# Anomalous Transport: Kubo Formulae and Fluid/Gravity Correspondence

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[1]Supported by the Juan de la Cierva Program of the Spanish MINECO.

International Nuclear Physics Conference (INPC 2013)
4 June, 2013, Firenze, Italy.

E. Megías et al. PRL 107(2011), JHEP 1109:121(2011), 1110.3615[hep-ph], J.Phys.Conf.Ser. 343(2012), Fortsch.Phys. 60(2012), Acta Phys.Polon.Supp. 6(2013), JHEP 1305:115(2013).

# Issues

1. Anomalous Transport and Kubo Formulae
   - Chiral Magnetic Effect in Heavy Ion Collisions
   - Hydrodynamics of Relativistic Fluids
   - Kubo Formulae
2. Transport Theory and Holography
   - Fluid/Gravity Correspondence
   - Fluid Frame and Boundary Conditions
   - 2nd Order Hydrodynamics
3. Conclusions

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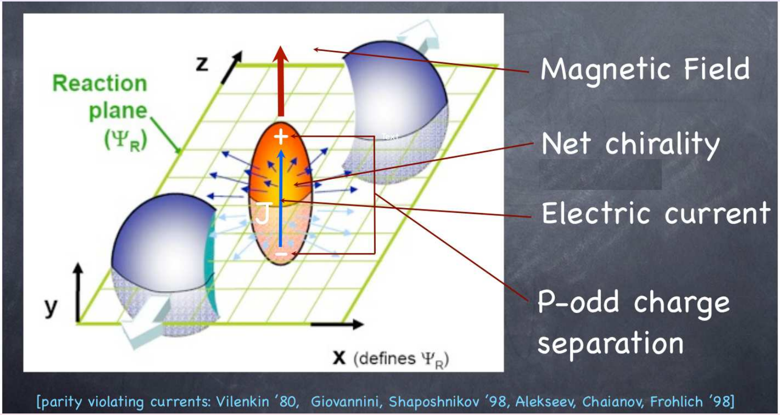
# Chiral Magnetic Effect in Heavy Ion Collisions

[Kharzeev, McLerran, Warringa '07]




Strong Magnetic field induces a P-odd charge separation $\Longrightarrow$
$\Longrightarrow$ Electric current: $\vec{J} = \sigma^{\mathcal{B}} \vec{B}$.

# Issues

# Hydrodynamics of Relativistic Fluids

**[Son,Surowka], [Eling,Neiman,Oz], [Erdmenger et al.], [Banerjee et al.], [Loganayagam], [Kharzeev, Yee], [Sadovyev et al.]**

$$\langle T^{\mu\nu}\rangle = \underbrace{(\epsilon + P)u^\mu u^\nu + P g^{\mu\nu}}_{\text{Ideal Hydro}} + \underbrace{\langle T^{\mu\nu}\rangle_{\text{diss \& anom}}}_{\text{Dissipative \& Anomalous}} ,$$

$$\langle J^\mu\rangle = \underbrace{n u^\mu}_{\text{Ideal Hydro}} + \underbrace{\langle J^\mu\rangle_{\text{diss \& anom}}}_{\text{Dissipative \& Anomalous}} .$$

- Landau frame: $\langle T^{0i}\rangle \sim u^i$

$$\langle T^{\mu\nu}\rangle_{\text{diss \& anom}} = -\eta P^{\mu\alpha}P^{\nu\beta}\left(D_\alpha u_\beta + D_\beta u_\alpha - \frac{2}{3} g_{\alpha\beta} D^\lambda u_\lambda\right) - \zeta P^{\mu\nu} D^\alpha u_\alpha + \cdots$$

$$\langle J^\mu\rangle_{\text{diss \& anom}} = -\sigma T P^{\mu\nu} D_\nu\left(\frac{\mu}{T}\right) + \sigma E^\mu + \xi^{\mathcal{B}} B^\mu + \xi^{\mathcal{V}}\omega^\mu + \cdots$$

where $P^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$, and vorticity: $\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\lambda} u_\nu D_\rho u_\lambda$.

