Proton-proton weak capture in chiral effective field theory

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Outline

Collaborators

- L.E. Marcucci - *INFN & Pisa University, Pisa (Italy)*
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Theoretical studies of *pp* fusion (1)

Historical perspective

- $p + p \to {}^2H + e^+ + \nu_e$ fusion rate at $E \approx 10$ keV
  - first estimate: Bethe & Critchfield, 1938
  - "Standard Nuclear Physics Approach" (SNPA): → Schiavilla *et al.*, 1998
  - phenomenological potentials (AV18, CD-Bonn) & currents
  - $\to \pi-$, $\rho-$, ... exchanges
- No consistency between potentials and currents
- No control over the accuracy of the calculations

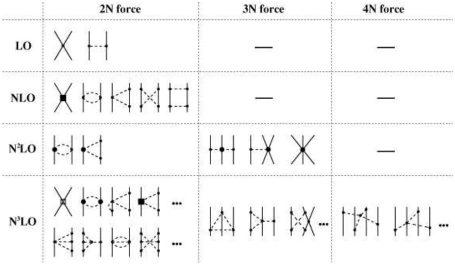
EFT approach

- Effective field theory: $N - \pi$ interactions "dictated" by chiral symmetry [Weinberg (1990), Bernard, Kaiser, & Meissner, (1995); Ordonéz, Ray, & U. van Kolck (1996), ...]
- Potentials and currents can be derived from the same Lagrangian
- The contributions can be organized in powers of $(Q/\Lambda\chi)^\nu$ (Chiral Perturbation Theory)

Theoretical studies of *pp* fusion (2)

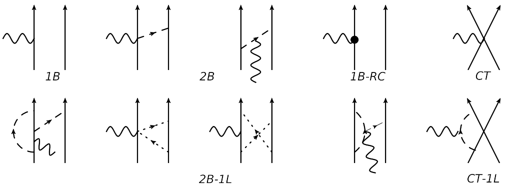
EFT approach (contd)

- NN interaction:
  - N3LO [Entem & Machleidt, (2003)]
  - Jülich N3LO [Epelbaum *et al.*, (2003)]
- 3N interaction:
  - N2LO [Epelbaum *et al.*, (2002)]
- Currents $\mathcal{J}_\mu^{(h)} = \mathcal{V}_\mu^{(h)} - \mathcal{A}_\mu^{(h)}$
  - $\mathcal{V}_\mu^{(h)} \equiv \{\rho^V, \mathbf{J}^V\}$ (from $\mathcal{J}_\mu^{(EM)}$)
  - $\mathcal{A}_\mu^{(h)} \equiv \{\rho^A, \mathbf{J}^A\}$
  - N3LO: Park *et al.*, PRC **67**, 055206 (2003)]
- Low energy constants (LECS) + cutoff parameter Λ



charge & current operators in the "pionfull" EFT

J^μ	LO (Q^{-3})	NLO (Q^{-2})	N^2LO (Q^{-1})	N^3LO (Q^0)	N^4LO (Q^1)
$\mathbf{j}^A$	1B	–	1B-RC	2B	1B-RC, 2B-1L, CT, 3B
ρ^A	–	1B	2B	1B-RC	1B-RC, 2B-1L
$\mathbf{j}^V$	–	1B	2B	1B-RC	1B-RC, 2B-1L, CT
ρ^V	1B	–	–	2B	1B-RC, 2B-1L, 3B



Current interest

- "Hybrid" calculations: AV18 + EFT currents [Park *et al.*, (2003)]
- Pionless EFT at N^2LO: [Wei *et al.*, (2012)]
- → new estimate using both EFT potential and currents

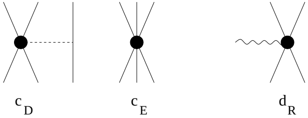
Low energy constants (LECs)

Most of the LECs in $\mathcal{L}$ are fixed by comparing with the abundant NN database

The N3LO currents by [Park *et al.*, (2003)] includes three additional unknown LECs

Two LEC in the vector part: using CVC $\Rightarrow$ fitted to the $A = 3$ magnetic moments

One LEC in the axial part – related to the 3N potential



$$d_R = \frac{M_N}{\Lambda_\chi g_A} c_D + \frac{1}{3} M_N (c_3 + 2c_4) + \frac{1}{6}$$

