Semi-empirical calculation of quenching factors in scintillators

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Contents:

Based on V.I. Tretyak, Astropart. Phys. 33 (2010) 40 + new results

1. Introduction

Motivations

1. Searches for dark matter:

Universe – usual matter ~4%, DM ~23%, DE ~73%;
WIMPs scatter on nuclei in a detector creating recoil ions;
Amount of light produced in scintillator by ions is lower than that produced by electrons of the same energy (experimental fact);
Thus, in scintillators calibrated with electrons or γ quanta, signals from ions will be seen at lower energies than their real values (up to ~40 times);
Evidently, knowledge of these transformation coefficients – quenching factors (QF) – is extremely important in searches for WIMPs, predicting where the signal should be seen;
Many experimental efforts (sometimes very sophisticated) to measure QFs.

It would be very nice if one would be able to calculate QFs.

2. Investigation of rare ($T_{1/2}=10^{18}$-10^{19} yr) alpha decays with scintillators or scintillating bolometers:

Observations:

^{180}W in $CdWO_4$ – F.A. Danevich et al., PRC 67 (2003) 014310;
in $CaWO_4$ – C. Cozzini et al., PRC 70 (2004) 064606;
– Yu.G. Zdesenko et al., NIMA 538 (2005) 657;
in $ZnWO_4$ – P. Belli et al., NIMA 626 (2011) 31;

^{151}Eu in CaF_2(Eu) – P. Belli et al., NPA 789 (2007) 15;

^{209}Bi in $Bi_4Ge_3O_{12}$ – P. de Marcillac et al., Nature 422 (2003) 876;
– J.W. Beeman et al., PRL 108 (2012) 062501;

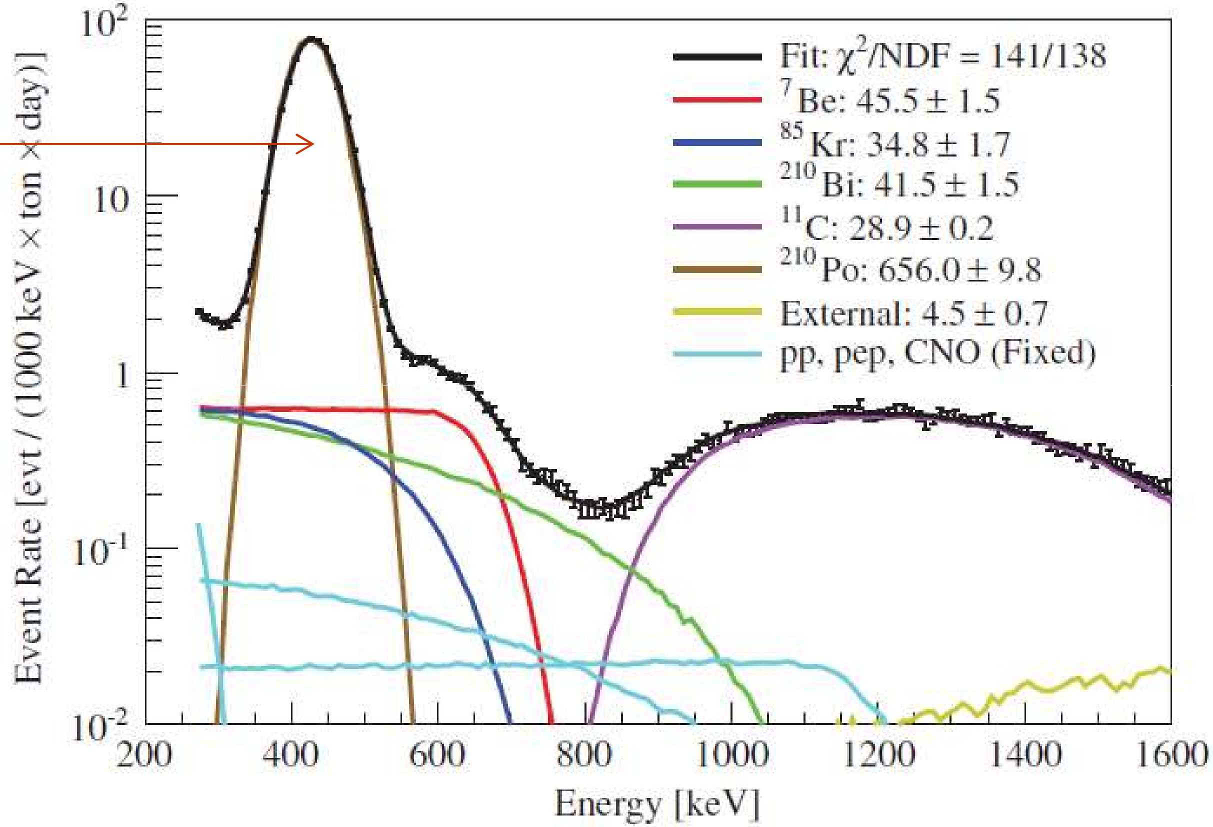
Limits:

^{204,206,207,208}Pb in $PbWO_4$ – J.W. Beeman et al., arXiv:1212.2422.

BOREXINO liquid scintillator (pseudocumene, C_9H_{12}), G. Bellini et al., PRL 107 (2011) 141302

^{210}Po

Q_α=5407 keV
Visible at ~430 keV
$QF_\alpha \approx 430/5407 \approx 1/13$



Knowledge of QF is important in all experiments which use scintillators

2. Outlines of the method

In calculation of QFs, we follow Birks approach in description of quenching of the light yield for highly ionizing particles [J.B. Birks, Proc. Phys. Soc. A 64 (1951) 874; The Theory and Practice of Scintillation Counting, Pergamon Press, Oxford, 1964]

Light yield (LY) of highly ionizing particles in scintillating material depends not only on its energy *E* but also on its stopping power *dE/dr*

Examples of stopping powers (SP) in $CaWO_4$:

for electrons calculated with the ESTAR code [M.J. Berger et al., Stopping-Power and Range Tables for Electrons, Protons, and Helium Ions] and

for different ions calculated with the SRIM code [J.F. Ziegler et al., SRIM. The Stopping and Range of Ions in Matter, SRIM Co., 2008]:

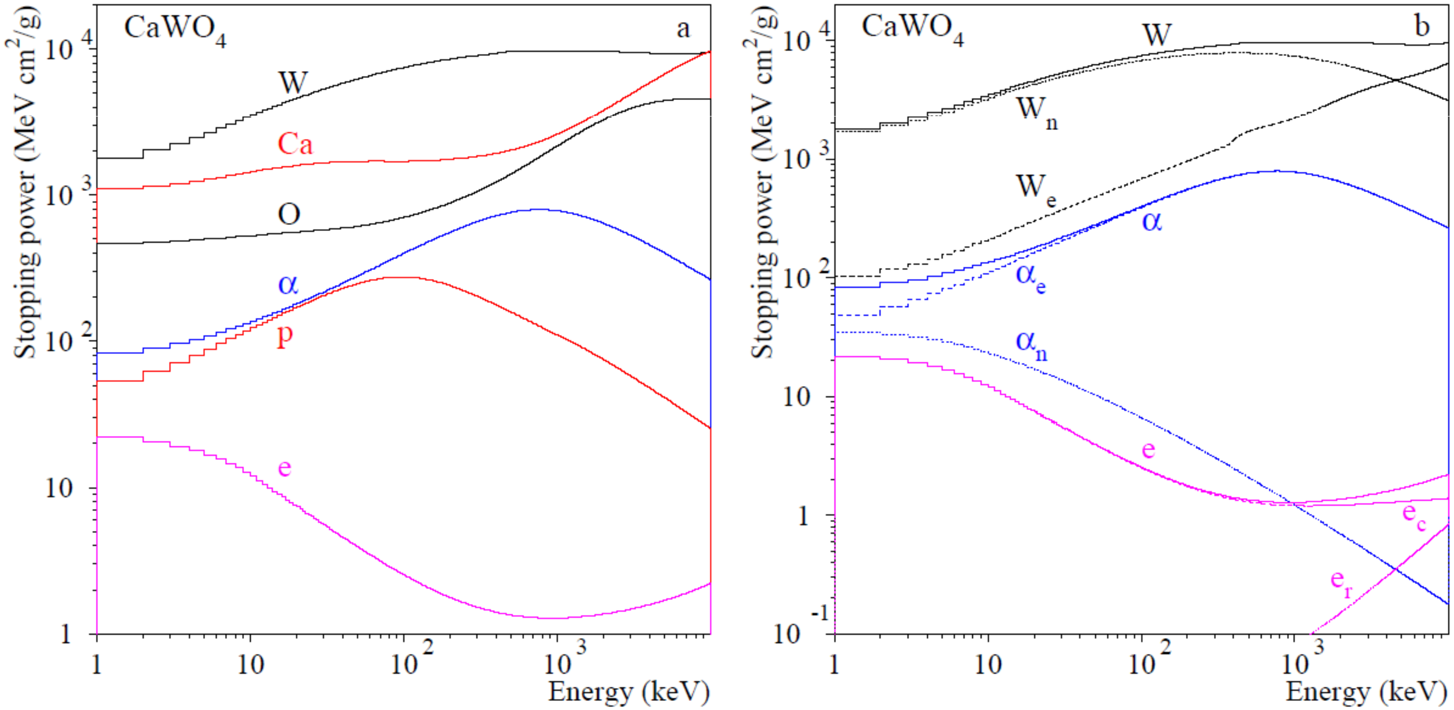


Fig. 1: (a) Total SP and (b) nuclear and electronic parts of SP

For particles with low SP (e$^-$ at *E*>~100 keV; *S* is absolute scintillation factor):

$$dL = SdE \qquad \frac{dL}{dr} = S\frac{dE}{dr}$$

To account for suppression of LY for highly ionizing particles (ions) Birks proposed: (*kB* – Birks factor)

$$\frac{dL}{dr} = \frac{S\frac{dE}{dr}}{1 + kB\frac{dE}{dr}}$$

Approximations of light yield for electrons and ions:

$$L_e(E) = SE \qquad L_i(E) = \frac{Sr}{kB}$$

but in general:

$$L(E) = \int_0^E dL = \int_0^E \frac{SdE}{1 + kB\frac{dE}{dr}}$$

Let us suppose S_e=S_i, and *S* and *kB* do not depend on energy

Quenching factor (often for α particles: "α/β ratio") is ratio of LY of ions to LY of electrons:

$$Q_i(E) = \frac{L_i(E)}{L_e(E)} = \frac{\int_0^E \frac{dE}{1+kB(\frac{dE}{dr})_i}}{\int_0^E \frac{dE}{1+kB(\frac{dE}{dr})_e}} \qquad \textbf{(1)}$$

S* disappeared, and we have only 1 parameter: *kB

Sometimes instead of QF, a relative LY (ratio of ion's LY to energy, normalized to that of electron at some energy E_0) is used:

$$R_i(E) = \frac{L_i(E)/E}{L_e(E_0)/E_0} \qquad (2)$$

Relation between Q and R: (R is practically equal to Q if electron energies E and E_0 are in energy range where $L \sim E$)

$$R_i(E) = \frac{L_i(E)}{L_e(E)} \frac{L_e(E)/E}{L_e(E_0)/E_0} = Q_i(E) \frac{L_e(E)/E}{L_e(E_0)/E_0}$$

With approximations: $dL_e/dE = S \qquad dL_i/dE = \frac{S}{kB} \frac{1}{(dE/dr)_i}$

we obtain the following approximation for QF:

$$Q_i(E) = \frac{L_i(E)}{L_e(E)} = \frac{L_i(E)/E}{L_e(E)/E} \simeq \frac{dL_i/dE}{dL_e/dE} \simeq \frac{1}{kB(dE/dr)_i}$$

It gives the following important features of QF:

- QF depends on E (in many papers constant QF was supposed);

- QF is minimal when dE/dr is maximal;

- QF increases at low E (because of decrease of dE/dr, see Fig. 1).

In the following calculations:

1. All results are obtained with Eq. (1) or Eq. (2):

$$Q_i(E) = \frac{L_i(E)}{L_e(E)} = \frac{\int_0^E \frac{dE}{1+kB(\frac{dE}{dr})_i}}{\int_0^E \frac{dE}{1+kB(\frac{dE}{dr})_e}} \qquad R_i(E) = \frac{L_i(E)/E}{L_e(E_0)/E_0}$$

2. *dE/dr* are calculated with the ESTAR code for electrons and the SRIM code for ions (sometimes with the ASTAR code for α particles);

3. Total *dE/dr* is used.

Some precautions when comparing experimental and calculated QF values:

1. QF depends on kind and amount of dopant in doped scintillators (like NaI(Tl)) but also could depend on impurities and defects in "pure" scintillators (change could be tens of %);

2. QF depends on temperature (up to tens of %, see APP'2010);

3. QF depends on such a technical parameter as time Δt during which scintillation signal is collected (see Fig. 2); if signals are not collected during proper time, it is possible to obtain wrong conclusions on QF values (enhancement instead of quenching);

4. It is better to use measurements of QFs when ions' energies are exactly known (e.g. not with non-monoenergetic neutron sources like Am-Be with spectrum up to ~11 MeV – they give mixture of QFs for different *E* and different ions constituting scintillator).

