

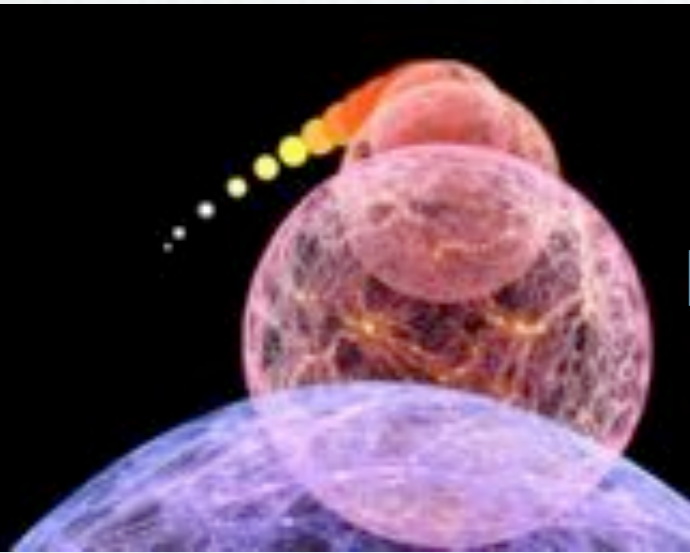
Understanding basic physics from the Large Scale Structure of the Universe

Massimo Pietroni - INFN Padova

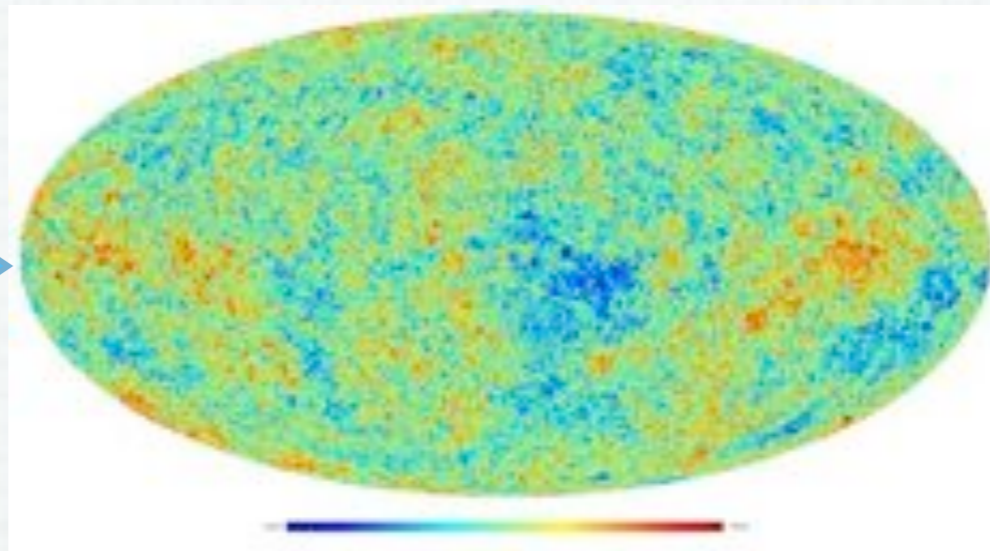
LNGS, dec. 11, 2012

Understanding the LSS of the Universe

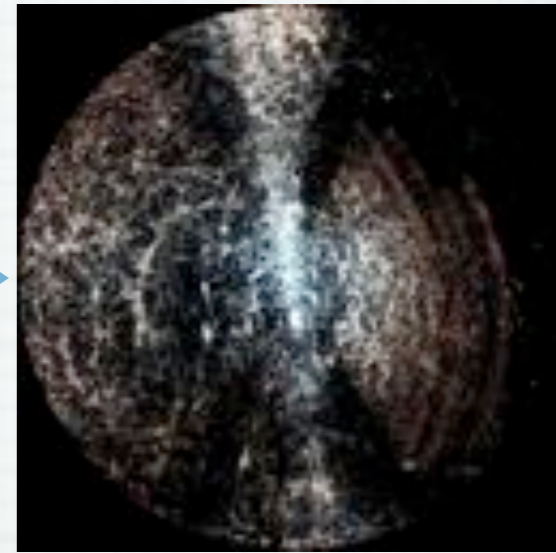
Inflation



Decoupling



Today



Linear, Gaussian

$$\left(\frac{\delta\rho}{\rho} \simeq 10^{-5}\right)$$

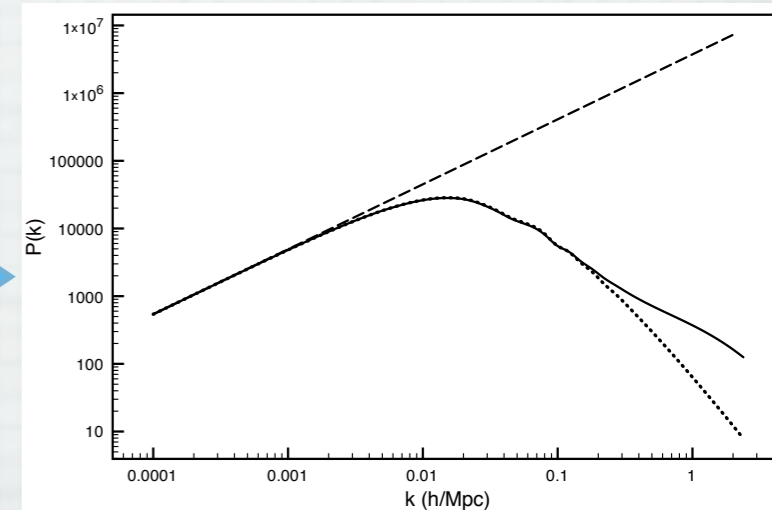
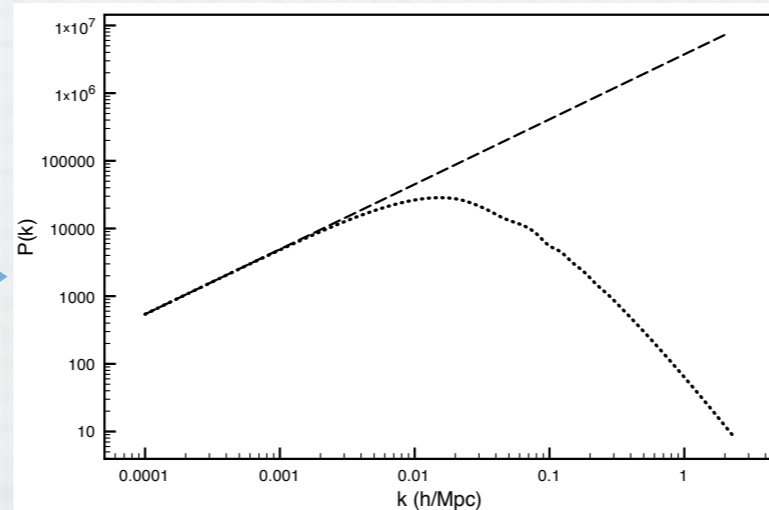
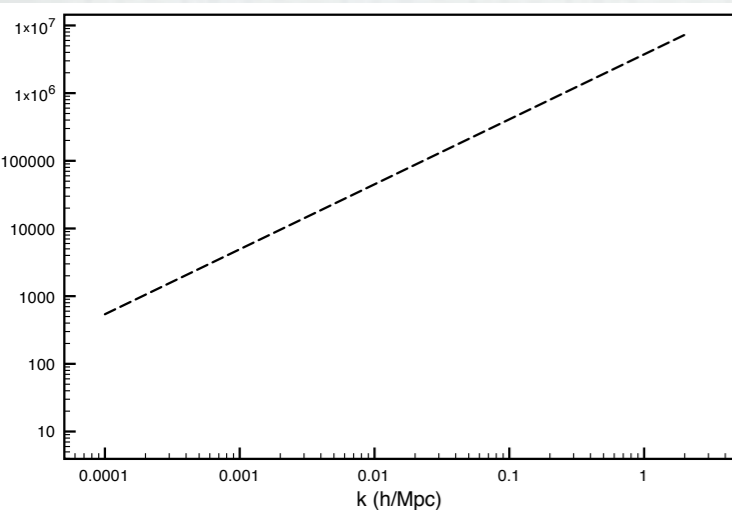
Linear, Gaussian

non-Linear,
non-Gaussian

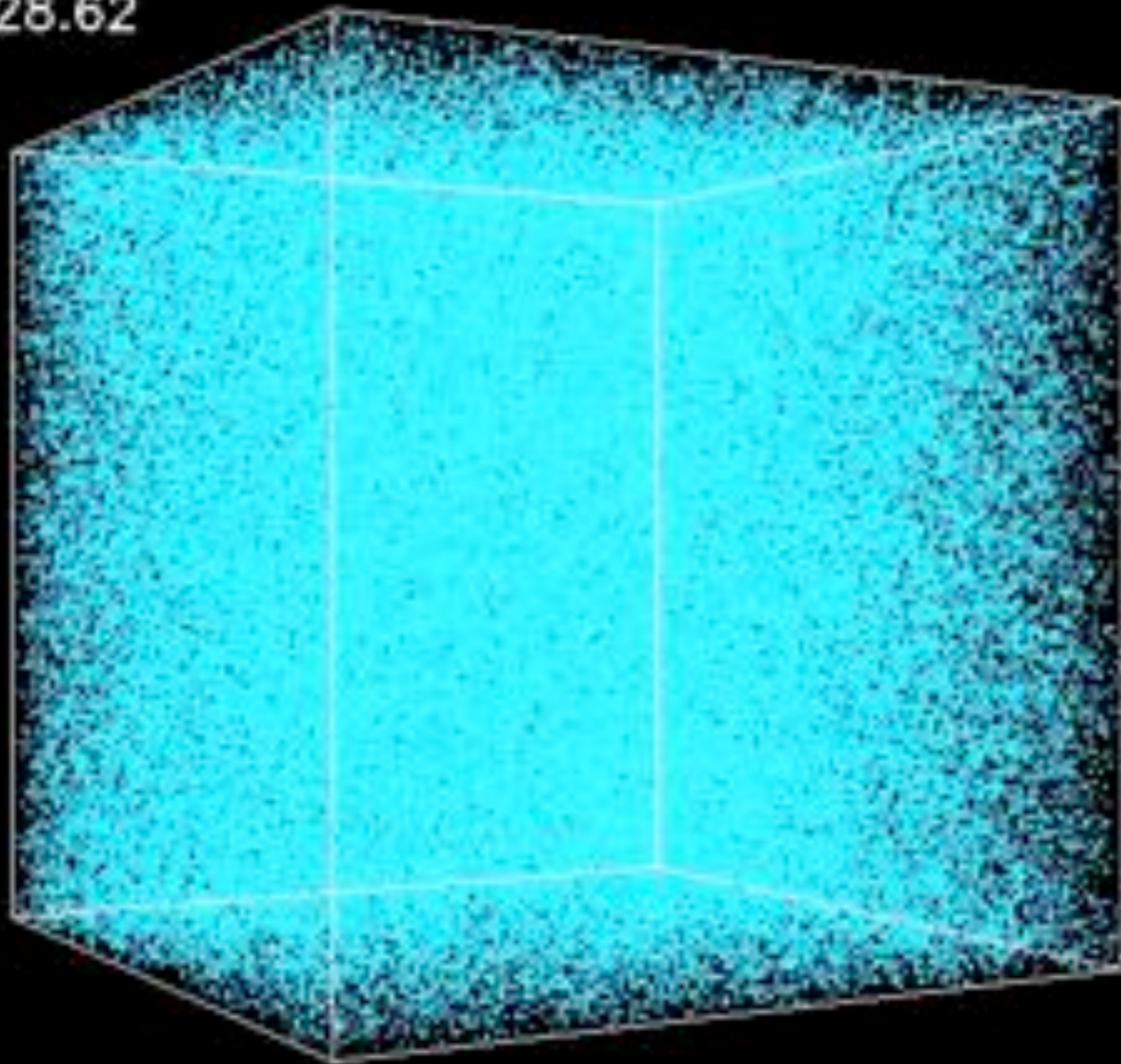
primordial density perturbations

photon-baryon-DM-neutrino...fluid

non-rel. matter



$Z=28.62$



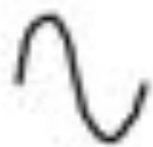
Movie by Kravtsov, Klypin
(National Center for Supercomputer applications)

Nonlinear Evolution (Qualitative)

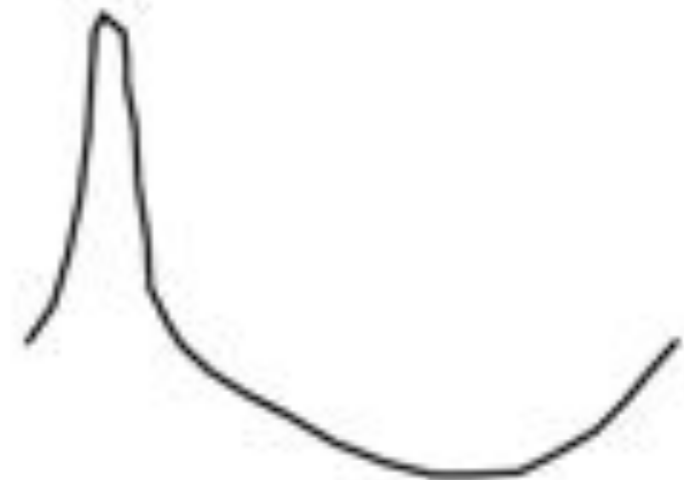
- Nonlinearity modifies the power spectrum
- Creates Non-Gaussianity

Growth of perturbations: gravity vs. the expansion of the universe

- underdense regions: expansion wins
- overdense regions: gravity wins



Small Gaussian Fluctuations



Non-Gaussian Fluctuations

Movie by R. Scoccimarro

Why do we need to study the late (and non-linear) evolution?

