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The role of transverse momenta and spins in QCD at high energies

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ABSTRACT

The role of transverse momenta and spins in QCD at high energies

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The standard Parton Distribution Functions (PDFs) used to describe high-energy scattering processes encode probabilities for finding specific quarks and gluons (partons) carrying fractions x of the parent hadron's momentum (soft collinear part). The interactions of the partons (hard part) are calculated using perturbation theory in the Standard Model. The Transverse Momentum Dependent (TMD) distribution functions also take into account the transverse momentum of the partons. Is this dependence a useful addition? Can it be measured? Can the formalism be set up and used in the same successful way as collinear PDFs, which are related to expectation values of field operators using the Operator Product Expansion in QCD? The answer is yes, but … . But after accounting for the complications, quark and gluon TMDs offer new insight into spin and orbital substructure of hadrons while they also may provide new tools to explore physics beyond the Standard Model.

Content

- High energies: soft x hard
  - Soft = hadron info (probabilities), hard = partonic cross section
  - Probabilities include spin-spin correlations
- Are TMDs relevant and can they be measured?
  - Yes, there are more spin-spin and also spin-orbit correlations
  - Yes, they can be measured (DY, SIDIS, …)
- But there are complications!
  - Gauge links, universality, factorization
- Theoretical framework: QCD
  - Extension of OPE resummed into PDFs to TMDs (definite rank)
  - Distribution and fragmentation functions (time reversal)
- The reward
  - Novel hadronic info on spin and orbital structure
  - Possible use of proton as tool (playing with partons)

Separating Soft and Hard Physics at high energies

High energy processes

- Collinear approach:

  The common procedure is to relate the hard partonic amplitude squared to the imaginary part of the forward elastic amplitude and expand this time ordered product, which is lightcone dominated into local operators (operator product expansion), of which only leading twist operators are needed at high energies.

- Hadronic correlators: beyond the collinear approach

  Using hadronic correlators is at first sight more heuristic, but is easily seen to incorporate all features of the collinear approach.

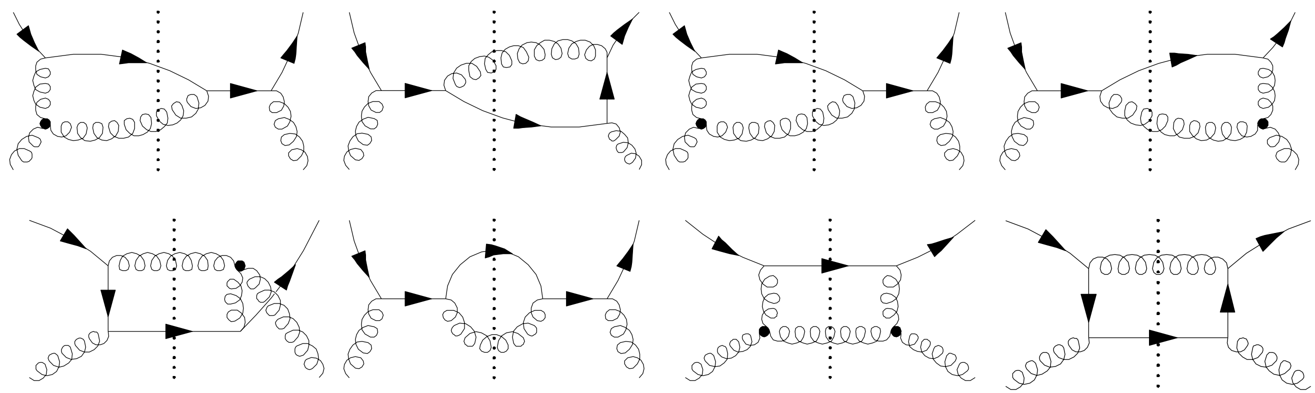
- Parton model for distribution and fragmentation functions:

  Heuristic approach with parton probabilities and decay functions with natural link to hadronic correlators

QCD & Standard Model

- QCD framework (including electroweak theory) provides the machinery to calculate cross sections, e.g. $\gamma^* q \to q$, $q\bar{q} \to \gamma^*$, $\gamma^* \to q\bar{q}$, $qq \to qq$, $qg \to qg$, etc.
- E.g.

  $qg \to qg$



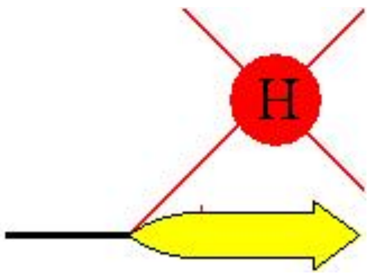
- Calculations work for plane waves

$$\langle 0 | \psi_i^{(s)}(\xi) | p, s \rangle = u_i(p,s)\, e^{-ip.\xi}$$

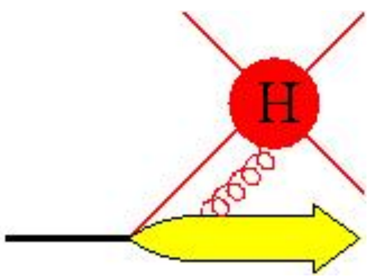
Hadron correlators

- Hadronic correlators establish the diagrammatic link between hadrons and partonic hard scattering amplitude
- Quark, quark + gluon, gluon, …

$$\left\langle 0 \left| \psi_i(\xi) \right| p,s \right\rangle = u_i(p,s)\, e^{-ip.\xi}$$

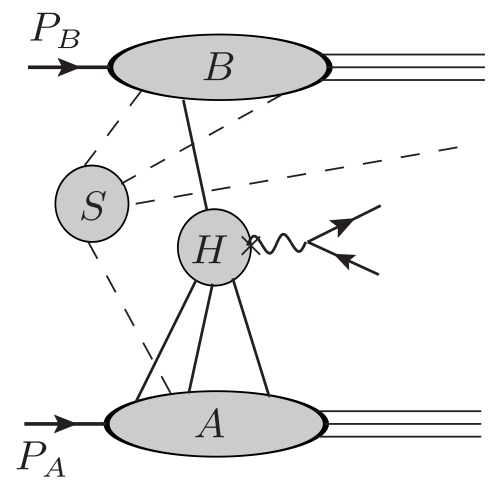


$$\left\langle X \left| \psi_i(\xi) \right| P \right\rangle e^{+ip.\xi}$$



$$\left\langle X \left| \psi_i(\xi) A^\mu(\eta) \right| P \right\rangle e^{+i(p-p_1).\xi + ip_1.\eta}$$

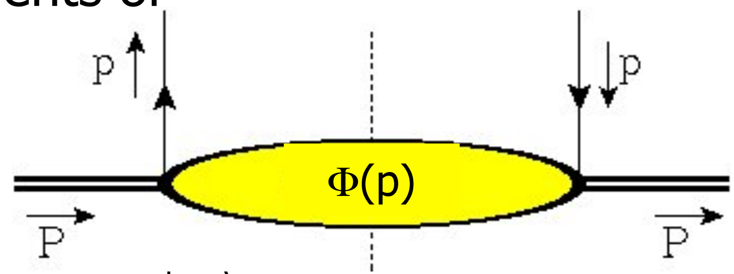
- Disentangling a hard process into parts involving hadrons, hard scattering amplitude and soft part is non-trivial



J.C. Collins, Foundations of Perturbative QCD, Cambridge Univ. Press 2011

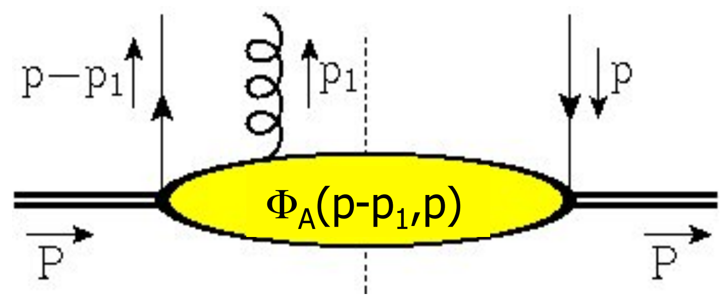
Hadron correlators

- At high energies interference terms suppressed and the soft parts combine into forward matrix elements of parton fields describing distribution (and fragmentation) parts



$$\Phi_{ij}(p;P) = \Phi_{ij}(p\,|\,p) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\,p.\xi} \left\langle P \left| \overline{\psi}_j(0)\,\psi_i(\xi) \right| P \right\rangle$$