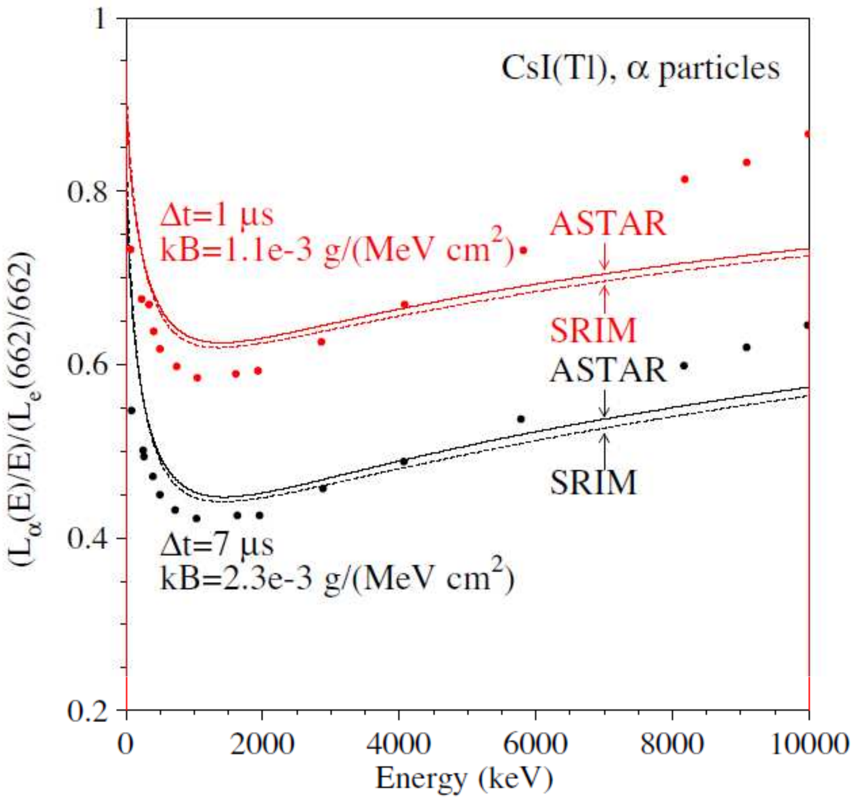


Fig. 2:
Data from R. Gwin, R.B. Murray, Phys. Rev. 131 (1963) 501;
Fit by eq. (2)

Change of Δt from 1 μs to 7 μs resulted in change of QF (~30%) and in change of kB from 1.1e-3 to 2.3e-3 g/(MeV cm^2)

And: scintillation signal for p is faster than that for e^-; at 662 keV $LY_p > LY_e$ with Δt=1 μs (thus $QF_p > 1$ – enhancement) while $LY_p < LY_e$ with Δt=7 μs (thus $QF_p < 1$ – quenching)

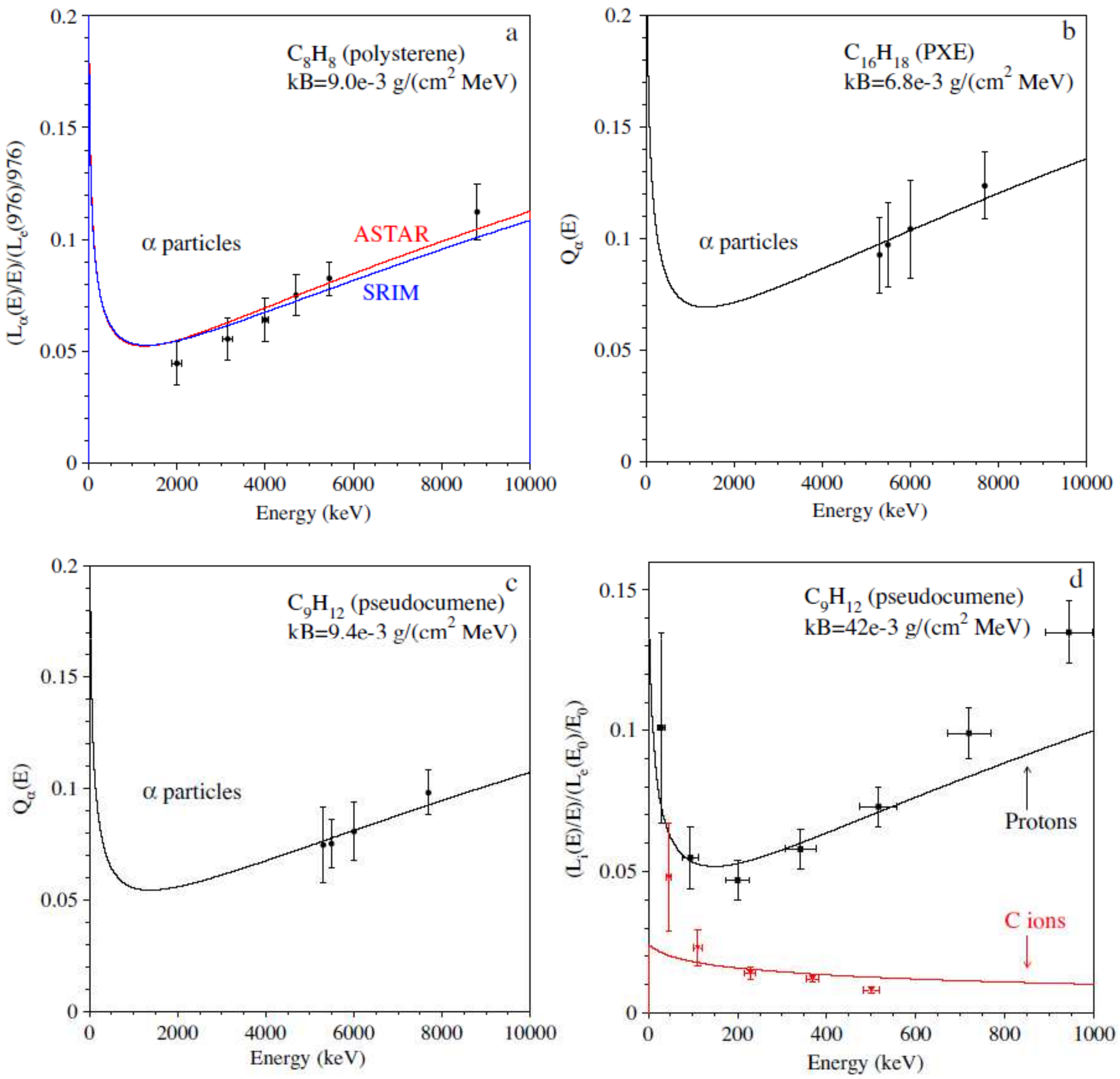
(and sometimes Δt could be even not mentioned in a paper …)

In the following, we will not expect that *kB* is some fundamental constant of scintillating material; its value could be different in different experimental conditions (including data treatment).

However, we will suppose that if experimental conditions and data treatment are fixed, *kB* is the same for all particles (e^- and different ions).

Below we will check this hypothesis.

3. Calculation of QFs for different scintillators



Organic scintillators:

C_8H_8 (polysterene, solid)

$C_{16}H_{18}$ (PXE, liquid)

C_9H_{12} (pseudocumene, liquid)

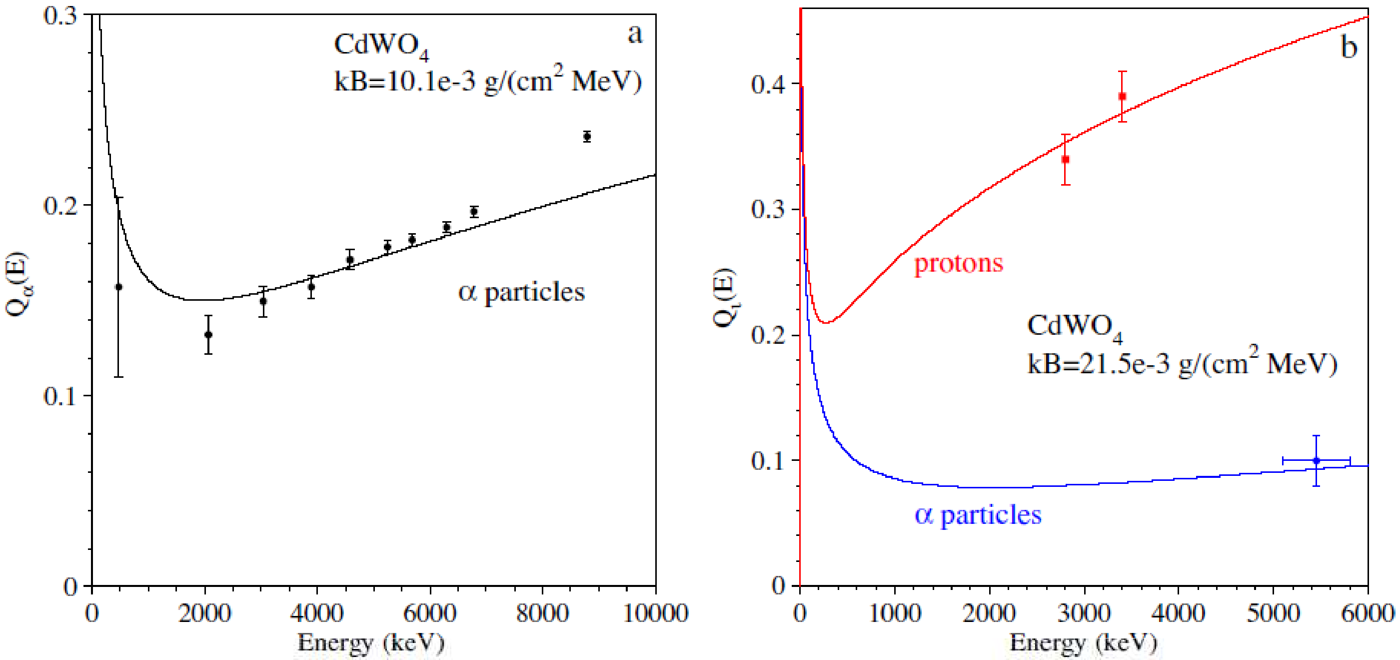
For C_9H_{12}, *kB* values in (c) and (d) are very different – not surprise (different conditions)

In (d), *kB* is obtained by fitting data for protons; after this, curve for C ions is calculated

(a): M. Bongrand (SuperNEMO), AIPCP 897 (2007) 14

(b,c): H.O. Back et al. (BOREXINO), NIMA 584 (2008) 98

(d) J. Hong et al., APP 16 (2002) 333

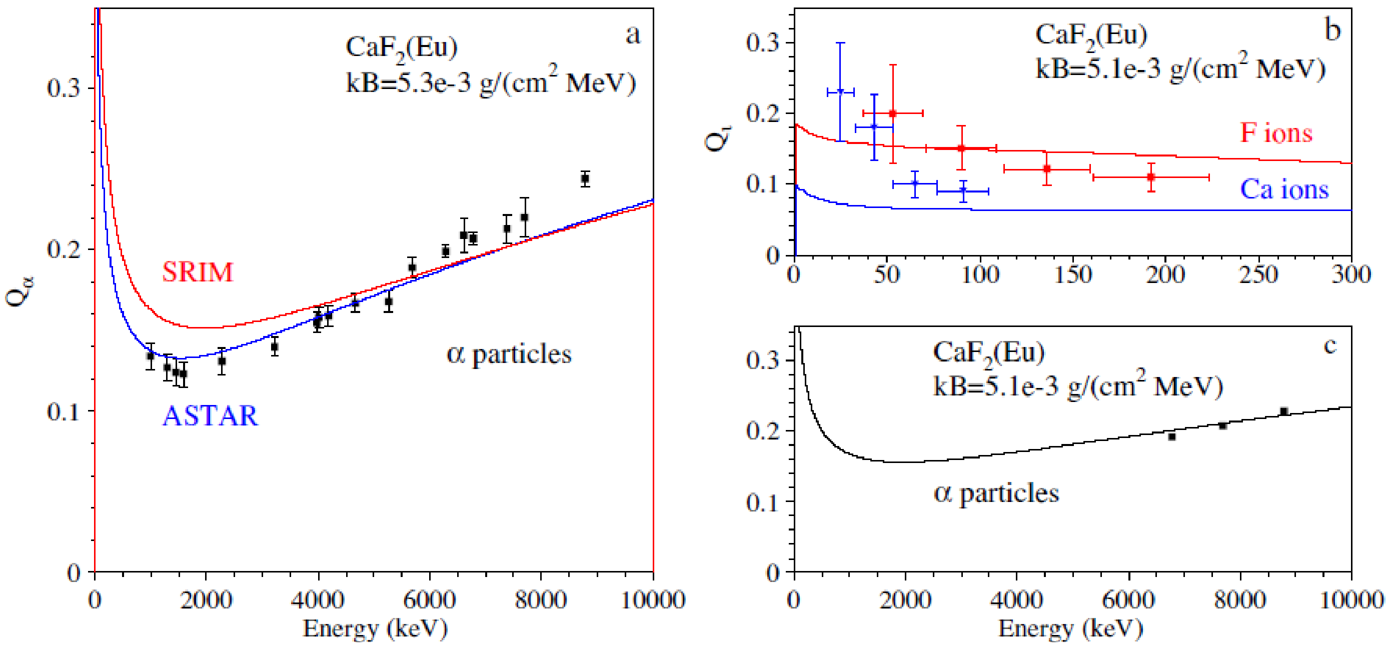


Crystal scintillators: $CdWO_4$

Once more, *kB* values in (a) and (b) are different for the same material due to different experimental conditions.

In (b), *kB* value is obtained fitting data for protons; then, curve for α particles was calculated

(a): F.A. Danevich et al., PRC 67 (2003) 014310
(b): T. Fazzini et al., NIMA 410 (1998) 213

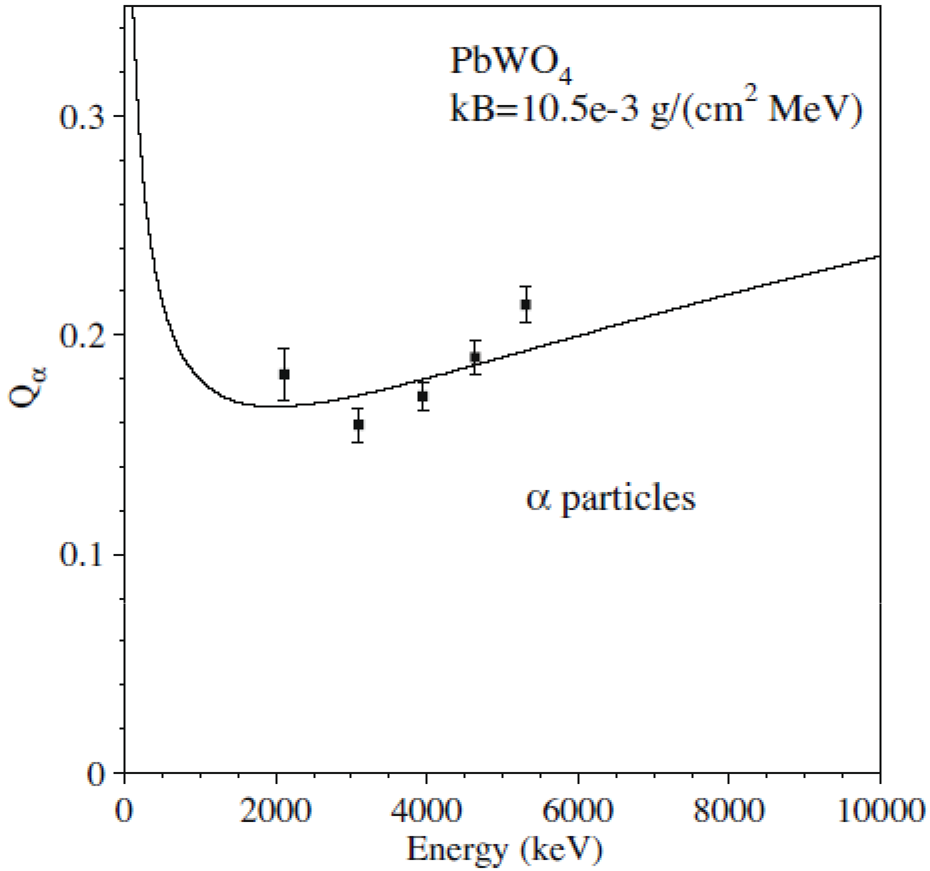


Crystal scintillators: CaF_2(Eu)

Near the same kB value for data of (a) and (b,c).

In (b), fit of data for F ions was used to draw curve for Ca ions (not so good agreement).