- * Dark Energy (Baryonic Acoustic Oscillations)
- * Neutrino masses
- * Primordial non-Gaussianity
- * Weak gravitational lensing
- * ...

The future of precision cosmology: non-linear scales

matter density

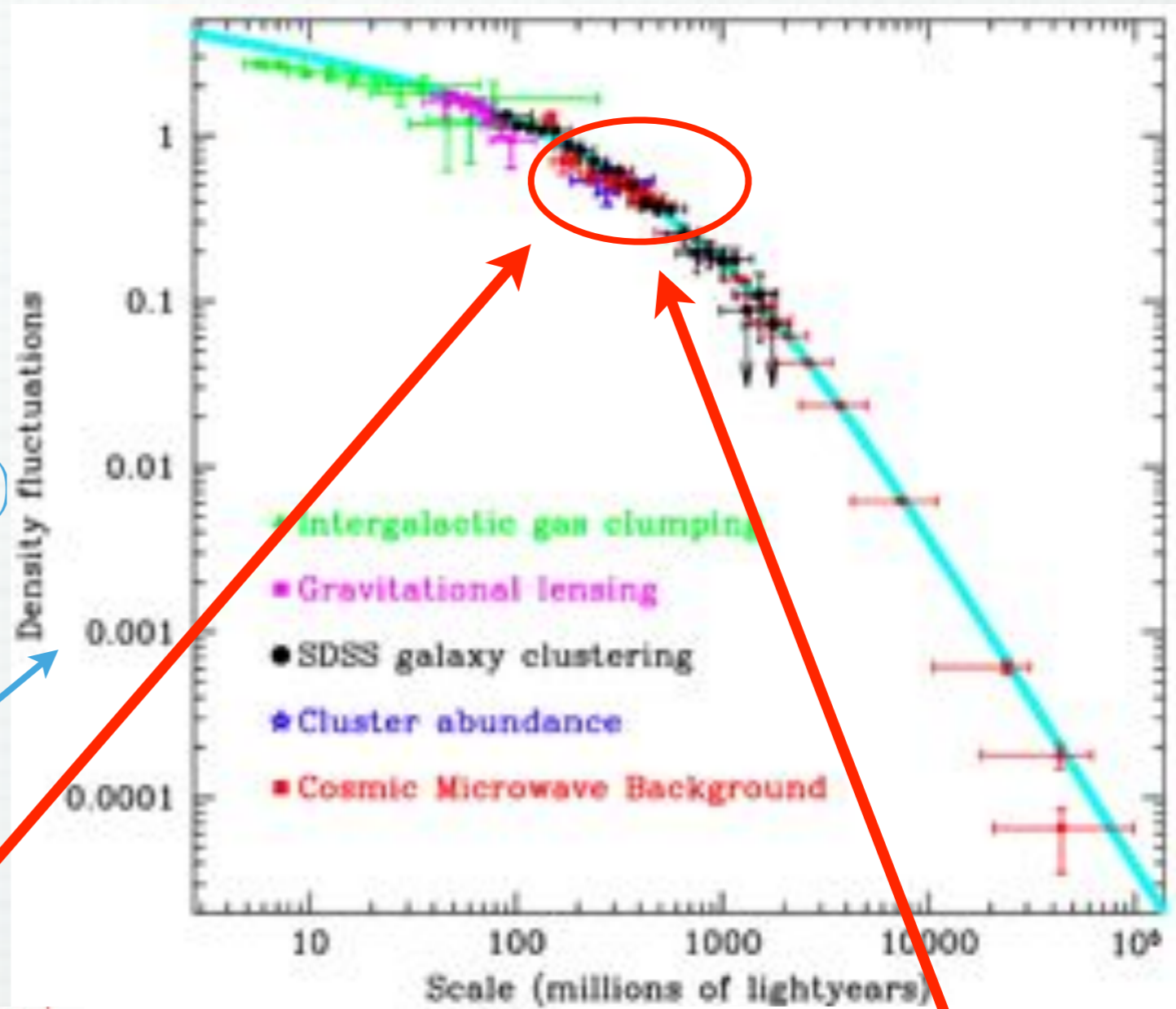
$$\rho(\mathbf{x}, \tau) \equiv \bar{\rho}(\tau)[1 + \delta(\mathbf{x}, \tau)]$$

power spectrum

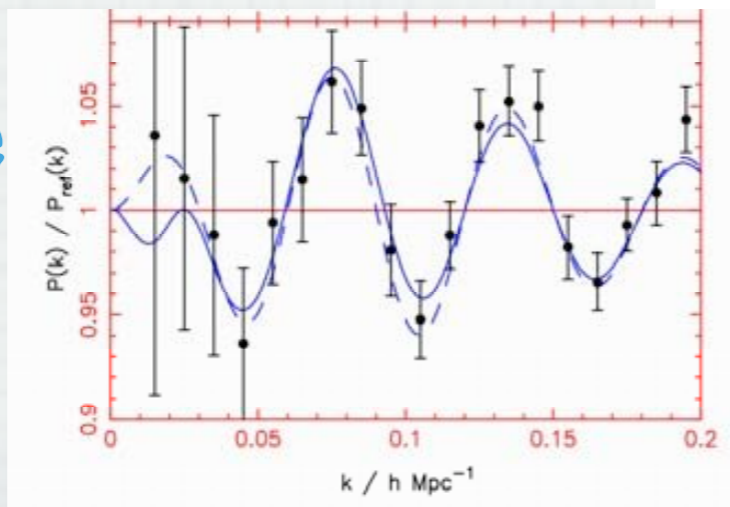
$$\langle \delta(\mathbf{k}, \tau) \delta(\mathbf{k}', \tau) \rangle = P(k, \tau) \delta^{(3)}(\mathbf{k} + \mathbf{k}')$$

'size' of the fluctuations at different scales/epochs:

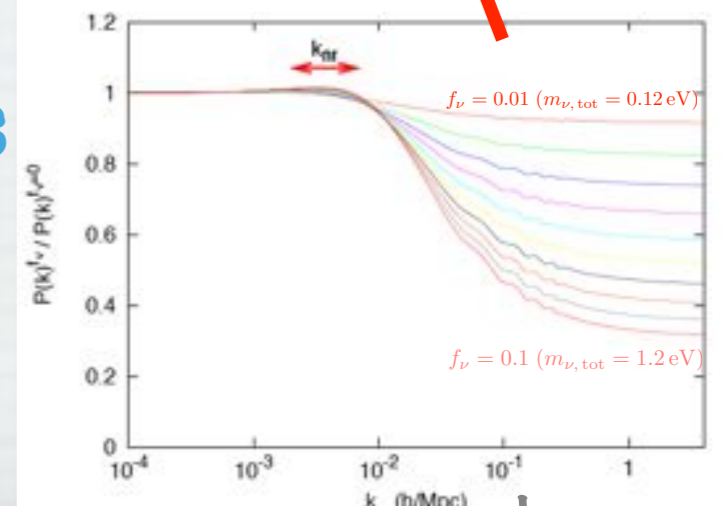
$$\Delta^2(k, \tau) = 4\pi k^3 P(k, \tau)$$

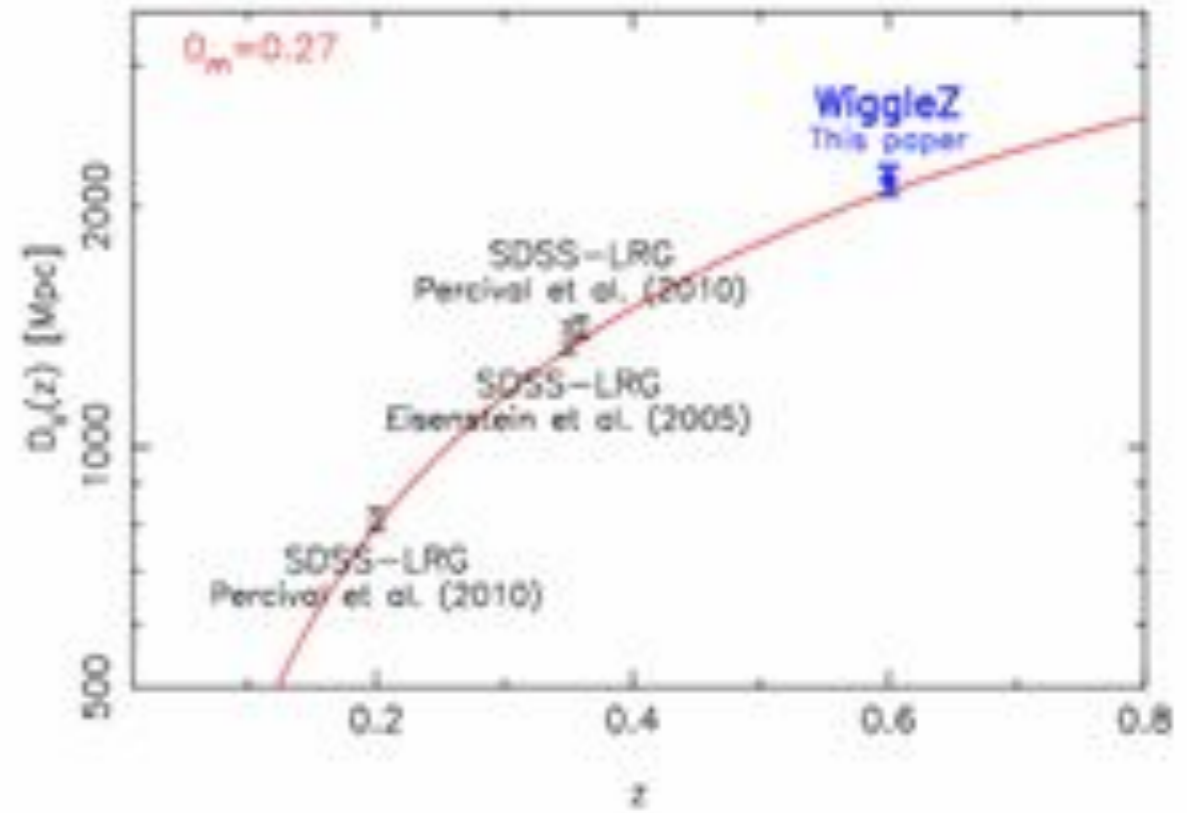
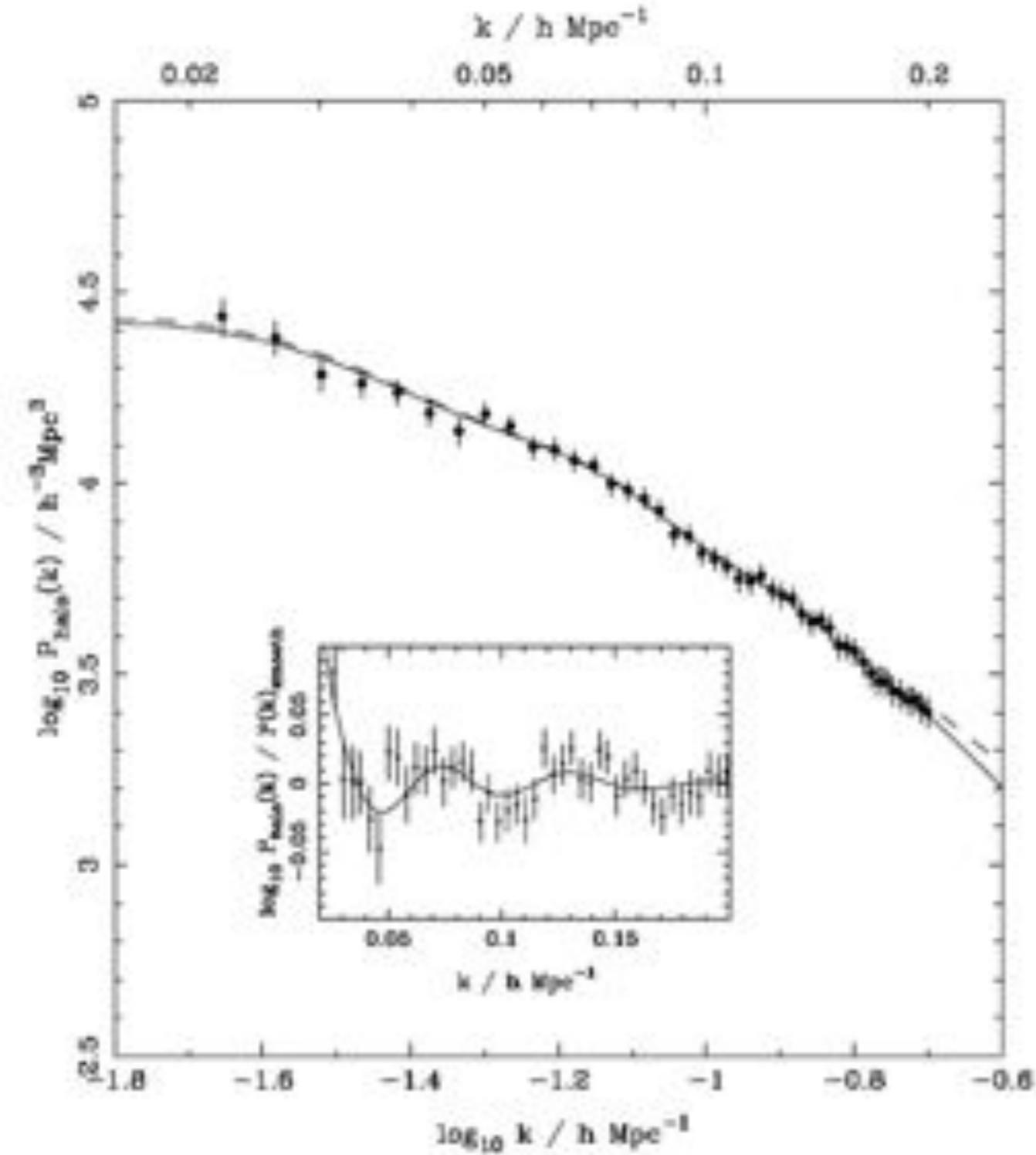


