Towards first principles in particle cosmology

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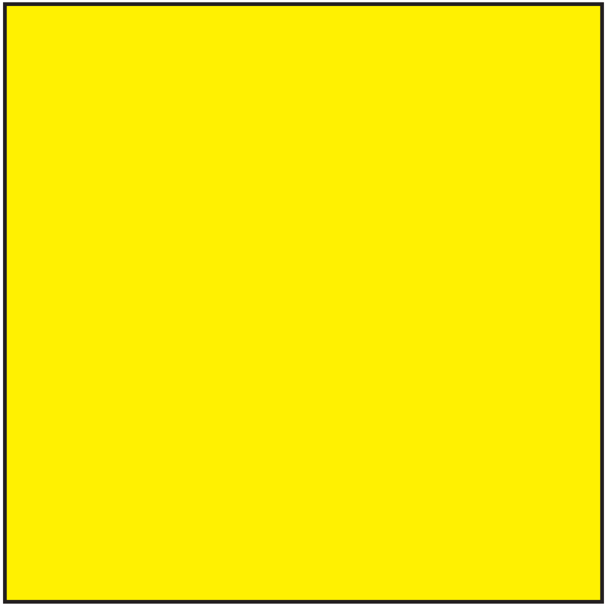
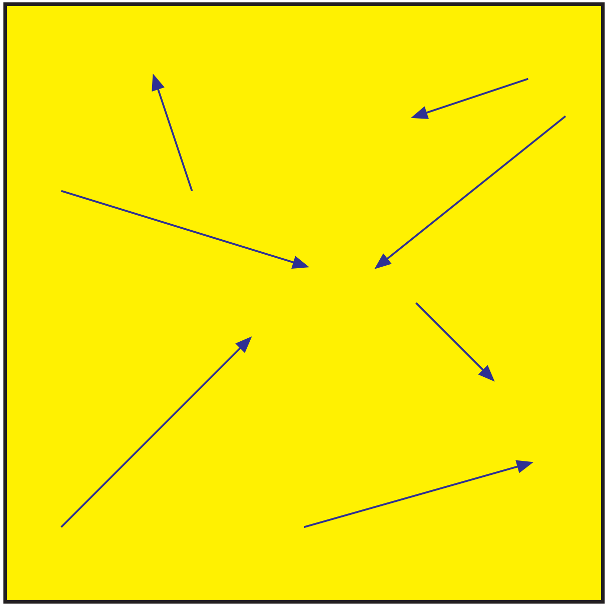
Introduction: basic cosmology

A small but important difference:



Statistical physics



Cosmology

$$n_B/n_\gamma = 6.2 \times 10^{-10}$$

Cosmology relies on deviations from thermal equilibrium.

Big Bang Nucleosynthesis:

weak interactions out of eq. → neutrons can decay

electromagnetism out of eq. → no deuterium photodissociation

strong interactions out of eq. → no heavy elements

Dark Matter:

weak (or superweak) interactions out of eq.

Baryon Asymmetry:

anomalous interactions out of eq.

Large-Scale Structure:

all particle physics interactions out of eq.

There are many ways to be "out of equilibrium".

Chemical non-equilibrium: number density not fixed by T, μ.

Kinetic non-equilibrium: average momentum not fixed by T, μ.

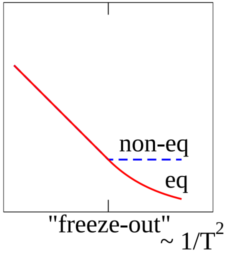
For instance, Dark Matter may first fall out of chemical equilibrium (fixing number density), much later out of kinetic equilibrium (fixing distance scale for structure formation).

First order phase transition: locally T, μ, but discontinuities.

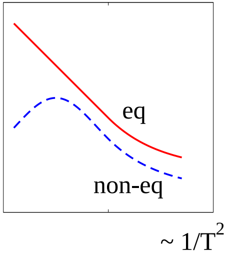
Inflationary dynamics: no T, μ at all.

There are also several possible time lines.