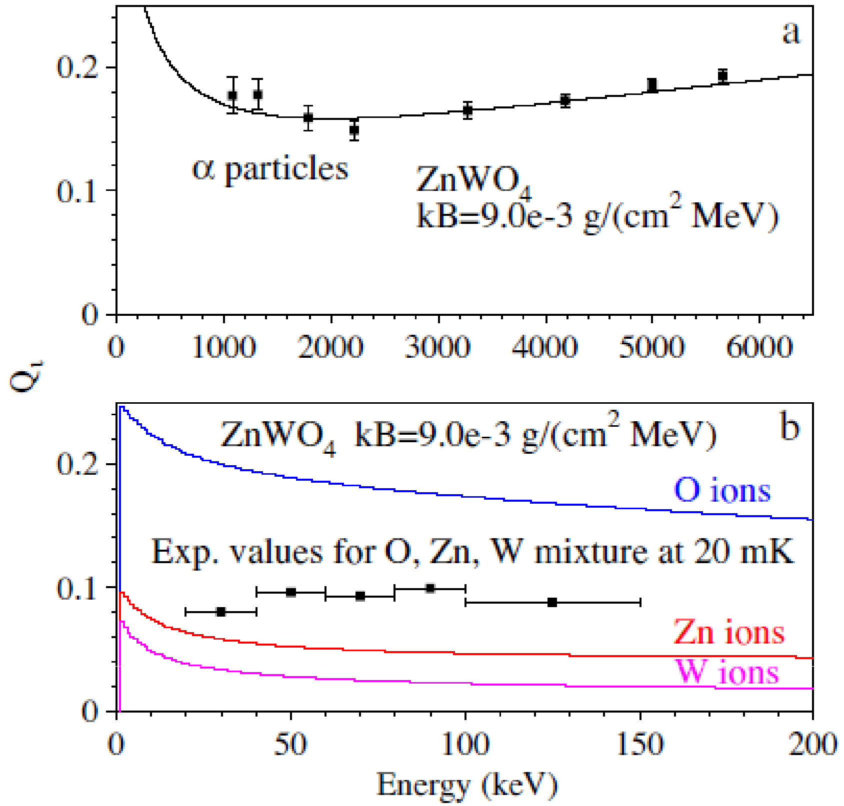
(a): P. Belli et al., NPA 789 (2007) 15
(b): R. Hazama et al., NIMA 482 (2002) 297
(c): S. Umehara et al., PRC 78 (2008) 058501



Crystal scintillators:

$PbWO_4$

Data: F.A. Danevich et al., NIMA 556 (2006) 259



Crystal scintillators: $ZnWO_4$

(a): I. Bavykina et al., IEEE TNS 55 (2008) 1449

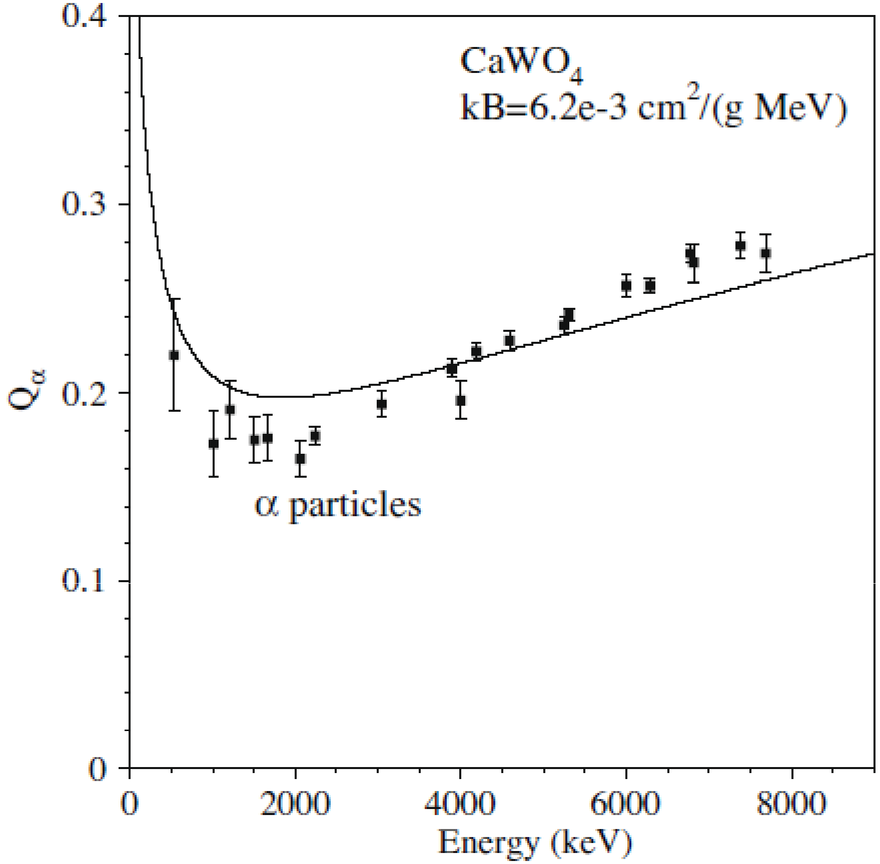
(b): F.A. Danevich et al., NIMA 544 (2005) 553

In (b), curves for O, Zn and W ions are calculated with *kB* value obtained by fitting data for α particles of (a). Not good agreement with data (b) is explained by:

(1) different temperatures – room *T* in (a) and 20 mK in (b);

(2) non-monoenergetic neutron source in (b), thus mixture of QFs for different energies and ions;

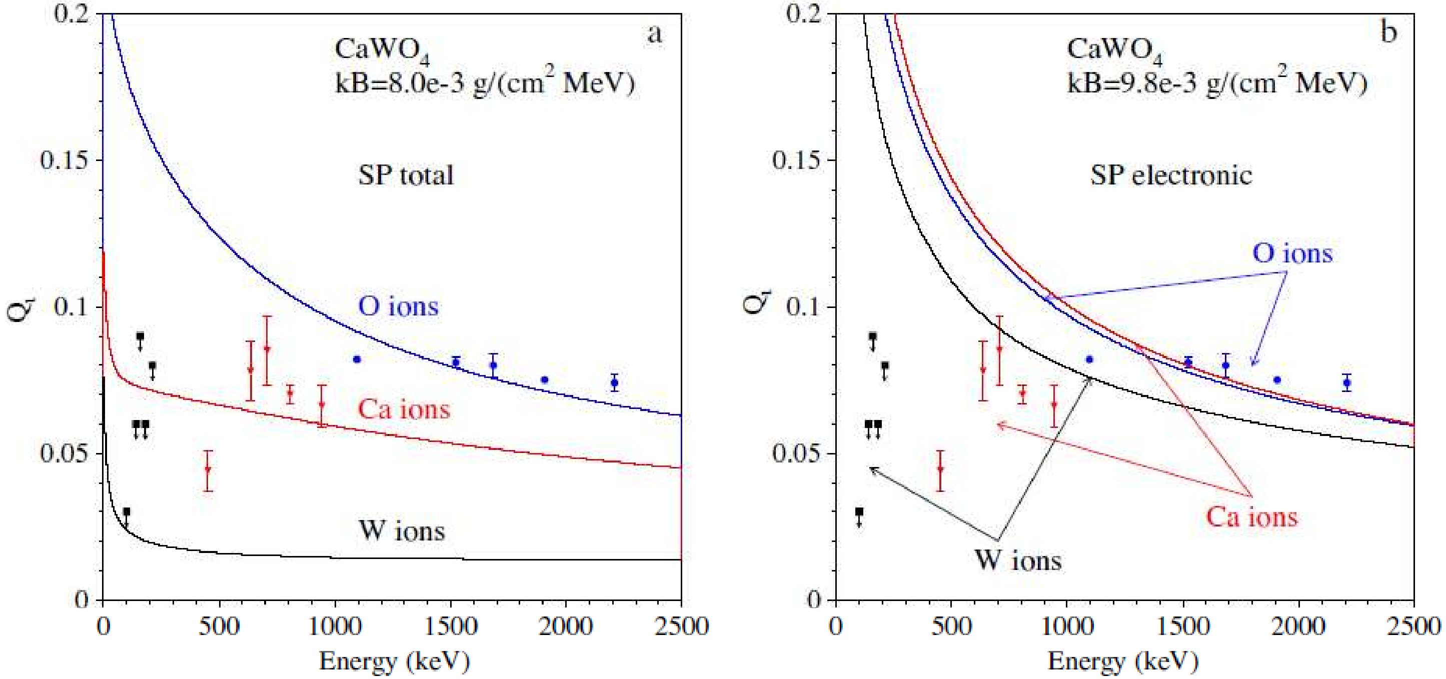
(3) different data treatment.



Crystal scintillators: $CaWO_4$ – 1

Data: Yu.G. Zdesenko et al., NIMA 538 (2005) 657

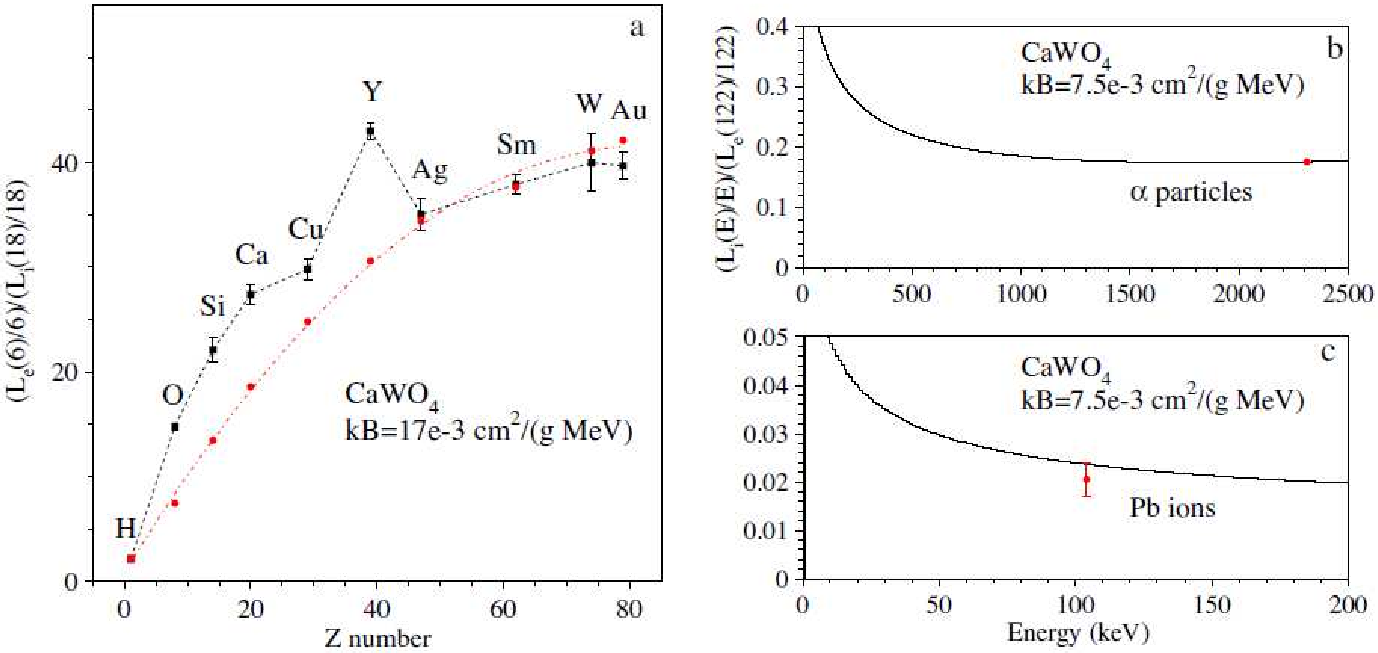
Crystal scintillators: $CaWO_4$ – 2



kB **was obtained from fit of data for O ions; then curves for Ca and W ions were calculated. In (a), the total *dE/dr* was used; in (b) – only electronic part of *dE/dr***

Total *dE/dr* is much more preferable in describing the QF behaviour.

Data: Th. Jagemann et al., APP 26 (2006) 269



Crystal scintillators: $CaWO_4$ – 3

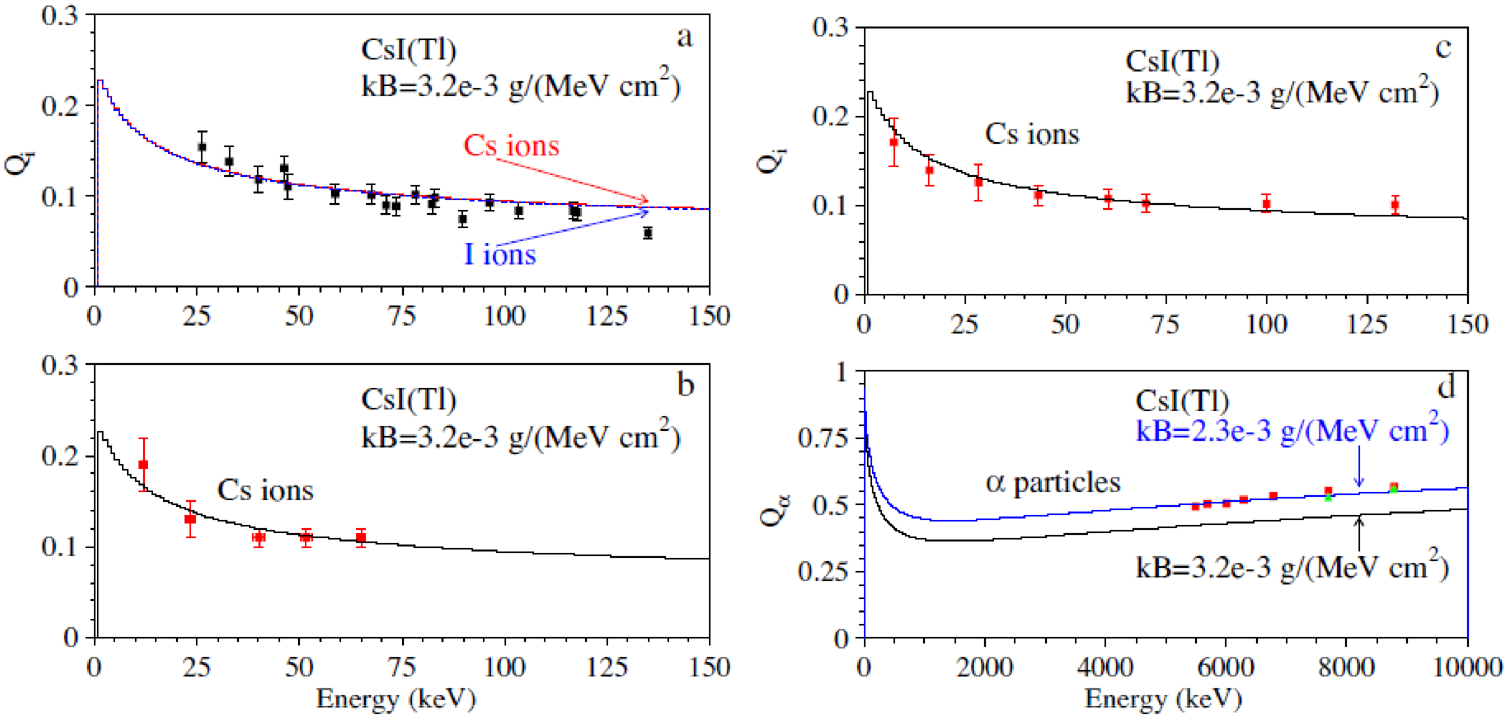
(a): J. Ninkovic et al., NIMA 564 (2006) 567.
(b): G. Angloher et al., APP 23 (2005) 325.