- Also needed are multi-parton correlators



$$\Phi^{\alpha}_{A;ij}(p-p_1,p_1\,|\,p) = \int \frac{d^4\xi\, d^4\eta}{(2\pi)^8}\, e^{i(p-p_1).\xi + ip_1.\eta} \left\langle P \left| \overline{\psi}_j(0)\,A^{\alpha}(\eta)\,\psi_i(\xi) \right| P \right\rangle$$

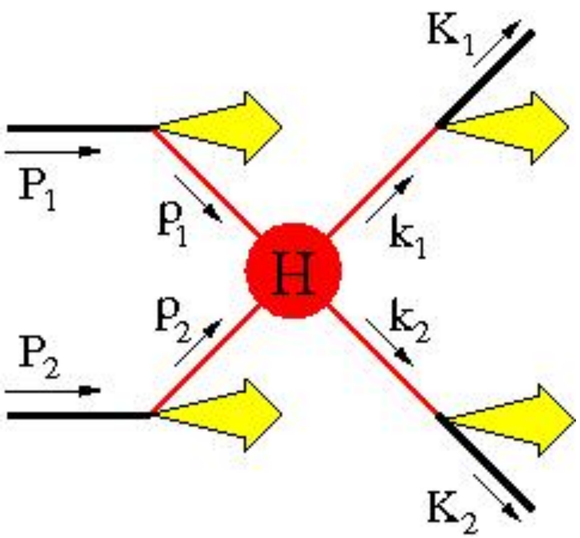
- Correlators usually just will be parametrized in terms of PDFs (nonperturbative physics)

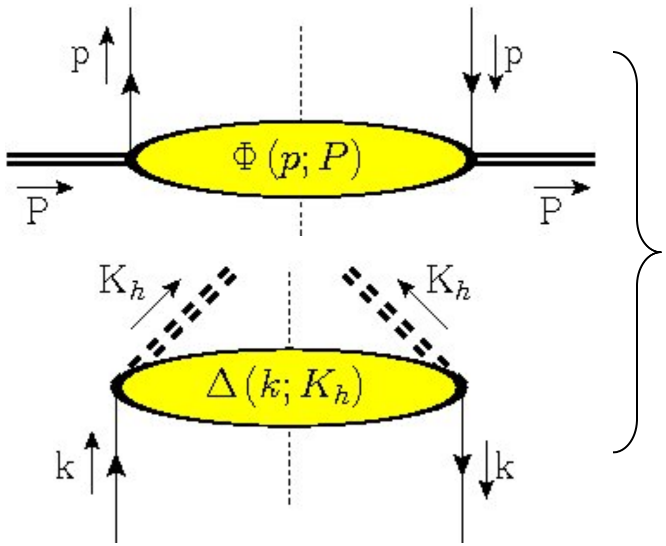
PDFs and PFFs

Basic idea of PDFs and PFFs is to get a full factorized description of high energy scattering processes

$$\hat{\sigma} = |\,H(p_1, p_2, \ldots)\,|^2$$

calculable





defined (!)
&
portable

$$\sigma(P_1, P_2, \ldots) = \iiint \ldots dp_1 \ldots \Phi_a(p_1, P_1; \mu) \otimes \Phi_b(p_2, P_2; \mu)$$

$$\otimes\, \hat{\sigma}_{ab,c\ldots}(p_1, p_2, \ldots; \mu) \otimes \Delta_c(k_1, K_1; \mu) \ldots .$$

Give a meaning to integration variables!

Hadron correlators

- Parton scattering

$$u(p,s)\overline{u}(p,s) = \not{p} + m$$

- Parton model: $p = x P^{\mu} + \ldots.$

$$u(p,s)\overline{u}(p,s) \Rightarrow f(x)\,(\not{p} + m)$$

$$= f(x)\; x\not{P} + m\; f(x) + \ldots$$

- Correlators:

$$\Phi(p,P) = x f(x)\,\not{P} + Mx\; e(x) + \ldots.$$

Role of the hard scale

- In high-energy processes other momenta are available, such that P.P′ ~ s with a hard scale s >> M^2
- Additional scale accessible through non-collinearities, e.g. in SIDIS γ^*+p is not aligned with produced hadron, or momenta inside a jet
- Employ light-like vectors P and n, such that P.n = 1 (e.g. n = P′/P.P′) to make a Sudakov expansion of parton momentum (write s = Q^2)

$$p = \underbrace{x P^\mu}_{\sim Q} + \underbrace{p_T^\mu}_{\sim M} + \underbrace{\sigma n^\mu}_{\sim M^2/Q}$$

$$x = p^+ = p.n \sim 1$$

$$\sigma = p.P - xM^2 \sim M^2$$

- Enables importance sampling (twist analysis) for integrated correlators,

$$\Phi(p) = \Phi(x, p_T, p.P) \;\Rightarrow\; \Phi(x, p_T) \;\Rightarrow\; \Phi(x) \;\Rightarrow\; \Phi$$

Twist analysis in PDF parametrization

- Dimensional analysis to determine importance in an expansion in inverse hard scale (smaller dimensions preferred)
- Maximize contractions with n

$$\dim[\bar{\psi}(0)\not{n}\,\psi(\xi)] = 2$$

$$\dim[F^{n\alpha}(0)F^{n\beta}(\xi)] = 2$$

$$\dim[\bar{\psi}(0)\not{n}\,A_T^{\alpha}(\eta)\,\psi(\xi)] = 3$$

- ... or maximize # of P's in parametrization of Φ

$$\boxed{\Phi_{ij}^{q}(x) = f_1^{q}(x)\frac{(\not{P})_{ij}}{2} \quad \Leftrightarrow \quad f_1^{q}(x) = \int \frac{d\lambda}{(2\pi)} e^{ix\lambda} \left\langle P \middle| \bar{\psi}(0)\not{n}\,\psi(\lambda n) \middle| P \right\rangle}$$

- In addition any number of collinear n.A(ξ) = $A^n(x)$ fields (dimension zero!), but of course in color gauge invariant combinations

dim 0: $i\partial^{n} \longrightarrow iD^{n} = i\partial^{n} + gA^{n}$

dim 1: $i\partial_T^{\alpha} \longrightarrow iD_T^{\alpha} = i\partial_T^{\alpha} + gA_T^{\alpha}$

(Un)integrated correlators

$$\Phi(x, p_T, p.P) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\,p.\xi} \left\langle P \middle| \overline{\psi}(0)\, \psi(\xi) \middle| P \right\rangle$$

- unintegrated

$$\Phi(x, p_T; n) = \int \frac{d(\xi.P)\, d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi} \left\langle P \middle| \overline{\psi}(0)\, \psi(\xi) \middle| P \right\rangle_{\boxed{\xi.n=0}}$$

- TMD (light-front)

- Time-ordering automatic, allowing interpretation as forward anti-parton – target scattering amplitude
- Involves operators of twists starting at a lowest value (which is usually called the 'twist' of a TMD)

$$\Phi(x) = \int \frac{d(\xi.P)}{(2\pi)}\, e^{i\,p.\xi} \left\langle P \middle| \overline{\psi}(0)\, \psi(\xi) \middle| P \right\rangle_{\boxed{\xi.n=\xi_T=0}\ or\ \boxed{\xi^2=0}}$$

- collinear (light-cone)

- Involves operators of a definite twist. Evolution via splitting functions (moments are anomalous dimensions)

$$\Phi = \left\langle P \middle| \overline{\psi}(0)\, \psi(\xi) \middle| P \right\rangle_{\boxed{\xi=0}}$$

- local

- Local operators with calculable anomalous dimension

relevance and measurability of TMDs

Access to transverse momenta: f(x) → f(x,p_T)

- SIDIS: $\gamma^* + H(P) \rightarrow h(K) + X$

  Underlying hard process: $\gamma^* + q(p) \rightarrow q(k)$

- Include transverse components in quark momenta

$$p \approx xP + p_T$$
$$k \approx z^{-1}K + k_T$$

- Sufficiently high energies to identify fractions x and z:

$$x = p.n / P.n = Q^2 / 2P.q = x_B$$
$$z = K_h.n / k.n = P.K_h / P.q = z_h$$

up to $1/Q^2$ corrections!

