5)	0.0042(1)	0.1312(27)	0.0007(14)
4	0.1177(27)	0.0058(3)	0.0014(0)	0.1249(27)	0.0027(54)

Different relative importance of resonances vs. continuum for $n = 1, 2, 3, 4$.

Strong impact of $\mathcal{M}_n^{\mathrm{cont}}$!

theory confirmed by experiment

4.8 GeV (BESS), 9 GeV (MD-1), 10.52 GeV (CLEO)

to better than 7%, 4.5%, 2%

$\sqrt{s}$ (GeV)	2.00	3.65	3.732	4.80	9.00	10.52
$R^{\mathrm{th}}(s)$	2.209(91)	2.161(18)	2.160(17)	3.764(64)	3.564(17)	3.548(12)
$R^{\mathrm{exp}}(s)$	2.18(7)(18)	2.157(35)(86)	2.156(86)(86)	3.66(14)(19)	3.62(7)(14)	3.56(1)(7)
Experiment	BES	BES	BES	BES	MD-1	CLEO

Comparison of the theory predictions for $R(s)$ with the experimental results at a few selected values for $\sqrt{s}$.

Potential experimental improvements

1.) narrow resonances ($J/\Psi, \Psi'$) dominate:

$\Gamma_e(1S) = 5.55 \pm 0.14 \pm 0.02$ keV; $\Gamma_e(2S) = 2.35 \pm 0.04$ keV

improvement? correlations?

2.) threshold region: improvement at BESS?

subtraction of $u\bar{u}$, $d\bar{d}$, $s\bar{c} \Rightarrow$ precise reference point below charm threshold needed

3.) continuum above 4.8 GeV:

missing data substituted by theory $\Rightarrow$ small error

excellent agreement with data

improved calibration at ~ 4.5 GeV desirable

$\rightarrow$ crucial information over wide range

Results (m_c)

PRD 80: (2009) 074010

n	$m_c(3\,\text{GeV})$	exp	α_s	μ	np	total
1	986	9	9	2	1	13
2	976	6	14	5	0	16
3	978	5	15	7	2	17
4	1004	3	9	31	7	33