# Issues

# Kubo Formulae

- Constitutive relation for the current:

$$\langle J^\mu \rangle = \underbrace{n u^\mu}_{\text{Ideal Hydro}} + \underbrace{\sigma^{\mathcal{B}} B^\mu + \sigma^{\mathcal{V}} \omega^\mu}_{\text{Anomalous}} + \cdots .$$

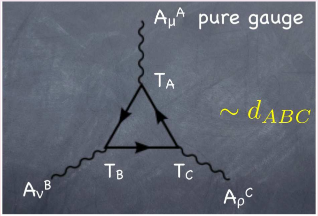
- Chiral Magnetic and Vortical Conductivities **[Kharzeev'09, Amado'11]**

$$\sigma^{\mathcal{B}} = \lim_{k_c \to 0} \frac{i}{2k_c} \sum_{a,b} \epsilon_{abc} \langle J^a J^b \rangle |_{\omega=0} , \qquad \sigma^{\mathcal{V}} = \lim_{k_c \to 0} \frac{i}{2k_c} \sum_{a,b} \epsilon_{abc} \langle J^a T^{0b} \rangle |_{\omega=0} .$$

- 1-loop calculation **[Kharzeev, Warringa '09], [Megías et al. '11]**

$$(\sigma^{\mathcal{B}})_{AB} = \frac{1}{4\pi^2} \underbrace{d_{ABC}}_{\text{Chiral Anom Coef}} \mu^C$$

$$d_{ABC} = \frac{1}{2} \mathrm{tr}(T_A \{T_B, T_C\})$$

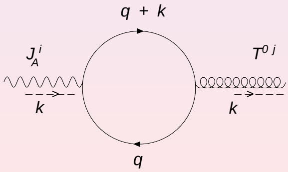


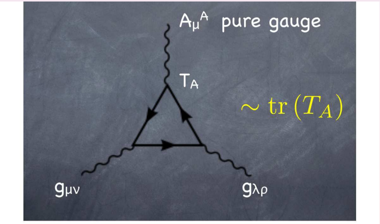

**Chiral Magnetic Conductivity induced by the Chiral Anomaly.**

# Chiral Vortical Effect

- Theory of $N$ Free Chiral fermions $\Longrightarrow$ 1 loop calculation
  **[Landsteiner, Megías, Pena-Benitez, PRL107 '11]**:

$$
\begin{aligned}
(\sigma^{\mathcal{V}})_A &= \lim_{k_n \to 0} \sum_{i,j} \epsilon_{ijn} \frac{i}{2k_n} \langle J^i_A T^{0j} \rangle |_{\omega=0} \\
&= \underbrace{\frac{1}{8\pi^2} \sum_{B,C} d_{ABC}\, \mu^B \mu^C}_{\text{Chiral Anomaly}} + \underbrace{\frac{T^2}{24} \mathrm{tr}\,(T_A)}_{\text{Gauge-Gravitational Anomaly}}
\end{aligned}
$$




**Chiral Vortical Conductivity induced by the Chiral and Gauge-Gravitational Anomalies.**

# Issues

# Holographic model

**[K. Landsteiner, E. Megías, L. Melgar, F. Pena-Benitez, JHEP 1109:121(2011)]**
**[E. Megías, F. Pena-Benitez, JHEP 1305:115(2013)]**

- Holographic model in 5 dim:

$$S = \frac{1}{16\pi G}\int d^5x\sqrt{-g}\left[R + 2\Lambda - \frac{1}{4}F_{MN}F^{MN} + \epsilon^{MNPQR}A_M\left(\frac{\kappa}{3}F_{NP}F_{QR} + \lambda R^A{}_{BNP}R^B{}_{AQR}\right)\right] + S_{GH} + S_{CSK}$$

Includes gauge and gauge-gravitational Chern Simons terms.

