Vus from τ decays: Status and perspectives at new facilities

Emilie Passemar

Los Alamos National Laboratory

Laboratori Nazionali di Frascati

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Outline :

1. Introduction and Motivation
2. V_{us} from semi-leptonic decays
3. V_{us} from leptonic decays
4. V_{us} from inclusive hadronic τ decays
5. New determination of V_{us} from predicting τ strange BRs
6. Prospects for τ at the new flavour factories

1. Introduction and Motivation

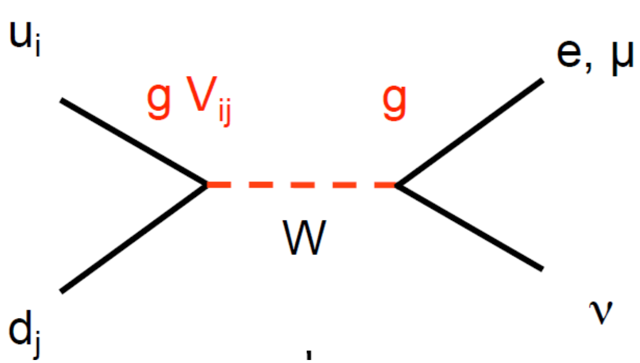
1.1 Test of New Physics : Vus

- Extraction of the Cabibbo-Kobayashi-Maskawa matrix element V_{us}
  - Fundamental parameter of the Standard Model
    Check unitarity of the first row of the CKM matrix:
    ⇒ *Cabibbo Universality*

$$\boxed{|V_{ud}|^2 + |V_{us}|^2 + \underbrace{|V_{ub}|^2}_{\text{Negligible (B decays)}} \stackrel{?}{=} 1}$$

  - Input in UT analysis

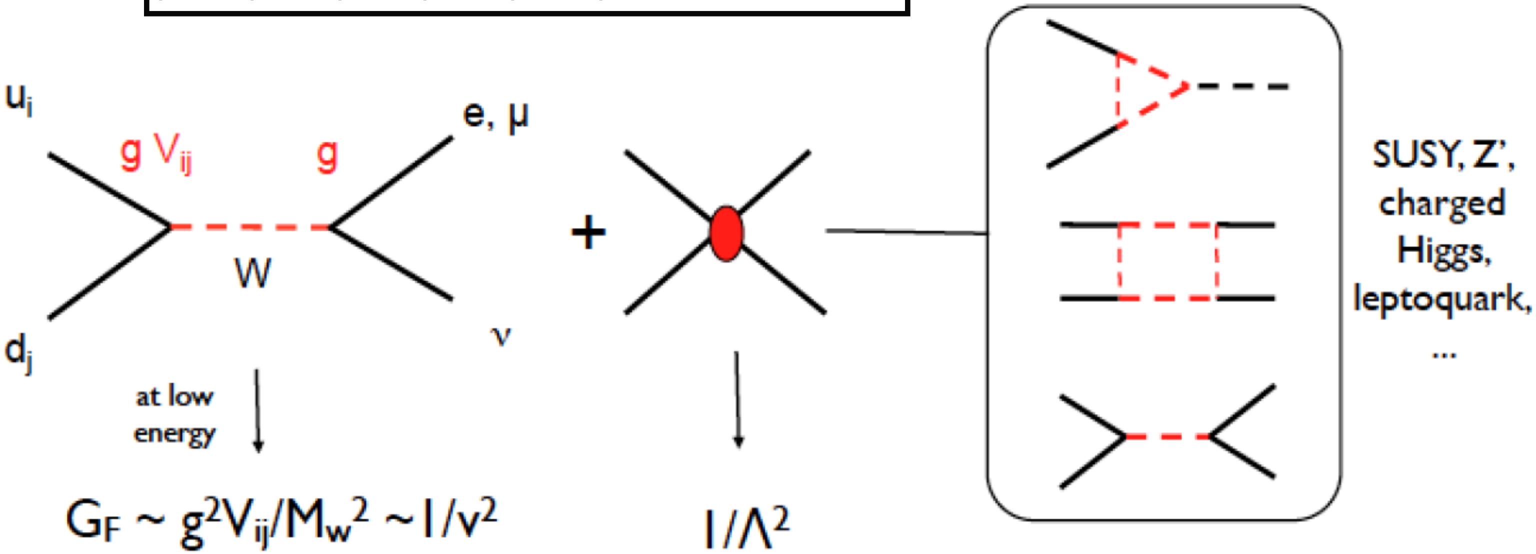
- Look for *new physics*
  - In the Standard Model : W exchange ⇒ only V-A structure



1.1 Test of New Physics : Vus

- BSM: sensitive to tree-level and loop effects of a large class of models

$$\boxed{|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \Delta_{CKM}}$$



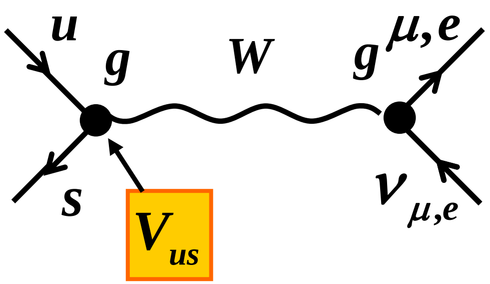
BSM effects : $\boxed{\Delta_{CKM} \sim (v/\Lambda)^2}$

- Look for new physics by comparing the extraction of V_{us} from different processes: helicity suppressed $K_{\mu 2}$, helicity allowed K_{l3}, hadronic τ decays

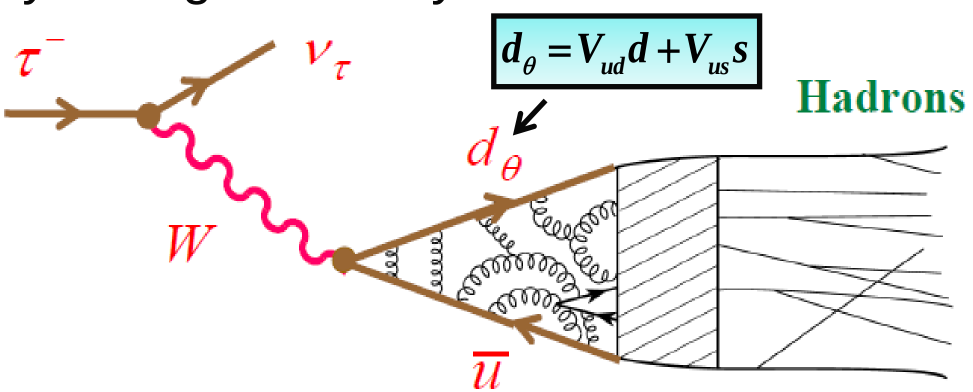
1.2 Paths to V_{ud} and V_{us}

- From kaon, pion and nuclear decays

V_{ud}	$0^+ \to 0^+$ $\pi^{\pm} \to \pi^0 e \nu_e$	$n \to p e \nu_e$	$\pi \to \ell \nu_\ell$
V_{us}	$K \to \pi \ell \nu_\ell$	$\Lambda \to p e \nu_e$	$K \to \ell \nu_\ell$



- From τ decays: only lepton heavy enough to decay into hadrons
  - ➢ Very rich phenomenology :
    - – α_S
    - – V_{us}, m_s



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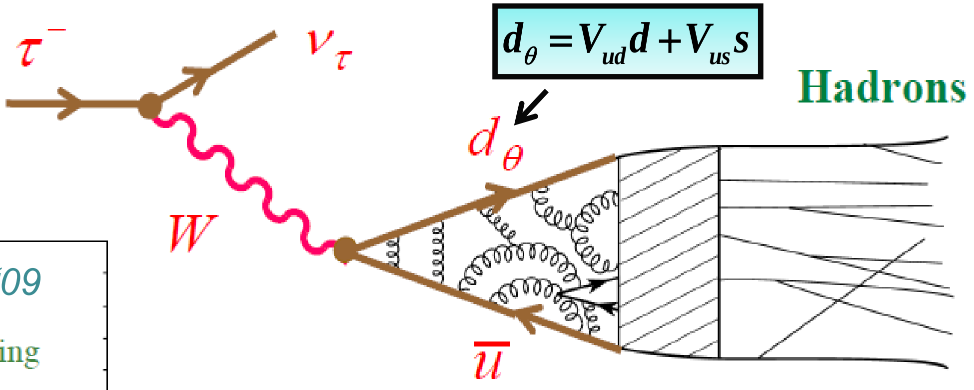

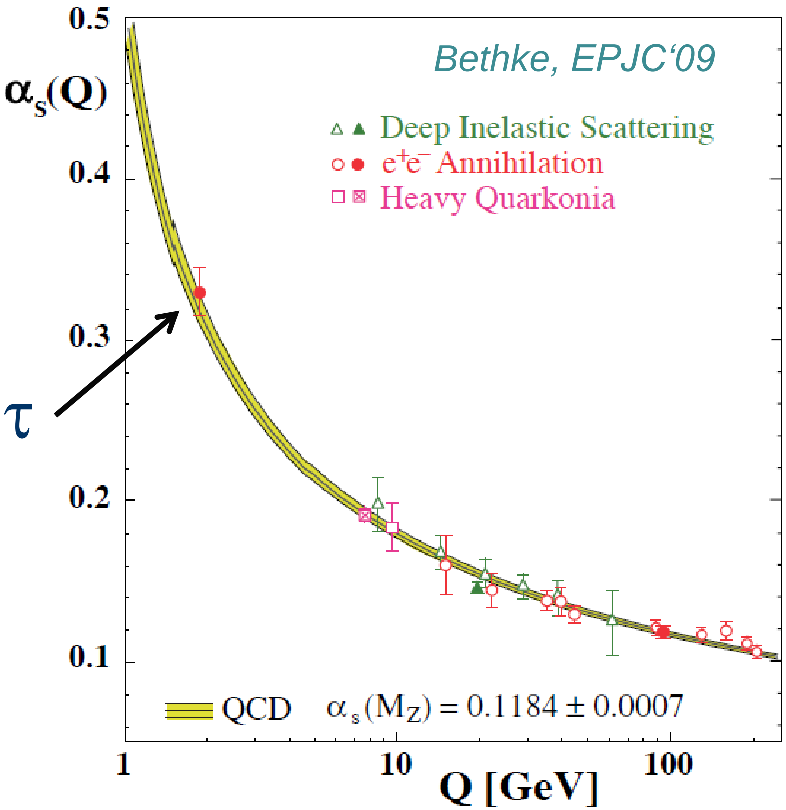

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- From kaon, pion, baryon and nuclear decays

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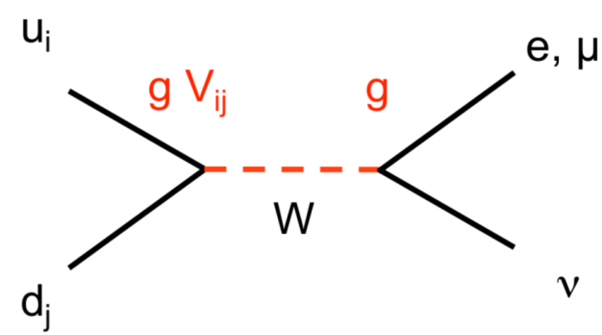

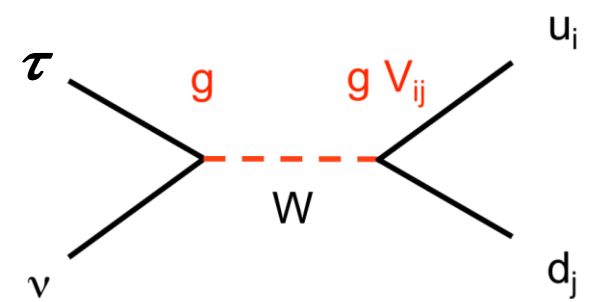

- From τ decays (crossed channel)

V_{ud}			$\tau \to \pi \nu_\tau$	$\tau \to h_{NS} \nu_\tau$
V_{us}	$\tau \to K \pi \nu_\tau$		$\tau \to K \nu_\tau$	$\tau \to h_S \nu_\tau$ (inclusive)

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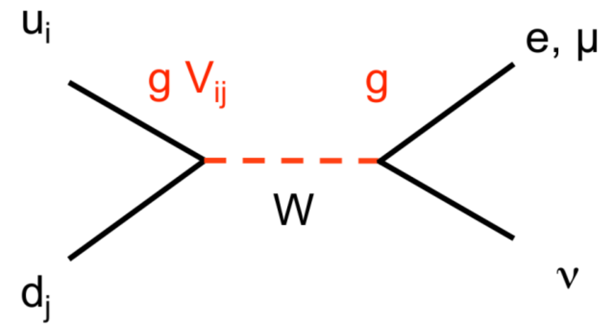

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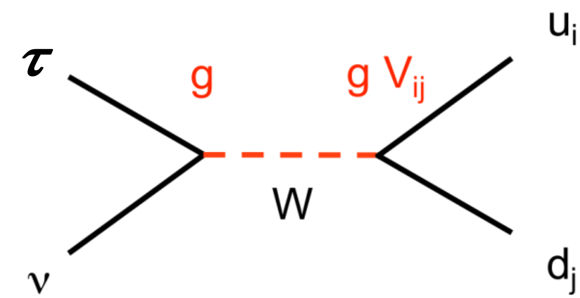

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1.2 Paths to V_{ud} and V_{us}

- These are the *golden modes* to extract V_{ud} and V_{us}
  - Only the *vector current* contributes $\langle A(p_A)|\bar{q}^i\gamma_\mu q^j|B(p_B)\rangle$
  - Normalization known in SU(2) [SU(3)] symmetry limit
  - Corrections start at 2nd order in SU(2) [SU(3)] breaking

Ademollo & Gato, Berhands & Sirlin

- Currently the most precise determination of V_{ud} and V_{us}

⇨ V_{ud} (*0.02 %*) and V_{us} (*0.5 %*)

1.2 Paths to V_{ud} and V_{us}

- From kaon, pion, baryon and nuclear decays

V_{ud}	$0^+ \to 0^+$ $\pi^\pm \to \pi^0 e \nu_e$	$n \to p e \nu_e$	$\pi \to \ell \nu_\ell$
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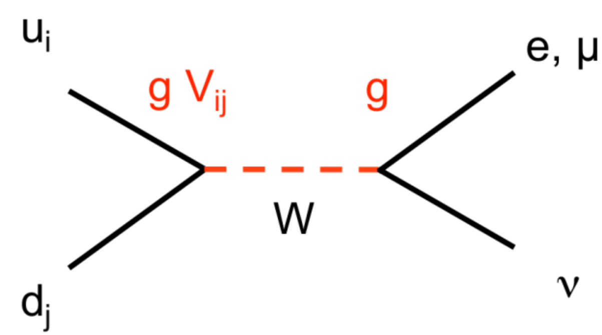

- $n \to p e \nu_e$:
  - Both *V* and *A currents* contribute ⇨ need experimental information on A (e.g. β asymmetry ($r_A = g_A/g_V$))
  - Free of nuclear uncertainties
  - Probe different combinations of BSM operators (e.g. right-handed currents, etc…)

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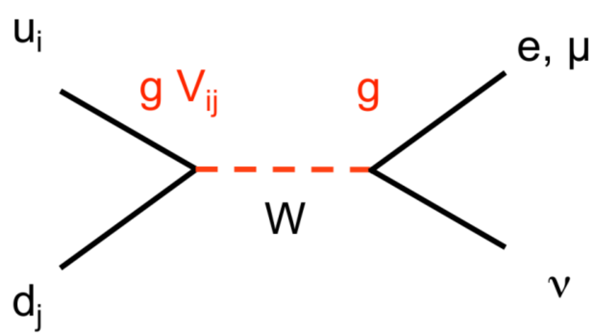

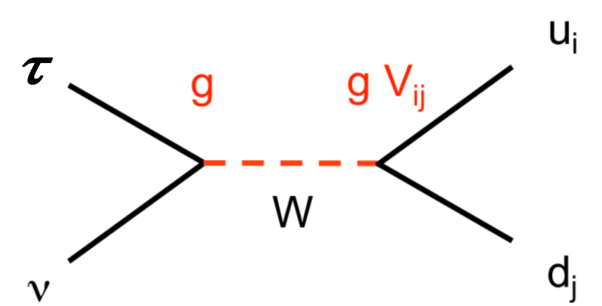

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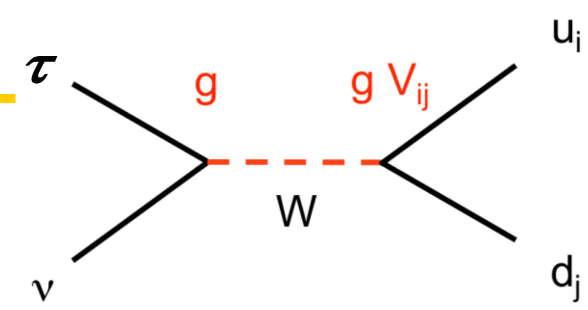
1.2 Paths to V_{ud} and V_{us}

- K_{l2}/π_{l2} and $\tau \to K/\pi\nu_\tau$
  - ➢ Only the *axial current* contributes
  - ➢ Need to know the decay constants F_K, F_π
    ⇒ *Lattice QCD*
  - ➢ Probe different BSM operators than from the vector case

- Input on F_K/F_π ⇒ V_{us}/V_{ud} very precisely

1.2 Paths to V_{ud} and V_{us}



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- Possibility to determine V_{ud}, V_{us} from *inclusive τ decays*
  - Use *OPE* to calculate the inclusive BRs
  - Different test of BSM operators *inclusive* vs. *exclusive*

2. V_{us} from semi-leptonic decays

2.1 Introduction

- From kaon, pion and nuclear decays

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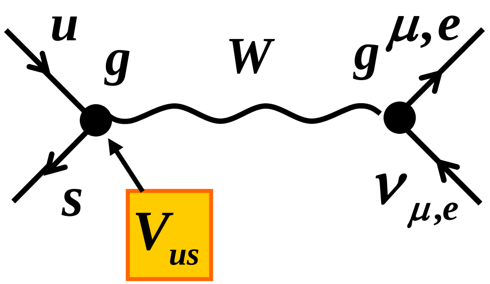

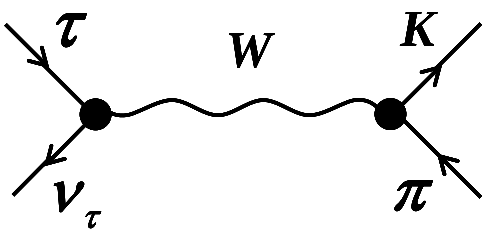

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2.2 K_{l3} decays

- Master formula for K $\rightarrow \pi l \nu_l$:

$$\boxed{\Gamma\left(K \rightarrow \pi l \nu [\gamma]\right) = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 S_{EW}^K \left|V_{us}\right|^2 \left| f_+^{K^0\pi^-}(0)\right|^2 I_K^l \left(1 + \delta_{\mathrm{EM}}^{Kl} + \delta_{\mathrm{SU(2)}}^{K\pi}\right)^2}$$

