Wilson loops and the generalized quark-antiquark potential in ABJM theory

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arXiv:1209.4032 [hep-th], V.Cardinali, L.Griguolo, G.M., D.Seminara

Summary

Summary

1. History
2. Motivations
3. Wilson Loop
4. Wilson Loops in $\mathcal{N} = 4$ SYM
5. Wilson Loops in ABJM theory
6. Conclusions

Duality between quantum gauge field theory-string theory

- 1997 Maldacena conjecture$\rightarrow$ AdS/CFT (realization of t'Hooft's idea)
- Duality between CFT in D dimensions and String theory in (AdS_{D+1} $\times$ internal spaces) dimensions

The conjecture in the large $\lambda = g_{YM}^2 N$ limit ($\alpha' \sim \frac{1}{\sqrt{\lambda}}$)

• large λ $\mathcal{N} = 4$ SYM for $N \rightarrow \infty$ • Expansion in $\lambda^{-\frac{1}{2}}$	$\Longleftrightarrow$	• Classical IIB Supergravity on the background $AdS_5 \times S^5$ • Expansion in α'

- AdS_4/CFT_3 $\rightarrow$ 't Hooft's limit of ABJM ($\mathcal{N} = 6$) $\rightarrow$ Classical Supergravity IIA on $AdS_4 \times \mathbb{CP}_3$
- find observables in QFT exactly calculable: anomalous dimension of composite operator, Wilson Loops etc...
- Phenomenological applications
- AdS/QCD (Sakai, Sugimoto, 2004) inserting branes to introduce flavors $\Rightarrow$ 5 $\Rightarrow$ 4 dimensions
- AdS/Condensed-Matter (Hartnoll, 2008), strong coupling problem, Quantum phase transition$\Rightarrow$ 4 $\Rightarrow$ 3 dimensions

Conformal Field Theory

- scale invariance: no asymptotic states, no particles
- what is the spectrum of a conformal field theory?
- 2-3-points functions are fixed by conformal symmetry
- the spectrum is the anomalous dimensions of the operators of the theory and the structure constant of three points-functions

$$< \mathcal{O}_1 \mathcal{O}_2 >= \frac{C_{12}}{(x_1 - x_2)^{2\Delta}}$$

$$< \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 >= \frac{C_{123}}{(x_{12})^{2(\Delta_1+\Delta_2-\Delta_3)}(x_{13})^{2(\Delta_1-\Delta_2+\Delta_3)}(x_{23})^{2(-\Delta_1+\Delta_2+\Delta_3)}}$$

- consistency condition: four points-funtions in term of three points-functions and so on...conformal bootstrap
- particular case: CFT in 2 dimensions$\rightarrow$ conformal algebra= Virasoro algebra $\rightarrow$ infinite symmetries $\rightarrow$ integrability
- deep connection between CFT in 2 dimensions and supersymmetry in dimensions more than 2: AdS/CFT and the AGT conjecture: N=2 theories in 4 dimensions mapped in the Liouville theory

AdS/CFT as a test of Integrability

- Integrable theories??? $\Rightarrow \mathcal{N} = 4$ SYM and $\mathcal{N} = 6$ SCS (ABJM)
- Integrable structure in 4 and 3 dimensions $\Rightarrow$ for the first time we have strong indications about Integrability in dimensions more than 2
- classical sigma model on $AdS_5 \times S_5$ is integrable (BPR, 2004)
- one loop mixing matrix for anomalous dimensions in $\mathcal{N} = 4$ SYM $\Rightarrow$ matrix identified with the Hamiltonian of an INTEGRABLE spin chain with vector sites $\Rightarrow$ Bethe Ansatz (Minahan, Zarembo 2003) $\rightarrow$ all-loop Bethe Ansatz (Beisert, Staudacher 2005)
- Integrable theory $\Rightarrow$ infinite symmetries: what the hidden symmetry in $\mathcal{N} = 4$? $\Rightarrow$ Yangian symmetry from gluon scattering amplitudes in planar limit (DHP 2009)
- also in ABJM there is a Yangian symmetry from scattering amplitudes (BLM 2010,Caron-Huot,Huang 2012) and all-loop Bethe Ansatz (Gromov, Vieira, 2009)

Tools for exact calculations

- all-loop Bethe Ansatz for NON-BPS observables
- localization for BPS observables

Summary

TBA and Localization

- there is a quantity calculable both with Bethe Ansatz and both with localization?
- the Bremsstrahlung function $B(\lambda)$ $\to$ energy emitted by a small velocity moving quark ($\cosh\phi = -\frac{p_1 \cdot p_2}{\sqrt{p_1^2 p_2^2}}$)

- as in the electrodynamics case with the replacement $\frac{2e^2}{3} \to 2\pi B$ (Lienard-Wiechert-fields and Larmor-formula)

$$\Delta E = 2\pi B \int dt (\dot{v})^2$$

- through Wilson Loops Maldacena at al. calculated in $\mathcal{N} = 4$ SYM the Bremsstrahlung function $B(\lambda)$ with a new set of integral equations of TBA type
- the same quantity is related to cusp anomalous dimension
- in a particular limit cusp anomalous dimension $\to$ BPS Wilson Loop
- BPS Wilson Loop $\to$ localization
- also in ABJM is possible a similar connection? An open problem...

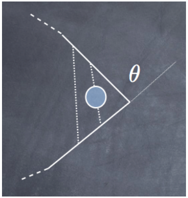
Summary

The Wilson Loop

$$W_f(C) = \frac{1}{\dim(f)} \mathrm{Tr}_f \left(P \exp \left(i \oint_C A_\mu dx^\mu \right) \right)$$

- W gives the potential between a static pair of quark-antiquark
- renormalization of Wilson Loop with cusp and cusp anomaly: infrared divergence in QCD and scattering of gluons in planar limit

$$W(C) = Z_C f(g_{ren}, C)$$



- equation for the renormalization of wave function (L perimeter and ϵ UV cut-off)

$$\ln Z_C \simeq -\gamma \frac{L}{\epsilon} + \Gamma_{cusp}(g,\theta) \log \frac{L}{\epsilon} + finite \qquad \Gamma_{cusp}(g,\theta) \simeq \frac{\alpha_{strong} N_c}{2\pi}(\theta \cot\theta - 1)$$

