A parametrization for the elastic differential *pp* cross section at LHC

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Brief Introduction of the Campinas Group

Hadronic Physics Group (Grupo de Física Hadrônica - GFH)
- 5 Researchers and 10 students

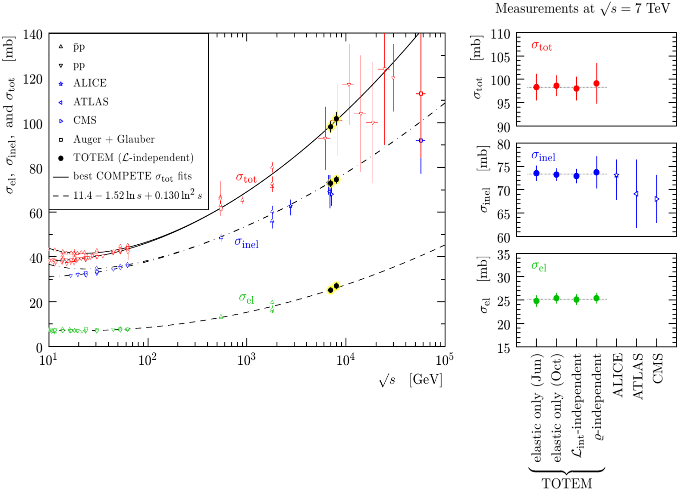
Main members:

- A.C. Aguilar - Nonperturbative QCD and Dyson-Schwinger Equations
- C.D. Chinellato and J.A. Chinellato - Hadronic interactions at UHECR and model-building in MC codes (members of the **Auger Collaboration**)
- J. Takahashi - Nuclear physics at high-energies (member of the **ALICE Collaboration**)
- M.J. Menon - Phenomenology of elastic and diffractive scattering at high-energies (group leader)

Outline

- Recent LHC results on *pp* Elastic Scattering
- Open Problems and Guidelines
- Our Strategies
- The *Dynamical Gluon Mass* (DGM) Approach
- Parametrizing Elastic Scattering Data
- Summary and Outlook

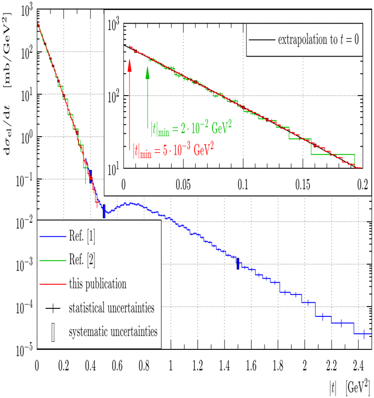
Total, elastic and inelastic cross sections at $\sqrt{s} = 7, 8$ TeV

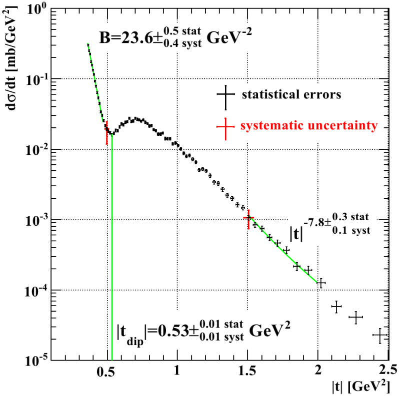


First precise measurements at cosmic ray energies through different techniques

Recent LHC results on *pp* Elastic Scattering

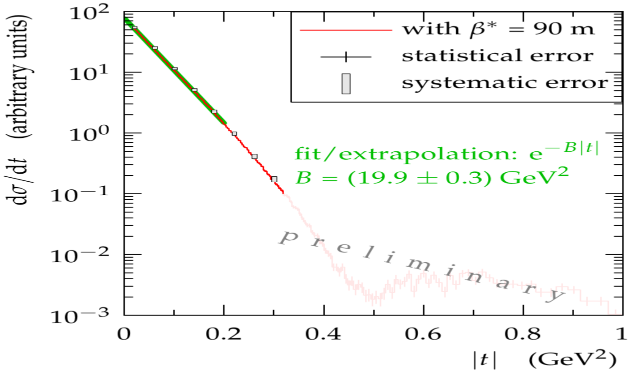
First measurements of the Differential Elastic Cross section at $\sqrt{s} = 7$ TeV





