I 25 GeV Higgs and the Scale of New Physics

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Everything in agreement with $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge theory

$$\mathcal{L}_{kinetic} = -\frac{1}{4g_1^2} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4g_2^2} W^i_{\mu\nu} W^{i\mu\nu} - \frac{1}{4g_3^2} G^a_{\mu\nu} W^{a\mu\nu} + i \sum_{j=1}^3 (\bar{\Psi}^j \bar{\sigma}^\mu \partial_\mu \Psi^j + h.c.)$$

 $\Psi = (3,2)_{\frac{1}{6}} \oplus (\bar{3},1)_{-\frac{2}{3}} \oplus (\bar{3},1)_{\frac{1}{3}} \oplus (1,2)_{\frac{1}{2}} \oplus (1,1)_1 \qquad (3 \text{ parameters})$

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Unbroken gauge symmetry forbids mass terms: vacuum must respect a smaller symmetry

 $SU(3)_c \otimes U(1)_Q$

Mass terms can be written,

$$\mathcal{L}_{mass} = \sum_{i,j=1}^{3} \left[\bar{u}_{L}^{i} M_{i,j}^{u} u_{R} + \bar{d}_{L}^{i} M_{i,j}^{d} d_{R} + \bar{e}_{L}^{i} M_{i,j}^{e} e_{R} \right] + h.c.$$

+ $m_{W}^{2} W^{2} + \frac{1}{2} m_{Z}^{2} Z^{2}$ O(20) parameters

Mass for gauge bosons implies new degrees of freedom



Mass for gauge bosons implies new degrees of freedom



The extra degrees of freedom are Goldstone Bosons $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_Q$

They become longitudinal polarizations of W & Z

Conceptually identical to superconductivity.

In the SM the spontaneous breaking is due a doublet scalar

$$V(H) = \lambda \left(|H|^2 - v^2 \right)^2$$

$$H(x) = U(x) \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}, \qquad v = 174 \, GeV$$

VEV breaks symmetry spontaneously. The Goldstone Bosons in U(x) are eaten giving mass to W & Z. Scalar h(x) is what we call the "Higgs". In the SM the spontaneous breaking is due a doublet scalar

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If SM is correct only unknown is the quartic/mass

$$m_h = \sqrt{\lambda} \, v$$

Can SM be the whole story?

$$\mu \frac{d\lambda}{d\mu} = \frac{1}{16\pi^2} (24\lambda^2 - 6y_t^4 + \dots)$$





 $115\,\mathrm{GeV} < m_h < 160\,\mathrm{GeV}$





Indirect tests: $m_h < 150 \text{ GeV}$ Direct search: $m_h > 114 \text{ GeV}$

July 31, 2012 Phys. Lett. B716





$m_h \approx 125 \,\mathrm{GeV}$

SM HIGGS?





 $SM: \quad \kappa_F = \kappa_V = 1$

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HIERARCHY PROBLEM

SM is an effective theory valid up to Λ

$$\mathcal{L} = \mathcal{L}_{kin} + g A_{\mu} \bar{\psi} \gamma^{\mu} \psi + y \bar{\psi} H \psi - \lambda |H|^4 \qquad D = 4$$

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Irrelevant interactions:

$$\frac{1}{\Lambda} (lH^c)^2 \qquad \frac{1}{\Lambda^2} (\bar{\psi}\psi)^2 \qquad \frac{1}{\Lambda} \bar{\psi} \,\sigma^{\mu\nu} \psi F_{\mu\nu} \qquad D > 4$$

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One relevant operator

$$[H^2] \approx 2$$

Without tuning:

 $m_h \sim \Lambda$

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 $m_h \sim \Lambda$

No obvious experimentalist:



New physics will be seen at the LHC



FLAVOR HAS FOUND NOTHING

$\Lambda > 10^5 \, {\rm TeV}$

LEP HAS FOUND NOTHING

$\Lambda > 5 - 10 \,\mathrm{TeV}$

LHC HAS FOUND THE HIGGS + NOTHING

 $\Lambda > \text{few} \times \text{TeV}$

$\Lambda \gg \text{TeV}?$

- Explains why we have not seen anything

$\Lambda \gg \text{TeV}?$

- Explains why we have not seen anything

- Higgs could be tuned anthropically

HINTS

• Running:



$$V(h) = m^2 h^2 / 2 + \lambda h^4 / 4$$

De Grassi et al.'12

Quartic almost zero at high scale for 125 GeV Higgs

• Strong CP problem:

 $\frac{\theta}{32\pi^2} \int d^4x \,\epsilon^{\mu\nu\rho\sigma} \,Tr[G_{\mu\nu}G_{\rho\sigma}]$

