Perturbative QCD for the LHC

Giancarlo Ferrera

giancarlo.ferrera@mi.infn.it

Università di Milano

IFAE2013 – Cagliari – April 4th 2013

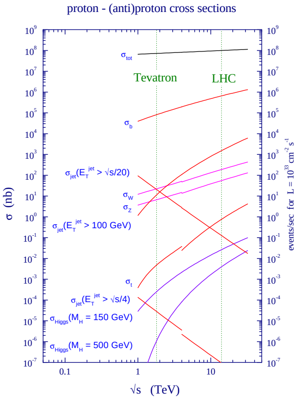
Outline

1. PDFs and α_S
2. Higher orders: NLO and NNLO
3. Resummation
4. Shower Monte Carlo
5. Conclusions

Motivations

The LHC is a (large) hadron collider machine: all the interesting high-p_T new reactions initiated by QCD hard scattering of partons.

To claim for new-physics signals a good control of the QCD processes is necessary.

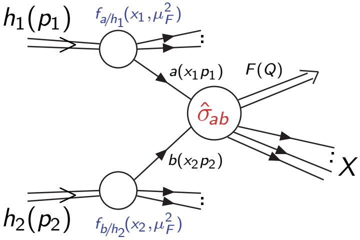


To fully exploit the information contained in the LHC experimental data, precise theoretical predictions are needed

$\Longrightarrow$ computation of higher-order pQCD corrections.

QCD Factorization

$$h_1(p_1) + h_2(p_2) \;\to\; F(Q) + X$$



The framework: QCD factorization formula

$$\sigma_{h_1 h_2}(p_1, p_2) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2\, f_{a/h_1}(x_1, \mu_F^2)\, f_{b/h_2}(x_2, \mu_F^2)\, \hat{\sigma}_{ab}(x_1 p_1, x_2 p_2; \mu_F^2) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{Q}\right)^p$$

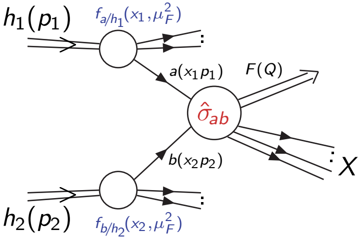
- $f_{a/h}(x, \mu_F^2)$: Non perturbative universal parton densities (PDFs), $\mu_F \sim Q$. Measured from experiments at a given scale μ_0, Evolution to μ_F calculable in pQCD through DGLAP equation.
- $\hat{\sigma}_{ab}$: Hard scattering cross section. Process dependent, calculable with a perturbative expansion i the strong coupling $\alpha_S(Q) \sim 1/(\beta_0 \ln Q^2/\Lambda^2_{QCD}) \sim 0.1$ (for $Q = m_H, m_W, m_Z, m_t, p_T^{jet}, \cdots$).

$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \alpha_S(\mu_R^2)\, \hat{\sigma}_{ab}^{(1)} + \alpha_S^2(\mu_R^2)\, \hat{\sigma}_{ab}^{(2)} + \mathcal{O}(\alpha_S^3)\,.$$

- $\left(\frac{\Lambda_{QCD}}{Q}\right)^p$ (with $p \leq 1$): Non perturbative power-corrections (higher-twist).

Precise predictions for σ depend on good knowledge of both $\hat{\sigma}_{ab}$ and $f_{a/h}(x, \mu_F^2)$

QCD Factorization



$$h_1(p_1) + h_2(p_2) \;\to\; F(Q) + X$$

The framework: QCD factorization formula

$$\sigma_{h_1 h_2}(p_1,p_2) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2\, f_{a/h_1}(x_1,\mu_F^2)\, f_{b/h_2}(x_2,\mu_F^2)\, \hat{\sigma}_{ab}(x_1 p_1, x_2 p_2; \mu_F^2) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{Q}\right)^p$$

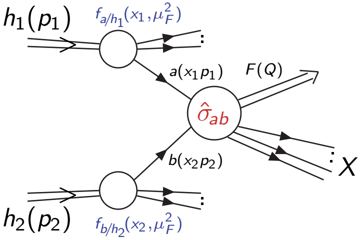
- $f_{a/h}(x,\mu_F^2)$: Non perturbative universal parton densities (PDFs), $\mu_F \sim Q$. Measured from experiments at a given scale μ_0, Evolution to μ_F calculable in pQCD through DGLAP equation.
- $\hat{\sigma}_{ab}$: Hard scattering cross section. Process dependent, calculable with a perturbative expansion i the strong coupling $\alpha_S(Q) \sim 1/(\beta_0 \ln Q^2/\Lambda^2_{QCD}) \sim 0.1$ (for $Q = m_H, m_W, m_Z, m_t, p_T^{jet}, \cdots$).

$$\hat{\sigma}_{ab} = \hat{\sigma}^{(0)}_{ab} + \alpha_S(\mu_R^2)\,\hat{\sigma}^{(1)}_{ab} + \alpha_S^2(\mu_R^2)\,\hat{\sigma}^{(2)}_{ab} + \mathcal{O}(\alpha_S^3)\,.$$

- $\left(\frac{\Lambda_{QCD}}{Q}\right)^p$ (with $p \le 1$): Non perturbative power-corrections (higher-twist).