- in Minkowski space when $i\theta \to \infty$ factorization of the angle

$$\Gamma_{cusp}(g,\theta) = \gamma_{cusp}(g)\theta$$

Summary

Construction of Wilson Loop in $\mathcal{N} = 4$ SYM

- no field transforms in the fundamental representation of the gauge group
- to have particles as massive-quarks → break the original U(N + 1) gauge symmetry to $U(N) \times U(1)$
- the massive W-bosons have a mass proportional to the VEV of a Higgs fields and transform in the fundamental representation of U(N)
- equation of motion of W

$$(\partial_0 - iA_0 - i\theta_I\phi^I)W = 0$$

- A_0, ϕ^I are matrices in the adjoint of U(N)

$$\langle W(C)\rangle = \frac{1}{N}\left\langle TrP \exp i \oint_C ds \left(A_\mu \dot{x}^\mu(s) + \phi_I\theta^I\sqrt{\dot{x}^2(s)}\right)\right\rangle$$

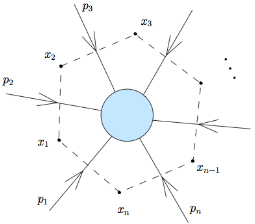
The Wilson Loop in $\mathcal{N} = 4$ SYM

$$\langle W(C) \rangle = \frac{1}{N} \left\langle TrP \exp i \oint_C ds \left(A_\mu \dot{x}^\mu (s) + \phi_I \theta^I \sqrt{\dot{x}^2(s)} \right) \right\rangle$$

- circular Maldacena-Wilson Loop 1/2 BPS$\Rightarrow \langle W(\text{circle}) \rangle = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda})$ exactly computable
- conjecture: sum of the ladder diagrams (ESZ)
- captured by a Matrix Model: confirmed by localization (Pestun) and strong coupling calculation (Drukker, Gross)

$$W_{\text{ladder}} = \frac{1}{Z} \int \mathcal{D}M \frac{1}{N} \mathrm{Tr}\ \exp(M) \exp\left(-\frac{2}{g^2} \mathrm{Tr} M^2 \right)$$

- duality WL/Scattering Amplitudes $\log M_n^{MHV} = \log W_n + cost \rightarrow$ dual space $p_i^\mu = x_i^\mu - x_{i+1}^\mu$



Families of Wilson Loops in $\mathcal{N}=4$ SYM

$$\langle W(C)\rangle = \frac{1}{N}\left\langle TrP\exp i\oint_C ds\left(A_\mu \dot{x}^\mu(s) + \phi_I\theta^I\sqrt{\dot{x}^2(s)}\right)\right\rangle$$

Zarembo's type Wilson loops

- only Poincaré supercharges

$$\theta^I(s) = M^I_\mu \frac{\dot{x}^\mu}{|\dot{x}|}, \qquad M^I_\mu M^I_\nu = \delta_{\mu\nu}.$$

- coupling to 4 scalars, generic curve in $\mathbb{R}^4$, 1/16 BPS
- all the Zarembo's loops are trivial and *VEV* is 1

DGRT's type Wilson loops

- Poincaré and conformal supercharges

$$\theta^I(s) = \sigma^\alpha_{\mu\nu} M^{\alpha I} x^\mu \frac{\dot{x}^\nu}{|\dot{x}|}, \qquad MM^T = 1.$$

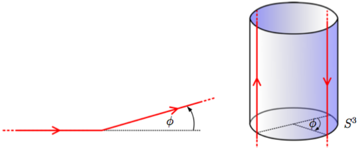
- coupling to 3 scalars, generic curve in S^3, 1/16 BPS
- rescaling of the coupling $\Rightarrow W = 1 + \frac{g^2_{4d}N}{4\pi}\frac{\mathcal{A}_1\mathcal{A}_2}{2\mathcal{A}^2} + \mathcal{O}(g^4_{4d})$
- interesting connection with YM in D=2

Wilson Loops as the quark-antiquark potential of the theory

- calculate the potential between two antiparallel lines

$$< W > \equiv \exp[-\frac{T}{L} V(\lambda)]$$

- we know only the first terms both at weak and strong coupling
- Try to interpolate the two expansions, the resummation of ladder gluon-diagrams fails



- cylinder map $\Gamma_{cusp}(\lambda)$ is identical with the energy of a static q and $\bar{q}$ ($V(\lambda)$), sitting on S^3 at an angle $\pi - \varphi$: scattering of quarks $\rightarrow$ quark-antiquark potential on the sphere on S^3

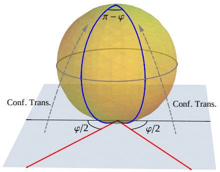
$$< W_{cusp} > \equiv \exp[-\log(\frac{L}{\epsilon})\Gamma_{cusp}(\lambda, \varphi)]$$

- Is it possible to connect protected and non-protected observables?

IDEA

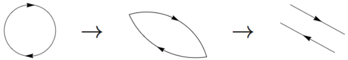
Drukker, Forini in $\mathcal{N} = 4$ SYM

- Deform smoothly the contour to interpolate between BPS (wedge) and non BPS configurations (potentials)
- Deform scalar couplings: $\theta^1 = \cos(\frac{\vartheta}{2})$ and $\theta^2 = \sin(\frac{\vartheta}{2})$



Euclidean cusp

Parameters: angle of R-symmetry ϑ and a space-time angle φ



Conformal mapping: wedge

- Circle: $\varphi = 0$ BPS
- Antiparallel lines: $\varphi = \pi, \vartheta = 0$ NON BPS

The generalized quark-antiquark potential in $\mathcal{N} = 4$ SYM

- BPS condition $\vartheta = \pm\varphi$ idea: expand a NONBPS observable near the BPS point
- recover the correct limits
- cusp anomalous dimension

$$V^{(1)}(i\varphi, \vartheta) = 2\varphi + O(e^{-\varphi}) \qquad V^{(2)}(i\varphi, \vartheta) = -\frac{2\pi^2}{3}\varphi - 4\,\zeta(3) + O(\varphi^{-1})$$

$$V(i\varphi, \vartheta) = \left(\frac{\lambda}{8\pi^2} - \frac{\lambda^2}{384\pi^2} + O(\lambda^3)\right)\varphi + O(u^0)$$

- potential between antiparallel lines calculated for $\vartheta = 0$ and $(\pi - \varphi) \to L$

$$V_{||}(0) = -\frac{\lambda}{4\pi L} + \frac{\lambda^2}{8\pi^3 L}\log\frac{T}{L} + O(\lambda^3)$$

- first term $\Rightarrow$ conformal theory
- second term $\Rightarrow$ infrared effects, including higher order soft–gluon graphs
- and now how can we use DGRT WL to generalize these results for all value of coupling constant???