- Equations of motion

$$G_{MN} - \Lambda g_{MN} = \frac{1}{2}F_{ML}F_N{}^L - \frac{1}{8}F^2 g_{MN} + 2\lambda\epsilon_{LPQR(M}\nabla_B\left(F^{PL}R^B{}_{N)}{}^{QR}\right)$$

$$\nabla_N F^{NM} = -\epsilon^{MNPQR}\left(\kappa F_{NP}F_{QR} + \lambda R^A{}_{BNP}R^B{}_{AQR}\right).$$

# Fluid/Gravity Correspondence

**[Erdmenger et al.], [Bhattacharyya et al.], [Banerjee et al.], [Megías,Pena-Benitez'13]**

- Boosted black branes:

$$ds^2 = -r^2 f(r) u_\mu u_\nu dx^\mu dx^\nu + r^2 P_{\mu\nu} dx^\mu dx^\nu - 2u_\mu dx^\mu dr$$

  where

$$f(r) = 1 - \frac{M}{r^4} + \frac{Q^2}{r^6}\,, \quad A_r = 0\,, \quad A_\mu = A_\mu^{(b)} - \frac{\sqrt{3}Q}{r^2} u_\mu\,,$$

$$\left[ T = \frac{(2r_+^2 M - 3Q^2)}{2\pi r_+^5}\,, \quad \mu = \frac{\sqrt{3}Q}{r_+^2} \right].$$

  $\Longrightarrow$ Solution of e.o.m. when $M, Q, \cdots \neq f(x^\mu)$.
- Consider $M(x), Q(x), u_\mu(x), A_\mu^{(b)}(x)$ are slowly varying in $x^\mu$. This means $\partial X \ll X/\ell_{mfp}$.
- Then one can compute the metric and the gauge field in a derivative expansion:

$$g_{AB} = g_{AB}^{(0)} + g_{AB}^{(1)} + g_{AB}^{(2)} + \cdots$$
$$A_M = A_M^{(0)} + A_M^{(1)} + A_M^{(2)} + \cdots$$

# Fluid/Gravity Correspondence

- Metric and gauge field in Weyl-invariant form. Decompose the fields into scalar, vector and tensor sectors:

$$
\begin{aligned}
ds^2 &= -2W_1 u_\mu dx^\mu (dr + r\mathcal{A}_\nu dx^\nu) \\
&\quad + \left[r^2(W_2\eta_{\mu\nu} + W_3 u_\mu u_\nu) + 2rW^4_\sigma P^\sigma_\mu u_\nu + W^5_{\mu\nu}\right] dx^\mu dx^\nu \,, \\
A_r &= 0\,, \qquad A_\mu = a^{(b)}_\mu + r_+ c\, u_\mu + a_\nu P^\nu_\mu \,.
\end{aligned}
$$

- Compute $W_1(\rho), W_2(\rho), \ldots \quad [\rho \equiv r/r_+]$ in a derivative expansion of $Q(x)$, $M(x)$, $u_\mu(x)$ and $a^{(b)}_\mu(x)$.
- 0-th order is trivial (no $x^\mu$ dependence):

$$
c^{(0)}(\rho) = -\frac{\sqrt{3}Q}{r_+\rho^2}\,, \qquad a^{(0)}_\mu = 0\,,
$$

$$
W^{(0)}_1 = 1 = W^{(0)}_2\,, \quad W^{(0)}_3(\rho) = 1 - f(\rho)\,, \quad W^{(0)}_{4\mu} = 0 = W^{(0)}_{5\mu\nu}\,.
$$

# Fluid/Gravity Correspondence

- At *n*-th order the Einstein-Maxwell equations write (*vector sector*)

$$\begin{aligned} \mathbb{J}_\mu^{(n)}(\rho) &= \partial_\rho\left(\rho^5\partial_\rho W_{4\mu}^{(n)} + 2\sqrt{3}Qa_\mu^{(n)}(\rho)\right) , \\ \underbrace{\mathbb{A}_\mu^{(n)}(\rho)}_{\text{Sources}} &= \underbrace{\partial_\rho\left(\rho^3 f(\rho)\partial_\rho a_\mu^{(n)}(\rho) + 2\sqrt{3}Q\partial_\rho W_{4\mu}^{(n)}(\rho)\right)}_{\text{Kinetic terms}} , \end{aligned}$$