Precise predictions for σ depend on good knowledge of both $\hat{\sigma}_{ab}$ and $f_{a/h}(x,\mu_F^2)$

QCD Factorization



$$h_1(p_1) + h_2(p_2) \;\rightarrow\; F(Q) + X$$

The framework: QCD factorization formula

$$\sigma_{h_1 h_2}(p_1,p_2) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 \; f_{a/h_1}(x_1,\mu_F^2)\, f_{b/h_2}(x_2,\mu_F^2)\, \hat{\sigma}_{ab}(x_1 p_1, x_2 p_2; \mu_F^2) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{Q}\right)^p$$

- $f_{a/h}(x,\mu_F^2)$: Non perturbative universal parton densities (PDFs), $\mu_F \sim Q$. Measured from experiments at a given scale μ_0, Evolution to μ_F calculable in pQCD through DGLAP equation.
- $\hat{\sigma}_{ab}$: Hard scattering cross section. Process dependent, calculable with a perturbative expansion i the strong coupling $\alpha_S(Q) \sim 1/(\beta_0 \ln Q^2/\Lambda_{QCD}^2) \sim 0.1$ (for $Q = m_H, m_W, m_Z, m_t, p_T^{jet}, \cdots$).

$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \alpha_S(\mu_R^2)\, \hat{\sigma}_{ab}^{(1)} + \alpha_S^2(\mu_R^2)\, \hat{\sigma}_{ab}^{(2)} + \mathcal{O}(\alpha_S^3)\,.$$

- $\left(\frac{\Lambda_{QCD}}{Q}\right)^p$ (with $p \leq 1$): Non perturbative power-corrections (higher-twist).

Precise predictions for σ depend on good knowledge of both $\hat{\sigma}_{ab}$ and $f_{a/h}(x,\mu_F^2)$

PDFs and α_S

Fit of PDFs

- Method: typical parameterization of parton densities at input scale $\mu_0^2 \sim 1 \div 4\ GeV^2$:

$$x\, f_a(x, \mu_0^2) \;=\; A_a\, x^{\lambda_a}\, (1-x)^{\eta_a}\, (1 \,+\, \epsilon_a \sqrt{x} \,+\, \gamma_a\, x + \cdots).$$

  Parameters constrained by imposing momentum sum rules:$\sum_a \int_0^1 dx\, x\, f_a(x, \mu_0^2) = 1$, then adjust parameters to fit data.

- Typical constraining process:
  - DIS (fixed target exp. and HERA): sensitive to quark densities.
  - Jet data (HERA and Tevatron): sensitive to high-x gluon density.
  - Drell-Yan (low energy and Tevatron data): sensitive to (anti-)quark densities.
- Evolution $\mu_0 \to \mu$ using DGLAP equations:

$$\frac{\partial f_a(x,\mu^2)}{\partial \ln \mu^2} = \frac{\alpha_S(\mu^2)}{2\pi} \int_x^1 \frac{d\xi}{\xi} P_{ab}(x/\xi) f_b(\xi, \mu^2)$$

  AP kernels calculable in pQCD

$$P_{ab}(z) = P_{ab}^{(0)}(z) + \frac{\alpha_S(\mu^2)}{2\pi} P_{ab}^{(1)}(z) + (\frac{\alpha_S(\mu^2)}{2\pi})^2 P_{ab}^{(2)}(z) + \cdots$$

Fit of PDFs

- Method: typical parameterization of parton densities at input scale $\mu_0^2 \sim 1 \div 4 \; GeV^2$:

$$x\, f_a(x,\mu_0^2) \;=\; A_a\, x^{\lambda_a}\, (1-x)^{\eta_a}\, (1 \,+\, \epsilon_a\, \sqrt{x} \,+\, \gamma_a\, x + \cdots).$$

  Parameters constrained by imposing momentum sum rules:$\sum_a \int_0^1 dx\, x\, f_a(x,\mu_0^2) = 1$, then adjust parameters to fit data.

- Typical constraining process:
  - DIS (fixed target exp. and HERA): sensitive to quark densities.
  - Jet data (HERA and Tevatron): sensitive to high-x gluon density.
  - Drell-Yan (low energy and Tevatron data): sensitive to (anti-)quark densities.

- Evolution $\mu_0 \to \mu$ using DGLAP equations:

$$\frac{\partial f_a(x,\mu^2)}{\partial \ln \mu^2} = \frac{\alpha_S(\mu^2)}{2\pi} \int_x^1 \frac{d\xi}{\xi} P_{ab}(x/\xi) f_b(\xi,\mu^2)$$

  AP kernels calculable in pQCD

$$P_{ab}(z) = P_{ab}^{(0)}(z) + \frac{\alpha_S(\mu^2)}{2\pi} P_{ab}^{(1)}(z) + (\frac{\alpha_S(\mu^2)}{2\pi})^2 P_{ab}^{(2)}(z) + \cdots$$

Fit of PDFs

- Method: typical parameterization of parton densities at input scale $\mu_0^2 \sim 1 \div 4 \ GeV^2$:

$$x\, f_a(x,\mu_0^2) \;=\; A_a\, x^{\lambda_a}\, (1-x)^{\eta_a}\, (1 \,+\, \epsilon_a\, \sqrt{x} \,+\, \gamma_a\, x + \cdots).$$

  Parameters constrained by imposing momentum sum rules: $\sum_a \int_0^1 dx\, x\, f_a(x,\mu_0^2) = 1$, then adjust parameters to fit data.