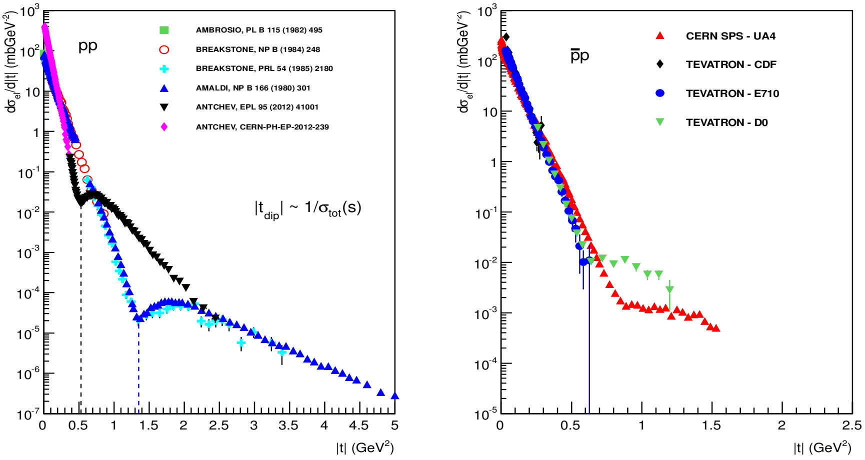
Essential to unravel dynamical aspects of small and large $|t|$ phenomena at high energies

...and preliminary results at $\sqrt{s} = 8$ TeV*



* from Jan Kašpar talk *"Total, elastic and diffractive cross sections with TOTEM"*, *CERN, December 4th, 2012*

Elastic $pp/\bar{p}p$ scattering at ISR, SPS, TEVATRON and LHC



presence of a 'dip' in *pp* channel and a 'shoulder' in $\bar{p}p$ one

Open Problems

1. the energy dependence of $\sigma_{tot}(s)$ $\leftrightarrow$ dynamics at large impact parameters $\leftrightarrow$ probing confinement region

2. unified description of hadronic interactions $\leftrightarrow$ 'soft' vs. 'hard' interactions $\leftrightarrow$ link betwen Reggeon Field Theory ('soft' Pomeron dynamics/interactions) and QCD (partonic) approaches

3. global description of the differential elastic cross section $\leftrightarrow$ interplay between small and large $|t|$ phenomena $\leftrightarrow$ $2g$ exchange (simplest gluon ladder) vs. $3g$ exchange (point-like interaction)

4. understanding the 'dip' (in $d\sigma_{el}{}^{pp}/d|t|$)/'shoulder' (in $d\sigma_{el}{}^{\bar{p}p}/d|t|$) region on a fundamental basis

5. interpreting diffractive excitation in partonic grounds $\leftrightarrow$ relationship with the dynamics of elastic scattering

6. asymptotic behaviour of physical quantities: is the simple black disk limit attainable?

(i) $s-$channel unitarity (in $b-$space): $\boxed{G_{ine}(s,b) = 2Re\ \Gamma(s,b) - |\Gamma(s,b)|^2}$

(ii) Optical Theorem: $\boxed{\sigma_{tot}(s) = 4\pi\ Im\ F(s,t=0)}$

(iii) analyticity and crossing symmetry $\leftrightarrow$ dispersion relations (integral/derivative)

(iv) Froissart-Martin Bound: $\boxed{\sigma_{tot} \leqslant \frac{1}{m_\pi^2} \ln^2(s/s_0)}$

(v) Pumplim Bound: $\boxed{\sigma_{el} + \sigma_{diff} \lesssim \sigma_{tot}/2}$

The 'dip'/'shoulder' occur through cancellations in elastic amplitude due to $t - channel$ processes:

$$A^{pp,\bar{p}p}(s,t) = \frac{A^{+}(s,t) \pm A^{-}(s,t)}{2},$$

where $A^{\pm}(s,t)$ are even/odd amplitudes related to $C = \pm 1$ exchange in $t - channel$. In *Regge Phenomenology*, they are called "Pomeron" and "Odderon" terms, which can be translated into QCD (LO) language as $2g$-exchange[1] and $3g$-exchange[2]. Eventually, the nonleading contribution of secondary *Reggeons* make their relative phase $\phi \neq \pi$.

