QCD Mini-jet contribution to the total cross section

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MPI@LHC'08

Outline

- **Mini-jet cross section**
- **Asymptotic rise of Mini-jet cross section**
- **Implementation of the Mini-jet cross section in our eikonal model**
- **Asymptotic rise of the total cross section in our model**

Mini-jet Cross-section

The rise of the total cross section is due to production of jets from high-energy partonic collisions. These are typical perturbative processes and we can describe them through QCD:

$$\sigma_{\rm jet}(s,p_{T\,\rm min}) = \int_{p_{T\,\rm min}}^{\sqrt{s}/2} dp_T \int_{4p_T^2/s}^{1} dx_1 \int_{4p_T^2/(x_1 s)}^{1} dx_2 \sum_{i,j,k,l} f_{i/A}(x_1,p_T^2) f_{j/B}(x_2,p_T^2) \frac{d\hat\sigma_{ij}^{kl}(\hat s)}{dp_T}$$

$f(x,p_T^2)$:PDF's (MRST, GRV, CTEQ all at LO)

p_{Tmin}: minimum transverse momentum of the jet

p_{Tmin}

- p_{Tmin} is a parameter of the mini-jet model. It parameterizes the transition from perturbative to non-perturbative QCD.

- From p_{Tmin} depends the asymptotic rise of σ_{jet}

- In mini-jet models typically p_{Tmin}~1-2 GeV/c (lower than the usual experimental values observable for jet produced in colliders)

Asymptotic Rise of σ_{jet}

- As the parton flux increases with energy, integrated jet cross-sections increase rapidly with energy

$$\sigma_{\text{jet}}(s, p_{T\,\text{min}}) = \int_{p_{T\,\text{min}}}^{\sqrt{s}/2} dp_T \int_{4p_T^2/s}^{1} dx_1 \int_{4p_T^2/(x_1 s)}^{1} dx_2 \sum_{i,j,k,l} f_{i/A}(x_1, p_T^2) f_{j/B}(x_2, p_T^2) \frac{d\hat{\sigma}_{ij}^{kl}(\hat{s})}{dp_T}$$

$$\frac{d\hat{\sigma}_{ij}^{kl}}{dp_T} \propto \frac{1}{p_T^3}$$

- If $\sqrt{(s)} \gg p_{Tmin}$ the integral receives its dominant contribution from $x_{1,2} \ll 1$.

- The relevant parton densities can then be approximated by a simple power law, $f \sim x^{-J}$ (J>1).