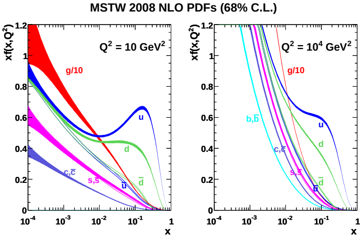
- Typical constraining process:
  - DIS (fixed target exp. and HERA): sensitive to quark densities.
  - Jet data (HERA and Tevatron): sensitive to high-x gluon density.
  - Drell-Yan (low energy and Tevatron data): sensitive to (anti-)quark densities.

- Evolution $\mu_0 \to \mu$ using DGLAP equations:

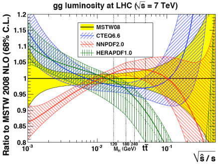
$$\frac{\partial f_a(x,\mu^2)}{\partial \ln \mu^2} = \frac{\alpha_S(\mu^2)}{2\pi} \int_x^1 \frac{d\xi}{\xi} P_{ab}(x/\xi) f_b(\xi,\mu^2)$$

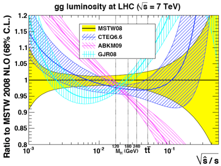
  AP kernels calculable in pQCD

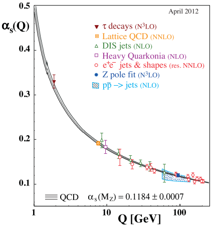
$$P_{ab}(z) = P_{ab}^{(0)}(z) + \frac{\alpha_S(\mu^2)}{2\pi} P_{ab}^{(1)}(z) + \left(\frac{\alpha_S(\mu^2)}{2\pi}\right)^2 P_{ab}^{(2)}(z) + \cdots$$

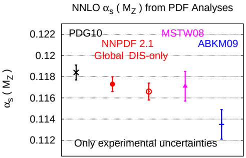


PDFs and α_S



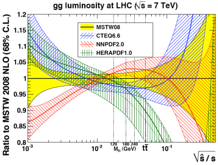


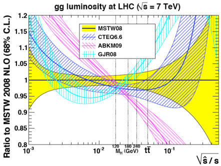


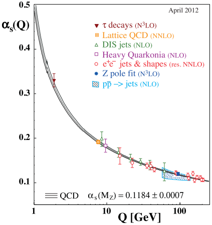


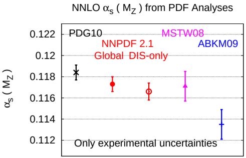
- Several PDFs sets available:
  MSTW, NNPDF, CTEQ/CT, GJR, ABKM, HERAPDF.
  MSTW: ∼ 3000 data pts., ∼ 50 free param.
  NNPDF: ∼ 3000 data pts., ∼ 250 free param.
- Differences among sets include:
  data set in the fit, parton parametrization, statistical treatment, perturbative accuracy (NLO,NNLO), value of α_S.
- The PDFs sets can be combined using the "PDF4LHC recommendation" to obtain a central value and an estimate of the uncertainty.
- Simultaneous extraction of $\alpha_S(m_Z)$ from NNLO fits lead to some tension:
  World avg.'12 $\alpha_S(m_Z) = 0.1184 \pm 0.0007$
  ABKM11 $\alpha_S(m_Z) = 0.1135 \pm 0.0014$
  MSTW08 $\alpha_S(m_Z) = 0.1171 \pm 0.0014$
  NNPDF2.1 $\alpha_S(m_Z) = 0.1173 \pm 0.0011$
  JR09 $\alpha_S(m_Z) = 0.124 \pm 0.002$

PDFs and α_S



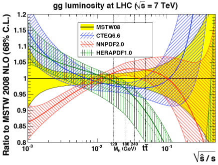


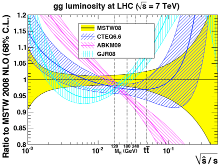


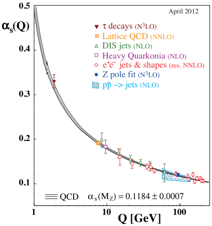


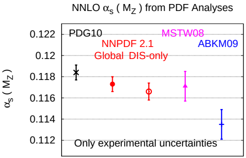
- Several PDFs sets available: MSTW, NNPDF, CTEQ/CT, GJR, ABKM, HERAPDF.
  MSTW: ∼ 3000 data pts., ∼ 50 free param.
  NNPDF: ∼ 3000 data pts., ∼ 250 free param.
- Differences among sets include: data set in the fit, parton parametrization, statistical treatment, perturbative accuracy (NLO,NNLO), value of α_S.
- The PDFs sets can be combined using the "PDF4LHC recommendation" to obtain a central value and an estimate of the uncertainty.
- Simultaneous extraction of $\alpha_S(m_Z)$ from NNLO fits lead to some tension:
  World avg.'12 $\alpha_S(m_Z) = 0.1184 \pm 0.0007$
  ABKM11 $\alpha_S(m_Z) = 0.1135 \pm 0.0014$
  MSTW08 $\alpha_S(m_Z) = 0.1171 \pm 0.0014$
  NNPDF2.1 $\alpha_S(m_Z) = 0.1173 \pm 0.0011$
  JR09 $\alpha_S(m_Z) = 0.124 \pm 0.002$