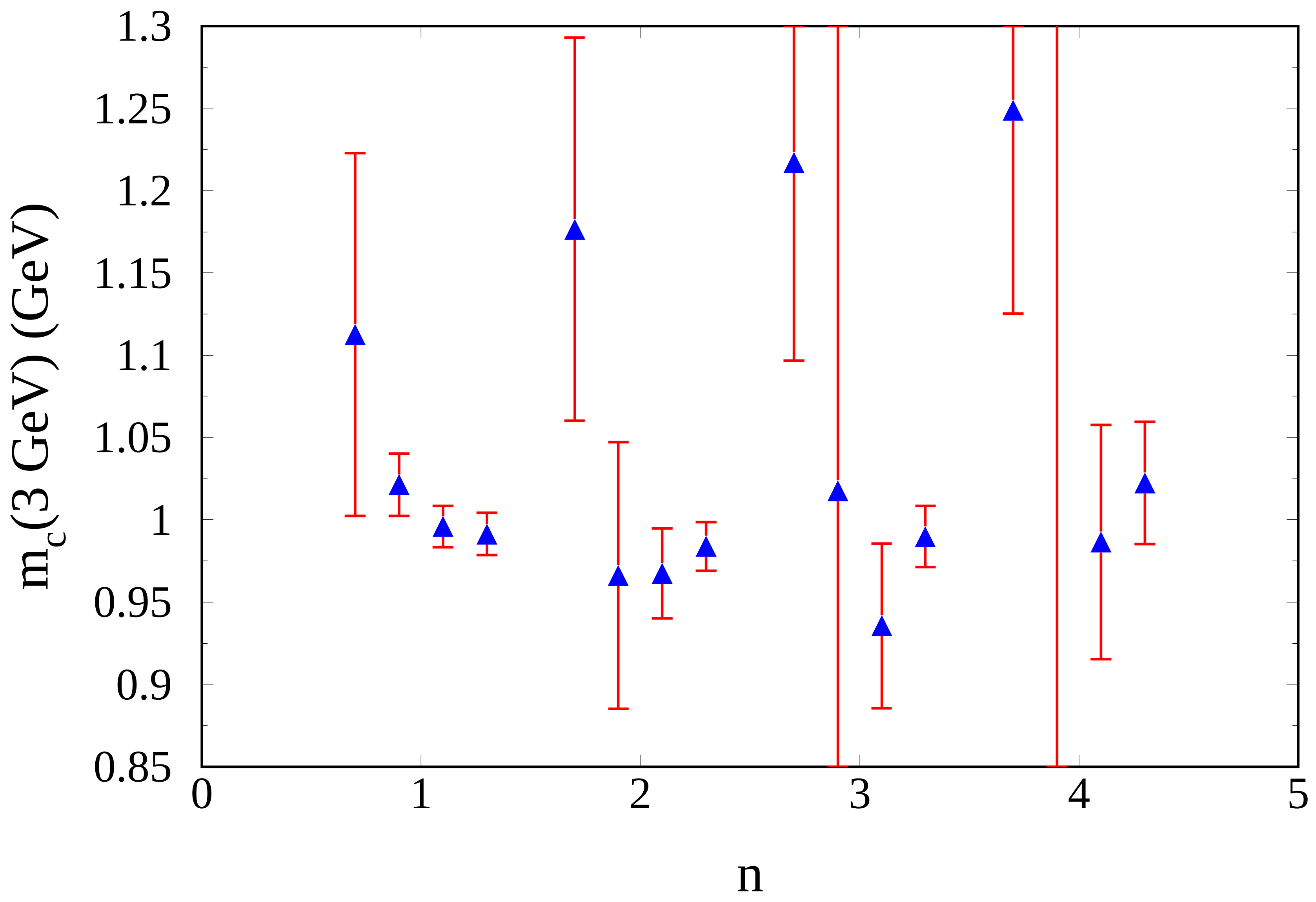
Remarkable consistency between $n = 1, 2, 3, 4$

and stability ($\mathcal{O}(\alpha_s^2)$ vs. $\mathcal{O}(\alpha_s^3)$);

prefered scale: $\mu = 3\,\text{GeV}$,

conversion to $m_c(m_c)$:

- $m_c(3\,\text{GeV}) = 986 \pm 13\,\text{MeV}$
- $m_c(m_c) = 1279 \pm 13\,\text{MeV}$



1.3
1.25
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0
1
2
3
4
5
n
m_c(3 GeV) (GeV)

Perturbative stability

$$m_c = \frac{1}{2}\left(\frac{9Q_c^2}{4}\frac{\bar{C}_n^{\rm Born}}{\mathcal{M}_n^{\rm exp}}\right)^{\frac{1}{2n}}\left(1+r_n^{(1)}\alpha_s+r_n^{(2)}\alpha_s^2+r_n^{(3)}\alpha_s^3\right)$$
$$\propto 1-\begin{pmatrix}0.328\\0.524\\0.618\\0.662\end{pmatrix}\alpha_s-\begin{pmatrix}0.306\\0.409\\0.510\\0.575\end{pmatrix}\alpha_s^2-\begin{pmatrix}0.262\\0.230\\0.299\\0.396\end{pmatrix}\alpha_s^3, \qquad (1)$$

