Neutron-rich hypernuclei: ${}^{6}_{\Lambda}$H & beyond

(arXiv:1305.6716 nucl-th)

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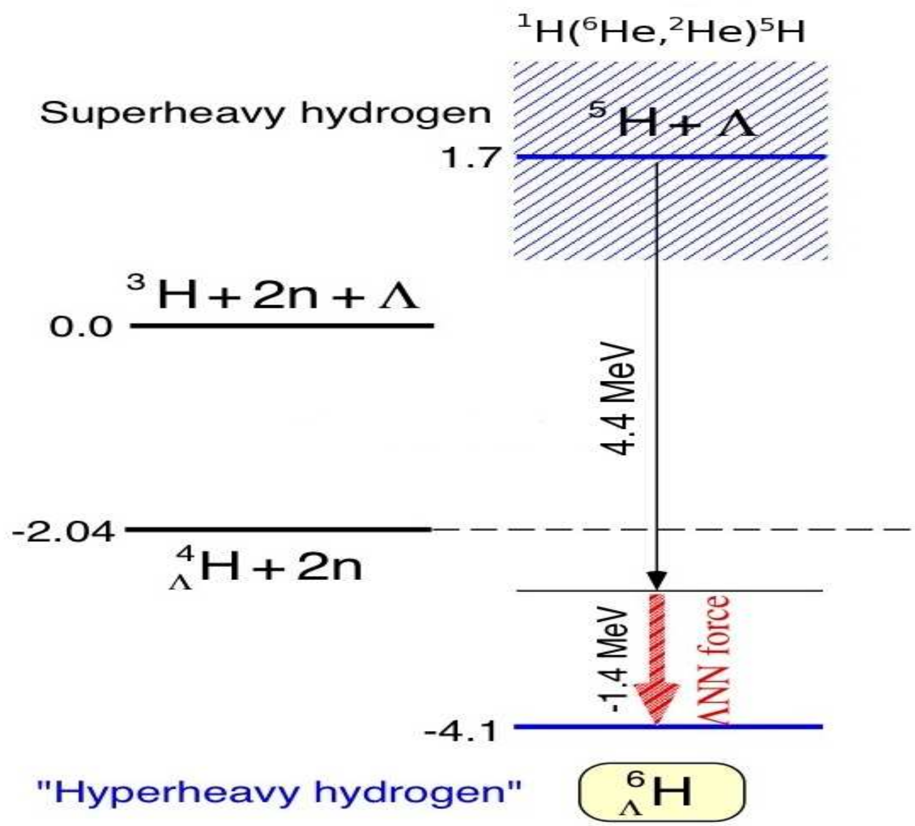
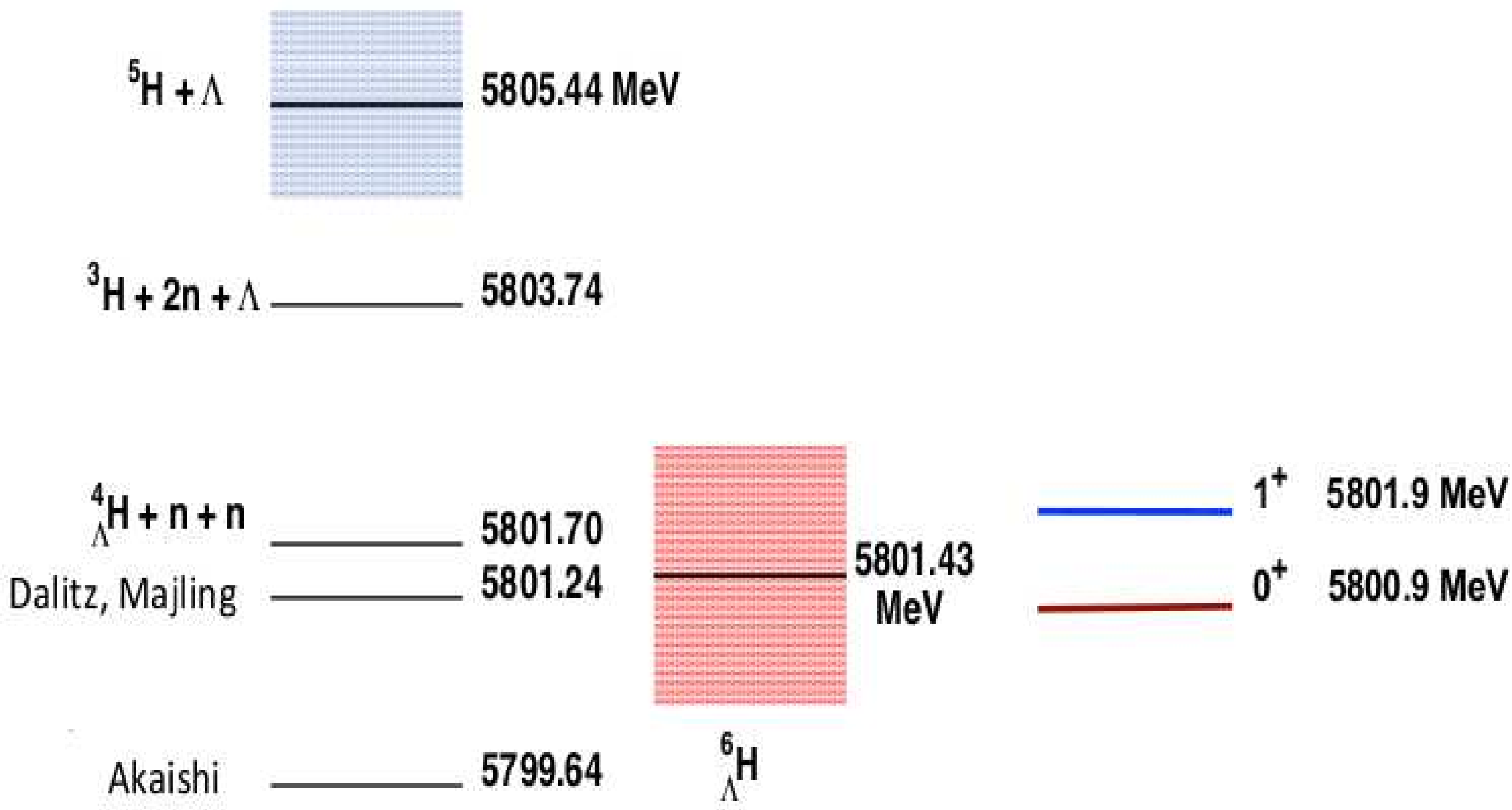
- Shell-model study of the recently observed ${}^{6}_{\Lambda}$H hypernucleus and other neutron-rich Λ hypernuclei that can be formed in (π^-, K^+) or in (K^-, π^+) reactions on stable p-shell nuclear targets.
- Study of $\Lambda\Sigma$ coupling effects in ${}^{49}_{\Lambda}$Ca & ${}^{209}_{\Lambda}$Pb.
- None of the large effects conjectured by Akaishi to arise from $\Lambda\Sigma$ coupling is borne out, neither by realistic p-shell calculations nor by estimates in heavy hypernuclei with substantial neutron excess.