 $\theta < 10^{-9}$

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Elegantly solved by axions

$$\theta \to \frac{a(x)}{f}$$

Axions are Goldstones of a symmetry anomalous under QCD

$$m_a \sim \frac{\Lambda_{QCD}^2}{f}$$



 $f > 10^9 \,\mathrm{GeV}$

Axions can be dark matter

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Axions can be dark matter

$$\frac{\rho_a}{\rho_{\rm DM}} \approx \theta_i^2 \left(\frac{f}{2 - 3 \times 10^{11} \,{\rm GeV}} \right) \longrightarrow \qquad f \approx 10^{11} \,{\rm GeV}$$

• Neutrino masses

$$\frac{1}{\Lambda} (lH^c)^2 \qquad \qquad m_\nu \propto \frac{v^2}{\Lambda}$$

• Unification

COMPOSITE HIGGS

Higgs could a be a remnant of strong dynamics



 $\sim \frac{1}{m_{\rho}}$

COMPOSITE HIGGS

Higgs could a be a remnant of strong dynamics



If $m_{\rho} = \Lambda \sim \text{TeV}$ theory natural

$$\delta m_h^2 = \frac{g_{SM}^2}{16\pi^2} \Lambda^2$$

Most compelling scenario Higgs is Goldstone boson

Georgi, Kaplan '80



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Main difference from technicolor is that f is not linked to v.

Deviation from SM:

$$\mathcal{O}\left(\frac{v^2}{f^2}\right)$$

Despite smart theorists difficulties remain:

- flavor

 $m_{\rho} > 10 \,\mathrm{TeV}$

- precision tests

 $m_{\rho} > 3 \,\mathrm{TeV}$

- direct exclusion

 $m_f > 0.7 \,\mathrm{TeV}$ $m_\rho > 1.5 \,\mathrm{TeV}$

- Higgs mass



Redi, Tesi '12

$\Lambda \sim 10^{11} \, {\rm GeV}$

AXION-HIGGS

Basic idea: Axion and Higgs are GBs from common dynamics. f is fixed by dark matter and the electro-weak scale is tuned.

Higgs + singlet

 $\frac{G}{H} \xrightarrow{f \approx 10^{11} \,\mathrm{GeV}}$

AXION-HIGGS

Basic idea: Axion and Higgs are GBs from common dynamics. f is fixed by dark matter and the electro-weak scale is tuned.



Singlet is axion candidate if SM interactions do not break its shift symmetry.

A SIMPLE MODEL

Kim-Shifman-Vainstein-Zakharov axion: Add new colored fermions + complex scalar

$$\Psi_Q \to e^{i\alpha_Q\gamma_5}\Psi_Q, \qquad \sigma \to e^{-2i\alpha_Q}\sigma$$

$$L = L_{\rm SM} + \bar{\Psi}_Q \partial \Psi_Q + |\partial_\mu \sigma|^2 + (\lambda \sigma \,\bar{\Psi}_Q \Psi_Q + \text{h.c.}) - V(\sigma)$$

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Spontaneous U(I) symmetry breaking

 $f \approx \langle \sigma \rangle$ $a = \sqrt{2} \operatorname{Im}[\sigma]$

U(I) symmetry anomalous under QCD



$$\frac{G}{H} = \frac{SU(6)_L \times SU(6)_R}{SU(6)_{L+R}}$$

Under $SU(5)_{SM}$

 $\mathbf{35} = \mathbf{24} \oplus \mathbf{5} \oplus \mathbf{ar{5}} \oplus \mathbf{1}$

One Higgs doublet. Two massless singlets are axion candidates.

Under SM 33 charged scalars acquire mass.

$$m \approx \frac{g_{SM}}{4\pi} \Lambda$$

UV realization: SU(n) gauge theory with 6 flavors

Fermions	$\mathrm{U}(1)_Y$	$SU(2)_L$	$SU(3)_{c}$	$SU(n)_{\rm TC}$
D	$\frac{1}{3}$	1	$\overline{3}$	n
L	$-\frac{1}{2}$	2	1	n
N	$\overline{0}$	1	1	n
$ar{D}$	$-\frac{1}{3}$	1	3	$ar{n}$
\overline{L}	$\frac{1}{2}$	$\overline{2}$	1	$ar{n}$
$ar{N}$	$\overset{2}{0}$	1	1	\bar{n}

 $\langle D\bar{D}\rangle = \langle L\bar{L}\rangle = \langle N\bar{N}\rangle \approx \Lambda^3$