Baryonic Acoustic Oscillations (BAO)



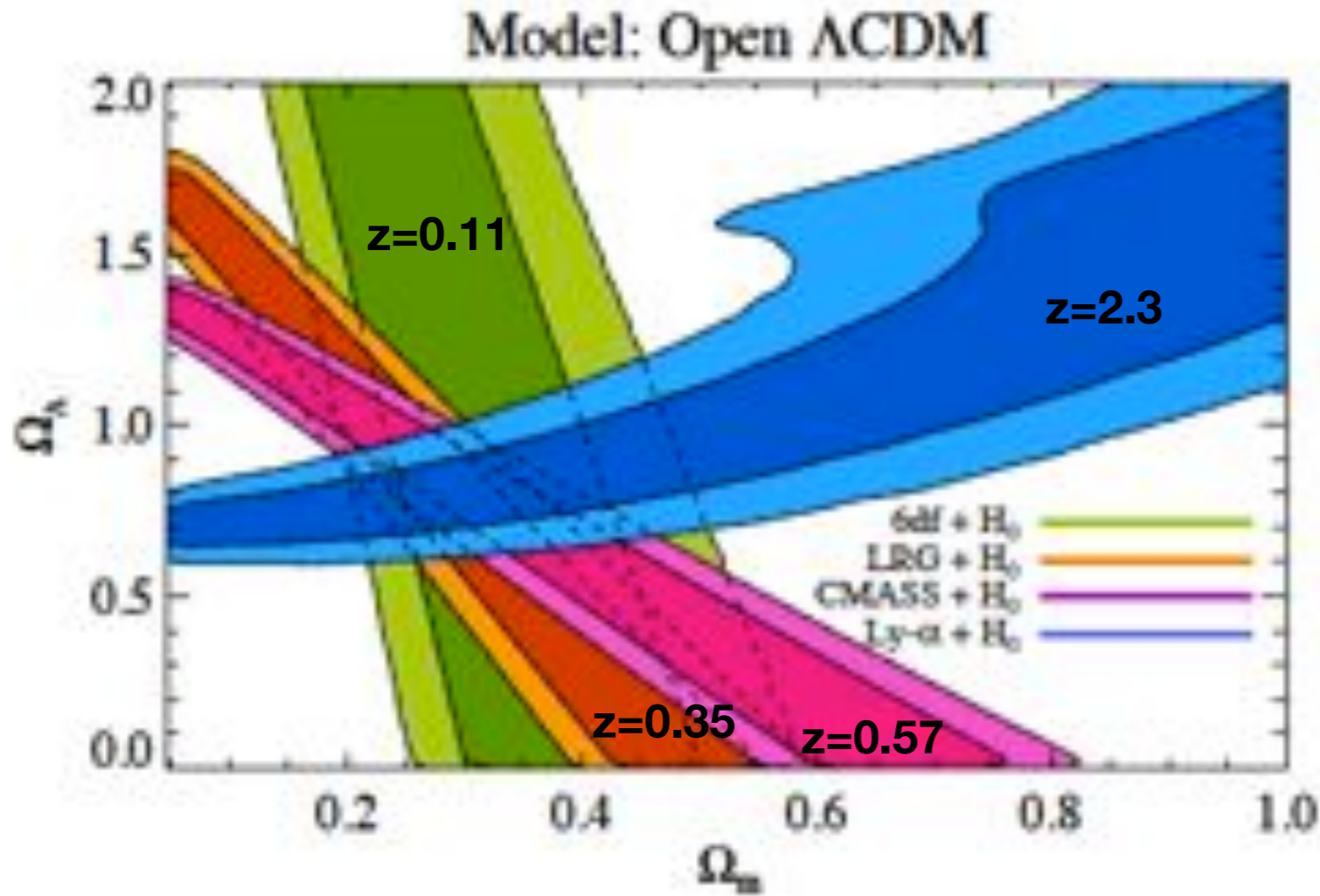
Neutrino mass bounds



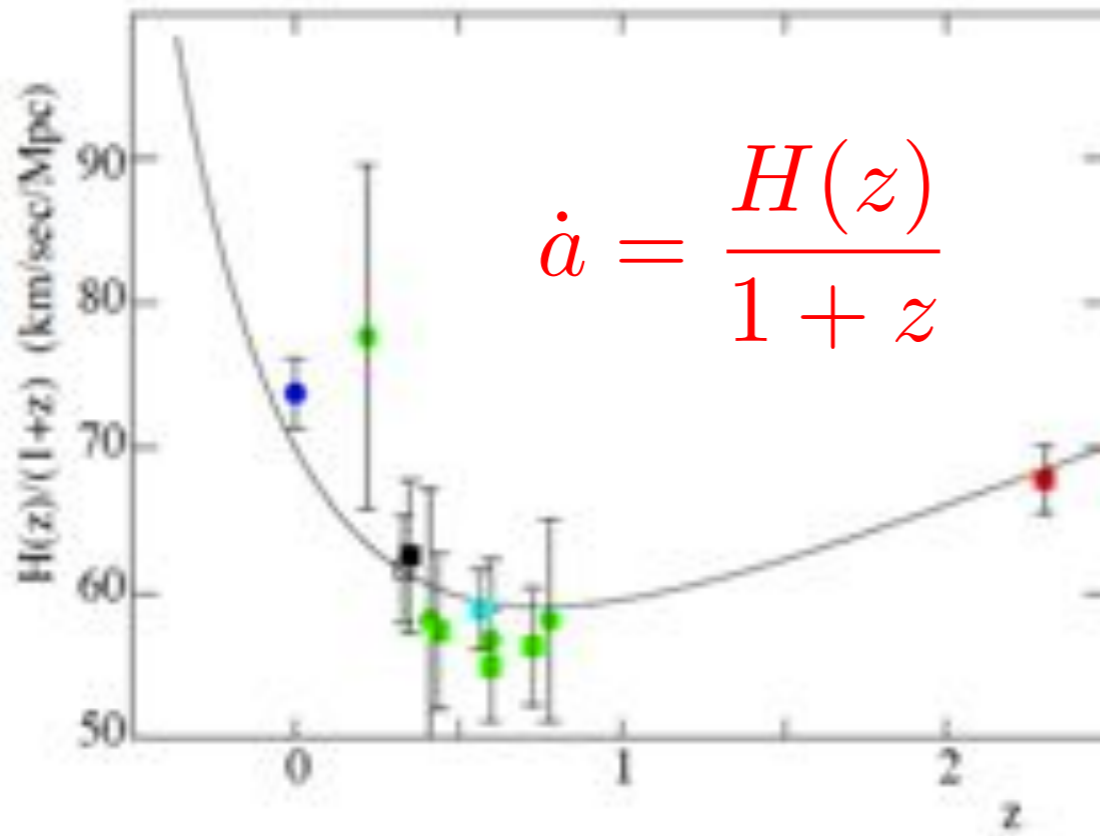


To be dramatically
 improved in the future:
BOSS, EUCLID, ...

WiggleZ 1105.2862



Measurement of Dark Energy from LSS "alone" (+ H_0)!

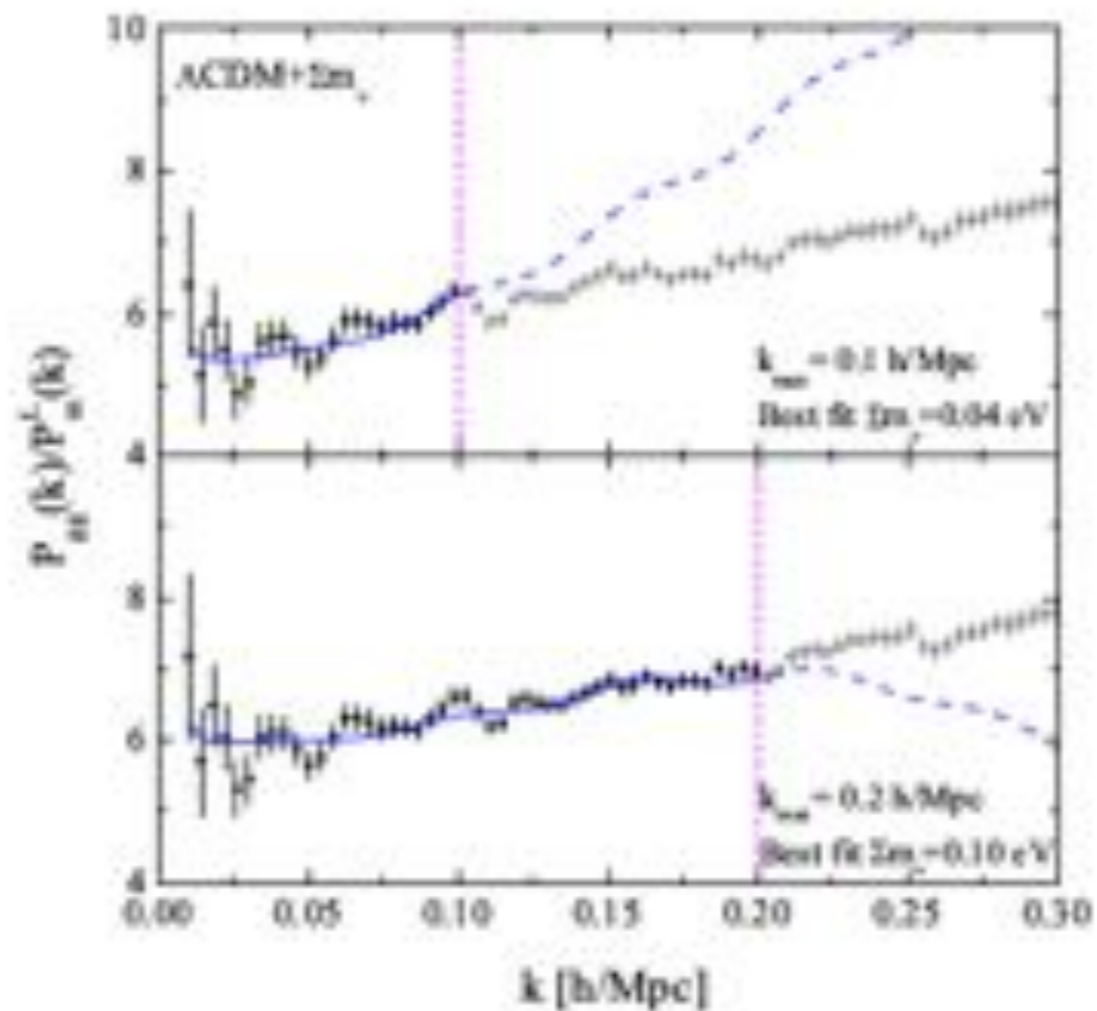
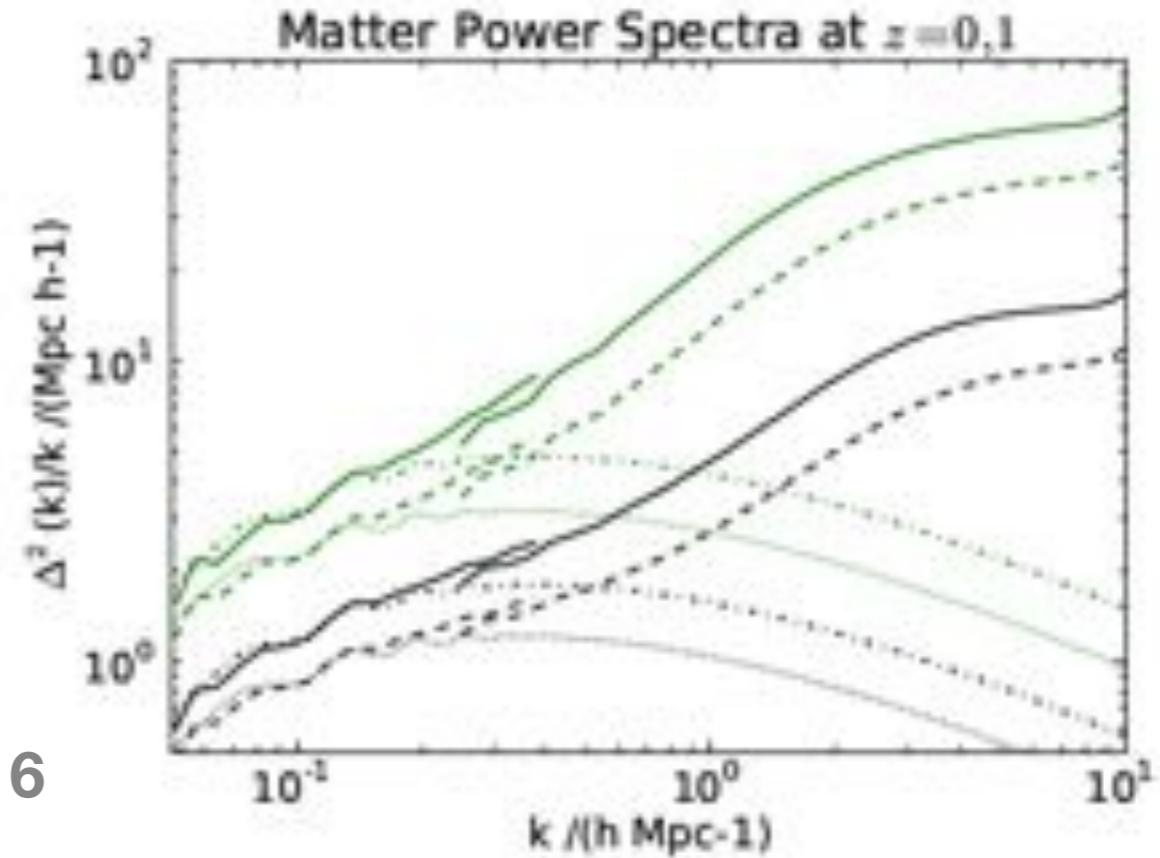


Deceleration to acceleration transition seen!!