Akaishi's ${}^{6}_{\Lambda}$H [Frascati Phys. Ser. XVI (1999) 59]



$\Lambda - \Sigma$ coupling, resulting in "ΛNN force", lowers $0^+_{\rm g.s.}$ by 1.4 MeV and increases $\Delta E(0^+_{\rm g.s.} - 1^+)$ from 1 MeV in ${}^{4}_{\Lambda}$H to 2.4 MeV in ${}^{6}_{\Lambda}$H.

FINUDA+Gal [PRL 108 (2012) 042501, NPA 881 (2012) 269]

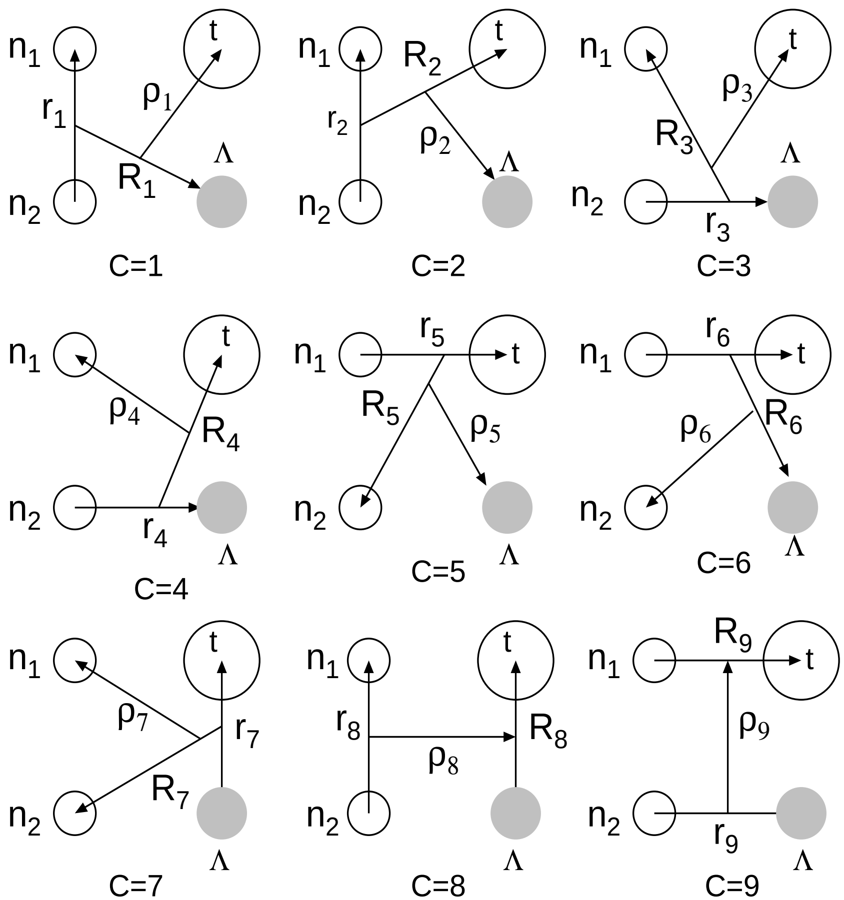


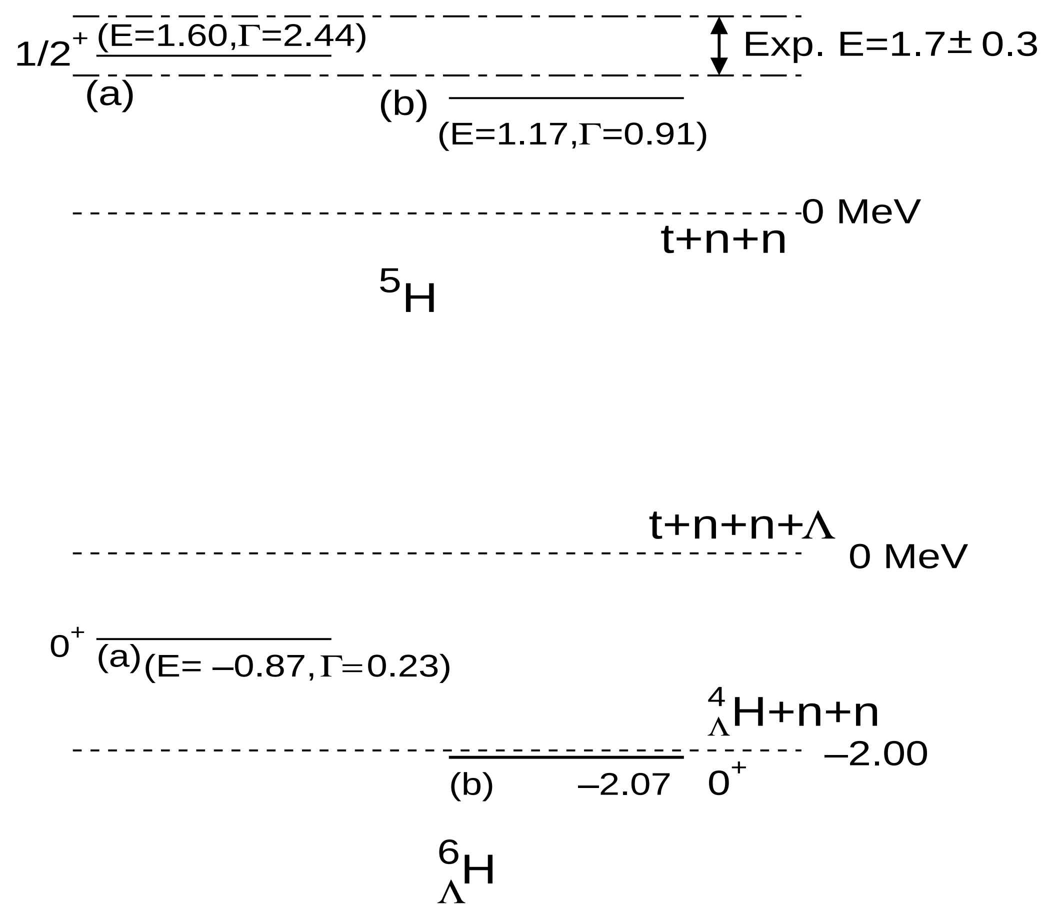
Note: $\Delta E(0^+_{\text{g.s.}} - 1^+) = 1.0 \pm 0.7$ MeV (2σ away from Akaishi).

