Multijet Production and s-channel unitarity

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pp collisions: Impact Parameter Picture

- *Profile function:*

$$\Gamma(s,b) = \frac{1}{2is(2\pi)^2} \int d^2\mathbf{q}\, e^{i\mathbf{q}\cdot\mathbf{b}} A(s,t), \qquad t \approx -\mathbf{q}^2$$

$A(s,t)$ → *pp elastic amplitude*

(small angle scattering)

- *s-channel unitarity:*

$$\sigma_{tot} = 2\int d^2\mathbf{b}\, \Re\Gamma(s,b),$$

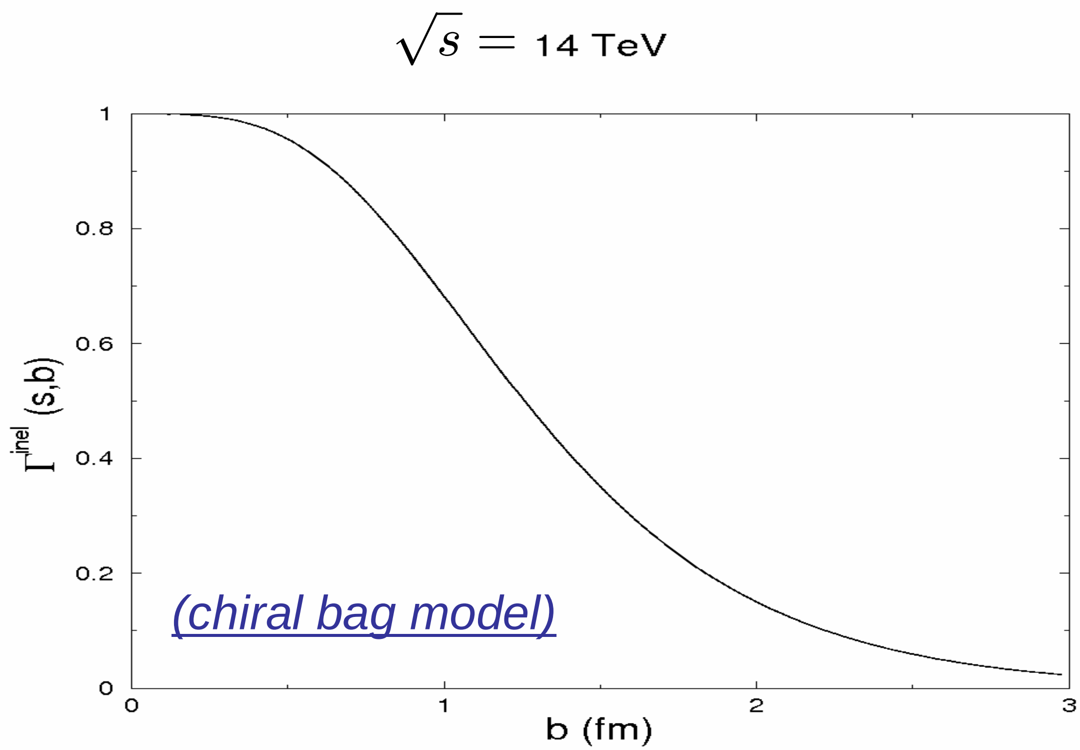
$$\sigma_{el} = \int d^2\mathbf{b}\, |\Gamma(s,b)|^2,$$

$$\sigma_{inel} = \int d^2\mathbf{b}\, \underbrace{\left(2\Re\Gamma(s,b) - |\Gamma(s,b)|^2\right)}_{\Gamma^{inel}(s,b)}.$$

Given Γ, we get all of these.

Modelling pp cross-sections at very high energies

- Unitarity constraint on (dominantly real) profile: $\Gamma(s,b) \leq 1$
- Extrapolation to very high energies: $(\sqrt{s} \approx 14\,\mathrm{TeV}, \mathrm{GZK\ energies})$
  - ***Use extrapolation of elastic differential cross section.***



(example extrapolation: Islam, Luddy, Prokudin (2003))

$$\Gamma = \Gamma^{inel} = 1$$

$$\Rightarrow \sigma_{el} = \sigma_{inel}$$

Modelling pp x-sections at very high energies

- Both hard and soft contributions:
  - Typically modelled in an eikonal/parton picture:

(e.g., R. Engel review (2003))

$$\Gamma(s,b) = 1 - \exp\left[-\underbrace{\chi_h(s,b)}_{\text{hard}} - \underbrace{\chi_s(s,b)}_{\text{soft}} + \cdots\right]$$

(DPMJET, QGSJET, SIBYLL…)

- Hard part modeled by pQCD dijet factorization formula.
  - Impact parameter dependence usually modelled / obtained from fits.
- Soft part modelled with Regge theory.
- Eikonal model unitarizes profile function. (Many versions.)
- Can use profile function to compare to total cross sections.

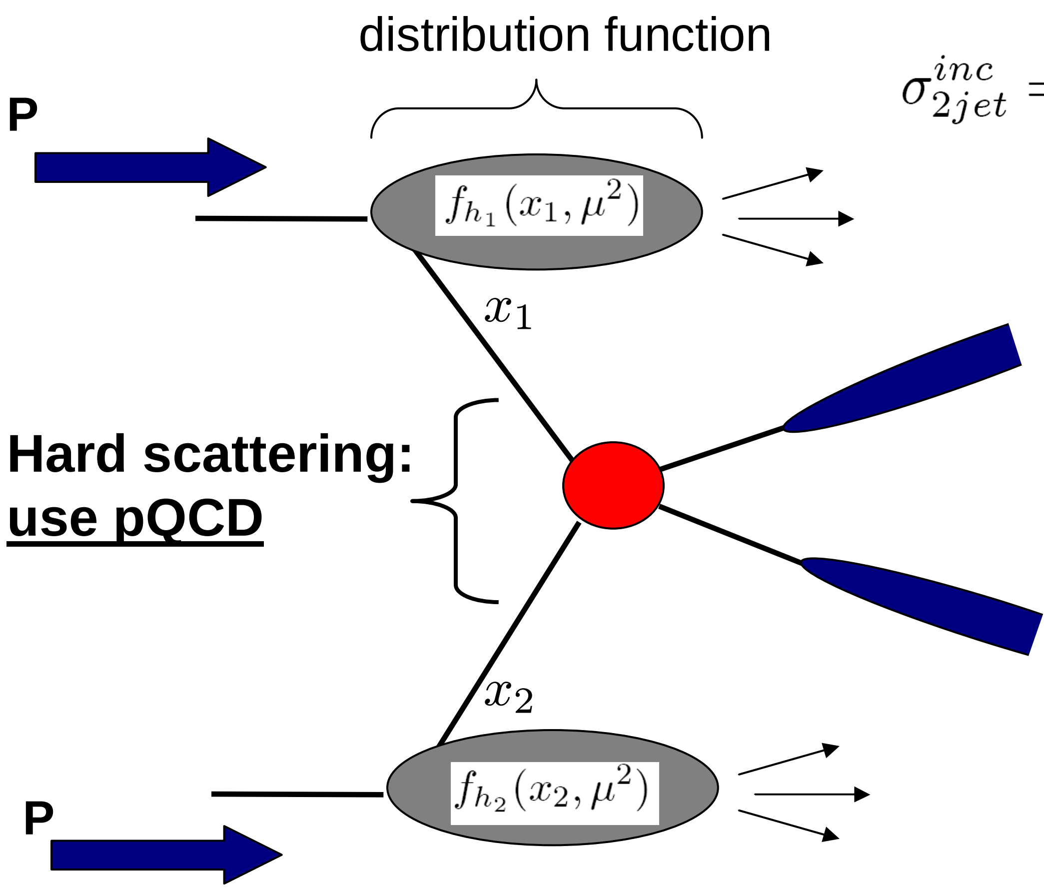
Focus: Hard Scattering

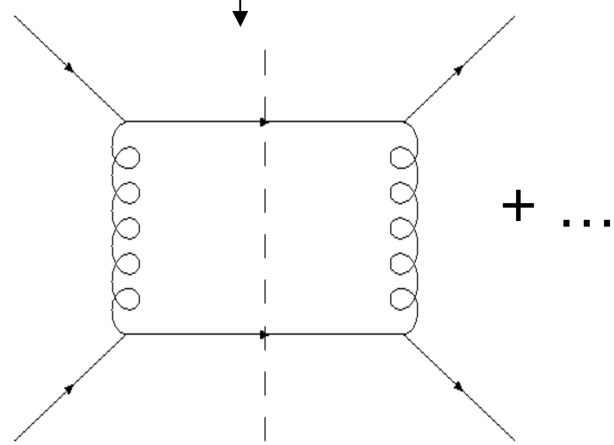
- At least one (semi-)hard jet pair.

