Gravitational-like interactions in a cloud of cold atoms?

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Statistical mechanics of self-gravitating systems

• A lot of theory and numerical simulations + astrophysical observations...

• **What about lab experiments?**

An effective interaction mimicking gravity is needed...

Ultimate goal: a tabletop analog of a galaxy or globular cluster...

More accessible goals: find signatures of the special phase transitions of self gravitating matter, and/or uncover new phenomena with long-range attractive interactions

Dynamics and thermodynamics of self-gravitating systems

Setting: a large number of classical particles interacting through (Newtonian) gravitation
$+$ Hamiltonian or Langevin dynamics.
Question: What is the long time behavior of such a system?
Difficulties: long-range interaction, short range singularity
$\rightarrow$ Special features:

- no usual thermodynamic limit: properly defined scaling parameters needed
- peculiar phase transitions: gravothermal catastrophe, isothermal collapse
- regimes with negative specific heat
- slow relaxation to equilibrium...

Langevin equations for self-gravitating particles

$$\dot{\mathbf{x}}_i = \mathbf{v}_i$$
$$m\dot{\mathbf{v}}_i = \mathbf{F}_i - m\gamma\mathbf{v}_i + \sqrt{2D}\eta_i(t)$$

Potential:

1D: $$V = G_1 m^2 \sum_{i<j} |x_i - x_j|$$

2D: $$V = G_2 m^2 \sum_{i<j} \ln |\mathbf{x_i} - \mathbf{x_j}|$$

3D: $$V = -G_3 m^2 \sum_{i<j} \frac{1}{|\mathbf{x_i} - \mathbf{x_j}|}$$

- $\gamma = D = 0$: Hamiltonian dynamics; of interest in astrophysics
- $\gamma \to +\infty$ (overdamped limit): $\sim$ a discrete version of the Patlack-Keller-Segel model in chemotaxy.

Kinetic equations

• In the appropriate scaling limit, one expects a Vlasov-Fokker-Planck equation (physicists usually do not pay much attention to the validity of this limiting procedure...);
$f(\mathbf{x}, \mathbf{v}, t) =$ probability density for one particle:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} f + \frac{1}{m} \mathbf{F}_{int}[f] \cdot \nabla_{\mathbf{v}} f \quad = \quad \nabla_{\mathbf{v}} \cdot (\gamma \mathbf{v} f + D \nabla_{\mathbf{v}} f)$$

$$\mathbf{F}_{int} = -\nabla \phi_{int} \quad ; \quad \Delta \phi_{int} = c_D m \rho$$

• Vanishing noise and friction $\rightarrow$ Vlasov equation
• Large friction limit $\rightarrow$ Smoluchowski equation

$$\frac{\partial \rho}{\partial t} = \nabla \cdot \left(\frac{1}{m\gamma} \mathbf{F}_{int}[\rho] \rho + \frac{D}{\gamma^2} \nabla \rho \right)$$

A heuristic computation

- Assume a gaussian density profile, and compute the rms size $\lambda(t)$

$$\rho = \frac{1}{(\lambda(t)\sqrt{2\pi})^D} e^{-\frac{x^2}{2\lambda^2(t)}}$$

1D:

$$\dot{\lambda} = \frac{2D}{\gamma} - C_1\lambda \quad \rightarrow \quad \text{stable fixed point}$$

2D:

$$\dot{\lambda} = \frac{4D}{\gamma} - C_2 \quad \rightarrow \quad \text{critical case}$$

3D:

$$\dot{\lambda} = \frac{6D}{\gamma} - \frac{C_3}{\lambda} \quad \rightarrow \quad \text{unstable fixed point}$$

- Note that qualitatively, this does not depend on the profile
- Experimental signature?

Outline

- Trapped cold atoms and long-range laser induced interactions
- A quasi 1D experiment
- Towards a 2D experiment?
  - Theory
  - Simulations
  - Experimental challenges

Cold atoms

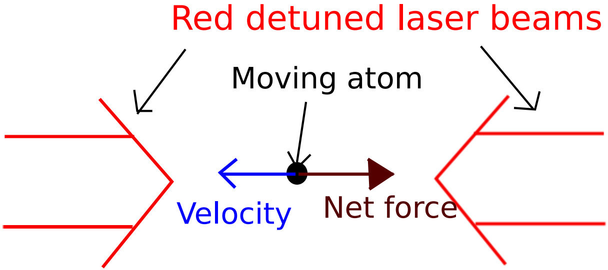
• 80's-90's : manipulating (trapping, cooling) atomic vapors with lasers:
Goal: reach Bose-Einstein condensation.