- Experimental inputs from FLAVIAnet review *Antonelli et al.'10,*

  ⇨ Update by *M. Moulson at CIPANP 2012*

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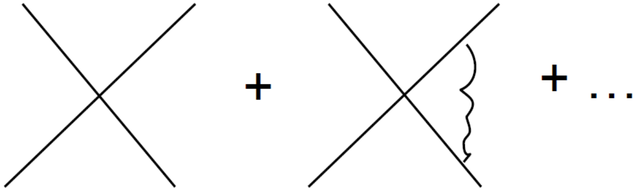
- Theoretical inputs :
  - S_{ew} : Short distance electroweak correction



$$\boxed{S_{\mathrm{ew}} = 1 + \frac{2\alpha}{\pi}\left(1 + \frac{\alpha_S}{4\pi}\right)\log\frac{m_Z}{m_\rho} + O\left(\frac{\alpha\alpha_S}{\pi^2}\right)}$$

$\Rightarrow$ $\boxed{S_{\mathrm{ew}} = 1.0232}$ Sirlin'82

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- Theoretical inputs :
  - S_{ew} : Short distance electroweak correction ⇒ $\boxed{S_{ew} = 1.0232}$
  - $f_+(0)$: vector form factor at zero momentum transfer:
    Hadronic matrix element:

$$\left\langle \pi(p_\pi) \right| \bar{s}\gamma_\mu u \left| \mathrm{K}(p_K) \right\rangle = \left[\left(p_K + p_\pi\right)_\mu - \frac{\Delta_{K\pi}}{t}\left(p_K - p_\pi\right)_\mu \right] \underbrace{f_+(t)}_{\text{vector}} + \frac{\Delta_{K\pi}}{t}\left(p_K - p_\pi\right)_\mu \underbrace{f_0(t)}_{\text{scalar}}$$

$$t = q^2 = (p_\mu + p_{\nu_\mu})^2 = (p_K - p_\pi)^2, \quad \bar{f}_{0,+}(t) = \frac{f_{0,+}(t)}{f_+(0)}$$

In chiral limit $f_+(0) = 1$, calculation of SU(3) breaking crucial

⇒ *ChPT* with resonances or *lattice*

2.2 K_{l3} decays

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- Theoretical inputs :
  - S_{ew} : Short distance electroweak correction ⇨ $\boxed{S_{ew} = 1.0232}$
  - $f_+(0)$: vector form factor at zero momentum transfer
    ⇨ *ChPT* with resonances or *lattice*
  - I_K : Phase space integral ⇨ need a *parametrization* for the normalized form factors to fit the experimental distributions
    Taylor expansion :

$$\bar{f}_{+,0}(s) = 1 + \lambda'_{+,0} \frac{s}{m_\pi^2} + \frac{1}{2}\lambda''_{+,0}\left(\frac{s}{m_\pi^2}\right)^2 + \ldots$$

⇨ *Dispersive parametrization*

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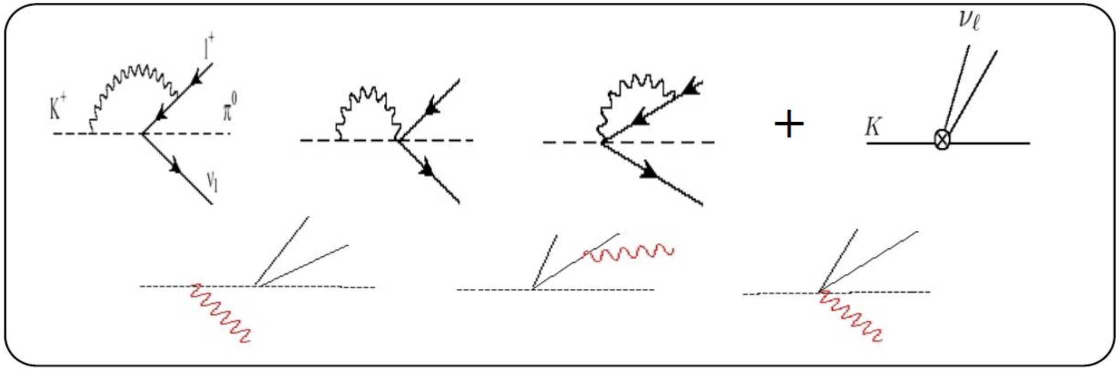
  - I_K : Phase space integral ⇨ *Dispersive parametrization* for the FFs
  - $\delta_{\mathrm{EM}}^{Kl}$: Long-distance electromagnetic corrections



→ ChPT to O(p^2e^2)

→ Fully inclusive prescription for real photons

→ Uncertainties: LECs (100%) + higher orders

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⇨

Mode	$\delta_{\rm EM}^{K\ell}$ (%)
K_{e3}^0	0.495 ± 0.110
$K_{e3}^\pm$	0.050 ± 0.125
$K_{\mu3}^0$	0.700 ± 0.110
$K_{\mu3}^\pm$	0.008 ± 0.125

Cirigliano, Giannotti, Neufeld'08

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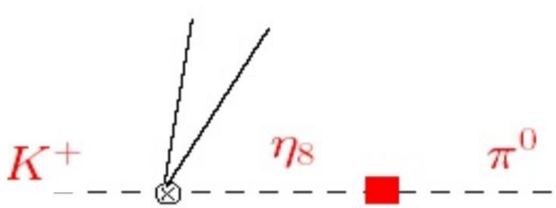
  - $\delta_{\rm SU(2)}^{K\pi}$: Isospin breaking corrections $\boxed{\delta_{\rm SU(2)}^{K\pi} = \frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} - 1}$



+ IB in one loop graphs + CT

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In ChPT at O(p⁴) :

$$\boxed{\delta_{\mathrm{SU(2)}}^{K\pi} = \frac{3}{4}\frac{1}{Q^2}\left[\frac{m_K^2}{m_\pi^2} + \frac{\chi_{p^4}}{2}\left(1 + \frac{m_s}{m}\right)\right]} \quad \text{with} \quad Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2} \quad \text{and} \quad m \equiv \frac{m_u + m_d}{2}$$

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⇨ $\boxed{\delta_{\rm SU(2)}^{K\pi} = (2.4 \pm 0.3)\%}$ *FLAG'10*

2.3 $\tau \to K\pi\nu_\tau$ decays

- Master formula for $\tau \to K\pi\nu_\tau$ (crossed channel) :

$$\boxed{\Gamma\left(\tau \to \overline{K}\pi\nu_\tau[\gamma]\right) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 S_{EW}^\tau \left|V_{us}\right|^2 \left| f_+^{K^0\pi^-}(0)\right|^2 I_K^\tau \left(1+\delta_{\mathrm{EM}}^{K\tau}+\delta_{\mathrm{SU(2)}}^{K\pi}\right)^2}$$

- Experimental inputs from HFAG *Banerjee et al.'12*

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- Theoretical inputs :
  - S_{ew} : Short distance electroweak correction ⇨ τ scale

$$\boxed{S_{\mathrm{ew}} = 1 + \frac{2\alpha}{\pi}\left(1 + \frac{\alpha_S}{4\pi}\right)\log\frac{m_Z}{m_\tau} + O\left(\frac{\alpha\alpha_S}{\pi^2}\right)}$$

⇨ $\boxed{S_{\mathrm{ew}} = 1.0201}$

Marciano &Sirlin'88, Braaten & Li'90, Erler'04

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  - $f_+(0)$: vector form factor at zero momentum transfer:
    Hadronic matrix element: Crossed channel

$$\boxed{\langle \mathbf{K}\pi |\, \bar{\mathbf{s}}\gamma_\mu \mathbf{u}\, | 0 \rangle = \left[\left(p_K - p_\pi\right)_\mu + \frac{\Delta_{K\pi}}{s}\left(p_K + p_\pi\right)_\mu \right] \underbrace{f_+(s)}_{\text{vector}} - \frac{\Delta_{K\pi}}{s}\left(p_K + p_\pi\right)_\mu \underbrace{f_0(s)}_{\text{scalar}}}$$

with $s = q^2 = (p_K + p_\pi)^2$, $\overline{f}_{0,+}(t) = \dfrac{f_{0,+}(t)}{f_+(0)}$

⇨ determined from *ChPT* with resonances or *lattice*

2.3 $\tau \to K\pi\nu_\tau$ decays

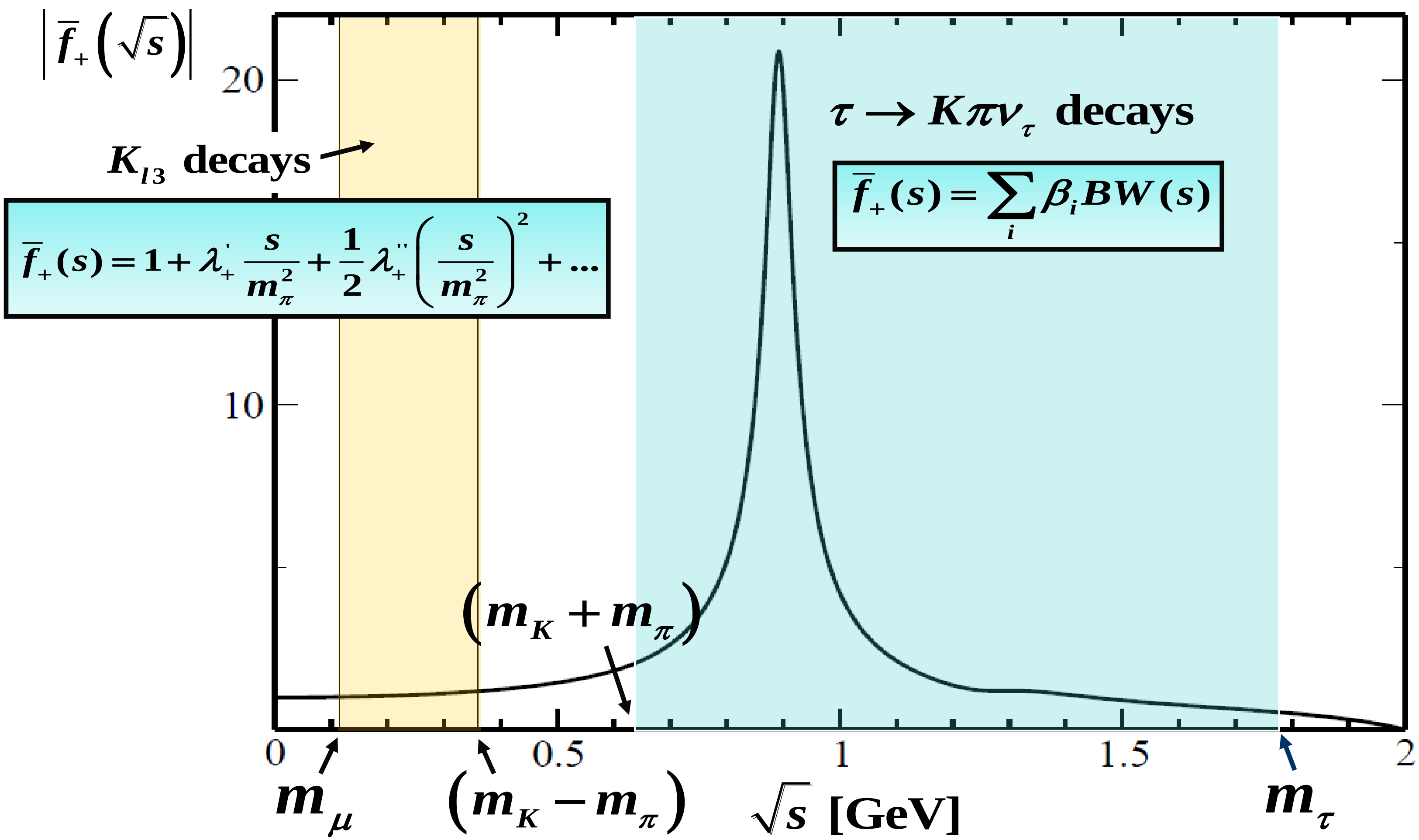
- Master formula for $\tau \to K\pi\nu_\tau$ (crossed channel) :

$$\boxed{\Gamma\left(\tau \to \overline{K}\pi\nu_\tau[\gamma]\right) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 S_{EW}^\tau \left|V_{us}\right|^2 \left| f_+^{K^0\pi^-}(0)\right|^2 \boxed{I_K^\tau} \left(1 + \delta_{\mathrm{EM}}^{K\tau} + \delta_{\mathrm{SU(2)}}^{K\pi}\right)^2}$$

- Theoretical inputs :
  - S_{ew} : Short distance electroweak correction ⇨ τ scale
  - $f_+(0)$: vector form factor at zero momentum transfer:
    ⇨ *ChPT* with resonances or *lattice*
  - I_K : Phase space integral ⇨ need a *parametrization* for the normalized form factors to fit the experimental distributions
    ⇨ Use a *dispersive parametrization* to combine with K_{l3} analysis

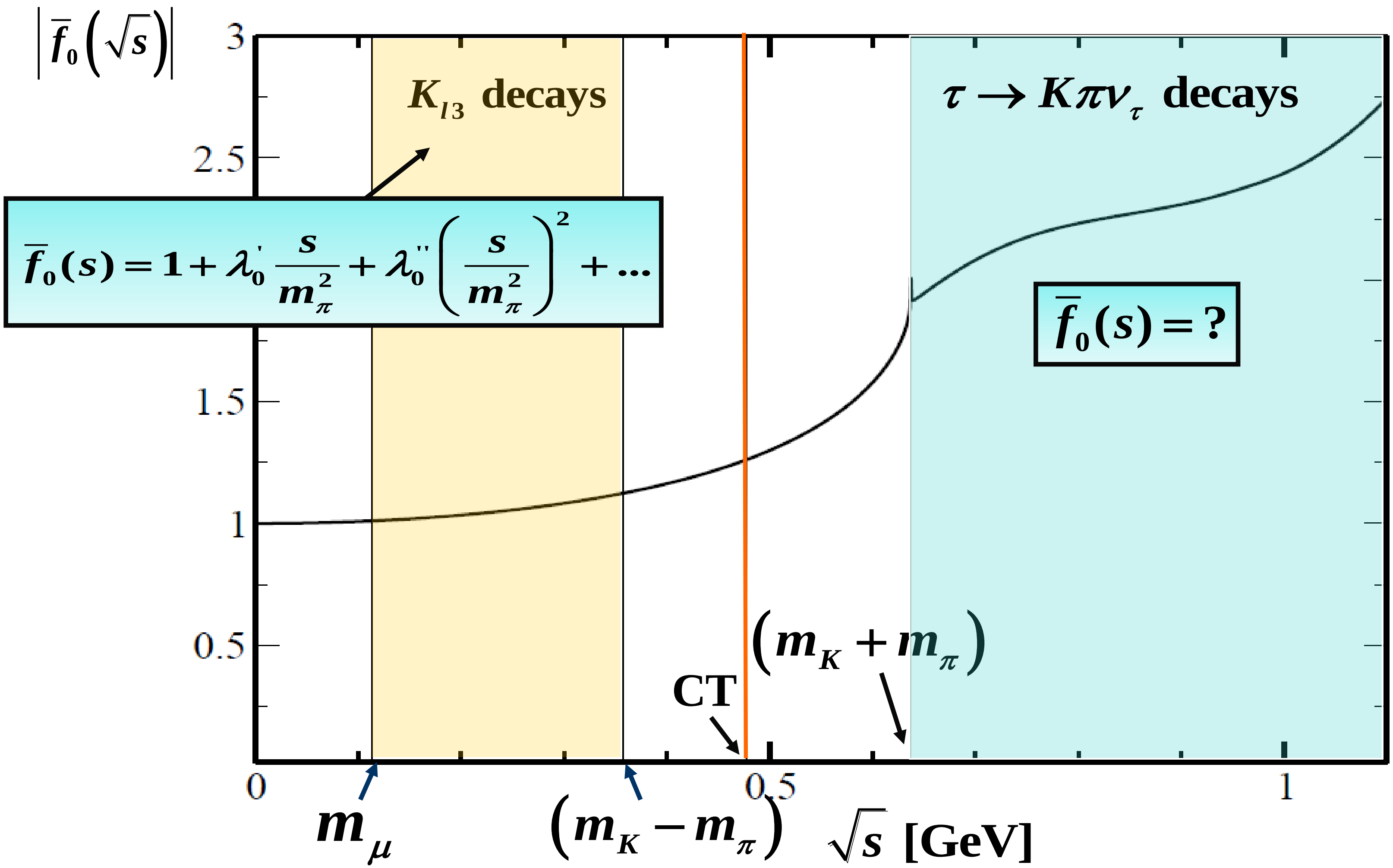
Determination of the Kπ form factors

- Parametrization to analyse both K_{l3} and τ decays
  - Vector form factor: ⇨ Dominance of K*(892) resonance



Determination of the Kπ form factors

- Parametrization to analyse both K_{l3} and τ decays
  - Scalar form factor: ⇨ No obvious dominance of a resonance

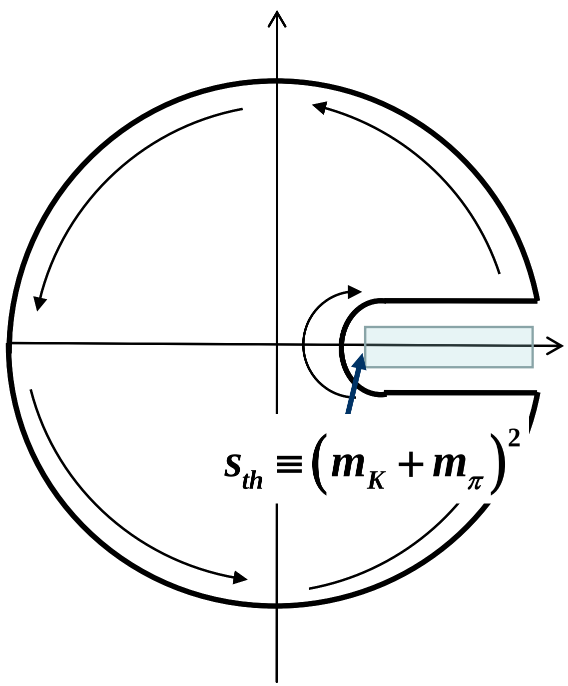


Determination of the Kπ form factors

- Parametrization to analyse both K_{l3} and τ decays

  ⇨ Use dispersion relations

- Omnès representation: ⇨ $$\bar{f}_{+,0}(s) = \exp\left[\frac{s}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{\phi_{+,0}(s')}{s'-s-i\varepsilon}\right]$$



$\phi_{+,0}$ (s): phase of the form factor

- $s < s_{in}$: $\phi_{+,0}(s) = \delta_{K\pi}(s)$

  Kπ scattering phase

- $s \geq s_{in}$: $\phi_{+,0}(s)$ unknown

  ⇨ $\phi_{+,0}(s) = \phi_{+,0as}(s) = \pi \pm \pi \ \left(\bar{f}_{+,0}(s) \to 1/s\right)$

[Brodsky&Lepage]

- Subtract dispersion relation to weaken the high energy contribution of the phase. Improve the convergence but sum rules to be satisfied!