factorization formula

$$\sigma_{2jet}^{inc} = \int_{p_t^c}^{\infty} dp_t^2 \, \frac{d\hat{\sigma}}{dp_t^2} \, f_{h_1}(x_1,\mu^2) \otimes f_{h_2}(x_2,\mu^2)$$

$$\mu^2 \sim p_t^2$$





Hard Scattering: Issues

$$\sigma_{2jet}^{inc}(s, p_t^c) = \sum \frac{K}{1+\delta_{kl}} \int_{p_t^c} d\,p_t^2 \frac{d\hat{\sigma}_{ij\to kl}}{dp_t^2} f_i(x_1, \mu^2) \otimes f_j(x_2, \mu^2)$$

$$(\mu^2 = p_t^2)$$

- Shape of profile?

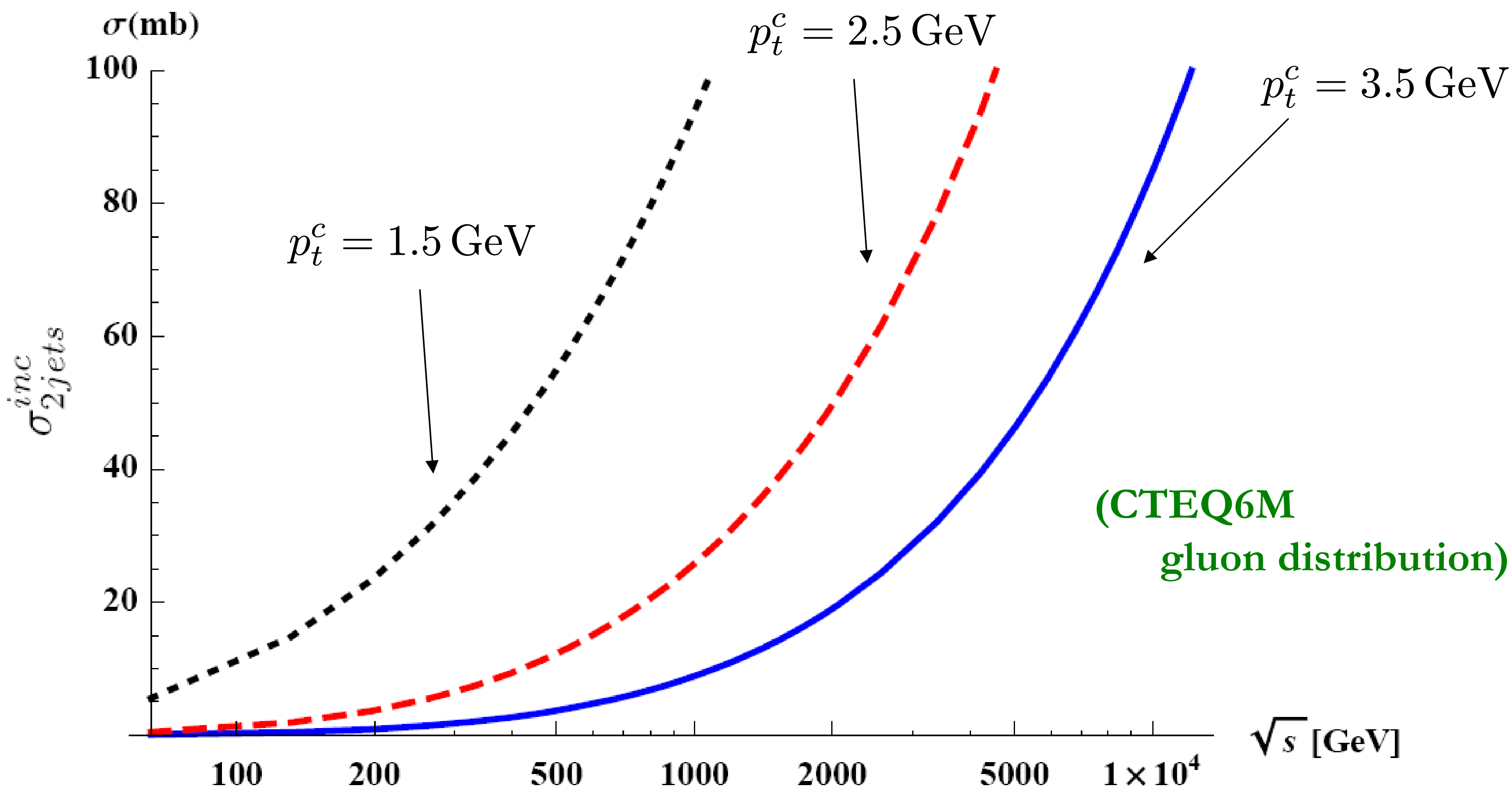
$$\chi_h(b) = \frac{\sigma_h(s)}{8\pi R_0^2} \exp\left\{-b^2/4R_0^2\right\}$$

  - Can we obtain profile from pQCD?

- How to extrapolate f(x,p_t^2) to very small x?
  - $f(x, \mu^2) \sim \frac{1}{x^{\Delta+1}}, \qquad \Delta = ??$

- Sensitive to precise value of p_t^c
  - Varies between ~ 1.5 GeV & 6 GeV in different approaches. *(sometimes grows with energy)*
  - How large p_t cutoff for factorization formula to be taken literally?

Hard Scattering



Rapid Increase!

How to constrain minimum p_t and/or cross section?

- Directly imposing unitarity constraints on inclusive process is not possible.

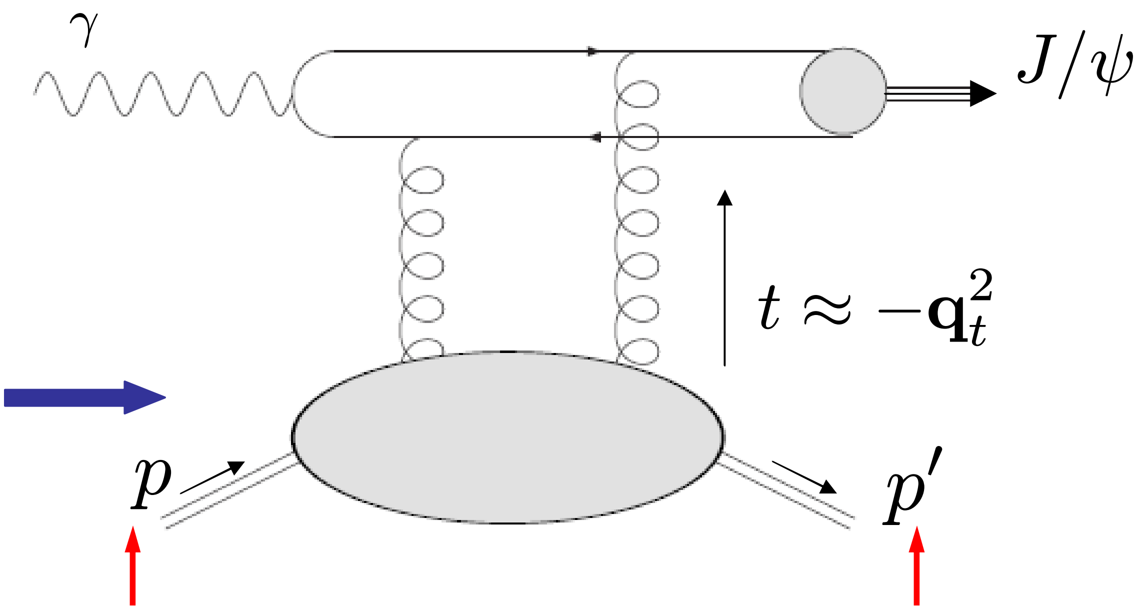
Strategy

- Reconstruct total inelastic cross section from pQCD dijet cross section (with some assumptions).
- Obtain pQCD description of impact parameter dependence from GPDs – J/ψ photo-production.
- Check consistency between reconstructed cross section with model/measurement.