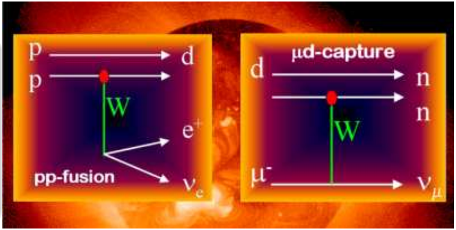
Gardestig and Phillips, PRL **96**, 232301 (2006)
Gazit *et al.*, PRL **103**, 102502 (2009)

fit c_D (d_R) to GT_{exp} and c_E to $B(A=3)$ (using the N3LO/N2LO model)
$\Rightarrow$ $\{c_D; c_E\}_{\rm MAX}$ and $\{c_D; c_E\}_{\rm MIN}$: related to the uncertainty of GT_{exp}

Test of the theory: muon capture

Muon catpures:

- $\mu^- + d \to n + n + \nu_\mu$
- $\mu^- + {}^3\mathrm{He} \to {}^3\mathrm{H} + \nu_\mu$ (70%)
- $\mu^- + {}^3\mathrm{He} \to n + d + \nu_\mu$ (20%)
- $\mu^- + {}^3\mathrm{He} \to n + n + p + \nu_\mu$ (10%)



- $\Gamma(\mu - d) = 300 - 500\ \mathrm{s}^{-1}$ →[MuSun Experiment (PSI)]
- $\Gamma(\mu - {}^3\mathrm{He}) = 1496(4)\ \mathrm{s}^{-1}$ [Ackerbauer *et al.*, (1998)]

Stringent test of the nuclear wave functions/transition operators
Extraction of the pseudoscalar form factor of the nucleon factor

$$j^\mu = \overline{u_p}\left[F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N} - G_A(q^2)\gamma^\mu\gamma^5 - G_{PS}(q^2)\frac{q^\mu\gamma^5}{2M_N}\right] u_n$$

Results: $\Gamma^D(\mu^- + d)$ and $\Gamma_0(\mu^- + {}^3\mathrm{He})$

	$^1S_0$	$^3P_0$	$^3P_1$	$^3P_2$	Γ^D	Γ_0
IA – $\Lambda =$ 500 MeV	238.8	21.1	44.0	72.4	381.7	1362
IA – $\Lambda =$ 600 MeV	238.7	20.9	43.8	72.0	380.8	1360
FULL – $\Lambda =$ 500 MeV	254.4(9)	20.5	46.8	72.1	399.2(9)	1488(9)
FULL – $\Lambda =$ 600 MeV	255(1)	20.3	46.6	71.6	399(1)	1499(9)

Γ^D =399(3) s^{-1} & Γ_0 =1494(21) s^{-1}

vs. Γ^D(exp) $\cdots$ & Γ_0(exp)=1496(4) s^{-1}

Comparison between Γ_0 and Γ_0(exp) $\rightarrow$

G_{PS} =8.2 $\pm$0.7 vs. $G_{PS}^{\chi\mathrm{PT}}$ =7.99$\pm$0.20

Marcucci *et al.*, PRL **108**, 052502 (2012)

$p + p \rightarrow d + e^+ + \nu_e$ astrophysical factor

$$\sigma(E) = \frac{1}{(2\pi)^3} \frac{G_V^2}{v} m_e^5 f(E) \sum_M |\langle d, M|\mathbf{A}_-|pp\rangle|^2 \qquad S(E) = S(0) + S'(0)E + \frac{1}{2} S''(0)E^2 + \dots$$

Goal: $< 1\%$ accuracy

- Dominant contribution from the $^1S_0$ wave
- P-wave contribution: $\sim 1\%$
- Two-body contribution: $\sim 1\%$

pp wave function

- EM interaction: $V_{C1} + V_{C2} + V_{DF} + V_{VP} + \cdots$
- $V_{VP} \sim \exp(-2m_e r)$: sizeable effect at low energies
- Necessity to solve the Schroedinger equation up to $1,000$ fm
- 1% effect

SNPA: [Schiavilla *et al.*, 1998], χEFT*: [Park *et al.*, 2003]
$S(0) = 3.94(1 \pm 0.0015 \pm 0.0010 \pm \epsilon)$ (only the $^1S_0$ wave)
errors from uncertainties in g_A, fit of the tritium β-decay, etc; ϵ "systematic error" (?)

