A parametrization for the elastic differential $pp$ cross section at LHC

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Brief Introduction of the Campinas Group

Hadronic Physics Group (Grupo de Física Hadrônica - GFH)
- 5 Researchers and 10 students

Main members:

- A.C. Aguilar - Nonperturbative QCD and Dyson-Schwinger Equations

- C.D. Chinellato and J.A. Chinellato - Hadronic interactions at UHECR and model-building in MC codes (members of the Auger Collaboration)

- J. Takahashi - Nuclear physics at high-energies (member of the ALICE Collaboration)

- M.J. Menon - Phenomenology of elastic and diffractive scattering at high-energies (group leader)
Outline

- Recent LHC results on \( pp \) Elastic Scattering
- Open Problems and Guidelines
- Our Strategies
- The *Dynamical Gluon Mass* (DGM) Approach
- Parametrizing Elastic Scattering Data
- Summary and Outlook
Recent LHC results on $pp$ Elastic Scattering

Total, elastic and inelastic cross sections at $\sqrt{s} = 7, 8$ TeV

First precise measurements at cosmic ray energies through different techniques
Recent LHC results on $pp$ Elastic Scattering

First measurements of the Differential Elastic Cross section at $\sqrt{s} = 7$ TeV

Essential to unravel dynamical aspects of small and large $|t|$ phenomena at high energies
Recent LHC results on $pp$ Elastic Scattering

...and preliminary results at $\sqrt{s} = 8$ TeV*

* from Jan Kašpar talk “Total, elastic and diffractive cross sections with TOTEM”, CERN, December 4th, 2012
Recent LHC results on \( pp \) Elastic Scattering

\textbf{Elastic \( pp/\bar{p}p \) scattering at ISR, SPS, TEVATRON and LHC}

\begin{itemize}
  \item presence of a ‘dip’ in \( pp \) channel and a ‘shoulder’ in \( \bar{p}p \) one
\end{itemize}
1. the energy dependence of $\sigma_{tot}(s) \leftrightarrow$ dynamics at large impact parameters $\leftrightarrow$ probing confinement region

2. unified description of hadronic interactions $\leftrightarrow$ ‘soft’ vs. ‘hard’ interactions $\leftrightarrow$ link between Reggeon Field Theory (‘soft’ Pomeron dynamics/interactions) and QCD (partonic) approaches

3. global description of the differential elastic cross section $\leftrightarrow$ interplay between small and large $|t|$ phenomena $\leftrightarrow$ 2$g$ exchange (simplest gluon ladder) vs. 3$g$ exchange (point-like interaction)

4. understanding the ‘dip’ (in $d\sigma_{el}^{pp}/d|t|$) / ‘shoulder’ (in $d\sigma_{el}^{\bar{p}p}/d|t|$) region on a fundamental basis

5. interpreting diffractive excitation in partonic grounds $\leftrightarrow$ relationship with the dynamics of elastic scattering

6. asymptotic behaviour of physical quantities: is the simple black disk limit attainable?
(i) $s$–channel unitarity (in $b$–space):
\[ G_{\text{ine}}(s, b) = 2 \text{Re} \, \Gamma(s, b) - |\Gamma(s, b)|^2 \]

(ii) Optical Theorem:
\[ \sigma_{\text{tot}}(s) = 4\pi \, \text{Im} \, F(s, t = 0) \]

(iii) analyticity and crossing symmetry $\leftrightarrow$ dispersion relations (integral/derivative)

(iv) Froissart-Martin Bound:
\[ \sigma_{\text{tot}} \leq \frac{1}{m^2} \ln^2 \left( \frac{s}{s_0} \right) \]

(v) Pumplim Bound:
\[ \sigma_{\text{el}} + \sigma_{\text{diff}} \lesssim \sigma_{\text{tot}} / 2 \]
The ‘dip’/‘shoulder’ occur through cancellations in elastic amplitude due to $t$ – channel processes:

$$A^{pp,pp}(s, t) = \frac{A^+(s, t) \pm A^-(s, t)}{2},$$

where $A^{\pm}(s, t)$ are even/odd amplitudes related to $C = \pm 1$ exchange in $t$ – channel. In Regge Phenomenology, they are called “Pomeron” and “Odderon” terms, which can be translated into QCD (LO) language as $2g$-exchange$^1$ and $3g$-exchange$^2$. Eventually, the nonleading contribution of secondary Reggeons make their relative phase $\phi \neq \pi$.