Information from pQCD:

- Factorization for vector meson production:

(universal object)



- Non-diagonal transition in *Generalized* PDF
  - – gives information about transverse distribution of partons.

Information from pQCD: GPDs

- The generalized gluon PDF from J/ψ photo-production.

$$x f_g(x,t,\mu) = x f_g(x,\mu)\, F_g(x,t,\mu)$$

Non-diagonal transitions.

$$F_g(x, t=0, \mu) = 1$$

- Impact parameter space gluon distribution function.

$$\mathcal{F}_g(x,\rho,\mu) = \int d^2\mathbf{\Delta}\, \underbrace{F_g(x,t,\mu)}_{\text{2 gluon form factor}}\, e^{-i\mathbf{\Delta}\cdot\rho}, \qquad t = -\Delta^2$$

(universal object)

2 gluon form factor

Information from pQCD: GPDs

- Frankfurt, Strikman, Weiss (2004): fit to 2-gluon form factor from J/ψ production:

$$F_g(x,t,\mu) = \frac{1}{\left(1 - \underbrace{\frac{t}{m_g^2(x,\mu)}}\right)^2}$$

mass scale varies to take into account evolution/small-x

- In impact parameter space:

$$\mathcal{F}_g(x,\rho,\mu) = \frac{m_g^3(x,\mu)\rho}{4\pi} K_1(m_g(x,\mu)\rho)$$

Minijet cross section: overlap function

- pp $\longrightarrow$ 2 jet + X cross section in impact parameter space.

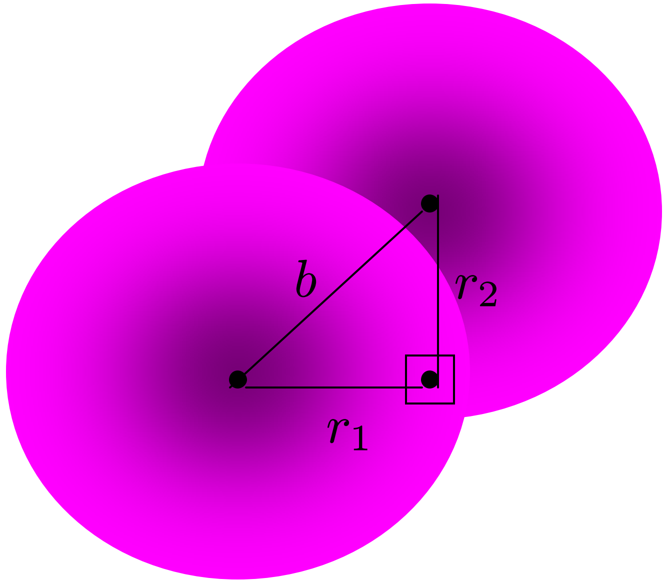
$$\sigma_{2jet}^{inc}(s, p_t^c) = \int d^2\mathbf{b}\, \mathcal{N}_2(b, s, p_t^c)$$

$$\mathcal{N}_2(b, s, p_t^c) \equiv \sum_{k,l} C_{k,l} \int_{p_t^{c\,2}}^{\infty} d\,p_t^2 \frac{d\hat{\sigma}_{ij\to kl}}{dp_t^2} f_g(x_1, p_t^2) \otimes f_g(x_2, p_t^2) P_2(b, x_1, x_2, p_t)$$

Overlap function

$$P_2(b, x_1, x_2, \mu) = \int d^2\mathbf{r_1} \int d^2\mathbf{r_2}\, \mathcal{F}_g(x_1, r_1, \mu)\, \mathcal{F}_g(x_2, r_2, \mu) \delta^2(\mathbf{b} - \mathbf{r}_1 - \mathbf{r}_2)$$

Overlap Function



Geometry of hard collision

- Evaluate using approximation: $x_1 \sim x_2 \sim 2p_t^c/\sqrt{s} \equiv \bar{x}$

Allows dijet cross section to be factored.

$$\mathcal{N}_2(b,s,p_t^c) \approx \sigma_{2jet}^{inc}(s,p_t^c)P_2(b,s,p_t^c)$$

Overlap can be directly evaluated.

$$P_2(b,s,p_t^c) = \frac{m_g^2(\bar{x},p_t^c)}{12\pi}\left(\frac{m_g(\bar{x},p_t^c)b}{2}\right)^3 K_3(m_g(\bar{x},p_t^c)b)$$

Reconstruct inelastic profile function

- Inclusive dijet cross section:
  - $$\mathcal{N}_2(b,s) = \sum_{n=1}^{\infty} n \tilde{\mathcal{N}}_{2n}(b,s)$$

    where,
  - $$\int d^2\mathbf{b}\, \tilde{\mathcal{N}}_{2n}(b,s) = \sigma_{2k}^{ex}(s)$$

- In general:

$$\mathcal{N}_{2k}(b,s) = \sum_{n\geq k}^{\infty} \binom{n}{k} \tilde{\mathcal{N}}_{2n}(b,s)$$

Invert:

$$\tilde{\mathcal{N}}_{2k}(b,s) = \sum_{n\geq k}^{\infty} \binom{n}{k} (-1)^{n-k} \mathcal{N}_{2n}(b,s)$$

Reconstruct inelastic profile function

- Total inelastic cross section:

$$\Gamma^{inel}_{dijets}(s,b) \equiv \sum_{k=1}^{\infty} \tilde{\mathcal{N}}_{2k}(b,s) = \sum_{n=1}^{\infty} (-1)^{n-1} \mathcal{N}_{2n}(b,s)$$

$$\tilde{\mathcal{N}}_{2k}(b,s) = \sum_{n\geq k}^{\infty} \binom{n}{k} (-1)^{n-k} \mathcal{N}_{2n}(b,s)$$

- Consistency requirement:

$$\Longrightarrow \quad \Gamma^{inel}_{dijets}(s,b) \leq \Gamma^{inel}(x,b)$$

- Compare with inelastic profile obtained from unitarity relation models/extrapolation of elastic profile:

$$\Gamma^{inel}(s,b) \equiv 2\Gamma(s,b) - |\Gamma(s,b)|^2$$

Reconstruct inelastic profile function

- Simplifying assumption valid for large impact parameters: neglect correlations.

$$\mathcal{N}_{2k}(s,b) \approx (\mathcal{N}_2(b,s,p_t^c))^k = (\sigma_{2jet}^{inc} P_2(b,\bar{x},p_t^c))^k$$

use the FSW expression:

❑ Non-identical partons:

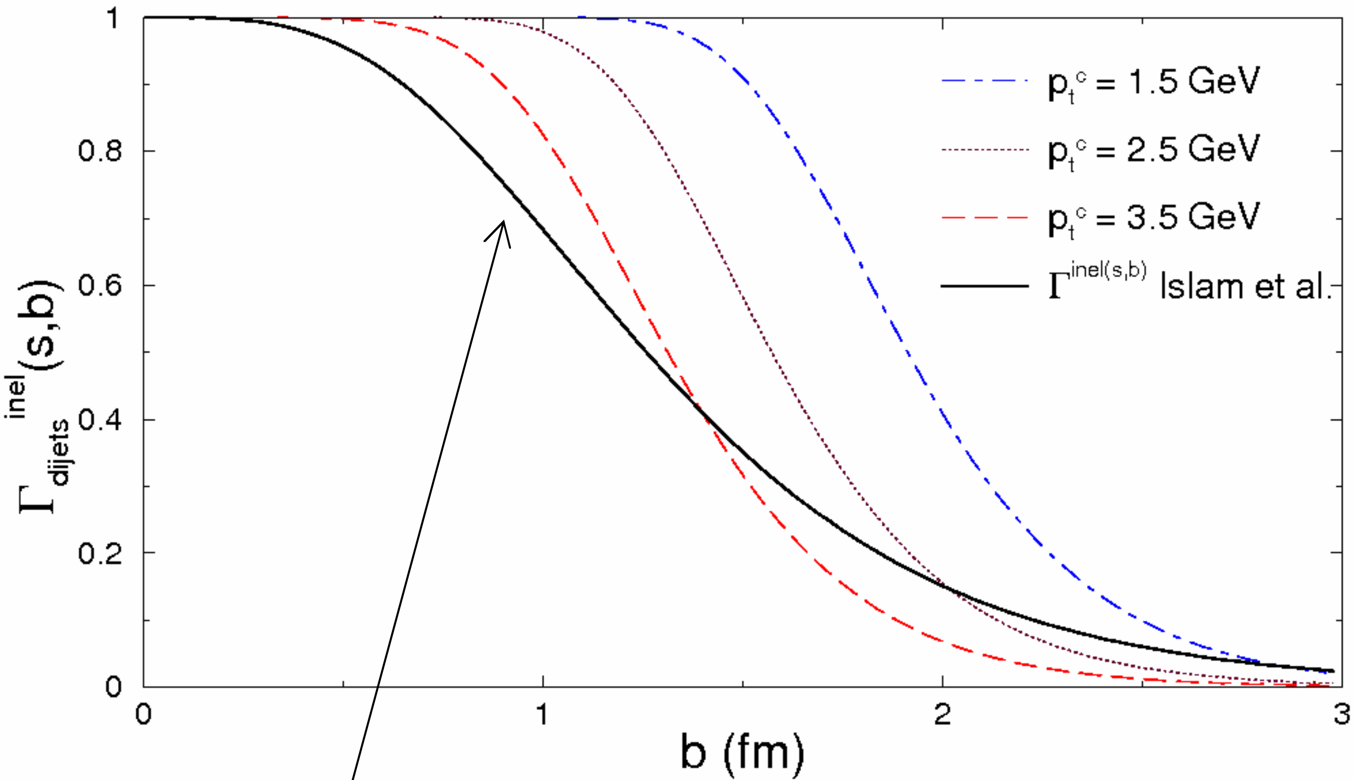
$$\Gamma_{dijets}^{inel}(s,b) = \sum_{n=1}^{\infty} (-1)^{n-1} \mathcal{N}_{2n}(b,s) = \frac{\sigma_{2jet}^{inc} P_2(b,\bar{x},p_t^c)}{1+\sigma_{2jet}^{inc} P_2(b,\bar{x},p_t^c)}$$

❑ Identical partons:

$$\Gamma_{dijets}^{inel}(s,b) = 1 - \exp\left[-\sigma_{2jet}^{inc} P_2(b,\bar{x},p_t^c)\right]$$

Check that: $\Gamma_{dijets}^{inel}(s,b) \leq \Gamma^{inel}(x,b)$

Compare reconstructed profile with model extrapolation.



- Identical partons,
- CTEQ6M gluon PDF

$\sqrt{s} = 14\ \mathrm{TeV}$

extrapolated profile function from elastic cross section

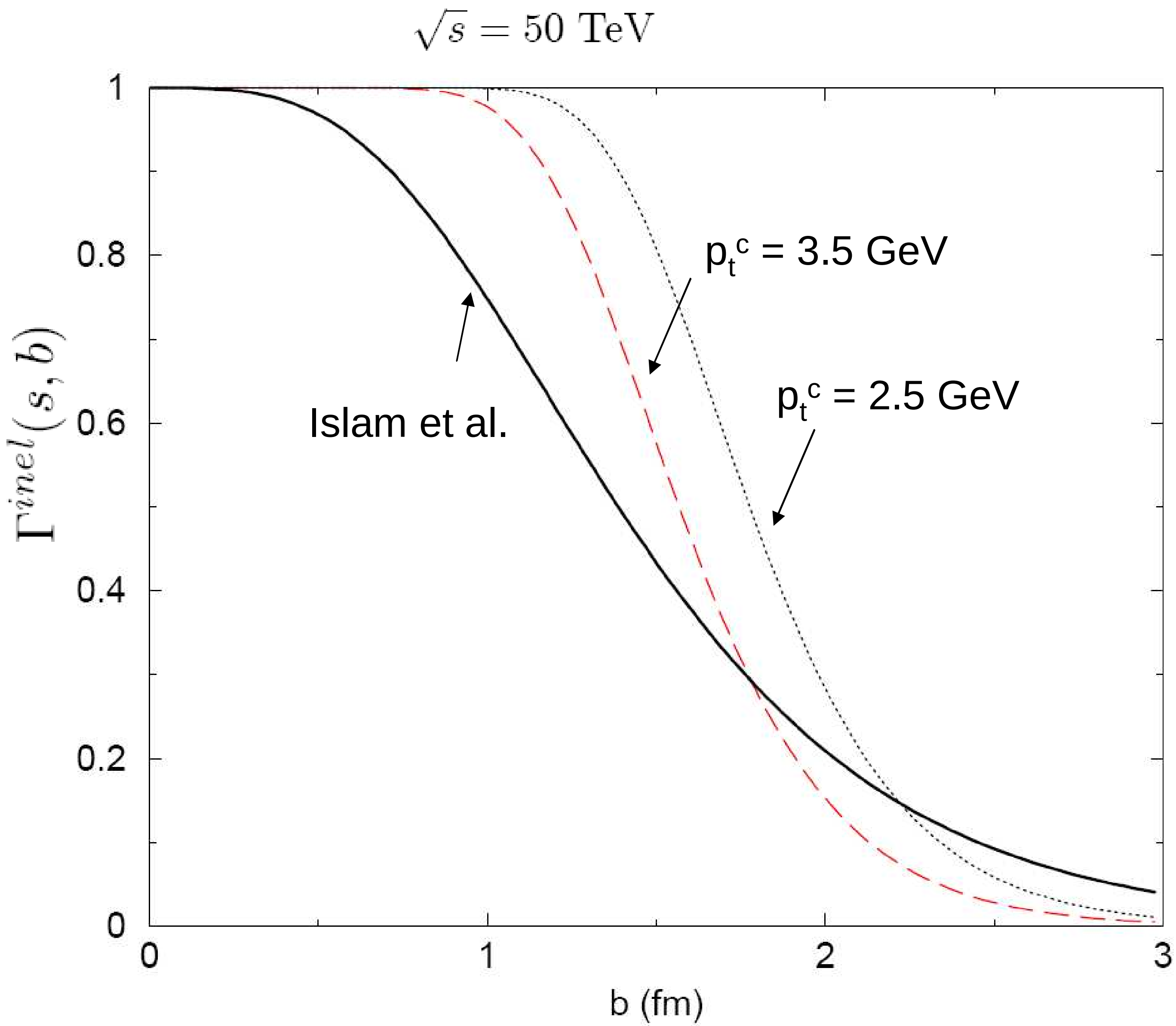
Small effect from correlations

- **Mismatch in description at large impact parameters where we expect small effect from correlations.**

(other models also compared.)