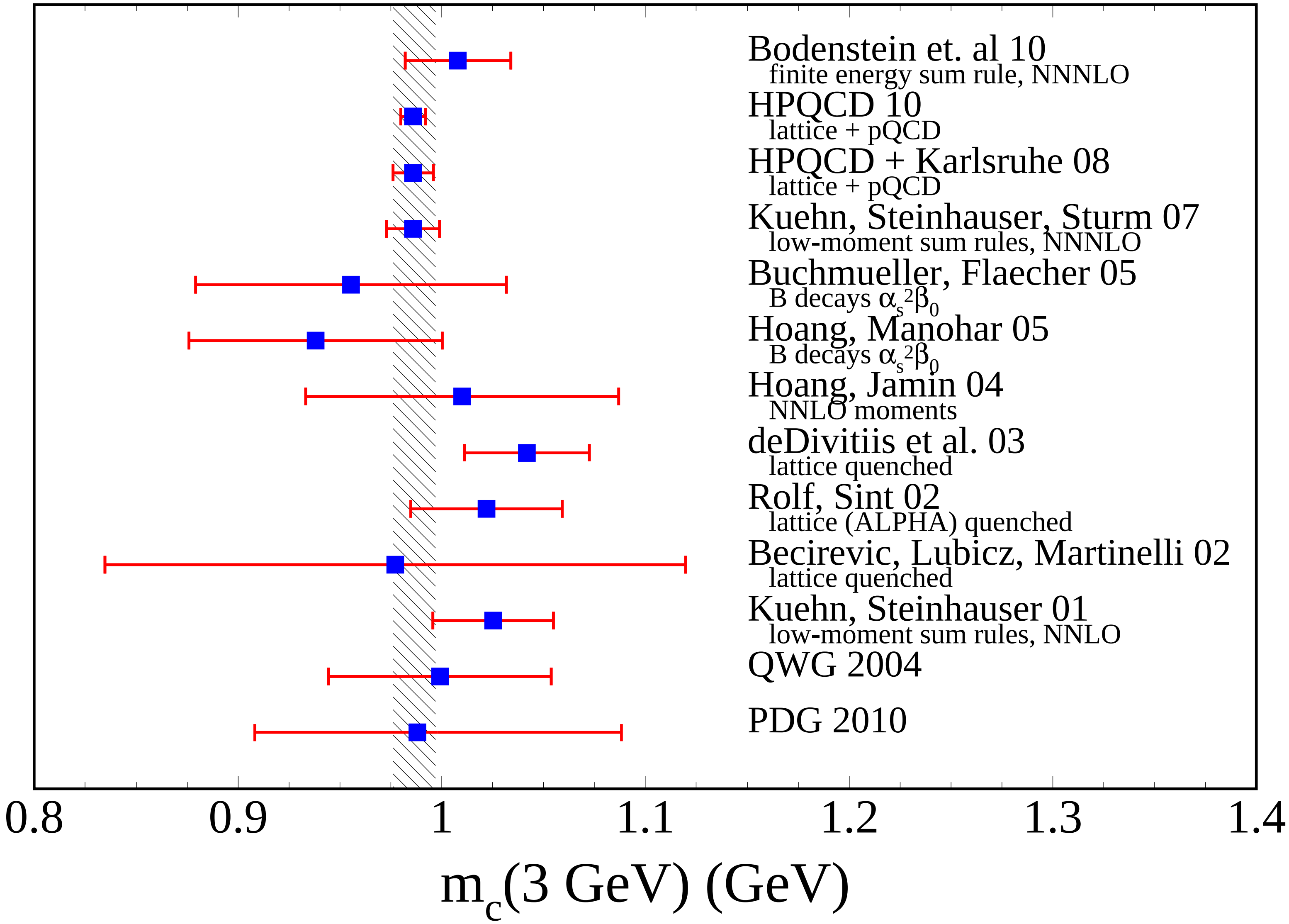
error from next order $\leq r_n^{max}\alpha_s^4 < 2$ to 3 permille

(smaller than μ-variation!)

potential improvement:

combined fit to three (or four) lowest moments

error correlations? (theory & experiment)