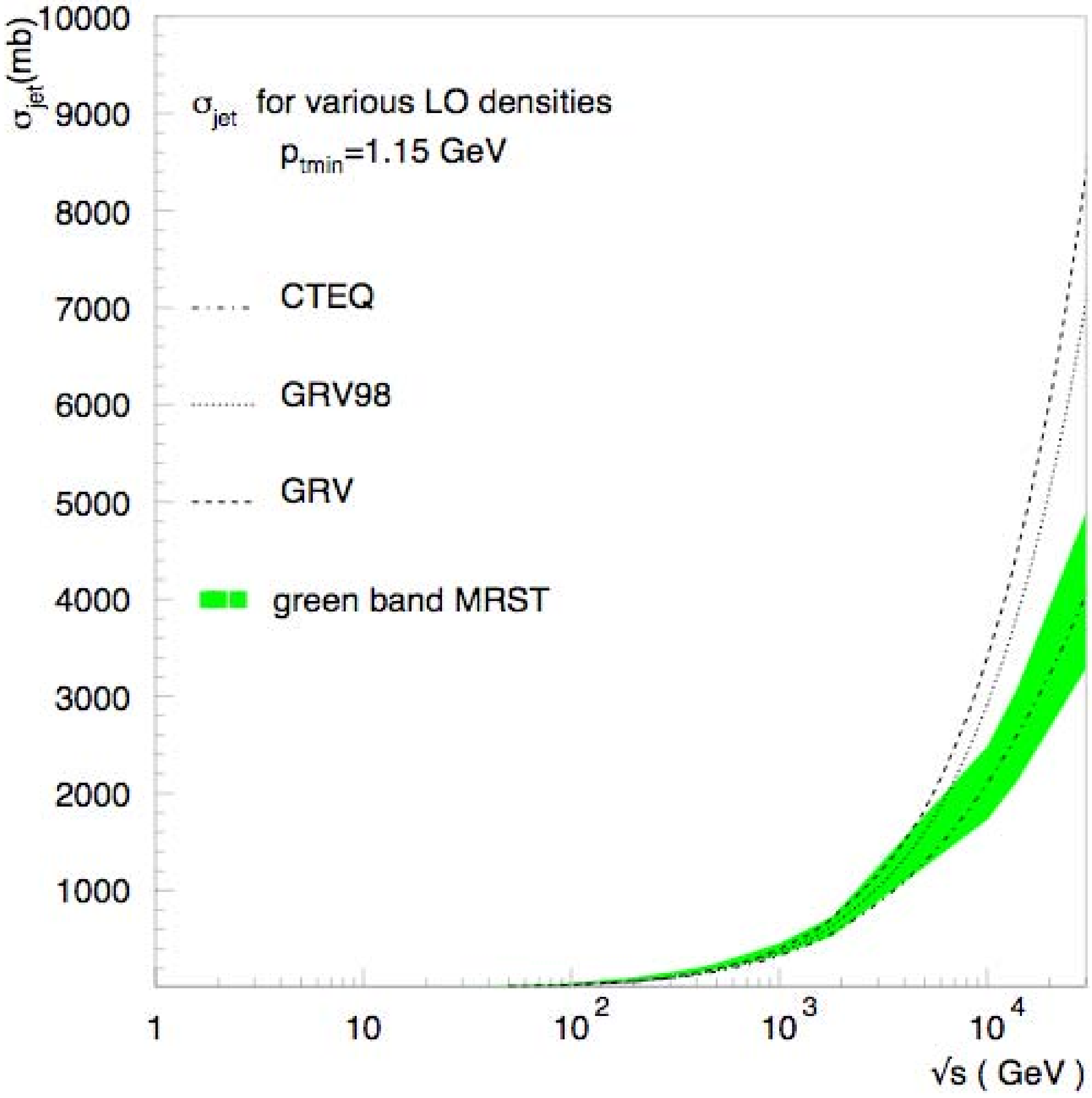
Asymptotic Rise of σ_{jet}

- Experimentally J~1.3
- For p-p scattering f_A=f_B and:

$$\sigma_{jet} \propto \frac{1}{p_{T\,\min}} \left[\frac{s}{4p_{T\,\min}^2} \right]^{J-1}$$

- Power law growth

$$\sigma_{jet} \propto \left(\frac{s}{GeV^2}\right)^{J-1}$$



Asymptotic behavior of σ_{jet} for different PDF (LO):

CTEQ: J~1.3

GRV98: J~1.4

GRV: J~1.4

MRST: J~1.3

Asymptotic Rise of σ_{jet}

- Violation of Froissart Bound
- Consequence of infinite range of QCD
- One needs to introduce a finite-distance interaction
- Eikonal does it through the hadron finite size

Multiplicity Factor in σ_{jet}

- Since partonic scattering always produces a pair of mini-jets:

$$\sigma_{jet} = n_{jet}\sigma_{inelastic}$$

- n_{jet} is the number of mini-jet pairs produced per inelastic partonic collision.
- on average each inelastic event contains more than one hard partonic scatter.
- The simplest possible assumption about these multiple partonic interactions is that they occur completely independently of each other.