[1] Low, *Phys.Rev. D12 (1975) 163* and Nussinov, *Phys.Rev.Lett. 34 (1975) 1286*
[2] Donnachie and Landshoff, *Z.Phys. C2 (1979) 55*

Strategies

Our ways to treat elastic *pp* scattering:

- Minimal model-dependence in data analyses (forward quantities) $\rightarrow$ 'unbiased' study of possible scenarios of saturation and asymptotic bahaviour of physical quantities (*Braz. J. Phys. 42 (2012) 452, Nucl.Phys. A 880 (2012) 1*)

- The Dynamical Gluon Mass (DGM) approach[3] $\rightarrow$ minijet model ($s-$channel model) with gluon mass as infrared cutoff and low-x parton interactions (*Nucl.Phys. A886 (2012) 48*)

- A parametrization of elastic *pp* scattering data from very small $|t|$ to past the dip[4]

[3] in collaboration with Dr. E.G.S. Luna (UFRGS) and Dr. A.A. Natale (UFABC)
[4] work in progress in collaboration with Dr. Giulia Pancheri

DGM Approach

Low-x parton interaction drive the growth of total cross section, essentially $gg \to gg$ [5]

$$\sigma_{gg} = C_{gg} \int_{M^2/s}^{1} d\tau F_{gg}(\tau) \hat{\sigma}_{gg}(\hat{s})$$

$$F_{gg}(\tau) = g(x) \otimes g(x) = \int_{\tau}^{1} \frac{dx}{x} g(x) g(\frac{\tau}{x})$$

- $\hat{s} = \tau s$ partonic squared c.m. energy
- $\hat{\sigma}$ - parton-parton cross section
- $g(x, Q^2) \sim x^{-J}$ - low-x gluon distribution function
- M^2 = mass scale separating pQCD e npQCD sectors

[5] along the lines of the *'Aspen Model'* by Block et al., *Phys.Rev. D60 (1999) 054024* and *Phys.Rept. 436 (2006) 71*

DGM Approach

Gluon mass as natural cutoff for infrared region

$$M^2 \to 4m_g^2,$$

with a frozen coupling constant

$$\bar{\alpha}_s(\hat{s}) = \frac{4\pi}{\beta_0 \ln\left[(\hat{s} + 4M_g^2(\hat{s}))/\Lambda^2\right]},$$

and dynamical gluon mass generation from solutions of Dyson-Schwinger Equations (DSE) for the gluon propagator[6]

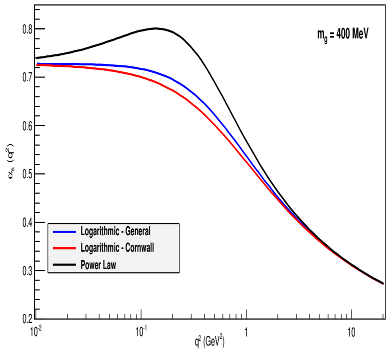
$$M_g^2(\hat{s}) = m_g^2 \left[\frac{\ln\left(\frac{\hat{s}+\rho {m_g}^2}{\Lambda^2}\right)}{\ln\left(\frac{\rho m_g^2}{\Lambda^2}\right)}\right]^{-(1+\gamma_1)}$$

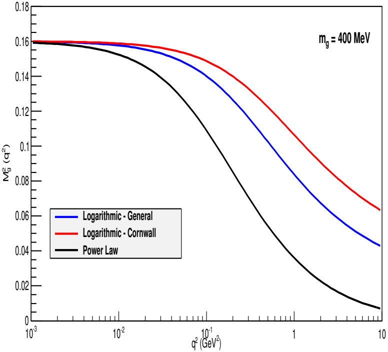
[6] Cornwall, *Phys. Rev. D 26 (1982) 1453* and Aguilar and Papavassiliou, *Eur. Phys. J. A 35 (2008) 189*