Neutrino masses

nonlinearities crucial to
increase the sensitivity!!

Bird et al 1109.4416



Zhao et al
1211.3741

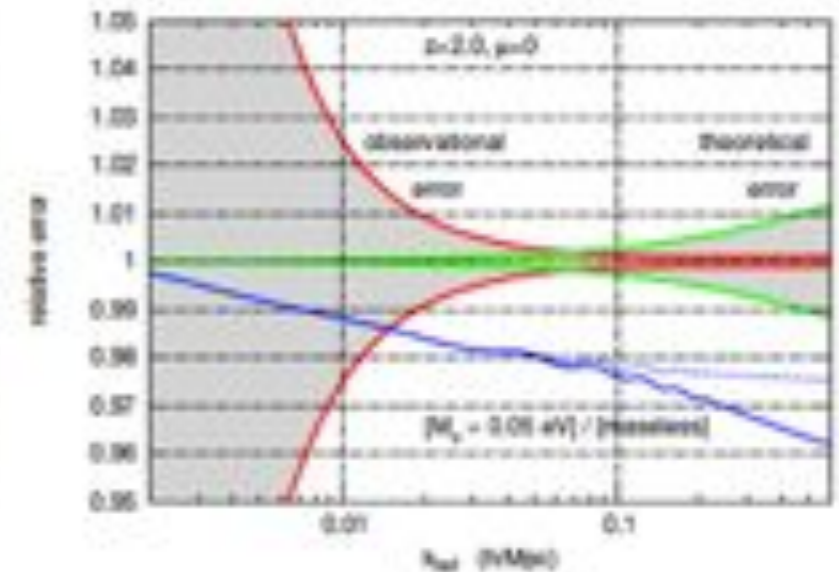
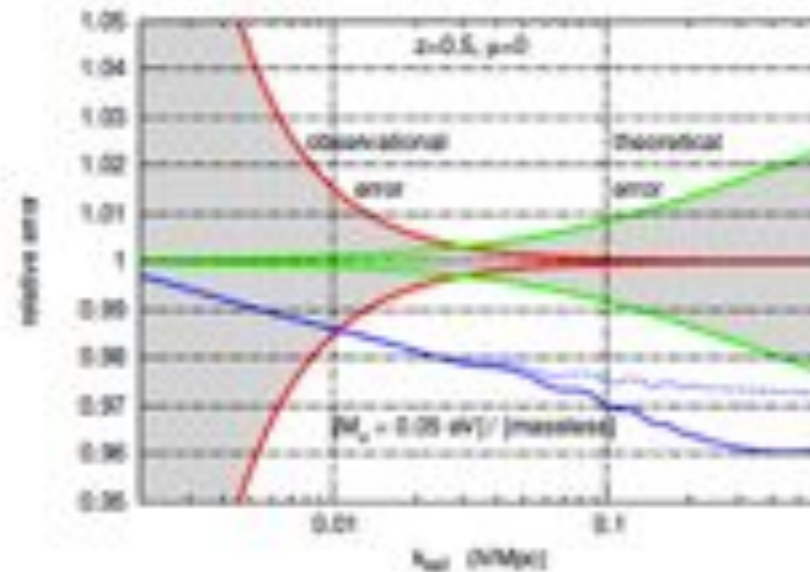
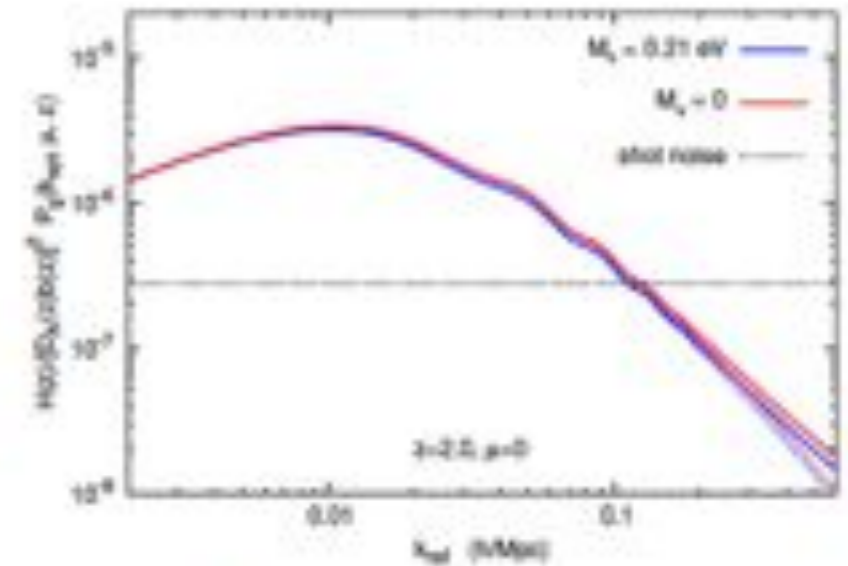
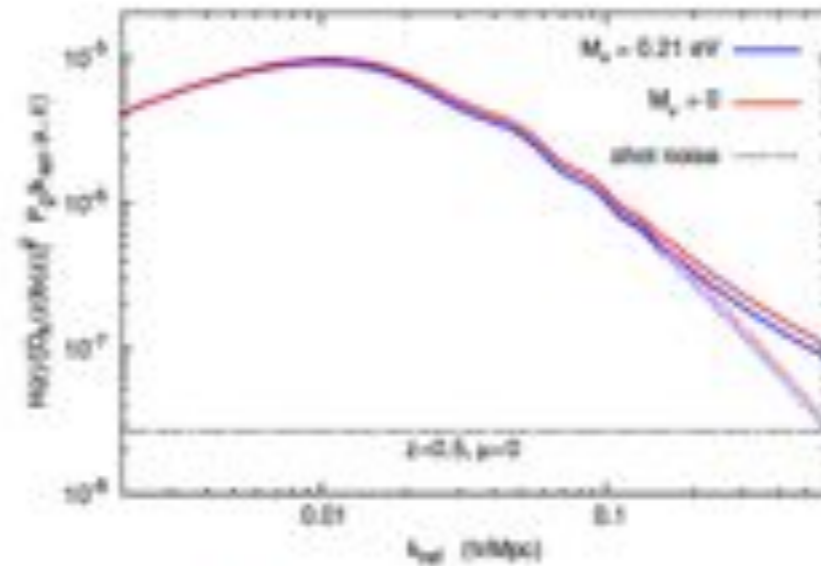
Now: LSS+CMB

$$M_\nu = \sum m_\nu < 0.3 \text{ eV}$$

Planck+Euclid

$$\Delta M_\nu < 0.032 \text{ eV}$$

Audren et al. 1210.2194



The LSS mantra



... and fast

→ scan over different cosmologies

not trivial even for Nbody

- * Initial conditions, large volumes, mass resolution, time-stepping (Heitmann et al 2010)
- * non- Λ CDM models: (massive neutrinos, coupled quintessence, $f(R)$, primordial NG, clustering DE,...)
- * not fast!

The Eulerian way

$$\frac{\partial \delta}{\partial \tau} + \nabla \cdot [(1 + \delta)\mathbf{v}] = 0,$$

$$\frac{\partial \mathbf{v}}{\partial \tau} + \mathcal{H}\mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla\phi$$

$$\nabla^2\phi = \frac{3}{2}\Omega_M\mathcal{H}^2\delta$$

subhorizon scales, newtonian gravity

Compact Perturbation Theory

Crocce, Scoccimarro '05

$$\frac{\partial \delta}{\partial \tau} + \nabla \cdot [(1 + \delta)\mathbf{v}] = 0,$$

$$\frac{\partial \mathbf{v}}{\partial \tau} + \mathcal{H}\mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla\phi,$$

define $\begin{pmatrix} \varphi_1(\eta, \mathbf{k}) \\ \varphi_2(\eta, \mathbf{k}) \end{pmatrix} \equiv e^{-\eta} \begin{pmatrix} \delta(\eta, \mathbf{k}) \\ -\theta(\eta, \mathbf{k})/\mathcal{H} \end{pmatrix}$ with $\eta = \log \frac{D^+(\tau)}{D^+(\tau_i)}$

$$\Omega = \begin{pmatrix} 1 & -1 \\ -3/2 & 3/2 \end{pmatrix}$$

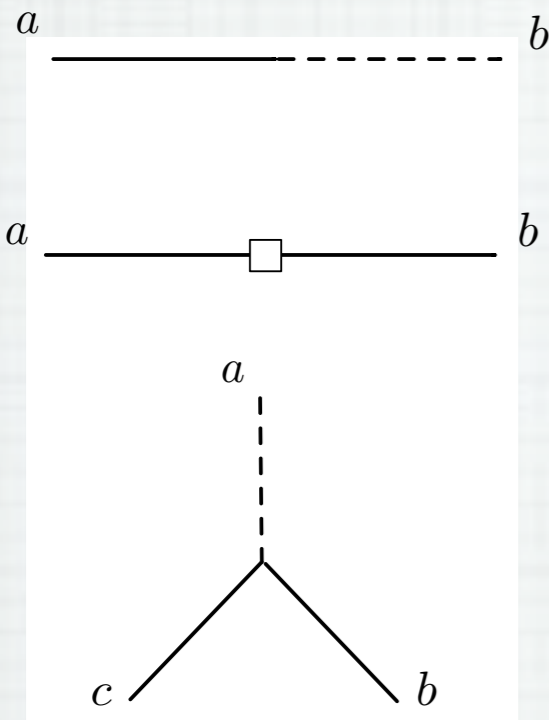
then we can write:

$$(\delta_{ab}\partial_\eta + \Omega_{ab})\varphi_b(\eta, \mathbf{k}) = e^\eta \gamma_{abc}(\mathbf{k}, -\mathbf{k}_1, -\mathbf{k}_2)\varphi_b(\eta, \mathbf{k}_1)\varphi_c(\eta, \mathbf{k}_2)$$

linear

nonlinear

Perturbation Theory: Feynman Rules

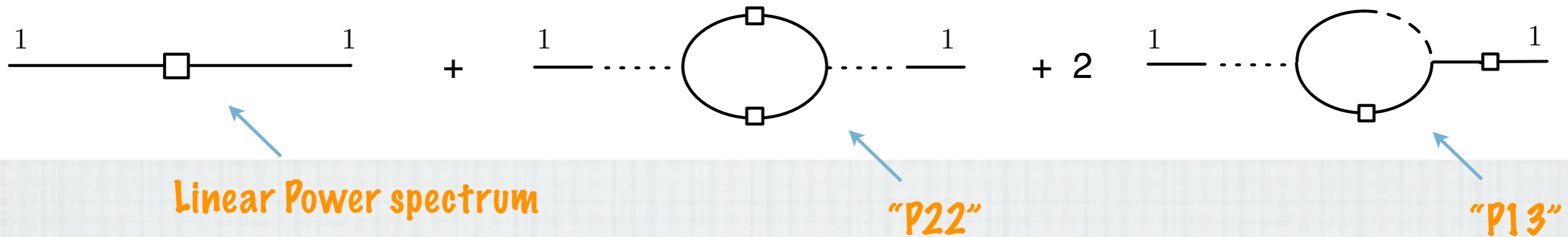


propagator (linear growth factor): $-i g_{ab}(\eta_a, \eta_b)$

power spectrum: $P_{ab}^L(\eta_a, \eta_b; \mathbf{k})$

interaction vertex: $-i e^\eta \gamma_{abc}(\mathbf{k}_a, \mathbf{k}_b, \mathbf{k}_c)$

Example: 1-loop correction to the density power spectrum:



All known results in cosmological perturbation theory are expressible in terms of diagrams in which only a trilinear fundamental interaction appears

PT in the BAO range

1-loop propagator
@ large k:



$$G_{ab}(k; \eta_a, \eta_b) = g_{ab}(\eta_a, \eta_b) \left[1 - k^2 \sigma^2 \frac{(e^{\eta_a} - e^{\eta_b})^2}{2} \right] + O(k^4 \sigma^4)$$

$$\left(\sigma^2 \equiv \frac{1}{3} \int d^3 q \frac{P^0(q)}{q^2} \right) (\sigma e^{\eta_a})^{-1} \simeq 0.15 \text{ h Mpc}^{-1}$$

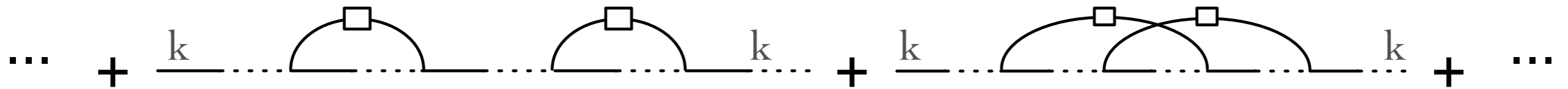
in the BAO range!

2-loop

the PT series blows up in the BAO range

But it can be resummed!!