Eikonal derivation

- If the number of jet pairs obeys a Poisson distribution, the probability of having k jet pairs is:

$$P(k, < n_{jet} >) = \frac{<n_{jet}>^k e^{-<n_{jet}>}}{k!}$$

- Then the inelastic cross-section is:

$$\sigma_{inelastic} = \int d^2b \sum_{k=1} P(k, < n(b,s) >) = \int d^2b [1 - e^{-<n(b,s)>}]$$

- Which is the usual eikonal expression with:

$$< n(b,s) >= 2\mathrm{Im}\chi(b,s)$$

Eikonal mini-jet Model

It implies multiple scattering and requires impact parameter distributions

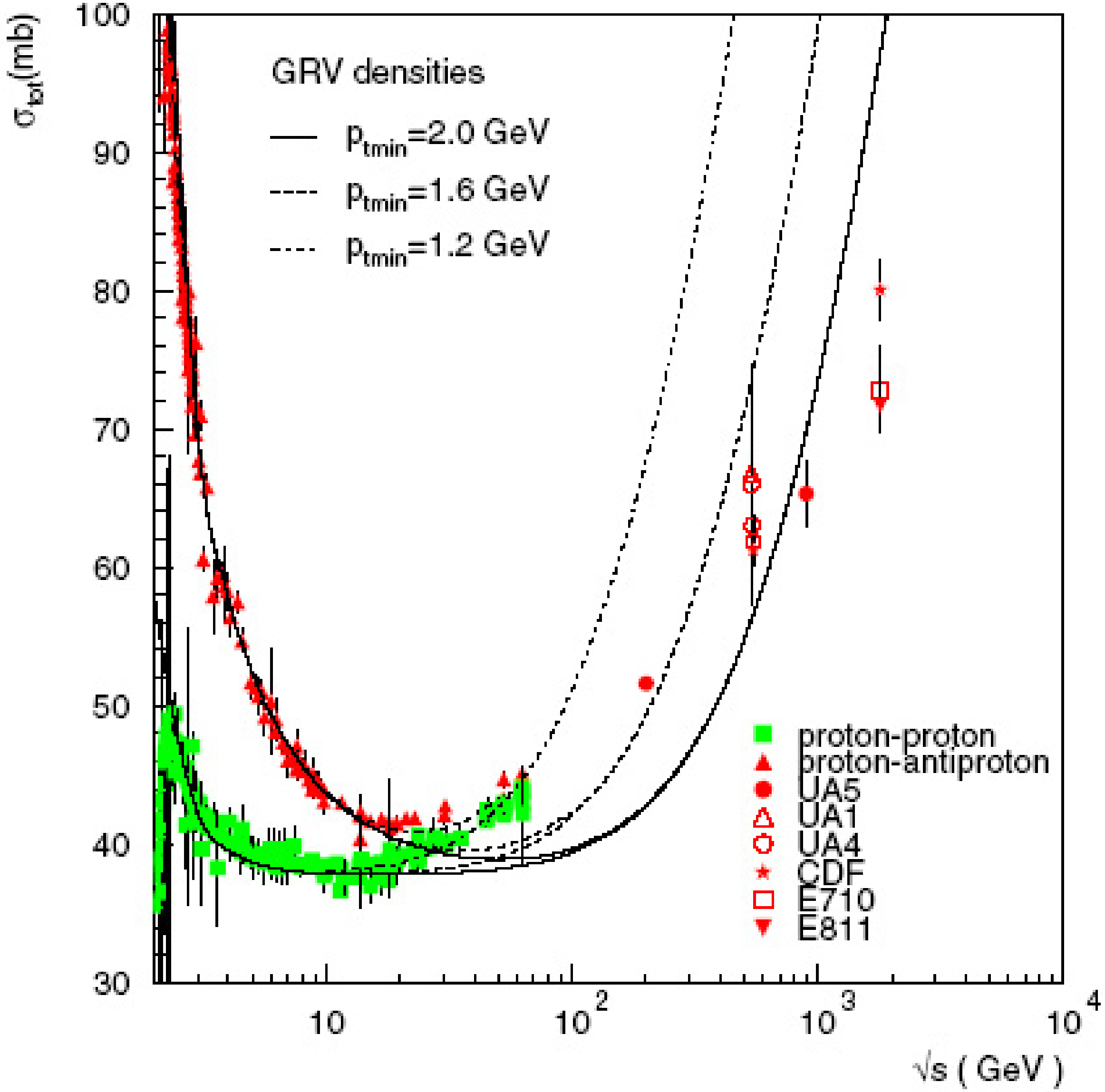
High energies: Reχ(b,s) ~ 0 $\sigma_{tot} = 2\int d^2b[1 - e^{-n(b,s)/2}]$

$$n(b,s) = n_{soft}(b,s) + n_{hard}(b,s)$$

$$n(b,s)_{hard} = A(b,s)\sigma_{jet}(s)$$

Spatial distribution of matter inside of the colliding hadrons

Electromagnetic Form Factor Model



Factorization between s and b dependence:

n(b,s)=A(b)σ(s)