• Cold atomic vapors interacting with quasi resonant lasers $\sim$ systems of stochastic interacting particles

$\rightarrow$ systems also interesting for themselves

Trapped cold atoms

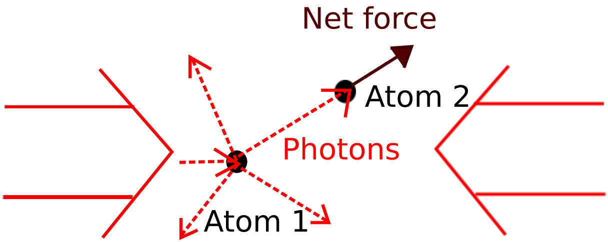
- Techniques developed in the 80's, now routinely used.



Doppler effect $\rightarrow$ a friction
Spatial trapping: through a magnetic field gradient, or a dipolar trap.

Trapped cold atoms - Multiple diffusion

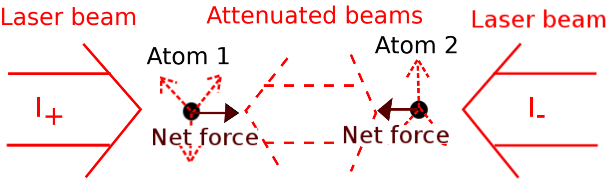
• Multiple diffusion $\rightarrow$ "Coulomb-like" repulsion (Walker, Sesko, Wieman 90).



$\rightarrow$ A research program: instead of considering the repulsion as a limitation, take advantage of it to study "plasma-like" effects in a cloud of cold atoms.

Shadow effect

Laser attenuation → laser unbalance → effective attraction. This effect has been known since the 80's (Dalibard)



Hypothesis: **small optical thickness** (weak attenuation)

$$I_+(z) = I_0 e^{-\bar{\sigma}\int_{-\infty}^{z}\rho(s)ds} \simeq I_0\left(1 - \bar{\sigma}\int_{-\infty}^{z}\rho(s)ds\right)$$

$$\vec{F}_{shadow} \propto I_+ - I_- \;\Rightarrow\; \mathrm{div}(\vec{F}_{shadow}) \propto -\rho$$

→ a "gravity-like" interaction...

Problem: the repulsive force is stronger...

Repulsive vs attractive

• Under normal circumstances, the repulsive force dominates
$\rightarrow$ a kind of dissipative plasma (R. Kaiser, T. Mendonça, H. Tercas)...
But the most spectacular collective effects are expected for attractive forces...

• For specific geometries (cigar- or pancake shaped cloud), the attractive force should dominate
$\rightarrow$ something that looks like a self-gravitating system in the lab??

• Brownian self-gravitating particles in 2D: critical case, with a finite time blow up possible...
$\rightarrow$ an experimental realization of the collapse ??

Simplifying assumptions

- Photon absorption and reemission time scale very short ($\pm$ OK ?)
  $\rightarrow$ we can average over this short time scale
- Small optical width hypothesis ($\pm$ OK)
  $\rightarrow$ the laser intensities disappear, replaced by an effective interaction
- The radiation pressure force is linearized in v_z (Dangerous!)
  $\rightarrow$ it is decomposed into
  1. A linear friction $\propto -v_z$
  2. The shadow effect
- Diffusion coefficient taken to be space independent (Probably wrong, but not crucial).

$\rightarrow$ a system of SDE for interacting particles

The validity of the assumptions depend of course on the experimental realization.

Vlasov-Fokker-Planck description

• System of interacting particles → a non linear Fokker-Planck equation taken as starting point
Discrete effects are neglected here (validity? Not completely clear)

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} f + \frac{1}{m}(\mathbf{F}_{trap} + \mathbf{F}_{int}[f]) \cdot \nabla_{\mathbf{v}} f = \nabla_{\mathbf{v}} \cdot (\gamma \mathbf{v} f + D \nabla_{\mathbf{v}} f)$$

with $\mathbf{F}_{int}$ = interaction force: "Coulomb-like" multiple diffusion + "gravitation-like" shadow effect

Reduction to $D = 1$ for a cigar shaped cloud

Further simplifying assumption:
Fast transverse equilibration (± OK ?)
→ Transverse degrees of freedom integrated out
→ an effective 1D Vlasov Fokker Planck equation