Determination of the Kπ form factors

Bernard, Boito, E.P., in progress

- Dispersion relation with n subtractions in $\bar{s}$:

$$\boxed{\bar{f}_{+,0}(s)=\exp\left[P_{n-1}(s)+\frac{(s-\bar{s})^n}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{(s'-\bar{s})^n}\frac{\phi_{+,0}(s')}{s'-s-i\varepsilon}\right]}$$

➢ $\bar{f}_0(s)$ ⇨ dispersion relation with 3 subtractions: 2 in s=0 and 1 in s=$\Delta_{K\pi}$

[Callan-Treiman]

$$\boxed{\bar{f}_0(s)=\exp\left[\frac{s}{\Delta_{K\pi}}\ln C+\frac{s}{\Delta_{K\pi}}(s-\Delta_{K\pi})\left(\frac{\ln C}{\Delta_{K\pi}}-\frac{\lambda_0'}{m_\pi^2}\right)+\frac{s^2(s-\Delta_{K\pi})}{\pi}\int_{(m_K+m_\pi)^2}^{\infty}\frac{ds'}{s'^2}\frac{\phi_0(s')}{(s'-\Delta_{K\pi})(s'-s-i\varepsilon)}\right]}$$

For s<s_{in}:Kπ scattering phase extracted from the data

Buettiker, Descotes-Genon, & Moussallam'02

2 parameters to fit to the data $\boxed{\ln C=\ln\bar{f}(\Delta_{K\pi})}$ and $\boxed{\lambda_0'}$

Determination of the Kπ form factors

Bernard, Boito, E.P., in progress

- Dispersion relation with n subtractions in $\bar{s}$:

$$\boxed{\bar{f}_{+,0}(s) = \exp\left[P_{n-1}(s) + \frac{(s-\bar{s})^n}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{(s'-\bar{s})^n}\frac{\phi_{+,0}(s')}{s'-s-i\varepsilon}\right]}$$

➢ $\bar{f}_+(s)$ ⇨ dispersion relation with 3 subtractions in s=0

Boito, Escribano, Jamin'09,'10

$$\boxed{\bar{f}_+(s) = \exp\left[\lambda_+'\frac{s}{m_\pi^2} + \frac{1}{2}\left(\lambda_+'' - \lambda_+'^2\right)\left(\frac{s}{m_\pi^2}\right)^2 + \frac{s^3}{\pi}\int_{(m_K+m_\pi)^2}^{\infty}\frac{ds'}{s'^3}\frac{\phi_+(s')}{(s'-s-i\varepsilon)}\right]}$$

Extracted from a model including
2 resonances K*(892) and K*(1414)

Jamin, Pich, Portolés'08

7 parameters to fit to the data:

– $\boxed{\lambda_+'}$ and $\boxed{\lambda_+''}$ ⇨ can be combined with K_{l3} fits

– Resonance parameters: $\boxed{m_{K^*}, \Gamma_{K^*}, m_{K^{*'}}, \Gamma_{K^{*'}}, \beta}$

Mixing parameter

Determination of the Kπ form factors

- Fit to the $\tau \to K\pi\nu_\tau$ decay data
  - from *Belle [Epifanov et al'08]* (*BaBar?*)

$$\boxed{N_{events} \propto N_{tot}\, b_w\, \frac{1}{\Gamma_{K\pi}} \frac{d\Gamma_{K\pi}}{d\sqrt{s}}} \;\Rightarrow\; \boxed{\chi^2_\tau = \sum_{bins} \left(\frac{N_{events} - N_\tau}{\sigma_{N_\tau}} \right)^2}$$

with

Number of events/bin

bin width

$$\boxed{\frac{d\Gamma_{K\pi}}{d\sqrt{s}} = \frac{G_F^2 m_\tau^3}{32\pi^3 s} C_K^2 S_{EW} \left| f_+(0) V_{us} \right|^2 \left(1 - \frac{s}{m_\tau^2} \right)^2 \left[\left(1 + \frac{2s}{m_\tau^2} \right) q_{K\pi}^3(s) \left| \bar{f}_+(s) \right|^2 + \frac{3\Delta_{K\pi}^2}{4s} q_{K\pi}(s) \left| \bar{f}_0(s) \right|^2 \right]}$$

- Normalization disappears by taking the ratio $\frac{1}{\Gamma_{K\pi}} \frac{d\Gamma_{K\pi}}{d\sqrt{s}}$

  ⇨ fit independant of V_{us}

3.1 Kπ form factors from $\tau \to K\pi\nu_\tau$ and K_{l3} decays

- Fit to the $\tau \to K\pi\nu_\tau$ decay data
  - from *Belle [Epifanov et al'08]* (*BaBar?*)

$$N_{events} \propto N_{tot}\, b_w \frac{1}{\Gamma_{K\pi}} \frac{d\Gamma_{K\pi}}{d\sqrt{s}} \quad \Rightarrow \quad \chi^2_\tau = \sum_{bins} \left(\frac{N_{events} - N_\tau}{\sigma_{N_\tau}} \right)^2 \quad \text{with}$$

Number of events/bin

bin width

$$\frac{d\Gamma_{K\pi}}{d\sqrt{s}} = \frac{G_F^2 m_\tau^3}{32\pi^3 s} C_K^2 S_{EW} \left| f_+(0) V_{us} \right|^2 \left(1 - \frac{s}{m_\tau^2} \right)^2 \left[\left(1 + \frac{2s}{m_\tau^2} \right) q_{K\pi}^3(s) \left| \bar{f}_+(s) \right|^2 + \frac{3\Delta_{K\pi}^2}{4s} q_{K\pi}(s) \left| \bar{f}_0(s) \right|^2 \right]$$

- Possible combination with K_{l3} decay data fits *Flavianet Kaon WG'10*

$$\chi^2 = \chi^2_\tau + \begin{pmatrix} \lambda_+ - \lambda_+^{K_{l3}} \\ \ln C - \ln C^{K_{l3}} \end{pmatrix}^T V^{-1} \begin{pmatrix} \lambda_+ - \lambda_+^{K_{l3}} \\ \ln C - \ln C^{K_{l3}} \end{pmatrix}$$

+ sum-rules

Determination of the Kπ form factors

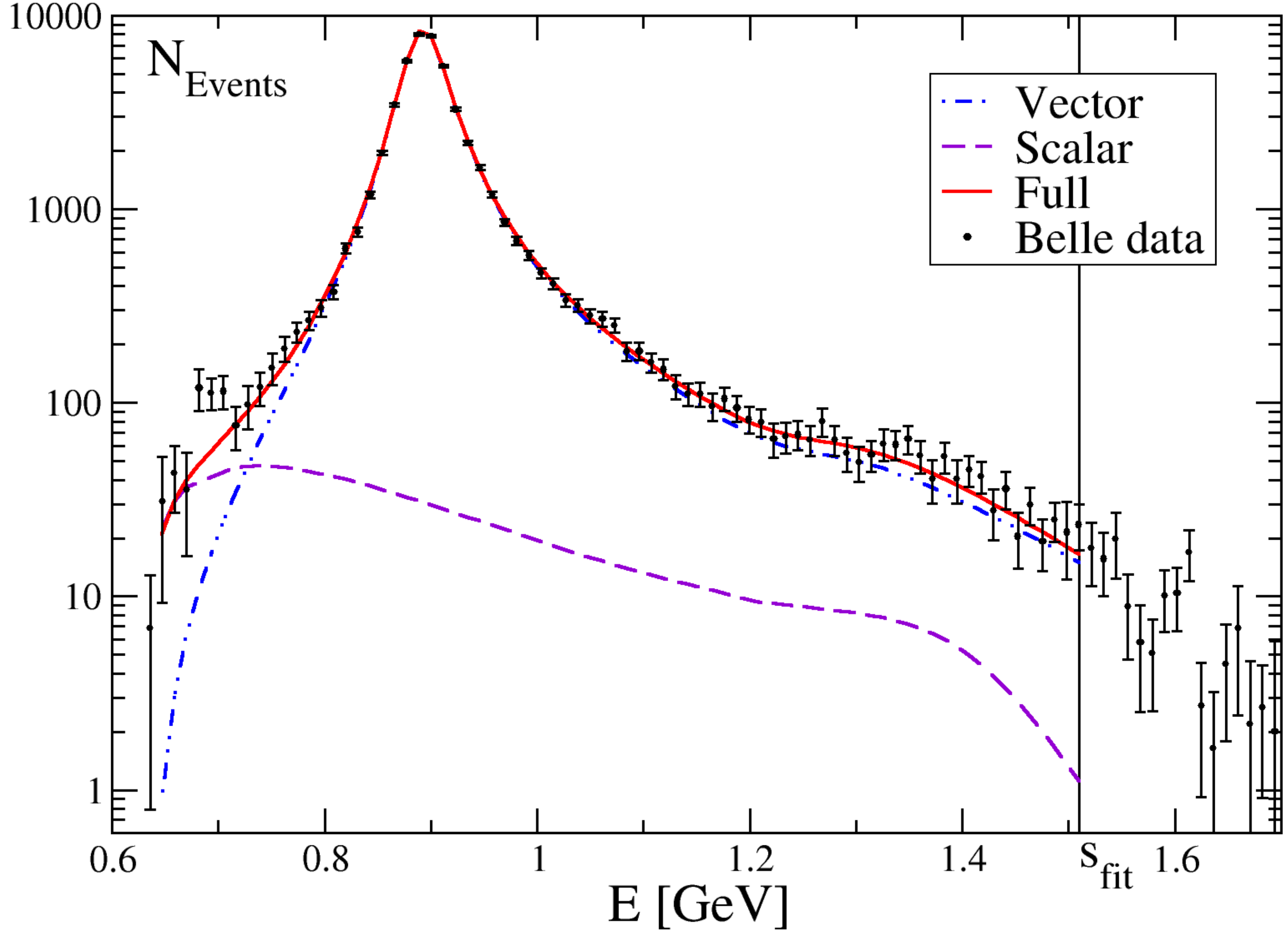
- Preliminary results :

Bernard, Boito, E.P., in progress

	$\tau \to K\pi\nu_\tau$ & $K_{\ell 3}$ Belle
$\ln C$	0.20340 ± 0.00894
$\lambda'_0 \times 10^3$	13.820 ± 0.824
m_{K^*} [MeV]	892.02 ± 0.21
Γ_{K^*} [MeV]	46.300 ± 0.426
$m_{K^{*\prime}}$ [MeV]	1282.7 ± 34.8
$\Gamma_{K^{*\prime}}$ [MeV]	217.29 ± 101.59
β	-0.0364 ± 0.0213
$\lambda'_+ \times 10^3$	25.613 ± 0.409
$\lambda''_+ \times 10^3$	1.2222 ± 0.0183
$\chi^2/d.o.f$	$60.4/68$

Determination of the Kπ form factors

Bernard, Boito, E.P., in progress



Phase space integrals

- From the results of the fit to the Belle + K_{l3} data :

Integral	result	error	exp	theo
$I^{\tau}_{K^0}$	0.50432	0.01721	0.01646	0.00501
$I^{e}_{K^0}$	0.15472	0.00022	0.00022	0.00000
$I^{\tau}_{K^0}/I^{e}_{K^0}$	3.25959	0.10875	0.10381	0.03240
$I^{\tau}_{K^+}$	0.52400	0.01929	0.01859	0.00516
$I^{e}_{K^+}$	0.15909	0.00025	0.00025	0.00000
$I^{\tau}_{K^+}/I^{e}_{K^+}$	3.29378	0.11874	0.11423	0.03240

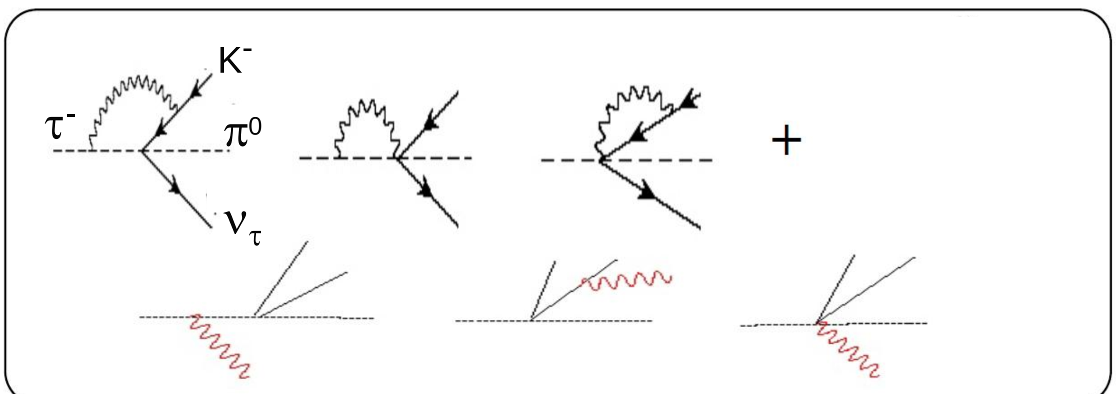
Precision : $I^{\tau}_{K^0}$ 3.4%, $I^{\tau}_{K^+}$ 3.7%
To be compared to the precision on I^{l}_{K} : 0.14 %
⇨ Should be improved with more *precise measurements*!

2.3 $\tau \to K\pi\nu_\tau$ decays

- Master formula for $\tau \to K\pi\nu_\tau$ (crossed channel) :

$$\boxed{\Gamma\left(\tau \to \overline{K}\pi\nu_\tau [\gamma]\right) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 S_{EW}^\tau \left|V_{us}\right|^2 \left| f_+^{K^0\pi^-}(0)\right|^2 I_K^\tau \left(1 + \boxed{\delta_{\mathrm{EM}}^{K\tau}} + \delta_{\mathrm{SU(2)}}^{K\pi}\right)^2}$$

- Theoretical inputs :
  - S_{ew} : Short distance electroweak correction ⇨ τ scale
  - $f_+(0)$: vector form factor at zero momentum transfer:
    ⇨ *ChPT* with resonances or *lattice*
  - I_K : Phase space integral ⇨ *dispersive parametrization*
  - $\delta_{\mathrm{EM}}^{Kl}$: Long-distance electromagnetic corrections



→ ChPT to $O(p^2e^2)$

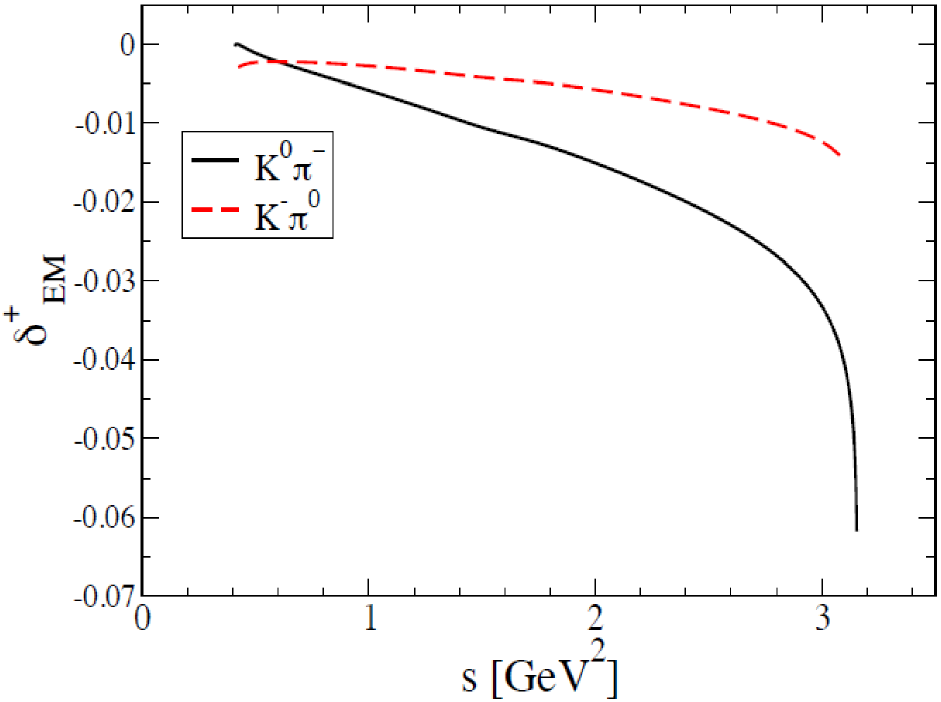
→ Counter-terms neglected

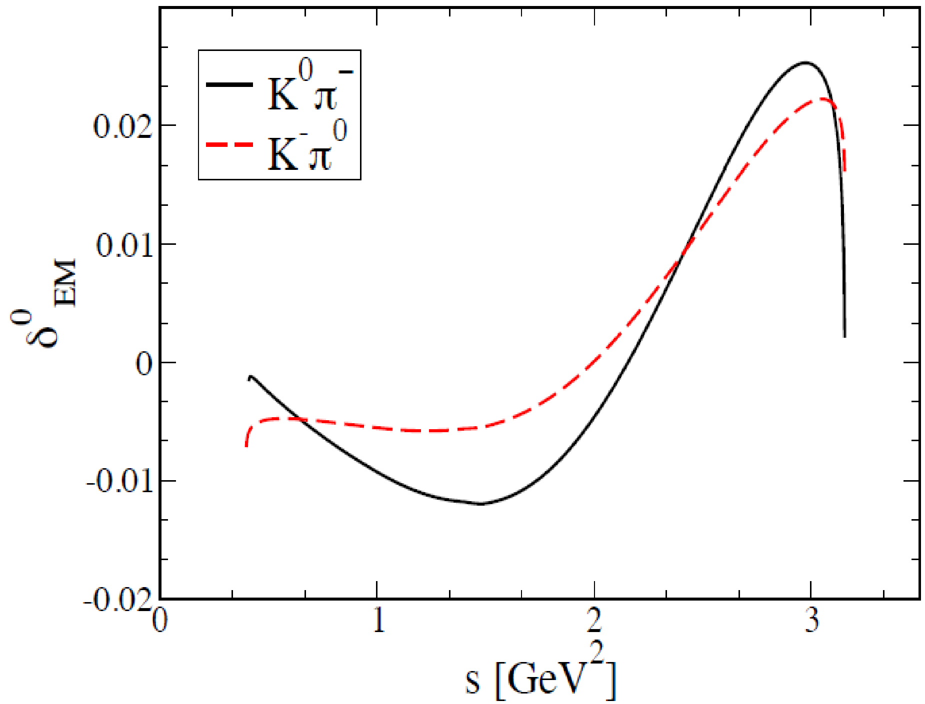
based on $\tau^- \to \pi^- \pi^0 \nu_\tau$