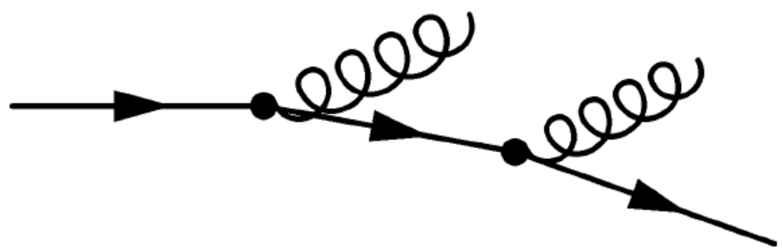
The overlap function depends only on the impact parameter

Excessive high energy rise

Soft Gluon resummation effects

- Soft processes for which $p_T < p_{Tmin}$ are not included into the mini-jet cross-section

- Soft gluon emission changes the parton co-linearity giving the impact parameter distribution



- The number of soft emissions and so also the degree of acolinearity increases with the energy and introduces an energy dependence in A(b)

Soft Gluon resummation effects

$$A_{BN}(b,s) = \frac{1}{2\pi}\int d^2K_\perp e^{-i\vec{K}_\perp\cdot\vec{b}}\,\frac{d^2P(K_\perp)}{d^2K_\perp}$$

Using a Bloch-Nordsieck inspired formalism we obtain the distribution of colliding partons as function of the transverse momentum of the soft gluons emitted

$$\frac{d^2P(K_\perp)}{d^2K_\perp} = \frac{1}{(2\pi)^2}\int d^2b' e^{i\vec{K}_\perp\cdot\vec{b'} - h(b',s)}$$

Average number of soft gluon emitted

$$h(b,s) = \int d^3\bar{n}_g(k)[1 - e^{-i\mathbf{k}_\perp\cdot\mathbf{b}}]$$

Soft Gluon resummation effects

$$A(b,s) = \frac{e^{-h(b,s)}}{\int d^2b\, e^{-h(b,s)}}$$

$$h(b,s) \approx \int_0^{q_{\max}(s)} \frac{dk_T}{k_T} \alpha_s(k_T) \log\left(\frac{2q_{\max}(s)}{k_T}\right) [1 - J_0(k_T)]$$

Phenomenological form modelling the infrared behaviour

Alpha strong parameterization

•Singular but integrable expression

•Requirements:

Asymptotic freedom QCD expression for $k_T >> \Lambda_{QCD}$

Confining potential in the infrared limit

$$\alpha_s(k_T) = \frac{12\pi}{(33-2N_f)} \frac{p}{\log(1+p(k_T^2/\Lambda_{QCD}^2)^p)}$$