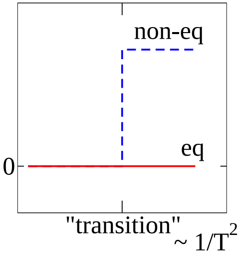
e.g. “freeze out” for dark matter, and “strong washout” regime for leptogenesis.



e.g. non-thermal dark matter, and “weak washout” regime for leptogenesis



e.g. phase transition for baryogenesis



Non-equilibrium is necessary but not sufficient.

non-eq $\longrightarrow$ dark matter through freeze-out

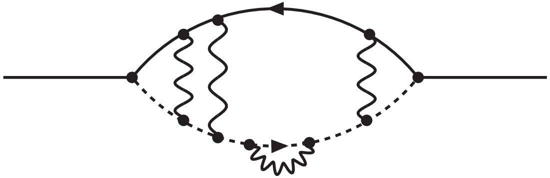
non-eq $+ \not{C}$, C$\not{P}$ $\longrightarrow$ e.g. kaon-antikaon asymmetry

non-eq $+ \not{C}$, C$\not{P}$ $+ \not{B}$ $\longrightarrow$ p-$\bar{p}$ asymmetry [Sakharov 1967]

non-eq $+ \not{C}$, C$\not{P}$ $+ \not{B}$ + IR $\longrightarrow$ p-$\bar{p}$ asymmetry reliably

Here IR refers to being able to systematically handle the **infrared problem** of relativistic thermal field theory.

The IR problem stands for the breakdown of the loop expansion in the ultrarelativistic regime (cf. later).



A possible avenue for progress is to *factorize* the different ingredients and solve the IR problem in a simpler setting.

In the “non-eq” case: there is a “rate” at which one is *approaching* equilibrium; if this is much smaller than all other rates, and we are close to equilibrium, then we may expect

$$\dot{n} + \underset{\substack{\uparrow \\ \text{Hubble rate}}}{3Hn} \overset{n \approx n_{\text{eq}}}{\approx} \underset{\substack{\uparrow \\ \text{“Chemical equilibration rate”}}}{-\Gamma}(n - n_{\text{eq}}) + \mathcal{O}(n - n_{\text{eq}})^2 \, .$$

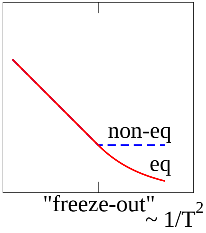
Γ can be determined arbitrarily close to equilibrium; the form of the macroscopic equation captures non-equilibrium.

Example: CP-conserving rates

What is really computed?

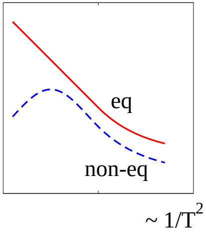
Setting $H \to 0$, the previous equation can be viewed as

$$\frac{\mathrm{d}N}{\mathrm{d}^4\mathcal{X}\mathrm{d}^3\mathbf{k}} = -\Gamma_1(\mathcal{K})\left[\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{x}\mathrm{d}^3\mathbf{k}} - f_{\mathrm{eq}}\right] + \mathcal{O}[\quad]^2 .$$



Another possibility is $X\,|\mathbf{0}\rangle \to Y\,|\mathbf{k}\rangle$, viz.

$$\frac{\mathrm{d}N}{\mathrm{d}^4\mathcal{X}\mathrm{d}^3\mathbf{k}} = -\Gamma_2(\mathcal{K})\left[\cancel{\frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{x}\mathrm{d}^3\mathbf{k}}} - \frac{2f_{\mathsf{F}}(k^0)}{(2\pi)^3}\right] .$$



The Boltzmann equation predicts that $\Gamma_1 = \Gamma_2$, but this may not be true in general. In the following, consider $\Gamma \equiv \Gamma_2$.

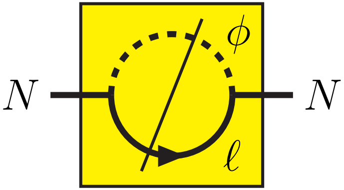
We wish to compute Γ for right-handed neutrinos.

$$\mathcal{L} \quad = \quad \mathcal{L}_{\mathsf{SM}} + \frac{1}{2}\bar{\tilde{N}}[i\partial\!\!\!/ - M]\tilde{N} - [h_\nu\,\bar{\ell}\,a_R\,\tilde{\phi}\,\tilde{N} + \mathrm{H.c.}]\ .$$