${}^{6}_{\Lambda}$H **Phenomenology**

- Spin flip is forbidden in production at rest:
  $K^-_{\rm stop} \; + \; {}^6{\rm Li}(L_i = 0, \; S = 1) \; \rightarrow \; {}^6_{\Lambda}{\rm H}(L_f, \; S = 1) \; + \; \pi^+$
  Here, $L_f = 0$, so only ${}^6_{\Lambda}{\rm H}(1^+_{\rm exc.})$ is produced, followed by
  (i) ${}^6_{\Lambda}{\rm H}(1^+_{\rm exc.}) \; \rightarrow \; \gamma \; + \; {}^6_{\Lambda}{\rm H}(0^+_{\rm g.s.}) \quad (\sim 10^{-13}\ {\rm s})$
  (ii) ${}^6_{\Lambda}{\rm H}(0^+_{\rm g.s.}) \; \rightarrow \; \pi^- \; + \; {}^6{\rm He}(0^+_{\rm g.s.}) \quad (\sim 10^{-10}\ {\rm s})$
- If so, $B_{\Lambda}({}^6_{\Lambda}{\rm H}) = (4.5 \pm 1.2)$ MeV; Is $(1^+_{\rm exc.})$ particle stable?
- Shell-model estimate for Λ interaction with p-shell neutrons:
  $B_{\Lambda}({}^7_{\Lambda}{\rm He})$-$B_{\Lambda}({}^5_{\Lambda}{\rm He})$=(5.68±0.25)-(3.12±0.02)=(2.56±0.25) MeV
  and additional $\Lambda N \leftrightarrow \Sigma N$: $\Delta V_{\Lambda N \leftrightarrow \Sigma N}$=0.15 MeV.
  Add to $B_{\Lambda}({}^4_{\Lambda}{\rm H})$=2.04±0.04 MeV $\Rightarrow$ $B^{\rm SM}_{\Lambda}({}^6_{\Lambda}{\rm H})$=4.75±0.25 MeV.
  Scale $\langle V_{\Lambda n} \rangle$ to fit halo n in ${}^6_{\Lambda}$He: $B_{\Lambda}({}^6_{\Lambda}{\rm H}) \approx 4.0 \pm 0.2$ MeV
  $\Rightarrow B_{2n}({}^6_{\Lambda}{\rm H}) \approx 0.3 \pm 0.2$ MeV, hardly bound w.r.t. ${}^4_{\Lambda}$H+2n.