- Momentum conservation p + q = k tells us that transverse momentum can be accessed [via transverse momentum $K_{h\perp(P,q)}$]

$$\boxed{q_T = q + x_B P - z_h^{-1} K_h = k_T - p_T}$$

Second scale!

Access to transverse momenta: f(x) → f(x,p_T)

$$p_1 \approx x_1 P_1 + p_{1T}$$
$$p_2 \approx x_2 P_2 + p_{2T}$$

- Also in hadroproduction at high energies fractional parton momenta are fixed by external kinematics up to M^2/Q^2

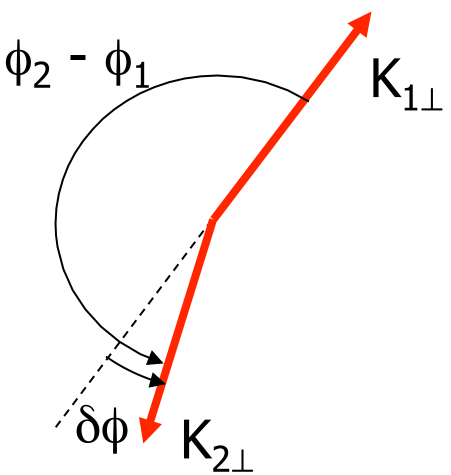
$$x_1 = p_1.n = \frac{p_1.P_2}{P_1.P_2} = \frac{q.P_2}{P_1.P_2}$$

- Measure for mismatch for transverse momenta of partons

DY: $$\boxed{q_T = q - x_1 P_1 - x_2 P_2 = p_{1T} + p_{2T}}$$

2-particle inclusive hadron-hadron scattering:

$$\boxed{\begin{aligned} q_T &= z_1^{-1} K_1 + z_2^{-1} K_2 - x_1 P_1 - x_2 P_2 \\ &= p_{1T} + p_{2T} - k_{1T} - k_{2T} \end{aligned}}$$



pp-scattering

Boer & Vogelsang

It shows that care is needed: we need more than one hadron and knowledge of hard process(es)!

Second scale!

New information in TMD's: $f(x,p_T)$

- Quarks in polarized nucleon: $S = S_L\left(\frac{P}{M} + Mn\right) + S_T$ $\qquad S_L^2 + S_T^2 = -1$

$$\Phi^q(p;P,S) \propto xf_1^q(x,p_T^2)\,\not{P} + S_L x g_{1L}^q(x,p_T^2)\,\not{P}\gamma_5$$
$$+xh_{1T}^q(x,p_T^2)\not{S}_T\not{P}\gamma_5 \quad + \quad \ldots$$

unpolarized quarks

chiral quarks in L-polarized N

T-polarized quarks in T-polarized N

compare

$$u(p,s)\bar{u}(p,s) = \tfrac{1}{2}(\not{p}+m)(1+\gamma_5\not{s})$$

- … but also

$$\Phi(p;P,S) \propto \ldots + \frac{(p_T\cdot S_T)}{M} x g_{1T}^q(x,p_T^2)\not{P}\gamma_5 \quad + \quad \ldots$$

spin ←→ spin

chiral quarks in T-polarized N

New information in TMD's: f(x,p_T)

- … and T-odd functions

$$\Phi^q(p;P,S) \propto \ldots + i h_1^{\perp q}(x,p_T^2)\frac{\not p_T}{M}\not P + i\frac{(p_T \times S_T)}{M} x f_{1T}^{\perp q}(x,p_T^2)\not P + \ldots$$

T-polarized quarks in unpolarized N (Boer-Mulders)

unpolarized quarks in T-polarized N (Sivers)