DGM Approach

...in another scenario

$$M_g^2(\hat{s}) = \frac{m_g^4}{m_g^2 + \hat{s}} \left[\frac{\ln\left(\frac{\hat{s}+\rho {m_g}^2}{\Lambda^2}\right)}{\ln\left(\frac{\rho m_g^2}{\Lambda^2}\right)} \right]^{-(1-\gamma_2)}$$





we use Cornwall's solution ($\rho = 4$ e $\gamma_1 = 1/11$)

Three level amplitudes and cross sections (gg)

$$\frac{d\hat{\sigma}^{DGM}}{d\hat{t}}(\hat{s},\hat{t}) = \frac{9\pi\bar{\alpha}_s^2}{2\hat{s}}\left\{3 - \frac{\hat{s}[4M_g^2(\hat{s}) - \hat{s} - \hat{t}]}{[\hat{t} - M_g^2(\hat{s})]^2} - \frac{\hat{s}\hat{t}}{[3M_g^2(\hat{s}) - \hat{s} - \hat{t}]^2} - \frac{\hat{t}[4M_g^2(\hat{s}) - \hat{s} - \hat{t}]}{[\hat{s} - M_g^2(\hat{s})]^2}\right\},$$

obtained with massive propagators. And its pQCD partner:

$$\frac{d\hat{\sigma}^{pQCD}}{d\hat{t}}(\hat{s},\hat{t}) = \frac{9\pi\alpha_s^2}{2\hat{s}}\left\{3 + \frac{\hat{s}[\hat{s}+\hat{t}]}{\hat{t}^2} - \frac{\hat{s}\hat{t}}{[\hat{s}+\hat{t}]^2} + \frac{\hat{t}[\hat{s}+\hat{t}]}{\hat{s}^2}\right\}$$

DGM Approach

At high-energies ($\hat{s} \gg \Lambda^2_{QCD}$)

$$M_g^2 \to 0 \text{ (massless vector boson)}$$

$$\bar{\alpha}_s \to \alpha_s \text{ (LO coupling constant)}$$

$$\Downarrow$$

$$\frac{d\hat{\sigma}^{DGM}}{d\hat{t}}(\hat{s},\hat{t}) \to \frac{d\hat{\sigma}^{pQCD}}{d\hat{t}}(\hat{s},\hat{t})$$

pQCD result recovered

DGM Approach

Integrated gg cross section:

$$\hat{\sigma}_{gg}(\hat{s}) = \left(\frac{3\pi\bar{\alpha}_s^2}{\hat{s}}\right)\left\{\frac{12\hat{s}^4 - 55M_g^2\hat{s}^3 + 12M_g^4\hat{s}^2 + 66M_g^6\hat{s} - 8M_g^8}{4M_g^2\hat{s}[\hat{s} - M_g^2]^2} - \left[3\ln\left(\frac{\hat{s} - 3M_g^2}{M_g^2}\right)\right]\right\}$$

Implemented in our eikonalized (unitarized) approach

DGM Approach

Eikonal model for pp e $\bar{p}p$ scattering:

$$A^{pp,\bar{p}p}(s,t) = i\int bdbJ_0(qb)[1 - e^{i\chi_h(s,b)\pm i\chi_s(s,b)}]$$

$\chi_{h/s}$ stand for 'semi-hard' and 'soft' contributions (even and odd under crossing symmetry)

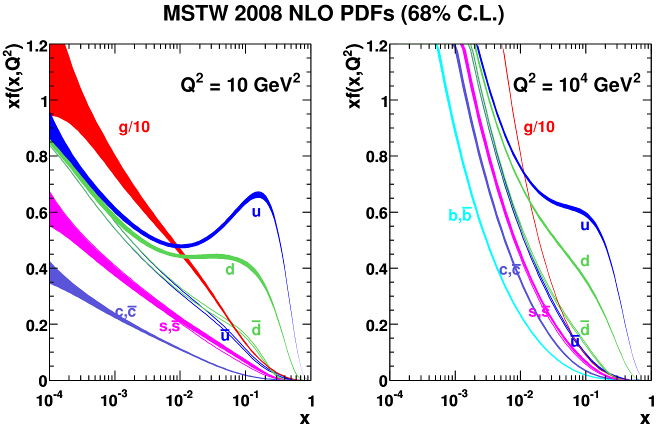
$$\begin{aligned}
\chi_h(s,b) &= \chi_{qq}(s,b) + \chi_{qg}(s,b) + \chi_{gg}(s,b) \\
&= i[\sigma_{qq}(s)W(b;\mu_{qq}) + \sigma_{qg}(s)W(b;\mu_{qg}) \\
&+ \sigma_{gg}(s)W(b;\mu_{gg})] \\
\chi_s(s,b) &= kC_o\frac{m_g}{\sqrt{s}}\, e^{i\pi/4}\, W(b;\mu^-)
\end{aligned}$$