LY/*E* of e^- (for *E*=6 keV) to LY/*E* of different ions (for *E*=18 keV)

In (a), *kB* was obtained normalizing th. and exp. *R'* values for protons; after this, *R'* values for all other ions were calculated.
Range of *A*, *Z* values: *A*=1, *Z*=1 for p, and *A*=197, *Z*=79 for Au.

In (a) – room *T*, in (b,c) *T*=7 mK – so, different *kB*.

In (b), *kB* was obtained to reproduce *R* for 2.3 MeV α particle (*A*=4, *Z*=2). In (c), calculated curve with this kB is in agreement with point for 104 keV Pb ions (*A*=206, *Z*=82).

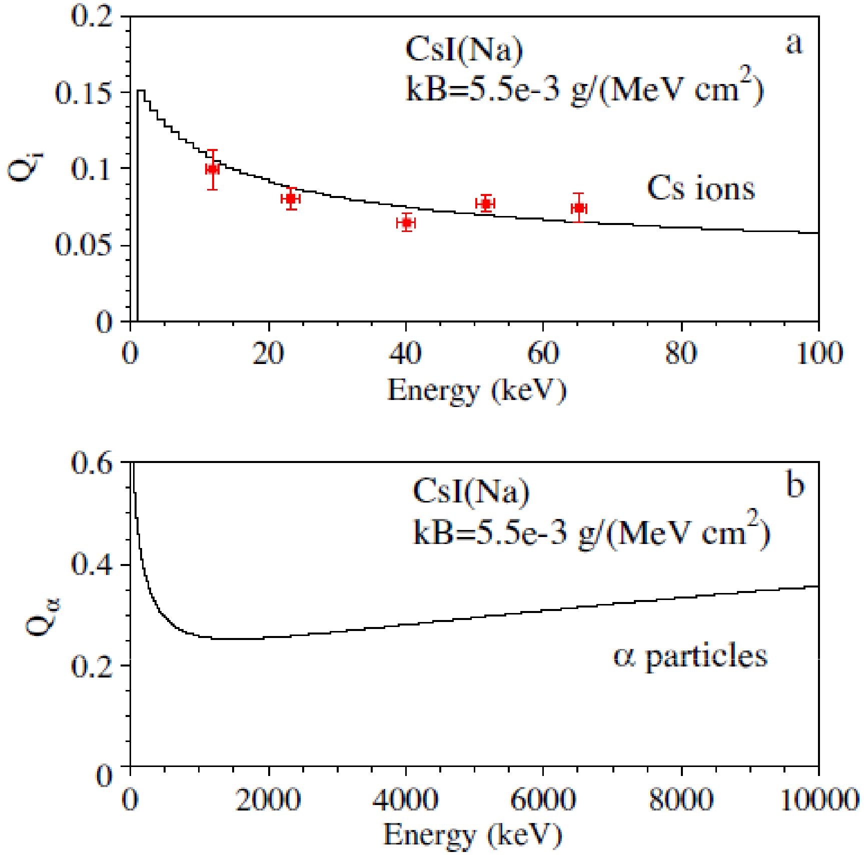


Crystal scintillators: CsI(Tl)

QF values for Cs and I ions are practically the same.
***kB* is the same for measurements by different groups (this could be just coincidence).**

***kB* for α particles is different in (d) but data were measured in different conditions.**

(a): S. Pecourt et al., APP 11 (1999) 457
(b): H. Park et al., NIMA 491 (2002) 460
(c): M.Z. Wang et al., PLB 536 (2002) 203
(d): T.Y. Kim et al., NIMA 500 (2003) 337
Y.F. Zhu et al., NIMA 557 (2006) 490

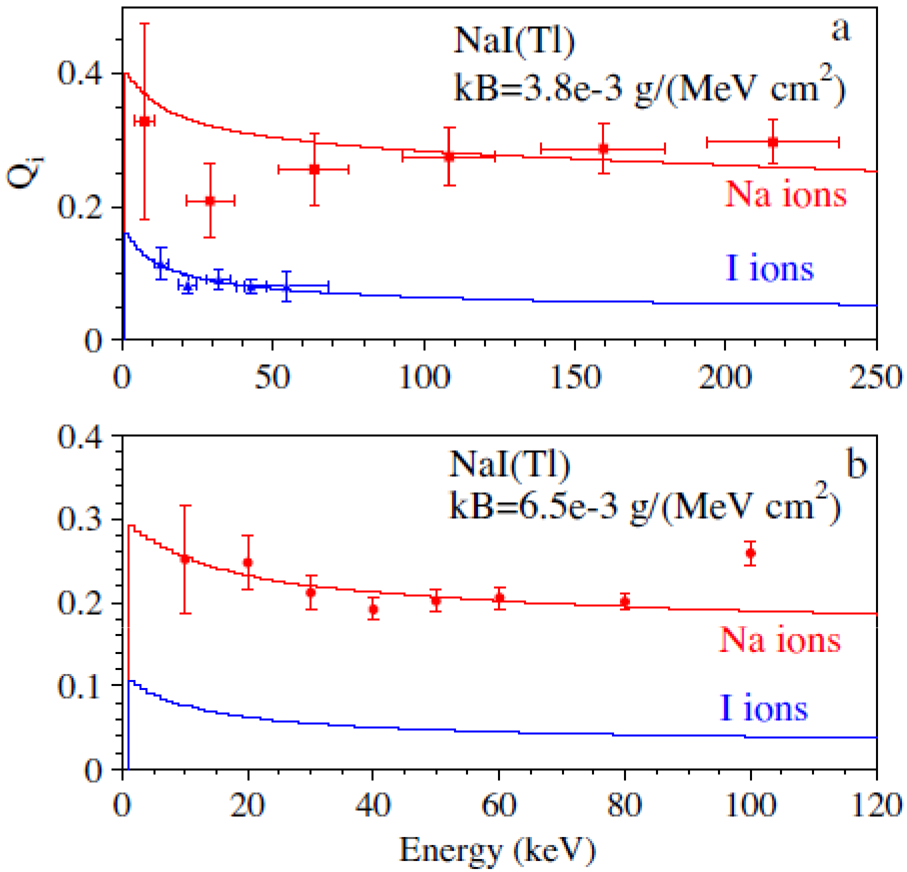


Crystal scintillators:
CsI(Na)

In (a), QF for Cs (or I) ions are described.

In (b), QF for α particles with *kB*=5.5e-3 is predicted.

Data: H. Park et al., NIMA 491 (2002) 460

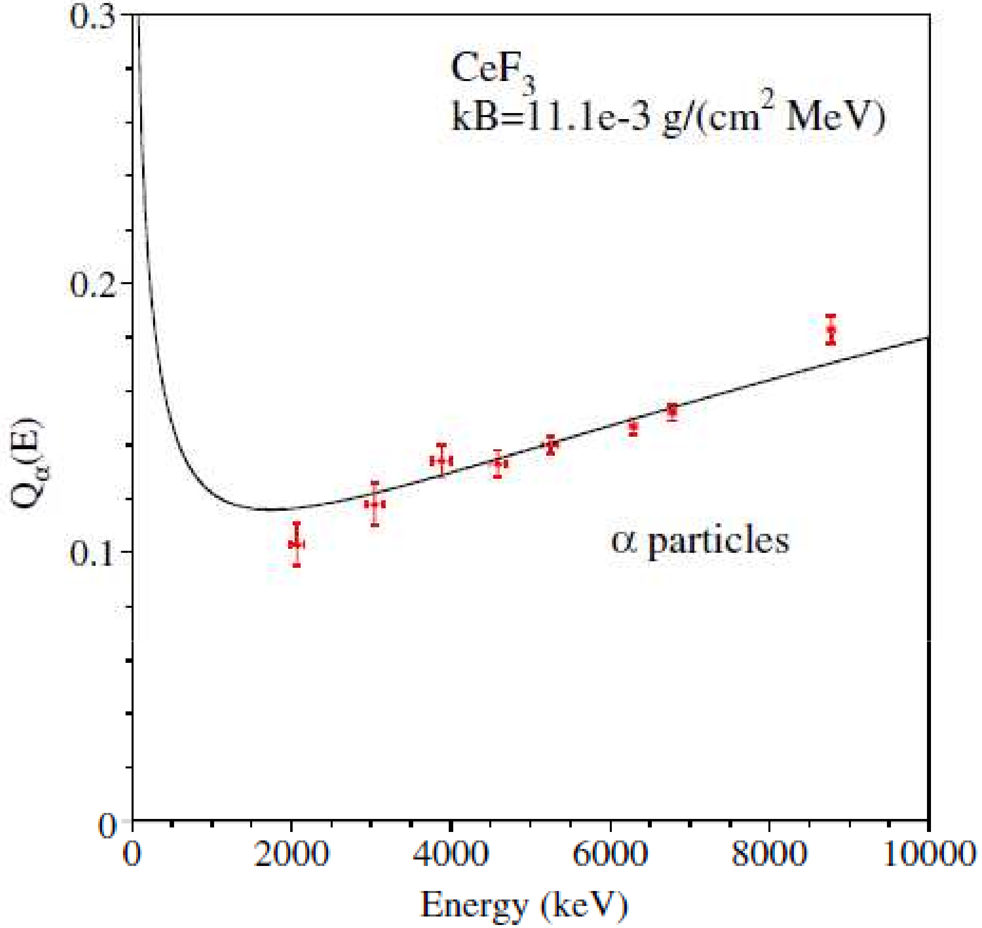


Crystal scintillators: NaI(Tl)

(a): D.R. Tovey et al., PLB 433 (1998) 150
(b): H. Chagani et al., JINST 3 (2008) P06003

In (a), *kB* was obtained by describing data for Na ions and then used to calculate curve for I ions (nice agreement).

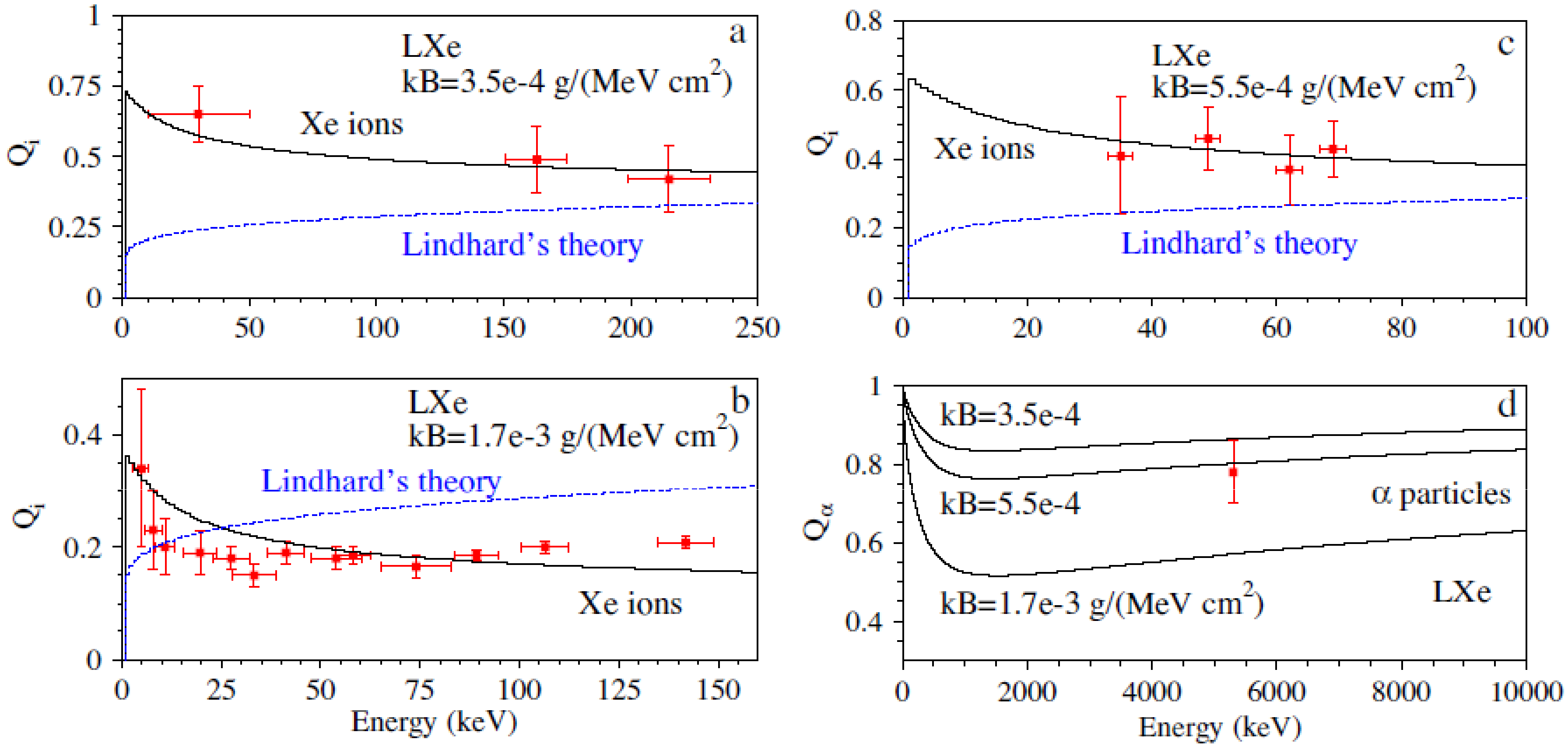
***kB* in (a) and in (b) are different due to different experimental conditions.**