- Use the energy conservation equation (constraint):

$$\left(D_\mu T^\mu_\nu = F_{\nu\alpha}J^\alpha\right)^{(n-1)} ,$$

- Sources at 1st order in the derivative expansion:

$$\begin{aligned} \mathbb{J}_\mu^{(1)} &= -\lambda\frac{96}{\rho^3}\left(\frac{5Q^2}{\rho^2} - M\right)\frac{B_\mu}{r_+} - \sqrt{3}Q\lambda\left(\frac{1008Q^2}{\rho^7} - \frac{320M}{\rho^5}\right)\omega_\mu , \\ \mathbb{A}_\mu^{(1)} &= -\frac{\sqrt{3}\pi T}{Mr_+\rho^2}P_\mu^\nu\mathcal{D}_\nu Q - \left(1 + \frac{9Q^2}{2M\rho^2}\right)\frac{E_\mu}{r_+} - \frac{16\sqrt{3}\kappa Q}{\rho^3}\frac{B_\mu}{r_+} - \frac{48\kappa Q^2}{\rho^5}\omega_\mu \\ &\quad - \frac{48\lambda}{\rho^{11}}\left(15Q^4 - 16MQ^2\rho^2 + 4M^2\rho^4\right)\omega_\mu . \end{aligned}$$

# Issues

# Fluid Frame and Boundary Conditions

**[E. Megías, K. Landsteiner, F. Pena-Benitez, Acta Phys. Polon. Supp 6, 45 (2013)]**

Boundary conditions when solving Einstein-Maxwell equations:

1. Field theory metric not modified → $W_{4\sigma}(\rho \to \infty) = 0.$
2. Background field not modified → $a_\mu(\rho \to \infty) = 0.$
3. Regularity up to the outer horizon → $a_\mu(r_h) = \text{finite}.$
4. There is some freedom.

4th b.c. related to definition of the fluid velocity $u_\mu$, i.e. to the frame.

$$\langle T_{\mu\nu}\rangle_{\text{diss \& anom}} = \frac{r_+^3}{4\pi G} W_{4\sigma}^{(\bar{4})}(P_\mu^\sigma u_\nu + P_\nu^\sigma u_\mu) + \dots , \quad \langle J_\mu \rangle_{\text{diss \& anom}} = \frac{r_+^2}{8\pi G} a_\mu^{(\bar{2})} + \dots$$

$$\text{where:} \quad W_{4\sigma} \simeq \dots + \frac{W_{4\sigma}^{(\bar{4})}}{\rho^4} + \dots , \qquad a_\mu \simeq \dots + \frac{a_\mu^{(\bar{2})}}{\rho^2} + \dots , \qquad (\rho \to \infty)$$

- Landau frame: $\langle T^{0i}\rangle = (\epsilon + P)u^i$ → $\langle T^{0i}\rangle_{\text{diss \& anom}} = 0.$
- Eckart frame: $\langle J^i \rangle = n u^i$ → $\langle J^i \rangle_{\text{diss \& anom}} = 0.$
- Laboratory frame: $W_{4\sigma}(r_h) = 0$ → $\langle J_S^i \rangle_{\text{anom}} = 0.$

**See [Loganayagam '11], [Oz '12] for $\langle J_S^i \rangle$.**

# 1st order results

- Constitutive relations:

$$\langle T_{\mu\nu}\rangle_{\text{anom}} = (\sigma^{\epsilon,\mathcal{B}})_{\text{F}}(B_\mu u_\nu + B_\nu u_\mu) + (\sigma^{\epsilon,\mathcal{V}})_{\text{F}}(\omega_\mu u_\nu + \omega_\nu u_\mu)\,,$$
$$\langle J_\mu\rangle_{\text{anom}} = (\sigma^{\mathcal{B}})_{\text{F}} B_\mu + (\sigma^{\mathcal{V}})_{\text{F}}\omega_\mu\,.$$