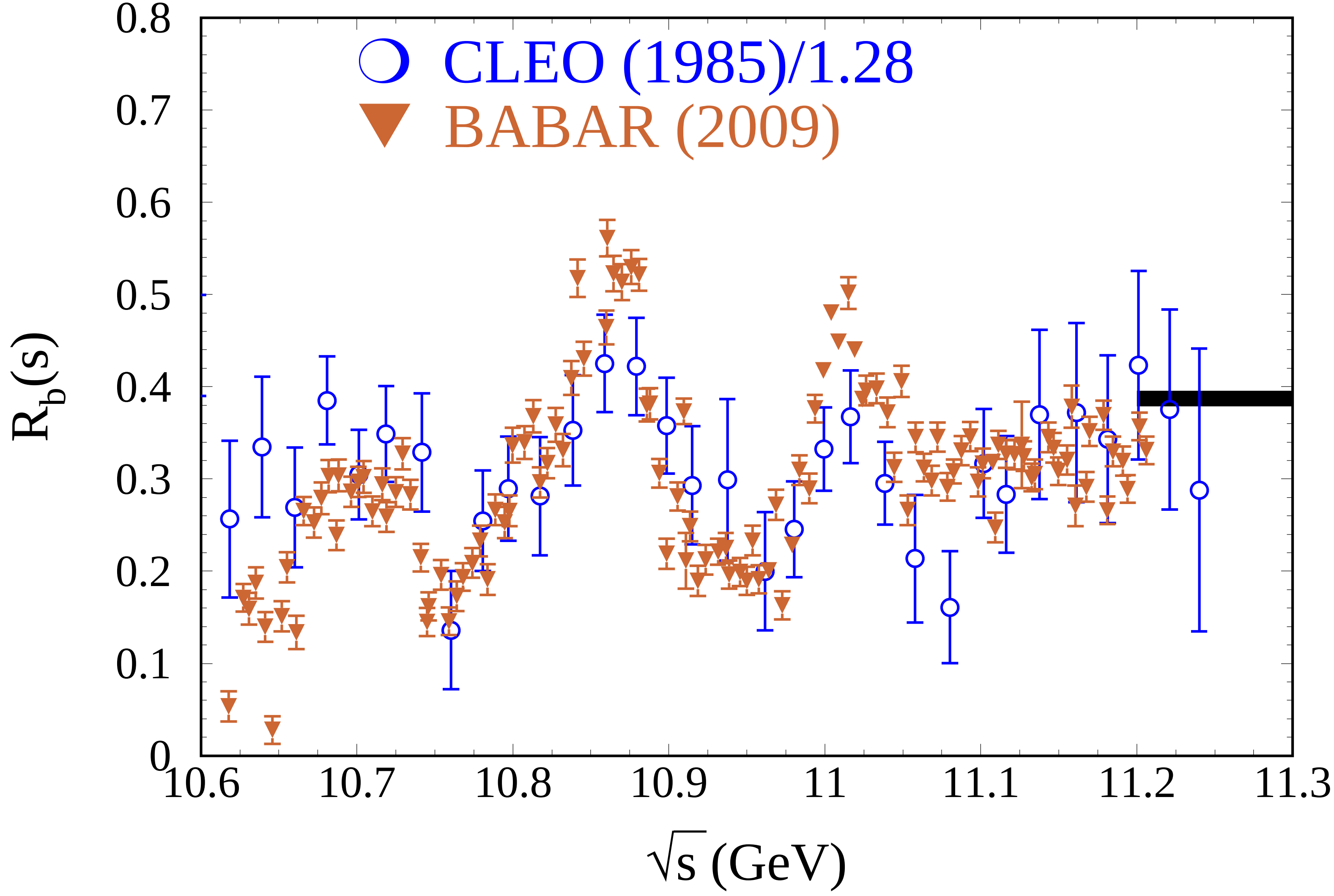

Bodenstein et. al 10
finite energy sum rule, NNNLO
HPQCD 10
lattice + pQCD
HPQCD + Karlsruhe 08
lattice + pQCD
Kuehn, Steinhauser, Sturm 07
low-moment sum rules, NNNLO
Buchmueller, Flaecher 05
B decays $\alpha_s^2\beta_0$
Hoang, Manohar 05
B decays $\alpha_s^2\beta_0$
Hoang, Jamin 04
NNLO moments
deDivitiis et al. 03
lattice quenched
Rolf, Sint 02
lattice (ALPHA) quenched
Becirevic, Lubicz, Martinelli 02
lattice quenched
Kuehn, Steinhauser 01
low-moment sum rules, NNLO
QWG 2004
PDG 2010
0.8
0.9
1
1.1
1.2
1.3
1.4
m_c(3 GeV) (GeV)

Experimental Ingredients for m_b

Contributions from

- narrow resonances ($\Upsilon(1S)$ – $\Upsilon(4S)$) (PDG)
- threshold region (10.618 GeV – 11.2 GeV) (BABAR 2009)
- perturbative continuum ($E \geq 11.2$ GeV) (Theory)
- different relative importance of resonances vs. continuum for $n = 1, 2, 3, 4$

n	$\mathcal{M}_n^{\mathrm{res},(1S-4S)}$ $\times 10^{(2n+1)}$	$\mathcal{M}_n^{\mathrm{thresh}}$ $\times 10^{(2n+1)}$	$\mathcal{M}_n^{\mathrm{cont}}$ $\times 10^{(2n+1)}$	$\mathcal{M}_n^{\mathrm{exp}}$ $\times 10^{(2n+1)}$
1	1.394(23)	0.287(12)	2.911(18)	4.592(31)
2	1.459(23)	0.240(10)	1.173(11)	2.872(28)
3	1.538(24)	0.200(8)	0.624(7)	2.362(26)
4	1.630(25)	0.168(7)	0.372(5)	2.170(26)



CLEO (1985)/1.28
BABAR (2009)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$R_b(s)$
10.6
10.7
10.8
10.9
11
11.1
11.2
11.3
$\sqrt{s}$ (GeV)

BELLE?