(Crocco-Scoccimarro '06)



$$G(k; \eta, \eta_{in}) = \frac{\langle \delta(k, \eta) \delta(k, \eta_{in}) \rangle}{\langle \delta(k, \eta_{in}) \delta(k, \eta_{in}) \rangle} \sim e^{-\frac{k^2 \sigma^2}{2}} e^{2\eta}$$

physically, it represents the effect of multiple interactions of the k-mode with the surrounding modes: **memory loss**

'coherence momentum' $k_{ch} = (\sigma e^\eta)^{-1} \simeq 0.15 \text{ h Mpc}^{-1}$
↑ damping in the BAO range!

RPT: use G , and not g , as the linear propagator
(beware of Galileian invariance!! Peloso, MP, in progress)

Partial (!) list of contributors to the field

- * “traditional” P.T.: see Bernardeau et al, Phys. Rep. 367, 1, (2002), and refs. therein; Jeong-Komatsu; Saito et al; Sefusatti;...
- * resummation methods: Valageas; Crocce-Scoccimarro; McDonald; Matarrese-M.P.; Matsubara; M.P.; Taruya-Hirataamatsu; Bernardeau-Valageas; Bernardeau-Crocce-Scoccimarro; Tassev-Zaldarriaga,...

More General Cosmologies

$$\frac{\partial \delta}{\partial \tau} + \nabla \cdot [(1 + \delta)\mathbf{v}] = 0,$$

deviation from geodesic
(e.g. DM-scalar field interaction)

$$\frac{\partial \mathbf{v}}{\partial \tau} + \mathcal{H}(1 + A(\vec{x}, \tau))\mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla\phi,$$

$$\nabla^2 \phi = 4\pi G (1 + B(\vec{x}, \tau)) \rho a^2 \delta$$

deviation from Poisson
(e.g. scale-dep. growth factor)

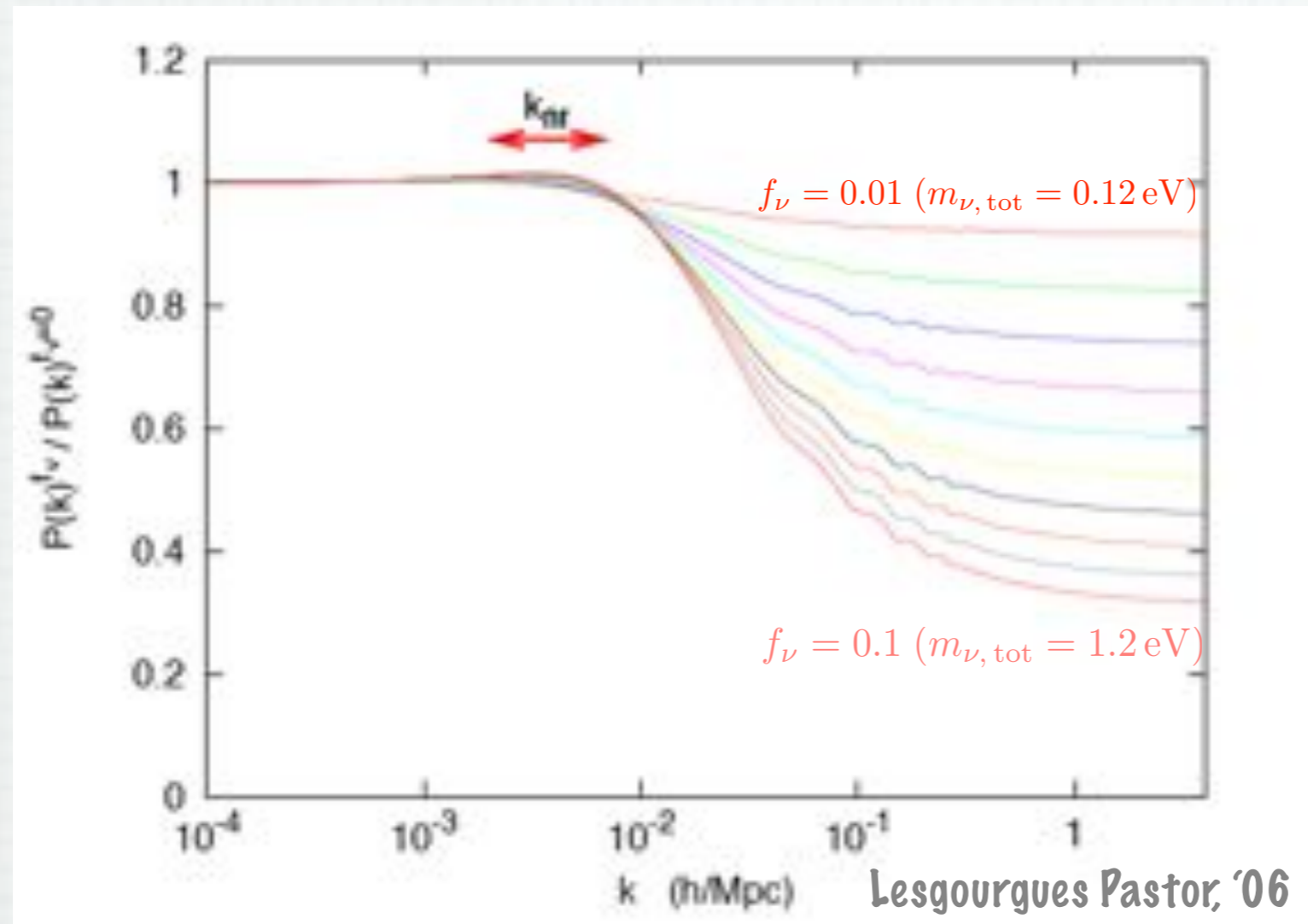


$$(\delta_{ab}\partial_\eta + \Omega_{ab}(\eta, \mathbf{k})) \varphi_b(\eta, \mathbf{k}) = e^\eta \gamma_{abc}(\mathbf{k}, -\mathbf{k}_1, -\mathbf{k}_2) \varphi_b(\eta, \mathbf{k}_1) \varphi_c(\eta, \mathbf{k}_2)$$

$$\Omega_{ab} = \begin{pmatrix} 1 & -1 \\ -\frac{3}{2}\Omega_M(1 + B(\eta, \mathbf{k})) & 2 + \frac{\mathcal{H}'}{\mathcal{H}} + A(\eta, \mathbf{k}) \end{pmatrix} \quad (\eta = \log a)$$

Ex: Scalar-Tensor: $A = \alpha d\varphi/d \log a$ $B = 2\alpha^2$ $\alpha^2 = 1/(2\omega + 3)$

Massive Neutrinos and the Power Spectrum



$$f_\nu = \frac{\Omega_\nu}{\Omega_m} = \frac{m_{\nu, \text{tot}}}{94.1 \Omega_m h^2 \text{ eV}}$$

$$k_{nr} \simeq 0.018 \Omega_m^{1/2} \left(\frac{m_\nu}{1 \text{ eV}} \right) h \text{ Mpc}^{-1}$$

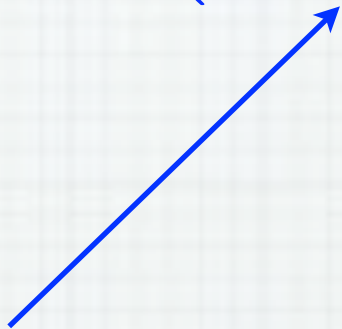
$$\frac{\Delta P}{P f_{\nu=0}} \sim -8 f_\nu$$

The linear growth factor

is scale dependent: $\delta_m(k \ll k_{nr}) \sim a$, $\delta_m(k \gg k_{nr}) \sim a^{1-3/5 f_\nu}$

Poisson equation for Cold DM + Neutrinos

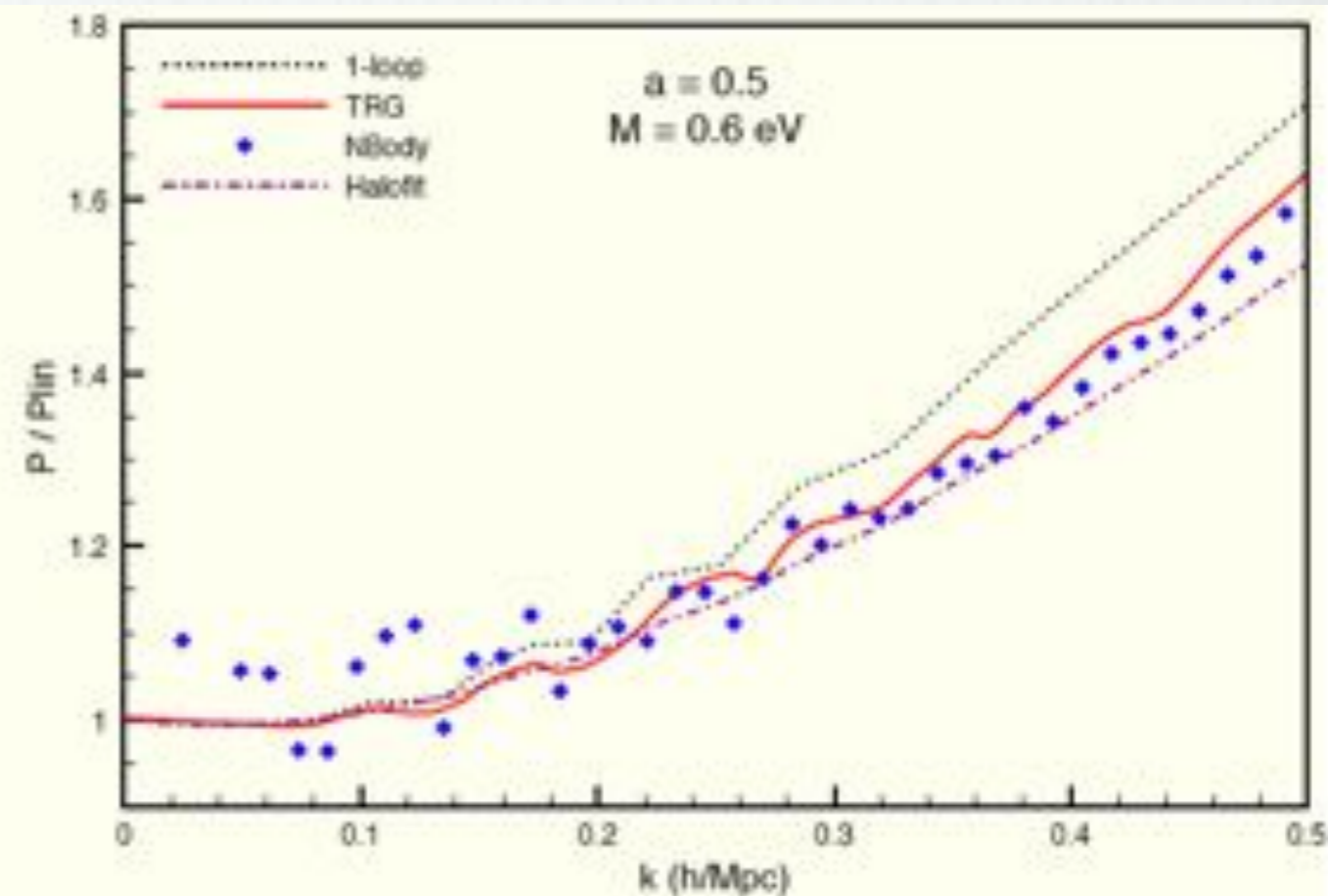
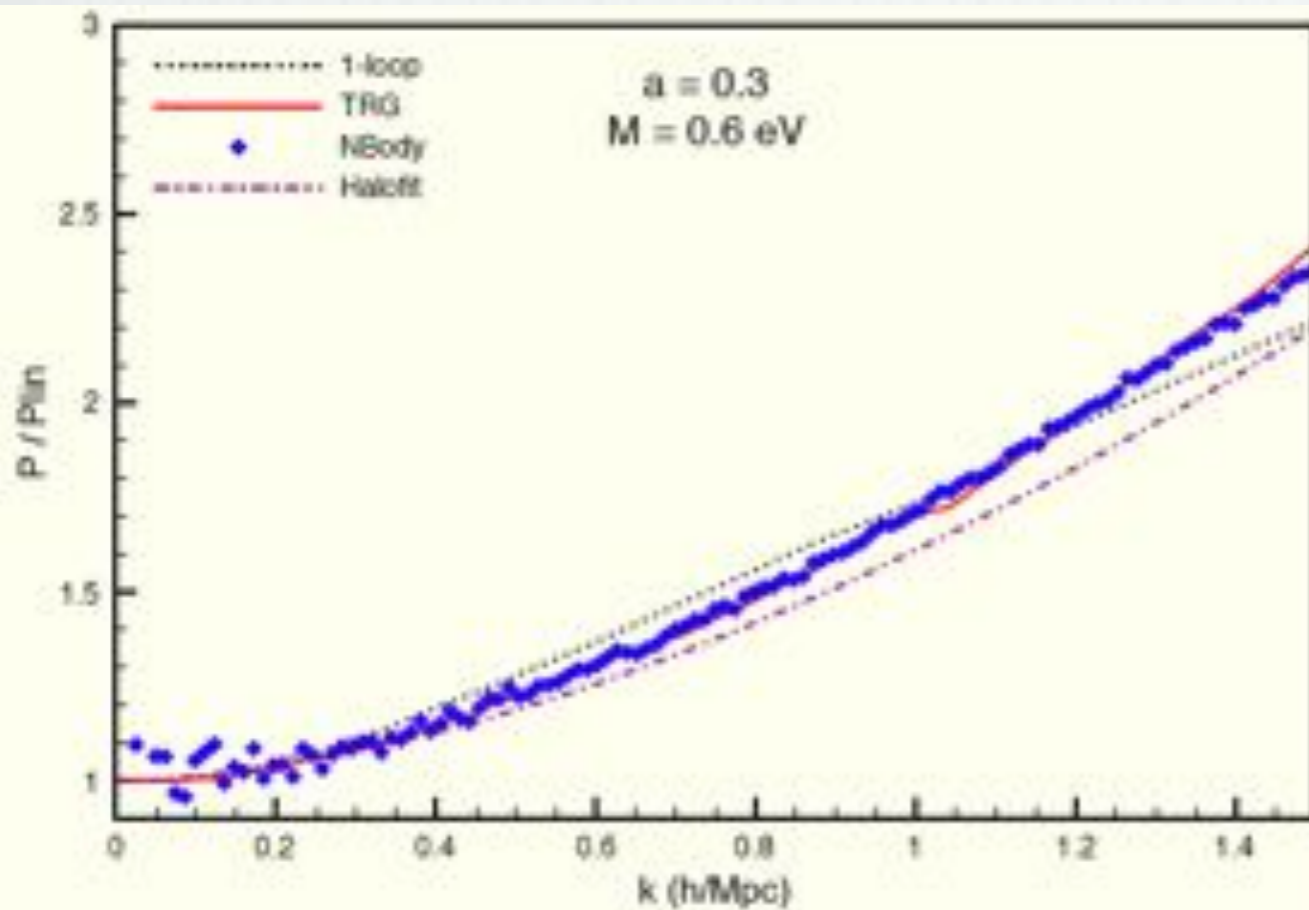
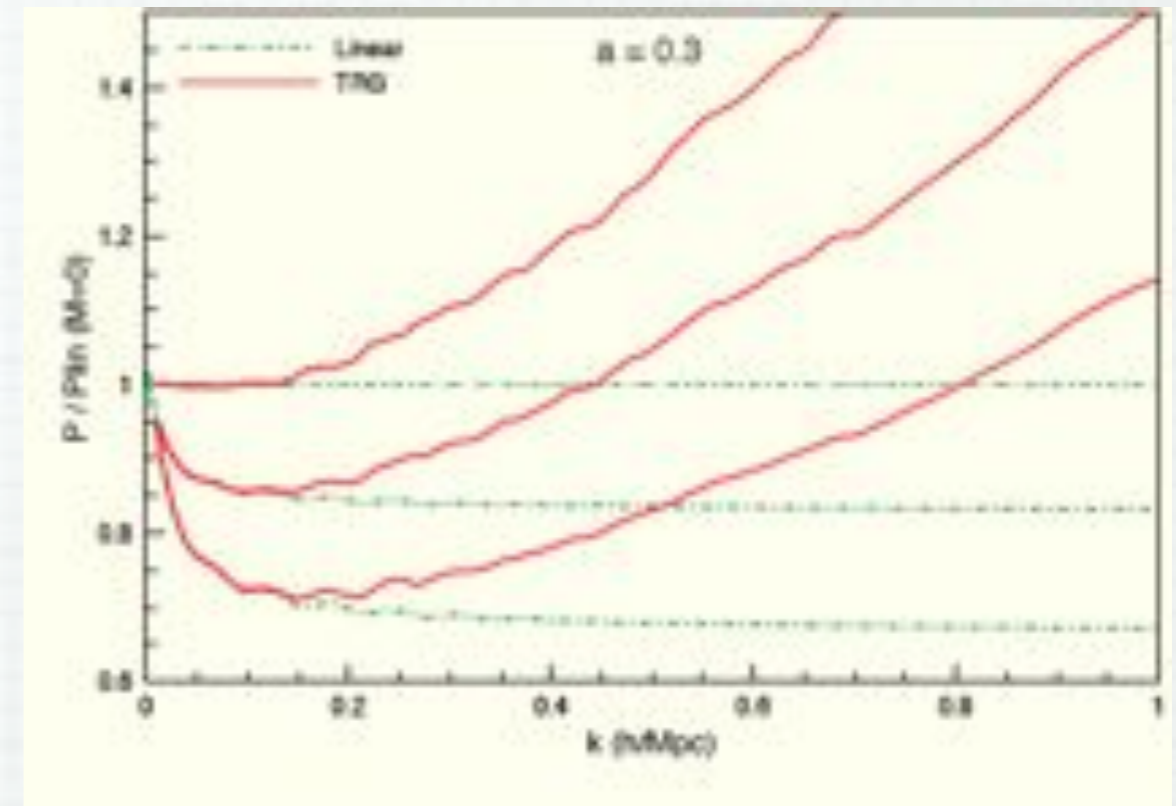
$$\nabla^2 \phi = 4\pi G a^2 (\rho_c \delta_c + \rho_\nu \delta_\nu) = 4\pi G a^2 \left(1 + \frac{\rho_\nu \delta_\nu}{\rho_c \delta_c} \right) \rho_c \delta_c$$