Crystal scintillators:
CeF_3

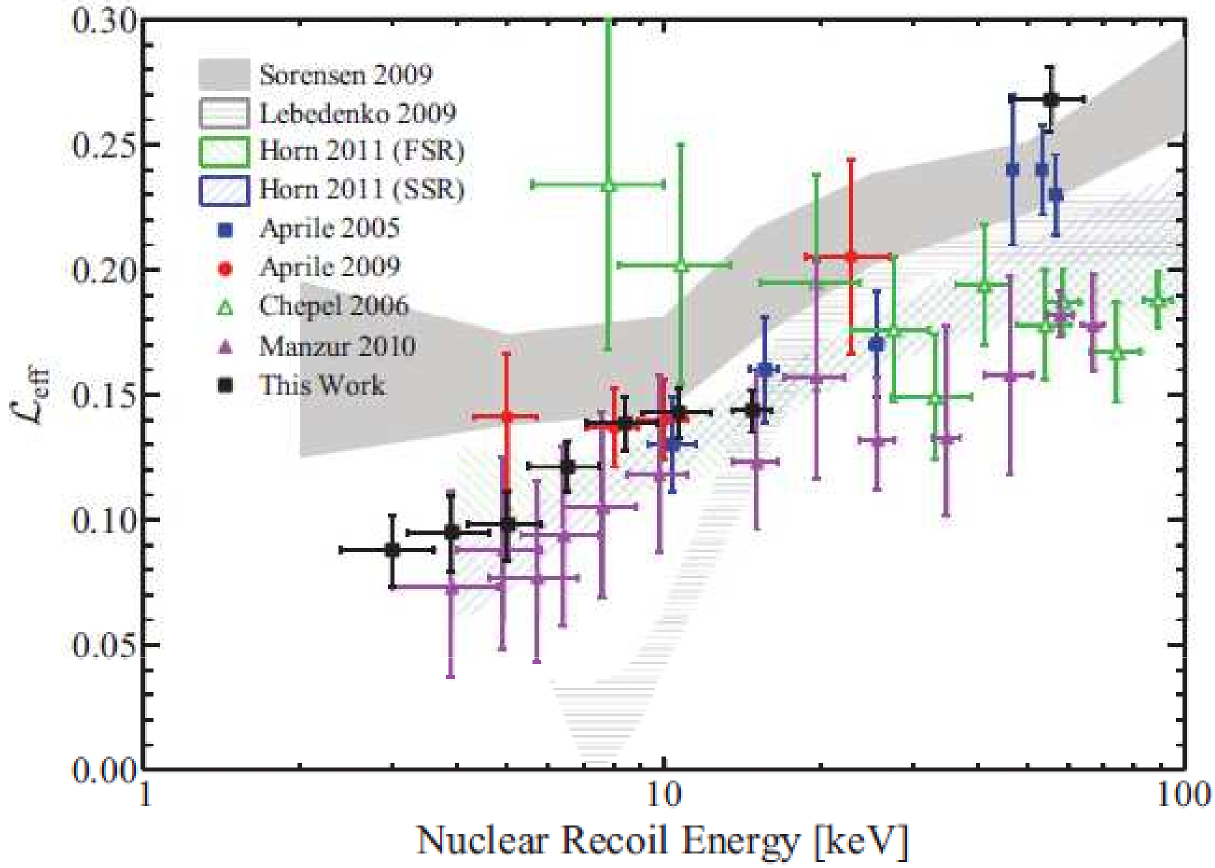
Data: P. Belli et al., NIMA 498 (2003) 352

Liquid noble gases: LXe – 1



Some experimental data are well described in the current approach; different *kB* values are due to different conditions.

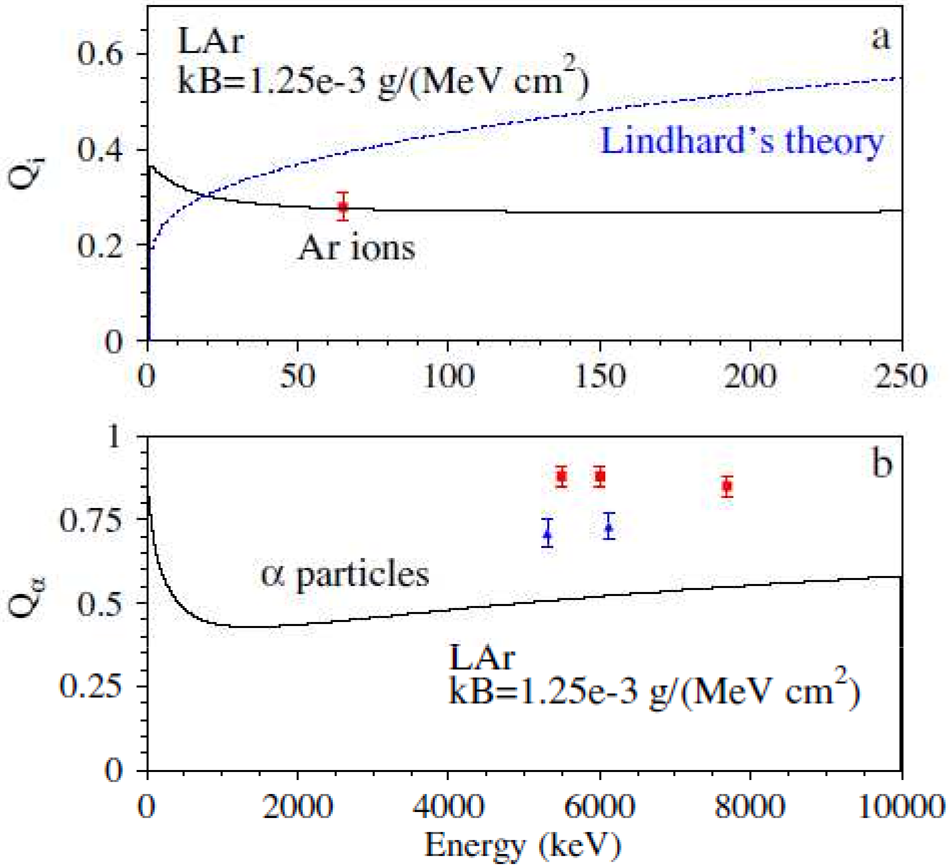
(a): R. Bernabei et al., PLB 436 (1998) 379
(b): V. Chepel et al., APP 26 (2006) 58
(c): R. Bernabei et al., EPJCdir 11 (2001) 1
(d): M. Tanaka et al., NIMA 457 (2001) 454



Liquid noble gases: LXe – 2

However, for Xe ions in LXe exist also other experimental data which are not described in the current approach.

Data (and summary): G. Plante et al., PRC 84 (2011) 045805



Liquid noble gases:

LAr

On the time of publication of the APP'2010 paper, only 1 point was known for Ar ions in LAr (see later new data).

(a): R. Brunetti et al., New Astron. Rev. 49 (2005) 265; WARP Proposal
(b): A. Hitachi et al., PRA 35 (1987) 3956
P. Peiffer et al., JINST 3 (2008) P08007

Few calculations fulfilled

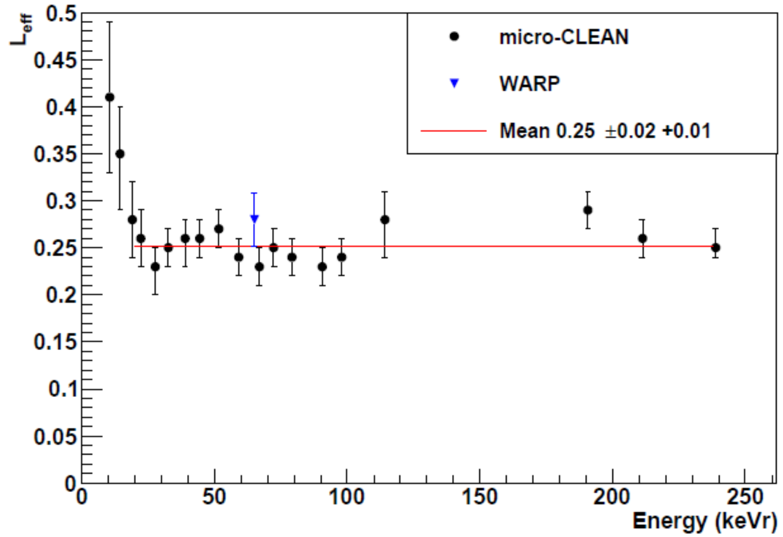
after the article: V.I. Tretyak, Astropart. Phys. 33 (2010) 40

are given below

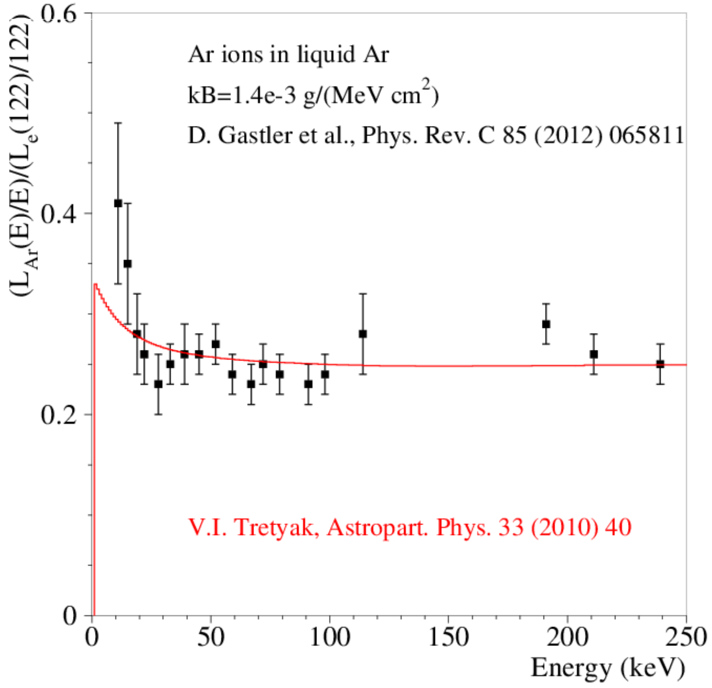
Ar ions in liquid Ar - D. Gastler et al., PRC 85 (2012) 065811:

Description of the data just by constant after 20 keV

Page 7: "An observed upturn in the scintillation efficiency below 20 keVr is currently unexplained."

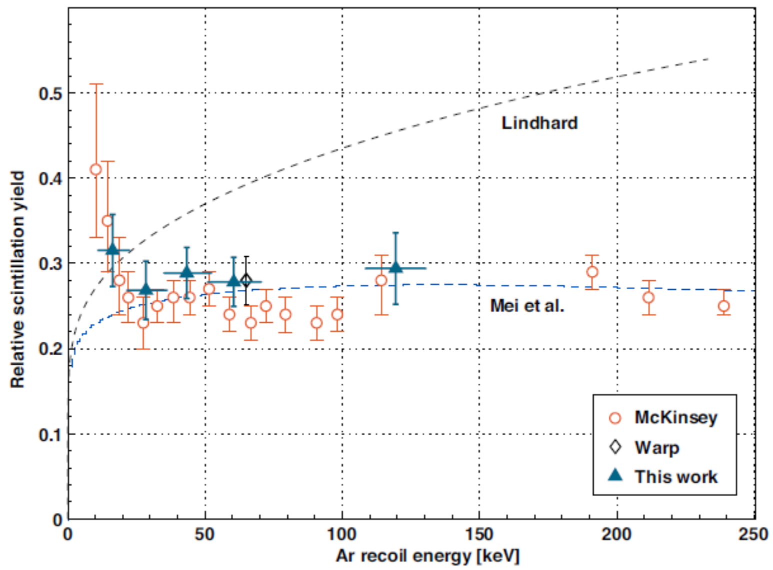


Description here in accordance with Eq. (2):

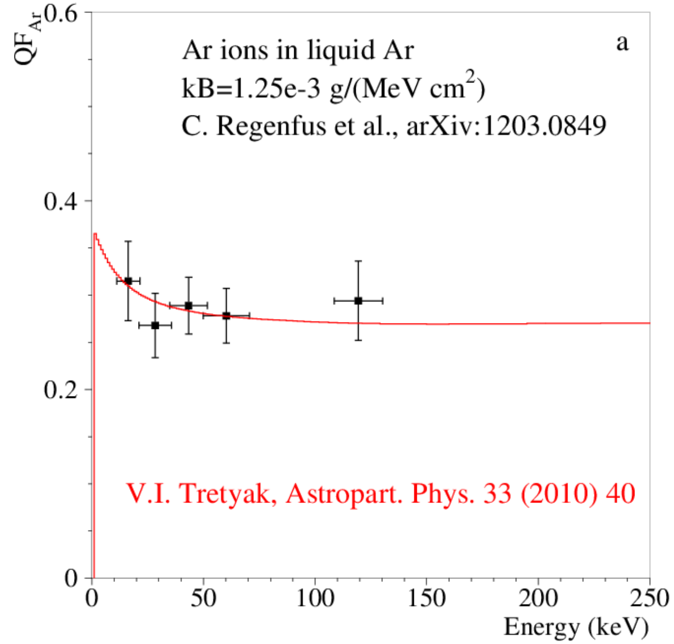


Ar ions in liquid Ar – C. Regenfus et al., arXiv:1203.0849 = JPCS 375 (2012) 012019

Experimental data with
2 models:
1. J. Lindhard et al.,
   Mat. Fys. Medd. Dan.
   Vid. Selsk. 33/14 (1963) 1
2. D. Mei et al., APP 30 (2008) 12



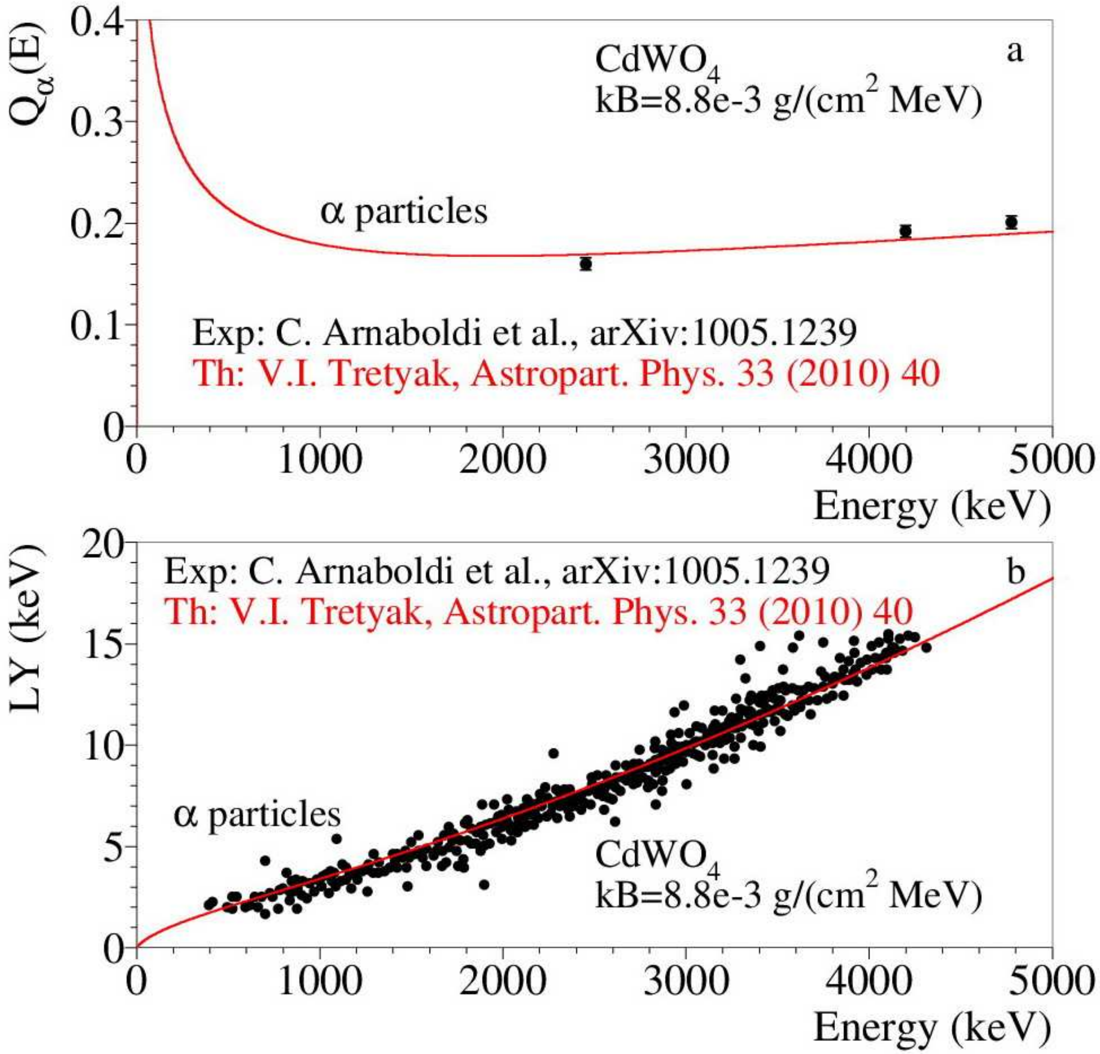
Description here in
accordance with Eq. (2):