PDFs and α_S









- Several PDFs sets available:
  MSTW, NNPDF, CTEQ/CT, GJR, ABKM, HERAPDF.
  MSTW: ∼ 3000 data pts., ∼ 50 free param.
  NNPDF: ∼ 3000 data pts., ∼ 250 free param.
- Differences among sets include:
  data set in the fit, parton parametrization, statistical treatment, perturbative accuracy (NLO,NNLO), value of α_S.
- The PDFs sets can be combined using the "PDF4LHC recommendation" to obtain a central value and an estimate of the uncertainty.
- Simultaneous extraction of $\alpha_S(m_Z)$ from NNLO fits lead to some tension:
  World avg.'12 $\alpha_S(m_Z) = 0.1184 \pm 0.0007$
  ABKM11 $\alpha_S(m_Z) = 0.1135 \pm 0.0014$
  MSTW08 $\alpha_S(m_Z) = 0.1171 \pm 0.0014$
  NNPDF2.1 $\alpha_S(m_Z) = 0.1173 \pm 0.0011$
  JR09 $\alpha_S(m_Z) = 0.124 \pm 0.002$

Higher orders: NLO and NNLO

Higher orders: NLO

- Calculations at LO give the order of magnitude of cross sections and distributions, NLO corrections provide reliable estimate
- Experiments have finite acceptance important to provide exclusive theoretical predictions.
- At NLO infrared singularities in *real* and *virtual* corrections prevent the straightforward implementation of Monte Carlo numerical techniques (especially for fully exclusive quantities).
- NLO subtraction method: introduction of auxiliary QCD cross section *in a general way* exploiting the universality of the soft and collinear emission `[Frixione,Kunszt,Signer('96) (FKS), Catani,Seymour('97) (CS)]`. It allows (relatively) straightforward calculations, **once the QCD amplitudes are available**

$$\sigma^{NLO} = \int_{m+1} d\sigma^R(\epsilon) \; + \; \int_m d\sigma^V(\epsilon)$$

$$= \int_{m+1}\left[d\sigma^R(\epsilon) - d\sigma^A(\epsilon)\right]_{\epsilon=0} \; + \; \int_m \left[d\sigma^V(\epsilon) + \int_1 d\sigma^A(\epsilon)\right]_{\epsilon=0}$$

Higher orders: NLO

- Calculations at LO give the order of magnitude of cross sections and distributions, NLO corrections provide reliable estimate
- Experiments have finite acceptance important to provide exclusive theoretical predictions.
- At NLO infrared singularities in *real* and *virtual* corrections prevent the straightforward implementation of Monte Carlo numerical techniques (especially for fully exclusive quantities).
- NLO subtraction method: introduction of auxiliary QCD cross section *in a general way* exploiting the universality of the soft and collinear emission [Frixione,Kunszt,Signer('96) (FKS), Catani,Seymour('97) (CS)]. It allows (relatively) straightforward calculations, **once the QCD amplitudes are available**

$$\sigma^{NLO} = \int_{m+1} d\sigma^R(\epsilon) \ + \int_m d\sigma^V(\epsilon)$$

$$= \int_{m+1} \Big[d\sigma^R(\epsilon) - d\sigma^A(\epsilon) \Big]_{\epsilon=0} + \int_m \Big[d\sigma^V(\epsilon) + \int_1 d\sigma^A(\epsilon) \Big]_{\epsilon=0}$$

Higher orders: NLO

- Calculations at LO give the order of magnitude of cross sections and distributions, NLO corrections provide reliable estimate
- Experiments have finite acceptance important to provide exclusive theoretical predictions.
- At NLO infrared singularities in *real* and *virtual* corrections prevent the straightforward implementation of Monte Carlo numerical techniques (especially for fully exclusive quantities).
- NLO subtraction method: introduction of auxiliary QCD cross section *in a general way* exploiting the universality of the soft and collinear emission `[Frixione,Kunszt,Signer('96) (FKS), Catani,Seymour('97) (CS)]`. It allows (relatively) straightforward calculations, **once the QCD amplitudes are available**

$$\sigma^{NLO} = \int_{m+1} d\sigma^{R}(\epsilon) \; + \; \int_{m} d\sigma^{V}(\epsilon)$$

$$= \int_{m+1} \Big[d\sigma^{R}(\epsilon) - d\sigma^{A}(\epsilon) \Big]_{\epsilon=0} \; + \; \int_{m} \Big[d\sigma^{V}(\epsilon) + \int_{1} d\sigma^{A}(\epsilon) \Big]_{\epsilon=0}$$