An exact formula for the radiation of a moving quark in $\mathcal{N} = 4$ SYM

$$\Gamma_{cusp} = -B(\varphi^2 - \vartheta^2) \qquad \varphi, \theta \ll 1 \qquad B = \frac{1}{4\pi^2} \frac{\sqrt{\lambda} I_2(\sqrt{\lambda})}{I_1(\sqrt{\lambda})} + o(1/N^2)$$

- we use 1/4 BPS DGRT Wilson Loop $\lambda' = \lambda \cos^2 \theta_0$
- we develop in θ_0: or second derivative in θ_0 or logarithmic derivative in λ $\rightarrow B = \frac{1}{2\pi^2} \lambda \partial_\lambda \log \langle W_{circle} \rangle$
- relation with the two points scalar function $\Gamma_{cusp} = \theta^2 I_c$, $I_c = -\frac{1}{2} \int_{-\infty}^{\infty} d\tau \, \langle \Phi(\tau) \Phi(0) \rangle$
- weak and strong coupling results (Correa et al.)

$$B = \frac{\lambda}{16\pi^2} - \frac{\lambda^2}{384\pi^2} + \frac{\lambda^3}{6144\pi^2} - \frac{\lambda^4}{92160\pi^2} + O\left(\lambda^5\right)$$

$$B = \frac{\sqrt{\lambda}}{4\pi^2} - \frac{3}{8\pi^2} + \frac{3}{32\pi^2\sqrt{\lambda}} + \frac{3}{32\pi^2\lambda} + O\left(\lambda^{-3/2}\right)$$

- agrees with the Drukker-Forini results
- agrees with the three loops expansion of TBA equation (Correa et al.)

The same pattern in ABJ(M) theories

- return to the initial problem
- renormalized operators in general are linear combinations of bare operators $\mathcal{O}^{B}_{ren} = Z^{B}_{A}\mathcal{O}^{A}_{bare}$
- find the hamiltonian of spin-chain which reproduces mixing matrix for anomalous dimensions at a given order in a loop expansion
- find the eigenvalues of this hamiltonian $\rightarrow$ find the anomalous dimensions
- infinite chain that preserves a centrally extended SU(2|2) superconformal algebra (Central charge $\rightarrow$ energy of the excitations)

$$Z = h(\lambda)(1 - e^{2\pi i p})$$

- p is the momentum of the excitations and about $h(\lambda)$???
- $h(\lambda)$ appears in the magnon dispersion relation and in the all-loop Bethe Ansatz

$$\epsilon = \frac{1}{2}\sqrt{1 + 8h^2(\lambda)\sin^2\frac{p}{2}},$$

- in $\mathcal{N} = 4$ SYM $h^2(\lambda) = \frac{\lambda}{8\pi^2}$ fixed by the symmetries (Beisert), but not yet fixed in ABJM

$$h^2(\lambda) \simeq \lambda, \quad \lambda \gg 1, \qquad h^2(\lambda) \simeq 2\lambda^2, \quad \lambda \ll 1.$$

- idea: $B(\lambda)$ through localization $= B_{\mathcal{N}=4}(h(\lambda))$ through TBA $\rightarrow$ find $h(\lambda)$

Summary

Short review of ABJ(M) theories

$$L_{ABJM} = L_{CS} + \widehat{L}_{CS} + L_{matter};$$

$$L_{CS} = \frac{k}{4\pi}\epsilon^{\mu\nu\rho}[Tr(A_\mu \partial_\nu A_\rho - \frac{2i}{3}A_\mu A_\nu A_\rho)];$$

$$\widehat{L}_{CS} = -\frac{k}{4\pi}\epsilon^{\mu\nu\rho}[Tr(\widehat{A}_\mu \partial_\nu \widehat{A}_\rho - \frac{2i}{3}\widehat{A}_\mu \widehat{A}_\nu \widehat{A}_\rho)];$$

$$L_{matter} = Tr(D_\mu C^I D^\mu \bar{C}_I) + iTr(\bar{\psi}^I \not{D}\psi_I) + L_{int};$$

- dual to M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$
- 't Hooft limit $\rightarrow$ string theory IIA on $AdS_4 \times \mathbb{CP}_3$
- CFT in 3-d, with N=6 Susy, Dim(A_μ)=Dim(ψ)=1, Dim(C_I)=1/2
- gauge group $U(N)_k \times U(N)_{-k}$ for ABJM theory
- gauge group $U(N)_k \times U(M)_{-k}$ for ABJ theory
- two coupling constant $\lambda = N/k$ and $\lambda' = M/k$

Wilson Loop in ABJ(M) theories

- bosonic Wilson Loop $\rightarrow$ 1/6 BPS and $SU(2) \times SU(2)$ R-symmetry $\rightarrow$ no matching with strong coupling (Drukker, Plefka, Young)

$$W_{\mathcal{R}} = Tr_{\mathcal{R}} \mathcal{P} \exp \left(i \int A_\mu \dot{x}^\mu + \frac{2\pi}{k} |\dot{x}| M^I_J C_I \bar{C}^J \, d\tau \right)$$

- Need to coupling fermions to have 1/2 BPS WL (Drukker, Trancanelli)

$$W_{\mathcal{R}} = Tr_{\mathcal{R}} \mathcal{P} \exp \left(-i \int L \, d\tau \right)$$

$$L = \begin{pmatrix} A_\mu \dot{x}^\mu + \frac{2\pi}{k} |\dot{x}| M^I_J C_I \bar{C}^J & \sqrt{\frac{2\pi}{k}} \, |\dot{x}| \, \eta^\alpha_I \bar{\psi}^I_\alpha \\ \sqrt{\frac{2\pi}{k}} \, |\dot{x}| \, \psi^\alpha_I \bar{\eta}^I_\alpha & \widehat{A}_\mu \dot{x}^\mu + \frac{2\pi}{k} |\dot{x}| \widehat{M}^I_J \bar{C}^J C_I \end{pmatrix}$$

- To obtain 1/2 BPS solution we must replace $\delta_{SUSY} W = 0$ with the weaker condition $\delta_{SUSY} L(\tau) = \mathcal{D}_\tau G = \partial_\tau G + i\{L, G\}$ $\rightarrow$ For the straight line we have $M^I_J = diag(1, -1, -1, -1)$ $\rightarrow$ SU(3) R-symmetry
- the low-energy Lagrangian of heavy W-bosons enjoys a larger gauge invariance, given by the supergroup $U(N|M)$
- interactions of fermions with heavy W-bosons are described by $\eta^\alpha_I, \bar{\eta}^I_\alpha$
- Susy conditions $\bar{\eta}^I_\alpha \eta^\beta_I = i(1 + \dot{x}^\mu \sigma_\mu)^\beta_\alpha$ $\qquad M^I_J = \delta^I_J + i\eta^\beta_J \bar{\eta}^I_\beta$ for generic circuits