spin ←→ orbit

compare

$$u(p,s)\bar{u}(p,s) = \tfrac{1}{2}(\not p + m)(1+\gamma_5 \not s)$$

- Note that there are parts that lack simple partonic interpretation

$$\Phi(p;P,S) \propto \ldots + Mx\; e^q(x,p_T^2) + \ldots$$

Higher-twist

parton mass? But these are linked to quark-gluon correlators via EQM

Fermionic structure of TMDs

unpolarized quark distribution

with p_T

T-odd

helicity or chirality distribution

with p_T

$f_1^q(x) = q(x)$

$g_1^q(x) = \Delta q(x)$

$h_1^q(x) = \delta q(x)$

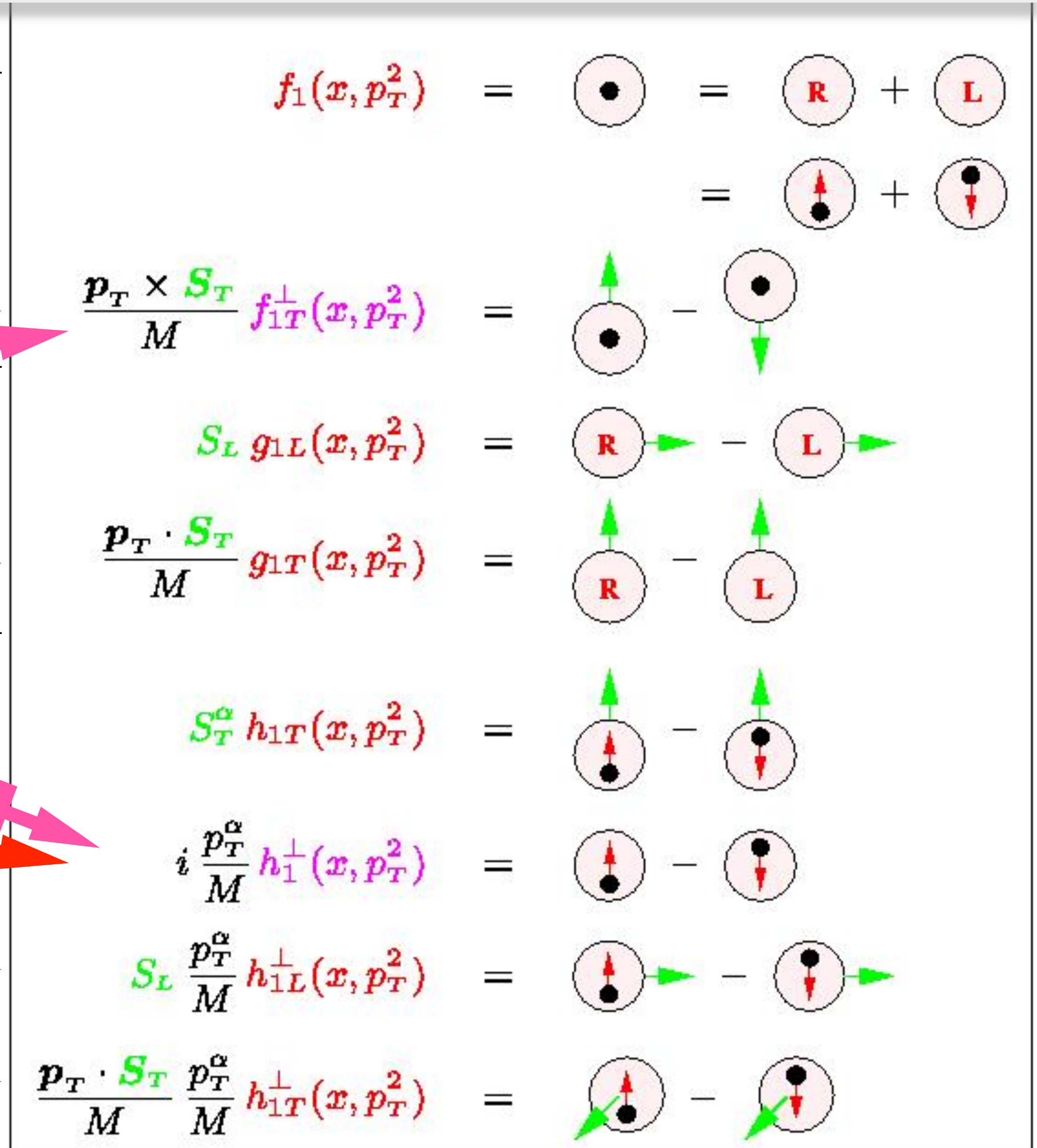
T-odd

with p_T

transverse spin distr. or transversity

with p_T

with p_T



New information in TMD's: $f(x,p_T)$

- Also for gluons there are new features in TMD's

circularly polarized gluons in L-pol. N

spin ←→ spin

$$\Phi^{g\ \mu\nu}(p;P,S) \propto -g_T^{\mu\nu}\, x f_1^g(x,p_T^2) + iS_L \varepsilon_T^{\mu\nu}\, x g_{1L}^g(x,p_T^2)$$

$$+ \left(\frac{p_T^\mu p_T^\nu}{M^2} - g_T^{\mu\nu}\frac{p_T^\mu}{2M^2}\right) x h_1^{\perp g}(x,p_T^2) \quad + \quad \ldots$$

Unpolarized gluons in unpol. N quarks

linearly polarized gluons in unpol. N (Gluon Boer-Mulders)

compare

$$\varepsilon^\mu(p,\lambda)\varepsilon^{\nu *}(p,\lambda) = -g_T^{\mu\nu} + \ldots$$

spin ←→ orbit

Time reversal invariance

- TMD-correlators are not T-invariant (allowing specific spin-orbit correlations)
- QCD is T-invariant
- T-odd observables ←→ T-odd TMDs
- Example of T-odd observable: single spin asymmetry

  E.g. left-right asymmetry in $p(P_1)\,p_\uparrow(P_2) \to \pi(K)X$
- Collinear hard T-odd contribution zero ($\sim\alpha_s^2$, $\alpha_s m_q$), p_T-contributions remain

$$p \approx xP + p_T$$
$$k \approx z^{-1}P + k_T$$

$$\varepsilon^{P_1 P_2 S_{2T} K_\pi} \approx \frac{z_\pi}{x_1 x_2}\Big(\cancel{\varepsilon^{p_1 p_2 S_{2T} k}} - \varepsilon^{p_{1T} p_2 S_{2T} k} - \varepsilon^{p_1 p_{2T} S_{2T} k} + \varepsilon^{p_1 p_2 S_{2T} k_T}\Big)$$

$\sim s^{3/2}$

$\sim s$

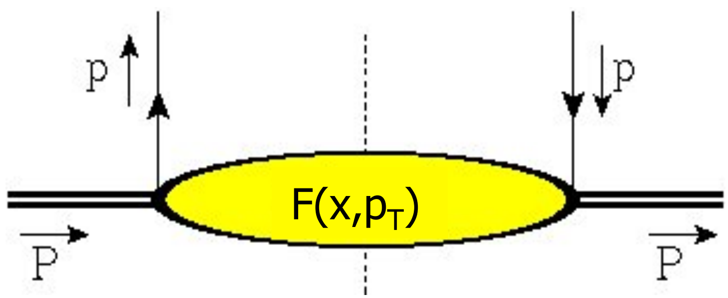
… + ‘normal’ twist three stuff (FF)

Qiu & Sterman, 1997

Complications for TMDs

Large p_T

- p_T-dependence of TMDs



$$\int^{\mu} d^2 p_T \, \Phi(x, p_T) = \Phi(x; \mu^2)$$

Fictitious measurement

Large μ^2 dependence governed by anomalous dim (i.e. splitting functions)

- $$\Phi(x, p_T) \xrightarrow[p_T^2 > \mu^2]{} \frac{1}{\pi p_T^2} \frac{\alpha_s(p_T^2)}{2\pi} \int_x^1 \frac{dy}{y} P\left(\frac{x}{y}\right) \Phi(y; p_T^2)$$

- Consistent matching to collinear situation: CSS formalism

JC Collins, DE Soper and GF Sterman, NP B 250 (1985) 199
A Bacchetta, D Boer, M Diehl, PJM, JHEP 0808 (2008) 023

Color gauge invariance

- Gauge invariance in a nonlocal situation requires a gauge link U(0,ξ)

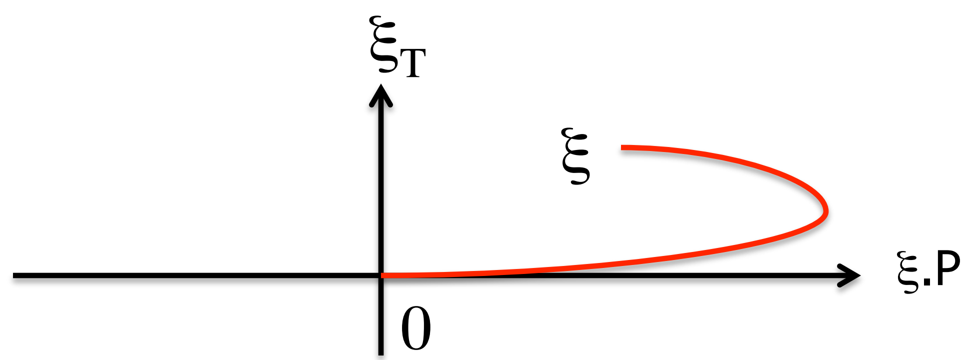
$$\bar{\psi}(0)\psi(\xi) = \sum_n \frac{1}{n!}\,\xi^{\mu_1}\ldots\xi^{\mu_N}\,\bar{\psi}(0)\,\partial_{\mu_1}\ldots\partial_{\mu_N}\,\psi(0)$$

$$U(0,\xi) = \mathcal{P}\,\exp\left(-ig\int_0^\xi ds^\mu A_\mu\right)$$

$$\bar{\psi}(0)\,{\color{red}U(0,\xi)}\,\psi(\xi) = \sum_n \frac{1}{n!}\,\xi^{\mu_1}\ldots\xi^{\mu_N}\,\bar{\psi}(0)\,D_{\mu_1}\ldots D_{\mu_N}\,\psi(0)$$

- Introduces path dependence for $\Phi(x,p_T)$

$$\Phi^{[U]}(x,p_T) \;\Rightarrow\; \Phi(x)$$

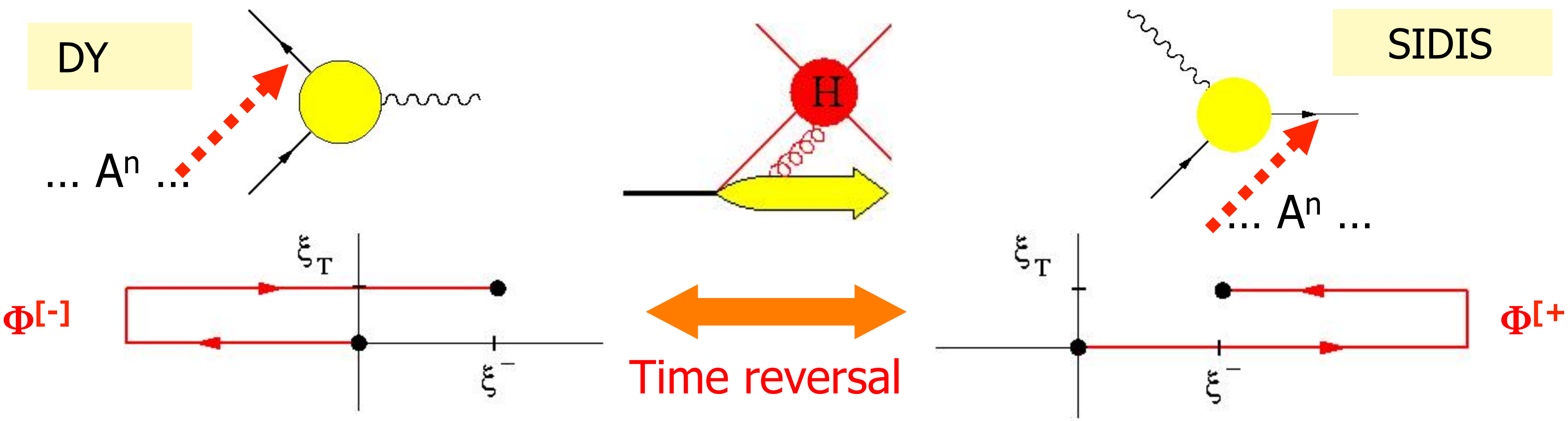


Which gauge links?