Higher orders: NLO

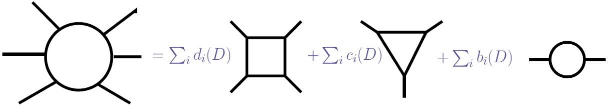
- Calculations at LO give the order of magnitude of cross sections and distributions, NLO corrections provide reliable estimate
- Experiments have finite acceptance important to provide exclusive theoretical predictions.
- At NLO infrared singularities in *real* and *virtual* corrections prevent the straightforward implementation of Monte Carlo numerical techniques (especially for fully exclusive quantities).
- NLO subtraction method: introduction of auxiliary QCD cross section *in a general way* exploiting the universality of the soft and collinear emission `[Frixione,Kunszt,Signer('96) (FKS), Catani,Seymour('97) (CS)].` It allows (relatively) straightforward calculations, **once the QCD amplitudes are available**

$$\sigma^{NLO} = \int_{m+1} d\sigma^R(\epsilon) \;+\; \int_m d\sigma^V(\epsilon)$$

$$= \int_{m+1} \left[d\sigma^R(\epsilon) - d\sigma^A(\epsilon) \right]_{\epsilon=0} \;+\; \int_m \left[d\sigma^V(\epsilon) + \int_1 d\sigma^A(\epsilon) \right]_{\epsilon=0}$$

NLO: virtual amplitudes

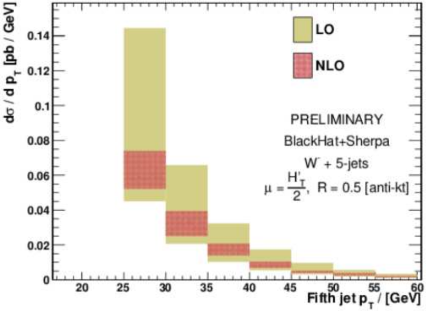
- The paradigm for the calculation of one-loop diagram is [Passarino,Veltman('79)].
- Any one-loop amplitude can be written as a linear sum of scalar box-, triangle-, bubble- and tadpole-integrals.



- Analytic results for these scalar integrals are known [Ellis,Zanderighi('08)].
- The traditional approach is not adequate when the number of external legs increase 2 → 3, 4, 5, ... (factorial growth of diagrams).
- Recently advances in multi-leg one-loop amplitudes calculations thanks to
  - New semi-numerical methods based on on-shell recursion relations and unitarity: isolate coefficients by cutting propagators [Bern,Dixon,Dunbar, Kosower('94)],[Britto,Cachazo,Feng('04)].
  - Tensor integrals to scalar master integrals reduction performed numerically at the integrand level in a algorithmic way [Ossola,Papadopoulos, Pittau('06)].

NLO: automation

- NLO calculations are now highly automated. Virtual corrections can be combined with real corrections (based on CS or FKS subtraction formalism).
    - HELAC-NLO [Bevilacqua,Czakon,Garzelli,van Hameren,Kardos,Papadopoulos, Pittau,Worek].
    - BlackHat+Sherpa [Berger,Bern,Dixon,Cordero,Forde,Gleisberg,Ita, Kosower,Maitre].
    - MadLoop+MadFKS [Hirschi,Frederix, Frixione,Garzelli,Maltoni,Pittau].
    - Rocket [Ellis,Giele,Kunszt, Melnikov,Zanderighi].
    - GoSam [Cullen,Greiner,Heinrich, Luisoni,Mastrolia,Ossola,Reiter, Tramontano].
    - OpenLoops [Cascioli,Maierhöfer, Pozzorini].
- NLO dedicated calculations also available: MCFM, VBFNLO,NLOJet++,. . . .



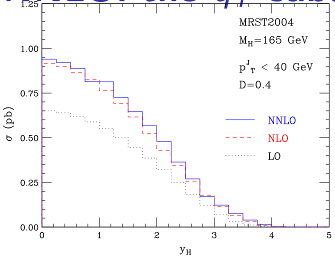
Higher orders: NNLO

- NNLO computation very cumbersome until few years ago results were known only for few highly-inclusive reactions. E.g. anomalous dimensions: LO 18 diagrams (1977), NLO 350 diagrams (1980), NNLO 9607 diagrams (2004).
- NNLO corrections important to have a good control of theoretical uncertainties especially:
  (i) When NLO corrections are large (e.g. Higgs production in gluon fusion).
  (ii) For benchmark process measured with high precision (e.g. DY).
- At NNLO in hadronic collisions only few fully exclusive calculations exist:
  - Sector decomposition: `[Binoth,Heinrich('00)]`
    $gg \to H$ `[Anastasiou,Melnikov,Petriello('04)]`→FEHIP
    Drell-Yan `[Melnikov,Petriello('06)]`→FEWZ
  - q_T-subtraction:
    $gg \to H$ `[Catani,Grazzini('07)]`→HNNLO
    Drell-Yan `[Catani,Cieri,de Florian,G.F.,Grazzini('09)]`→DYNNLO
    Associated WH production `[G.F.,Grazzini,Tramontano('11)]`→ WNNLO
    Diphoton prod. `[Catani,Cieri,de Florian,G.F.,Grazzini('11)]`→2γNNLO