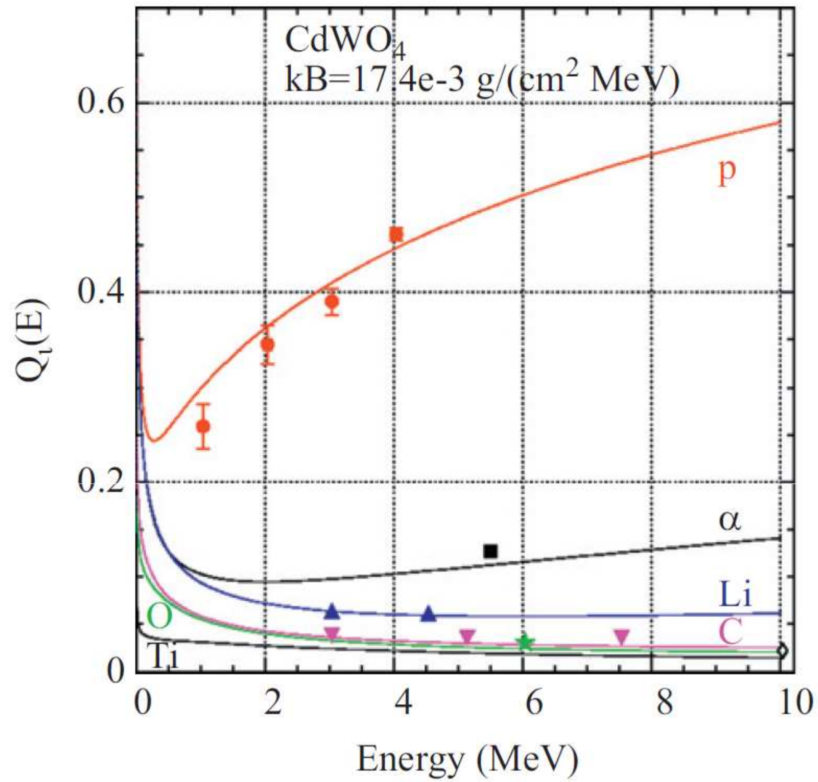
Alpha particles in $CdWO_4$ - C. Arnaboldi et al., APP 34 (2010) 143

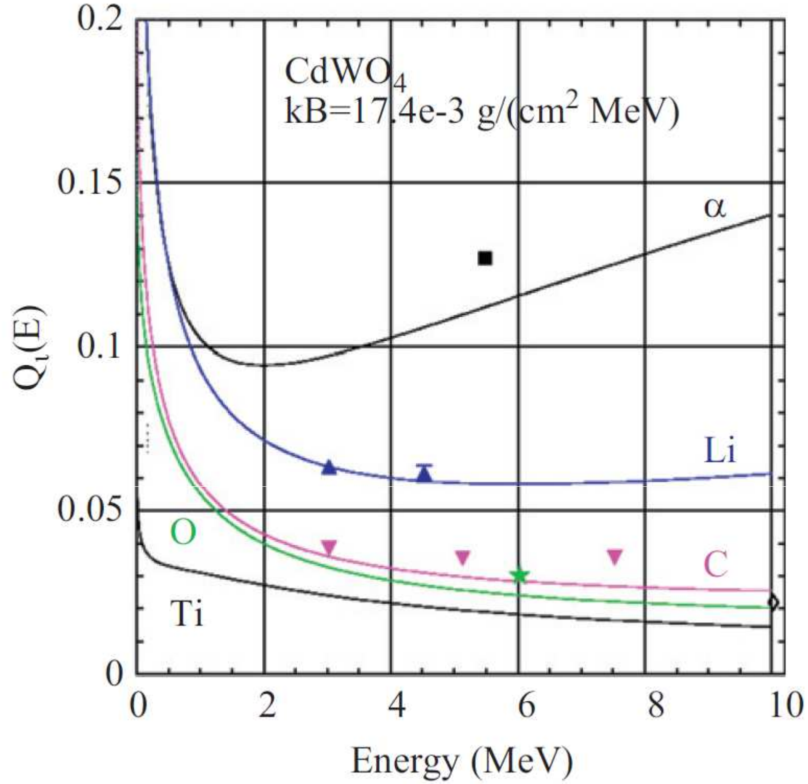
Data presented in (a) were used to obtain the *kB* value.

LY curve for continuous spectrum of alpha particles was calculated with this *kB*.



Different ions in $CdWO_4$ - P.G. Bizzeti et al., NIMA 696 (2012) 144





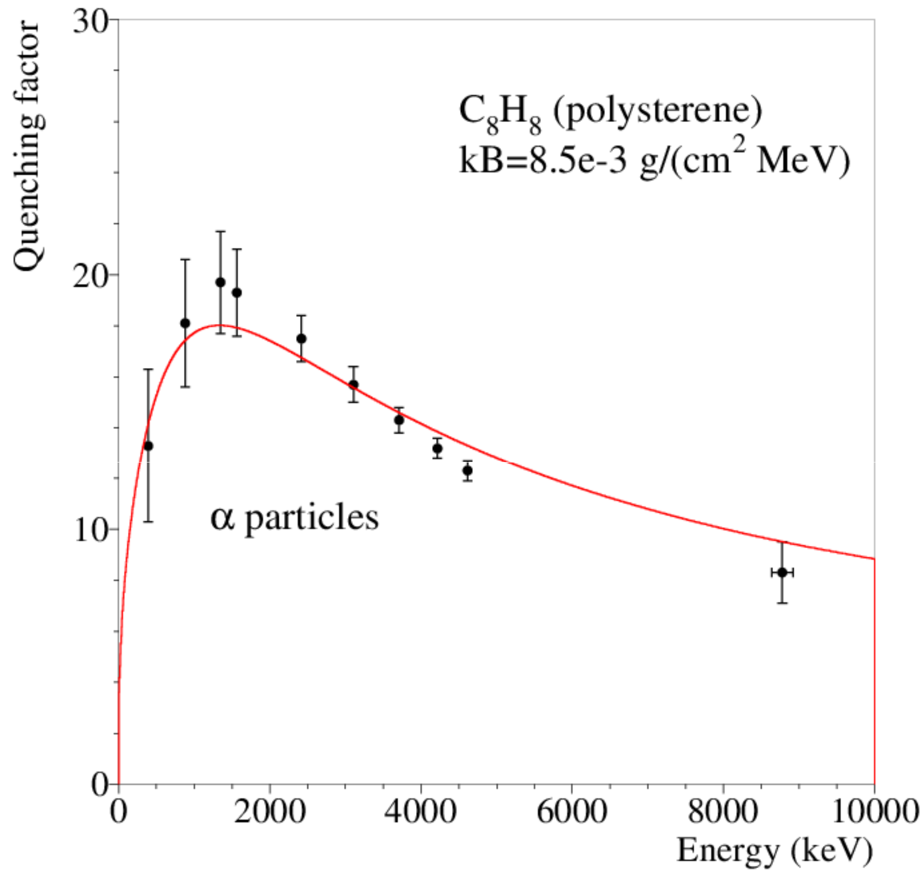
Value of *kB* was obtained by fitting data for protons.

Quenching curves for other ions (α, Li, C, O, Ti) were calculated with this *kB*, in quite good agreement with the experimental data.

Alpha particles in plastic - X. Sarazin, Memoire d’habilitation, 2012

BiPo-3: C_8H_8 (polysterene)

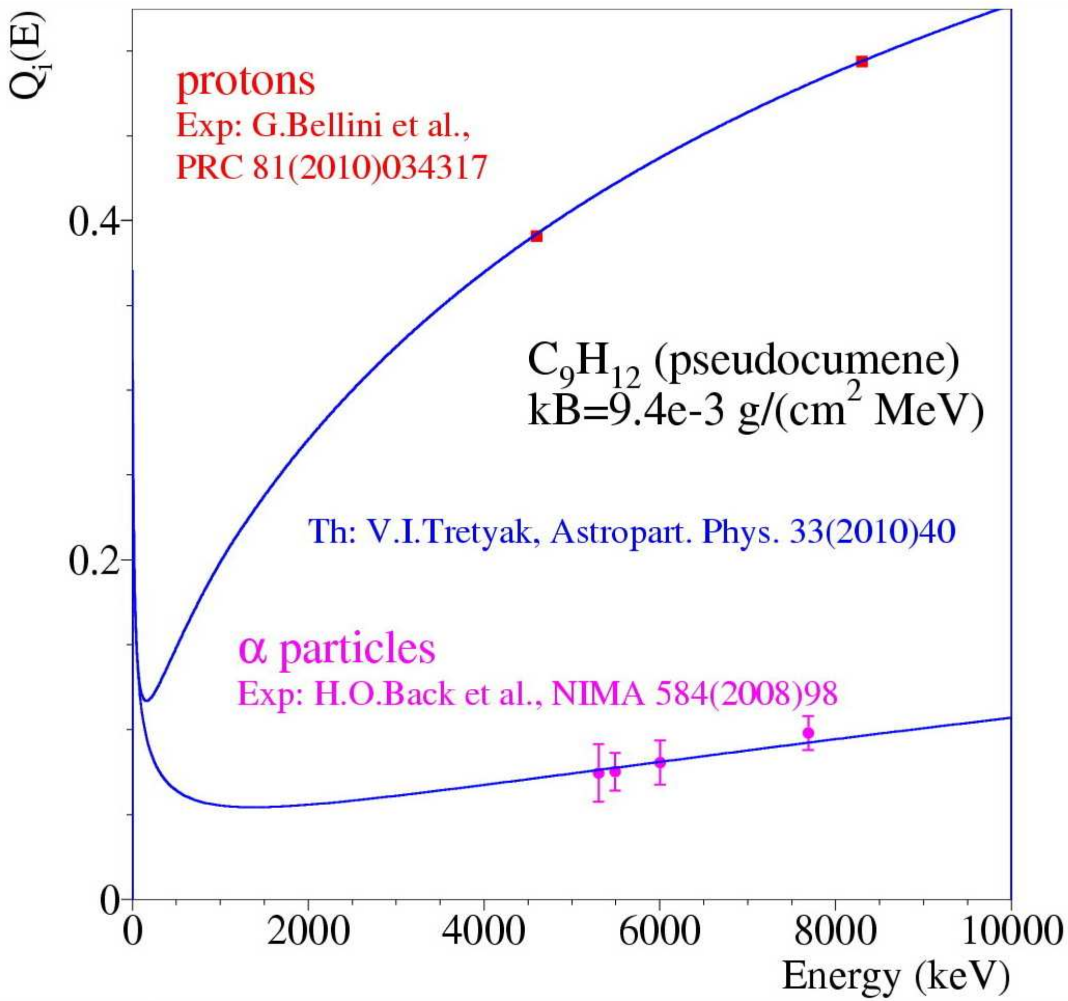
Red line is calculation of 1/QF



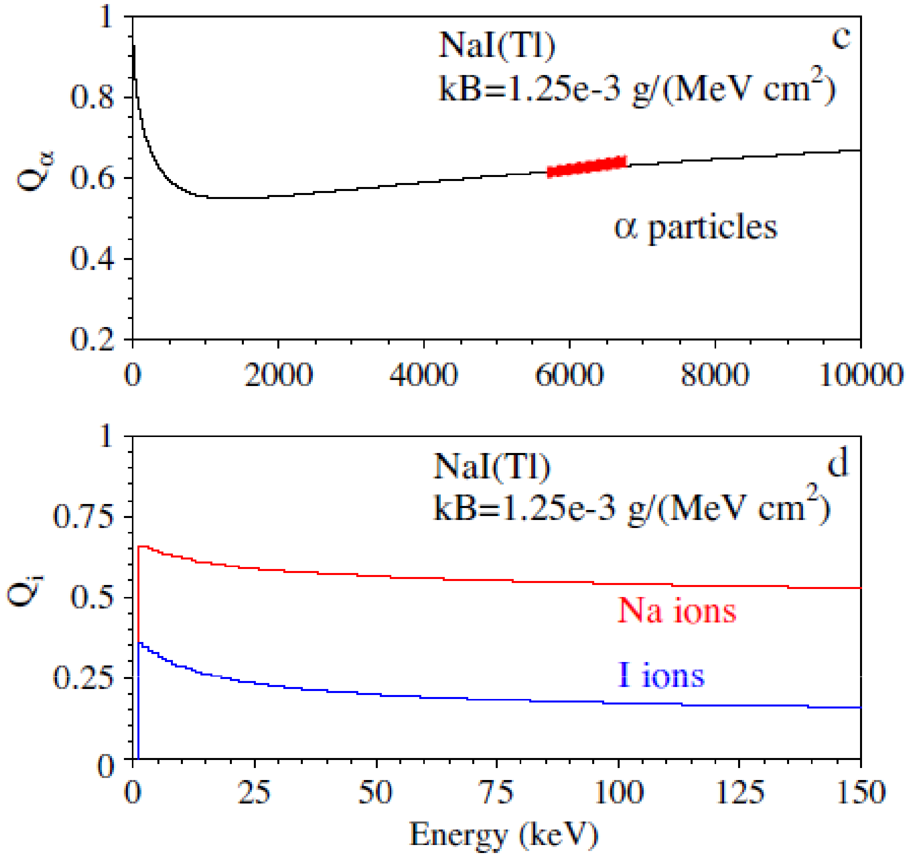
Protons in pseudocumene (C_9H_{12}) – G. Bellini et al., PRC 81 (2010) 034317

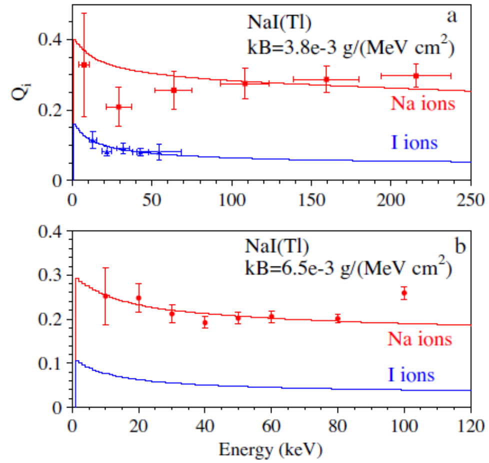
Data for alpha particles are drawn as in APP'2010

Data for protons appeared after APP'2010.
Curve QF_p for protons is calculated with the same *kB* value as for alpha's.
Agreement is excellent.