Results for Wilson Loops in ABJ(M) theories (1)

- weak coupling results for 1/6 BPS Wilson Loop for a circular circuit

$$\langle W\rangle = 1 + \frac{\pi^2 NM}{k^2} - \frac{N^2\pi^2}{6k^2} + \ldots$$

- first term ⇒ matter contribution, second term ⇒ topological contribution
- exact result ⇒ localization (Kapustin et al.)
- the partition function localizes to the following matrix integral

$$Z = \int (\prod_i e^{-ik\pi(b_i^2 - \hat{b}_i^2)} db_i d\hat{b}_i) \frac{\prod_{i\neq j}(2\sinh(\pi(b_i - b_j))2\sinh(\pi(\hat{b}_i - \hat{b}_j)))}{\prod_{i,j}(2\cosh(\pi(b_i - \hat{b}_j)))^2}$$

- *VEV* of WL is given by inserting $\sum_i e^{2\pi b_i}$
- the CS and the matter contributions are factorized

$$\langle W\rangle = 1 + (\frac{5}{6} + \frac{1}{6N^2})\frac{\pi^2 N^2}{k^2} + \ldots$$

- large N limit
- confirmed the prediction of strong coupling (Marino et al.) $\langle W\rangle \sim e^{\pi\sqrt{2\lambda}}$

Results for Wilson Loops in ABJ(M) theories (2)

- 1/6 BPS WL and 1/2 BPS WL are in the same cohomological class (Drukker, Trancanelli)

$$W_{1/2} - W_{1/6} = Tr_R \mathcal{P}\left(e^{i\int L_{1/2}} - e^{i\int L_{1/6}}\right) = QV, \qquad Q = Q_{12}^{+} + Q_{34+}.$$

$$V = iTr_R\mathcal{P}\left[\int_{-\infty}^{+\infty} d\tau\, e^{i\int_{-\infty}^{\tau} L_{1/6}(\tau_1)d\tau_1}\Lambda(\tau) e^{i\int_{\tau}^{\infty} L_{1/2}(\tau_2)d\tau_2}\right]$$

$$\Lambda = \sqrt{\frac{\pi}{2k}}\begin{pmatrix} 0 & -\eta C_2 \\ \bar{\eta}\bar{C}^2 & 0 \end{pmatrix}$$

- insertion in Z of the operator $\sum_a^N e^{2\pi b_a} + \sum_{\hat{a}}^M e^{2\pi \hat{b}_{\hat{a}}}$

$$\langle W_{1/2}\rangle = e^{\pi i(\lambda-\hat{\lambda})} 2\pi i(\lambda+\hat{\lambda})[1 - \frac{\pi^2}{6}(\lambda^2 - 4\lambda\hat{\lambda} + \hat{\lambda}^2) + \mathcal{O}(\lambda^4)].$$

- weak coupling result very subtle, difficult to obtain $\langle W_{1/2}\rangle = 1 - \frac{\pi^2}{6}(\lambda^2 - 4\lambda\hat{\lambda} + \hat{\lambda}^2) + \cdots$ (Penati et al.)

New families of Wilson Loops in ABJ(M) theories (1)

arXiv:1209.4032 [hep-th], V.Cardinali, L.Griguolo, G.M., D.Seminara

- general condition for the couplings

$$\eta_I^\alpha(\tau) = n_I(\tau)\eta^\alpha(\tau), \quad \bar{\eta}^I_\alpha(\tau) = \bar{n}^I(\tau)\bar{\eta}_\alpha(\tau),$$

$$M_J{}^I(\tau) = p_1(\tau)\delta^I_J - 2p_2(\tau)n_J(\tau)\bar{n}^I(\tau), \quad \widehat{M}_J{}^I(\tau) = q_1(\tau)\delta^I_J - 2q_2(\tau)n_J(\tau)\bar{n}^I(\tau).$$

- study the susy variation

$$\delta^\beta_\alpha = \frac{1}{2i}(\eta^\beta\bar{\eta}_\alpha - \eta_\alpha\bar{\eta}^\beta) \qquad (\dot{x}^\mu\gamma_\mu)^\beta_\alpha = \frac{l}{2i}|\dot{x}|(\eta^\beta\bar{\eta}_\alpha + \eta_\alpha\bar{\eta}^\beta)$$

- differential equations for the vector n and η

$$\epsilon_{IJKL}(\eta\bar{\Theta}^{IJ})\bar{n}^K = 0 \qquad n_I(\bar{\eta}\bar{\Theta}^{IJ}) = 0$$

$$\bar{\Theta}^{IJ}\partial_\tau\bar{\eta}^K\epsilon_{IJKL} = 0 \qquad \bar{\Theta}^{IJ}\partial_\tau\eta_I = 0$$

- Θ is the generic supercharge

$$\bar{\Theta}^{IJ} = \bar{\theta}^{IJ} - (x\cdot\gamma)\bar{\epsilon}^{IJ}$$

New families of Wilson Loops in ABJ(M) theories (2)

- setting $\bar{\epsilon}^{IJ} = 0$
- "Zarembo"-like loop operators $\Rightarrow$ Wilson Loops on $\mathbb{R}^3$
- Ansatz: $\bar{s}^I_\alpha$ are four $\tau-$indepent spinors

$$\bar{n}^I = (\eta \bar{s}^I) \qquad n_I = (s_I \bar{\eta})$$

- after tedious calculation we get the couplings

$$\eta_I^\alpha = n_I \eta^\alpha = s_I^\beta \bar{\eta}_\beta \eta^\alpha = i s_I^\beta \left(\mathbb{1} + \frac{\dot{x} \cdot \gamma}{|\dot{x}|} \right)_\beta^{\ \alpha}$$

$$\bar{\eta}^I_\alpha = \bar{n}_I \bar{\eta}_\alpha = \bar{\eta}_\alpha \eta^\beta \bar{s}^I_\beta = i \left(\mathbb{1} + \frac{\dot{x} \cdot \gamma}{|\dot{x}|} \right)_\alpha^{\ \beta} \bar{s}^I_\beta$$

$$M_K^J(\tau) = \widehat{M}_K^J(\tau) = \left(\delta_K^J - 2 i s_K \bar{s}^J - 2i \frac{\dot{x}^\mu}{|\dot{x}|} s_K \gamma_\mu \bar{s}^J \right)$$