Cirigliano, Neufeld, Ecker'02

Long-distance electromagnetic corrections

- Form factors corrections:





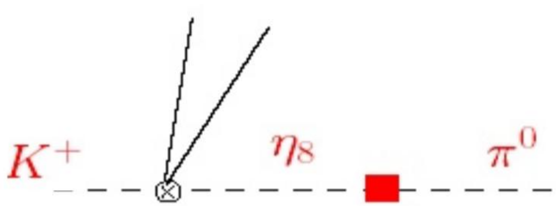
$\Rightarrow$ $\delta_{\mathrm{EM}}^{\bar{K}^0\tau} = (-0.15 \pm 0.2)\,\%$ and $\delta_{\mathrm{EM}}^{K^-\tau} = (-0.2 \pm 0.2)\,\%$

2.3 $\tau \to K\pi\nu_\tau$ decays

- Master formula for $\tau \to K\pi\nu_\tau$ (crossed channel) :

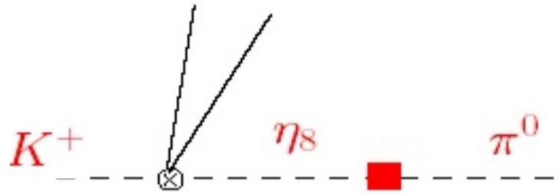
$$\boxed{\Gamma\left(\tau \to \overline{K}\pi\nu_\tau[\gamma]\right) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 S_{EW}^\tau \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_K^\tau \left(1 + \delta_{\mathrm{EM}}^{K\tau} + \boxed{\delta_{\mathrm{SU(2)}}^{K\pi}}\right)^2}$$

- Theoretical inputs :
  - S_{ew} : Short distance electroweak correction ⇨ τ scale
  - $f_+(0)$: vector form factor at zero momentum transfer:
    ⇨ *ChPT* with resonances or *lattice*
  - I_K : Phase space integral ⇨ *dispersive parametrization*
  - $\delta_{\mathrm{EM}}^{Kl}$: Long-distance electromagnetic corrections
  - $\tilde{\delta}_{\mathrm{SU(2)}}^{K\pi}$: Isospin breaking corrections $\boxed{\delta_{\mathrm{SU(2)}}^{K\pi} = \frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} - 1}$



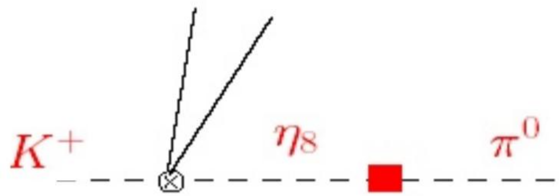
+ IB in one loop graphs + CT

Isospin breaking corrections



+ IB in one loop graphs + CT

approximated by



+ IB in the K^{*-} to $K\pi$ coupling

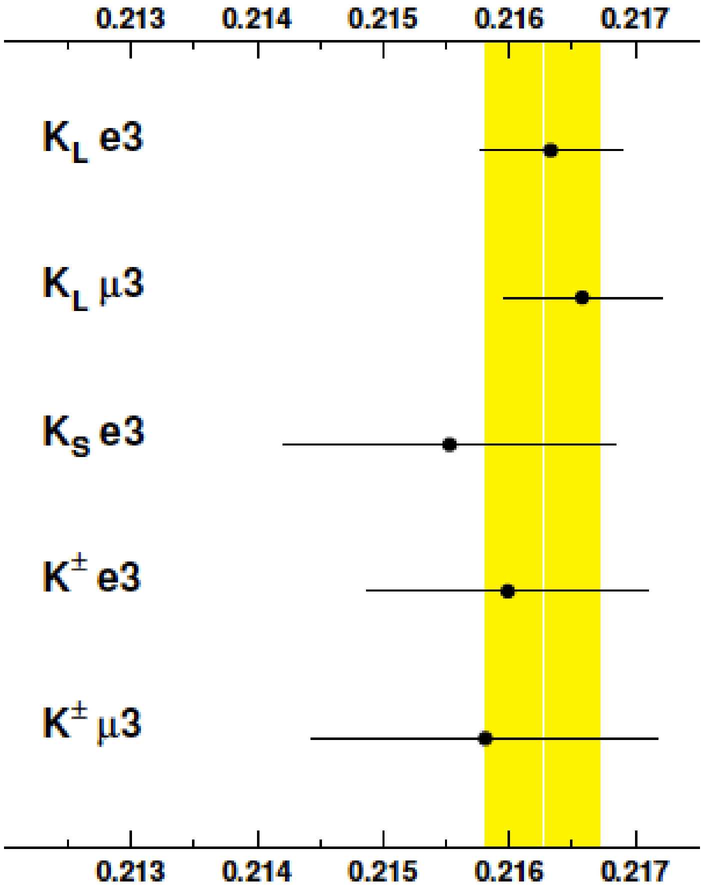
$$\Rightarrow \boxed{\frac{f_+^{K^-\pi^0}(s)}{f_+^{K^0\pi^-}(s)} = \left(1+\sqrt{3}\varepsilon\right)\left(1+\tilde{g}\frac{m_K^2}{(4\pi F_\pi)^2}\frac{s}{m_{K^*}^2}\varepsilon\right)} \quad \text{with} \quad \boxed{\varepsilon = \frac{\sqrt{3}}{4}\frac{m_d - m_u}{m_s - \hat{m}}}$$

$$\tilde{g} \in [-2, 2] \Rightarrow \tilde{\delta}^{K\pi}_{\mathrm{SU(2)}} = \pm 0.5\%$$

ε from *FLAG* $\Rightarrow$ $\boxed{\tilde{\delta}^{K\pi}_{\mathrm{SU(2)}} = \left(2.9 \pm 0.4_{\mathrm{mixing}} \pm 0.5\right)\%}$

2.4 Extraction of $f_+(0)$ |Vus|

- Results for K_{l3}: *FLAVIAnet Kaon WG*



$$f_+(0)|V_{us}| = 0.2163 \pm 0.0005$$

Very good precision : *0.23%!*

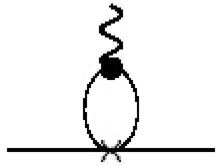
- Result for $\tau \to K\pi\nu_\tau$: $f_+(0)|V_{us}| = 0.2140 \pm 0.0041_{I_K} \pm 0.0031_{exp}$

2.5 $f_+(0)$

- CVC + Ademollo-Gato theorem ⇨ $\boxed{f_+^{K^0\pi^-}(0) - 1 = O\left((m_s - m_u)^2\right)}$

- Chiral expansion : $\boxed{f_+^{K^0\pi^-}(0) = 1 + f_{p^4} + f_{p^6} + \ldots}$

  ➢ At $O(p^4)$: One loop graphs



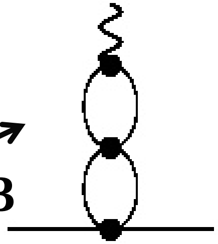
  1^{rst} order in m_q and second in $(m_s - m_u)$ ⇨ $\boxed{f_{p^4} \sim \frac{(m_s - m_u)^2}{m_s}}$

  Computed exactly: no local operators, UV finite, free of uncertainties

  ⇨ $\boxed{f_{p^4} = -0.027}$ *Gasser & Leutwyler'85*

  ➢ At $O(p^6)$: Two-loop graphs + One loop graphs x L_i + tree p^6 (C_i)

$f_{p^6}^{2-loops}(m_\rho) = 0.0113$



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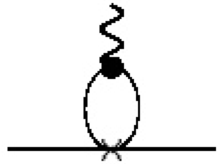


+

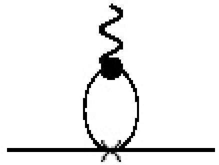
Bijnens & Talavera'03

2.5 $f_+(0)$

- CVC + Ademollo-Gato theorem ⇨ $f_+^{K^0\pi^-}(0) - 1 = O\left((m_s - m_u)^2\right)$

- Chiral expansion : $f_+^{K^0\pi^-}(0) = 1 + f_{p^4} + f_{p^6} + \ldots$

  - At $O(p^4)$: One loop graphs



    1rst order in m_q and second in $(m_s - m_u)$ ⇨ $f_{p^4} \sim \frac{(m_s - m_u)^2}{m_s}$

    Computed exactly: no local operators, UV finite, free of uncertainties

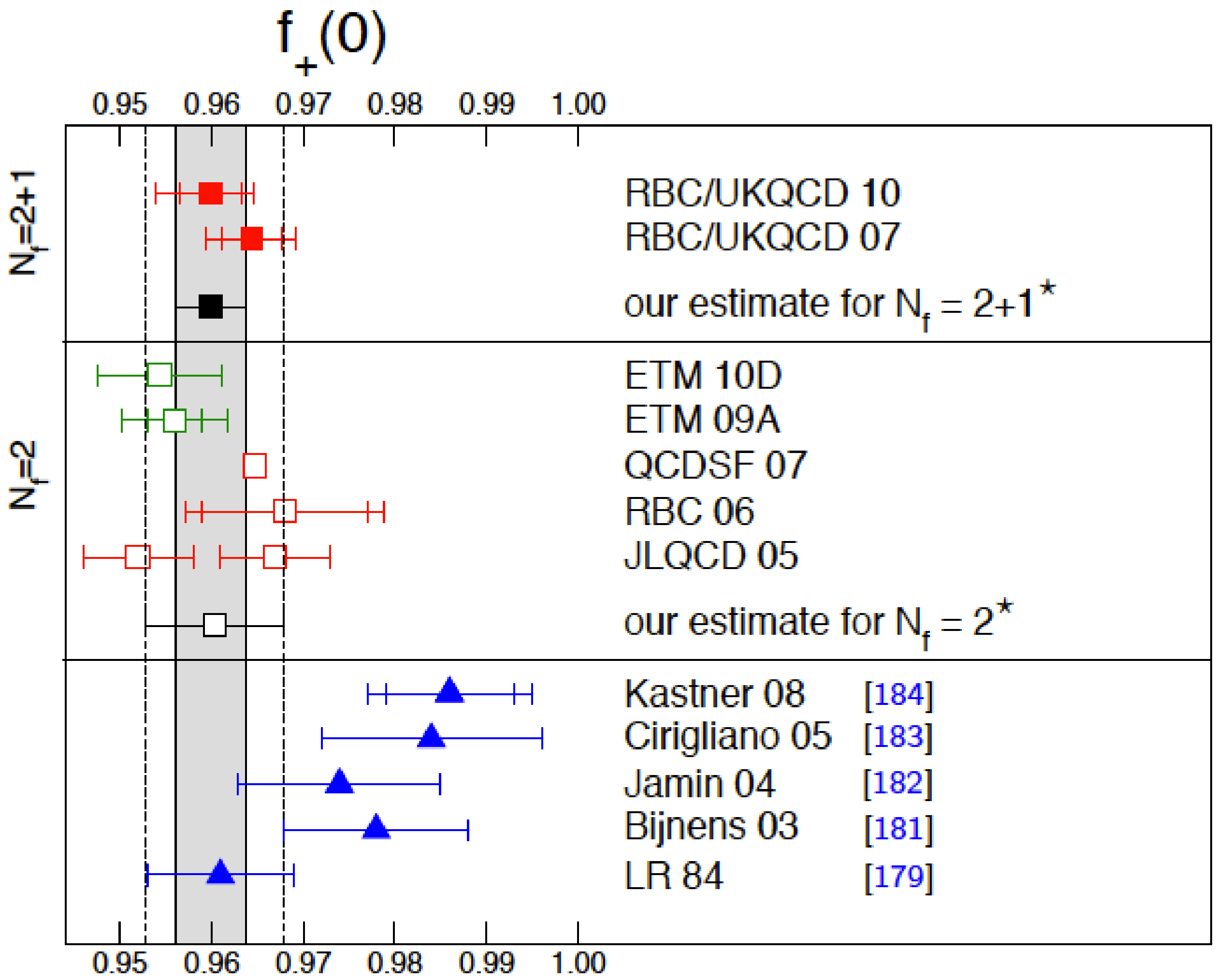
    ⇨ $f_{p^4} = -0.027$

  - At $O(p^6)$: Two-loop graphs + One loop graphs x L_i + tree p^6 (C_i)

⇨ Difficulty: LECs not fixed by theory, rely on models

2.5 $f_+(0)$

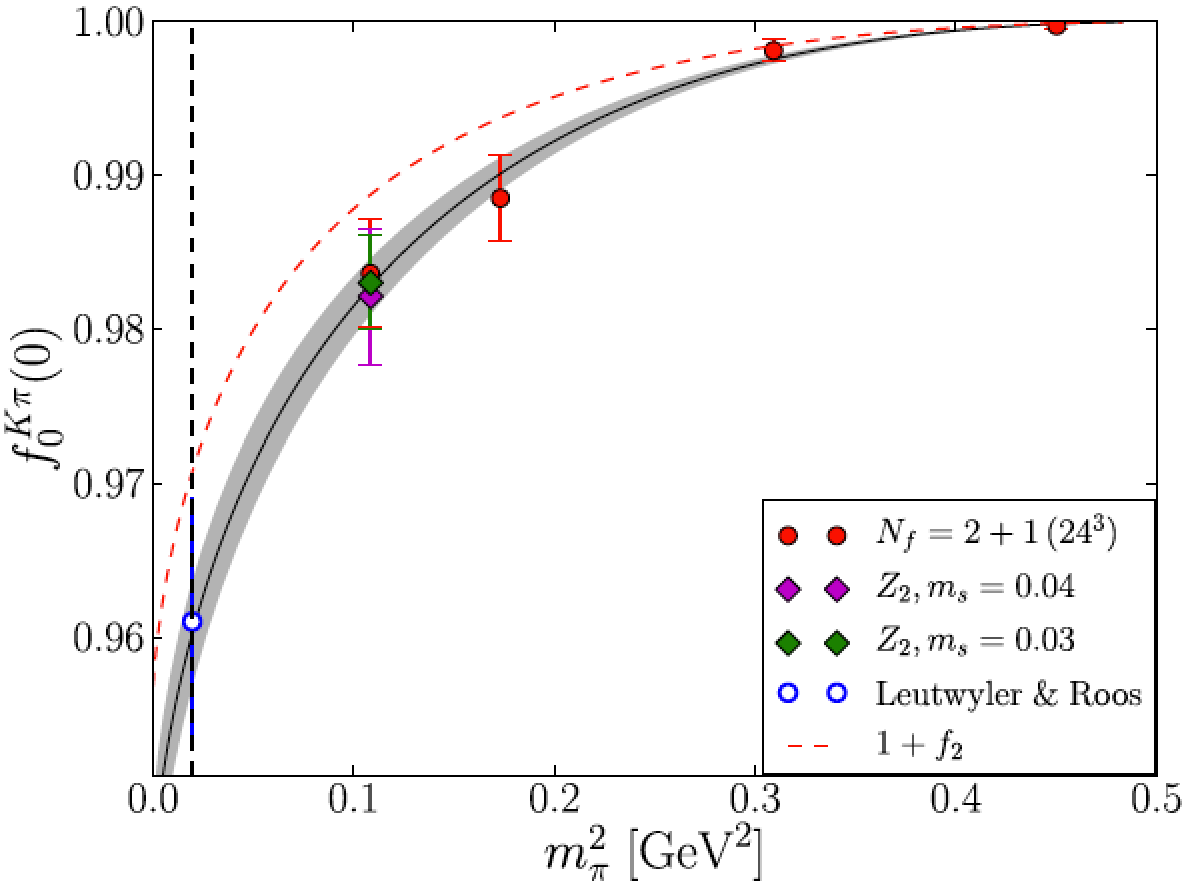
- Comparison of lattice QCD results with ChPT + models



FLAG'10

2.6 $f_+(0)$ and Vus

- N_f=2+1 lattice result *RBC-UKQCD'10*

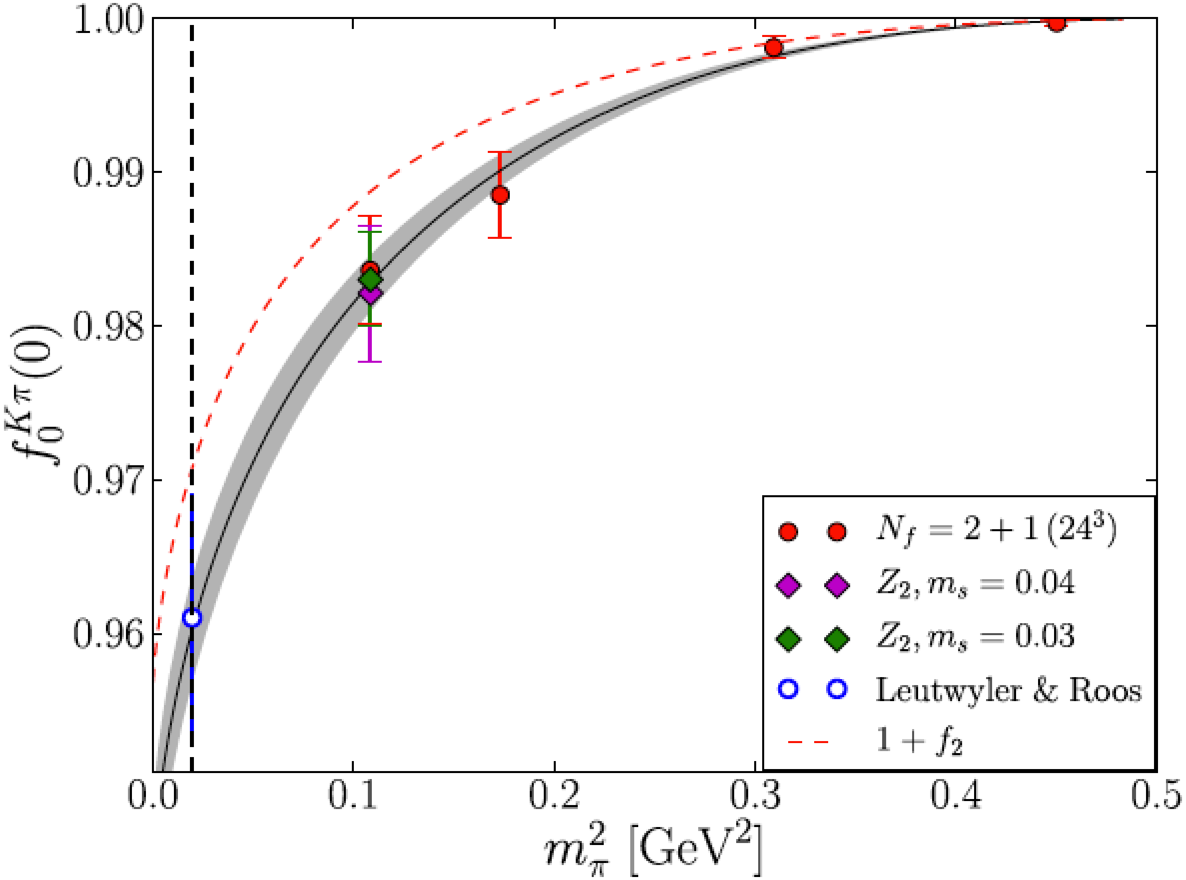


$\Rightarrow$ $\boxed{f_+(0) = 0.959(5)}$

- K_{l3} decays: $\boxed{f_+(0)|V_{us}| = 0.2163 \pm 0.0005}$ $\Rightarrow$ $\boxed{|V_{us}| = 0.2255 \pm 0.0013}$