4-body calc. $^{6}_{\Lambda}$H: E. Hiyama et al. NPA 908 (2013) 29





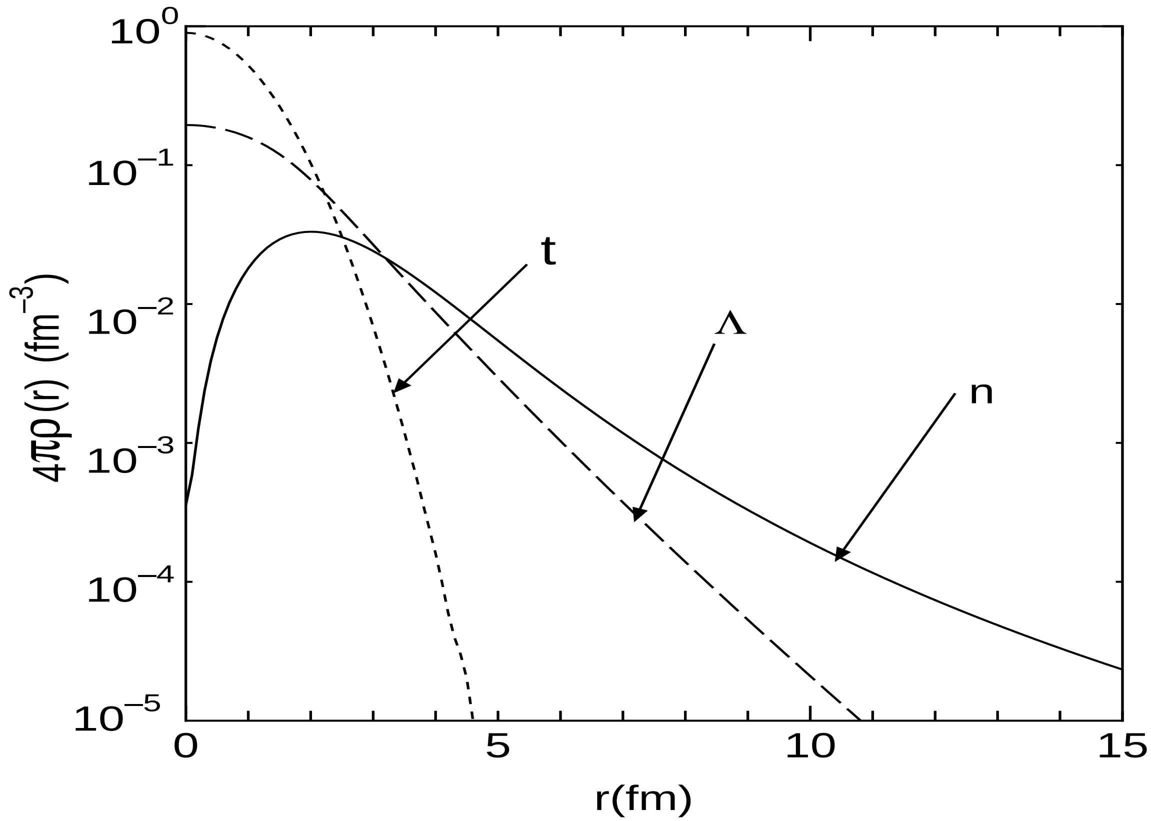
Jacobi coordinates for tnnΛ

All subsystems accounted for

Calculated levels of ^{5}H & $^{6}_{\Lambda}$H

$^{6}_{\Lambda}$H: (a) unbound (b) just bound

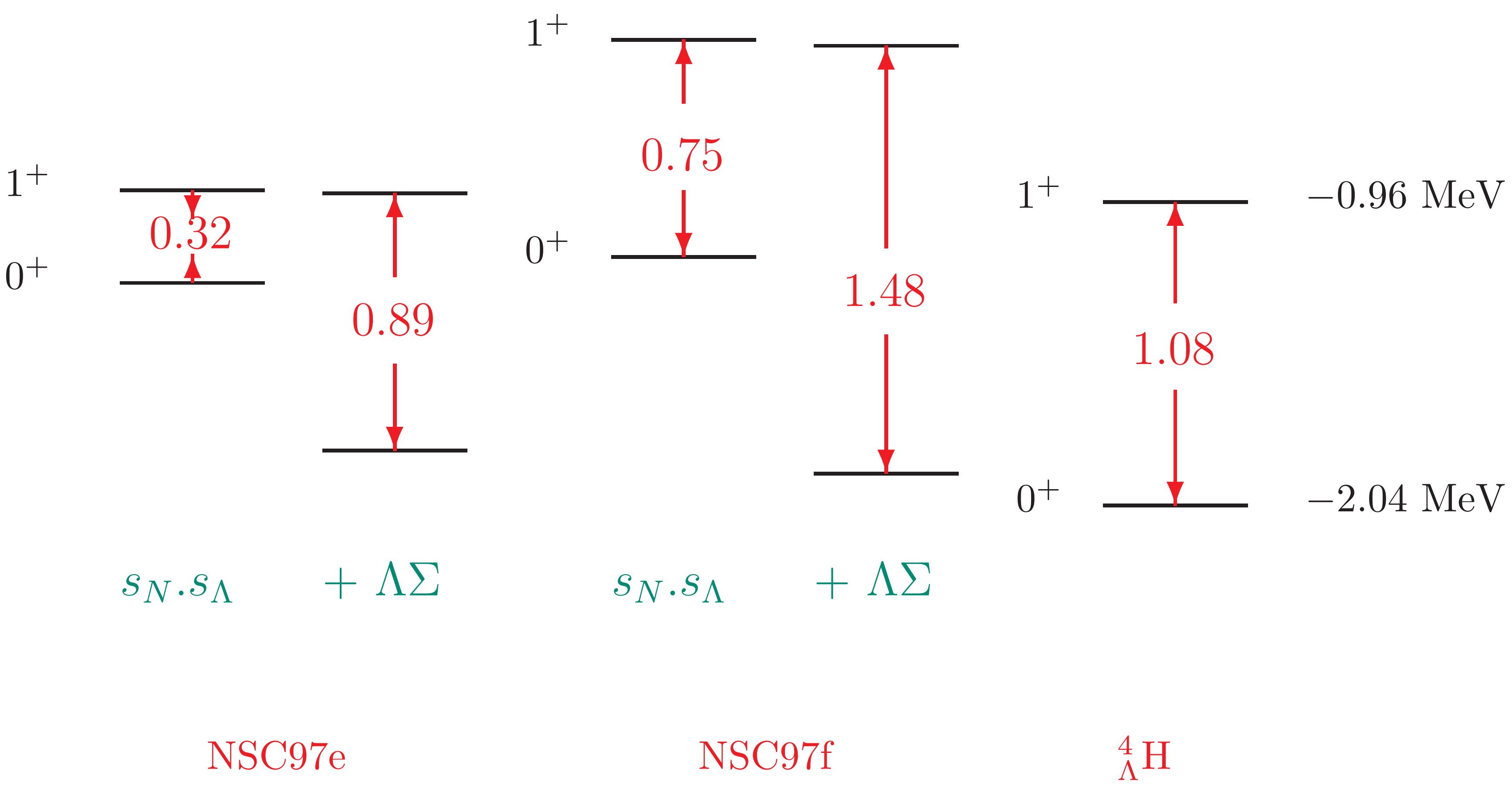
Hiyama's single-particle densities in a just-bound ${}^{6}_{\Lambda}$H



Halo neutrons: effect on SM Λn matrix elements?

Extend Hiyama's calc. from tnnΛ to tnnΛ–tnnΣ calc.

$\Lambda - \Sigma$ Nijmegen model dependence



$\Lambda - \Sigma$ coupling for ${}^4_\Lambda$H and ${}^4_\Lambda$He

Y. Akaishi et al., PRL 84 (2000) 3539

$$|{}^4_\Lambda\mathrm{H}(\mathrm{T}=1/2)\rangle = \alpha s^3 s_\Lambda + \beta s^3 s_\Sigma$$

From ΛN$-\Sigma$N g matrix for 0s orbits:

$v = \langle s^3 s_\Lambda | g | s^3 s_\Sigma \rangle$, $\Delta E \sim 80$ MeV ${}^3g_{ss} = 4.8$ ${}^1g_{ss} = -1.0$

0^+	$v = \frac{3}{2}\,{}^3g_{ss} - \frac{1}{2}\,{}^1g_{ss}$	Admixture $\sim -v/\Delta E$
1^+	$v = \frac{1}{2}\,{}^3g_{ss} + \frac{1}{2}\,{}^1g_{ss}$	$E^{\mathrm{shift}} \sim v^2/\Delta E$

NSC97f: for 0^+ $v \sim 7.6$ MeV $\Rightarrow$ $E^{\mathrm{shift}} \sim 0.72$ MeV,
comparable to genuine ΛN splitting $\Delta_s = 0.75$ MeV.
(s-shell ΛN interaction: $\bar{V}_s + \Delta_s s_N . s_\Lambda$)

$\Lambda - \Sigma$ Coherent Coupling

Coherent: $1s_\Lambda \to 1s_\Sigma$ & no change in nucleon orbital wavefunction.