$$\mathcal{B}(k, \tau) \simeq \frac{\rho_\nu \delta_\nu}{\rho_c \delta_c} \Big|_{\text{linear th.}}$$


Non-Linear effects on v_{mass} bounds: RG vs. 1-loop approximation

Lesgourgues, Matarrese, M.P., Riotto, '09

Cosmological Neutrino mass bounds should take non-linear effects into account!



Improving resummed PT methods: 2012 achievements

- * PS established at % in the BAO range down to $z=0$
- * fast implementations: from O(hrs) to O(mins) a $P(k,z)$
- * scales smaller than BAO's

Exact time-evolution equations I: the propagator

Anselmi, Matarrese, MP
10114477

a) start from the exact expression

$$G_{ab}(k; \eta, \eta') = g_{ab}(\eta, \eta') + \int ds ds' g_{ac}(\eta, s) \Sigma_{cd}(k; s, s') G_{cb}(k; s', \eta')$$

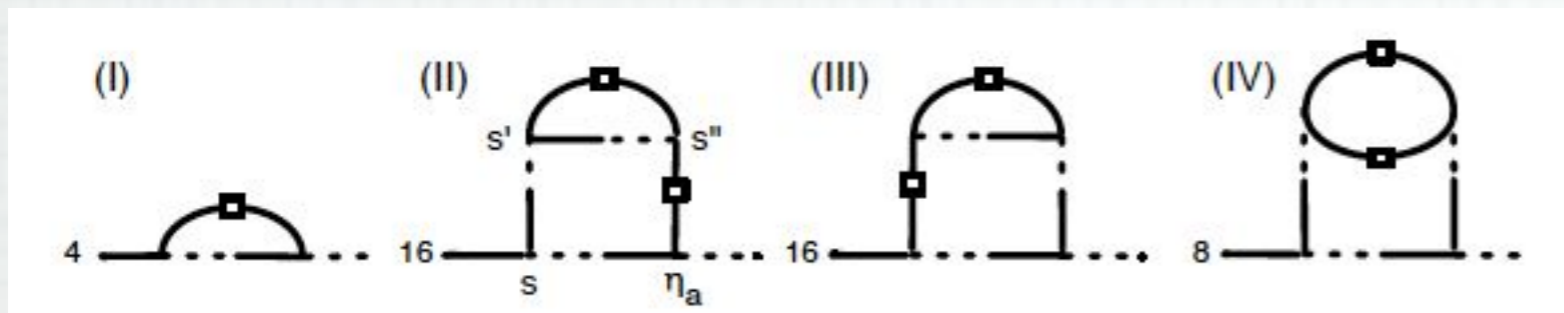
b) take the time derivative...

$$\partial_{\eta} G_{ab}(k; \eta, \eta') = \delta_{ab} \delta_D(\eta - \eta') - \Omega_{ac} G_{cb}(k; \eta, \eta') + \Delta G_{ab}(k; \eta, \eta')$$

$$\Delta G_{ab}(k; \eta, \eta') \equiv \int ds' \Sigma_{ad}(k; \eta, s') G_{db}(k; s', \eta')$$

c) compute Σ ...

$$\Sigma_{ab}(k; s, s')$$



+ higher loops...

small k :

$$\Sigma_{ab}(k; s, s') \simeq \Sigma_{ab}^{1-loop}(k; s, s') \longrightarrow G_{ab} \longrightarrow G_{ab}^{1-loop}$$

large k : $\int ds' \Sigma_{ad}(k; \eta, s') G_{db}(k; s', \eta') \simeq -k^2 \sigma^2 e^\eta (e^\eta - e^{\eta'}) G_{ab}(k; \eta, \eta')$



$$G_{ab} \longrightarrow \exp\left(\frac{k^2 \sigma^2 (e^\eta - e^{\eta'})^2}{2}\right) g_{ab}$$

1-loop evaluation of Σ gives the propagator at all loops!!

Exact time-evolution equations II: the power spectrum

Anselmi, MP,
1205.2235

a) exact expression

$$P_{ab}(k; \eta, \eta') = G_{ac}(k; \eta, \eta_{in}) G_{bd}(k; \eta', \eta_{in}) P_{cd}(k; \eta_{in}, \eta_{in}) \\ + \int ds ds' G_{ac}(k; \eta, s) G_{bd}(k; \eta', s') \Phi_{cd}(k; s, s')$$

b) time derivative

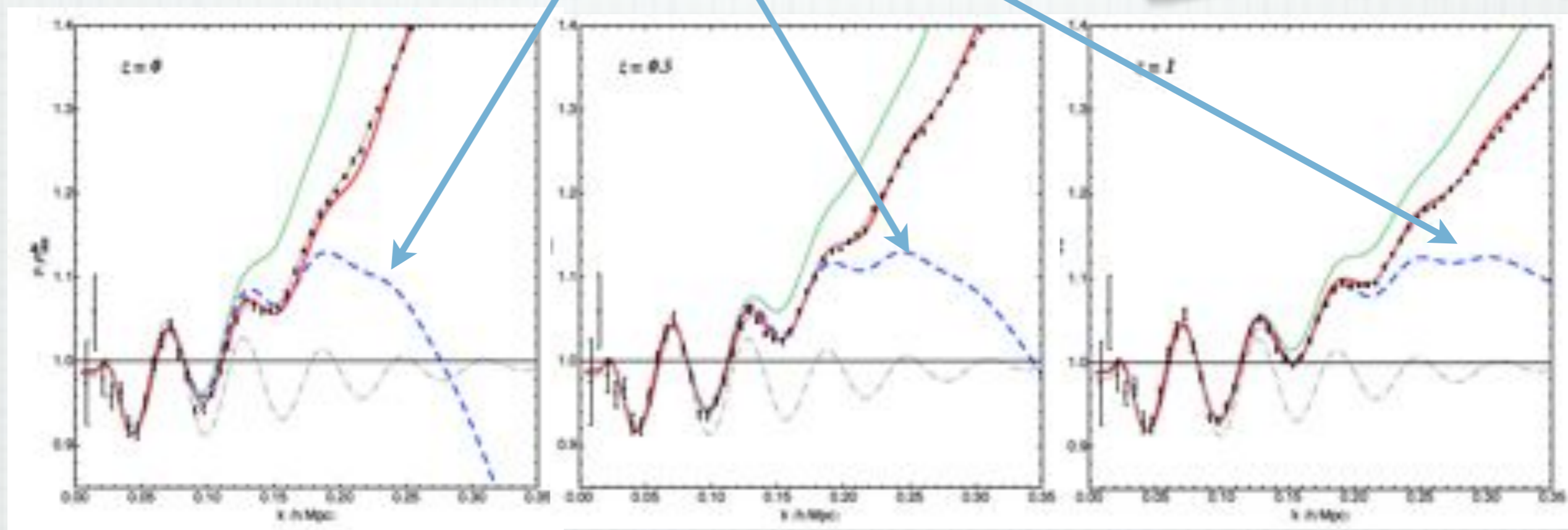
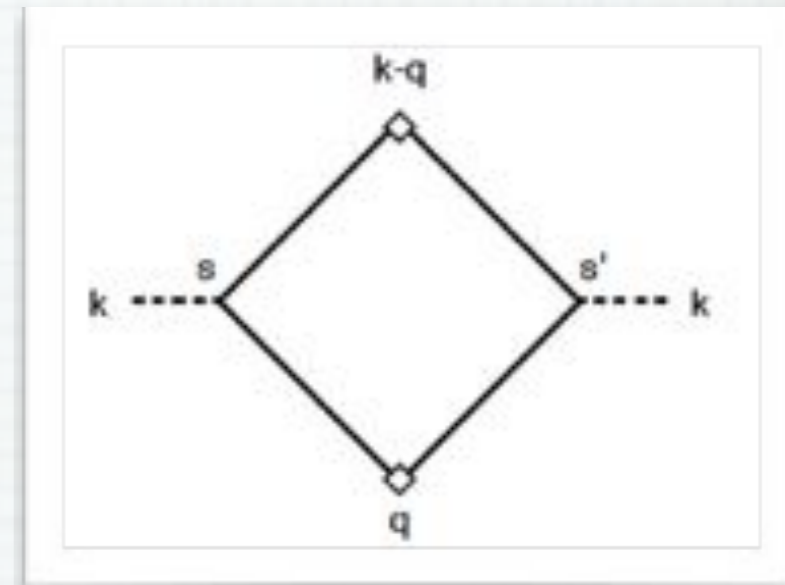
$$\partial_{\eta} P_{ab}(k; \eta) = -\Omega_{ac} P_{cb}(k; \eta) - \Omega_{bc} P_{ac}(k; \eta) \\ + H_a(k; \eta, \eta_{in}) P_{ab}(k; \eta) + H_b(k; \eta, \eta_{in}) P_{ab}(k; \eta) \\ + \int ds [\tilde{\Phi}_{ad}(k; \eta, s) \bar{G}_{bd}(k; \eta, s) + \bar{G}_{ad}(k; \eta, s) \tilde{\Phi}_{db}(k; s, \eta)]$$

$$H_a(k; \eta, s) \equiv \int_s^{\eta} ds'' \Sigma_{ae}^{(1)}(k; \eta, s'') u_e$$