2.6 $f_+(0)$ and Vus

- N_f=2+1 lattice result *RBC-UKQCD'10*



$$f_+(0) = 0.959(5)$$

- τ decays: $f_+(0)|V_{us}| = 0.2140 \pm 0.0051$ ⇒ $|V_{us}| = 0.2233 \pm 0.0055$

3. V_{us} from leptonic decays

3.1 Introduction

- From K_{l2}/π_{l2}:

$$\boxed{\frac{\Gamma\left(K\to\mu\nu[\gamma]\right)}{\Gamma\left(\pi\to\mu\nu[\gamma]\right)}=\frac{m_{K^\pm}}{m_{\pi^\pm}}\frac{\left(1-m_\mu^2/m_{K^\pm}^2\right)}{\left(1-m_\mu^2/m_{\pi^\pm}^2\right)}\frac{f_K^2}{f_\pi^2}\frac{|V_{us}|^2}{|V_{ud}|^2}\left(1+\delta_{\mathrm{EM}}\right)}$$

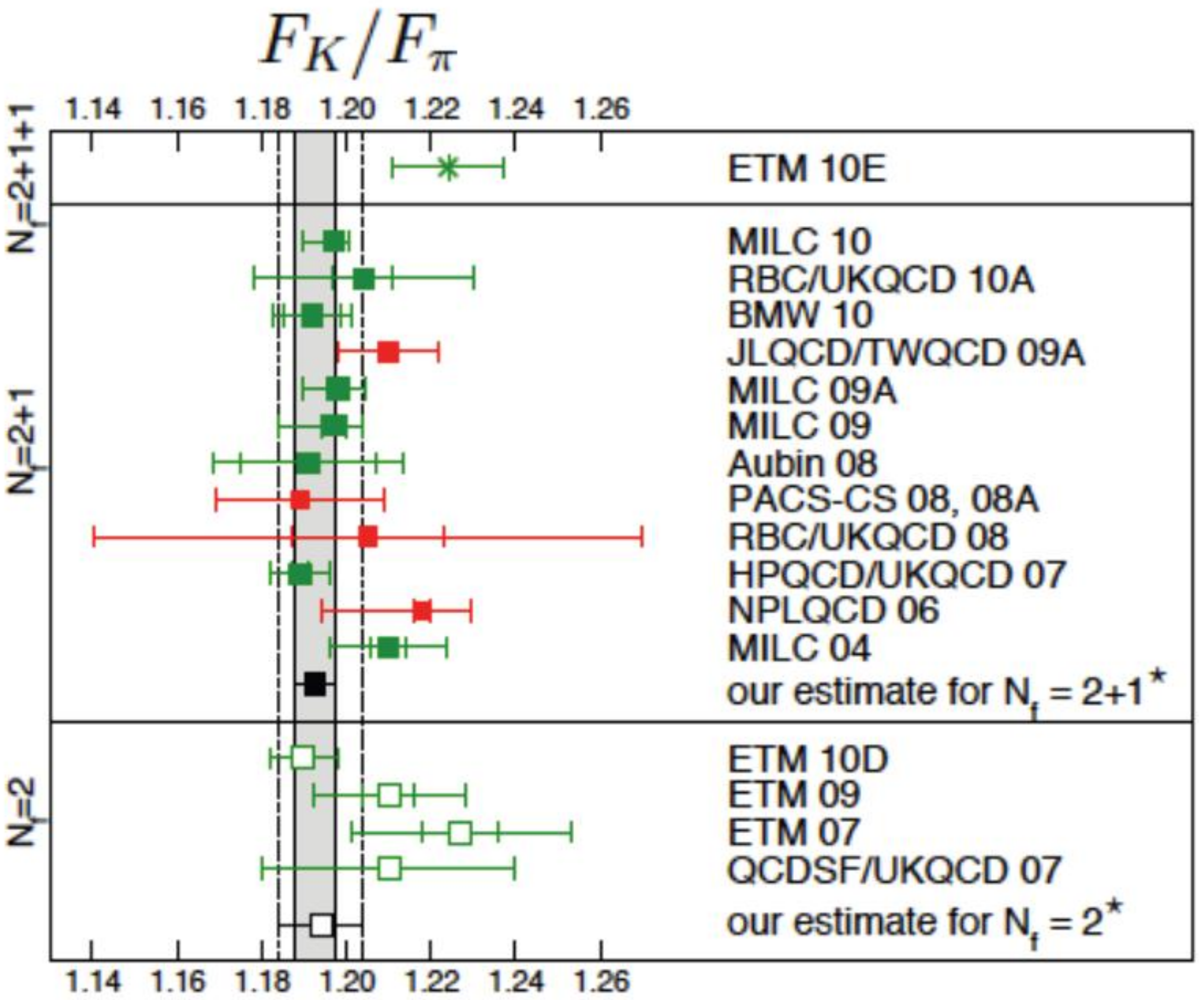
- From $\tau \to K/\pi\nu_\tau$

$$\boxed{\frac{\Gamma\left(\tau\to K\nu[\gamma]\right)}{\Gamma\left(\tau\to\pi\nu[\gamma]\right)}=\frac{\left(1-m_{K^\pm}^2/m_\tau^2\right)}{\left(1-m_{\pi^\pm}^2/m_\tau^2\right)}\frac{f_K^2}{f_\pi^2}\frac{|V_{us}|^2}{|V_{ud}|^2}\left(1+\delta_{\mathrm{LD}}\right)}$$

➡ Inputs needed :

→ Experimental BRs
→ F_K/F_π
→ Electromagnetic and isospin breaking corrections

3.2 F_K/F_π from lattice QCD



$$\frac{F_K}{F_\pi} = 1.193 \pm 0.006$$

FLAG'10

3.3 Results

- From K_{l2}/π_{l2}:

$$\boxed{\frac{\Gamma\left(K\to\mu\nu[\gamma]\right)}{\Gamma\left(\pi\to\mu\nu[\gamma]\right)}=\frac{m_{K^\pm}}{m_{\pi^\pm}}\frac{\left(1-m_\mu^2/m_{K^\pm}^2\right)}{\left(1-m_\mu^2/m_{\pi^\pm}^2\right)}\frac{f_K^2}{f_\pi^2}\frac{\left|V_{us}\right|^2}{\left|V_{ud}\right|^2}\left(1+\delta_{\text{EM}}\right)}$$

- δ_{EM} : Long-distance electromagnetic corrections
  Computed to $O(p^2e^2)$ in ChPT, UV finite and no LECs
  Uncertainties due to higher orders

$\Rightarrow$ $\boxed{\delta_{\text{EM}}=-0.0069\pm0.0017}$ *Knecht et al.'06, Cirigliano & Neufeld'11*

- Brs from *Flavianet Kaon WG'10*
- F_K/F_π from lattice *FLAG'10*
- V_{ud} : $\left|V_{ud}\right|=0.97425(22)$ *Towner & Hardy'08*

$\Rightarrow$ $\left|\frac{V_{us}}{V_{ud}}\right|=0.2312(13)$ $\Rightarrow$ $\boxed{\left|V_{us}\right|=0.2252\pm0.0013}$

3.3 Results

- $\tau \to K/\pi\nu_\tau$:

$$\boxed{\frac{\Gamma\left(\tau \to K\nu[\gamma]\right)}{\Gamma\left(\tau \to \pi\nu[\gamma]\right)} = \frac{\left(1-m_{K^\pm}^2 / m_\tau^2\right)}{\left(1-m_{\pi^\pm}^2 / m_\tau^2\right)} \frac{f_K^2}{f_\pi^2} \frac{|V_{us}|^2}{|V_{ud}|^2} \left(1+\delta_{\mathrm{LD}}\right)}$$

  - δ_{LD} : Long-distance radiative corrections

    ⇨ $\boxed{\delta_{\mathrm{LD}} = 1.0003 \pm 0.0044}$

  - Brs from *HFAG'12*
  - F_K/ F_π from lattice *FLAG'10*
  - V_{ud} : $|V_{ud}| = 0.97425(22)$ *Towner & Hardy'08*

⇨ $\boxed{|V_{us}| = 0.2229 \pm 0.0021}$

3.3 Results

- $\tau \to K\nu_\tau$ absolute :

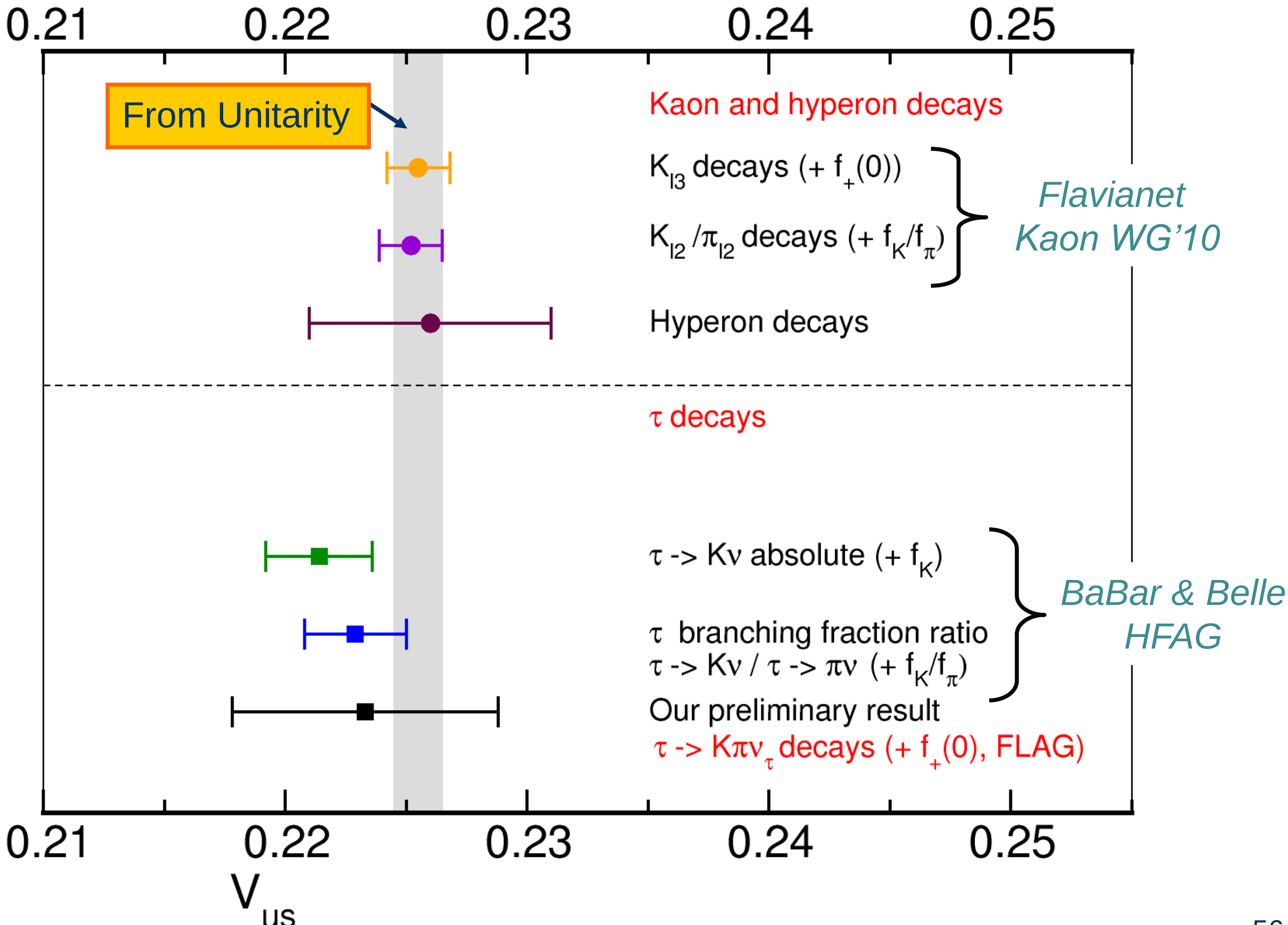
$$\boxed{BR\left(\tau \to K\nu[\gamma]\right) = \frac{G_F^2 m_\tau^3 S_{EW} \tau_\tau}{16\pi h}\left(1 - \frac{m_{K^\pm}^2}{m_\tau^2}\right) f_K^2 \left|V_{us}\right|^2}$$

In principle less precise than ratios

- Inputs from *HFAG*
- F_K from lattice average $\boxed{F_K = (1.561 \pm 0.001)\ \mathrm{MeV}}$

Laiho, Lunghi, Van de Water

$\Rightarrow$ $\boxed{\left|V_{us}\right| = 0.2214 \pm 0.0022}$


0.21 0.22 0.23 0.24 0.25
From Unitarity
Kaon and hyperon decays
K_{l3} decays (+ $f_+(0)$)
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Flavianet Kaon WG'10
Hyperon decays
τ decays
τ -> Kν absolute (+ f_K)
τ branching fraction ratio
τ -> Kν / τ -> πν (+ f_K/f_π)
BaBar & Belle HFAG
Our preliminary result
τ -> Kπν_τ decays (+ $f_+(0)$, FLAG)
V_{us}

4. V_{us} from inclusive hadronic τ decays

4.1 Introduction

- Observable studied

$$\boxed{R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau\, h^-(\gamma)\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \overline{\nu}_e(\gamma)\right)}} \quad \text{and} \quad \frac{dR_\tau}{ds}$$

- Decomposition as a function of observed and separated final states

$$R_\tau = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

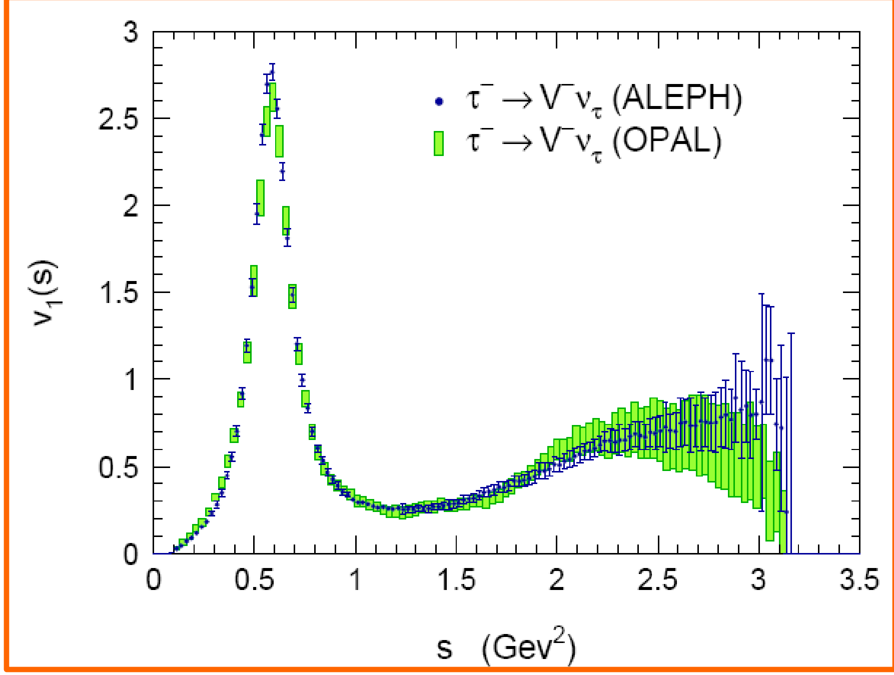
$$R_{\tau,V} \Rightarrow \boxed{\tau^- \to \nu_\tau + h_{v,s=0}}$$

(even number of pions)

$$R_{\tau,A} \Rightarrow \tau^- \to \nu_\tau + h_{A,s=0}$$

(odd number of pions)

$$R_{\tau,S} \Rightarrow \tau^- \to \nu_\tau + h_{V+A,s=1}$$



4.1 Introduction

- Observable studied

$$\boxed{R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau\, h^-(\gamma)\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e(\gamma)\right)}} \quad \text{and} \quad \frac{dR_\tau}{ds}$$

- Decomposition as a function of observed and separated final states

$$R_\tau = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

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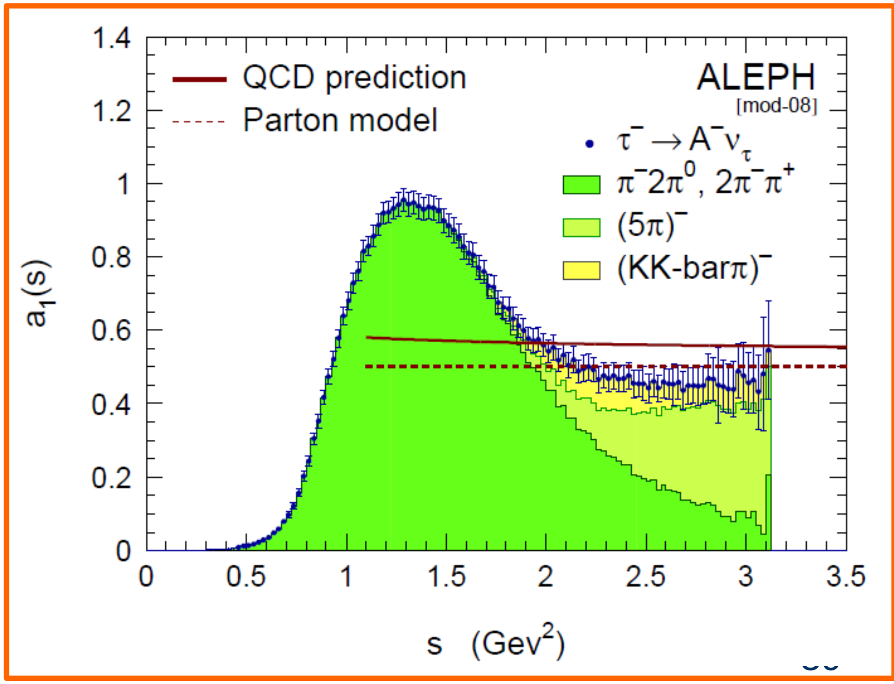
(even number of pions)

$R_{\tau,A} \Rightarrow \boxed{\tau^- \to \nu_\tau + h_{A,s=0}}$

(odd number of pions)