Write the central Λ-Σ coupling interaction as

$$\sqrt{4/3}\, t_N \cdot t_Y\, \overline{V}' + \sqrt{4/3}\, s_N \cdot s_Y\, t_N \cdot t_Y\, \Delta'$$

($\sqrt{4/3}$ arises from t_Y changing Λ into Σ)

It is a combination of Fermi & Gamow-Teller nuclear matrix elements:

Diagonal matrix element $\quad \sqrt{4/3}\sqrt{T(T+1)}\overline{V}' + a(J)\langle J_c|| \sum_i s_i t_i ||J_c\rangle \Delta'$

Off-diagonal matrix element $\quad b(J)\langle J_c'|| \sum_i s_i t_i ||J_c\rangle \Delta'$

The important Λ-Σ coupling matrix elements involve a Σ coupled to the same core state as the Λ and nuclear states connected by a large GT matrix element.

$p_N s_{\Lambda-\Sigma}$ coupling parameters from Nijmegen YN potentials

Source	Interaction	$\bar{V}'_p$	Δ'_p	S'_Λ	S'_N	T'
Akaishi (s-shell)	NSC97e/f	1.45	3.04	-0.09	-0.09	0.16
Yamamoto	NSC97f	0.96	3.62	-0.07	-0.07	0.31
Halderson	NSC97e	0.75	3.51	-0.45	-0.24	0.31
Halderson	NSC97f	1.10	3.73	-0.45	-0.23	0.30
Halderson	ESC08a	1.05	4.71	-0.07	0.02	0.32

Halderson, Phys. Rev. C 77, 034304 (2008)

- ${}^4_\Lambda$H/${}^4_\Lambda$He 0^+: $\bar{V}'_s + 3/4\ \Delta'_s$ 1^+: $\bar{V}'_s - 1/4\ \Delta'_s$
- Note: $\bar{V}'_p \approx 0.5\ \bar{V}'_s$ & $\Delta'_p \approx 0.5\ \Delta'_s$,
  hence p-shell effect is roughly 1/4 of s-shell effect.