Main ingredients

- $\chi_h(s,b) \sim \sigma_{gg}(s)W(b;\mu_{gg})$ - contribution from low-x cloud of size $r_{gg} \sim \mu_{gg}^{-1}$
- $\chi_s(s,b)$ - low energy sliplitting between pp e $\bar{p}p$ channels

Influence of low-x partons

From PDFs - e.g. MRSTW[7]



[7] A.D. Martin, W.J. Stirling, R.S. Thorne, G. Watt, *Eur. Phys. J. C* **63**, 189 (2009)

Phenomenological gluon distribution function

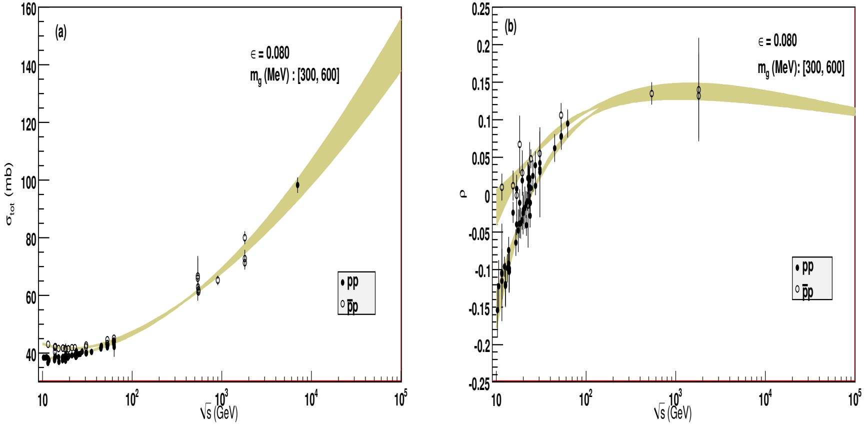
$$xg(x) = N_g\,(1-x)^5 x^{-\epsilon},$$

where ϵ stand for the 'soft' Pomeron intercept. Asymptotically,

$$\lim_{s\to\infty}\int_{4m_g^2/s}^{1} d\tau\, F_{gg}(\tau)\,\hat{\sigma}_{gg}(\hat{s}) \sim \left(\frac{s}{4{m_g}^2}\right)^{\epsilon} \ln\left(\frac{s}{4{m_g}^2}\right).$$

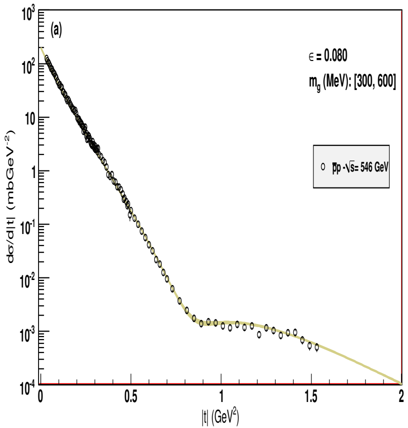
ϵ and m_g affect extrapolations to high-energies