$R_{\tau,S} \Rightarrow \tau^- \to \nu_\tau + h_{V+A,s=1}$



4.1 Introduction

- Observable studied

$$\boxed{R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau\, h^-(\gamma)\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e(\gamma)\right)}}$$ and $$\frac{dR_\tau}{ds}$$

- Decomposition as a function of observed and separated final states

$$R_\tau = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

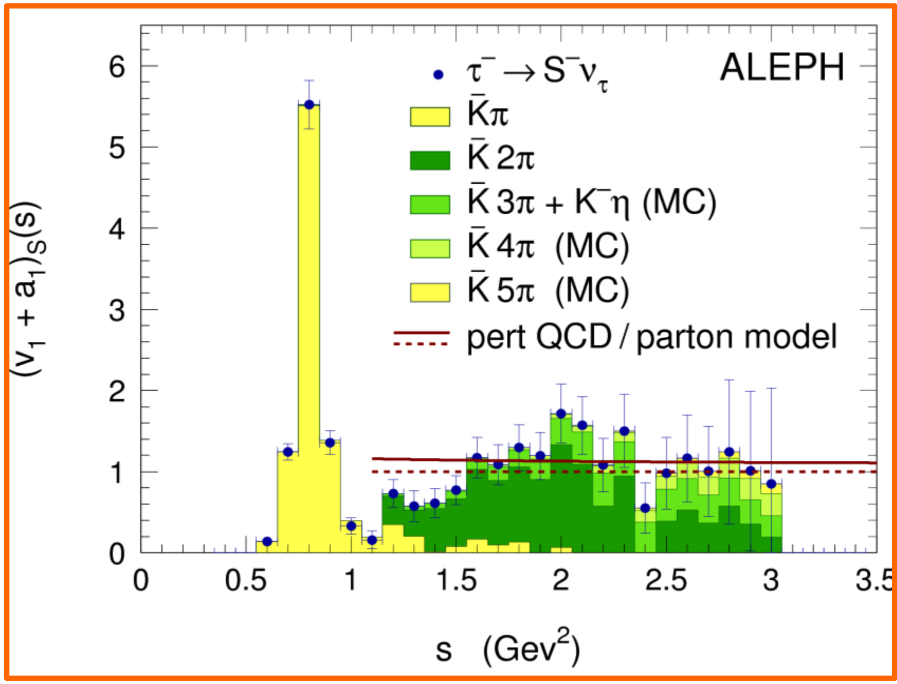
$R_{\tau,V}$ ⇨ $\tau^- \to \nu_\tau + h_{V,s=0}$

(even number of pions)

$R_{\tau,A}$ ⇨ $\tau^- \to \nu_\tau + h_{A,s=0}$

(odd number of pions)

$R_{\tau,S}$ ⇨ $\boxed{\tau^- \to \nu_\tau + h_{V+A,s=1}}$



4.1 Introduction

- Extraction of V_{us}

$$R_\tau^{kl} = N_C S_{EW} \left\{ \left(|V_{us}|^2 + |V_{ud}|^2 \right) \left[1 + \delta^{(0)} \right] + \sum_{D \geq 2} \left[|V_{ud}|^2 \delta_{ud}^{(D)} + |V_{us}|^2 \delta_{us}^{(D)} \right] \right\}$$

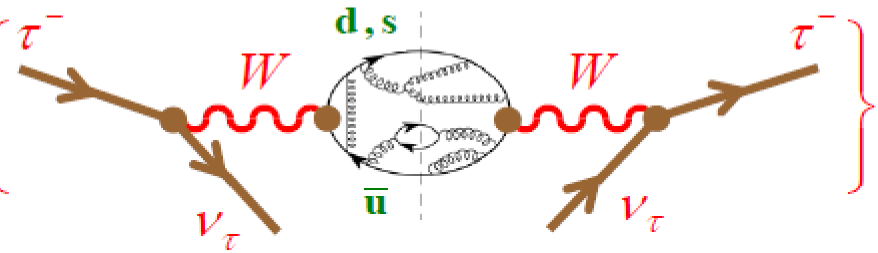
QCD part determined using *OPE*

- Use instead $$\delta R_\tau \equiv \frac{R_{\tau,V+A}}{|V_{ud}|^2} - \frac{R_{\tau,S}}{|V_{us}|^2}$$

SU(3) breaking quantity, strong dependence in m_s

⇨ $$|V_{us}|^2 = \frac{R_{\tau,S}}{\dfrac{R_{\tau,V+A}}{|V_{ud}|^2} - \delta R_{\tau,th}}$$

4.2 Theoretical Method

- Optical theorem: $\Gamma_{\tau\to\nu_\tau+\text{had}} \sim \mathrm{Im}\left\{ \right.$  $\left. \right\}$

$$\boxed{\Gamma \,\alpha\, \mathrm{Im}\,\Pi^{\mu\nu}(q)} \Rightarrow \Pi^{\mu\nu}(q) = i\int d^4x\, e^{iqx} \left\langle 0 \left| T\left\{ J^\mu(x) J^{\nu\dagger}(0)\right\}\right| 0 \right\rangle$$

- Lorentz decomposition: $\Pi^{\mu\nu}(q) = \left(-g_{\mu\nu}\, q^2 + q^\mu q^\nu\right) \Pi^1(q^2) + q^\mu q^\nu\, \Pi^0(q^2)$

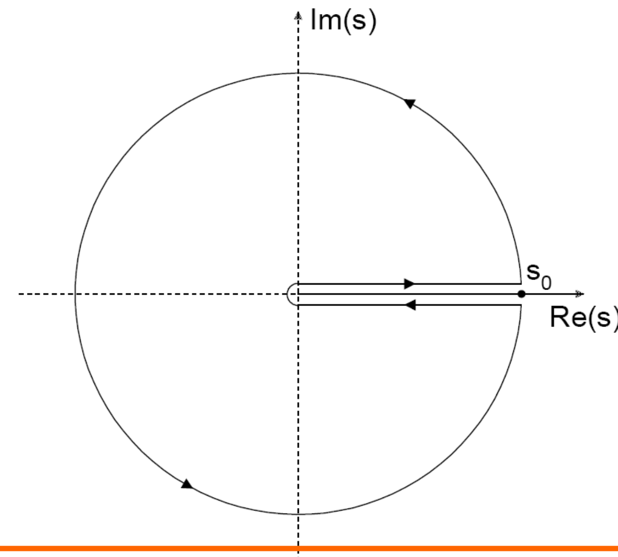
$$\Rightarrow \boxed{R_\tau(m_\tau^2) = 12\pi S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\mathrm{Im}\,\Pi^{(1)}(s+i\varepsilon) + \mathrm{Im}\,\Pi^{(0)}(s+i\varepsilon)\right]}$$

$$\boxed{\Pi^{(J)}(s) = \left|V_{ud}\right|^2 \left(\Pi^{(J)}_{ud,VV}(s) + \Pi^{(J)}_{ud,AA}(s)\right) + \left|V_{us}\right|^2 \left(\Pi^{(J)}_{us,VV}(s) + \Pi^{(J)}_{us,AA}(s)\right)}$$

4.3 Correlators

Braaten, Narison, Pich'92

- Analyticity: Π analytic in the entire complex plane except for s real positive



⇨ Cauchy theorem:

$$\frac{1}{\pi}\int_0^{s_0} ds\, g(s)\ \mathrm{Im}\,\Pi(s) = -\frac{1}{2\pi i}\oint_{|s|=s_0} ds\, g(s)\,\Pi(s)$$

⇨ $$R_\tau(m_\tau^2) = 6i\pi S_{EW}\oint_{|s|=m_\tau^2}\frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2\left[\left(1+2\frac{s}{m_\tau^2}\right)\Pi^{(1)}(s)+\Pi^{(0)}(s)\right]$$

- Sufficient high energy for *Operator Product Expansion*
Kinematic factor ⇨ decreases the weight close to the real axis where Π has a cut

4.4 Operator Product Expansion

Braaten, Narison, Pich'92

$$\Pi^{(J)}(s) = \sum_{D=0,2,4...} \frac{1}{(-s)^{D/2}} \sum_{\dim O=D} \mathcal{C}^{(J)}(s,\mu) \, \langle O_D(\mu) \rangle$$

Wilson coefficients

Operators

μ separation scale between short and long distances

- D=0: Perturbative contributions
- D=2: Quark mass corrections
- D=4: Non perturbative physics operators, $\left\langle \frac{\alpha_s}{\pi} GG \right\rangle$, $\left\langle m_j \bar{q}_i q_i \right\rangle$
- D=6: 4 quarks operators, $\left\langle \bar{q}_i \Gamma_1 q_j \bar{q}_j \Gamma_2 q_i \right\rangle$
- D≥8: Neglected terms, expected to be small…

$$\Rightarrow \quad R_{\tau,V+A}(s_0) = 3 |V^{ud}|^2 S_{EW} \left(1 + \delta^{(0)} + \sum_{D=2,4..} \delta_{ud}^{(D)} \right)$$

similar for $R_{\tau,S}(s_0)$

4.5 δR_τ

$$\boxed{\delta R_\tau \equiv \frac{R_{\tau,V+A}}{|V_{ud}|^2} - \frac{R_{\tau,S}}{|V_{us}|^2}} \approx N_C S_{EW} \sum_{D\geq 2} \left[\delta_{ud}^{(D)} - \delta_{us}^{(D)}\right]$$

- $\delta_{ij}^{(2)}$ known up to $O\left(\alpha_S^3\right)$ for both J=L and J=L+T

  Chetyrkin, Gorishry, Kataev, Larin, Sugurladze; Baikov, Chetyrkin,Kuehn
  Becchi, Narison, de Rafael; Bernreuther, Wetzel

- $\delta_{ij}^{(4)}$ fully included , e.g. m_j^4 / m_τ^4 , $\langle m_j \bar{q}_i q_i \rangle / m_\tau^4$
- $\delta_{ij}^{(6)}$ estimated (VSA) to be of order or smaller than errors on D=4
- D≥8: Neglected terms, expected to be small…

$\Rightarrow$ $$\boxed{\delta R_\tau \approx 24 \frac{m_s^2(m_\tau^2)}{m_\tau^2} \Delta\left(\alpha_s\right)}$$

but perturbatives series for L behave very badly!

4.6 Longitudinal contribution

- Longitudinal series does not converge fast enough!

  ⇨ Replace scalar and pseudoscalar QCD correlators with phenomenology

  Results: uncertainties very much reduced for J=L !

E. Gamiz, CKM'12

	$R_{us,A}^{00,L}$	$R_{us,V}^{00,L}$	$R_{ud,A}^{00,L}$
Theory:	-0.144 ± 0.024	-0.028 ± 0.021	$-(7.79 \pm 0.14) \cdot 10^{-3}$
Phenom:	-0.135 ± 0.003	-0.028 ± 0.004	$-(7.77 \pm 0.08) \cdot 10^{-3}$

4.7 Results

$$\boxed{|V_{us}|^2 = \frac{R_{\tau,S}}{\frac{R_{\tau,V+A}}{|V_{ud}|^2} - \delta R_{\tau,th}}}$$

- $\delta R_{\tau,\mathrm{theo}}$ determined from OPE (L+T) + phenomenology

$\Rightarrow$ $$\boxed{\delta R_{\tau,th} = \underbrace{(0.1544 \pm 0.0037)}_{J=0} + (9.3 \pm 3.4)\, m_s^2 + (0.0034 \pm 0.0028)}$$

Gamiz, Jamin, Pich, Prades, Schwab'07, Maltman'11

Input : m_s $\Rightarrow$ $m_s\left(2\ \mathrm{GeV}, \overline{\mathrm{MS}}\right) = 93.4 \pm 1.1$ lattice average

Laiho, Lunghi, Van de Water

- Tau data : $R_{\tau,S} = 0.1612(28)$ and $R_{\tau,V+A} = 3.4671(84)$ *HFAG'12*

- V_{ud} : $|V_{ud}| = 0.97425(22)$ *Towner & Hardy'08*

4.7 Results

$$\boxed{|V_{us}|^2 = \frac{R_{\tau,S}}{\frac{R_{\tau,V+A}}{|V_{ud}|^2} - \delta R_{\tau,th}}}$$

- $\delta R_{\tau,th} = 0.239(30)$

$$\Rightarrow \quad \boxed{|V_{us}| = 0.2173 \pm 0.0020_{\text{exp}} \pm 0.0010_{\text{th}}}$$

- Determination dominated by experimental uncertainties! Contrary to V_{us} from K_{l3}, dominated by uncertainties on $f_+(0)$

- 2.6σ away from unitarity!

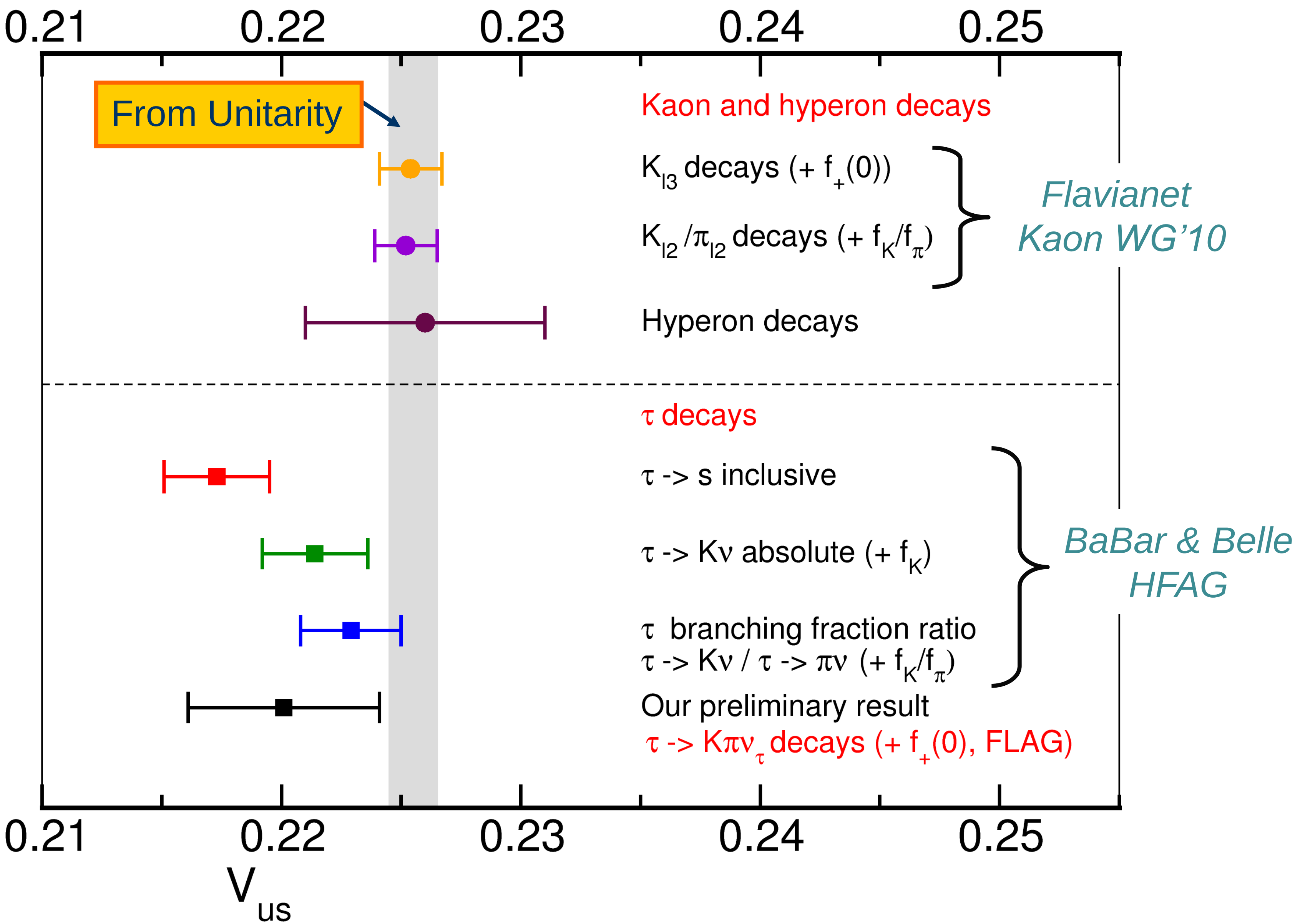
4.8 Experimental problem or hint of New Physics?

- Smaller $\tau \to$ K branching ratios $\Rightarrow$ smaller $R_{\tau,S}$ $\Rightarrow$ smaller $\boxed{V_{us}}$

$$\boxed{R_{\tau,S}\Big|_{\text{old}} = 0.1686(47)} \Rightarrow \boxed{R_{\tau,S}\Big|_{\text{new}} = 0.1612(28)}$$

$$\boxed{|V_{us}|_{\text{old}} = 0.2214 \pm 0.0031_{\text{exp}} \pm 0.0010_{\text{th}}} \Rightarrow \boxed{|V_{us}|_{\text{new}} = 0.2173 \pm 0.0020_{\text{exp}} \pm 0.0010_{\text{th}}}$$

Missing modes at B factories?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τ branching fraction ratio
τ -> Kν / τ -> πν (+ f_K/f_π)
Our preliminary result
τ -> Kπν_τ decays (+ $f_+(0)$, FLAG)
V_{us}

5. New determination of V_{us} from predicting τ strange BRs

Antonelli, Cirigliano, Lusiani, E.P. in progress

5.1 Introduction

- Modes measured in the strange channel for $\tau \to s$:

HFAG'12

Branching fraction	HFAG Winter 2012 fit
$\Gamma_{10} = K^-\nu_\tau$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$
$\Gamma_{16} = K^-\pi^0\nu_\tau$	$(0.4322 \pm 0.0149) \cdot 10^{-2}$
$\Gamma_{23} = K^-2\pi^0\nu_\tau$ (ex. K^0)	$(0.0630 \pm 0.0222) \cdot 10^{-2}$
$\Gamma_{28} = K^-3\pi^0\nu_\tau$ (ex. K^0, η)	$(0.0419 \pm 0.0218) \cdot 10^{-2}$
$\Gamma_{35} = \pi^-\overline{K}^0\nu_\tau$	$(0.8206 \pm 0.0182) \cdot 10^{-2}$
$\Gamma_{40} = \pi^-\overline{K}^0\pi^0\nu_\tau$	$(0.3649 \pm 0.0108) \cdot 10^{-2}$
$\Gamma_{44} = \pi^-\overline{K}^0\pi^0\pi^0\nu_\tau$	$(0.0269 \pm 0.0230) \cdot 10^{-2}$
$\Gamma_{53} = \overline{K}^0h^-h^-h^+\nu_\tau$	$(0.0222 \pm 0.0202) \cdot 10^{-2}$
$\Gamma_{128} = K^-\eta\nu_\tau$	$(0.0153 \pm 0.0008) \cdot 10^{-2}$
$\Gamma_{130} = K^-\pi^0\eta\nu_\tau$	$(0.0048 \pm 0.0012) \cdot 10^{-2}$
$\Gamma_{132} = \pi^-\overline{K}^0\eta\nu_\tau$	$(0.0094 \pm 0.0015) \cdot 10^{-2}$
$\Gamma_{151} = K^-\omega\nu_\tau$	$(0.0410 \pm 0.0092) \cdot 10^{-2}$
$\Gamma_{801} = K^-\phi\nu_\tau(\phi \to KK)$	$(0.0037 \pm 0.0014) \cdot 10^{-2}$
$\Gamma_{802} = K^-\pi^-\pi^+\nu_\tau$ (ex. K^0, ω)	$(0.2923 \pm 0.0068) \cdot 10^{-2}$
$\Gamma_{803} = K^-\pi^-\pi^+\pi^0\nu_\tau$ (ex. K^0, ω, η)	$(0.0411 \pm 0.0143) \cdot 10^{-2}$
$\Gamma_{110} = X_s^-\nu_\tau$	$(2.8746 \pm 0.0498) \cdot 10^{-2}$

5.1 Introduction

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$\Gamma_{130} = K^- \pi^0 \eta \nu_\tau$	$(0.0048 \pm 0.0012) \cdot 10^{-2}$
$\Gamma_{132} = \pi^- \overline{K}^0 \eta \nu_\tau$	$(0.0094 \pm 0.0015) \cdot 10^{-2}$
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$\Gamma_{801} = K^- \phi \nu_\tau (\phi \to KK)$	$(0.0037 \pm 0.0014) \cdot 10^{-2}$
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~70% of the decay modes crossed channels from Kaons!