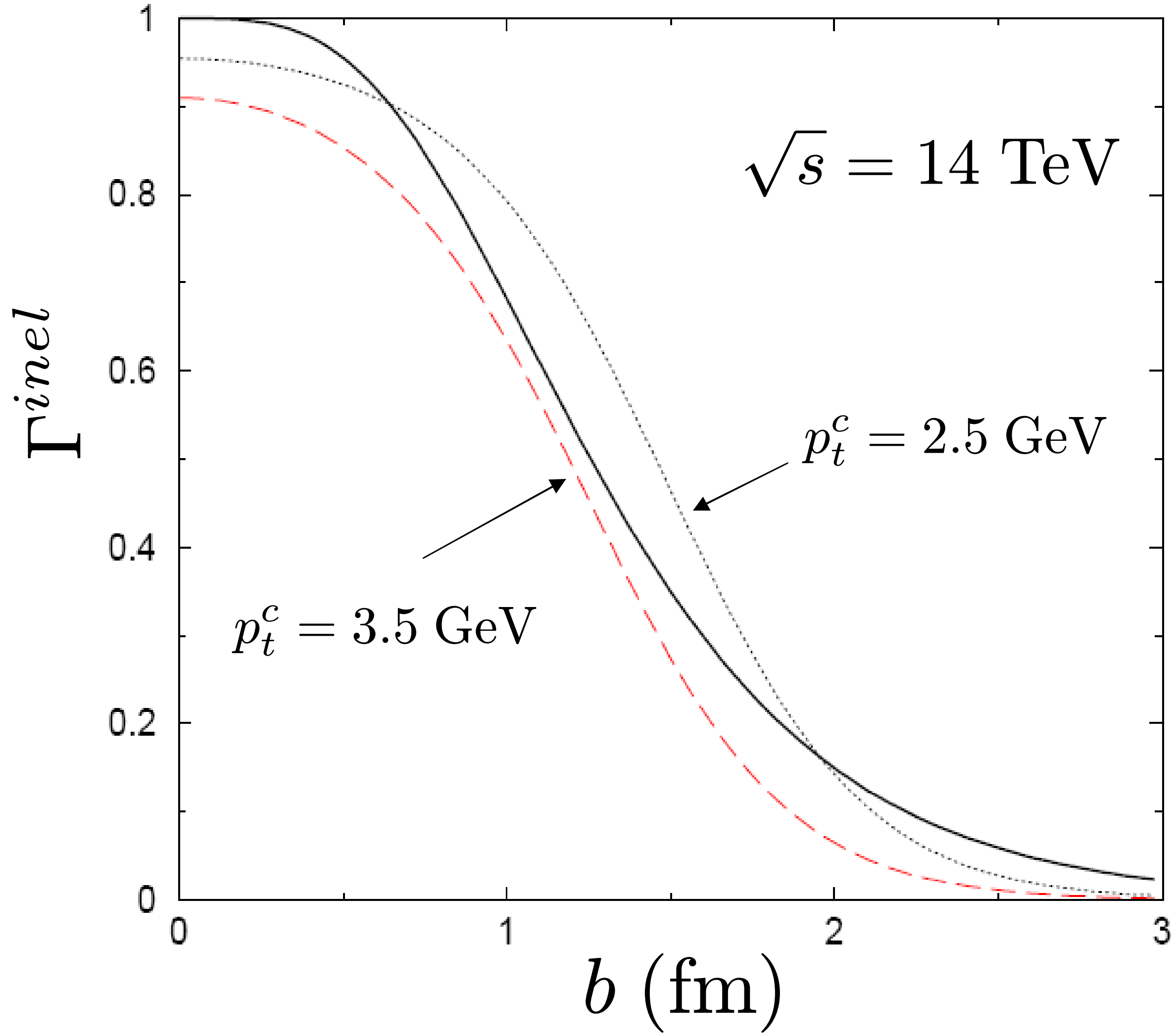
Comparison with extrapolation: UHE

(Cosmic ray energies.)



- Identical partons,
- CTEQ6M gluon PDF

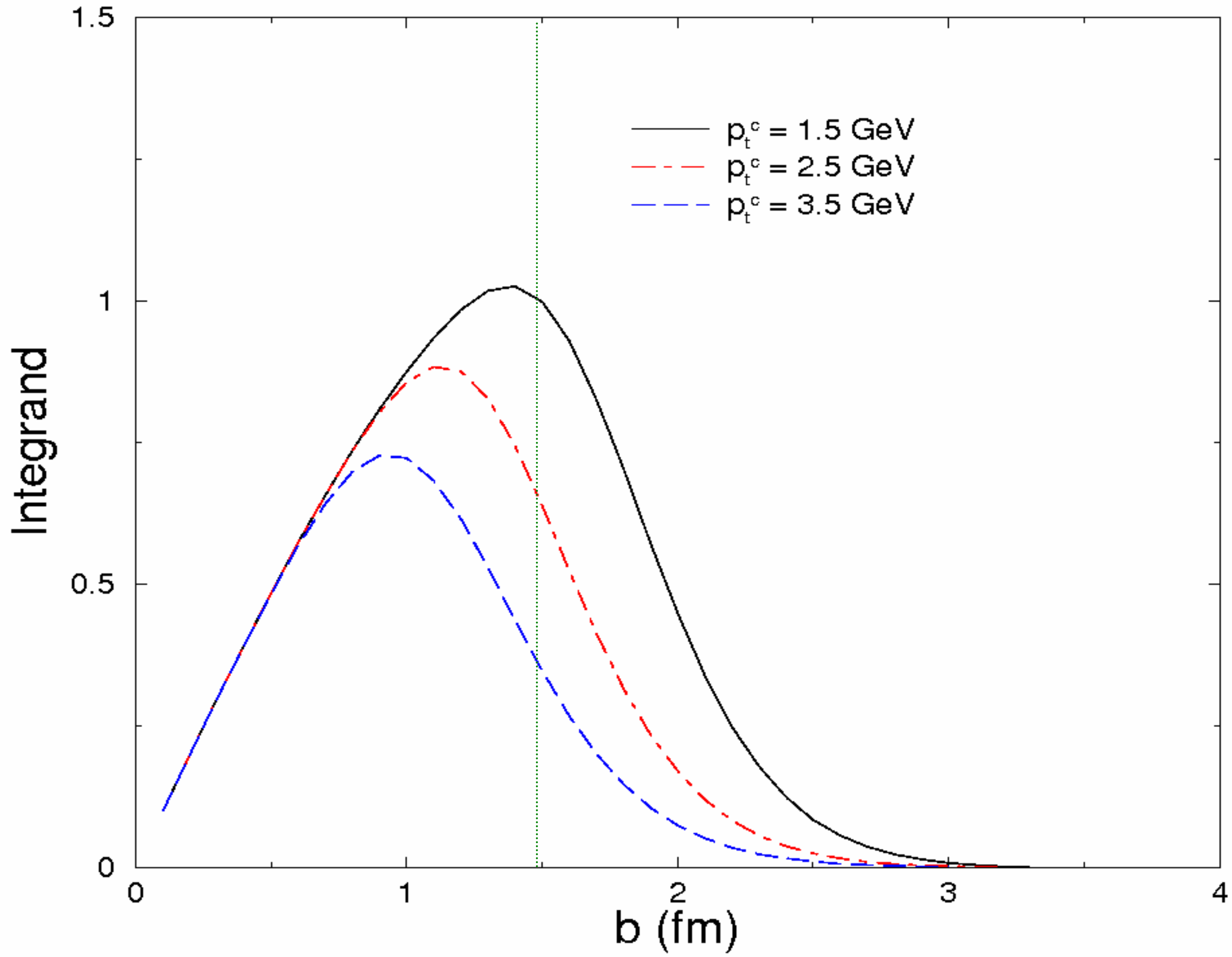
Comparison with extrapolation: non-identical partons



Comparison with extrapolation

- Sensitivity to large-b at $-t = .02\ \mathrm{GeV}^2$.