$\frac{1}{2} < p < 1$

q_{max}

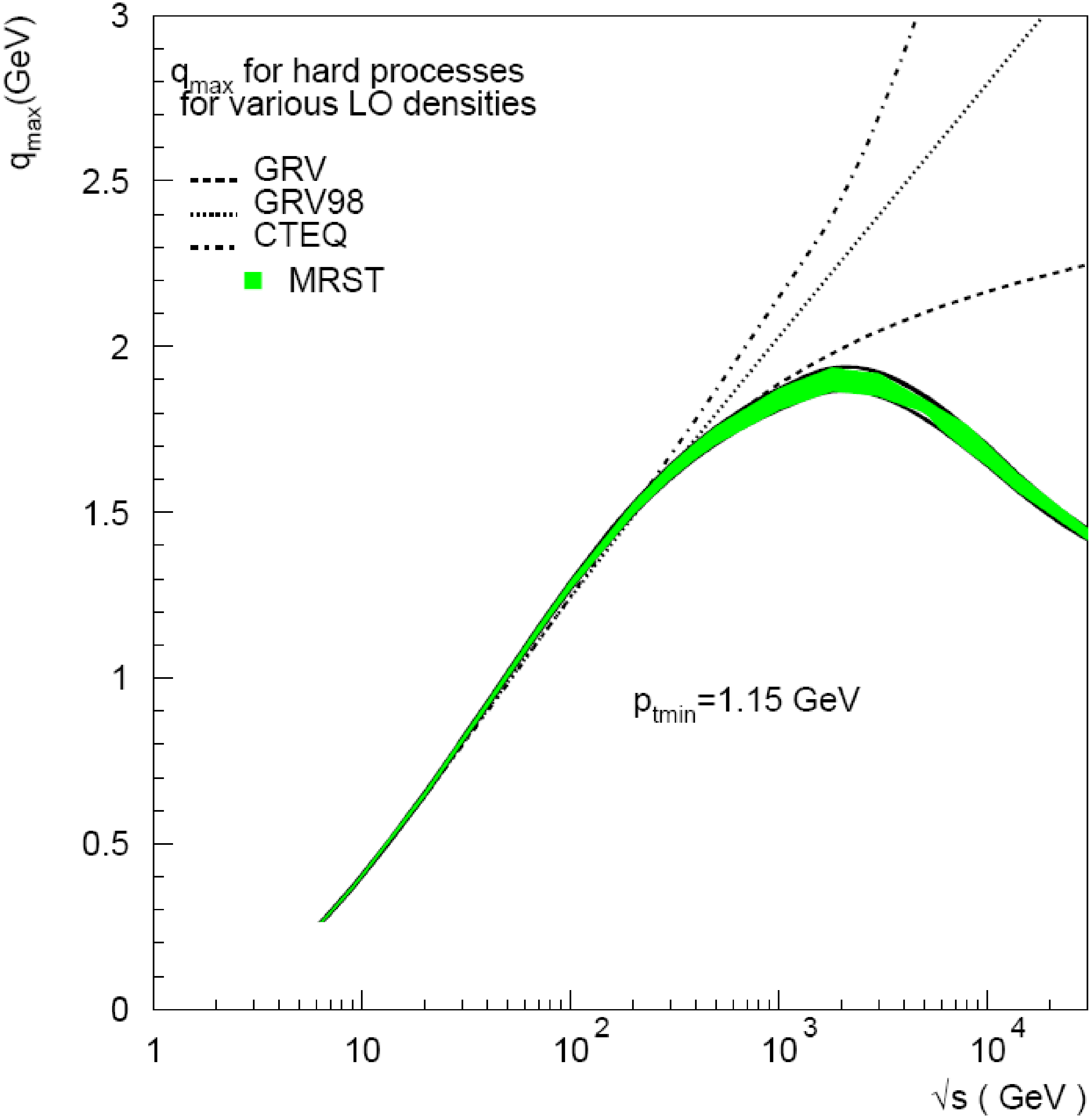
Linked to the maximum transverse momentum of a single gluon emitted in a hard collision allowed by kinematics:

$$q_{\max} = \sqrt{\frac{\hat{s}}{2}(1 - \frac{Q^2}{\hat{s}})} = \sqrt{\frac{s}{2}}\sqrt{x_1 x_2}(1 - z)$$

Averaged over same PDF used for σ_{jet}

$$< q_{\max}(s) >= \sqrt{\frac{s}{2}} \frac{\sum_{i,j} \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \int_{z_{\min}}^1 dz f_i(x_1) f_j(x_2) \sqrt{x_1 x_2}(1-z)}{\sum_{i,j} \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \int_{z_{\min}}^1 dz f_i(x_1) f_j(x_2)}$$

$z_{min} = 4(p_{Tmin})^2 / (sx_1x_2)$



q_{max} for hard processes
for various LO densities
GRV
GRV98
CTEQ
MRST
p_{tmin}=1.15 GeV
q_{max}(GeV)
3
2.5
2
1.5
1
0.5
0
1
10
10^2
10^3
10^4
$\sqrt{s}$ (GeV)