- Anomalous transport coefficients in three different frames:

| Conductivities | Laboratory rest frame | Landau frame | Eckart frame |
|---|---|---|---|
| $(\sigma^{\mathcal{B}})_{\text{F}}$ | $\sigma^{\mathcal{B}} = \frac{\mu}{4\pi^2}$ | $\sigma^{\mathcal{B}} - \frac{n}{\epsilon+P}\sigma^{\epsilon,\mathcal{B}}$ | 0 |
| $(\sigma^{\mathcal{V}})_{\text{F}}$ | $\sigma^{\mathcal{V}} = \frac{\mu^2}{8\pi^2} + \frac{T^2}{24}$ | $\sigma^{\mathcal{V}} - \frac{n}{\epsilon+P}\sigma^{\epsilon,\mathcal{V}}$ | 0 |
| $(\sigma^{\epsilon,\mathcal{B}})_{\text{F}}$ | $\sigma^{\epsilon,\mathcal{B}} = \sigma^{\mathcal{V}}$ | 0 | $\sigma^{\epsilon,\mathcal{B}} - \frac{\epsilon+P}{n}\sigma^{\mathcal{B}}$ |
| $(\sigma^{\epsilon,\mathcal{V}})_{\text{F}}$ | $\sigma^{\epsilon,\mathcal{V}} = \frac{\mu^3}{12\pi^2} + \frac{\mu T^2}{12}$ | 0 | $\sigma^{\epsilon,\mathcal{V}} - \frac{\epsilon+P}{n}\sigma^{\mathcal{V}}$ |

Different frames can be related by a redefinition of $u_\mu$: $u_\mu \to u_\mu + \delta u_\mu$.

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# Fluid/Gravity Correspondence up to 2nd order

**[Erdmenger et al.'09], [Bhattacharyya et al.'11], [Megías,Pena-Benitez'13]**

$$\begin{aligned} ds^2 &= -2W_1 u_\mu dx^\mu (dr + r\mathcal{A}_\nu dx^\nu) \\ &\quad + \left[r^2(W_2\eta_{\mu\nu} + W_3 u_\mu u_\nu) + 2rW^4_\sigma P^\sigma_\mu u_\nu + W^5_{\mu\nu}\right] dx^\mu dx^\nu \,, \\ A_r &= 0\,, \qquad A_\mu = a^{(b)}_\mu + r_+ c\, u_\mu + a_\nu P^\nu_\mu \,. \end{aligned}$$

- Solve exactly at 1st order:

$$\begin{aligned} W^{(1)}_{4\mu}(\rho) &= F_1(\rho) P^\nu_\mu \partial_\nu Q + F_2(\rho)\omega_\mu + F_3(\rho)\frac{E_\mu}{r_+} + F_4(\rho)\frac{B_\mu}{r_+}\,, \\ W^{(1)}_{5\mu\nu}(\rho) &= F_5(\rho)\sigma_{\mu\nu}\,, \\ a^{(1)}_\mu(\rho) &= F_6(\rho) P^\nu_\mu \partial_\nu Q + F_7(\rho)\omega_\mu + F_8(\rho)\frac{E_\mu}{r_+} + F_9(\rho)\frac{B_\mu}{r_+}\,. \end{aligned}$$

# Fluid/Gravity Correspondence up to 2nd order

**[Kharzeev, Yee, Phys. Rev. D84 (2011)], [Megías, Pena-Benitez, JHEP 1305:115(2013)]**

- Sources at 2nd order in the derivative expansion (*vector sector*):