New families of Wilson Loops in ABJ(M) theories (3)

- another ansatz to solving the equation with also superconformal charges

$$\bar{n}^I = r(\eta U \bar{s}^I) \qquad n_I = \frac{1}{r}(s_I U^\dagger \bar{\eta}) \qquad U = \cos\alpha \mathbb{1} + i \sin\alpha (x^\mu \gamma_\mu)$$

- "DGRT"-like loop operators $\Rightarrow$ Wilson Loops on S^2
- after tedious calculation we get the couplings

$$\eta_I^\beta = \frac{i}{r_0} e^{\frac{i}{2}(\sin 2\alpha)s} \left[s_I(\cos\alpha\mathbb{1} - i\sin\alpha(x^\mu\gamma_\mu)) \left(\mathbb{1} + \frac{\dot{x}\cdot\gamma}{|\dot{x}|}\right)\right]^\beta$$

$$\bar{\eta}^I_\beta = i r_0 e^{-\frac{i}{2}(\sin 2\alpha)s} \left[\left(\mathbb{1} + \frac{\dot{x}\cdot\gamma}{|\dot{x}|}\right)(\cos\alpha\mathbb{1} + i\sin\alpha(x^\mu\gamma_\mu))\,\bar{s}^I\right]_\beta$$

$$M_K^J(\tau) = \left[\delta_K^J - 2is_K\bar{s}^J - 2i\cos 2\alpha\left(s_K\frac{\dot{x}\cdot\gamma}{|\dot{x}|}\bar{s}^J\right) - 2i\sin 2\alpha\left(s_K\gamma^\lambda\bar{s}^J\right)\epsilon_{\lambda\mu\nu}x^\mu\frac{\dot{x}^\nu}{|\dot{x}|}\right]$$

- $M_K^J(\tau) = \widehat{M}_K^J(\tau)$, α-circle and latitude 1/6 BPS
- Cohomologically trivial fermionic couplings?

A technical point: supertrace or trace?

- supergroup $\rightarrow$ supertrace, but periodic (antiperiodic) conditions...

$$G = \begin{pmatrix} 0 & g_1 \\ \bar{g}_2 & 0 \end{pmatrix} \text{ with } g_1 \equiv 2\sqrt{\frac{2\pi}{k}}(\eta_I \bar{\Theta}^{IL} C_L) \text{ and } \bar{g}_2 \equiv \sqrt{\frac{2\pi}{k}}(\epsilon_{IJKL}(\bar{\eta}^K \bar{\Theta}^{IJ})\bar{C}^L)$$

- in general the functions g_1 and $\bar{g}_2$ for a closed loops are neither periodic nor anti-periodic, but if we take the range of τ to be $[0, 2\pi]$ and we denote with L the perimeter of the curve, G acquires a phase

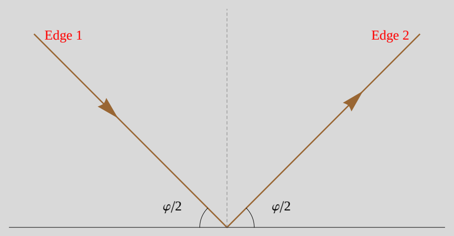
$$G(2\pi) = \begin{pmatrix} e^{\frac{i}{2}(\sin 2\alpha)L} & 0 \\ 0 & e^{-\frac{i}{2}(\sin 2\alpha)L} \end{pmatrix} G(0) = G(0) \begin{pmatrix} e^{-\frac{i}{2}(\sin 2\alpha)L} & 0 \\ 0 & e^{\frac{i}{2}(\sin 2\alpha)L} \end{pmatrix}$$

$$\mathcal{A} = \begin{pmatrix} e^{\frac{i}{4}(\sin 2\alpha)L} & 0 \\ 0 & e^{-\frac{i}{4}(\sin 2\alpha)L} \end{pmatrix}$$

- $G(2\pi) = \mathcal{A}G(0)\mathcal{A}^{-1}$, $\rightarrow U(2\pi) = \mathcal{A}U(0)\mathcal{A}^{-1}$ then $\mathrm{STr}(\mathcal{W}\mathcal{A})$ defines a supersymmetric operator
- in the case of the circle $\alpha = \frac{\pi}{4}$ and $L = 2\pi$, the twist matrix $\mathcal{A}$ is "$i\sigma_3$" which means that we have to take the trace

Construction of the generalized cusp(1)

arXiv:1208.5766 [hep-th], L.Griguolo, D.Marmiroli, G.M., D. Seminara



- basic ingredient: coupling of a straight line
- $M_I^J = \widehat{M}_I^J(-1, 1, 1, 1)$
- $\eta_I^\alpha = \eta \delta_I^1 \delta_1^\alpha$ and $\bar{\eta}_I^\alpha = \bar{\eta} \delta_I^1 \delta_1^\alpha$ with $\eta \bar{\eta} = 2i$
- $x_\mu = (s, 0, 0)$
- to generalized cusp: rotation in space-time and in the space of R-symmetry

Coordinates

- $x^0 = 0$, $x^1 = s \cos \frac{\varphi}{2}$, $x^2 = |s| \sin \frac{\varphi}{2}$

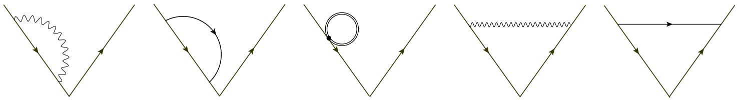
Construction of the generalized cusp(2)

- fermionic coupling: $\eta^{\alpha}_{I1} = in_{I1}\eta^{\alpha}_{1} = i\begin{pmatrix}\cos(\frac{\vartheta}{4})\\ \sin(\frac{\vartheta}{4})\\ 0\\ 0\end{pmatrix} \times \begin{pmatrix} e^{\frac{i\varphi}{4}} \\ e^{\frac{-i\varphi}{4}}\end{pmatrix}$ and $\bar{\eta}^{\alpha}_{I1} = n_{I1}\bar{\eta}^{\alpha}_{1}$
- fermionic coupling: $\eta^{\alpha}_{I2} = in_{I2}\eta^{\alpha}_{2} = i\begin{pmatrix}\cos(\frac{\vartheta}{4})\\ -\sin(\frac{\vartheta}{4})\\ 0\\ 0\end{pmatrix} \times \begin{pmatrix} e^{\frac{-i\varphi}{4}} \\ e^{\frac{i\varphi}{4}}\end{pmatrix}$ and $\bar{\eta}^{\alpha}_{I2} = n_{I2}\bar{\eta}^{\alpha}_{2}$
- scalar coupling: $M_1(\theta) = \begin{pmatrix} -\cos(\frac{\vartheta}{2}) & -\sin(\frac{\vartheta}{2}) & 0 & 0\\ -\sin(\frac{\vartheta}{2}) & \cos(\frac{\vartheta}{2}) & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$
- scalar coupling: $M_2(\theta) = \begin{pmatrix} -\cos(\frac{\vartheta}{2}) & \sin(\frac{\vartheta}{2}) & 0 & 0\\ \sin(\frac{\vartheta}{2}) & \cos(\frac{\vartheta}{2}) & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$