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial z} + (-\omega_0^2 z + F_{int}[f](z))\frac{\partial f}{\partial v} = \frac{\partial}{\partial v}\left(\gamma v f + D\frac{\partial f}{\partial v}\right)$$

→ in principle, equation identical to Vlasov-Fokker-Planck for 1D self gravitating Brownian particles

Theoretical predictions, 1D case

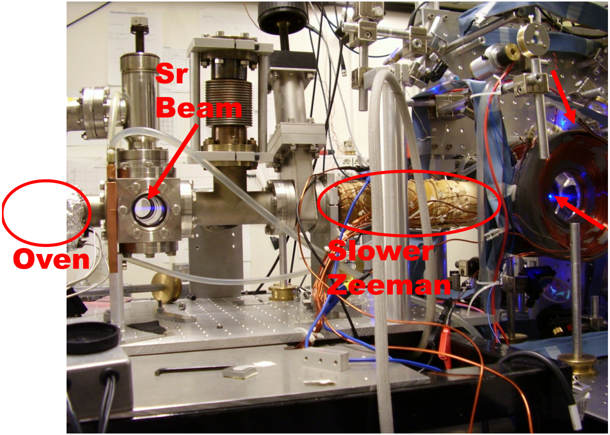
- No phase transition; equilibrium density profile

$$\rho(z) = \frac{c}{\cosh^2(z/L)}$$

- Asymptotic dynamics: convergence to the equilibrium profile
- The size $L \propto 1/N$

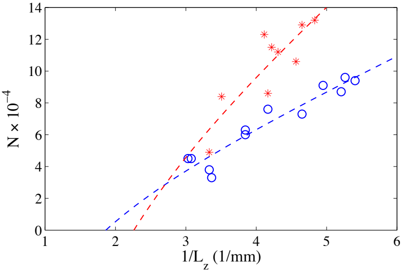
The experiment

- Experiment: Maryvonne Chalony, David Wilkowski (Institut Non Linéaire de Nice)
- Strontium; size $\sim$ 500μm; temperature $\sim 2\mu K$; number of atoms $N \sim 10^5$.



Experimental signatures: cloud's size

Theory, in the self gravitating limit (trap=negligible): $L \propto 1/N$
L = cloud's size; N = number of atoms



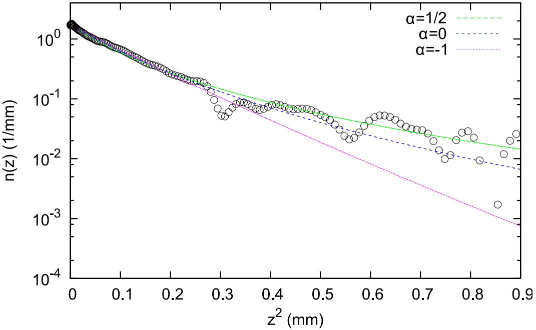
N vs $1/L$. *Red:* $T \simeq 1.5\mu K$, *Blue:* $T \simeq 2.1\mu K$; *the theory includes the trap.*

$\rightarrow$ qualitative agreement; difficult to be more precise...
At least, the size decreases when the number of particles increases.

Experimental signatures: density profile

Theory, in the self-gravitating limit (trap=negligible):

$$\rho(z) = \frac{N}{2L} \frac{1}{\cosh^2(z/L)}$$



Experimental profile vs theory with $1/r^{\alpha}$ *forces,* $\alpha = -1, 0, 1/2$.

Experimental signatures: breathing frequency

Effective interaction are turned on $\rightarrow$ relaxation to the new "self-gravitating" stationary state through breathing oscillations.

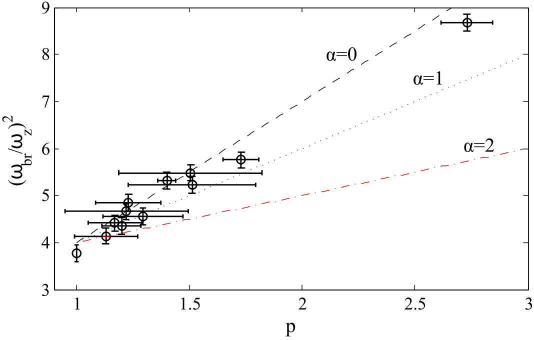
Theory: the breathing frequency depends on the compression factor c and the force exponent α

$$\omega_{br} = \omega_0 \sqrt{(3-\alpha)(c^2-1)+4}$$

c=cloud's size without interaction/ cloud's size with interaction
α= force exponent; $\alpha = 0$ for 1D gravity.