$$\boxed{\Phi_{ij}^{q[C]}(x, p_T; n) = \int \frac{d(\xi.P)\,d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi} \langle P | \bar{\psi}_j(0)\, U_{[0,\xi]}^{[C]}\, \psi_i(\xi) | P \rangle_{\xi.n=0}}$$ TMD

$$\Phi_{ij}^{q}(x; n) = \int \frac{d(\xi.P)}{(2\pi)}\, e^{i\,p.\xi} \langle P | \bar{\psi}_j(0)\, U_{[0,\xi]}^{[n]}\, \psi_i(\xi) | P \rangle_{\xi.n=\xi_T=0}$$ collinear

- Gauge links for TMD correlators process-dependent with simplest cases



AV Belitsky, X Ji and F Yuan, NP B 656 (2003) 165
D Boer, PJM and F Pijlman, NP B 667 (2003) 201

Which gauge links?

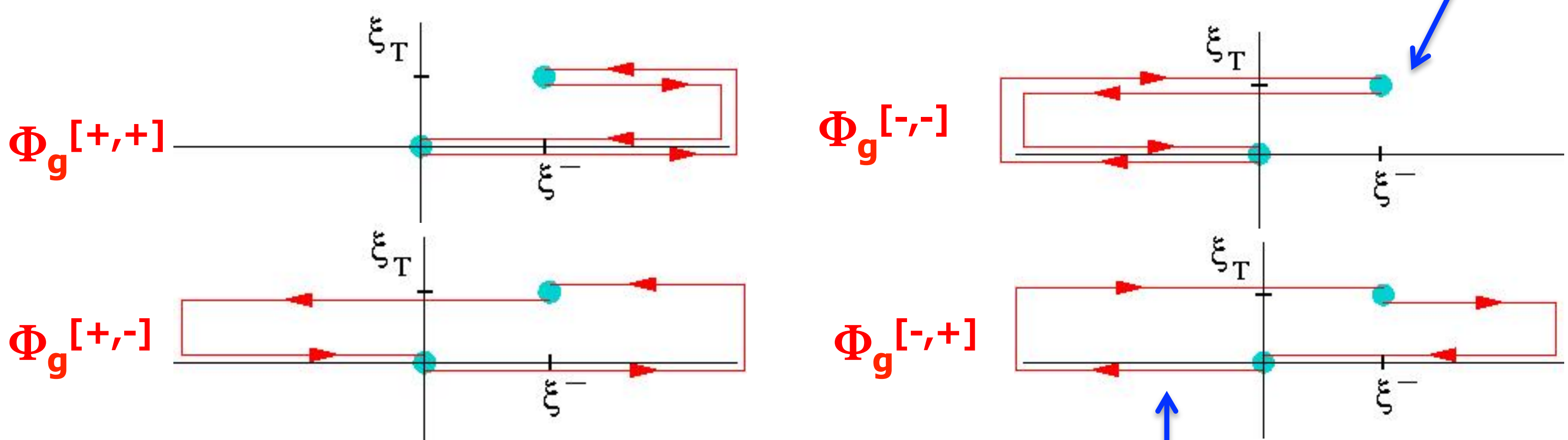
$$\Phi_g^{\alpha\beta[C,C']}(x,p_T;n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \left\langle P \left| U_{[\xi,0]}^{[C]} F^{n\alpha}(0) U_{[0,\xi]}^{[C']} F^{n\beta}(\xi) \right| P \right\rangle_{\xi.n=0}$$

- The TMD gluon correlators contain two links, which can have different paths. Note that standard field displacement involves C = C'

$$F^{\alpha\beta}(\xi) \to U_{[\eta,\xi]}^{[C]} F^{\alpha\beta}(\xi) U_{[\xi,\eta]}^{[C]}$$

- Basic (simplest) gauge links for gluon TMD correlators:

gg → H

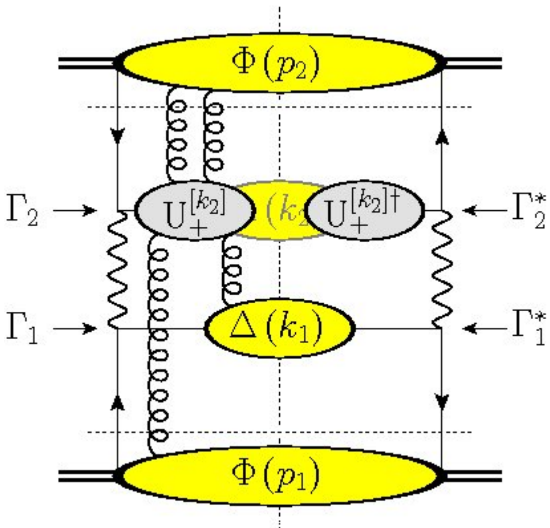


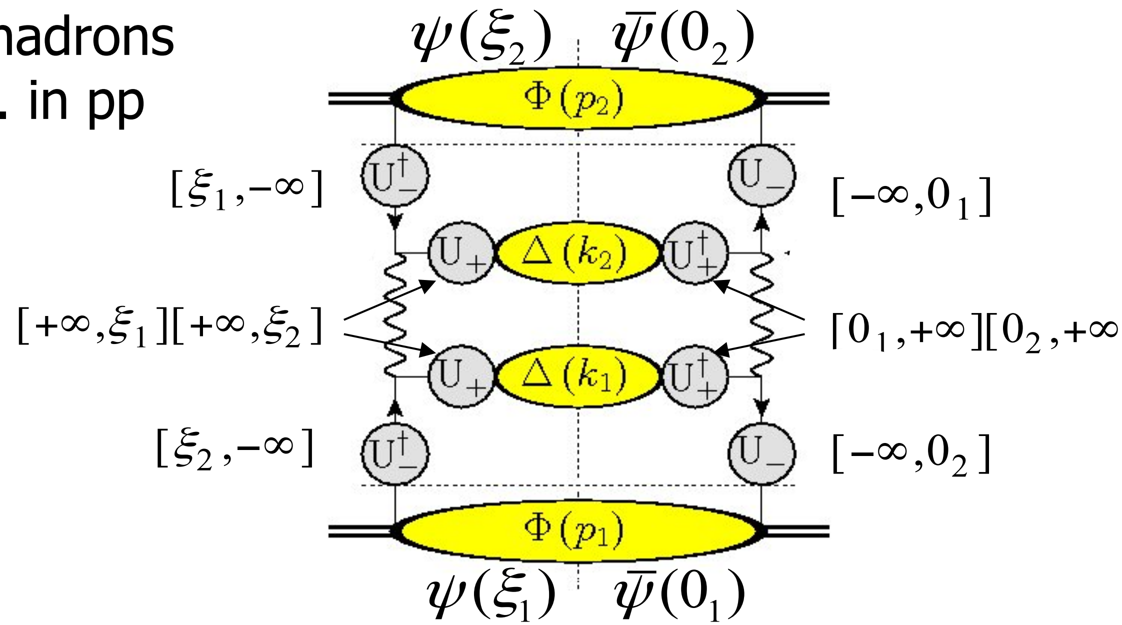
in gg → $Q\bar{Q}$

C Bomhof, PJM, F Pijlman; EPJ C 47 (2006) 147
F Dominguez, B-W Xiao, F Yuan, PRL 106 (2011) 022301

Which gauge links?