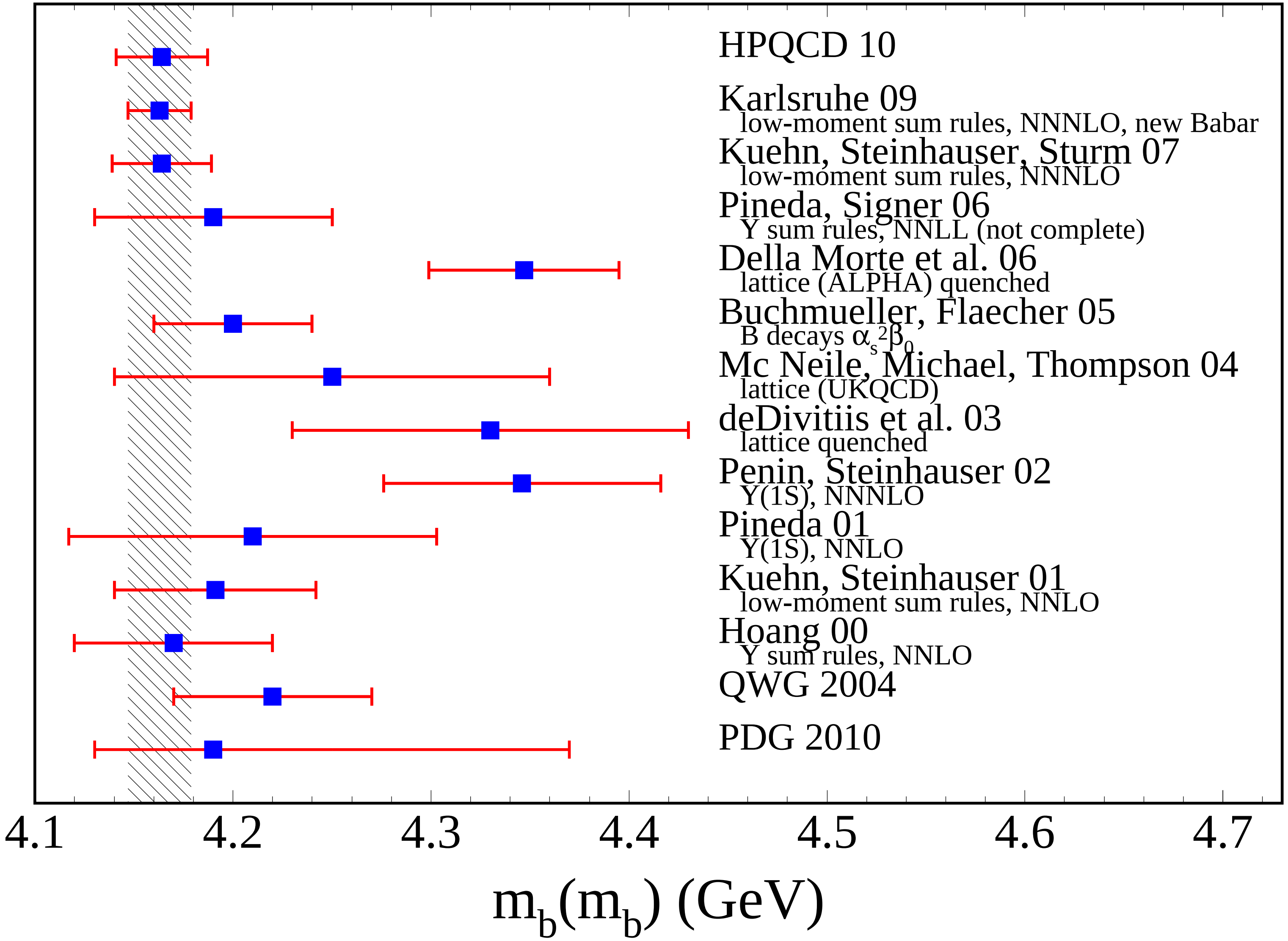
n	$m_b(10\,\text{GeV})$	exp	α_s	μ	total	$m_b(m_b)$
1	3597	14	7	2	16	4151
2	3610	10	12	3	16	4163
3	3619	8	14	6	18	4172
4	3631	6	15	20	26	4183

Consistency ($n = 1, 2, 3, 4$) and stability ($\mathcal{O}(\alpha_s^2)$ vs. $\mathcal{O}(\alpha_s^3)$);

(slight dependence on n could result from input into $\mathcal{M}^n_{\text{exp}}$)

- $m_\text{b}(10\,\text{GeV}) = 3610 \pm 16\,\text{MeV}$
- $m_\text{b}(m_\text{b}) = 4163 \pm 16\,\text{MeV}$

well consistent with KSS 2007


HPQCD 10
Karlsruhe 09
low-moment sum rules, NNNLO, new Babar
Kuehn, Steinhauser, Sturm 07
low-moment sum rules, NNNLO
Pineda, Signer 06
Υ sum rules, NNLL (not complete)
Della Morte et al. 06
lattice (ALPHA) quenched
Buchmueller, Flaecher 05
B decays $\alpha_s^2\beta_0$
Mc Neile, Michael, Thompson 04
lattice (UKQCD)
deDivitiis et al. 03
lattice quenched
Penin, Steinhauser 02
Υ(1S), NNNLO
Pineda 01
Υ(1S), NNLO
Kuehn, Steinhauser 01
low-moment sum rules, NNLO
Hoang 00
Υ sum rules, NNLO
QWG 2004
PDG 2010
4.1
4.2
4.3
4.4
4.5
4.6
4.7
$m_b(m_b)$ (GeV)