Other phenomenological models

Donnachie e Landshoff [Phys. Lett. B296 (1992) 227–232]

$$\sigma_{tot} = X(\frac{s}{s_0})^{\epsilon} + Y(\frac{s}{s_0})^{-\eta} \qquad \varepsilon \approx 0.08 \quad \eta \approx 0.5$$

Donnachie e Landshoff [Phys. Lett. B595 (2004) 393–399]

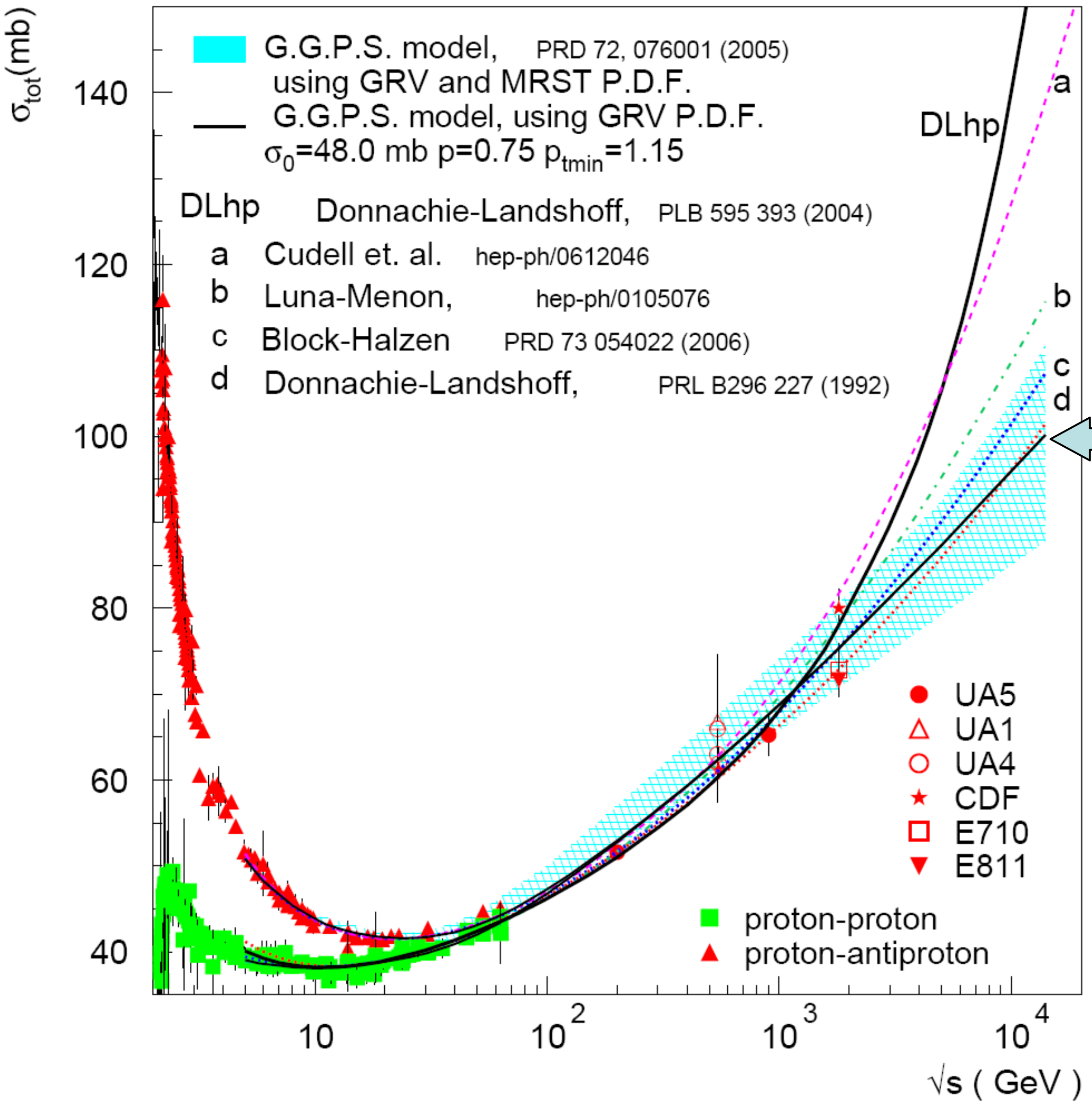
$$\sigma_{tot} = X_0(\frac{s}{s_0})^{\epsilon_D} + X(\frac{s}{s_0})^{\epsilon} + Y(\frac{s}{s_0})^{-\eta}$$