$^2$Donnachie and Landshoff, Z.Phys. C2 (1979) 55
Our ways to treat elastic $pp$ scattering:


- The Dynamical Gluon Mass (DGM) approach$^3$ $\rightarrow$ minijet model ($s$–channel model) with gluon mass as infrared cutoff and low-$x$ parton interactions ($Nucl. Phys. A 886 (2012) 48$)

- A parametrization of elastic $pp$ scattering data from very small $|t|$ to past the dip$^4$

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$^3$ in collaboration with Dr. E.G.S. Luna (UFRGS) and Dr. A.A. Natale (UFABC)

$^4$ work in progress in collaboration with Dr. Giulia Pancheri
DGM Approach

Low-x parton interaction drive the growth of total cross section, essentially $gg \rightarrow gg$  

$$\sigma_{gg} = C_{gg} \int_{M^2/s}^{1} d\tau F_{gg}(\tau) \hat{\sigma}_{gg}(\hat{s})$$

$$F_{gg}(\tau) = g(x) \otimes g(x) = \int_{\tau}^{1} \frac{dx}{x} g(x) g(\frac{\tau}{x})$$

- $\hat{s} = \tau s$ partonic squared c.m. energy
- $\hat{\sigma}$ - parton-parton cross section
- $g(x, Q^2) \sim x^{-J}$ - low-x gluon distribution function
- $M^2$ = mass scale separating pQCD e npQCD sectors

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Gluon mass as natural cutoff for infrared region

\[ M^2 \rightarrow 4m^2, \]

with a frozen coupling constant

\[ \bar{\alpha}_s(\hat{s}) = \frac{4\pi}{\beta_0 \ln \left[ (\hat{s} + 4M^2_g(\hat{s}))/\Lambda^2 \right]}, \]

and dynamical gluon mass generation from solutions of Dyson-Schwinger Equations (DSE) for the gluon propagator\(^6\)

\[ M^2_g(\hat{s}) = m^2 \left[ \frac{\ln \left( \frac{\hat{s} + \rho m^2_g}{\Lambda^2} \right)}{\ln \left( \frac{\rho m^2_g}{\Lambda^2} \right)} \right]^{-(1+\gamma_1)} \]

...in another scenario

\[ M_g^2(\hat{s}) = \frac{m_g^4}{m_g^2 + \hat{s}} \left[ \ln \left( \frac{\hat{s} + \rho m_g^2}{\Lambda^2} \right) \right] \left( \frac{\rho m_g^2}{\Lambda^2} \right)^{(1 - \gamma_2)} \]

we use Cornwall’s solution (\( \rho = 4 \) e \( \gamma_1 = 1/11 \))
Three level amplitudes and cross sections \((gg)\)

\[
\frac{d\hat{\sigma}^{DGM}}{d\hat{t}}(\hat{s}, \hat{t}) = \frac{9\pi \bar{\alpha}_s^2}{2\hat{s}} \left\{ 3 - \frac{\hat{s}[4M_g^2(\hat{s}) - \hat{s} - \hat{t}]}{[\hat{t} - M_g^2(\hat{s})]^2} - \frac{\hat{s}\hat{t}}{[3M_g^2(\hat{s}) - \hat{s} - \hat{t}]^2} \right\} 
\]

obtained with massive propagators. And its pQCD partner:

\[
\frac{d\hat{\sigma}^{pQCD}}{d\hat{t}}(\hat{s}, \hat{t}) = \frac{9\pi \alpha_s^2}{2\hat{s}} \left\{ 3 + \frac{\hat{s}[\hat{s} + \hat{t}]}{\hat{t}^2} - \frac{\hat{s}\hat{t}}{[\hat{s} + \hat{t}]^2} + \frac{\hat{t}[\hat{s} + \hat{t}]}{\hat{s}^2} \right\}
\]
DGM Approach