Properties

- BPS condition imposes that $\vartheta = \pm\varphi$

Perturbative analysis: one loop results



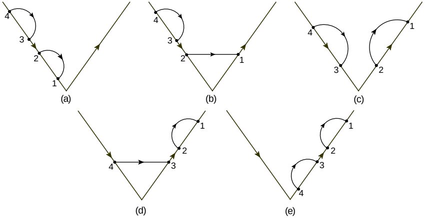
- the exchange of a single scalar is not permitted
- the exchange of a gluon is zero for any planar loop for the antisymmetry of the Levi-Civita tensor
- the only contribution goes from the exchange of a fermion

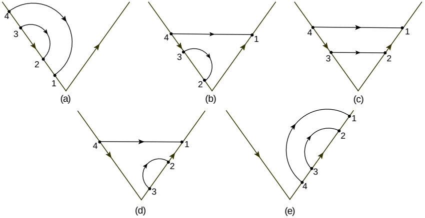
1-loop results: correct BPS condition

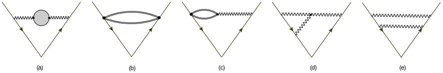
- $< W_{cusp} >^{1-loop} =$
$$\left(\frac{2\pi}{k}\right) MN\left(\frac{\Gamma(1/2-\epsilon)}{4\pi^{3/2-\epsilon}}\right)(\mu L)^{2\epsilon}\left(\frac{1}{\epsilon}\left(\frac{\cos\frac{\vartheta}{2}}{\cos\frac{\varphi}{2}}-2\right)-2\frac{\cos\frac{\vartheta}{2}}{\cos\frac{\varphi}{2}}\log(\sec\frac{\varphi}{2}+1)\right)$$
- after renormalization $< W_{cusp} >^{1-loop} =$
$$\left(\frac{2\pi}{k}\right) MN\left(\frac{\Gamma(1/2-\epsilon)}{4\pi^{3/2-\epsilon}}\right)(\mu L)^{2\epsilon}\left(\frac{1}{\epsilon}\left(\frac{\cos\frac{\vartheta}{2}}{\cos\frac{\varphi}{2}}-1\right)-2\frac{\cos\frac{\vartheta}{2}}{\cos\frac{\varphi}{2}}\log(\sec\frac{\varphi}{2}+1)\right)$$

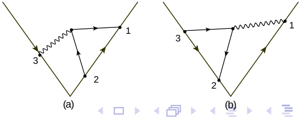
Diagrams at two loops

- proliferation of diagrams
- there are diagrams which contribute to exponentiation of 1-loop divergence and diagrams which contribute to the potential









About the renormalization of cusped Wilson Loop with fermions

- in 4D the only divergence is the cusp anomalous divergence, easily extracted from maximal non-abelian diagrams, not present Z_{open} in Feynman gauge in 4-D
- (Polyakov) analysis: the one-loop mass renormalization for a test particle guided along the trajectory C $\rightarrow [\mathcal{W}^{(1)}]_{sing.} \simeq g^2 \frac{\sqrt{\pi}}{(a^2)^{\frac{D-3}{2}}} \frac{\Gamma(\frac{D-3}{2})}{\Gamma(\frac{D-2}{2})} L[C]$

$$\mathcal{W}(g) = \exp\left[-\hat{\gamma}(g_{ren.})\frac{L[C]}{a}\right]\mathcal{W}(g_{ren.})$$

- 4D: precise distinction between the linear divergence (proportional to the length of the loop) and the cusp divergence (logarithmic in the cut-off)
- 3D: both logarithmic in the cut-off $\rightarrow \mathcal{W}^{(1)} \simeq g^2 L[C] \log\frac{L[C]}{a}$
- divergence from fermions $\rightarrow \log\left(\frac{L}{a}\right)\left[\frac{1}{\eta(1)\bar{\eta}(0)} + \frac{1}{\eta(1)\bar{\eta}(1)} + \frac{1}{\eta(0)\bar{\eta}(0)} + \frac{1}{\eta(1)\bar{\eta}(0)}\right]$
- interpretation: open contour divergence $\mathcal{W} = Z_{open}\mathcal{W}^{ren}$ with

$$Z_{open} = \exp\left[-\left(\frac{2\pi}{\kappa}\right)N\left(\frac{\Gamma(\frac{1}{2}-\epsilon)}{4\pi^{3/2-\epsilon}}\right)(\mu L)^{2\epsilon}(\frac{1}{\epsilon}+\log 4)\right]$$

Final results

- renormalized potential with the subtraction of the divergence of the straight line: open contour divergence

$$V_N^{2-loop} = \left(\frac{2\pi}{\kappa}\right)^2 N^2 \left(\frac{\Gamma\left(\frac{1}{2}-\epsilon\right)}{4\pi^{3/2-\epsilon}}\right)^2 (\mu L)^{4\epsilon} \left[\frac{1}{\epsilon}\log\left(\cos\frac{\varphi}{2}\right)^2 \left(\frac{\cos\frac{\theta}{2}}{\cos\frac{\varphi}{2}} - 1\right) + O(1)\right]$$

- again correct BPS condition
- we find a result compatible with this double exponentiation

$$< W >\equiv \frac{1}{N+M}(N\exp[-TV_M(\lambda',\vartheta,\varphi)] + M\exp[-TV_N(\lambda,\vartheta,\varphi)])$$

- we recover the quark-antiquark potential ($V_M \rightarrow$ exchange $M \leftrightarrow N$ and $k \leftrightarrow -k$)

$$V_N(L) = \frac{N}{k}\frac{1}{L} - \left(\frac{N}{k}\right)^2 \frac{1}{L}\log\left(\frac{T}{L}\right)$$

- we recover the cusp anomalous dimension

$$\gamma_{cusp} = \frac{N^2}{k^2}$$

Summary

Conclusions

Results

- new families of WL in ABJM theory: Wilson loops on $\mathbb{R}^3$ and S^2 (V.Cardinali,L. Griguolo, GM,D. Seminara)
- the generalized cusp at 1-loop and 2-loop (L.Griguolo, GM, D.Marmiroli,D. Seminara)
- give a possible way to calculate $h(\lambda)$

What is still missing to complete the project???