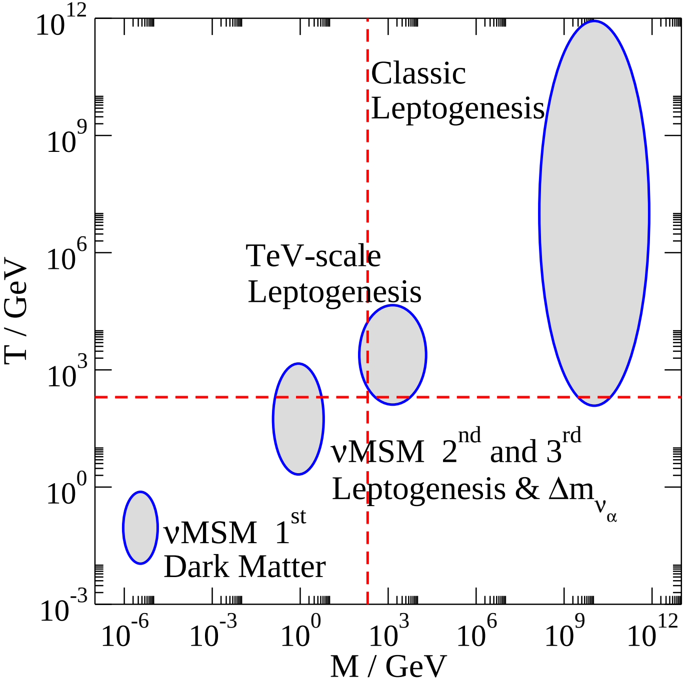
Let ρ be the "spectral function" corresponding to $\bar{\ell}\,a_R\,\tilde{\phi}$.

$$\ldots \quad \Rightarrow \quad \Gamma(\mathcal{K}) = \frac{|h_\nu|^2}{k^0}\mathrm{Tr}\{\mathcal{K}\!\!\!/\ \rho(\mathcal{K})\} + \mathcal{O}(|h_\nu|^4)\ .$$

The four-momentum is on-shell: $\mathcal{K} = (\sqrt{k^2 + M^2}, \mathbf{k})$.



Many different parameter values can be envisaged.



In this talk: $M, \pi T \gg 100$ GeV.

What are the scales of the problem?

- The properties of the plasma are characterized by the scales

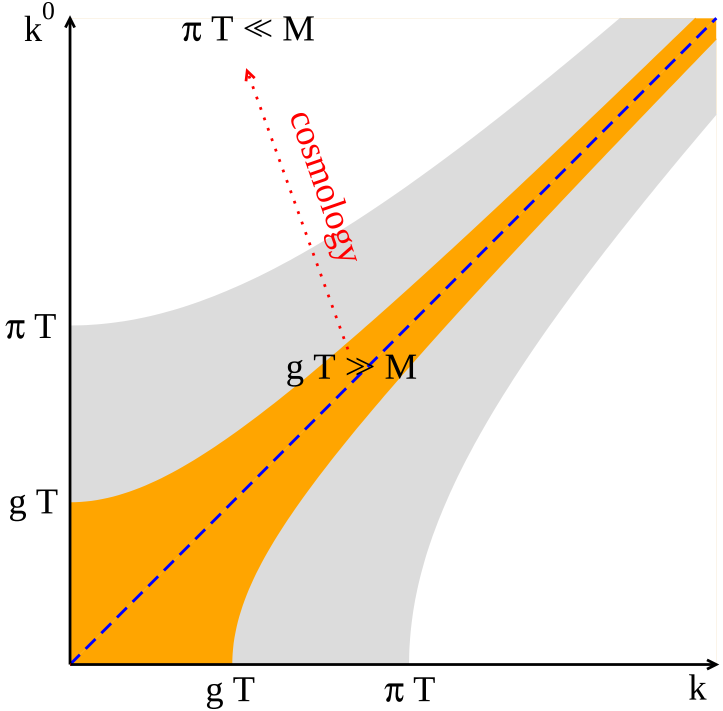
$$\pi T \;, \quad gT \qquad (\alpha \equiv \frac{g^2}{4\pi}) \;.$$

- Vacuum scales (m_Z, m_{top}) do not appear if $\pi T \gg 100$ GeV.

- To $\mathcal{O}(|h_\nu|^2)$, the scale M only appears "externally" in

$$k^0 = \sqrt{k^2 + M^2} \;.$$

Because there are three scales, there are different regimes.



$$(k^0, \mathbf{k}) = (\sqrt{k^2 + M^2}, \mathbf{k})$$

Simple case: late universe ("non-relativistic" regime)

$$\boxed{M \gg \pi T \gg 100 \text{ GeV}}$$

A. Salvio, P. Lodone and A. Strumia, *Towards leptogenesis at NLO: the right-handed neutrino interaction rate*, JHEP 08 (2011) 116 [1106.2814].