Higher orders: NNLO

- NNLO computation very cumbersome until few years ago results were known only for few highly-inclusive reactions. E.g. anomalous dimensions: LO 18 diagrams (1977), NLO 350 diagrams (1980), NNLO 9607 diagrams (2004).
- NNLO corrections important to have a good control of theoretical uncertainties especially:
  (i) When NLO corrections are large (e.g. Higgs production in gluon fusion).
  (ii) For benchmark process measured with high precision (e.g. DY).
- At NNLO in hadronic collisions only few fully exclusive calculations exist:
  - Sector decomposition: [Binoth,Heinrich('00)]
    $gg \to H$ [Anastasiou,Melnikov,Petriello('04)]→FEHIP
    Drell-Yan [Melnikov,Petriello('06)]→FEWZ
  - q_T-subtraction:
    $gg \to H$ [Catani,Grazzini('07)]→HNNLO
    Drell-Yan [Catani,Cieri,de Florian,G.F.,Grazzini('09)]→DYNNLO
    Associated WH production [G.F.,Grazzini,Tramontano('11)]→ WNNLO
    Diphoton prod. [Catani,Cieri,de Florian,G.F.,Grazzini('11)]→2γNNLO

NNLO: the q_T-subtraction formalism

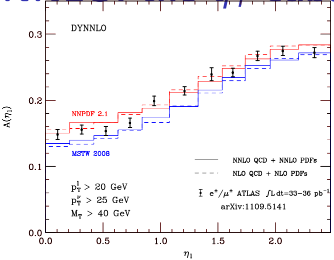


Rapidity distribution of the Higgs boson with $M_H = 165$ GeV. Final-state jets are required to have transverse momentum smaller than 40 GeV.

NNLO extension of the subtraction formalism valid for hadroproduction of colourless high-mass system `[Catani, Grazzini('07)]`: fully exclusive calculations implemented in (parton level) Monte Carlo numerical codes:

- Higgs prod. in gluon fusion: Main Higgs production mechanism at the LHC `[Catani,Grazzini('07)]`.
- Vector boson prod. (DY): Most "classical" process in hadron collisions (constrain for PDFs fits, measure of M_W, beyond the SM analysis) `[Catani,Cieri,de Florian,G.F.,Grazzini('09)]`.
- WH prod.: Important LHC channel through boosted analysis, direct information on Higgs-fermions coupling `[G.F.,Grazzini, Tramontano('11)]`.
- Diphoton prod.: Main irreducible background of $H \to \gamma\gamma$ `[Catani,Cieri,de Florian,G.F., Grazzini('11)]`.

NNLO: the q_T-subtraction formalism

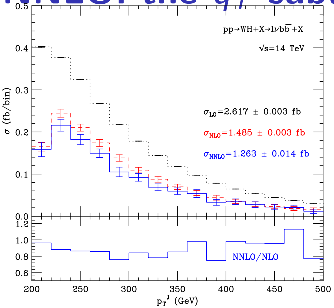


Lepton charge asymmetry from W decay in NLO and NNLO QCD with MSTW08 and NNPDF2.1 PDFs, compared with ATLAS data.

NNLO extension of the subtraction formalism valid for hadroproduction of colourless high-mass system `[Catani, Grazzini('07)]`: fully exclusive calculations implemented in (parton level) Monte Carlo numerical codes:

- Higgs prod. in gluon fusion: Main Higgs production mechanism at the LHC `[Catani,Grazzini('07)]`.
- Vector boson prod. (DY): Most "classical" process in hadron collisions (constrain for PDFs fits, measure of M_W, beyond the SM analysis) `[Catani,Cieri,de Florian,G.F.,Grazzini('09)]`.
- WH prod.: Important LHC channel through boosted analysis, direct information on Higgs-fermions coupling `[G.F.,Grazzini, Tramontano('11)]`.
- Diphoton prod.: Main irreducible background of $H \to \gamma\gamma$ `[Catani,Cieri,de Florian,G.F., Grazzini('11)]`.

NNLO: the q_T-subtraction formalism

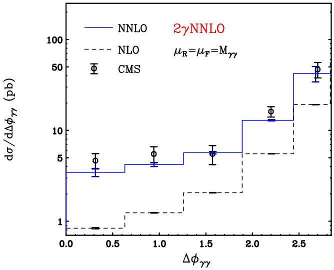


p_T spectra of the fat jet at the LHC@14TeV for $m_H = 120 GeV$ at LO (dots), NLO (dashes) and NNLO (solid).

NNLO extension of the subtraction formalism valid for hadroproduction of colourless high-mass system [Catani, Grazzini('07)]: fully exclusive calculations implemented in (parton level) Monte Carlo numerical codes:

- Higgs prod. in gluon fusion: Main Higgs production mechanism at the LHC [Catani,Grazzini('07)].
- Vector boson prod. (DY): Most "classical" process in hadron collisions (constrain for PDFs fits, measure of M_W, beyond the SM analysis) [Catani,Cieri,de Florian,G.F.,Grazzini('09)].
- WH prod.: Important LHC channel through boosted analysis, direct information on Higgs-fermions coupling [G.F.,Grazzini, Tramontano('11)].
- Diphoton prod.: Main irreducible background of $H \to \gamma\gamma$ [Catani,Cieri,de Florian,G.F., Grazzini('11)].

NNLO: the q_T-subtraction formalism



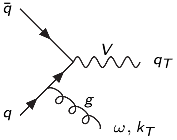
Azimuthal angle $\Delta\phi_{\gamma\gamma}$ spectrum measured by CMS compared with NLO and NNLO QCD corrections (CMS cuts but smooth cone isolation).