- localize the Wilson Loops on S^2 $\Rightarrow$ possible rescaling on the coupling constant as in SYM???(V.Cardinali,L. Griguolo, GM,D. Seminara, to appear)
- find an exact perturbative expression for the planar loops at two-loops (L.Griguolo, GM, M. Poggi, D.Seminara , to appear)
- a check: study the TBA to calculate the Bremsstrahlung function B

THANKS

Spin chains and long operator

- take some colored field Φ with $L \gg 1$

$$\mathcal{O}_L = Tr(\Phi D^L \Phi)$$

- under operator mixing we have the mixing matrix

$$\mathcal{O}_{L,m} = Tr(D^m \Phi D^{L-m} \Phi)$$

- by diagonalizing the mixing matrix we find again cusp anomaly

$$\Delta = L + \gamma_{cusp}(\lambda) \log L$$

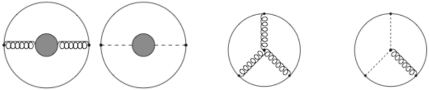
- $X^L = |\downarrow, \downarrow, ..., \downarrow>$ and $X^m Y X^{L-m-1} = |\downarrow, \downarrow, ..., \uparrow, \downarrow, ..., \downarrow>$
- find the hamiltonian which reproduces mixing matrix at a given order in a loop expansion
- find the eigenvalues of this hamiltonian
- this gives the anomalous dimensions

1/2 BPS Circular Wilson Loop

- the simplest example of BPS Wilson Loop
- an effective constant propagator at 1-loop

$$\langle (iA_\mu(x_1)\dot{x}_1^\mu + \phi_0(x_1))(iA_\mu(x_2)\dot{x}_2^\mu + \phi_0(x_2))\rangle = \frac{1}{4\pi^2}\frac{1-\dot{x}(\tau_1)\cdot\dot{x}(\tau_2)}{(x(\tau_1)-x(\tau_2))^2} =$$
$$= \frac{1}{4\pi^2}\frac{1-\cos(\tau_1-\tau_2)}{2(1-\cos(\tau_1-\tau_2))} = \frac{1}{8\pi^2}.$$

- interactions cancel at two loops (ESZ) $\Rightarrow$ only ladder diagrams contribute



- captured by an exact Matrix Model: confirmed by localization and strong coupling calculation

$$W_{\text{ladder}} = \frac{1}{Z}\int \mathcal{D}M \frac{1}{N}\mathrm{Tr}\ \exp(M)\exp\left(-\frac{2}{g^2}\mathrm{Tr}M^2\right)$$

Localization

- theory defined by an action S that is invariant under a fermionic symmetry Q

$$QS = 0.$$

- $Q^2 = \mathcal{L} \Rightarrow$ compact bosonic symmetry of the theory
- $\mathcal{T}$ a Q-closed operator ($Q\mathcal{T} = 0$)

$$S \to S + \, t\, QV.$$

- the expectation value of $\mathcal{T}$ doesn't change

$$\frac{d}{dt}\langle \mathcal{T} \rangle_t = \langle \mathcal{T}\{Q, V\}\rangle = \langle \{Q, \mathcal{T}\, V\}\rangle = 0.$$

$$\langle \mathcal{T} \rangle_t = \langle \mathcal{T} \rangle,$$

- f now t goes to infinity and the term added $\,t\,QV$ is semipositive defined, we observe that the theory has to *localize* (by semiclassical arguments) on some set of critical points of $Q\,V$ over which we have to sum $\Rightarrow$ one-loop saddle point approximation

Localization of 1/2 Circular BPS Wilson Loop(1)

- the euclidean version of the theory on a four sphere (with radius r) obtained via dimensional reduction of the euclidean $\mathcal{N}=1$ SYM 10d

$$S_{\mathcal{N}=4} = \frac{1}{2g_{YM}^2}\int_{S^4}\sqrt{g}d^4x\left(\frac{1}{2}F_{MN}F^{MN} - \overline{\Psi}\,\Gamma^M D_M\Psi + \frac{2}{r^2}\phi^A\phi_A\right)$$

$$\begin{aligned}\delta_\epsilon A_M &= \overline{\Psi}\,\Gamma_M\epsilon \\ \delta_\epsilon\Psi &= \frac{1}{2}F_{MN}\Gamma^{MN}\epsilon + \frac{1}{2}\Gamma_{\mu A}\phi^A\nabla^\mu\epsilon\end{aligned}$$

- off-shell closure of the fermionic subalgebra
- The number of auxiliary fields compensates the difference between the number of fermionic and bosonic off-shell degrees of freedom

$$\begin{aligned}\delta_\epsilon A_M &= \overline{\Psi}\,\Gamma_M\epsilon \\ \delta_\epsilon\Psi &= \frac{1}{2}F_{MN}\Gamma^{MN}\epsilon + \frac{1}{2}\Gamma_{\mu A}\phi^A\nabla^\mu\epsilon + K_i\nu^i \\ \delta_\epsilon K_i &= -\Psi\Gamma^M D_M\nu_i\end{aligned}$$

Localization of 1/2 Circular BPS Wilson Loop(2)

- choice the supercharge

$$\Gamma^{1234}\epsilon_0 = -\Gamma^{5678}\epsilon_0 = \epsilon_0 \quad \bar{\epsilon}_0\epsilon_0 = 1.$$

- the super-conformal transformation generated by this spinor with Q_ϵ
- $Q_\epsilon S_{SYM} = Q_\epsilon W_{\text{circle}} = 0$
- adding the following Q_ϵ-exact term, V invariant under the bosonic symmetry

$$V = (\Psi, \overline{Q\Psi})$$

- the critical points of S^Q_{bos} are given by

$$\phi_0 = 0, \qquad K_i = -w_i a \ \ (i = 1, 2, 3; \ \text{with } w_i w^i = 1/r^2) \qquad [\text{other fields}] = 0$$

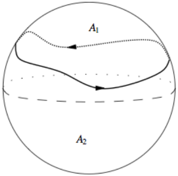
$$Z\Big|_{\substack{\phi_0=0, \ K_i=-w_i a \\ [\text{other fields}]=0}} = \int [da]\, e^{-\frac{4\pi^2 r^2}{g^2_{YM}}\text{Tr}[a^2]}\text{Tr}[e^{2\pi r a}]$$