 $H \sim (L\bar{N}) - (\bar{L}N)^*$

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\overline{L}	$\frac{1}{2}$	$\overline{2}$	1	$ar{n}$
$ar{N}$	$\tilde{0}$	1	1	\bar{n}

 $\langle D\bar{D}\rangle = \langle L\bar{L}\rangle = \langle N\bar{N}\rangle \approx \Lambda^3$

 $H \sim (L\bar{N}) - (\bar{L}N)^*$

FLAVOR:

 $(qu)(L\bar{N})$

 $(\bar{q}\bar{u})(\bar{L}N)$

Axions couple to photon and gluons through anomalies

$$\frac{a\,E}{32\pi^2 f}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$

$$\frac{aN}{32\pi^2 f} \epsilon^{\mu\nu\rho\sigma} Tr[G_{\mu\nu}G_{\rho\sigma}]$$

$$E = \sum Q_{PQ} Q_{em}^2$$

$$N = \sum Q_{PQ} T_{SU(3)}^2$$



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Experiments measure conversion of axion to photons





$$\frac{E}{N} < 1.92 + 3.5 \sqrt{\frac{0.3 \,\mathrm{GeV/cm^3}}{\rho_{DM}}}$$

$$(m_a = 1.9 - 3.55 \times 10^{-6} \,\mathrm{ev})$$

a) If UV interactions respect singlets symmetry

$$\frac{4D - 3L - 6N}{\sqrt{102}}, \qquad \frac{L - 2N}{\sqrt{3}} \qquad \qquad \frac{E}{N} = -\frac{5}{6}$$

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b) If SU(5) is gauged

D+L-5N	$E _ 8$
$\sqrt{30}$	$\overline{N} = \overline{3}$

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b) If SU(5) is gauged

$$\frac{D+L-5N}{\sqrt{30}} \qquad \qquad \frac{E}{N} = \frac{8}{3}$$

c) If all Yukawas allowed

D - 3L + 3N	$E _ 1$	16
$\sqrt{30}$	\overline{N} – – –	3

Incomplete SU(5) multiplets can improve unification



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Neutrino masses obtained from see-saw

$$\frac{1}{\Lambda^2}(l\nu_R^c)(LN) + \Lambda(\nu_R^c)^2$$

HIGGS MASS

Higgs potential is generated by the couplings that break the global symmetry. Minimally gauge and Yukawa couplings.

$$V(h) = \sum_{i} a_{i} \sin^{2i} \left(\frac{h}{f}\right)$$

Electro-weak scale:



 a_i must be tuned

Higgs mass is then "predicted".

Gauge contribution:

$$V(h)_{\text{gauge}} = \frac{9}{2} \int \frac{d^4p}{(2\pi)^4} \ln\left[1 + F(p^2)\sin^2\frac{h}{f}\right]$$

$$V(h)_{\text{gauge}} \approx \frac{9}{4} \frac{g^2}{16\pi^2} \frac{m_{\rho}^4}{g_{\rho}^2} \ln\left[\frac{m_{\rho}^2 + m_{a_1}^2}{2m_{\rho}^2}\right] \sin^2 \frac{h}{f} \qquad \qquad \lambda(m_{\rho})_{\text{gauge}}^{\text{leading}} \approx -3g^2 \log\frac{3}{2} \frac{g_{\rho}^2}{(4\pi)^2}$$





• Tuning with leading terms:

$$\lambda(\Lambda) \sim g_{\rm SM}^2 \frac{g_{\rho}^2}{(4\pi)^2} \sim \text{few} \, 10^{-2}$$

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Tuning with sub-leading terms

$$\lambda(\Lambda) \sim \frac{g_{\rm SM}^4}{(4\pi)^2} \sim 10^{-3}$$

Model I:

$$V_{\text{fermions}} \sim \frac{N_c \,\lambda_t^2}{16\pi^2} \Lambda^2 f^2 \sum_{\alpha=1}^2 |\text{Tr}[\Pi_t^{\alpha} \cdot U]|^2 \qquad \propto \sin^2 \frac{h}{f}$$

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• Giving up naturalness, strong CP, dark matter, Higgs mass can be explained. Unification and neutrino masses could also fit into the picture.