Already computed for the propagator

$$\Phi_{ab}(k; s, s')$$

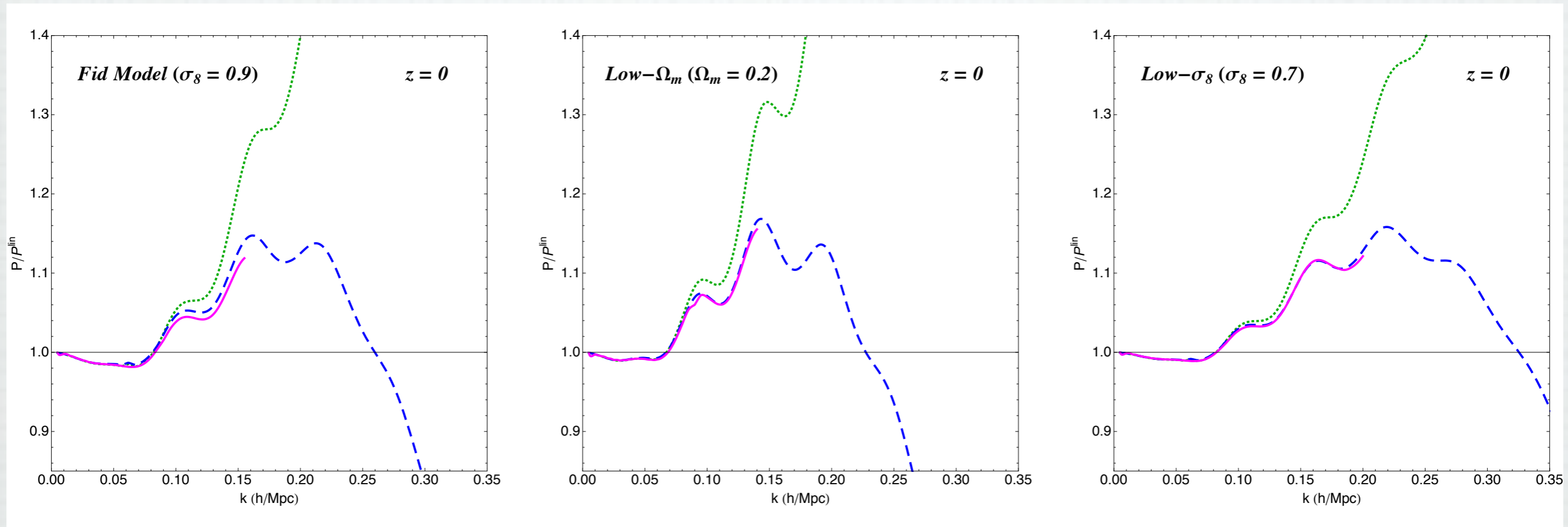
small k : $\Phi_{ab}(k; s, s') \simeq \Phi_{ab}^{1-loop}(k; s, s')$



1-loop evaluation of Σ and Φ reproduces the BAO's at the $\%$ level at all z

compare with PTbreeze

Crocce Scoccimarro Bernardeau 1207.1465



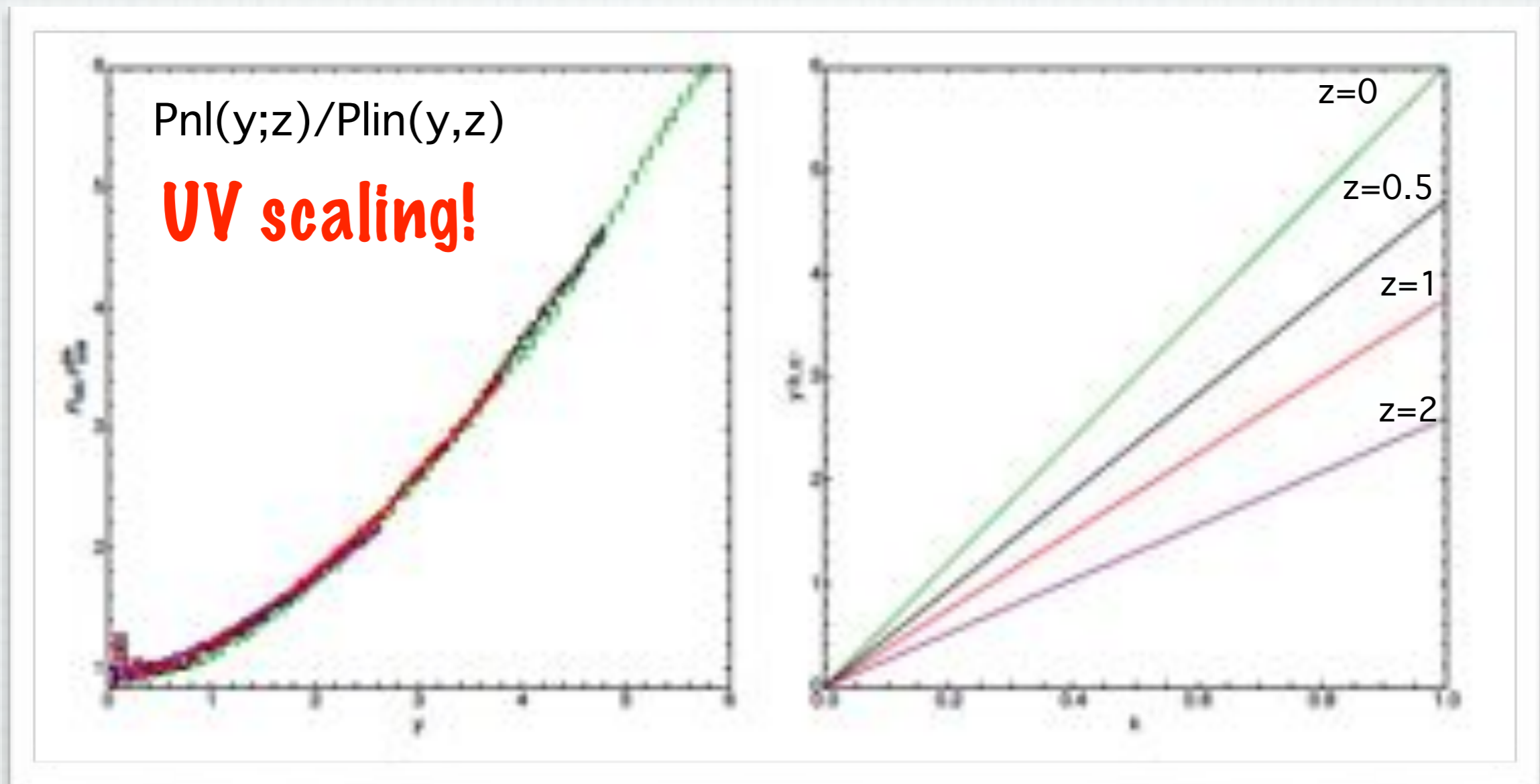
0(minutes) and 0(%) accuracy in the BAO region achieved!

The large k regime for the PS

Anselmi, MP,
1205.2235

for the nonlinear propagator, the relevant variable
in the large k regime is : $y \equiv k \sigma(e^\eta - e^{\eta_i})$

$$G(k; \eta, \eta_i) \sim \exp\left(-\frac{y^2}{2}\right)$$

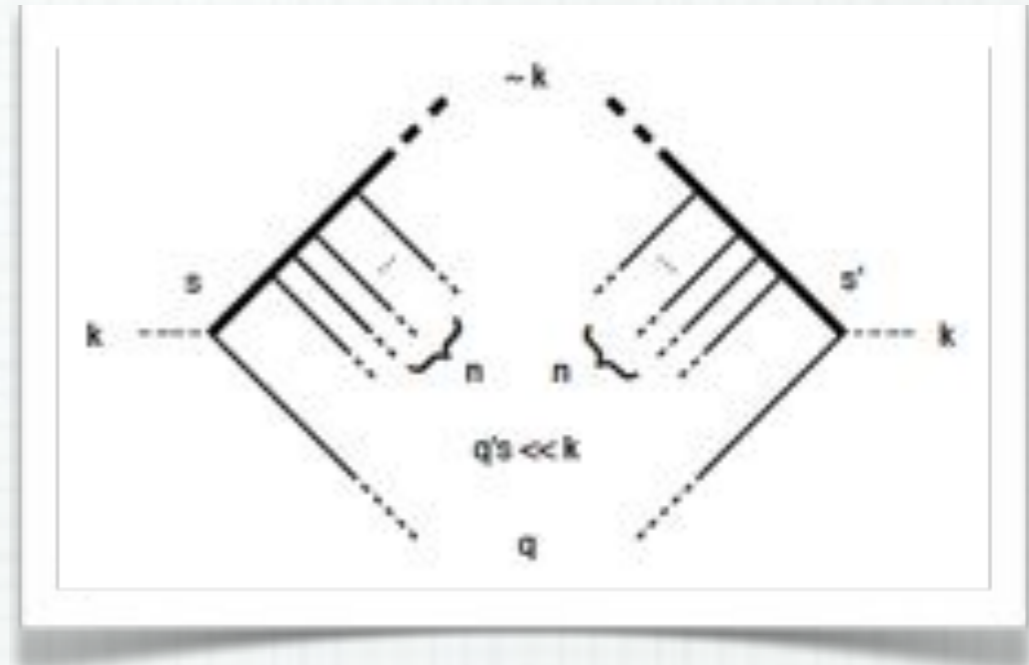


The non-linear PS is y -dependent too. resummation possible !!

$$\Phi_{ab}(k; s, s')$$

large k

leading contributions to Φ :