5.1 Introduction

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Branching fraction	HFAG Winter 2012 fit
$\Gamma_{10} = K^- \nu_\tau$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$
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$\Gamma_{110} = X_s^- \nu_\tau$	$(2.8746 \pm 0.0498) \cdot 10^{-2}$

~70% of the decay modes crossed channels from Kaons!

Up to ~90% Including the 2π modes

5.2 Prediction of the strange Brs

Antonelli, Cirigliano, Lusiani, E.P. in progress

- The Brs of these 3 modes can be predicted using Kaon Brs very precisely measured + form factor information

➢ $\tau \to K\nu_\tau$:

$$\boxed{\mathrm{BR}(\tau \to K\nu_\tau) = \frac{m_\tau^3}{2m_K m_\mu^2} \boxed{\frac{S_{\mathrm{EW}}^\tau}{S_{\mathrm{EW}}^K}} \left(\frac{1 - m_K^2/m_\tau^2}{1 - m_\mu^2/m_K^2} \right)^2 \boxed{\frac{\tau_\tau}{\tau_K}} \boxed{\delta_{\mathrm{EM}}^{\tau/K}} \boxed{\mathrm{BR}(K_{\ell 2})}}$$

➢ Inputs needed:
→ Experimental : $\mathrm{BR}(K_{l2})$, lifetimes

→ Theoretical : Short distance EW corrections
Long distance EM corrections

⇨ $$\boxed{BR\left(\tau^- \to K^-\nu_\tau\right) = (0.713 \pm 0.003)\%}$$

5.2 Prediction of the strange Brs

Antonelli, Cirigliano, Lusiani, E.P. in progress

- The Brs of these 3 modes can be predicted using Kaon Brs very precisely measured + form factor information

  - $\tau \rightarrow K\pi\nu_\tau$:

$$\mathrm{BR}(\tau \to \bar{K}\pi\nu_\tau) = \frac{2m_\tau^5}{m_K^5}\boxed{\frac{S_{\mathrm{EW}}^\tau}{S_{\mathrm{EW}}^K}}\boxed{\frac{I_K^\tau}{I_K^\ell}}\frac{\left(1+\boxed{\delta_{\mathrm{EM}}^{K\tau}}+\boxed{\tilde{\delta}_{\mathrm{SU(2)}}^{K\pi}}\right)^2}{\left(1+\boxed{\delta_{\mathrm{EM}}^{K\ell}}+\boxed{\delta_{\mathrm{SU(2)}}^{K\pi}}\right)^2}\boxed{\frac{\tau_\tau}{\tau_K}}\boxed{\mathrm{BR}(K \to \pi e \bar{\nu}_e)}$$

  - Inputs needed :
    - The K_{e3} branching ratios, lifetimes
    - Phase space integrals ⇨ use the dispersive parametrization for the form factors
    - The electromagnetic and isospin-breaking corrections

⇨ $\boxed{BR\left(\tau \to \overline{K}^0\pi^-\nu_\tau\right) = (0.8569 \pm 0.0293)\%}$ and $\boxed{BR\left(\tau \to K^-\pi^0\nu_\tau\right) = (0.4709 \pm 0.0178)\%}$

Preliminary

5.2 Prediction of the strange Brs and V_{us}

Mode	BR	% err	BR(K_{e3})	τ_K	τ_τ	I_K^τ / I_K^e	Δ_{EM}	$\Delta_{SU(2)}$
$\tau^- \to \bar{K}^0 \pi^- \nu_\tau$	0.8569 ± 0.0293	3.42	0.22	0.41	0.35	3.34	0.46	0
$\tau^- \to K^- \pi^0 \nu_\tau$	0.4709 ± 0.0178	3.79	0.06	0.12	0.34	3.60	0.47	1.00

Branching fraction	**HFAG Winter 2012 fit**	**Prediction (Preliminary)**
$\Gamma_{10} = K^- \nu_\tau$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$	$(0.713 \pm 0.003) \cdot 10^{-2}$
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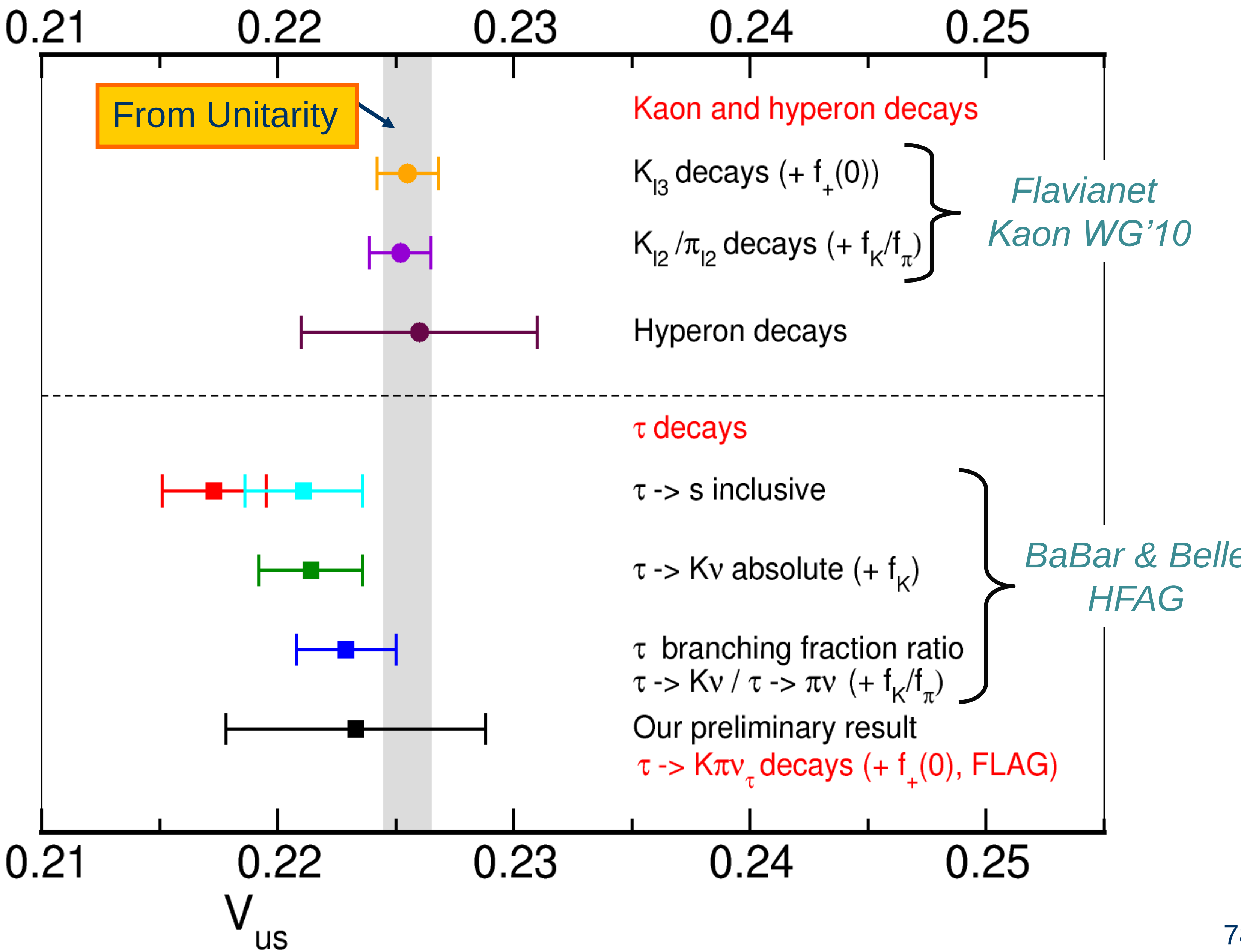
$\Gamma_{35} = \pi^- \bar{K}^0 \nu_\tau$	$(0.8206 \pm 0.0182) \cdot 10^{-2}$	$(0.8569 \pm 0.0293) \cdot 10^{-2}$
$\Gamma_{110} = X_s^- \nu_\tau$	$(2.8746 \pm 0.0498) \cdot 10^{-2}$	$(2.9714 \pm 0.0561) \cdot 10^{-2}$

$$\boxed{|V_{us}| = 0.2173 \pm 0.0022} \Rightarrow \boxed{|V_{us}| = 0.2211 \pm 0.0025}$$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τ branching fraction ratio
τ -> $K\nu$ / τ -> $\pi\nu$ (+ f_K/f_π)
Our preliminary result
τ -> $K\pi\nu_\tau$ decays (+ $f_+(0)$, FLAG)
V_{us}

6. Prospects for τ at the new flavour factories

6.1 Introduction

- Studying τ physics ⇒ very interesting tests of the Standard Model : we have entered a precision era
  - V_{us}
  - Strong coupling constant α_S
  - CP violating asymmetries

- Studying τ physics much more involved theoretically than kaon decays ⇒ much higher energies: perturbative and non-perturbative effects
  - Use OPE, moments
  - Use ChPT with resonances, dispersion relations, lattice QCD

- Experimentally:
  - OPAL/ALEPH measurements
  - A lot of data from B factories (BaBar, Belle) to be analysed
  - Tau charm factories

6.1 Introduction

- Studying τ physics ⇨ very interesting tests of the Standard Model :
  - V_{us}
  - Strong coupling constant α_S
  - CP violating asymmetries

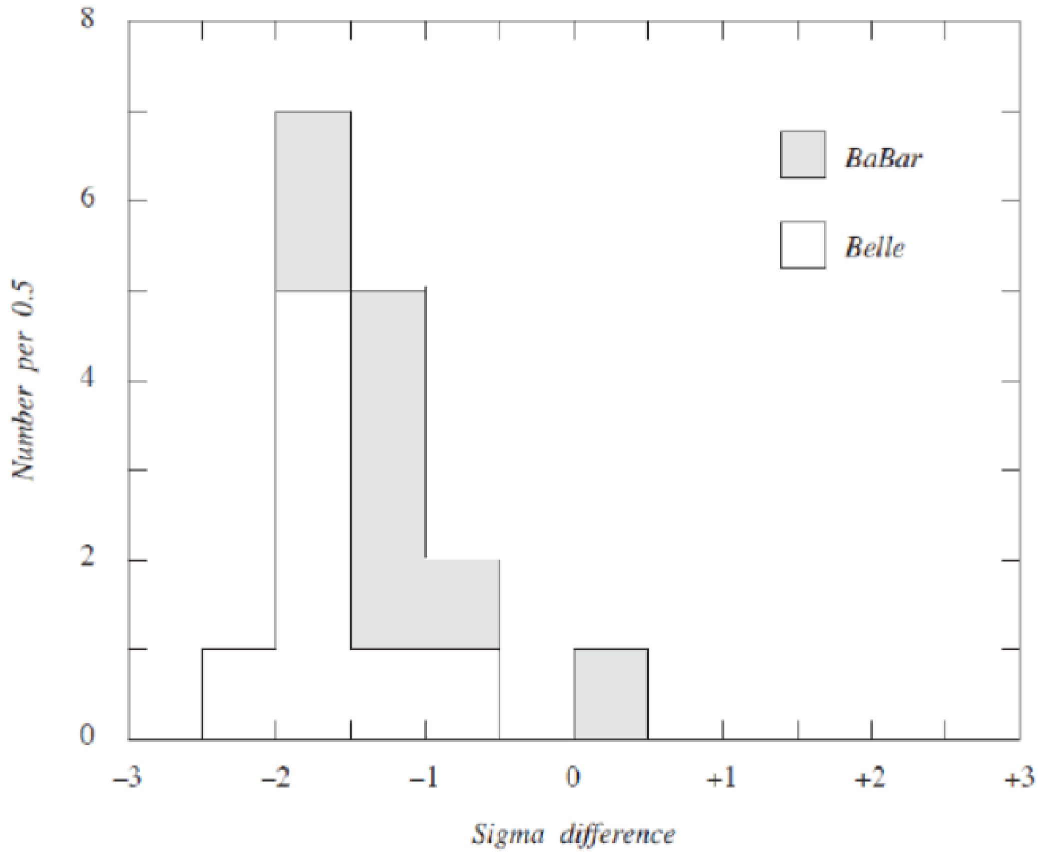
- Experimental Challenges:
  - measurements of the Brs
  - measurements of the spectral functions

- Theoretical challenges:
  - Having the hadronic uncertainties under control: OPE vs. Lattice QCD or ChPT
  - Isospin breaking
  - Electromagnetic corrections

6.2 Experimental Challenges

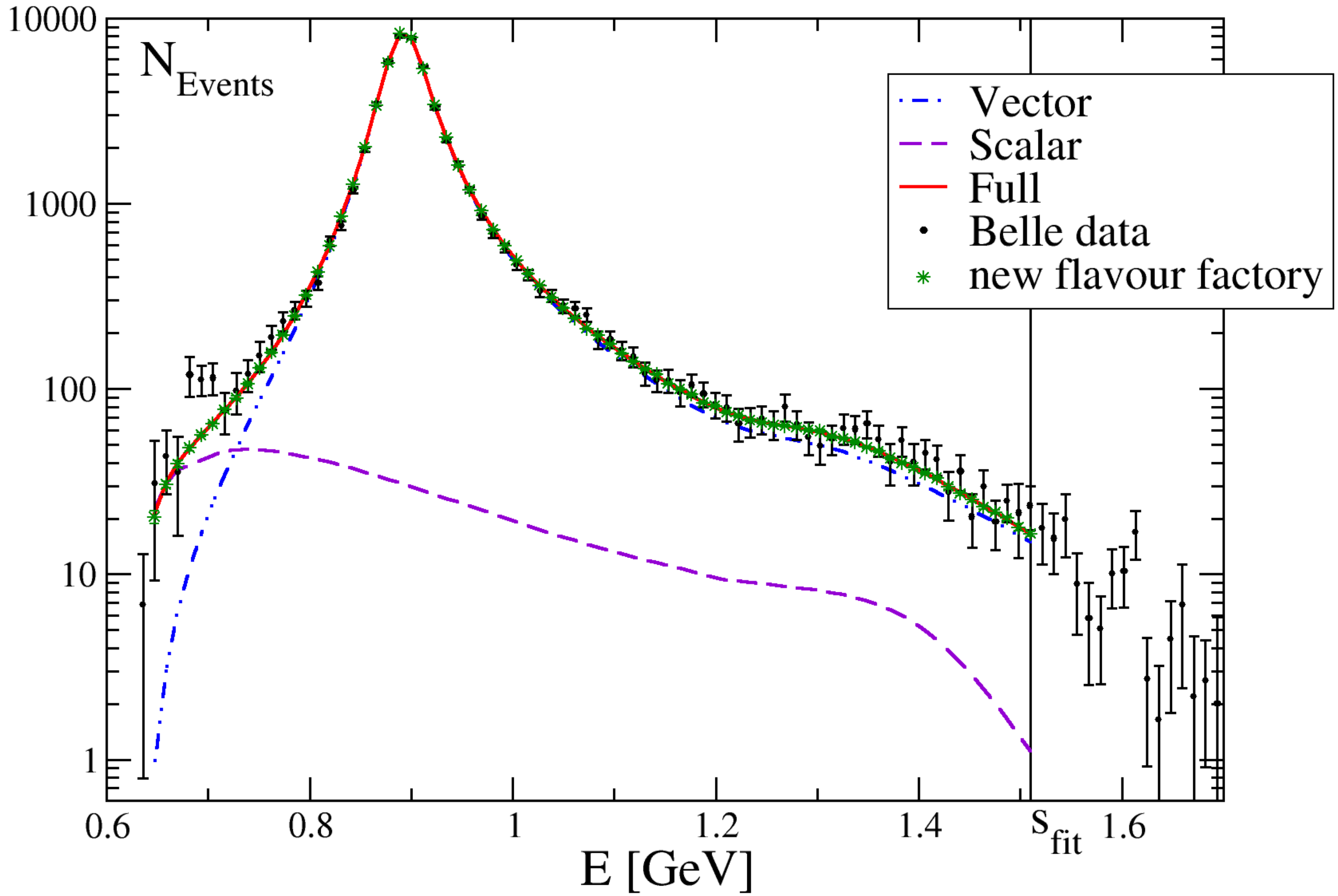
- τ strange Brs:
  PDG 2010: « Fifteen of the 16 *B*-factory branching fraction measurements are smaller than the non-*B*-factory values. The average normalized difference between the two sets of measurements is -1.36 »

  ⇨ Supported by predictions from kaon X channel measurements



Prospects for $\tau \rightarrow K\pi\nu_\tau$ analyses

- Simulated *New flavour factory* data from *Belle* data : *M. Antonelli*
  Same central values but uncertainties rescaled assuming 40 ab^{-1} luminosity



Prospects for inclusive τ decay analyses

- Simulated *New flavour factory* data from *Belle* data :
  Same central values but uncertainties rescaled assuming 40 ab^{-1} luminosity