HIGGS + DFSZ AXION

Dine-Fischler-Srednicki-Zhitnitsky: Two Higgs doublets and complex singlet

$$\sigma \to e^{4i\alpha}\sigma, \qquad q_{L,R} \to e^{i\alpha}q_{L,R} \qquad H_u \to e^{-2i\alpha}H_u, \qquad H_d \to e^{-2i\alpha}H_d$$
$$f = \sqrt{v_u^2 + v_d^2 + |\sigma|^2}$$

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$$f = \sqrt{v_u^2 + v_d^2 + |\sigma|^2}$$

Ex:

$$\frac{G}{H} = \frac{SU(6)}{SO(6)} \qquad \qquad SO(6) \supset SO(4) \otimes \mathrm{U}(1)_{\mathrm{PQ}}$$

 $\mathbf{20}' = (\mathbf{2}, \mathbf{2})_{\pm \mathbf{2}} \oplus (\mathbf{1}, \mathbf{1})_{\pm \mathbf{4}} \oplus (\mathbf{1}, \mathbf{1})_{\mathbf{0}} \oplus (\mathbf{3}, \mathbf{3})_{\mathbf{0}}$

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L	$-\frac{1}{2}$	2	1	n	0
\overline{L}	$\frac{1}{2}$	$\overline{2}$	1	n	0
N	$\tilde{0}$	1	1	n	2
$ar{N}$	0	1	1	n	-2

 $H_1 \sim LN$

$$\langle L\bar{L}\rangle = \langle N\bar{N}\rangle = \Lambda^3$$

 $H_2 \sim \bar{L}\bar{N}$

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$ar{L}$	$\frac{\overline{1}}{2}$	$\overline{2}$	1	n	0
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\bar{N}	0	1	1	n_{c}	-2

 $H_1 \sim LN$

 $\langle L\bar{L}\rangle = \langle N\bar{N}\rangle = \Lambda^3$ $H_2 \sim \bar{L}\bar{N}$

Yukawas must respect PQ

$$\frac{1}{\Lambda_t^2} (q_L t_R^c)^{\dagger} (LN) + \frac{1}{\Lambda_b^2} (q_L b_R^c)^{\dagger} (\bar{L}\bar{N}) + \text{h.c.}$$

Anomalies:

$$E_{TC} = 0 \qquad \qquad \frac{E}{N} = \frac{8}{3}$$

Neutrino masses can be generated by see-saw mechanism

$$\frac{1}{\Lambda_{\nu}^{2}} (L\nu_{R}^{c})^{\dagger} (\bar{L}\,\bar{N}) + m^{2} (\nu_{R}^{c})^{2} + h.c.$$

$$m_{\nu} \sim \frac{v}{m}^2$$

If no right-handed neutrinos

$$\frac{1}{\Lambda_{\nu}^4} \, (\ell \bar{L})^2 N^2 \to \frac{1}{\Lambda_{\nu}^3} (\ell H_u)^2 \sigma^2 + \cdots$$

Same order of magnitude.

PARTIAL COMPOSITENESS

$$\frac{G}{H} = \frac{SO(6)}{SO(5)} \simeq \frac{SU(4)}{\mathrm{Sp}(4)}$$

Gripaios, Pomarol, Riva, Serra '09 Redi, Tesi '12 Galloway et. al. '10

5 GBs:

5 = (2, 2) + 1

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5 = (2, 2) + 1

Gauging of SM gauge symmetry preserves

 $SU(2)_L \times U(1)_Y \times U(1)_{PQ}$

Under $U(1)_{PQ}$ singlet shifts.

Sp(n) theories with 4 flavors

Fermions	$\mathrm{U}(1)_Y$	$SU(2)_L$	$SU(3)_{\rm c}$	$\operatorname{Sp}(n)_{\mathrm{TC}}$	$U(1)_{PQ}$
D	0	2	1	n	+1
S	$+\frac{1}{2}$	1	1	n	-1
$ar{S}$	$-\frac{1}{2}$	1	1	n	-1

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Difficult to generate QCD anomaly

(qu)(DS) $(qu)(DS)(S\overline{S})$

We can be build models with partial compositeness

 $m\psi\Psi + M\Psi\Psi + g_{\rm TC}\Psi\Psi H$

Fermions can couple to $6=(2,2)+2 \times 1$

$$q_L \to \frac{1}{\sqrt{2}} \begin{pmatrix} b_L \\ -ib_L \\ t_L \\ it_L \\ 0 \\ 0 \end{pmatrix} \qquad \qquad t_R \to \begin{pmatrix} 0 \\ 0 \\ 0 \\ i\cos\theta t_R \\ \sin\theta t_R \end{pmatrix}$$

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For $\theta = \frac{\pi}{4}$ singlet becomes exact GB PQ symmetry is anomalous due to tR rotations

$$E = 2\left[\left(\frac{4}{9} + \frac{1}{9}\right)3 + 1\right]N_F + E_{\rm TC}$$

$$\frac{E}{N} = \frac{8}{3} + \frac{E_{\rm TC}}{6} \qquad \qquad E_{TC} \sim n$$