$$\begin{aligned}
\mathbb{J}^{(2)}_\mu(\rho) &= \sum_{a=1}^{10} s_a \mathcal{J}^{(a)\mu} + \sum_{a=1}^{5} \tilde{s}_a \tilde{\mathcal{J}}^{(a)\mu} = s_1 \sigma^{\mu\nu}\mathcal{D}_\nu\bar{\mu} + s_2\omega^{\mu\nu}\mathcal{D}_\nu\bar{\mu} \\
&+ s_3 P^{\mu\nu}\mathcal{D}^\alpha\sigma_{\nu\alpha} + s_4 P^{\mu\nu}\mathcal{D}^\alpha\omega_{\nu\alpha} + s_5\sigma^{\mu\nu}E_\nu \\
&+ s_6\omega^{\mu\nu}E_\nu + s_7 u^\nu\mathcal{D}_\nu E^\mu + s_8\epsilon^{\mu\nu\alpha\beta}u_\nu B_\alpha \mathcal{D}_\beta\bar{\mu} + s_9\epsilon^{\mu\nu\alpha\beta}u_\nu E_\alpha B_\beta \\
&+ s_{10}\epsilon^{\mu\nu\alpha\beta}u_\nu\mathcal{D}_\alpha B_\beta + \tilde{s}_1\sigma^{\mu\nu}\omega_\nu + \tilde{s}_2\sigma^{\mu\nu}B_\nu + \tilde{s}_3\omega^{\mu\nu}B_\nu \\
&+ \tilde{s}_4\epsilon^{\mu\nu\alpha\beta}u_\nu E_\alpha\mathcal{D}_\beta\bar{\mu} + \tilde{s}_5\epsilon^{\mu\nu\alpha\beta}u_\nu\mathcal{D}_\alpha E_\beta \,, \qquad \bar{\mu} \equiv \mu/T \,, \\
\mathbb{A}^{(2)}_\mu(\rho) &= \text{same structure} \,,
\end{aligned}$$

- Solve e.o.m. to get the constitutive relations at 2nd order:

$$\langle T^{\mu\nu}\rangle^{(2)}_{\text{diss \& anom}} = \sum_{a=1}^{15}\Lambda_a \mathcal{T}^{(a)\mu\nu} + \sum_{a=1}^{8}\tilde{\Lambda}_a\tilde{\mathcal{T}}^{(a)\mu\nu} \,,$$

$$\langle J^{\mu}\rangle^{(2)}_{\text{diss \& anom}} = \sum_{a=1}^{10}\xi_a \mathcal{J}^{(a)\mu} + \sum_{a=1}^{5}\tilde{\xi}_a\tilde{\mathcal{J}}^{(a)\mu} \,.$$

# Fluid/Gravity Correspondence up to 2nd order

**[E. Megías, F. Pena-Benitez, JHEP 1305:115(2013)]**

- Some anomalous results $(C,P)=(\pm 1,-1)$:

$$\langle T^{\mu\nu}\rangle^{(2)}_{\rm anom} = \tilde\Lambda_1 \Pi^{\mu\nu}{}_{\alpha\beta}\mathcal{D}^\alpha\omega^\beta + \tilde\Lambda_4\Pi^{\mu\nu}{}_{\alpha\beta}\mathcal{D}^\alpha B^\beta + \cdots ,$$

$$\langle J^{\mu}\rangle^{(2)}_{\rm anom} = \tilde\xi_5 \epsilon^{\mu\nu\alpha\beta}u_\nu \mathcal{D}_\alpha E_\beta + \cdots ,$$

$$\tilde\Lambda_1 \equiv -2\eta\tilde I_\omega = -\frac{4\pi\eta}{Gp}\left[\frac{\kappa\mu^3}{48\pi^2} + 64\mu\lambda\left(3r_+^2 - 2\mu^2 - \frac{\pi T\mu^2}{r_+}\right)\right],$$

$$\tilde\Lambda_4 \equiv -2\eta\tilde I_B = -\frac{\eta}{\pi Gp}\left[\frac{\kappa\mu^2}{8} + \lambda\pi^2T^2\right],$$

$$\tilde\xi_5 \equiv \sigma\tilde I_E = -\frac{8\sigma\bar\mu}{\pi^2 T}\left[\kappa\log 2 - 2\lambda(1+2\log 2)\right] + \mathcal{O}(\bar\mu^3).$$