NNLO extension of the subtraction formalism valid for hadroproduction of colourless high-mass system [Catani, Grazzini('07)]: fully exclusive calculations implemented in (parton level) Monte Carlo numerical codes:

- Higgs prod. in gluon fusion: Main Higgs production mechanism at the LHC [Catani,Grazzini('07)].
- Vector boson prod. (DY): Most "classical" process in hadron collisions (constrain for PDFs fits, measure of M_W, beyond the SM analysis) [Catani,Cieri,de Florian,G.F.,Grazzini('09)].
- WH prod.: Important LHC channel through boosted analysis, direct information on Higgs-fermions coupling [G.F.,Grazzini, Tramontano('11)].
- Diphoton prod.: Main irreducible background of $H \to \gamma\gamma$ [Catani,Cieri,de Florian,G.F., Grazzini('11)].

Sudakov resummation

An example: q_T-resummation



Drell-Yan production at small transverse-momentum: $q_T \ll M_V$.
The standard fixed-order QCD perturbative expansions gives:

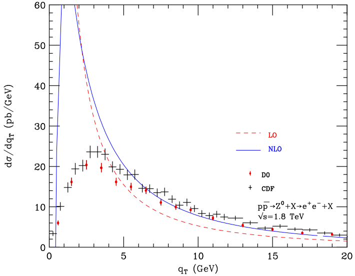
$$\int_0^{Q_T^2} dq_T^2 \frac{d\hat{\sigma}_{q\bar{q}}}{dq_T^2} \;\sim\; 1 + \alpha_S \left[c_{12} \log^2(M^2/Q_T^2) + c_{11} \log(M^2/Q_T^2) + c_{10}(Q_T) \right]$$

$$+\alpha_S^2 \left[c_{24} \log^4(M^2/Q_T^2) + \cdots + c_{21} \log(M^2/Q_T^2) + c_{20}(Q_T) \right] + \mathcal{O}(\alpha_S^3)$$

The logs are the residue of the cancellation of the real-virtual infrared singularities due to soft/collinear gluon emissions (recoiling radiation is forced to be soft/collinear).

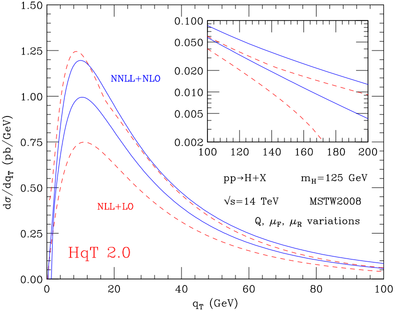
Fixed order calculation reliable only for $q_T \sim M_V$

For $q_T \to 0$, $\alpha_S^n \log^m(M^2/q_T^2) \gg 1$: need for resummation of logarithmic corrections.



DY q_T spectrum at the Tevatron.

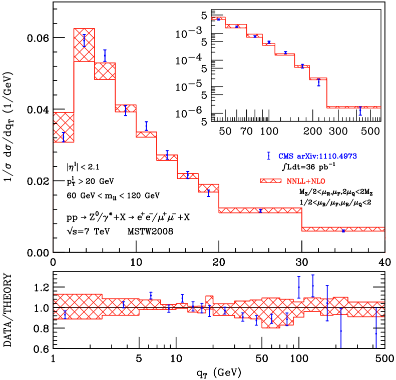
Resummed results



Higgs q_T spectrum for $m_H = 125\ GeV$ at LHC. (from HqT code [Bozzi, Catani, de Florian, G.F., Grazzini, Tommasini('06,'11)]).

- Transverse-momentum (small q_T) resummation: resum to all order large logarithmic corrections of the form $log(q_T/M)$ when $q_T \ll M$. [Parisi, Petronzio('79)], [Collins, Soper, Sterman('85)] After resummation perturbative results became predictive also for small q_T.
- Threshold resummation: resum to all order large logarithmic corrections of the form $log(1 - M^2/\hat{s})$ when $M^2/\hat{s} \to 1$ (large invariant mass limit). [Sterman('87)], [Catani, Trentadue('89)] Smaller quantitative effect of resummation, still reduction of perturbative uncertainty.

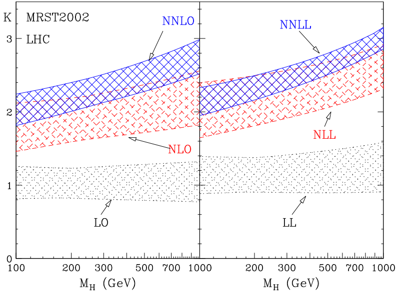
Resummed results



CMS data for the Z q_T spectrum compared with NNLL+NLO result. (from `[Catani,de Florian,G.F., Grazzini('13)]`).

- **Transverse-momentum (small q_T) resummation**: resum to all order large logarithmic corrections of the form $log(q_T/M)$ when $q_T \ll M$. `[Parisi,Petronzio('79)]`, `[Collins,Soper,Sterman('85)]` After resummation perturbative results became predictive also for small q_T.
- Threshold resummation: resum to all order large logarithmic corrections of the form $log(1-M^2/\hat{s})$ when $M^2/\hat{s} \to 1$ (large invariant mass limit). `[Sterman('87)]`, `[Catani,Trentadue('89)]` Smaller quantitative effect of resummation, still reduction of perturbative uncertainty.