p-shell Λ hypernuclei

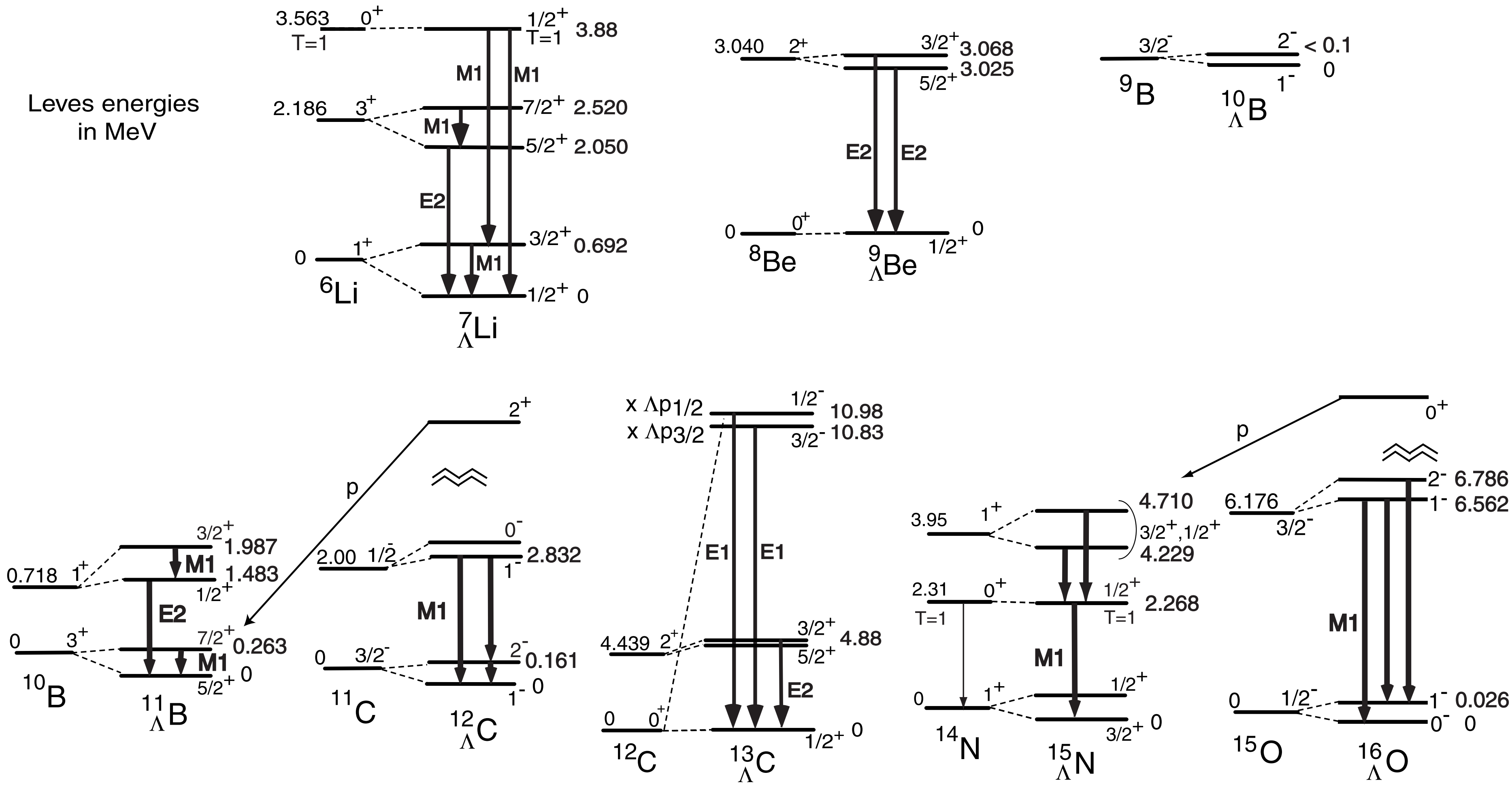
$$V_{\Lambda N} = V_0(r) + V_\sigma(r)\, s_N \cdot s_\Lambda + V_{LS}(r)\, l_{N\Lambda} \cdot (s_\Lambda + s_N) + V_{ALS}(r)\, l_{N\Lambda} \cdot (s_\Lambda - s_N) + V_T(r)\, S_{12}$$

$$V_0 = 1/4\,{}^1V_C + 3/4\,{}^3V_C \qquad V_\sigma = {}^3V_C - {}^1V_C$$

For $p_N s_Y$ $\quad V_{\Lambda N} = \bar{V} + \Delta\, s_N \cdot s_\Lambda + S_\Lambda\, l_N \cdot s_\Lambda + S_N\, l_N \cdot s_N + T\, S_{12}$

Parameters in MeV

	$\bar{V}$	Δ	S_Λ	S_N	T
$N\Lambda$-$N\Lambda$ $\;A = 7-\,?$		0.430	-0.015	-0.390	0.030
$A = 11 - 16$		0.330	-0.015	-0.350	0.024
$N\Lambda$-$N\Sigma$	1.45	3.04	-0.085	-0.085	0.157



Level schemes of Λ hypernuclei from recent γ-ray measurements

H. Tamura et al., Nucl. Phys. A 835 (2010) 3 [HYP-X]

Λ spin-orbit splitting: 150 keV in $^{13}_{\Lambda}$C & related 43 keV in $^{9}_{\Lambda}$Be.

Doublet spacings in p-shell hypernuclei (in keV)

D.J. Millener, NPA 881 (2012) 298

	J_u^π	J_l^π	$\Lambda\Sigma$	Δ	S_Λ	S_N	T	$\Delta E^{\rm th}$	$\Delta E^{\rm exp}$
$^{7}_{\Lambda}$Li	$3/2^+$	$1/2^+$	72	628	-1	-4	-9	693	692
$^{7}_{\Lambda}$Li	$7/2^+$	$5/2^+$	74	557	-32	-8	-71	494	471
$^{8}_{\Lambda}$Li	2^-	1^-	151	396	-14	-16	-24	450	(442)
$^{9}_{\Lambda}$Be	$3/2^+$	$5/2^+$	-8	-14	37	0	28	44	43
$^{11}_{\Lambda}$B	$7/2^+$	$5/2^+$	56	339	-37	-10	-80	267	264
$^{11}_{\Lambda}$B	$3/2^+$	$1/2^+$	61	424	-3	-44	-10	475	505
$^{12}_{\Lambda}$C	2^-	1^-	61	175	-22	-13	-42	153	161
$^{15}_{\Lambda}$N	$3/2_2^+$	$1/2_2^+$	65	451	-2	-16	-10	507	481
$^{16}_{\Lambda}$O	1^-	0^-	-33	-123	-20	1	188	23	26
$^{16}_{\Lambda}$O	2^-	1_2^-	92	207	-21	1	-41	248	224

$\Lambda\Sigma$ coupling contributions normally are below 100 keV

Beyond-mean-field $\Delta B_{\Lambda}^{\rm g.s.}$ shell-model contributions (in keV) to normal-parity g.s. of neutron-rich hypernuclei