$\sqrt{s} = 14$ TeV



- ❑ Integrand of Fourier transform to t-space.
- ❑ Integrate to b = 1.5 fm. Percent of integral;
- p_t^c = 1.5 GeV: 65 %
- p_t^c = 2.5 GeV: 80 %
- p_t^c = 3.5 GeV: 87 %

Width of hard profile

- In eikonal picture, decreased width of hard profile decreases total cross section:

$$\Gamma(s,b) = 1 - \exp\left[-\chi_h(s,b) - \chi_s(s,b) + \cdots\right]$$

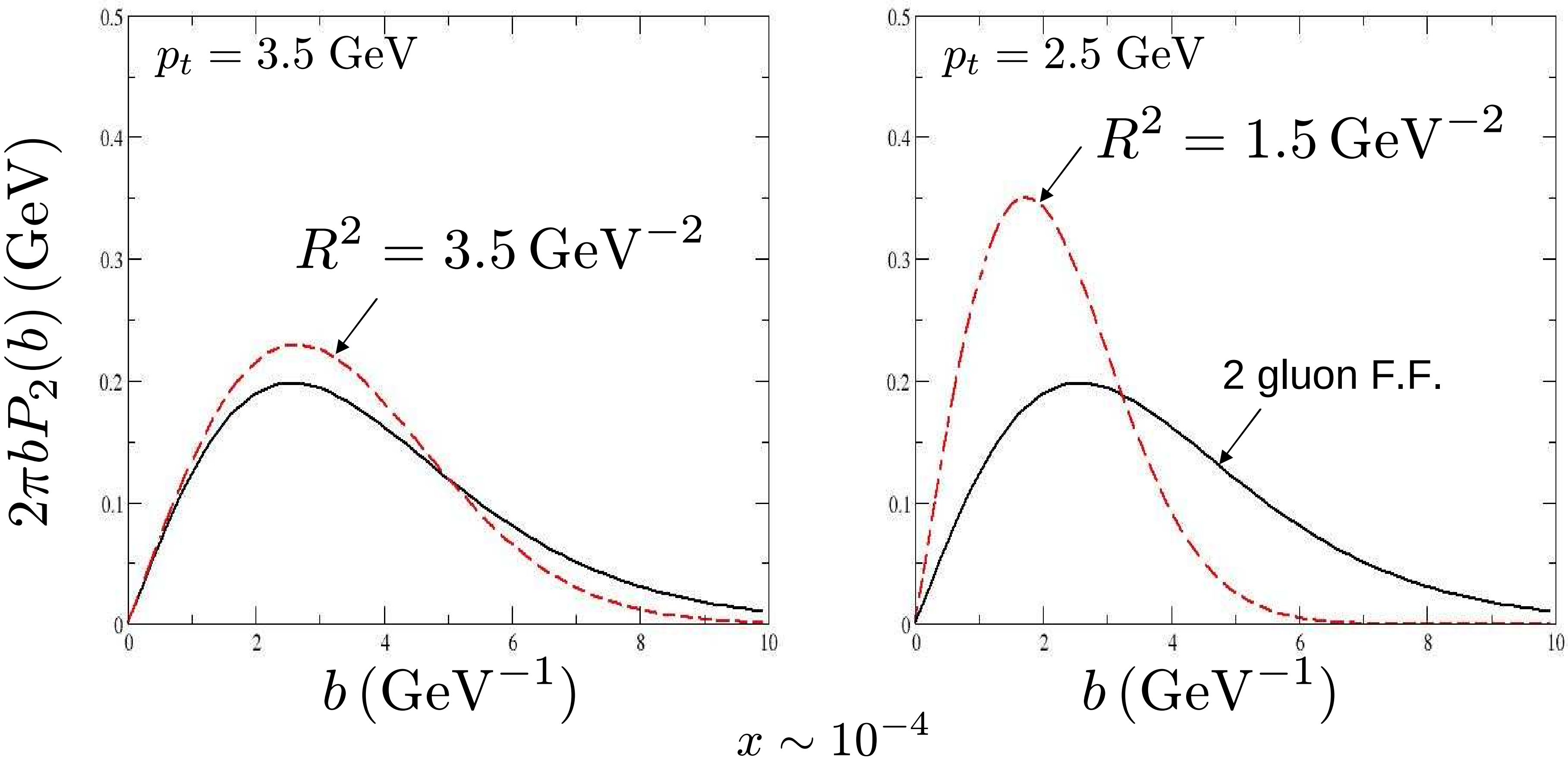
$$\chi_h(s,b) = \frac{\sigma_{2\,jet}^{inc}(s,p_t^c)}{8\pi R_0^2}\exp\left\{-b^2/4R_0^2\right\}$$

- Can be used to fit total/elastic cross sections using very low p_t.

- Effect is due to onset of saturation at *higher* p_t – does not correspond to increased range of applicability of dijet factorization formula.

Width of hard profile

- Compare 2-gluon overlap function with Gaussian model for different radii.
- Larger radius needed for consistency with 2-gluon F.F.



Summary

- Large uncertainty in inclusive dijet formula due to lack of knowledge about acceptable value of p_t cutoff.

- s-channel unitarity provides general consistency relations relating hard dijet component of the cross section, and model extrapolations to high energies. (Beyond usual $\Gamma < 1$ constraint.)

- Can use information about transverse distribution of partons obtained from vector meson production via generalized PDFs.

- p_t cutoff and model extrapolations should be chosen to avoid inconsistencies – precise t-dependence needed at LHC energies.

- How to consistently extend pt to smaller and smaller b:
  - Modify shape of distribution at very small pt due to soft effects?
  - Resummation of soft gluons?
  - Correlations?

Back-up Slides

Width of hard profile

- Compare with profile function for quark-antiquark scattering in DIS, obtained from 2-gluon form factor.

- Modify m_g to correspond to R^2 of 1.5 GeV^{-2} in Gaussian overlap function.

$$P_2(b,s,p_t^c) = \frac{m_g^2(\bar{x},p_t^c)}{12\pi}\left(\frac{m_g(\bar{x},p_t^c)b}{2}\right)^3 K_3(m_g(\bar{x},p_t^c)b)$$

- Check deviation of profile from what is found in DIS.

Consistency with DIS

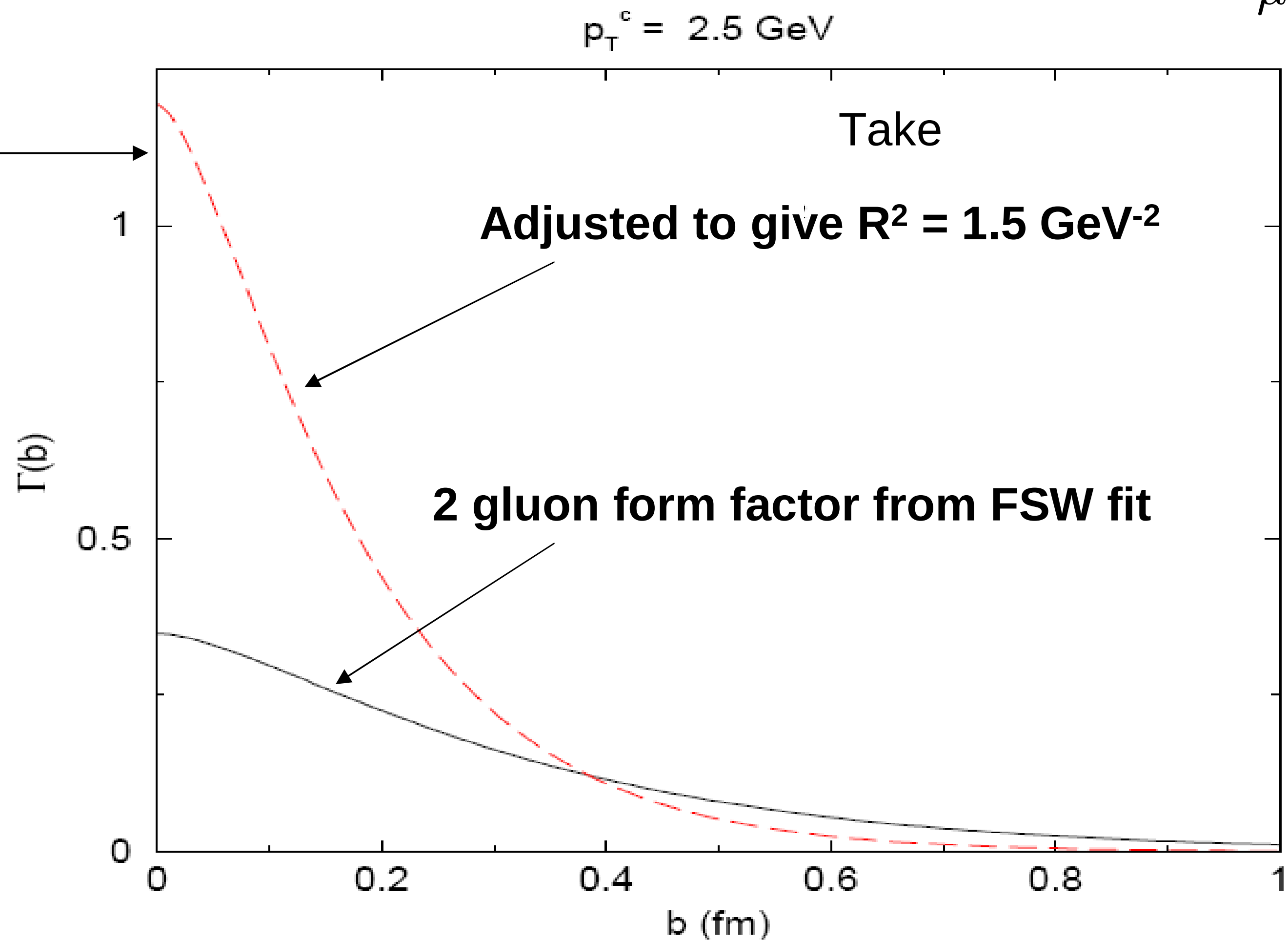
- Compare with DIS using the same 2-gluon F.F.