- the so-called one-loop determinants $\Rightarrow$ fermionic and bosonic contributions cancel and produce just 1 because of supersymmetry

Exact results for DGRT Wilson Loops

- we restrict on $S^2 \subset S^3 \Rightarrow \sigma_i^R = 2\epsilon_{ijk} x^j dx^k$

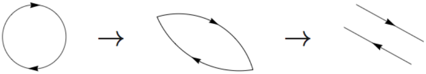
$$W = \frac{1}{N} Tr\mathcal{P} \exp \oint ds \left(iA_\mu x^\mu + \epsilon_{\mu\nu\rho}\, \dot{x}^\mu x^\nu \Phi^\rho \right)$$



- rescaling of the coupling $\Rightarrow W = 1 + \frac{g_{4d}^2 N}{4\pi} \frac{\mathcal{A}_1 \mathcal{A}_2}{2\mathcal{A}} + \mathcal{O}(g_{4d}^4)$
- connection with YM_2 in the zero instanton sector ($g_{2d}^2 = -\frac{g_{4d}^2}{4\pi}$)

$$\langle W \rangle = \frac{1}{N} L_{N-1}^1 \left(g_{2d}^2 \frac{\mathcal{A}_1 \mathcal{A}_2}{\mathcal{A}} \right) \exp\left[-\frac{g_{2d}^2}{2} \frac{\mathcal{A}_1 \mathcal{A}_2}{\mathcal{A}} \right]$$

The generalized quark-antiquark potential in $\mathcal{N} = 4$ SYM(1)



$$\langle W \rangle \equiv \exp[-\log(L/\epsilon) V(\lambda, \theta, \varphi)]$$

- weak coupling results

$$V^{(1)}(\phi, \theta) = -2 \frac{\cos\theta - \cos\varphi}{\sin\varphi} \varphi$$

$$V^{(2)}_{\text{lad}}(\varphi, \theta) = -4 \frac{(\cos\theta - \cos\varphi)^2}{\sin^2\varphi} \left[Li_3\left(e^{2i\varphi}\right) - \zeta(3) - i\varphi \left(Li_2\left(e^{2i\varphi}\right) + \frac{\pi^2}{6} \right) + \frac{i}{3}\varphi^3 \right]$$

$$V^{(2)}_{\text{int}}(\varphi, \theta) = \frac{4}{3} \frac{\cos\theta - \cos\varphi}{\sin\varphi} (\pi - \varphi)(\pi + \varphi)\varphi$$

- BPS condition $\theta = \varphi$

An exact formula for the radiation of a moving quark in $\mathcal{N} = 4$ SYM(2)

- confirmed the Drukker-Forini results
- confirmed with the three loops expansion of TBA equation, (Correa et al.): propagation of a magnon moving on a long strip with two boundaries associated to the Wilson Loop
- connection with the two points scalar function $\Rightarrow$ a particular limit

$$\lambda \to 0, \qquad e^{\imath\theta} \to \infty, \qquad \hat{\lambda} = \frac{\lambda e^{\imath\theta}}{4} = \text{fixed},$$

- resummation through a Bethe-Salpeter equation of scalar ladder diagrams (Correa et al.)

$$\Gamma_{cusp} \to \Gamma_{ladder}(\hat{\lambda}, \varphi)$$

- matches with strong coupling, remember the resummation of ladder diagrams for the circle BPS loop

Wilson Loop in SCS theories

$$W_{\mathcal{R}} = Tr_{\mathcal{R}} \mathcal{P} \exp \left(i \int A_\mu \dot{x}^\mu + \frac{2\pi}{k} |\dot{x}| M_J^I C_I \bar{C}^J \, d\tau \right)$$

Properties of this WL

- In D=3 WL is quadratic in the scalar coupling
- $\delta_{SUSY} W = 0$ implies $M_J^I = diag(1, 1, -1, -1)$
- Circle is 1/6 BPS

Problem

- the fundamental string in AdS_4 ending on a circle at the boundary is 1/2 BPS
- the fundamental string preserves SU(3) R-symmetry: 1/6 BPS WL has only SU(2)$\times$SU(2) R-symmetry
- 1/6 BPS WL exists in both of the group of ABJM and in $\mathcal{N} = 2$ SCS : no enhancement of SUSY from $\mathcal{N} = 2$ to $\mathcal{N} = 6$

Results for Wilson Loops in ABJ(M) theories

- weak coupling results for 1/6 BPS Wilson Loop

$$\langle W \rangle = 1 + \frac{\pi^2 NM}{k^2} - \frac{N^2\pi^2}{6k^2},$$

- first term $\Rightarrow$ matter contribution, second term $\Rightarrow$ topological contribution
- exact result $\Rightarrow$ localization, here $V = \mathrm{Tr}'(Q\lambda^\dagger, \lambda)$ (Kapustin et al.)
- λ is the gaugino present in the gauge supermultiplet when we write the theory in the language of $\mathcal{N} = 2$ off-shell supersymmetry

$$W = \frac{1}{\mathrm{dim} R} Tr_R(\mathcal{P}) \exp\left(\oint dt (iA_\mu \dot{x}^\mu + \sigma|\dot{x}|) \right)$$

- σ is the second real scalar field of the gauge $\mathcal{N} = 2$ multiplet
- the dominant contribution to the path integral will come from the region of field space where

$$Q\lambda = 0$$

Results for Wilson Loops in ABJ(M) theories (2)

- two gauge group $U(N)_k \times U(N)_{-k}$ and the matter sector, the partition function localizes to the following matrix integral

$$Z = \int (\prod_i e^{-ik\pi(\lambda_i^2 - \hat{\lambda}_i^2)} d\lambda_i d\hat{\lambda}_i) \frac{\prod_{i \neq j}(2\sinh(\pi(\lambda_i - \lambda_j))2\sinh(\pi(\hat{\lambda}_i - \hat{\lambda}_j)))}{\prod_{i,j}(2\cosh(\pi(\lambda_i - \hat{\lambda}_j)))^2}$$

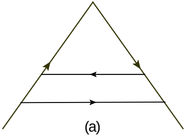
- *VEV* of WL is given by inserting $\sum_i e^{2\pi\lambda_i}$
- the CS and the matter contributions are factorized
- remove the phase due to the framing

$$\langle W \rangle = 1 + (\frac{5}{6} + \frac{1}{6N^2})\frac{\pi^2 N^2}{k^2} + ...$$

- large N limit
- confirmed the prediction of strong coupling(Marino et al.)

$$\langle W \rangle \sim e^{\pi\sqrt{2\lambda}}$$

Diagrams at two loops (2)



(a)