α_s-dependence

$$m_c(3\text{ GeV}) = \left(986 - \frac{\alpha_s - 0.1189}{0.002} \cdot 9 \pm 10\right)\text{ MeV}$$

$$m_b(10\text{ GeV}) = \left(3610 - \frac{\alpha_s - 0.1189}{0.002} \cdot 12 \pm 11\right)\text{ MeV}$$

$$m_b(m_b) = \left(4163 - \frac{\alpha_s - 0.1189}{0.002} \cdot 12 \pm 11\right)\text{ MeV}$$

$$m_b(M_Z) = \left(2835 - \frac{\alpha_s - 0.1189}{0.002} \cdot 27 \pm 8\right)\text{ MeV}$$

$$m_b(161.8\text{ GeV}) = \left(2703 - \frac{\alpha_s - 0.1189}{0.002} \cdot 28 \pm 8\right)\text{ MeV}$$

lattice & pQCD

(HPQCD + Karlsruhe, Phys. Rev. D78, 054513)

lattice evaluation of pseudoscalar correlator

$\Rightarrow$ replace experimental moments by lattice simulation

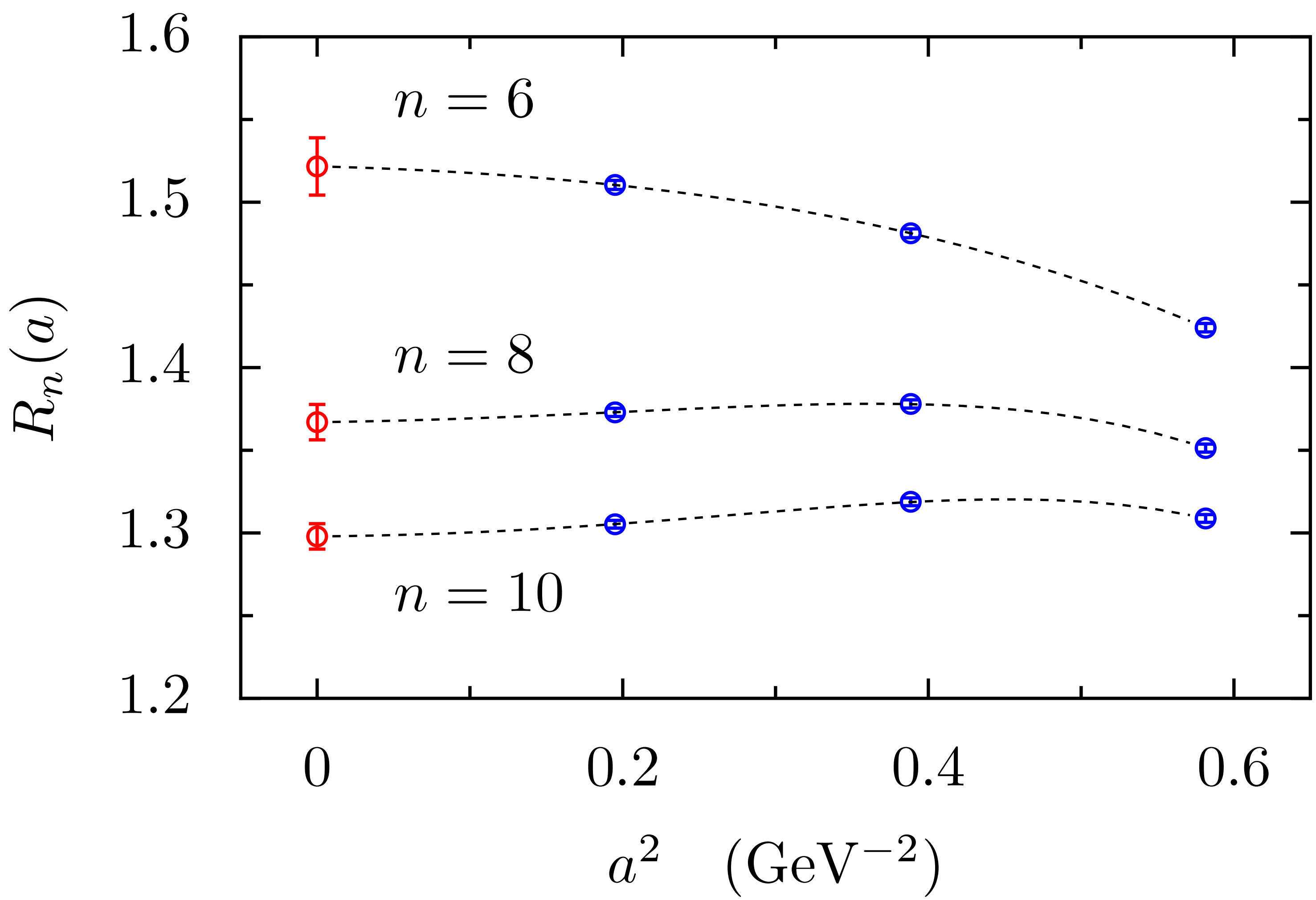
input: $M(\eta_c) \hat{=} m_c\,, \quad M(\Upsilon(1S)) - M(\Upsilon(2s)) \hat{=} \alpha_s$