- With more (initial state) hadrons color gets entangled, e.g. in pp





- Outgoing color contributes future pointing gauge link to $\Phi(p_2)$ and future pointing part of a loop in the gauge link for $\Phi(p_1)$

- Can be color-detangled if only p_T of one correlator is relevant (using polarization, …) but must include Wilson loops in final U

T.C. Rogers, PJM, PR D81 (2010) 094006

MGA Buffing, PJM, JHEP 07 (2011) 065

Summarizing: color gauge invariant correlators

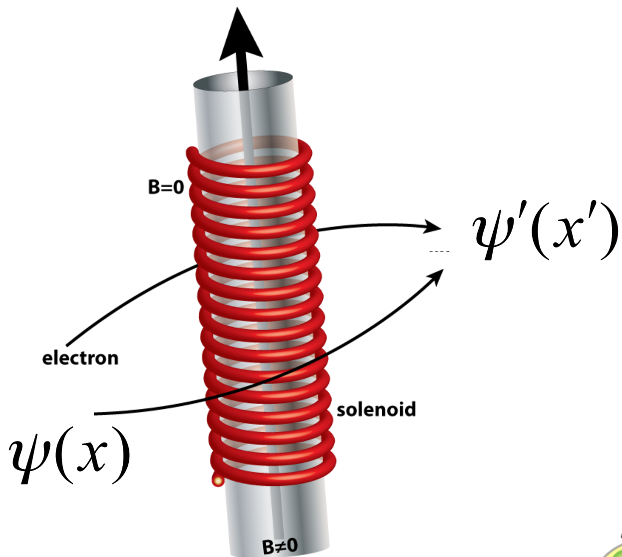
- So it looks that at best we have well-defined matrix elements for TMDs but including multiple possiblities for gauge links
- Leading quark TMDs:

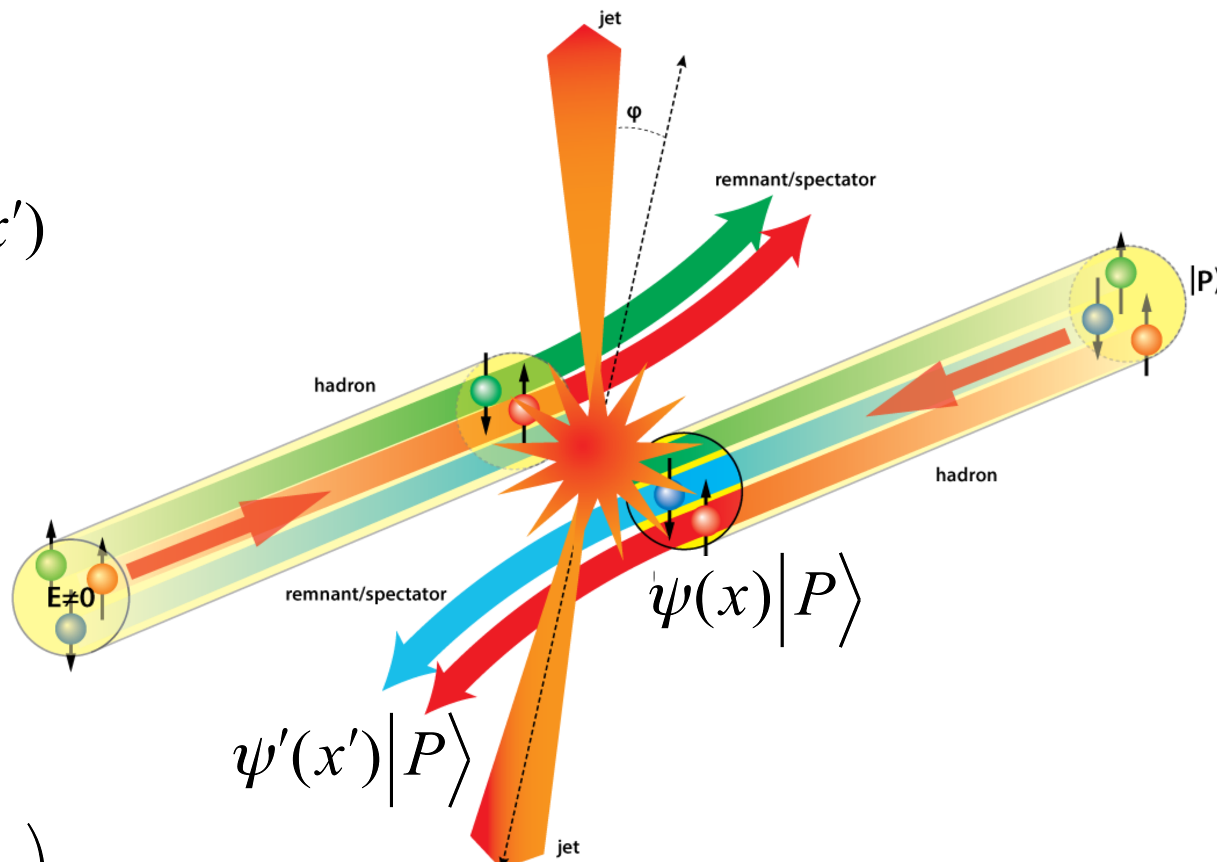
$$\Phi^{[U]}(x,p_T;n) = \Big\{ f_1^{[U]}(x,p_T^2) - f_{1T}^{\perp[U]}(x,p_T^2)\,\frac{\epsilon_T^{p_T S_T}}{M} + g_{1s}^{[U]}(x,p_T)\gamma_5 + h_{1T}^{[U]}(x,p_T^2)\,\gamma_5\,\not{S}_T + h_{1s}^{\perp[U]}(x,p_T)\,\frac{\gamma_5\,\not{p}_T}{M} + i h_1^{\perp[U]}(x,p_T^2)\,\frac{\not{p}_T}{M}\Big\}\frac{\not{P}}{2},$$

- Leading gluon TMDs:

$$2x\,\Gamma^{\mu\nu[U]}(x,p_T) = -g_T^{\mu\nu}\,f_1^{g[U]}(x,p_T^2) + g_T^{\mu\nu}\frac{\epsilon_T^{p_T S_T}}{M}\,f_{1T}^{\perp g[U]}(x,p_T^2) + i\epsilon_T^{\mu\nu}\,g_{1s}^{g[U]}(x,p_T) + \left(\frac{p_T^\mu p_T^\nu}{M^2} - g_T^{\mu\nu}\frac{p_T^2}{2M^2}\right) h_1^{\perp g[U]}(x,p_T^2) - \frac{\epsilon_T^{p_T\{\mu}\,p_T^{\nu\}}}{2M^2}\,h_{1s}^{\perp g[U]}(x,p_T) - \frac{\epsilon_T^{p_T\{\mu}\,S_T^{\nu\}} + \epsilon_T^{S_T\{\mu}\,p_T^{\nu\}}}{4M}\,h_{1T}^{g[U]}(x,p_T^2).$$

Opportunities to see color phases in QCD





$$\psi(\xi) = \boldsymbol{P}\exp\left(-ig\int_0^{\xi} ds^{\mu} A_{\mu}\right)\psi(0)$$

Figures by Kees Huyser

Next step


$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
$+ i \bar{\psi} \not{D} \psi + h.c.$
$+ \psi_i y_{ij} \psi_j \phi + h.c.$
$+ |D_\mu \phi|^2 - V(\phi)$

Operator structure in collinear case (reminder)

- Collinear functions and x-moments

$$\Phi^q(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P \middle| \overline{\psi}(0) U^{[n]}_{[0,\xi]} \psi(\xi) \middle| P \right\rangle_{\xi.n=\xi_T=0}$$

$$x^{N-1}\,\Phi^q(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P \middle| \overline{\psi}(0) (\partial^n)^{N-1} U^{[n]}_{[0,\xi]} \psi(\xi) \middle| P \right\rangle_{\xi.n=\xi_T=0}$$

$$= \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P \middle| \overline{\psi}(0) U^{[n]}_{[0,\xi]} (D^n)^{N-1} \psi(\xi) \middle| P \right\rangle_{\xi.n=\xi_T=0}$$