At high-energies ($\hat{s} \gg \Lambda_{QCD}^2$)

$$M_g^2 \rightarrow 0 \text{ (massless vector boson)}$$

$$\bar{\alpha}_s \rightarrow \alpha_s \text{ (LO coupling constant)}$$

$$\Downarrow$$

$$\frac{d\hat{\sigma}^{DGM}}{d\hat{t}}(\hat{s}, \hat{t}) \rightarrow \frac{d\hat{\sigma}^{pQCD}}{d\hat{t}}(\hat{s}, \hat{t})$$

pQCD result recovered
Integrated $gg$ cross section:

$$\hat{\sigma}_{gg}(\hat{s}) = \left(\frac{3\pi \bar{\alpha}_s^2}{\hat{s}}\right) \left\{ \frac{12\hat{s}^4 - 55M_g^2\hat{s}^3 + 12M_g^4\hat{s}^2 + 66M_g^6\hat{s} - 8M_g^8}{4M_g^2\hat{s}\left[\hat{s} - M_g^2\right]^2} \right\} - \left[3 \ln \left(\frac{\hat{s} - 3M_g^2}{M_g^2}\right)\right]$$

Implemented in our eikonalized (unitarized) approach
Eikonal model for $pp$ e $\bar{p}p$ scattering:

$$A^{pp,\bar{p}p}(s, t) = i \int b db J_0(qb) [1 - e^{i\chi_h(s,b) \pm i\chi_s(s,b)}]$$

$\chi_{h/s}$ stand for ‘semi-hard’ and ‘soft’ contributions (even and odd under crossing symmetry)

$$\chi_h(s, b) = \chi_{qq}(s, b) + \chi_{qg}(s, b) + \chi_{gg}(s, b)$$

$$\chi_h(s, b) = i[\sigma_{qq}(s) W(b; \mu_{qq}) + \sigma_{qg}(s) W(b; \mu_{qg}) + \sigma_{gg}(s) W(b; \mu_{gg})]$$

$$\chi_s(s, b) = kC_o \frac{m_g}{\sqrt{s}} e^{i\pi/4} W(b; \mu^-)$$

Main ingredients

- $\chi_h(s, b) \sim \sigma_{gg}(s) W(b; \mu_{gg})$ - contribution from low-x cloud of size $r_{gg} \sim \mu_{gg}^{-1}$
- $\chi_s(s, b)$ - low energy splitting between $pp$ e $\bar{p}p$ channels
DGM Approach

**Influence of low-\(x\) partons**

From PDFs - e.g. MRSTW$^7$

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Phenomenological gluon distribution function

\[ xg(x) = N_g (1 - x)^5 x^{-\epsilon}, \]

where \( \epsilon \) stand for the ‘soft’ Pomeron intercept. Asymptotically,

\[ \lim_{s \to \infty} \int_{4m_g^2/s}^{1} d\tau \ F_{gg}(\tau) \ \hat{\sigma}_{gg}(\hat{s}) \sim \left( \frac{s}{4m_g^2} \right)^{\epsilon} \ln \left( \frac{s}{4m_g^2} \right). \]

\( \epsilon \) and \( m_g \) affect extrapolations to high-energies
**Our best fit** - $\epsilon = 0.080$ (standard ‘bare’ Pomeron) - $\sigma_{tot}$ and $\rho$

Uncertainty band for variations of the cutoff $m_g$!
Our best fit - $\epsilon = 0.080$ (standard ‘bare’ Pomeron) - $d\sigma_{el}/d|t|$ $\bar{p}p$

at 546 GeV and $1.80 + 1.96$ TeV
**DGM Approach**

*Our best fit* - \( \epsilon = 0.080 \) (standard ‘bare’ Pomeron)- \( d\sigma_{el}/d|t| \) at 7.0 TeV and 14 TeV

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**Description of LHC7 data up to \(|t| \simeq 0.2 \text{ GeV}^2 \leftrightarrow\)** DGM accounts for 2g-exchange...
DGM Approach

...but misses the ‘dip’ structure and large $|t|$ region, as well as other representative approaches

How can one treat consistently global features of $d\sigma_{el}/d|t|$?
Before building/modifying models, we try a descriptive approach using the simplest parametrization for the elastic amplitude $^8$:

$$A(s, t) = i\left[\sqrt{A(s)} e^{-B(s)|t|/2} + \sqrt{C(s)} e^{i\phi(s)} e^{-D(s)|t|/2}\right]$$

Our new fit LHC7 data do not reproduce all $|t|$ range...