See also the review paper: [E. G. Adelberger *et al.*, Rev. Mod. Phys. **83**, 195 (2011) [arXiv:1004.2318]]

Results with the EFT

N3LO potential – d_R, g_{4S}, g_{4V} fixed using the N3LO/N2LO wave functions

$S(0)$ [$\times 10^{-25}$ MeV b]

		$^1S_0$	$^3P_0$	$^3P_1$	$^3P_2$
IA	Λ=500 MeV	3.961(2)			
FULL	Λ=500 MeV	4.008(5)	4.011(5)	4.020(5)	4.030(5)
	Λ=600 MeV	4.008(5)	4.010(5)	4.019(5)	4.029(5)

Effect of the EM interactions $V_{C2} + V_{DF} + V_{VP} + \cdots$

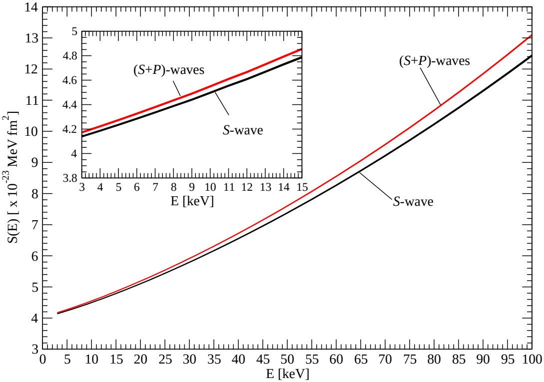
To be included only in the $^1S_0$ channels[Machleidt & Emtem, (2012)]

	$S(0) \times 10^{23}$ [MeV fm^2]	$S'(0)/S(0)$ [MeV^{-1}]	$S''(0)/S(0)$ [MeV^{-2}]
N3LO+V_{C1}	4.03	11.53	226
M3LO+V_{EM}	4.00	11.42	239

Summary: $S(0) = (4.030 \pm 0.006) \times 10^{-25}$ MeV b
Theoretical uncertainty reflects our knowledge of the LECs (g_A, g_{4S}, d_R, ...)

$S(E)$ calculated in the range $0 - 100$ keV

Effect of the inclusion of the P-waves



N3LO potential + N3LO current

Results with the EFT

Fit of the astrophysical factor $[S^{(n)}] = \mathrm{MeV}^{-n}$

$$S(E) = S(0)\left[1 + \sum_{n=1}^{N} \frac{1}{n!} S^{(n)} E^n\right]$$

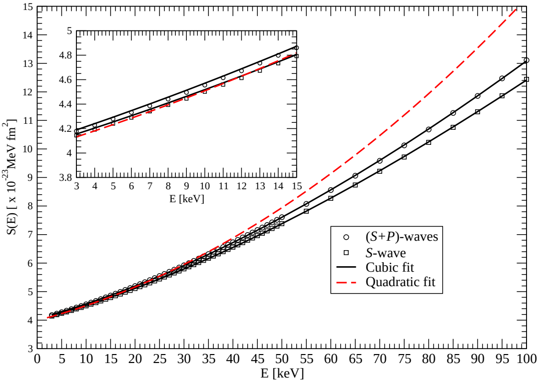
Result

n	$S^{(1)}$	$S^{(2)}$	$S^{(3)}$	$S^{(4)}$	χ^2
$N = 2$	12.59(1)	199.3(1)			8.8×10^{-4}
$N = 3$	11.94(1)	248.8(2)	–1183(8)		1.9×10^{-4}
$N = 4$	11.34(1)	327.1(5)	–5592(12)	99×10^{3}	2.0×10^{-5}
Only S-wave - $N = 2$	12.23(1)	178.4(3)			1.2×10^{-3}
Only S-wave - $N = 3$	11.42(1)	239.6(5)	–1464(5)		1.9×10^{-4}

Pionless EFT at N^2LO [Chen *et al.*, 2012]
$S(0) = (3.99 \pm 0.14)10^{-25}$ MeV b, $S^{(1)}(0) = (11.3 \pm 0.1)$ MeV^{-1}, $S^{(2)}(0) = (170 \pm 2)$ MeV^{-2}

$S(E)$ calculated in the range $0 - 100$ keV

Fit of the astrophyical factor



N3LO potential + N3LO current

Conclusions and outlook

- Test of the nuclear models using the muon capture
- $\Gamma_0(\mu^- + {}^3\mathrm{He})$: nice agreement theory vs. experiment
- $\Gamma^D(\mu^- + d)$:
  - more accurate experimental results $\rightarrow$ MuSun
- New refined calculation of the *pp* fusion up to 100 keV
- In the future:
  - χEFT $\rightarrow$ $\mu^- + {}^3\mathrm{He} \rightarrow n + d + \nu_\mu$
    $\mu^- + {}^3\mathrm{He} \rightarrow n + n + p + \nu_\mu$
  - χEFT $\rightarrow$ reactions of astrophysical interest

$$p + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + e^+ + \nu_e$$
$$p + d \rightarrow^3 \mathrm{He} + \gamma$$
$$d + d \rightarrow^4 \mathrm{He} + \gamma$$

Superkamiokande excess of high energy neutrinos