- Moments correspond to local matrix elements with calculable anomalous dimensions, that can be Mellin transformed to splitting functions

$$\Phi^{(N)} = \left\langle P \middle| \overline{\psi}(0) (D^n)^{N-1} \psi(0) \middle| P \right\rangle$$

- All operators have same twist since dim(D^n) = 0

Operator structure in TMD case

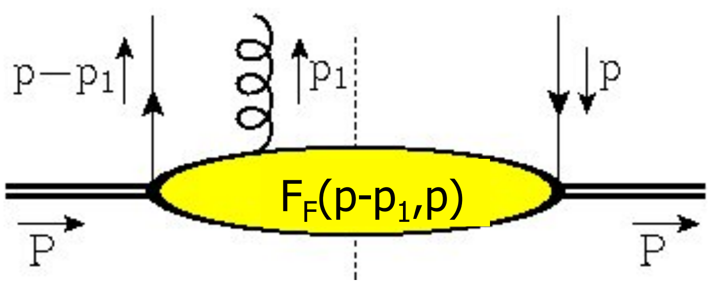
- For TMD functions one can consider transverse moments

$$\Phi(x,p_T;n)=\int\frac{d(\xi.P)d^2\xi_T}{(2\pi)^3}e^{i\,p.\xi}\left\langle P\left|\overline{\psi}(0)\,U^{[\pm]}\,\psi(\xi)\right|P\right\rangle_{\xi.n=0}$$

$$p_T^\alpha\Phi^{[\pm]}(x,p_T;n)=\int\frac{d(\xi.P)d^2\xi_T}{(2\pi)^3}e^{i\,p.\xi}\left\langle P\left|\overline{\psi}(0)UD_T^\alpha(\pm\infty)U\psi(\xi)\right|P\right\rangle_{\xi.n=0}$$

- Transverse moments involve collinear twist-3 multi-parton correlators Φ_D and Φ_F built from non-local combination of three parton fields

$$\Phi_F^\alpha(x-x_1,x_1\,|\,x)=\int\frac{d\xi.P\,d\eta.P}{(2\pi)^2}e^{i(p-p_1).\xi+ip_1.\eta}\left\langle P\left|\overline{\psi}(0)F^{n\alpha}(\eta)\,\psi(\xi)\right|P\right\rangle_{\xi.n=\xi_T=0}$$



$$\Phi_D^\alpha(x)=\int dx_1\,\Phi_D^\alpha(x-x_1,x_1\,|\,x)$$

$$\Phi_A^\alpha(x)=PV\int dx_1\,\frac{1}{x_1}\Phi_F^{n\alpha}(x-x_1,x_1\,|\,x)$$

T-invariant definition

Operator structure in TMD case

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$$\Phi(x,p_T;n)=\int\frac{d(\xi.P)d^2\xi_T}{(2\pi)^3}e^{i\,p.\xi}\left\langle P\left|\bar{\psi}(0)\,U^{[\pm]}\,\psi(\xi)\right|P\right\rangle_{\xi.n=0}$$

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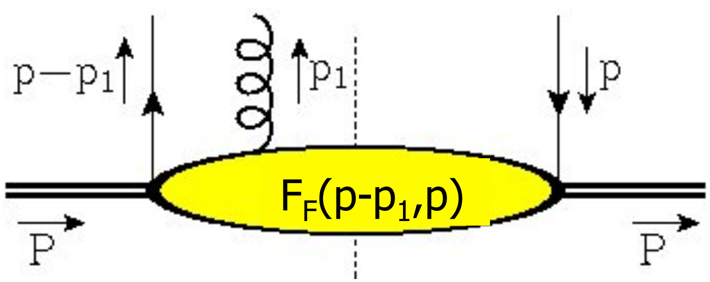

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T-invariant definition

Operator structure in TMD case

- Transverse moments can be expressed in these particular collinear multi-parton twist-3 correlators (which is NOT suppressed!)
- $$\Phi_{\partial}^{\alpha[U]}(x) = \int d^2p_T\; p_T^{\alpha}\,\Phi^{[U]}(x,p_T;n) = \tilde{\Phi}_{\partial}^{\alpha}(x) + C_G^{[U]}\pi\Phi_G^{\alpha}(x)$$

$$C_G^{[\pm]} = \pm 1$$

T-even

T-odd (gluonic pole or ETQS m.e.)

$$\boxed{\tilde{\Phi}_{\partial}^{\alpha}(x) = \Phi_D^{\alpha}(x) - \Phi_A^{\alpha}(x)} \qquad \boxed{\Phi_G^{\alpha}(x) = \Phi_F^{n\alpha}(x,0\,|\,x)}$$

- This gives rise to process dependence in PDFs, for unpolarized case

$$\frac{1}{M}\Phi_{\partial}^{\alpha[U]}(x) = \;\ldots\, h_1^{\perp(1)[U]}(x) = \;\ldots\, C_G^{[U]}\,h_1^{\perp(1)}(x)$$

  - Weightings defined as

$$h_1^{\perp(n)}(x) = \int d^2p_T\left(-\frac{p_T^2}{2M^2}\right)^n h_1^{\perp}(x,p_T^2)$$

Operator structure in TMD case

- Transverse moments can be expressed in these particular collinear multi-parton twist-3 correlators (which is NOT suppressed!)
- $\Phi_{\partial}^{\alpha[U]}(x) = \int d^2p_T\; p_T^\alpha \Phi^{[U]}(x,p_T;n) = \tilde{\Phi}_{\partial}^{\alpha}(x) + C_G^{[U]}\pi\Phi_G^\alpha(x)$

$C_G^{[\pm]} = \pm 1$

T-even T-odd (gluonic pole or ETQS m.e.)

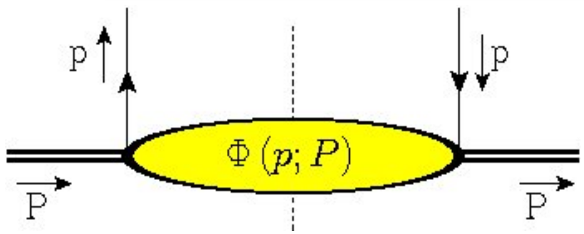
$$\boxed{\tilde{\Phi}_{\partial}^{\alpha}(x) = \Phi_D^\alpha(x) - \Phi_A^\alpha(x)} \qquad \boxed{\Phi_G^\alpha(x) = \Phi_F^{n\alpha}(x,0\,|\,x)}$$

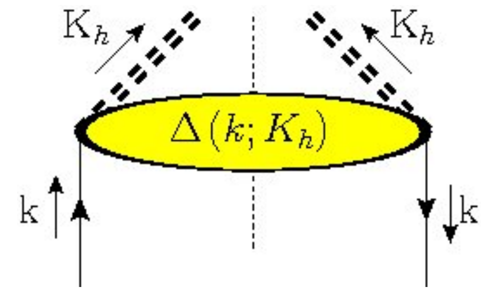
- For a polarized nucleon:

$$\frac{1}{M}\Phi_{\partial}^{\alpha[U]}(x) = \Big(\ldots g_{1T}^{\perp(1)}(x) + \ldots h_{1L}^{\perp(1)}(x)\Big) + \ldots C_G^{[U]}\, f_{1T}^{\perp(1)}(x)$$

T-even T-odd

Distributions versus fragmentation





- Operators:

$$\Phi^{[U]}(p\,|\,p) \sim \left\langle P\,|\,\bar\psi(0)U_{[0,\xi]}\psi(\xi)\,|\,P\right\rangle$$

$$\Phi_\partial^{\alpha[U]}(x) = \tilde\Phi_\partial^\alpha(x) + C_G^{[U]}\pi\,\Phi_G^\alpha(x)$$

T-even   T-odd (gluonic pole)

$$\boxed{\Phi_G^\alpha(x) = \Phi_F^{n\alpha}(x,0\,|\,x) \neq 0}$$

- Operators:

out state

$$\Delta(k\,|\,k) \sim \sum_X \left\langle 0\,|\,\psi(\xi)\,|\,K_h X\right\rangle\left\langle K_h X\,|\,\bar\psi(0)\,|\,0\right\rangle$$

$$\boxed{\Delta_G^\alpha(x) = \Delta_F^{n\alpha}\left(\tfrac{1}{Z},0\,|\,\tfrac{1}{Z}\right) = 0}$$

$$\Delta_\partial^{\alpha[U]}(x) = \tilde\Delta_\partial^\alpha(x)$$

T-even operator combination, but no T-constraints!