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...though is quite good through the ‘dip’ and at large $|t|$

![Graph showing elastic scattering data](image)

$|t|_{\text{min}} = 0.377 \text{ GeV}^2$

$\chi^2/\text{DOF} = 106/73 = 1.4$

$\sigma_{\text{tot}} = 63.2 \text{ mb}$

$\sigma_{\text{el}}^{2} = 204.6 \text{ mbGeV}$

but still we miss the optical point $\rightarrow$ need to modify BP amplitude at small $|t|$ region
Our first attempt - introduction of a square root threshold\(^9\) at small \(|t|\) (normalized):

\[
A(s, t) = i[\sqrt{A(s)}e^{-B(s)|t|/2}e^{-\gamma(s)(\sqrt{4m^2 + |t|} - 2m)} + \sqrt{C(s)}e^{i\phi(s)}e^{-D(s)|t|/2}]
\]

Provide a good fits to LHC7 and ISR53 (typical):

\[
\begin{align*}
A &= 565.3 \pm 2.0 \text{ mbGeV}^2 \\
B &= 13.69 \pm 0.16 \text{ GeV}^2 \\
C &= 0.969 \pm 0.036 \text{ mbGeV}^2 \\
D &= 4.425 \pm 0.033 \text{ GeV}^2 \\
\gamma &= 2.005 \pm 0.060 \text{ GeV}^{-1} \\
\phi &= 2.7030 \pm 0.0068 \text{ rad}
\end{align*}
\]

\(\chi^2/\text{DOF} = 502/155 = 3.2\)

\(\chi^2/\text{DOF} = 1490/313 = 4.8\)

\(^9\) motivated by the \textit{two - pion loop} insertion in the Pomeron trajectory. See e.g. the recent review by Fiore et al. \textit{Int.J.Mod.Phys.} A24 (2009) 2551
However, the new term do not behave as expected, with $\gamma(s) \sim \ln s$...

...instead it ‘swings’ with increasing c.m. energy → interpretation fails
Our second attempt - correcting with the EM proton's form factor (joining very small \(|t|\) region)

\[
A(s, t) = i \left[ \sqrt{A(s)} e^{-B(s)|t|/2} \frac{1}{\left(1 + \frac{|t|}{0.71 \text{ GeV}^2}\right)^2} + \sqrt{C(s)} e^{i\phi(s)} e^{-D(s)|t|/2} \right]
\]

Suitable for LHC7, but not for ISR energies...
Parametrizing Elastic Scattering Data

Next attempt - proton’s form factor with a free parameter

$$A(s, t) = i \left[ \sqrt{A(s)} e^{-B(s) |t|/2} \left(1 + \frac{|t|}{t_0}\right)^2 + \sqrt{C(s)} e^{i\phi(s)} e^{-D(s) |t|/2} \right]$$

Suitable for LHC7 and ISR energies...

...our best results so far, but still lacks clear physical interpretation
As far as we understand this model:

- amplitudes $\sqrt{A(s)}$ and $\sqrt{C(s)} \rightarrow$ may correspond to $C = +1$, leading 2g-exchange and $C = -1$, non-leading 3g-exchange

- relative phase $\phi \neq \pi \rightarrow$ possibly arise from non-leading Reggeon exchange cancellations

- exponentials $e^{B(s)t}$ and $e^{D(s)t} \rightarrow$ may be attributed to overall ressumation of soft gluons emitted from many multiple process, with $B(s) \neq D(s)$, depending on the underlying dynamics

- proton’s FF with free parameter $\rightarrow$ would indicate an average matter distribution denser than the charge one

But again, we still need to understand clearly all of that!
Summary and Outlook

What have we learned with this exercise?

1. the BP parametrization is able to reproduce essential features of LHC7 data and all ISR data sets

2. but it has to be modified to accommodate small $|t|$ phenomena

3. in simple terms, elastic scattering can be described with two exponentials and a relative phase

4. goodness of fit at LHC7 shows that it might be premature to claim a power-law behaviour $\sim |t|^{-8}$ (corresponding to a $3g$-exchange) for the present large $|t|$ data

5. the ‘old’ idea of implementing the proton’s form factor at the elastic amplitude remains efficient at present energies

6. however, the introduction of FFs and threshold singularities in the BP amplitude still lack clear physical understanding
Summary and Outlook

Perspectives for the near future

1. conclude our analyses of elastic scattering data with BP amplitude
   - understanding the role of FFs and threshold singularities
   - extracting the energy dependence of fit parameters
   - making predictions for LHC8 and LHC14

2. move to QCD-based models and study the implementation of these ideas; e.g. the *DGM approach* and the *Soft Gluon Ressumation Model* (by G. Pancheri and collaborators)
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