$$\varepsilon_0 = 0.452 \quad \varepsilon = 0.0667 \quad \eta = 0.476$$

Block e Halzen [Phys. Rev. D72 (2005) 036006]

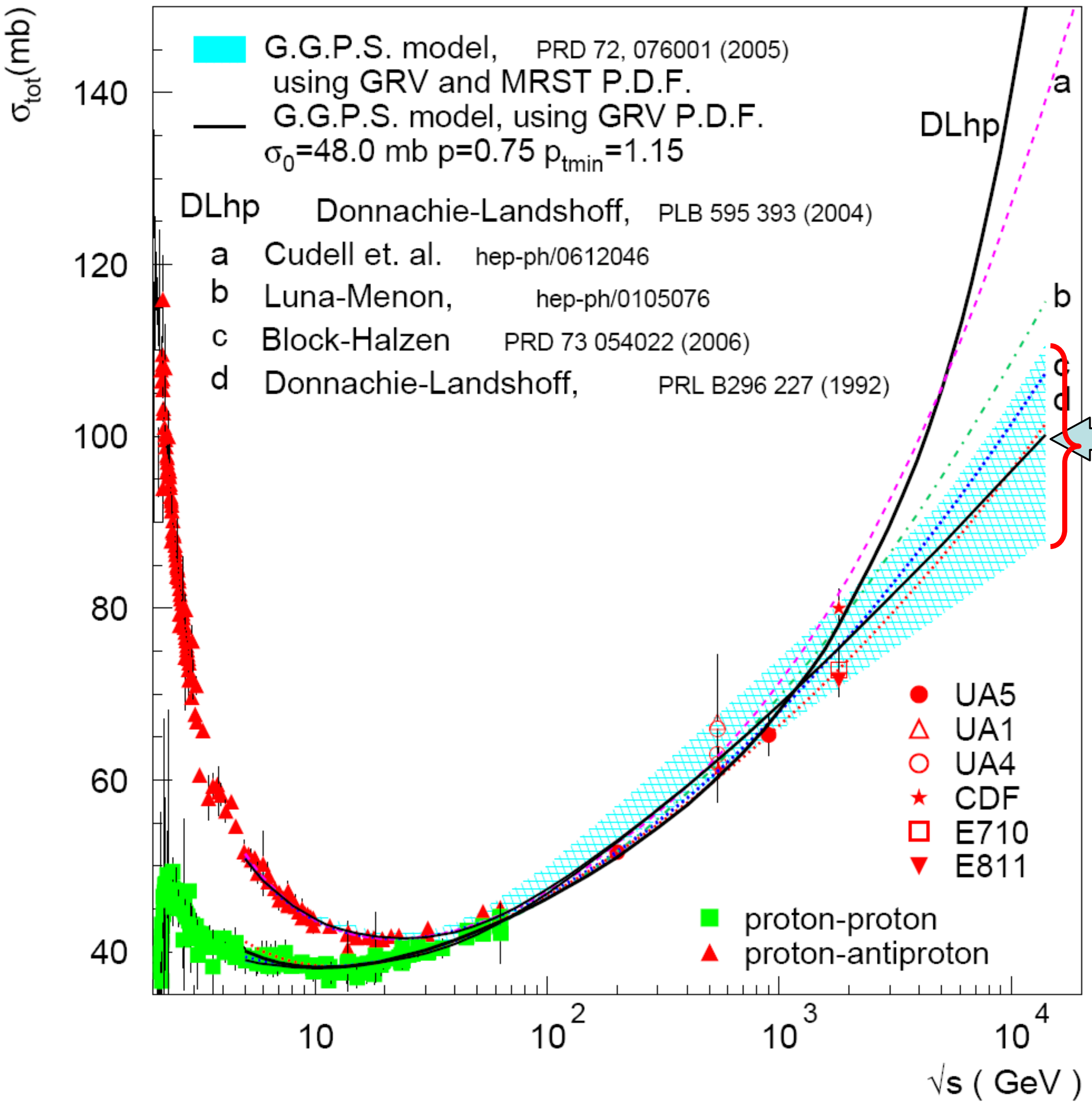
$$\sigma^{\pm} = c_0 + c_1 \ln\left(\frac{\nu}{m}\right) + c_2 \ln^2\left(\frac{\nu}{m}\right) + \beta_{P'}\left(\frac{\nu}{m}\right)^{\mu-1} \pm \delta\left(\frac{\nu}{m}\right)^{\alpha-1}$$