Experiment: the breathing frequency is measured and compared with the theory including the experimentally measured compression factor, varying α.

Frequency vs compression



c^2

Experimental frequency ratio $(\omega/\omega_0)^2$ *vs compression factor* c^2
Theory = dashed line. From top to bottom, force exponent $\alpha = 0, 1, 2$.

Some caveats

- Linearization in velocity
  $\rightarrow$ the force felt by an atom actually depends on its velocity! This is a serious problem: in the experiments that have been performed, the force is "gravitation-like" in a velocity averaged sense...
- The optical thickness is actually $0.2 \leq b \leq 0.6$; this is not very small... The optical thickness is also difficult to measure precisely.
- The theoretical predictions depend very sensitively on the laser detuning, which is difficult to set precisely.

$\rightarrow$ the analogy with a self-gravitating system is only qualitative, and the comparison with the theory cannot be really precise...

Reduction to $D = 2$ for a pancake shaped cloud

Same simplifying assumption as in 1D:

Fast transverse equilibration ($\pm$ OK ?)

$\rightarrow$ Transverse degrees of freedom integrated out

$\rightarrow$ An effective 2D Vlasov Fokker Planck equation

$$\frac{\partial f}{\partial t} + \mathbf{v}\nabla_{\mathbf{r}} f + (-\omega_0^2 \mathbf{r} + \frac{1}{m}\mathbf{F}_{int}[f](\mathbf{r}))\nabla_{\mathbf{v}} f = \nabla_{\mathbf{v}} \cdot (\gamma \mathbf{v} f + D\nabla_{\mathbf{v}} f)$$

$$\begin{aligned} F_{int}^{x}[\rho](x,y) &= -c \iint \rho(x',y')\delta(y-y')\mathrm{sgn}(x-x')dx'dy' \\ F_{int}^{y}[\rho](x,y) &= -c \iint \rho(x',y')\delta(x-x')\mathrm{sgn}(y-y')dx'dy' \end{aligned}$$

$\rightarrow$ $\nabla \cdot \mathbf{F}_{int} = -4c\rho$

$\rightarrow$ $\mathbf{F}_{int}$ similar to 2D gravity (or Keller-Segel), but $\mathbf{F}_{int}$ is not a gradient!

Theory, 2D self gravitating system

- Take **trapped** Brownian self gravitating particles in 2D.
- There is a critical temperature T_c such that:

$$\begin{array}{lcl} T \geq T_c & \rightarrow & \text{smooth equilibrium profile} \\ T < T_c & \rightarrow & \text{Dirac peak in finite time} \end{array}$$

- Analytical tool: there is a Lyapunov functional (free energy)

$$J = T \int f \ln f + \int \frac{1}{2} \mathbf{v}^2 f + \frac{1}{2} \int \phi[f] f$$

which decreases in time.

Theory, pancake shaped cloud

Overdamped case, to simplify $\rightarrow$ one dimensionless parameter Θ

$$\frac{\partial \rho}{\partial t} = \nabla \cdot (\mathbf{F}[\rho]\rho + \Theta \nabla \rho)$$

$$\begin{aligned} F_x[\rho](x,y) &= -c \iint \rho(x',y')\delta(y-y')\text{sgn}(x-x')dx'dy' \\ F_y[\rho](x,y) &= -c \iint \rho(x',y')\delta(x-x')\text{sgn}(y-y')dx'dy' \end{aligned}$$

Question: What happens to the blow-up?
Problem: No free energy; our main analytical tool disappears!

Heuristic idea

- Assume a gaussian profile

$$\rho(x, y, t) = \frac{1}{2\pi\lambda^2(t)} e^{-\frac{x^2+y^2}{2\lambda^2(t)}}$$

And look for an equation for λ

$$\dot{\lambda} = 4\Theta - \frac{2\sqrt{2}}{\pi}$$

$\rightarrow$ a critical parameter is predicted, like in 2D...

Heuristic computation, made rigorous...

• Classical and quick way to prove the existence of a finite time blow up for Keller-Segel (or overdamped 2D gravity)= compute the second moment $m_2(t) = \int \mathbf{x}^2 \rho$; here

$$\begin{aligned} \dot{m}_2 &= -2m_2 + 4\Theta - \int |x - x'| \rho(x, y, t) \rho(x', y, t) dx dx' dy \\ &\quad - \int |y - y'| \rho(x, y, t) \rho(x, y', t) dx dy dy' \end{aligned}$$

Red term is scale invariant. Can we prove that:

$$\text{red term} < -c < 0 \text{ ?}$$

No, there are special profiles ρ such that the red term tends to 0...