4. Specific case of QF for DAMA NaI(Tl)





Typical measurements for NaI(Tl):
QF_{Na} – 0.25–0.40
QF_{I} – 0.05–0.10
(see APP'2010)

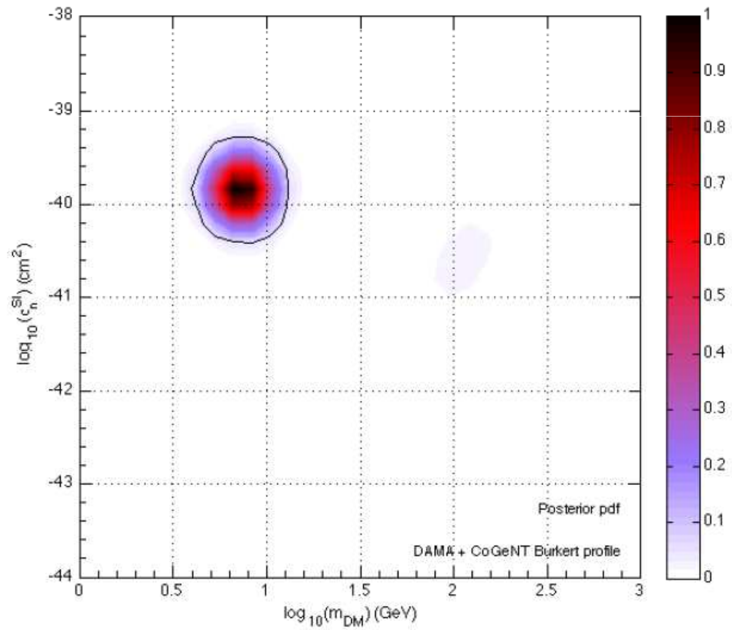
In (c), *kB* is obtained by fitting data for α particles (internal contamination) in LIBRA experiment (R. Bernabei et al., NIMA 592 (2008) 297) – in the same conditions as DM data. Predicted with this *kB* QFs for Na and I ions in (d) are much higher (QF_{Na}=0.64 at 5 keV) than those usually measured and used in NaI(Tl) dark matter experiments.

C. Arina, J. Hamann, Y.Y.Y. Wong, JCAP 09 (2011) 022:

(see also JPCS 375 (2012) 012009 and 1210.4011)

Limitation: $QF_{Na} \leq 0.6$

Combined fit of the DAMA and CoGeNT, QF_{Na} is free parameter. "If we demand compatibility between these experiments, then the inference process naturally concludes that a high value for the sodium quenching factor for DAMA is preferred."



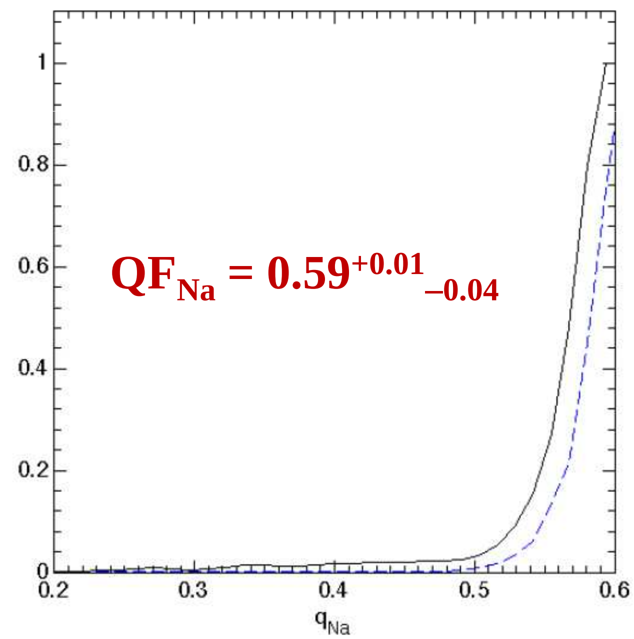
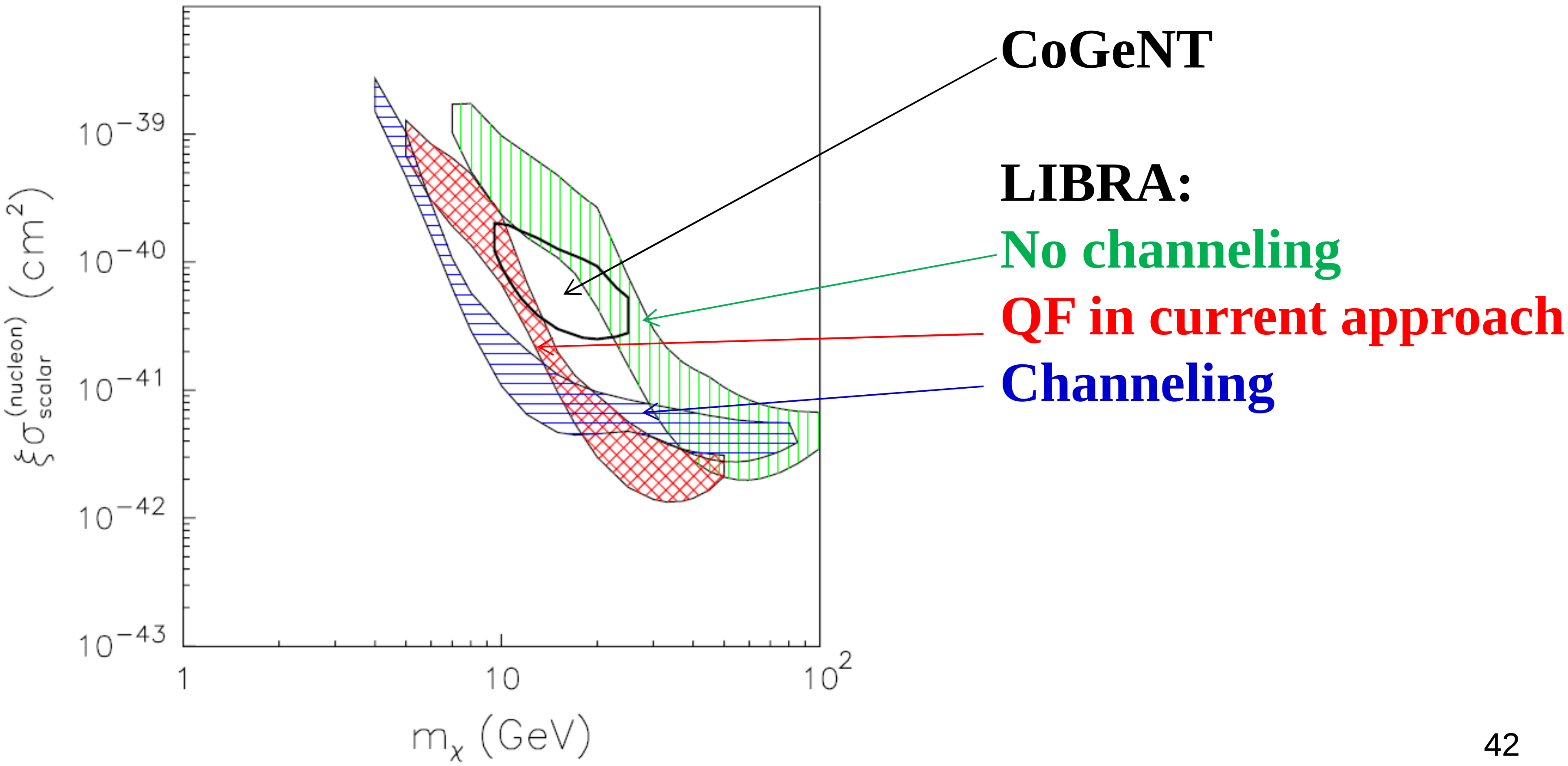


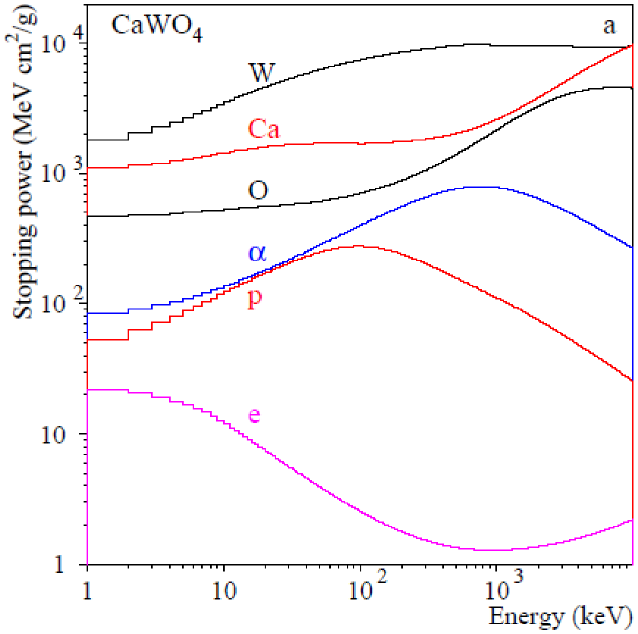
Figure 13. Same as figure 11, but for an extended prior range for the DAMA sodium quenching factor q_{Na} (up to $q_{Na} = 0.6$).

Consequence of bigger QF_{Na} and QF_I:
shift of WIMPs mass to lower values ~10 GeV.

P. Belli et al., PRD 84 (2011) 055014, Fig. 1 (for some set of parameters):



5. Possible relation between QF for low energy electrons (and γ quanta) and QF for ions



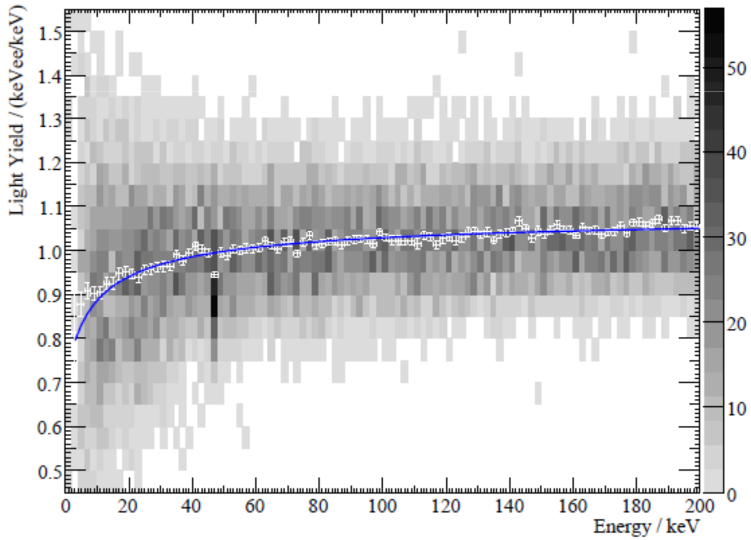
SP for electrons is higher at low energies: so, quenching (non-proportionality) for low energy electrons should exist?

Answer: yes, it exists, and this fact is known many years.

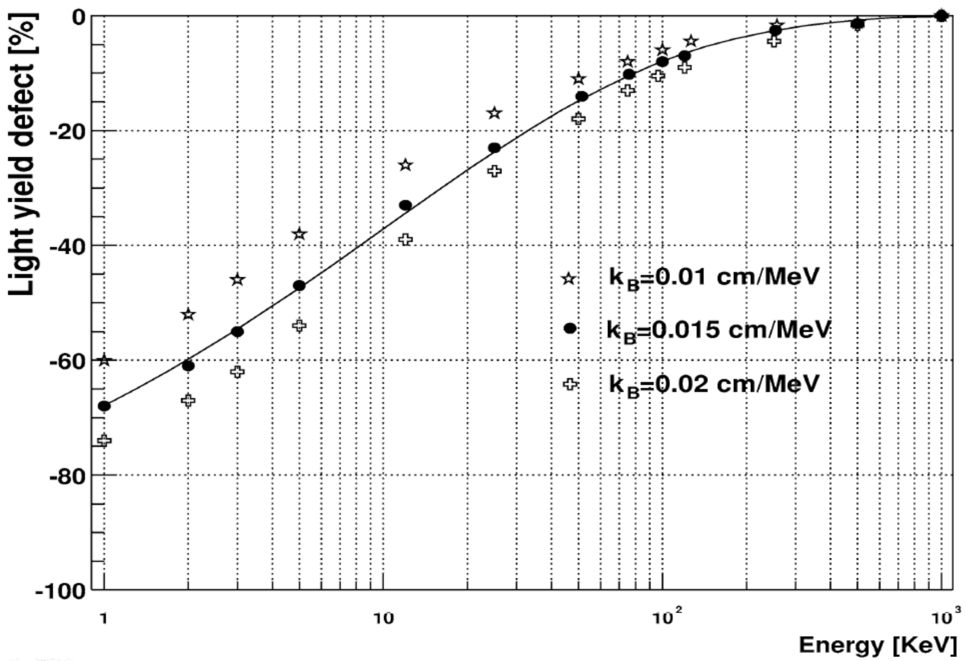
For some scintillators, it is possible to describe R_e on the basis of the Birks formula:

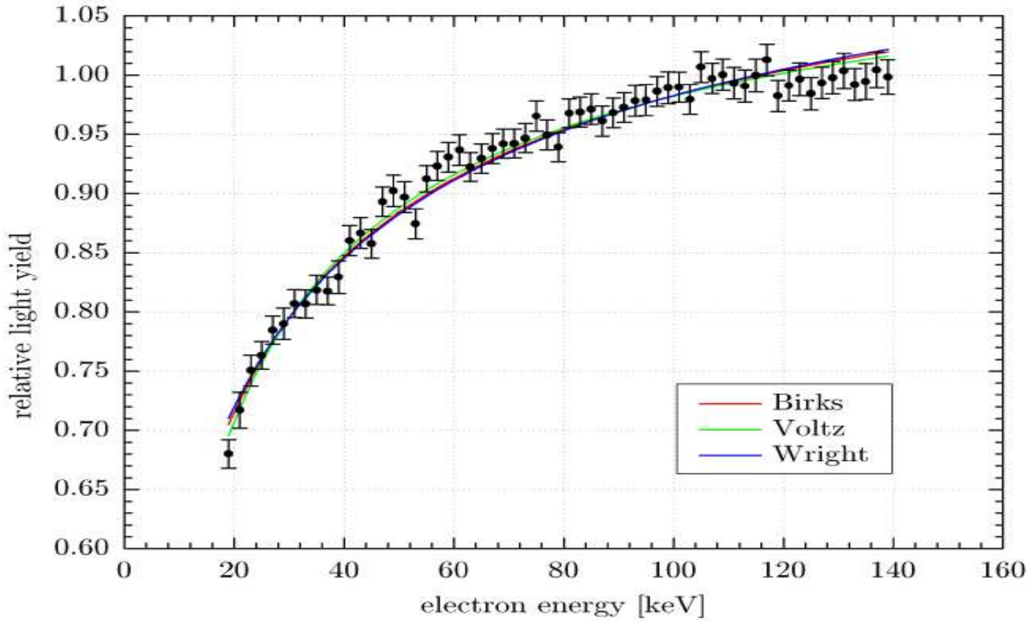
$$R_e(E) = \frac{L_e(E)/E}{L_e(E_0)/E_0}$$

$$L_e(E) = \int_0^E dL_e = \int_0^E \frac{SdE}{1 + kB(\frac{dE}{dr})_e}$$



$CaWO_4$ – R.F. Lang et al., arXiv: 0910.4414

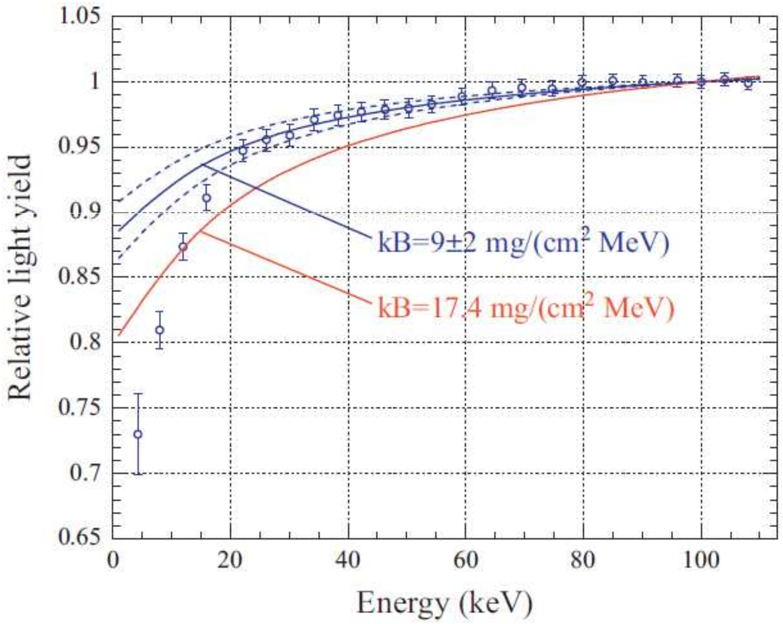


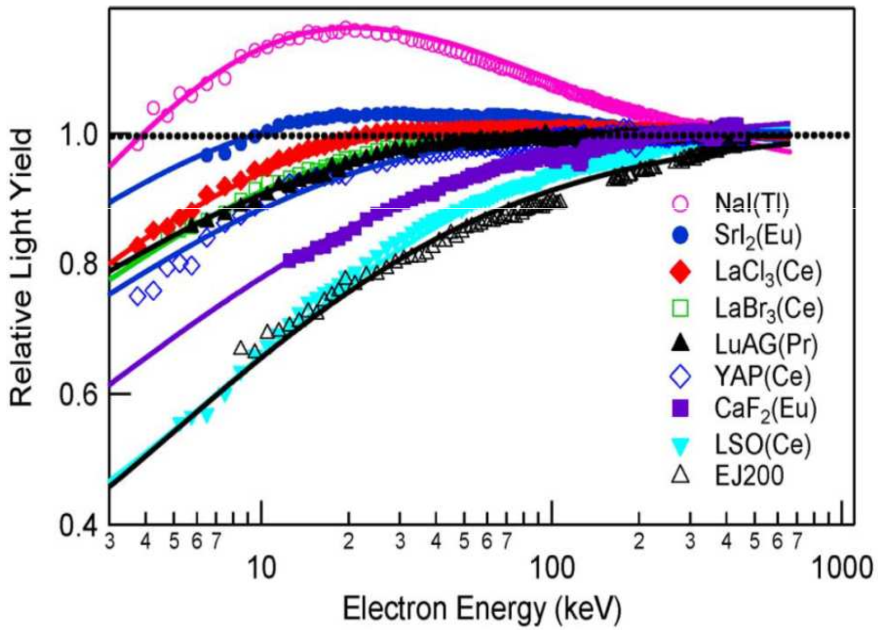


BOREXINO liquid scintillator – H.O. Back et al., PLB B 525 (2002) 29

Double Chooz liquid scintilator – C. Aberle et al., JINST 06 (2011) P11006

However, life is more rich: there are also scintillators for which R_e cannot be described with the Birks law. Moreover, with the Birks formula it is possible to obtain only quenching (at most, constant at *kB*=0), hovewer, there are scintillators where instead enhancement of R_e is observed at low energies.





$CdWO_4$ – P.G. Bizzeti et al., NIMA A 696 (2012) 144

Summary for different scintillators – S.A. Payne et al., IEEE TNS 58 (2011) 3392

So, in my current understanding, if even non-proportional behaviour of relative LY for electrons can be described on the basis of the Birks law, value of *kB* factor is not necessary equal to *kB* for description of QF for ions.

6. Conclusions

(1) Old Birks formula still gives nice description of QF for ions in many cases – if *total* SP for electrons and ions are used, and SP are calculated with the ESTAR and SRIM codes which are: (a) publicly available, (b) are ones of the best codes in this field.

(2) There is only one free parameter in the approach – the Birks *kB* factor. It is not considered as some fundamental constant for a given scintillating material but as a variable which depends on conditions of measurements and data treatment.

(3) There are experimental data which confirm the hypothesis that, once conditions of measurements and data treatment are fixed, the *kB* value is the same for different ions. Thus, if *kB* was determined by fitting data for particles of one kind (e.g. α particles of few MeV from internal contamination), it can be used to calculate QFs for particles of another kind and for another energies of interest (e.g. low energy recoils after scattering of DM particles).

(4) Quenching factors for ions calculated in the present approach in general increase at low energies, and this encourages experimental searches for DM particles.

(5) For the DAMA/LIBRA experiment, it was shown that, based on measured in DAMA/LIBRA QFs for α particles, QFs for Na and I ions should be ~2 higher than those typically used in NaI(Tl) DM experiments. It shifts the "evidence spot" of the DAMA/LIBRA observations to WIMPs' lower masses (~10 GeV) relaxing contradictions with other experiments which give only limits for WIMPs cross-sections.

(6) It seems, that *kB* value which could be extracted from data on non-proportionality of LY for low-energy electrons, is not necessary equal to *kB* for ions (additional investigations are needed).

Thank you for attention!

Some data on quenching of ionization signal in Ge detectors

1. α particles in Ge: S. Fiorucci et al., Astropart. Phys. 28 (2007) 143:
Q=0.30±0.02 at E_α=5.33 MeV and T=17 mK

2. Ge ions in Ge: A. Benoit et al., NIMA 577 (2007) 558

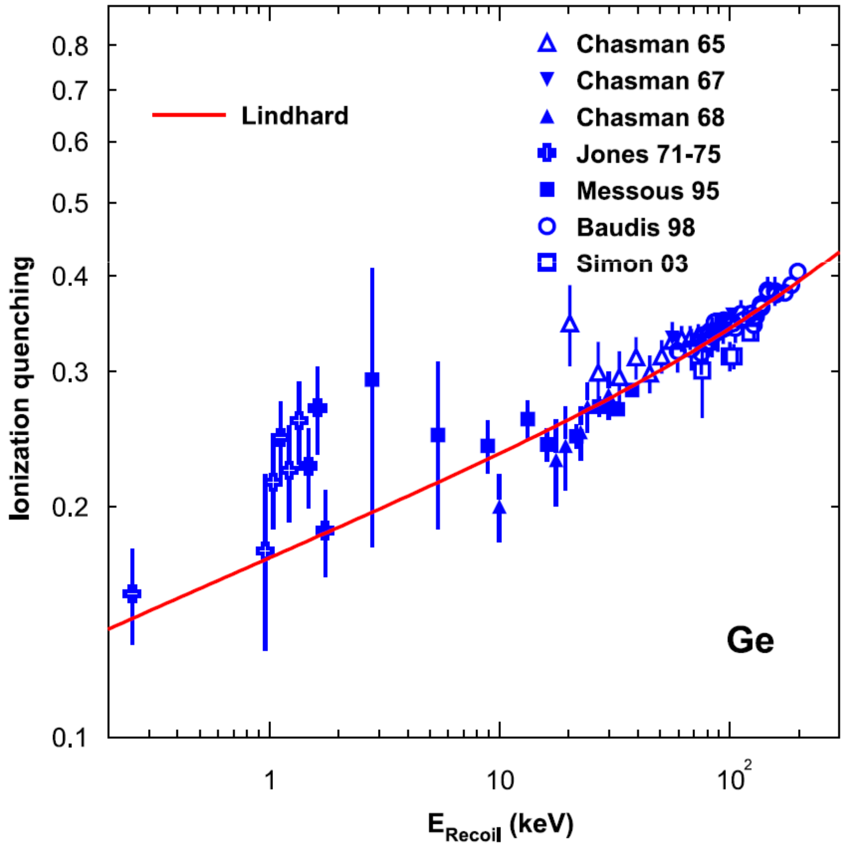


Fig. 2. Experimental results of the direct measurement of the ionization quenching for germanium recoils in germanium, from Refs. [5,7–11]. The line represents Eq. (1), with parameter values as of Eqs. (2)–(4).

3. α particles in Ge (at T=77 K):
Ph. Hubert et al., NIMA 252 (1986) 87
also Ph. Hubert, private comm. (2007)
also G. Heusser, private comm. (2007)
also our Ge measurements in LNGS
Q≅1 at E_α=5.33 MeV

Puzzle ? (different electric fields ?)

Some data on quenching of ionization signal in Si detectors

Si ions in Si:

1. G. Gerbier et al., PRD 42 (1990) 3211

2. A.R. Sattler, Phys. Rev. A 138 (1965) 1815

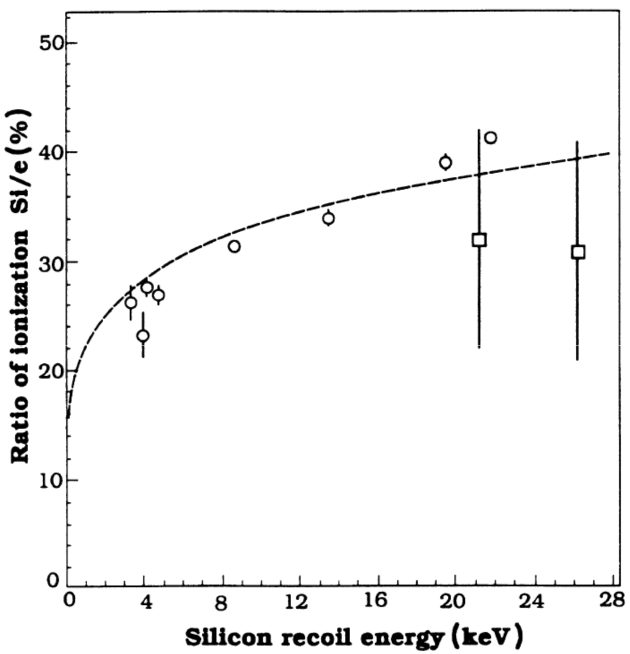


FIG. 4. Ratio between the observed energy [equivalent electron energy (EEE)] and the calculated recoil energy as a function of the silicon recoil energy. Circles are data points from the present experiment, squares are data points from Sattler's experiment (Ref. 8). The curve represents the result of the calculation of Lindhard *et al.* (Ref. 6).

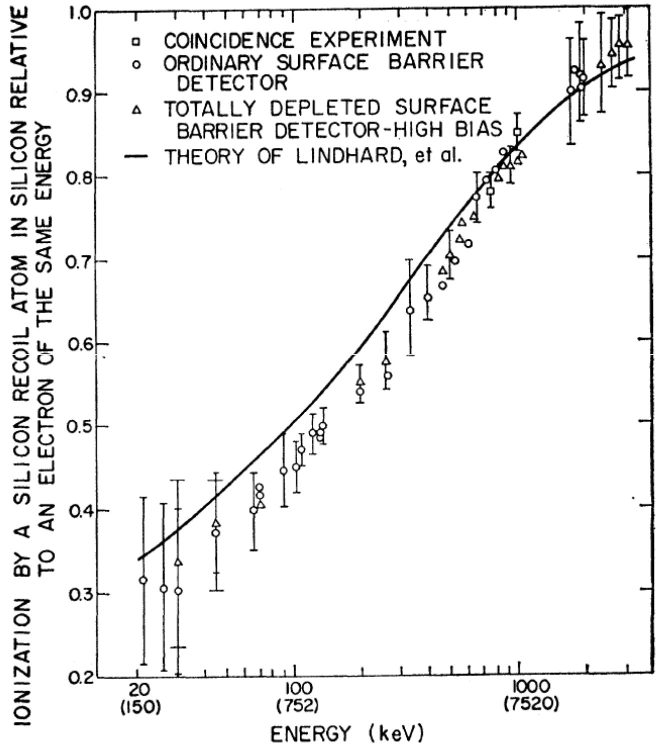


FIG. 5. Pulse height produced by a Si recoil atom relative to that of an electron of the same energy in Si as a function of Si recoil energy. The incident monoenergetic neutron energy necessary to produce the denoted recoil energy in a backscattering event is shown in parenthesis. Solid line denotes predictions of Lindhard *et al.*, in variables $\bar{\eta}/E$.

Theoretical attempts:

1. **J.B. Birks, Proc. Phys. Soc. A 64 (1951) 874;**
   ***The Theory and Practice of Scintillation Counting*, Pergamon Press, Oxford, 1964.**
2. **R.B. Murray, A. Meyer, Phys. Rev. 122 (1961) 815.**
3. **J. Lindhard et al., Mat. Fys. Medd. K. Dan. Vidensk. Selsk. 33 (1963) 1 (practical receipt is reproduced in: D.-M. Mei et al., Astropart. Phys. 30 (2008) 12).**
4. **A. Hitachi, Astropart. Phys. 24 (2005) 247;**
   J. Phys.: Conf. Ser. 65 (2007) 012013.