Collins, Metz; Meissner, Metz; Gamberg, M, Mukherjee, PR D 83 (2011) 071503

Double transverse weighting

- The double transverse weighted distribution function contains multiple 4-parton matrix elements

$$\Phi_{\partial\partial}^{\alpha\beta[U\}}(x) = \tilde{\Phi}_{\partial\partial}^{\alpha\beta}(x) + C_{GG}^{[U]}\pi^2\,\Phi_{GG}^{\alpha\beta}(x) + C_{G}^{[U]}\pi\left(\tilde{\Phi}_{\partial G}^{\alpha\beta}(x) + \tilde{\Phi}_{G\partial}^{\alpha\beta}(x)\right)$$

T-even T-even T-odd

$$\Phi_{\partial\partial}^{\alpha\beta[U\}}(x) = \ldots h_{1T}^{\perp(2)[U]}(x)$$

$$\boxed{h_{1T}^{\perp(2)[U]}(x) = h_{1T}^{\perp(2)(A)}(x) + C_{GG}^{[U]}h_{1T}^{\perp(2)(B1)}(x)}$$

- Note: "$\partial = D - A$"

- Separation in T-even and T-odd parts is no longer enough to isolate process dependent parts $\longrightarrow$ also Pretzelocity function is non-universal
- …. although $C_{GG}^{[+]} = C_{GG}^{[-]} = 1$ (so not different in DY and SIDIS)

MGA Buffing, A Mukherjee, PJM, PRD2012 , Arxiv: 1207.3221 [hep-ph]

Double transverse weighting

- Pretzelocity type of correlations come actually in three matrix elements and have to be parametrized using three functions

$$\Phi_{\partial\partial}^{\alpha\beta[U]}(x) = \tilde{\Phi}_{\partial\partial}^{\alpha\beta}(x) + C_{GG,c}^{[U]}\pi^2\,\Phi_{GG,c}^{\alpha\beta}(x) + C_G^{[U]}\pi\left(\tilde{\Phi}_{\partial G}^{\alpha\beta}(x) + \tilde{\Phi}_{G\partial}^{\alpha\beta}(x)\right)$$

$\mathrm{Tr}_c(GG\,\psi\overline{\psi})$ $\qquad$ $\mathrm{Tr}_c(GG)\,\mathrm{Tr}_c(\psi\overline{\psi})$

$$\boxed{h_{1T}^{\perp(2)[U]}(x) = h_{1T}^{\perp(2)(A)}(x) + C_{GG,1}^{[U]}h_{1T}^{\perp(2)(B1)}(x) + C_{GG,2}^{[U]}h_{1T}^{\perp(2)(B2)}(x)}$$

U	$U^{[\pm]}$	$U^{[+]}\,U^{[\square]}$	$\frac{1}{N_c}\,\mathrm{Tr}_c(U^{[\square]})\,U^{[+]}$
$\Phi^{[U]}$	$\Phi^{[\pm]}$	$\Phi^{[+\square]}$	$\Phi^{[(\square)+]}$
$C_G^{[U]}$	± 1	3	1
$C_{GG,1}^{[U]}$	1	9	1
$C_{GG,2}^{[U]}$	0	0	4

MGA Buffing, A Mukherjee, PJM, PRD2012 , Arxiv: 1207.3221 [hep-ph]

The next step: TMDs of definite rank

- Expansion into TMDs of definite rank

$$\Phi^{[U]}(x,p_T) = \tilde{\Phi}(x,p_T^2) + C_G^{[U]}\pi\, p_{Ti}\,\tilde{\Phi}_G^i(x,p_T^2) + C_{GG,c}^{[U]}\pi^2\, p_{Tij}\,\tilde{\Phi}_{GG,c}^{ij}(x,p_T^2) + \ldots$$
$$+\, p_{Ti}\,\tilde{\Phi}_\partial^i(x,p_T^2) + C_G^{[U]}\pi\, p_{Tij}\,\tilde{\Phi}_{\{\partial G\}}^{ij}(x,p_T^2) + \ldots$$
$$+\, p_{Tij}\,\tilde{\Phi}_{\partial\partial}^{ij}(x,p_T^2) + \ldots$$
$$+ \ldots$$

- Depending on spin and type of operators, only a finite number needed
- Example: quarks in an unpolarized target

$$\tilde{\Phi}(x,p_T^2) = \left(f_1(x,p_T^2)\right)\frac{\not{P}}{2} \qquad \pi\tilde{\Phi}_G^\alpha(x,p_T^2) = \left(i h_1^\perp(x,p_T^2)\frac{\gamma_T^\alpha}{M}\right)\frac{\not{P}}{2}$$

Summarizing quark TMDs up to spin 1/2 targets

GLUONIC POLE RANK			
0	1	2	3
$\Phi(x,p_T^2)$	$\pi C_G^{[U]}\,\Phi_G$	$\pi^2 C_{GG,c}^{[U]}\,\Phi_{GG,c}$	$\pi^3 C_{GGG,c}^{[U]}\,\Phi_{GGG,c}$
$\widetilde{\Phi}_\partial$	$\pi C_G^{[U]}\,\widetilde{\Phi}_{\{\partial G\}}$	$\pi^2 C_{GG,c}^{[U]}\,\widetilde{\Phi}_{\{\partial GG\},c}$	$\ldots$
$\widetilde{\Phi}_{\partial\partial}$	$\pi C_G^{[U]}\,\widetilde{\Phi}_{\{\partial\partial G\}}$	$\ldots$	$\ldots$
$\widetilde{\Phi}_{\partial\partial\partial}$	$\ldots$	$\ldots$	$\ldots$

PDFs FOR SPIN 0 HADRONS		
f_1	$h_1^\perp$	

+

PDFs FOR SPIN 1/2 HADRONS		
$g_1,\ h_1$	$f_{1T}^\perp$	$h_{1T}^{\perp(B1)},\ h_{1T}^{\perp(B2)}$
$g_{1T},\ h_{1L}^\perp$		
$h_{1T}^{\perp(A)}$		

PFFs FOR SPIN 0 HADRONS		
D_1		
$H_1^\perp$		

+

PFFs FOR SPIN 1/2 HADRONS		
$G_1,\ H_1$		
$G_{1T},\ H_{1L}^\perp,\ D_{1T}^\perp$		
$H_{1T}^\perp$		

MGA Buffing, A Mukherjee, PJM, PRD2012 , Arxiv: 1207.3221 [hep-ph]

Conclusions

- (Generalized) universality using definite rank functions: azimuthal dependence of transverse momentum multiplying functions $f(x,p_T^2)$.
- Rank 0 are the well-known collinear functions (three quark and two gluon spin distributions)
- Rank m is coupled to $\cos(m\phi)$ and $\sin(m\phi)$ azimuthal asymmetries. Leading azimuthal asymmetries with m up to $2(S_{hadron}+s_{parton})$.
- Multiple distribution functions showing up in azimuthal asymmetries (depending on color structure of operators), e.g. three pretzelocities.
- In principle distinguishable in different experiments (with different color flow in tree-level diagrams):

  gluon + gluon → colorless (distinguish CP+ from CP- Higgs)

  gluon-gluon → quark-antiquark pair.
- Novel information on hadron structure (comparison with lattice calc.)
- Factorization studies are a next step

Thank you