"*Previous partial results are extremely complicated because only some NLO effects have been computed, missing the great simplification that happens when including all NLO corrections...*"

"*In practice this means that 'gauge scatterings' and 'higgs scatterings' must be removed from codes for leptogenesis...*"

How to handle this situation?

From the viewpoint of the plasma, $\mathcal{K}^2 = M^2$ is a **hard** scale. This means that we are in an "ultraviolet" regime, and can make use of the Operator Product Expansion (OPE):[1]

$$\mathrm{Tr}\{\not{\mathcal{K}}\ \rho(\mathcal{K})\} \sim f^{(2)}_{T=0}(\mathcal{K}^2) + f^{(0)}(\mathcal{K}^2;\bar{\mu}^2)\langle\phi^\dagger\phi\rangle_T + \mathcal{O}\left(\frac{T^4}{\mathcal{K}^2}\right) .$$

IR problem affects $\langle\phi^\dagger\phi\rangle_T$ but it's suppressed by $\mathcal{O}(T^2/\mathcal{K}^2)$.

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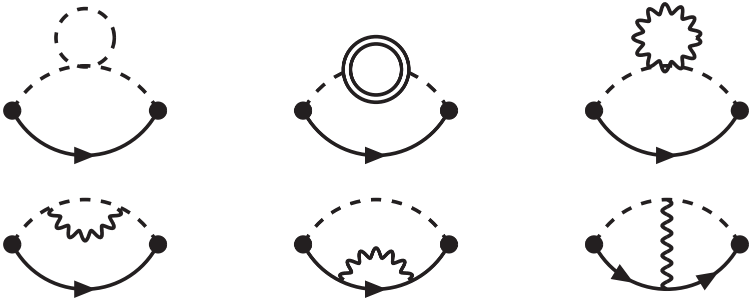
[1] S. Caron-Huot, *Asymptotics of thermal spectral functions*, Phys. Rev. D 79 (2009) 125009 [0903.3958].

Explicit computation at NLO:[2]



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[2] M. Laine and Y. Schröder, *Thermal right-handed neutrino production rate in the non-relativistic regime*, JHEP 02 (2012) 068 [1112.1205].

Complete self-energy: $(\rho = \mathrm{Im}\,\Sigma|_{K^2\to -\mathcal{K}^2})$

$$
\begin{aligned}
\mathcal{Z}_\nu\, \Sigma_E(K) \quad = \quad & a_L\, i \not{K}\, a_R \Big\{ \frac{1}{(4\pi)^2}\Big(\frac{1}{\epsilon} + \ln\frac{\bar\mu^2}{K^2} + 2\Big) \\
& + \frac{|h_t|^2 N_{\rm c}}{(4\pi)^4}\Big(\frac{1}{2\epsilon^2} - \frac{3}{4\epsilon} - \frac{1}{2}\ln^2\frac{\bar\mu^2}{K^2} - \frac{7}{2}\ln\frac{\bar\mu^2}{K^2} - \frac{57}{8}\Big) \\
& + \frac{g_1^2 + 3g_2^2}{(4\pi)^4}\Big(-\frac{3}{8\epsilon^2} + \frac{17}{16\epsilon} + \frac{3}{8}\ln^2\frac{\bar\mu^2}{K^2} + \frac{29}{8}\ln\frac{\bar\mu^2}{K^2} + \frac{275}{32} - 3\zeta(3)\Big) \\
& + \Big[1 + \frac{6\lambda}{(4\pi)^2}\Big(\ln\frac{\bar\mu^2}{K^2} + 1\Big)\Big]\, \frac{\mathcal{Z}_m\langle\phi^\dagger\phi\rangle_T}{K^2} \;+\; \mathcal{O}\Big(g^4, \frac{T^4}{K^4}\Big)\Big\}\,,
\end{aligned}
$$