Our best fit - $\epsilon = 0.080$ (standard 'bare' Pomeron) - σ_{tot} and ρ

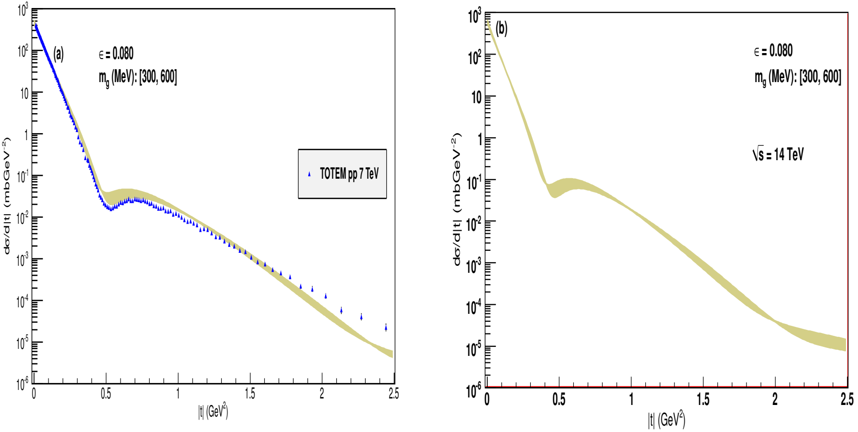


Uncertainty band for variations of the cutoff m_g!

Our best fit - $\epsilon = 0.080$ (standard 'bare' Pomeron) - $d\sigma_{el}/d|t|$ $\bar{p}p$ at 546 GeV and 1.80 + 1.96 TeV



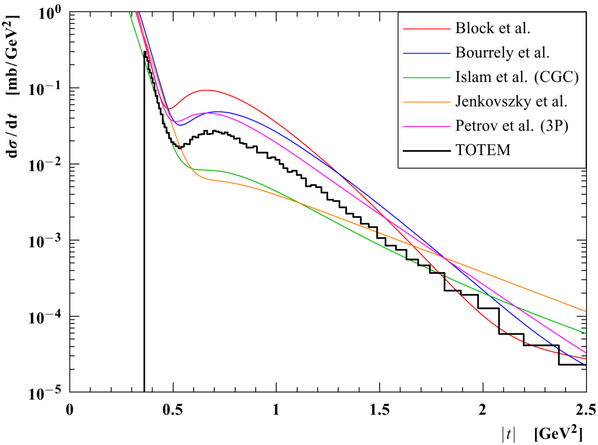
Our best fit - $\epsilon = 0.080$ *(standard 'bare' Pomeron)*- $d\sigma_{el}/d|t|$ at 7.0 TeV and 14 TeV



Description of LHC7 data up to $|t| \simeq$0.2 GeV^2 $\leftrightarrow$ DGM accounts for 2g-exchange...

DGM Approach

...but misses the 'dip' structure and large $|t|$ region, as well as other representative approaches

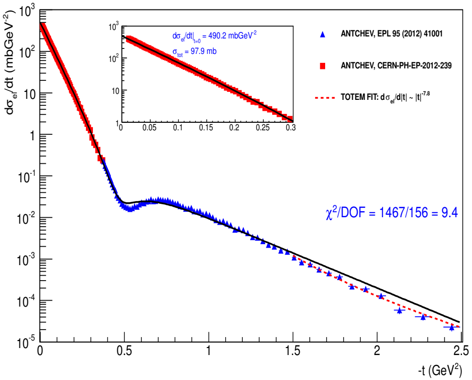


How can one treat consistently global features of $d\sigma_{el}/d|t|$?

Before building/modifying models, we try a *descriptive* approach using the simplest parametrization for the elastic amplitude [8]:

$$\mathcal{A}(s,t) = i[\sqrt{A(s)}e^{-B(s)|t|/2} + \sqrt{C(s)}e^{i\phi(s)}e^{-D(s)|t|/2}]$$

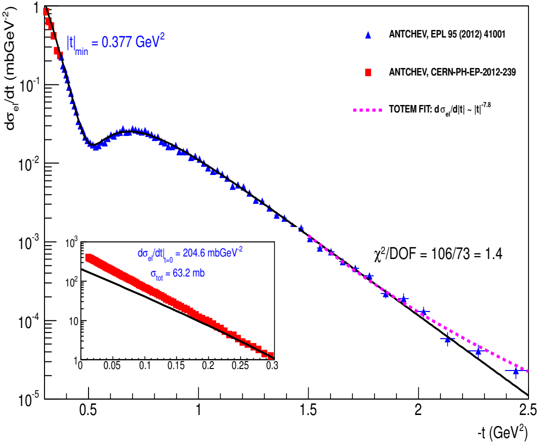
Our new fit LHC7 data do not reproduce all $|t|$ range...