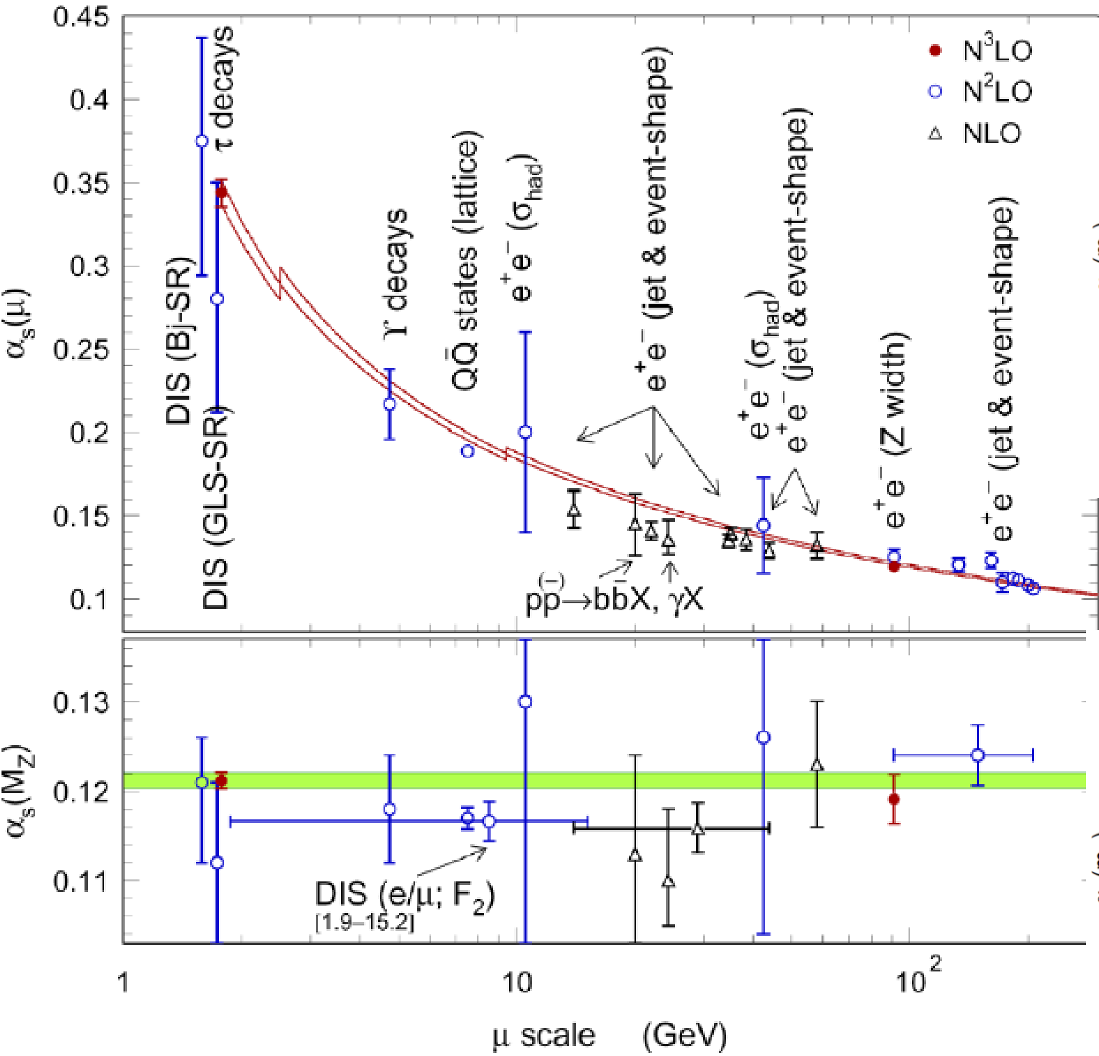
Mode	BR	% err	BR(K_{e3})	τ_K	τ_τ	I_K^τ/I_K^e	$\Delta_{\rm EM}$	$\Delta_{\rm SU(2)}$
$\tau^- \to \bar{K}^0\pi^-\nu_\tau$	0.8427 ± 0.0122	1.45	0.22	0.41	0.34	1.24	0.46	0
$\tau^- \to K^-\pi^0\nu_\tau$	0.4631 ± 0.0079	1.71	0.06	0.12	0.34	1.25	0.47	1.00

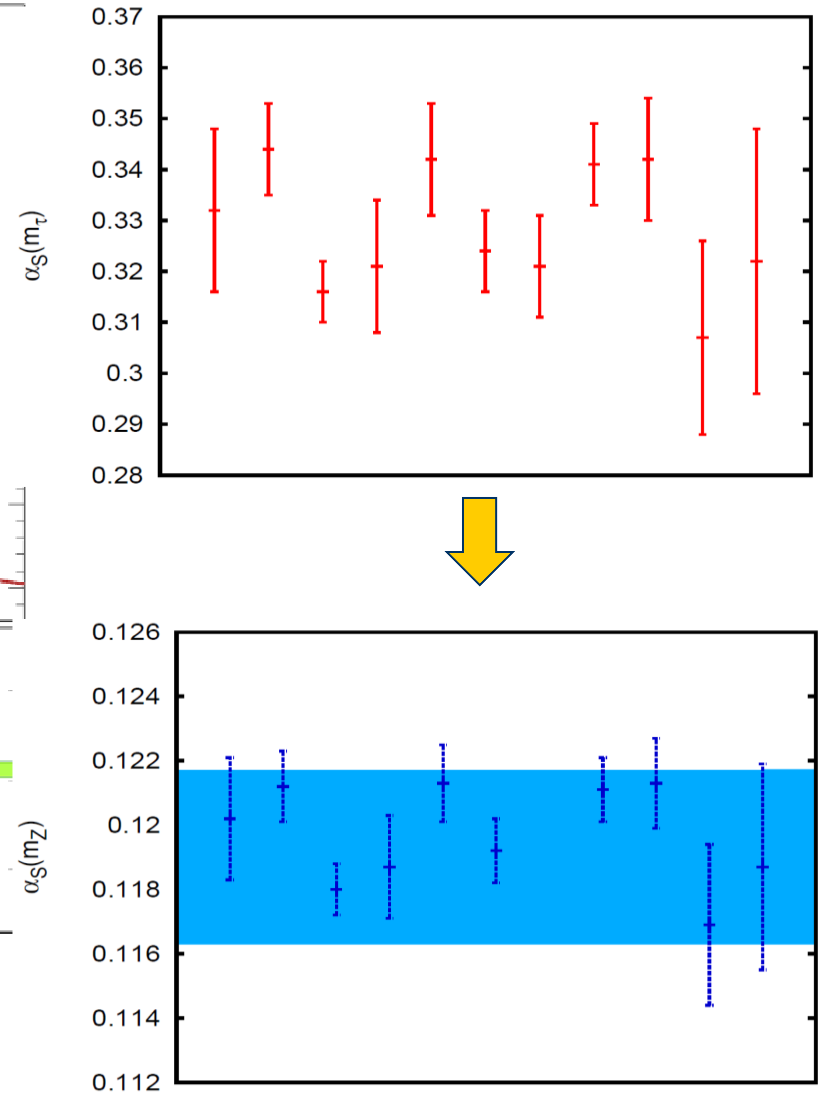
$$\boxed{|V_{us}| = 0.2173 \pm 0.0020_{\rm exp} \pm 0.0010_{\rm th}} \Rightarrow \boxed{|V_{us}| = 0.2211 \pm 0.0006_{\rm exp} \pm 0.0010_{\rm th}}$$

- *Promising!* Competitive with kaon physics!

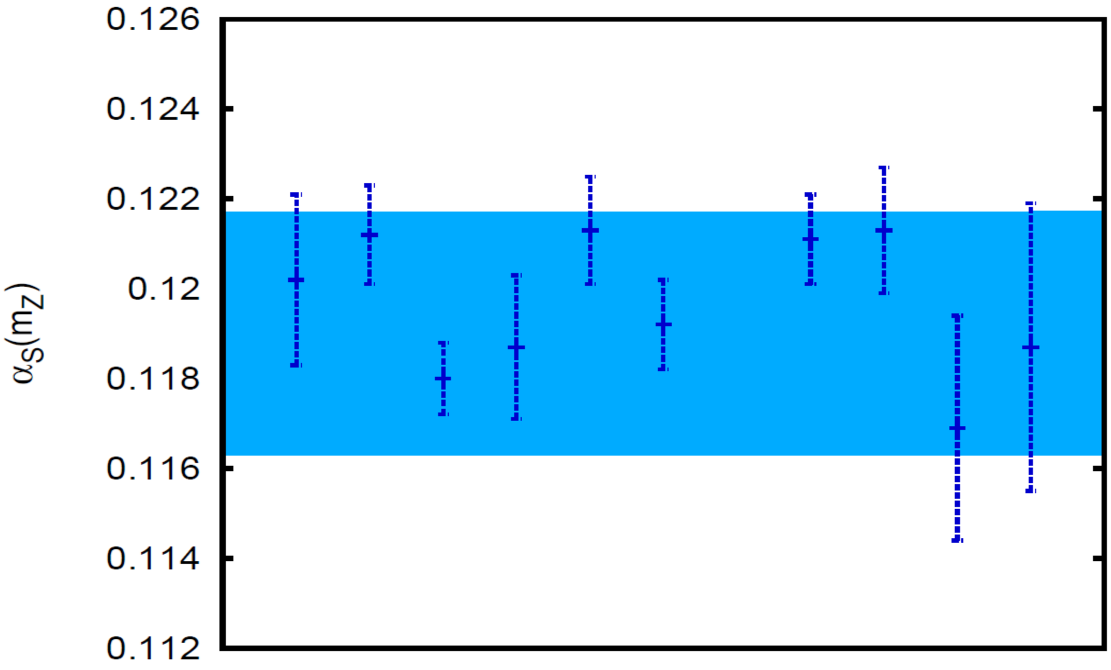
$\Rightarrow$ $\boxed{|V_{us}| = 0.2255 \pm 0.0013}$ (K_{l3} decays)

6.3 Strong coupling constant α_S





Strong coupling constant α_S



- *Extraction of α_S* from hadronic τ decays very *competitive*!
- If new data room for *improvement*!
  - Study of duality violation effects
  - Higher order condensates
  - New physics?

New Physics in R_τ

- Models with modifications of the couplings:
  - Tensor & scalar interactions ex: leptoquarks

$$
\begin{aligned}
R_\tau^{NS}(s_0) = 6\pi i \left|V^{ud}\right|^2 \oint_{|s|=m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \Bigg\{ & \left|\kappa_V\right|^2 \left[\left(1+\frac{2s}{m_\tau^2}\right)\Pi^{(1)}_{ud,VV}(s)+\Pi^{(0)}_{ud,VV}(s)\right] \\
& + \left|\kappa_A\right|^2 \left[\left(1+\frac{2s}{m_\tau^2}\right)\Pi^{(1)}_{ud,AA}(s)+\Pi^{(0)}_{ud,VV}(s)\right] \\
& + 2\,\mathrm{Re}\left(\kappa_V \kappa_S^*\right)\frac{\Pi_{ud,VS}(s)}{m_\tau} + 2\,\mathrm{Re}\left(\kappa_A \kappa_P^*\right)\frac{\Pi_{ud,AP}(s)}{m_\tau} \\
& + 12\,\mathrm{Re}\left(\kappa_V \kappa_T^*\right)\frac{\Pi_{ud,VT}(s)}{m_\tau}\Bigg\}\left[1-2\tilde{v}_L\right]
\end{aligned}
$$

- But also charged Higgs, little Higgs, SUSY…

$\tau \to K\pi\nu_\tau$ CP violating asymmetry

- CP violating asymmetry

$$\boxed{A_Q = \frac{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \bar{\nu}_\tau\right) - \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \bar{\nu}_\tau\right) + \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}}$$

$$\left|K_S^0\right\rangle = p\left|K^0\right\rangle + q\left|\overline{K}^0\right\rangle$$

$$\left|K_L^0\right\rangle = p\left|K^0\right\rangle - q\left|\overline{K}^0\right\rangle$$

$$\boxed{\left\langle K_L \middle| K_S \right\rangle = |p|^2 - |q|^2 \simeq 2\,\mathrm{Re}\left(\varepsilon_K\right)}$$

$= |p|^2 - |q|^2$ $\boxed{\approx \left(0.33 \pm 0.01\right)\%}$ in the Standard Model *Bigi & Sanda'05*

- Experimental measurement: $\boxed{A_{Q\,\mathrm{exp}} = \left(-0.45 \pm 0.24_{\mathrm{stat}} \pm 0.11_{\mathrm{syst}}\right)\%}$

BaBar'11

⇒ $\boxed{\sim 3\sigma}$ from the SM!

- New physics: Charged Higgs, leptoquarks or others?

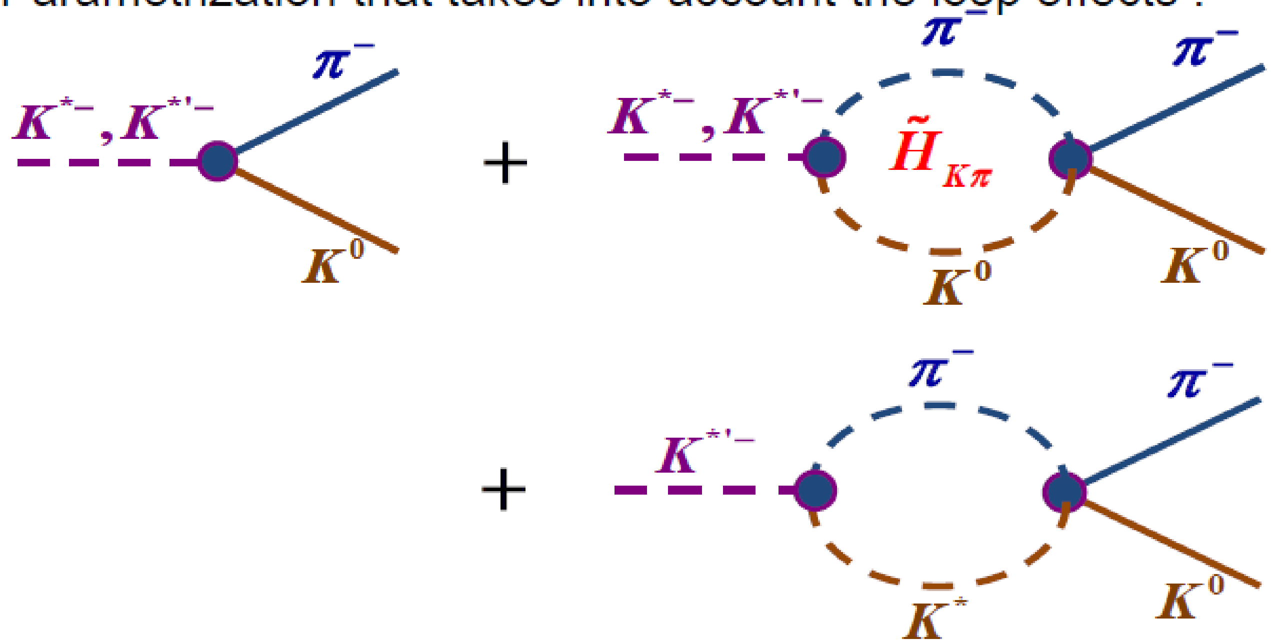
7. Back-up

- For ϕ_+ (s): In this case instead of the data, use of a parametrization including 2 resonances K*(892) and K*'(1414) : *Jamin, Pich, Portolés'08*

$$\bar{f}_+(s) = \left[\frac{m_{K^*}^2 - \kappa_{K^*}\left(\mathrm{Re}\,\tilde{H}_{K\pi}(0) + \mathrm{Re}\,\tilde{H}_{K\eta}(0)\right) + \beta s}{D(m_{K^*},\Gamma_{K^*})} - \frac{\beta s}{D(m_{K^{*\prime}},\Gamma_{K^{*\prime}})} \right] \Rightarrow \tan\delta_{K\pi}^{P,1/2} = \frac{\mathrm{Im}\,\bar{f}_+(s)}{\mathrm{Re}\,\bar{f}_+(s)}$$

with $$D(m_n,\Gamma_n) = m_n^2 - s - \kappa_n \sum \mathrm{Re}\,\tilde{H} - i m_n \Gamma_n(s)$$

  - Parametrization that takes into account the loop effects :



  - Loops with K*(892)π dominant decay channel of K*'(1410) (>40%) also included but not in *Jamin, Pich , Portolés '08, Boito, Escribano , Jamin'08 '10*

Determination of the $K\pi$ form factors

- $\bar{f}_+(t)$ accessible in K_{e3} and $K_{\mu 3}$ decays
- $\bar{f}_0(t)$ only accessible in $K_{\mu 3}$ (suppressed by m_l^2/M_K^2) + correlations
  ⇨ difficult to measure
- Data from *Belle* and *BaBar* on $\tau \to K\pi\nu_\tau$ decays (*Belle II*, *New flavour factories* soon)

  ⇨ Use them to constrain the form factors and especially $\bar{f}_0$

Inclusive τ decays

- $\Delta_{kl}(\alpha_s)$ known to order $O(\alpha_s^3)$: *Gámiz, Jamin, Pich, Prades,Schwab'03,'05*
  - *transverse* contribution (J=0+1) computed from *theory*
  - *longitudinal* contribution (J=0) divergent ⇨ determined from *data*
    - kaon pole ($K \to \mu\nu$)
    - Pion pole ($\pi \to \mu\nu$)
    - $(K\pi)_{J=0}$ (S-wave $K\pi$ scattering)
    - ….

	$R^{00,L}_{us,A}$	$R^{00,L}_{us,V}$	$R^{00,L}_{ud,A}$
Theory:	-0.144 ± 0.024	-0.028 ± 0.021	$-(7.79 \pm 0.14) \cdot 10^{-3}$
Phenom:	-0.135 ± 0.003	-0.028 ± 0.004	$-(7.77 \pm 0.08) \cdot 10^{-3}$

- Smaller uncertainties ⇨ $\boxed{\delta R^{00}_{\tau,th} = \underbrace{0.1544(37)}_{J=0} + \underbrace{0.062(15)}_{m_s(m_\tau) = 100 \pm 10\ \mathrm{MeV}} = 0.216(16)}$

4.6 Longitudinal contribution

- Longitudinal series does not converge!

  ➡ Replace scalar and pseudoscalar QCD correlators with phenomenology

  - Scalar spectral functions from S-wave Kπ scattering data

    Jamin, Oller, Pich'06

  - Dominant contribution: pseudoscalar us spectral function

$$\boxed{s^2 \frac{1}{\pi} \operatorname{Im} \Pi^L_{us,A} = 2 f_K^2 m_K^4 \delta\left(s - m_K^2\right) + 2 f_{K(1460)} m^4_{K(1460)} \mathrm{BW}(s)}$$

Kambor & Maltman'06

BW: normalized Breit-Wigner

  - Results: uncertainties very much reduced for J=L !

	$R^{00,L}_{us,A}$	$R^{00,L}_{us,V}$	$R^{00,L}_{ud,A}$
Theory:	-0.144 ± 0.024	-0.028 ± 0.021	$-(7.79 \pm 0.14) \cdot 10^{-3}$
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