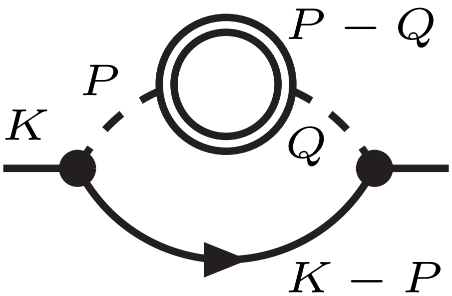
$$
\begin{aligned}
\mathcal{Z}_m\langle\phi^\dagger\phi\rangle_T \quad = \quad & \frac{T^2}{6} - \frac{T^2}{2\pi}\sqrt{\frac{m_H^2}{T^2} - \frac{g_1^2 m_{D1} + 3g_2^2 m_{D2}}{16\pi T}} \qquad \boxed{m_H^2 \sim g^2T^2} \\
& + \frac{T^2}{48\pi^2}\Big\{-6\lambda\Big[\ln\Big(\frac{\bar\mu e^{\gamma_E}}{4\pi T}\Big) - 3\Big] - |h_t|^2 N_{\rm c}\ln\Big(\frac{\bar\mu e^{\gamma_E}}{8\pi T}\Big) \\
& + \frac{3(g_1^2 + 3g_2^2)}{4}\Big[\ln\Big(\frac{\bar\mu e^{\gamma_E}}{4\pi T}\Big) - \frac{2}{3} - 2\gamma_E - 2\frac{\zeta'(-1)}{\zeta(-1)} + 4\ln\Big(\frac{2\pi T}{m_H}\Big)\Big]\Big\} + \ldots
\end{aligned}
$$

What changes in the “relativistic” regime $M \sim \pi T$?

Here no “expansion” is possible; just compute the full result.

Cancellation of divergences (soft, collinear, thermal) can be investigated order by order. [Soft and collinear divergences are guaranteed to cancel because this is a physical observable.]

Example: top loop



After cancelling numerator structures against propagators, the most non-trivial structure left over is

$$\widetilde{\mathcal{I}}_{\mathsf{h}} \equiv \lim_{\lambda\to0} \SumInt_{P\{Q\}} \frac{K^2}{Q^2P^2[(Q-P)^2+\lambda^2](P-K)^2}\,,$$

$$K^2 \equiv k_n^2+k^2\,.$$

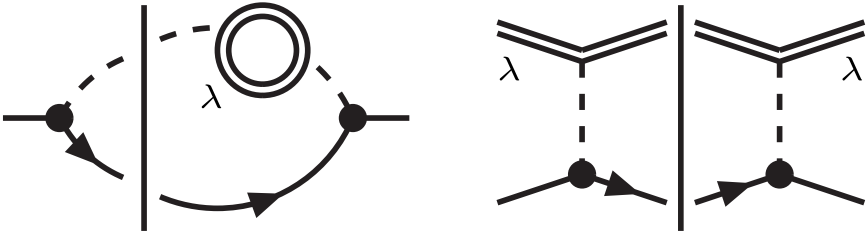
Spectral function

Take the cut:

$$\rho_{\widetilde{\mathcal{I}}_{\mathsf{h}}} \equiv \mathrm{Im}[\widetilde{\mathcal{I}}_{\mathsf{h}}]_{k_n \to -i[k_0 + i0^+]} \; ,$$

$$\mathcal{K}^2 \equiv k_0^2 - k^2 \; .$$

For $\lambda \to 0$ virtual and real processes contain soft, collinear and thermal divergences, which cancel in the sum.

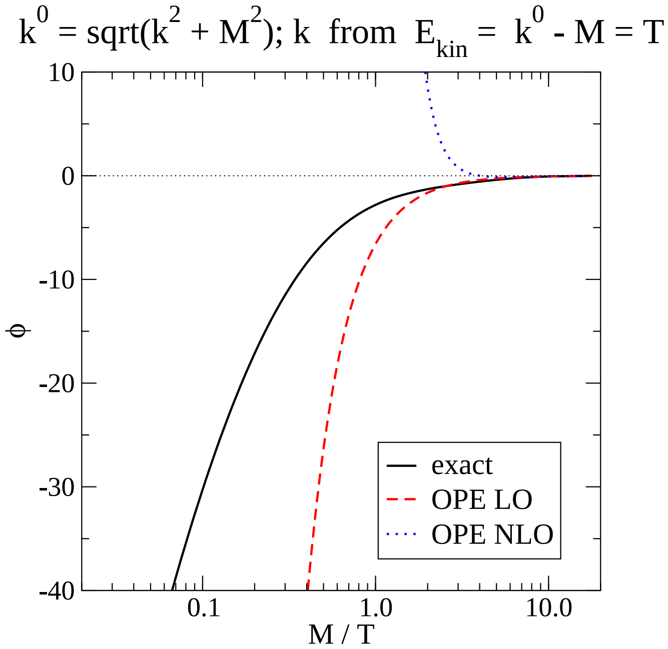


Summing together both processes, the limit $\lambda \to 0$ can be taken, and the result can be expressed as