[8] proposed a long time ago by Phillips and Barger, *Phys.Lett. B46 (1973) 412* and recently reviewed by Grau, Pacetti, Pancheri and Srivastava, *Phys.Lett. B714 (2012) 70*

Parametrizating Elastic Scattering Data

...though is quite good through the 'dip' and at large $|t|$

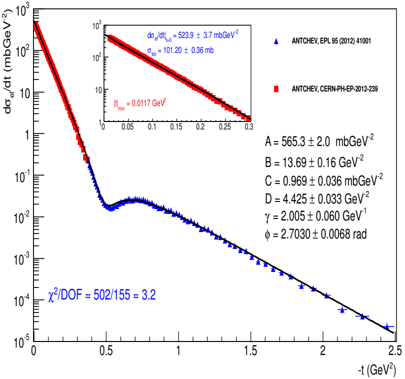


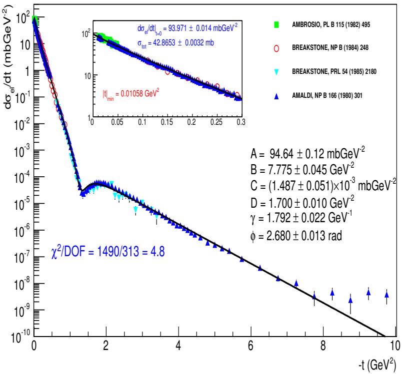
but still we miss the optical point → need to modify BP amplitude at small $|t|$ region

Our first attempt - introduction of a square root threshold[9] at small $|t|$ (normalized):

$$\mathcal{A}(s,t) = i[\sqrt{A(s)}e^{-B(s)|t|/2}e^{-\gamma(s)(\sqrt{4m_\pi^2+|t|}-2m_\pi)} + \sqrt{C(s)}e^{i\phi(s)}e^{-D(s)|t|/2}]$$

Provide a good fits to LHC7 and ISR53 (typical):

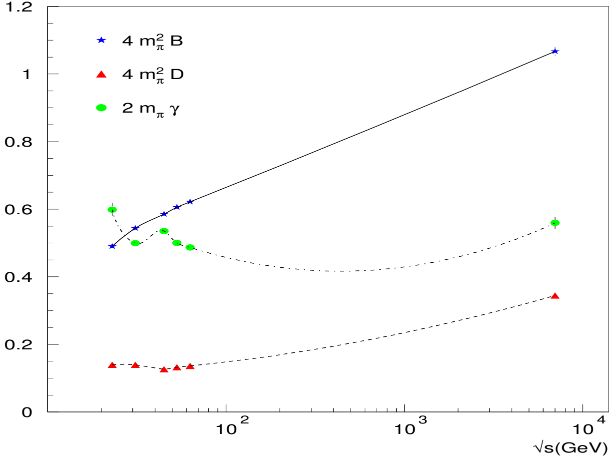




[9] motivated by the *two − pion loop* insertion in the Pomeron trajectory. See e.g. the recent review by Fiore et al. *Int.J.Mod.Phys. A24 (2009) 2551*

Parametrizating Elastic Scattering Data

However, the new term do not behave as expected, with $\gamma(s) \sim \ln s$...

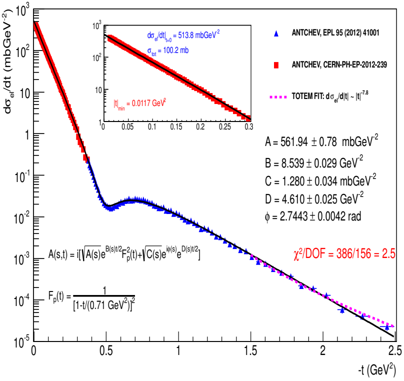


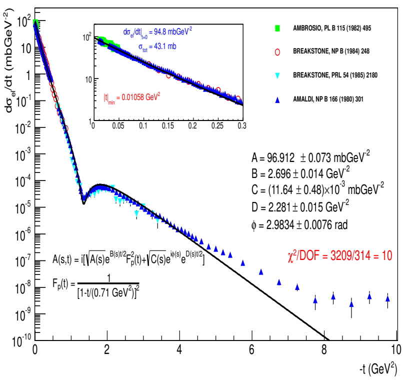
...instead it 'swings' with increasing c.m. energy → interpretation fails