Results of the model



100.2 mb (14 TeV)

Results of the model



100.2 mb (14 TeV)

Range:

~ 87-110 mb

Restoration of Froissart Bound

There is an interesting relation linking the high energy behaviour of the total cross section with the asymptotic rise of σ_{jet} and with the degree of infrared divergence of α:

$$\alpha_s(Q^2) \longrightarrow \left(\frac{1}{Q^2}\right)^p \quad \text{as} \quad Q^2 \longrightarrow 0$$

$$\sigma_{jet}(s; p_{T\,\min}) \longrightarrow \sigma_1\left(\frac{s}{GeV^2}\right)^\varepsilon \quad \text{as} \quad s \longrightarrow \infty$$

ε ~ 0.3

Restoration of Froissart Bound

$$\sigma_{tot} \approx 2\pi \int db^2 [1 - e^{-n_{hard}(b,s)/2}]$$

$$n_{hard}(b, s) \approx \sigma_{jet}(s) A_{hard}(b, s)$$

$$A_{hard}(b, s) \propto e^{-h(b,s)}$$

$$h(b, s) = \int d^3 \bar{n}_g(k) [1 - e^{-i\mathbf{k}_\perp \cdot \mathbf{b}}]$$

Restoration of Froissart Bound

If we take: $k_t \rightarrow 0$

$$d^3n(k) \propto \alpha_s(k_t^2)$$

$$\alpha_s(k_t) \approx (\frac{\Lambda}{k_t})^{2p}$$

$$h(b,s) \propto (b\bar{\Lambda})^{2p} \qquad p < 1;$$

$$A_{hard}(b) \propto e^{-(b\bar{\Lambda})^{2p}}$$

Restoration of Froissart Bound

$$n_{hard} = 2C(s)e^{-(b\bar{\Lambda})^{2p}}$$

$$C(s) = A_0\sigma_1\left(\frac{s}{GeV^2}\right)^{\varepsilon}$$

$$\sigma_{tot} \approx 2\pi\int_0^\infty db^2[1 - e^{-C(s)e^{-(b\bar{\Lambda})^{2p}}}]$$

$$\sigma_{tot}(s) \longrightarrow [\varepsilon\ln(s/s_0)]^{1/p}$$

Restoration of Froissart Bound

$$\alpha_s(Q^2) \longrightarrow \left(\frac{1}{Q^2}\right)^p \quad \text{as} \quad Q^2 \longrightarrow 0$$

$$\sigma_{jet}(s; p_{T\,\min}) \longrightarrow \sigma_1\left(\frac{s}{GeV^2}\right)^{\varepsilon} \quad \text{as} \quad s \longrightarrow \infty$$

$$\sigma_{tot}(s) \longrightarrow \left[\varepsilon \ln(s/s_0)\right]^{1/p}$$

Conclusion

- Mini-jet cross-section has an asymptotic power law rise that violates Froissart Bound

- Soft gluon emissions from colliding partons, restore the finite range of the hadronic interaction

- Inserting QCD mini-jet cross-section into an eikonal formalism which implements multi-particle interaction and resumming soft gluon emission effects it is possible reproduce the right asymptotic rise of σ_{tot}

- The infrared behaviour of α_s has a fundamental effect on the asymptotic rise of σ_{tot}