$$\rho_{\widetilde{\mathcal{I}}_\mathsf{h}} \;=\; -\frac{\mathcal{K}^2}{4(4\pi)^3}\left[\frac{1}{\epsilon} + 2\ln\frac{\bar{\mu}^2}{\mathcal{K}^2} + 5 + \phi_T(\mathcal{K})\right] \; .$$

The function ϕ_T vanishes for $T \to 0$ but, for $T \neq 0$, depends separately on k_0 and k.

A numerical evaluation, compared with low-T asymptotics:



So the NLO correction is perfectly finite for $\lambda \to 0$, but the *loop expansion* breaks down for $M \ll T$.

Difficult case: early universe ("ultrarelativistic" regime)

$$\boxed{\pi T \gg M \gg 100 \text{ GeV}}$$

A. Anisimov, D. Besak and D. Bödeker, *Thermal production of relativistic Majorana neutrinos: Strong enhancement by multiple soft scattering,* JCAP 03 (2011) 042 [1012.3784]; D. Besak and D. Bödeker, *Thermal production of ultrarelativistic right-handed neutrinos: Complete leading-order results,* JCAP 03 (2012) 029 [1202.1288].

Techniques ("LPM") inspired by the QCD computation

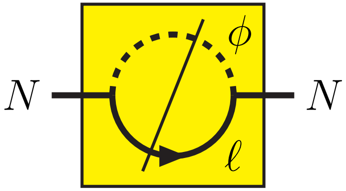
P.B. Arnold, G.D. Moore and L.G. Yaffe, *Photon emission from ultrarelativistic plasmas,* JHEP 11 (2001) 057 [hep-ph/0109064].

Concepts

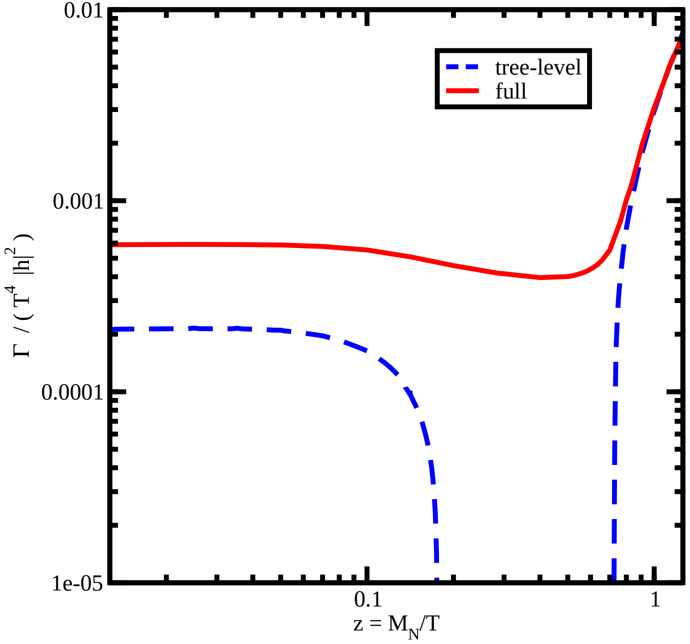
“tree-level”: give particles (“asymptotic”) thermal masses and compute rate from the lowest-order kinematically allowed diagram.

“consistent LO”: include all processes which contribute at the same order in *coupling constants* (not necessarily at the same order in the loop expansion).

$$\frac{\mathrm{d}N}{\mathrm{d}^4\mathcal{X}\,\mathrm{d}^3\mathbf{k}} \propto$$



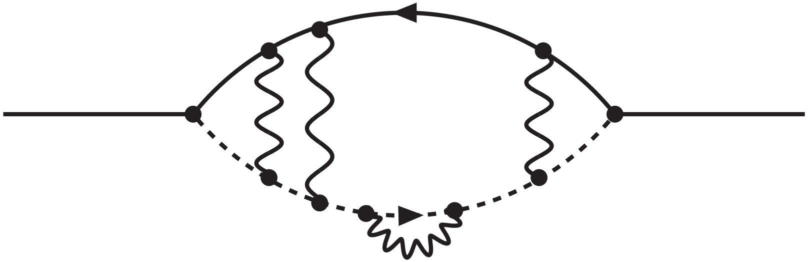
Difference of “tree-level” and “consistent LO” is substantial:



Anisimov et al

Why is this challenging?