pQCD for pseudoscalar correlator available:

“all” moments in $O(\alpha_s^2)$

three lowest moments in $O(\alpha_s^3)$.

lowest moment: dimensionless: $\sim \left(\bar{C}^{(0)} + \frac{\alpha_s}{\pi}\bar{C}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2\bar{C}^{(2)} + \left(\frac{\alpha_s}{\pi}\right)^3\bar{C}^{(3)} + \ldots\right)$



Reduced moments R_n from lattice simulations with different lattice spacings a. The dashed lines show the functions used to fit the lattice results. These extrapolation functions were used to obtain the $a = 0$ results shown in the plot.

	R_6	R_8	R_{10}	R_{12}
a^2 extrapolation	1.3%	0.9%	0.7%	0.5%
pert'n theory	0.4	0.9	1.3	1.6
$\alpha_{\overline{\mathrm{MS}}}$ uncertainty	0.3	0.6	1.0	1.3
gluon condensate	0.3	0.0	0.3	0.7
statistical errors	0.0	0.0	0.0	0.0
relative scale errors	0.4	0.4	0.3	0.3
overall scale errors	0.6	0.6	0.7	0.7
sea quarks	0.2	0.2	0.1	0.1
finite volume	0.1	0.1	0.3	0.4
Total	1.6%	1.6%	2.0%	2.4%

Sources of uncertainty in the determinations of $m_c(\mu = 3\,\mathrm{GeV})$ from different reduced moments R_n of the pseudoscalar correlator. The uncertainties listed are percentages of the final result 0.984 (16) GeV.

lowest moment:

$\Rightarrow$ $\alpha_s(3\text{GeV}) \Rightarrow \alpha_s(M_Z) = 0.1174(12)$

higher moments: $\sim m_c^2 \times \left(1 + \ldots \frac{\alpha_s}{\pi} \ldots\right)$

$\Rightarrow$ $m_c(3\text{GeV}) = 986(10)$ MeV

to be compared with 986(13) MeV from e^+e^- !

update: HPQCD 2010

$\alpha_s(3\text{GeV}) \Rightarrow \alpha_s(M_Z) = 0.1183(7)$

$m_c(3\text{GeV}) = 986(6)$ MeV

$m_b(10\text{GeV}) = 3617(25)$ MeV

note: lattice simulation leads directly to predictions for the moments ($\widehat{=}$ observables)

SUMMARY

$$m_c(3\,\text{GeV}) = 986(13)\ \text{MeV}$$

$e^+e^- +$ pQCD

$$m_c(3\,\text{GeV}) = 986(6)\ \text{MeV}$$

lattice + pQCD

$$m_b(10\,\text{GeV}) = 3610(16)\ \text{MeV}$$
$$m_b(m_b) = 4163(16)\ \text{MeV}$$

$e^+e^- +$ pQCD