$$\bar{x} = 2p_t^c/\sqrt{s}$$

$$\Gamma_h^{q\bar{q}}(b,x) = \frac{\sigma_{tot}^{q\bar{q}}(d,x)m_g^2(x,\mu)}{4\pi}\left(\frac{m_g(x,\mu)b}{2}\right)K_1(m_g(x,\mu)b)$$

$$\mu = \lambda/d = p_t^c$$

Larger problem with factorization formula for small radius.



Gluon saturation and low-p_t taming:

- Very rapid growth of gluon distribution at small-x.
- Below some value of x, non-linear effects come into play, growth is tamed.
- In pp $\longrightarrow$ 2 jets + X jets x-section, what is the role "saturation" of the gluon distribution?
- Look at jet rapidity distribution:

$$x_{1,2} = \frac{p_t}{\sqrt{s}}(e^{\pm y_1} + e^{\pm y_2})$$

- Use models of saturation in DIS to estimate "saturation" scale.

Saturation/Black Disk Scattering In deep inelastic scattering

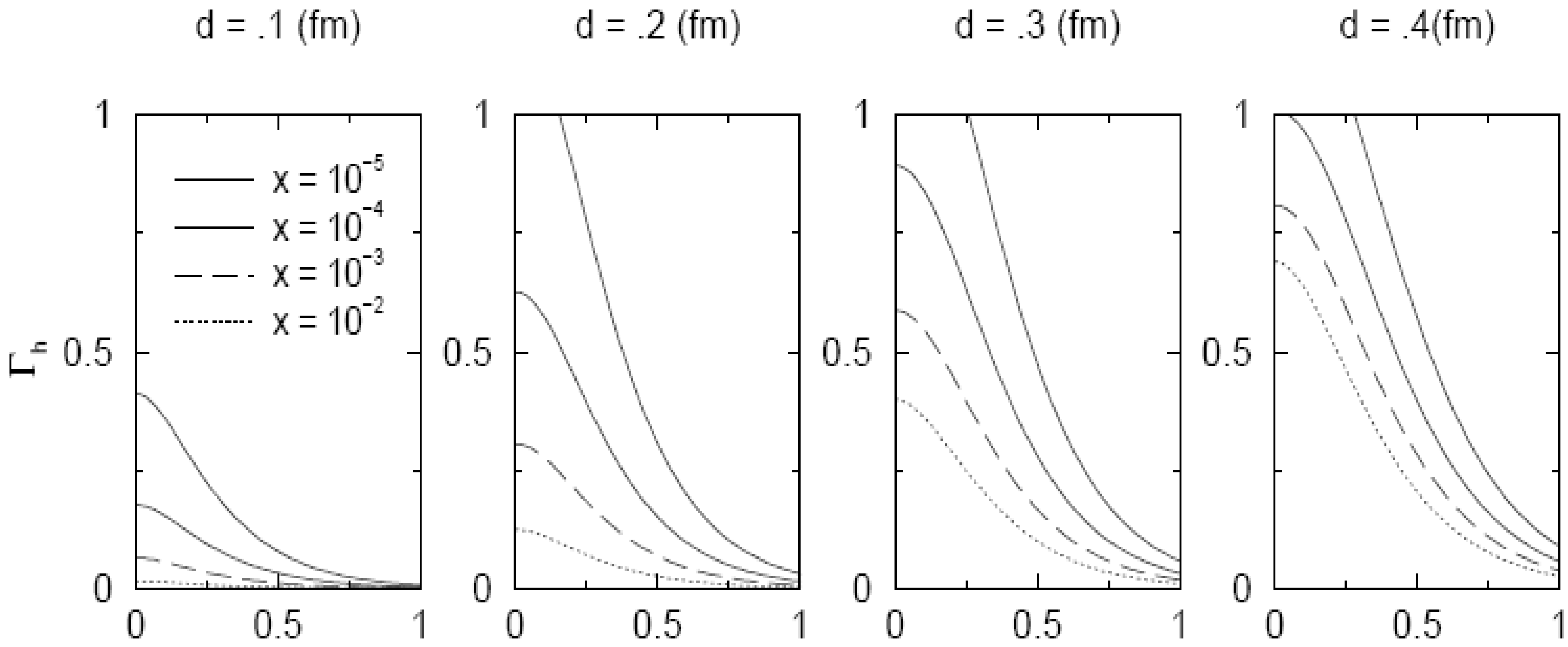
- Use 2-gluon form factor to obtain profile function for quark-antiquark proton total cross section.

$$\Gamma_h^{q\bar{q}}(b,x) = \frac{\sigma_{tot}^{q\bar{q}}(d,x) m_g^2(x,\mu)}{4\pi} \left(\frac{m_g(x,\mu) b}{2}\right) K_1(m_g(x,\mu) b)$$

- Can determine how close the cross section is to black disk limit – where gluon density grows large enough that s-channel unitarity is violated.

- At what values of x and hard scale does the gluon density become too large?

Saturation/Black Disk Scattering In deep inelastic scattering



- *Model that extrapolates directly between soft and hard regions.*

(TCR,Guzey,Strikman,Zu (2004))

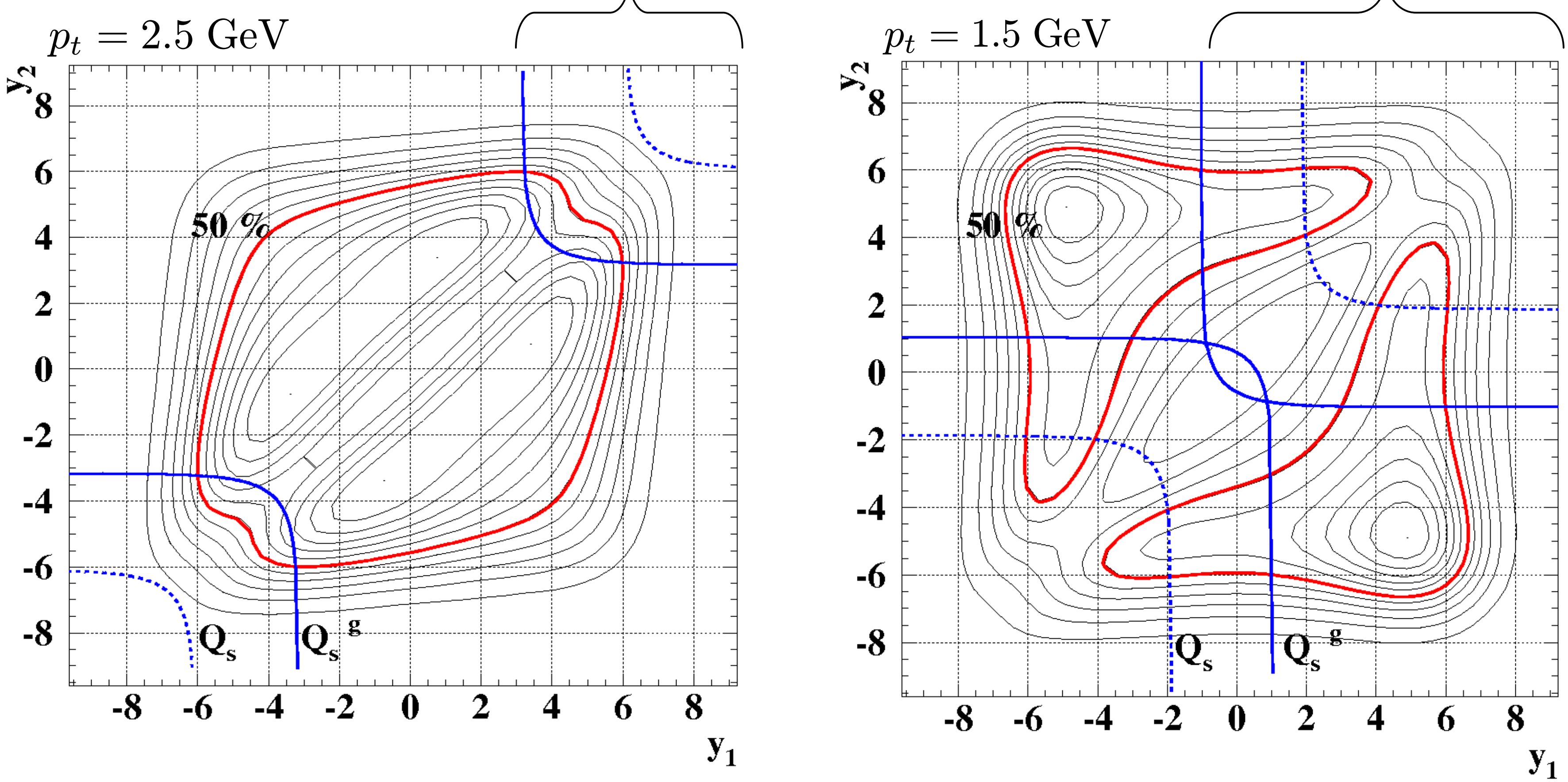
Role of saturation:

Integrand of dijet factorization formula

- *Apparently small effect from gluon saturation.*

Region where gluon PDF would lead to saturation in dipole-proton scattering

(solid line for octet dipoles)



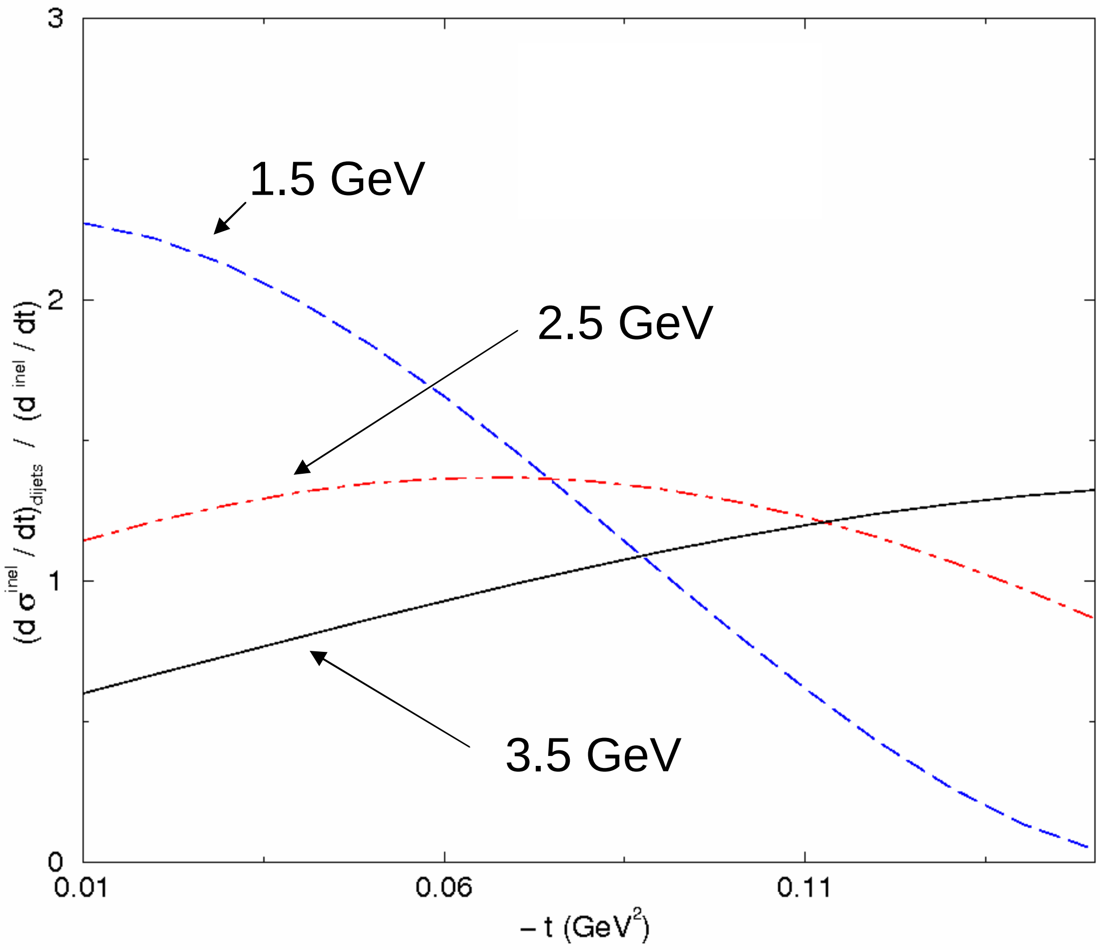
$\sqrt{s} = 14$ TeV

Blue curves – Golec-Biernat, Wustoff (GBW) model

Comparison with extrapolation

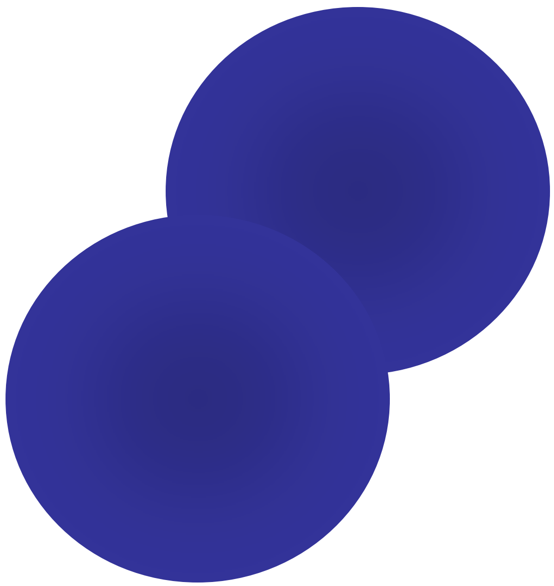
- Transition from small to large b, affects cross section at t = 0.

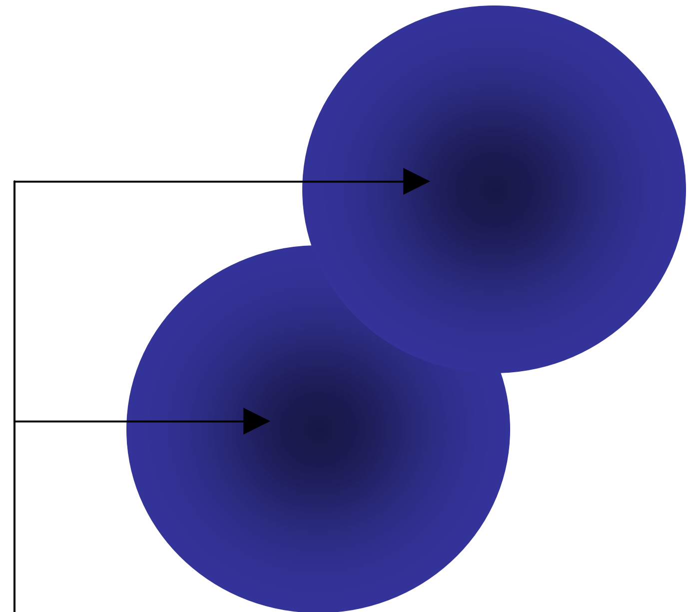
$\sqrt{s} =$ 14 TeV



- ❑ Fraction of differential inelastic cross section from dijet production to expectation (Islam,Luddy,Prokudin fit).

Width of hard profile





Large radius for hard collision

Small radius for hard collision

Profile function becomes black.