- Dispersion relation of chiral shear waves **[Kharzeev,Sahoo,Yee '10 '11]**

$$\omega \approx -i\frac{\eta}{\epsilon+p}k^2\left(1 \pm \underbrace{\tilde I_\omega k}_{\text{Anomalous}}\right).$$

# Fluid/Gravity Correspondence up to 2nd order

**[E. Megías, F. Pena-Benitez, JHEP 1305:115(2013)]**

- Some non anomalous results $(C, P) \neq (\pm 1, -1)$:

$$\begin{aligned} \langle T^{\mu\nu}\rangle^{(2)}_{\text{non anom}} &= \Lambda_{10}\Pi^{\mu\nu}{}_{\alpha\beta}B^\alpha B^\beta + \cdots , \\ \langle J^{\mu}\rangle^{(2)}_{\text{non anom}} &= \xi_6\omega^{\mu\nu}E_\nu + \cdots , \end{aligned}$$

$$\begin{aligned} \Lambda_{10} &= \frac{11}{96\pi G} + \kappa^2\Lambda_{10,\kappa^2}(\rho_2) + \kappa\lambda\Lambda_{10,\kappa\lambda}(\rho_2) + \lambda^2\Lambda_{10,\lambda^2}(\rho_2) , \\ \xi_6 &= \xi_{6,0}(\rho_2) + \kappa^2\frac{3\left(3+M^2\right)Q^2}{4\pi GM^3} + \kappa\lambda\xi_{6,\kappa\lambda}(\rho_2) + \lambda^2\xi_{6,\lambda^2}(\rho_2) \end{aligned}$$

where $\rho_2 = r_-/r_+$.

- Transport coefficients not affected by the presence of anomalies:

$$\begin{aligned} \langle T^{\mu\nu}\rangle^{(2)}_{\text{non anom}} &= \Lambda_1 u^\alpha\mathcal{D}_\alpha\sigma^{\mu\nu} + \Lambda_2\sigma^{\langle\mu}{}_\gamma\sigma^{\nu\rangle\gamma} + \Lambda_5\mathcal{D}^{\langle\mu}\mathcal{D}^{\nu\rangle}\bar{\mu} + \Lambda_6\mathcal{D}^{\langle\mu}\bar{\mu}\mathcal{D}^{\nu\rangle}\bar{\mu} \\ &+ \Lambda_7\mathcal{D}^{\langle\mu}E^{\nu\rangle} + \Lambda_8E^{\langle\mu}\mathcal{D}^{\nu\rangle}\bar{\mu} + \Lambda_9E^{\langle\mu}E^{\nu\rangle} + \ldots , \\ \langle J^{\mu}\rangle^{(2)}_{\text{non anom}} &= \xi_1\sigma^{\mu\nu}\mathcal{D}_\nu\bar{\mu} + \xi_3P^{\mu\nu}\mathcal{D}^\alpha\sigma_{\nu\alpha} + \xi_5\sigma^{\mu\nu}E_\nu + \xi_7u^\nu\mathcal{D}_\nu E^\mu + \ldots . \end{aligned}$$

# Conclusions

- We have studied the effects of external electromagnetic fields and vortices in a relativistic fluid.
- Anomalies $\Longrightarrow$ parity violating transport.
- Surprise: mixed gauge-gravitational anomaly contributes!!!
- Holography with gauge-gravitational Chern Simons term: Fluid/Gravity Correspondence.
- (Non)-renormalization of anomalous conductivities?
  **[Jensen,Loganayagan,Yarom], [Gokar,Son], [Hou,Liu,Ren], [Zakharov].**
- Currently working on the derivation of Kubo formulae for 2nd order hydrodynamics.
- Observable effects in heavy ion physics and cosmology?:
  - Enhanced production of high spin hadrons, especially $\Omega^-$ baryons, due to chiral separation effect **[Keren-Zur, Oz '10]**.
  - Lepton number separation in the early universe due to gravitational anomaly **[Alexander, Peskin, Sheikh-Jabbari '06]**.

  **[Kharzeev, Son '11], [Kharzeev, Yee '11].**