$\rightarrow$ Analytical tools??

Numerics, finite diference scheme

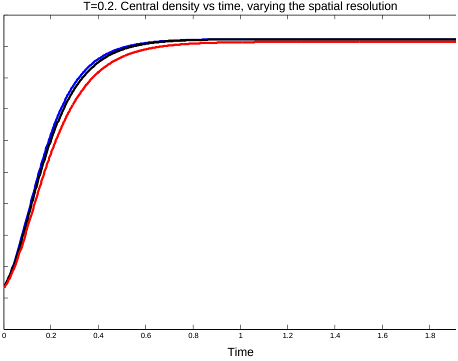
• *Method*: Finite difference scheme; implicit for the diffusion, explicit for the nonlinear term (inspired by Saito for chemotaxy)
• *Difficulty*: When concentration increases, the time-step necessary to ensure stability decreases...
• *Results*: There seems to be a critical parameter $\Theta_c \simeq 0.15$ such that

$$\Theta > \Theta_c \quad \to \quad \text{a smooth stationary profile}$$
$$\Theta < \Theta_c \quad \to \quad \text{blow up}$$

caveats: the spatial step is not very small; difficult to reach high densities...

Numerics, finite difference scheme

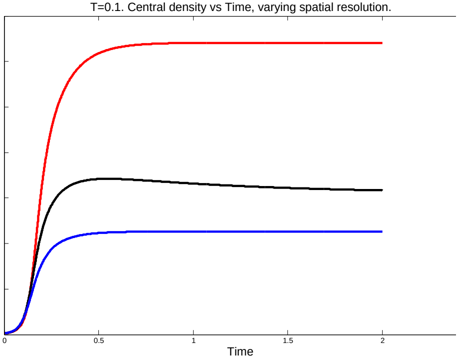
$\Theta = 0.2$; Central density vs Time, varying the grid size.



$\rightarrow$ There seems to be no collapse...

Numerics, finite difference scheme

$\Theta = 0.1$; Central density vs Time, varying the grid size.



No convergence when the step decreases; $\rightarrow$ Collapse?

Numerics, with particles

• A natural way to simulate the Smoluchowski equation: overdamped Langevin equations

$$\dot{\mathbf{r}}_i = \mathbf{F}_d(\mathbf{r}_i) + \sqrt{2\Theta}\eta_i(t)$$

• *Difficulty*: the two body interaction is not smooth at all

$$K_x = \delta(y - y')\mathrm{sgn}(x - x') \ , \ K_y = \delta(x - x')\mathrm{sgn}(y - y')$$

$\rightarrow$ numerically, one needs to regularize the δ, introducing a new length scale σ

$\rightarrow$ the convergence properties of the discrete process when $\sigma \rightarrow 0$ are not proved.

• With these caveats, the result is similar as above with $\Theta_c \simeq 0.12$

Back to experiments

- Using dipolar trap, a pancake shaped trap is "easy" to do.
- Using the experience of the 1D setting, it is possible to have an idea of the parameter regime reachable in a Strontium experiment. $\Theta \simeq \Theta_c$ is in principle attainable.
- A precise quantitative agreement between model and experiment is too optimistic (see the 1D experiment...); we may expect a qualitative agreement.
- More numerical work is needed to convince the experimentalists it is worth doing the experiment!

Other experimental proposals

- Some proposals in the literature (list probably not exhaustive!):
  - O'Dell et al. (2000): Bose-Einstein Condensate + intense off-resonant laser beams
  - Dominguez et al. (2010): capillary interactions between colloids at a fluid interface $\sim$ 2D gravity.
  - Golestanian (2012): colloids driven by temperature gradients; temperature field induced by the colloids

To my knowledge, no experimental implementation of these ideas...

Conclusions

- A cigar-shaped cold atomic cloud qualitatively described by a 1D self-gravitating model
- A finite time blow up is conjectured in a Smoluchowski equation similar to Keller-Segel or 2D self-gravity, but **without free energy**
- This equation models a cold atom experiment with a pancake-shaped cloud; the blow up regime seems close to be experimentally reachable.
- Outstanding theoretical questions remain:
  - does the blow-up exists? how to prove it?
  - if indeed it exists, what exactly does it look like?
  - is the particles approximation correct?