Our second attempt - correcting with the EM proton's form factor (joining very small $|t|$ region)

$$\mathcal{A}(s,t) = i[\sqrt{A(s)}e^{-B(s)|t|/2}\frac{1}{\left(1+\frac{|t|}{0.71\ \text{GeV}^2}\right)^2} + \sqrt{C(s)}e^{i\phi(s)}e^{-D(s)|t|/2}]$$

Suitable for LHC7, but not for ISR energies...



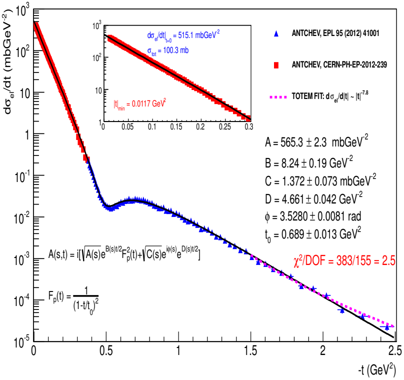


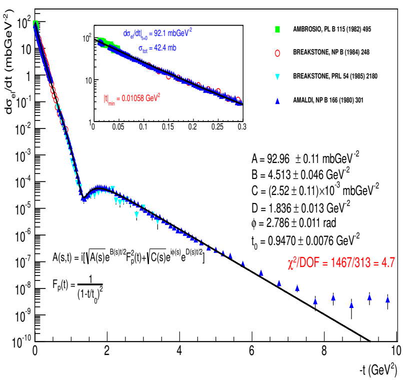
Parametrizating Elastic Scattering Data

Next attempt - proton's form factor with a free parameter

$$\mathcal{A}(s,t) = i[\sqrt{A(s)}e^{-B(s)|t|/2}\frac{1}{\left(1+\frac{|t|}{t_0}\right)^2} + \sqrt{C(s)}e^{i\phi(s)}e^{-D(s)|t|/2}]$$

Suitable for LHC7 and ISR energies...





...our best results so far, but still lacks clear physical interpretation

Parametrizating Elastic Scattering Data

On possible Interpretations

As far as we understand this model:

- amplitudes $\sqrt{A(s)}$ and $\sqrt{C(s)}$ $\rightarrow$ may correspond to $C = +1$, **leading** $2g$-exchange and $C = -1$, **non-leading** $3g$-exchange
- relative phase $\phi \neq \pi$ $\rightarrow$ possibly arise from **non-leading** *Reggeon* exchange cancellations
- exponentials $e^{B(s)t}$ and $e^{D(s)t}$ $\rightarrow$ may be attributed to overall ressumation of soft gluons emitted from many multiple process, with $B(s) \neq D(s)$, depending on the underlying dynamics
- proton's FF with free parameter $\rightarrow$ would indicate an average matter distribution denser than the charge one

But again, we still need to understand clearly all of that!

Summary and Outlook

What have we learned with this exercise?

1. the BP parametrization is able to reproduce essential features of LHC7 data and all ISR data sets
2. but it has to be modified to accommodate small $|t|$ phenomena
3. in simple terms, elastic scattering can be described with two exponentials and a relative phase
4. goodness of fit at LHC7 shows that it might be premature to claim a power-law behaviour $\sim |t|^{-8}$ (corresponding to a $3g$-exchange) for the present large $|t|$ data
5. the 'old' idea of implementing the proton's form factor at the elastic amplitude remains efficient at present enegies
6. however, the introduction of FFs and threshold singularities in the BP amplitude still lack clear physical understanding

Summary and Outlook

Perspectives for the near future

1. conclude our analyses of elastic scattering data with BP amplitude
   - understanding the role of FFs and threshold singularities
   - extracting the energy dependence of fit parameters
   - making predictions for LHC8 and LHC14
2. move to QCD-based models and study the implementation of these ideas; e.g. the *DGM approach* and the *Soft Gluon Ressumation Model* (by G. Pancheri and collaborators)

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Thank you!!!