D.J. Millener, NPA 881 (2012) 298, and Gal & Millener (2013)

target ^{A}Z	n–rich $^{A}_{\Lambda}(Z-2)$	$\Lambda\Sigma$ diag.	$\Lambda\Sigma$ total	$\Delta B_{\Lambda}^{\rm g.s.}$ total
^{9}Be	$^{9}_{\Lambda}$He$(\frac{1}{2}^{+})$	210	253	879
^{10}B	$^{10}_{\Lambda}$Li(1^{-})	202	275	1022
^{12}C	$^{12}_{\Lambda}$Be(0^{-})	184	158	748
^{14}N	$^{14}_{\Lambda}$B(1^{-})	189	255	785

- Production by $^{A}Z(K^{-},\pi^{+})^{A}_{\Lambda}(Z-2)$ or by $^{A}Z(\pi^{-},K^{+})^{A}_{\Lambda}(Z-2)$.
- Modest effects from $\Lambda\Sigma$ coupling due to neutron excess.
- Overall beyond-mean-field $\Delta B_{\Lambda}^{\rm g.s.}$ is generated by ΛN spin dependent terms, dominantly the induced $s_N \cdot \ell_N$ term.

Binding energy predictions (in MeV) for n-rich hypernuclei

A. Gal & D.J. Millener (arXiv:1305.6716)

n-rich $^{A}_{\Lambda}Z$	normal $^{A}_{\Lambda}Z'$	normal $B_{\Lambda}^{\rm g.s.}$	normal $\Delta B_{\Lambda}^{\rm g.s.}$	n-rich $\Delta B_{\Lambda}^{\rm g.s.}$	n-rich $B_{\Lambda}^{\rm g.s.}$
$^{9}_{\Lambda}$He$(\frac{1}{2}^{+})$	$^{9}_{\Lambda}$Li/$^{9}_{\Lambda}$B	8.44±0.10	0.952	0.879	8.37±0.10
$^{10}_{\Lambda}$Li(1^{-})	$^{10}_{\Lambda}$Be/$^{10}_{\Lambda}$B	8.94±0.11	0.518	1.022	9.44±0.11
$^{12}_{\Lambda}$Be(0^{-})	$^{12}_{\Lambda}$B	11.37±0.06	0.869	0.748	11.25±0.06
$^{14}_{\Lambda}$B(1^{-})	$^{14}_{\Lambda}$C	12.17±0.33	0.904	0.785	12.05±0.33

- Small binding-energy modifications induced by $\Lambda\Sigma$ coupling.
- Small effects persist also in the particle-stable portion of neutron-rich Λ hypernuclear spectra.

$\Lambda\Sigma$ matrix elements and contributions to binding energies (in MeV) across the periodic table

A. Gal & D.J. Millener (arXiv:1305.6716)

$N-Z$	${}^{A}_{\Lambda}Z$	$\bar{V}_{\Lambda\Sigma}$	$\Lambda\Sigma(\bar{V})$	$\Delta_{\Lambda\Sigma}$	$\Lambda\Sigma(\Delta)$	$\Delta B^{\rm g.s.}_{\Lambda}(\Lambda\Sigma)$
4	${}^{9}_{\Lambda}$He	1.194	0.143	4.070	0.104	0.246
8	${}^{49}_{\Lambda}$Ca	0.175	0.010	0.946	0.014	0.024
22	${}^{209}_{\Lambda}$Pb	0.0788	0.052	0.132	0.001	0.053

- $\Lambda\Sigma$ from Halderson, following NSC97f.
- $\Lambda\Sigma$ matrix elements $\bar{V}_{\Lambda\Sigma}$ & $\Delta_{\Lambda\Sigma}$ decrease drastically as the overlap between the $0s$ hyperon and the high-ℓ excess neutrons becomes poorer with A $(0f_{7/2}$ in ${}^{49}_{\Lambda}$Ca, $0h_{9/2}$ & $0i_{13/2}$ in ${}^{209}_{\Lambda}$Pb).

Conclusion

- Akaishi's substantial $\Lambda\Sigma$ coupling effects are not supported by the FINUDA observation of ${}^{6}_{\Lambda}$H and by realistic shell-model calculations which are rooted in Λ hypernuclear phenomenology.
- J-PARC Experiment E10, ^{6}Li$(\pi^-, K^+){}^{6}_{\Lambda}$H, is being analyzed to shed more light on the particle stability of ${}^{6}_{\Lambda}$H.
- Realistic shell-model calculations done for binding energies of neutron-rich Λ hypernuclei that could be formed in (π^-, K^+) or in (K^-, π^+) reactions on stable p-shell nuclear targets do not support the large effects conjectured by Akaishi to arise from $\Lambda\Sigma$ coupling. $\Lambda\Sigma$ contributions to binding energies of heavier neutron-rich hypernuclei are less than 100 keV.