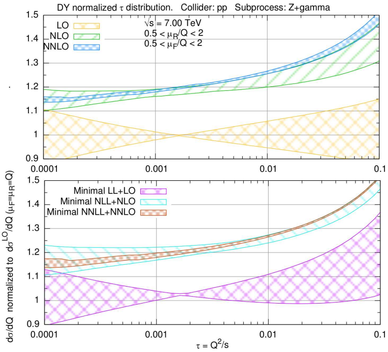
Resummed results



Fixed-order and resummed K-factors for Higgs production at the LHC. (from [Catani,de Florian,Grazzini,Nason ('03,'09,'12)]).

- Transverse-momentum (small q_T) resummation: resum to all order large logarithmic corrections of the form $log(q_T/M)$ when $q_T \ll M$. [Parisi,Petronzio('79)], [Collins,Soper,Sterman('85)] After resummation perturbative results became predictive also for small q_T.
- Threshold resummation: resum to all order large logarithmic corrections of the form $log(1 - M^2/\hat{s})$ when $M^2/\hat{s} \to 1$ (large invariant mass limit). [Sterman('87)], [Catani,Trentadue('89)] Smaller quantitative effect of resummation, still reduction of perturbative uncertainty.

Resummed results



Invariant mass distribution of neutral DY pairs at the LHC ($\sqrt{s} = 7\ TeV$), perturbative uncertainty band for fixed order and resummed results (from `[Bonvini,Forte,Ridolfi('10)]`).

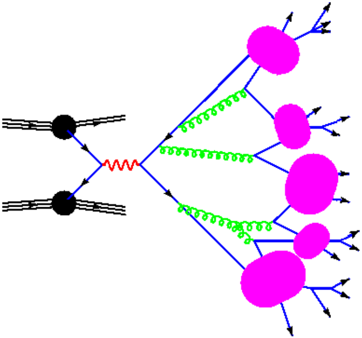
- Transverse-momentum (small q_T) resummation: resum to all order large logarithmic corrections of the form $log(q_T/M)$ when $q_T \ll M$. `[Parisi,Petronzio('79)]`, `[Collins,Soper,Sterman('85)]` After resummation perturbative results became predictive also for small q_T.
- Threshold resummation: resum to all order large logarithmic corrections of the form $log(1 - M^2/\hat{s})$ when $M^2/\hat{s} \to 1$ (large invariant mass limit). `[Sterman('87)]`, `[Catani,Trentadue('89)]` Smaller quantitative effect of resummation, still reduction of perturbative uncertainty.

Shower Monte Carlo

Shower Monte Carlo

Complete description of a hadron scattering event.

- QCD parton shower: Starting from LO QCD, inclusion of dominant collinear and soft-gluon emissions (by angular ordering thanks to color coherence) to all order in a approximate way as a Markov process (probabilistic picture).
- No analytic solution but simple iterative structure of coherent parton branching.
- QCD accuracy analogous to LL (plus part of NLL) Sudakov resummation.
- Implemented in numerical Monte Carlo programs.



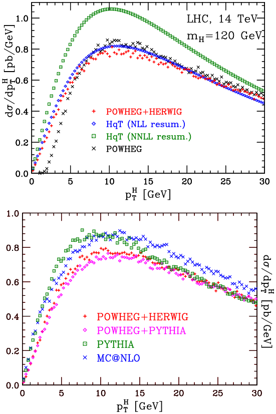
Scheme of QCD Parton Shower and hadronization from final states.

- QCD parton cascade matched with hadronization model for conversion of partons into hadrons (and model for resonance decay) ⇒ QCD event generators (Herwig/PYTHIA/Sherpa).
- Possible to consistently combine Parton Shower with high multiplicity tree-level matrix elements: CKKW/MLM matching.

Parton Shower+NLO

State of the art for QCD event generators: methods for combining Shower MC with perturbative calculation at NLO (avoiding double counting).
Two general method exist:

- MC@NLO [Frixione,Webber('02)]: method which implement in a given Shower Monte Carlo the NLO accuracy for total cross section. Hard emission exactly included at NLO, soft/collinear emission included to all order by the Shower Monte Carlo.
- POWHEG [Nason('04)]: method which realize a NLO+PS (with no negative weights) by a modification of existing shower.
- Present directions PS+NLO towards automation: aMC@NLO/POWHEGBox (automation for matrix elements and NLO matching).



Comparison of Higgs p_T from analytic resummation and PS+NLO.

Conclusions

- This talk is an overview on some selected topic in pQCD: PDFs and α_S, NLO and NNLO QCD computations, all order resummation, Shower Monte Carlo.
- Important omissions: Jets (see `[Cacciari,Salam]` reviews), Flavour Physics (see Heavy Flavour session), QCD infrared structure to all order, Top/W/Higgs physics, effective theories...
- Main message: pQCD is always involved in the description of the LHC processes.
  To fully exploit the information contained in the LHC data precise pQCD predictions are needed.