From the plasma viewpoint it is a "**soft / collinear**" situation, with light-cone physics playing an important role: loop expansion breaks down, and needs to be resummed to all orders.

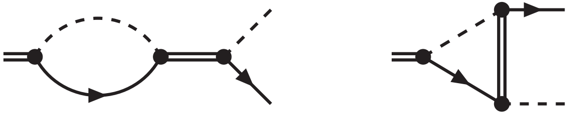


CP-violating rates (leptogenesis)

How to add CP-violation?

Initial ensemble of right-handed neutrinos produced (N_1, N_2, N_3) has no asymmetry, but then $\Gamma(N_i \rightarrow H\nu) \neq \Gamma(N_i \rightarrow H\bar{\nu})$.

Similarly to CP-violation in the kaon system, the origin might be "indirect" (related to oscillations) or "direct" (related to decays).



Oscillations and CP-violation are **quantum-mechanical** phenomena in which complex phases play an important role.

Attempts at factorization.

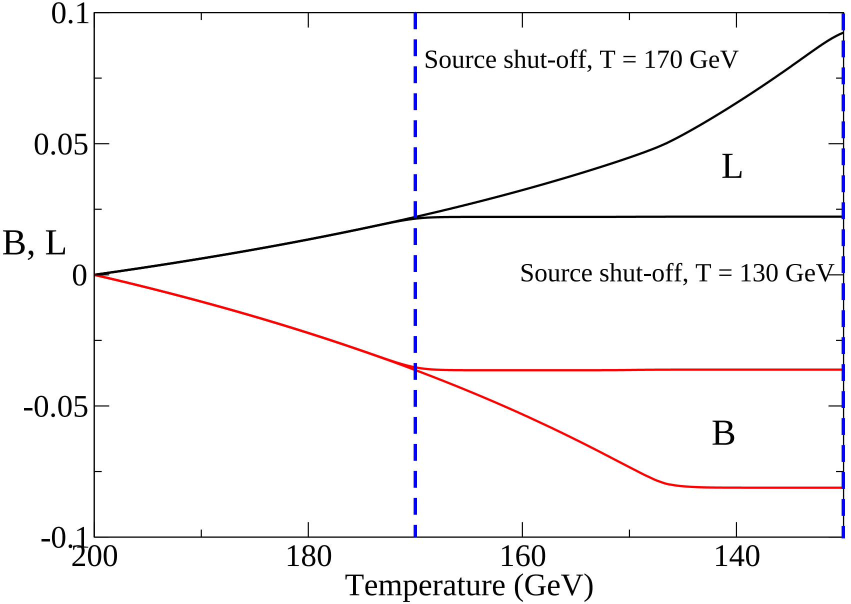
- “Canonical” density matrix with a hermitean part and decays (but have to choose a basis of CP-even and odd on-shell states):

$$i(\partial_t - Hk\,\partial_k)\hat{\rho} \simeq [\hat{\mathcal{H}}_{\text{eff}}, \hat{\rho}] - \frac{i}{2}\{\hat{\Gamma}_{\text{eff}}, \hat{\rho} - \hat{\rho}_{\text{eq}}\} \; .$$

- Kadanoff-Baym ($\sim$ Schwinger-Dyson) equations for full non-equilibrium propagators (but writing down equations does not solve the problem of factorizing the scales affecting the solution).

- lepton number production “from vacuum” – like before but at $\mathcal{O}(h_\nu^\dagger h_\nu)^2$ (however only works in the “weak washout” case).

Subsequently a part of lepton asymmetry is converted to baryons through non-perturbative "sphaleron" processes.[3]



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[3] Illustration from M. D'Onofrio, K. Rummukainen and A. Tranberg, *The Sphaleron Rate through the Electroweak Cross-over*, 1207.0685.

Summary

There has been much progress in recent years ...

... but important conceptual and technical challenges remain.