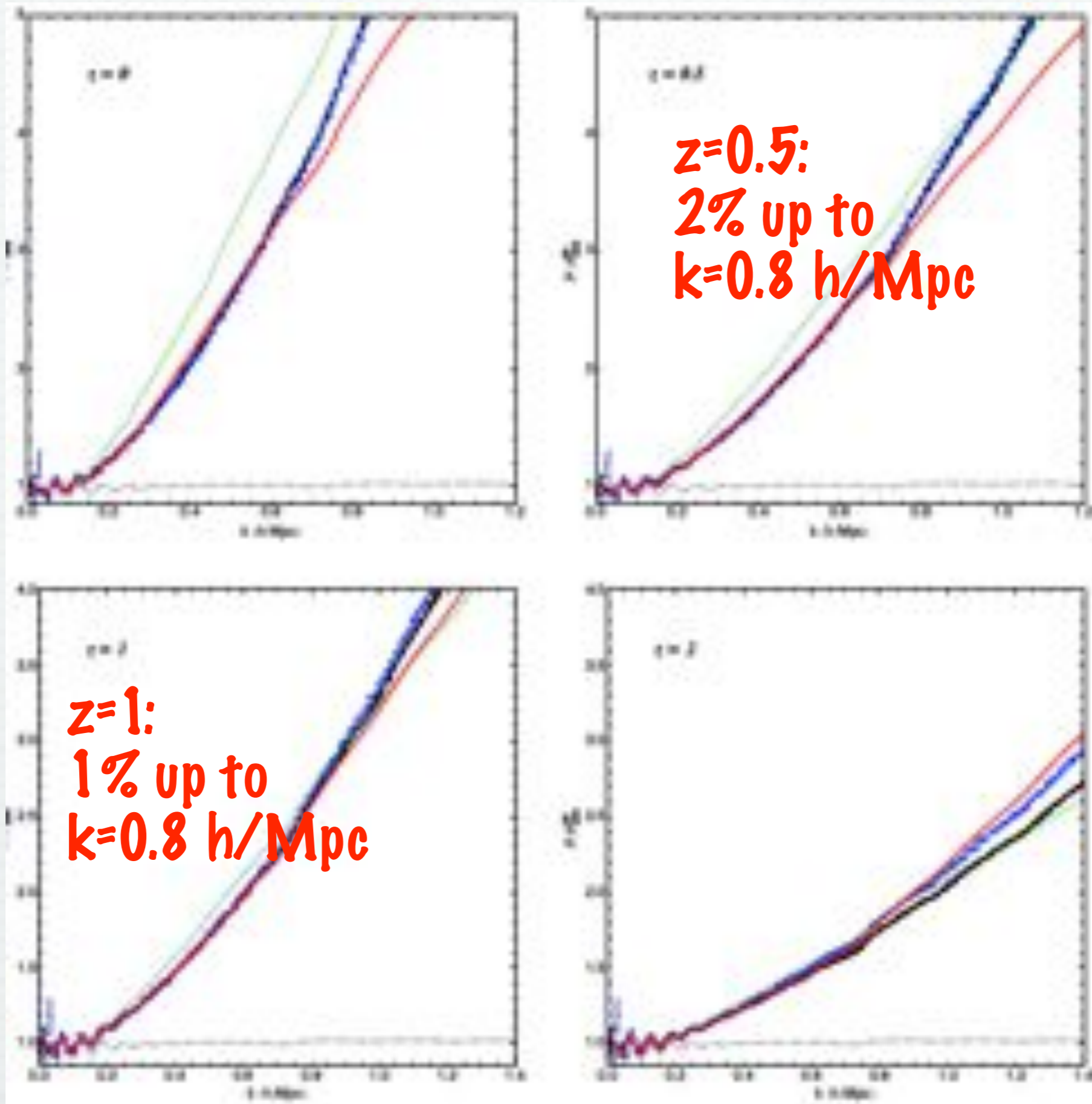


1 “hard” loop momentum, $n-1$ “soft” ones

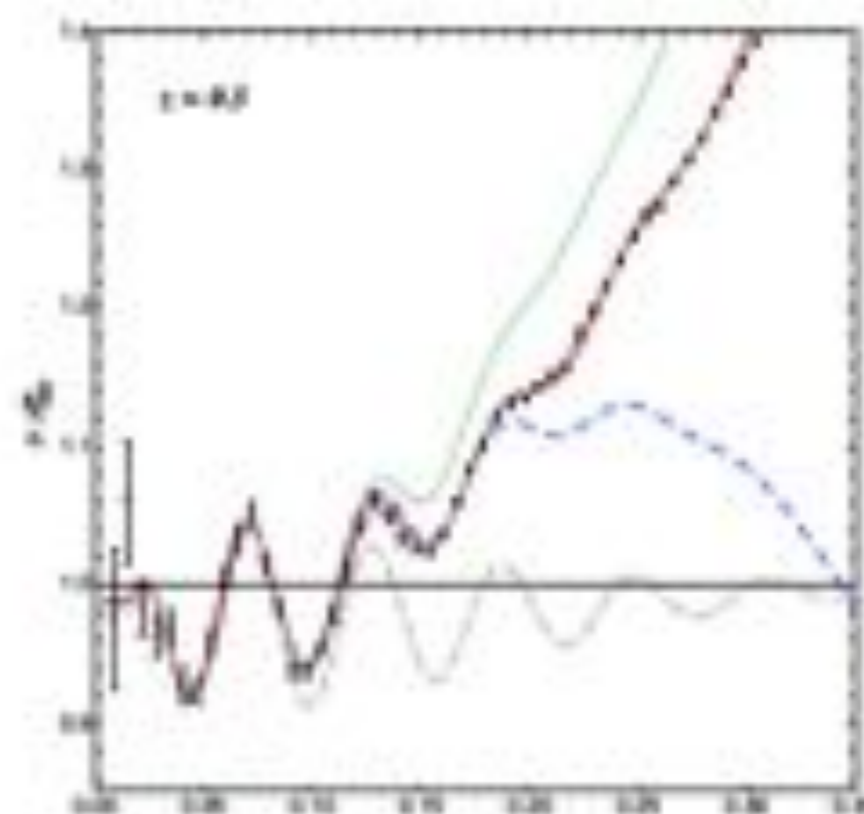
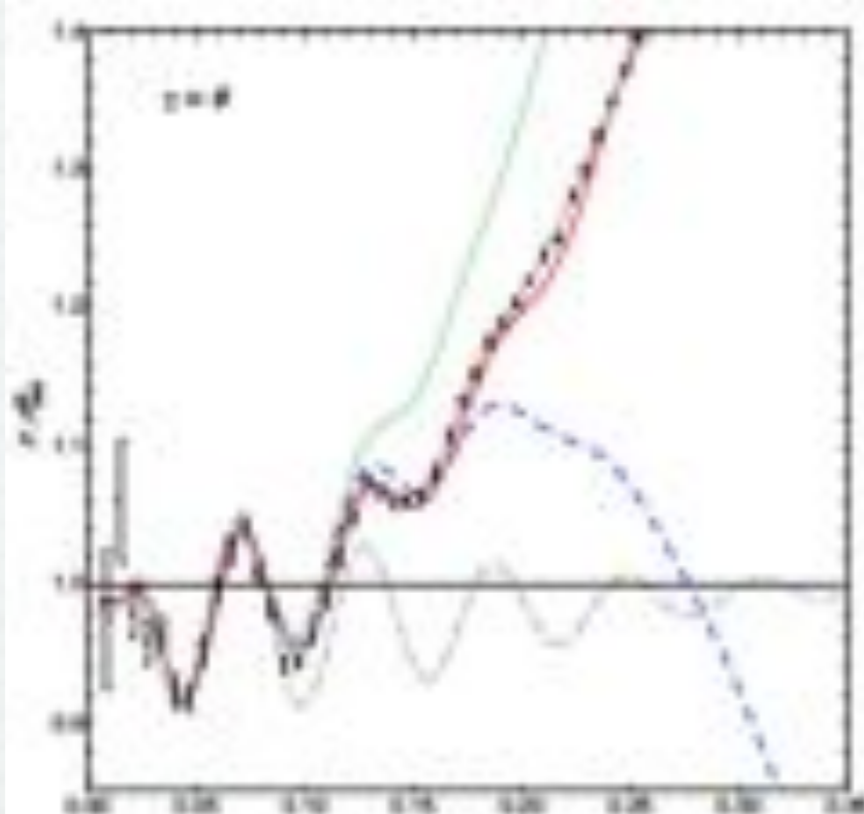
$$\tilde{\Phi}_{ab}(k; s, s') \rightarrow e^{-\frac{k^2 \sigma_v^2}{2} (e^s - e^{s'})^2} \left[\Phi_{ab}^{(1)}(k; s, s') + \left(k^2 \sigma_v^2 e^{s+s'} \right)^2 P(k) u_a u_b \right]$$

Can be obtained in eRPT: tree-level=UV limit

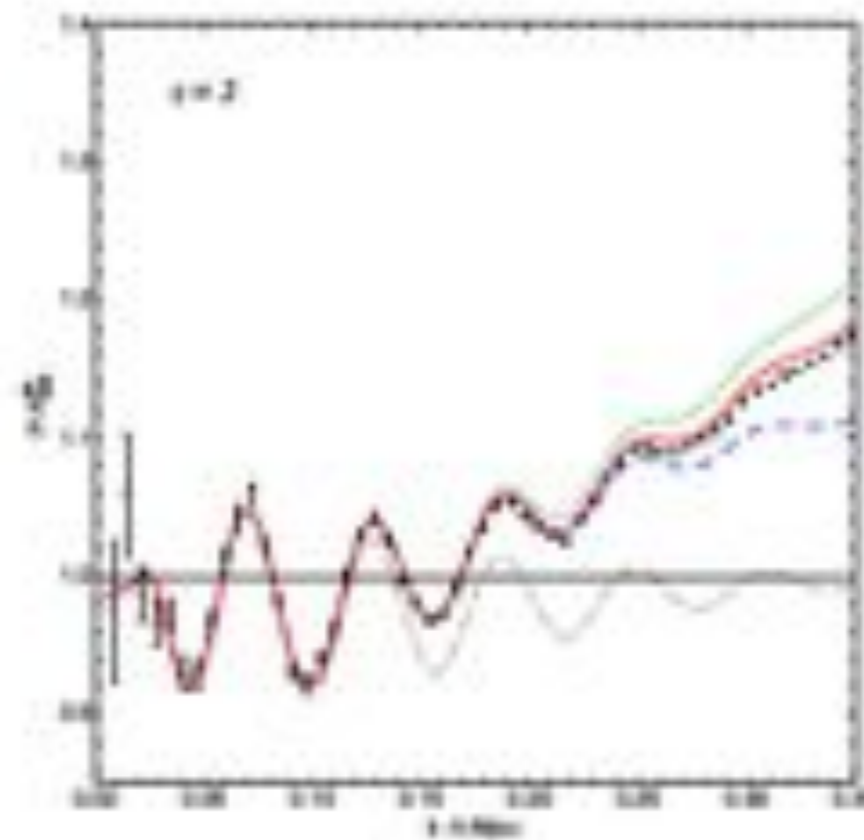
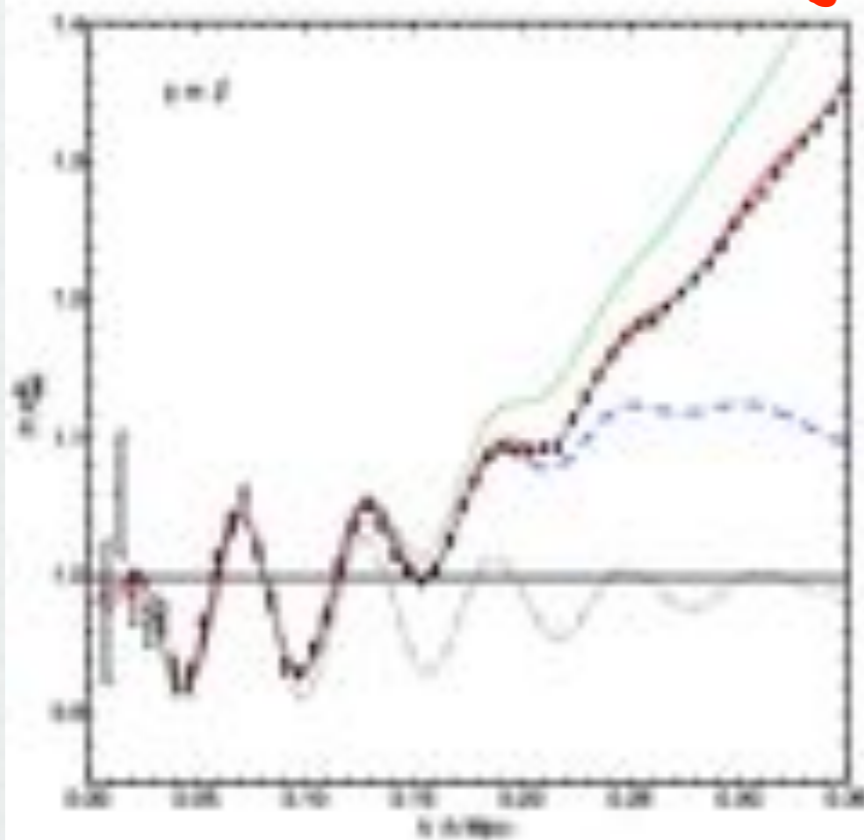
large k



BAO scales



1% in the BAO region at all redshifts!



Practical Implementation

- * linear PS at z_{in}
- * compute 5 momentum integrals:

$$H_1(k; \eta, -\infty) = -e^{2\eta} \frac{k^3 \pi}{21} \int dr \left[19 - 24r^2 + 9r^4 - \frac{9}{2r} (r^2 - 1)^3 \log \left| \frac{1+r}{1-r} \right| \right] P^0(kr)$$

$$\Phi_{11}^{(1)}(k; \eta, \eta') = e^{\eta+\eta'} \frac{\pi}{4k} \int_0^\infty dq \int_{|k-q|}^{k+q} dp \frac{[k^2(p^2 + q^2) - (p^2 - q^2)^2]^2}{p^3 q^3} P^0(q) P^0(p)$$

- * integrate the evolution equation from z_{in} to z : get P_{dd} , P_{dt} , P_{tt}
- * public code available soon



The intrinsic limit of (resummed) PT

The DM particle distribution function, $f(\mathbf{x}, \mathbf{p}, \tau)$, obeys the Vlasov equation:

$$\frac{\partial f}{\partial \tau} + \frac{\mathbf{p}}{am} \cdot \nabla f - am \nabla \phi \cdot \nabla_{\mathbf{p}} f = 0$$

with $\dot{\mathbf{x}} = am \frac{d\mathbf{x}}{d\tau}$ and $\nabla^2 \phi = \frac{3}{2} \Omega_M \mathcal{H}^2 \delta$ sub-horizon scales, Newtonian gravity

Taking moments,

$$\int d^3 \mathbf{p} f(\mathbf{x}, \mathbf{p}, \tau) \equiv \rho(\mathbf{x}, \tau) \equiv \bar{\rho} [1 + \delta(\mathbf{x}, \tau)]$$

$$\int d^3 \mathbf{p} \frac{p_i}{am} f(\mathbf{x}, \mathbf{p}, \tau) \equiv \rho(\mathbf{x}, \tau) v_i(\mathbf{x}, \tau)$$

$$\int d^3 \mathbf{p} \frac{p_i p_j}{a^2 m^2} f(\mathbf{x}, \mathbf{p}, \tau) \equiv \rho(\mathbf{x}, \tau) [v_i(\mathbf{x}, \tau) v_j(\mathbf{x}, \tau) + \sigma_{ij}(\mathbf{x}, \tau)]$$

...

$$\frac{\partial n}{\partial \tau} + \frac{\partial}{\partial x^i} (n v^i) = 0$$

$$\frac{\partial v^i}{\partial \tau} + \mathcal{H} v^i + v^k \frac{\partial}{\partial x^k} v^i + \frac{1}{n} \frac{\partial}{\partial x^k} (n \sigma^{ki}) = - \frac{\partial}{\partial x^i} \phi$$

source term

$$\frac{\partial \sigma^{ij}}{\partial \tau} + 2\mathcal{H} \sigma^{ij} + v^k \frac{\partial}{\partial x^k} \sigma^{ij} + \sigma^{ik} \frac{\partial}{\partial x^k} v^j + \sigma^{jk} \frac{\partial}{\partial x^k} v^i + \frac{1}{n} \frac{\partial}{\partial x^k} (n \omega^{ijk}) = 0$$

$$\frac{\partial \omega^{ijk}}{\partial \tau} + \dots = 0$$

...

$$\nabla^2 \phi = \frac{3}{2} \Omega_M \mathcal{H}^2 \delta$$

No sources for σ^{ij} , ω^{ijk} , ..., $\vec{\nabla} \times \vec{v}$, ...

$\sigma^{ij} = \omega^{ijk} = \dots = \vec{\nabla} \times \vec{v} = 0$ is a fixed point

neglecting σ_{ij} and higher moments...

$$\frac{\partial n}{\partial \tau} + \frac{\partial}{\partial x^i} (n v^i) = 0$$

continuity

$$\frac{\partial \mathbf{v}}{\partial \tau} + \mathcal{H}\mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla\phi$$

Euler

$$\nabla^2 \phi = \frac{3}{2} \Omega_M \mathcal{H}^2 \delta$$

Poisson

$$[n = n_0(1 + \delta)]$$

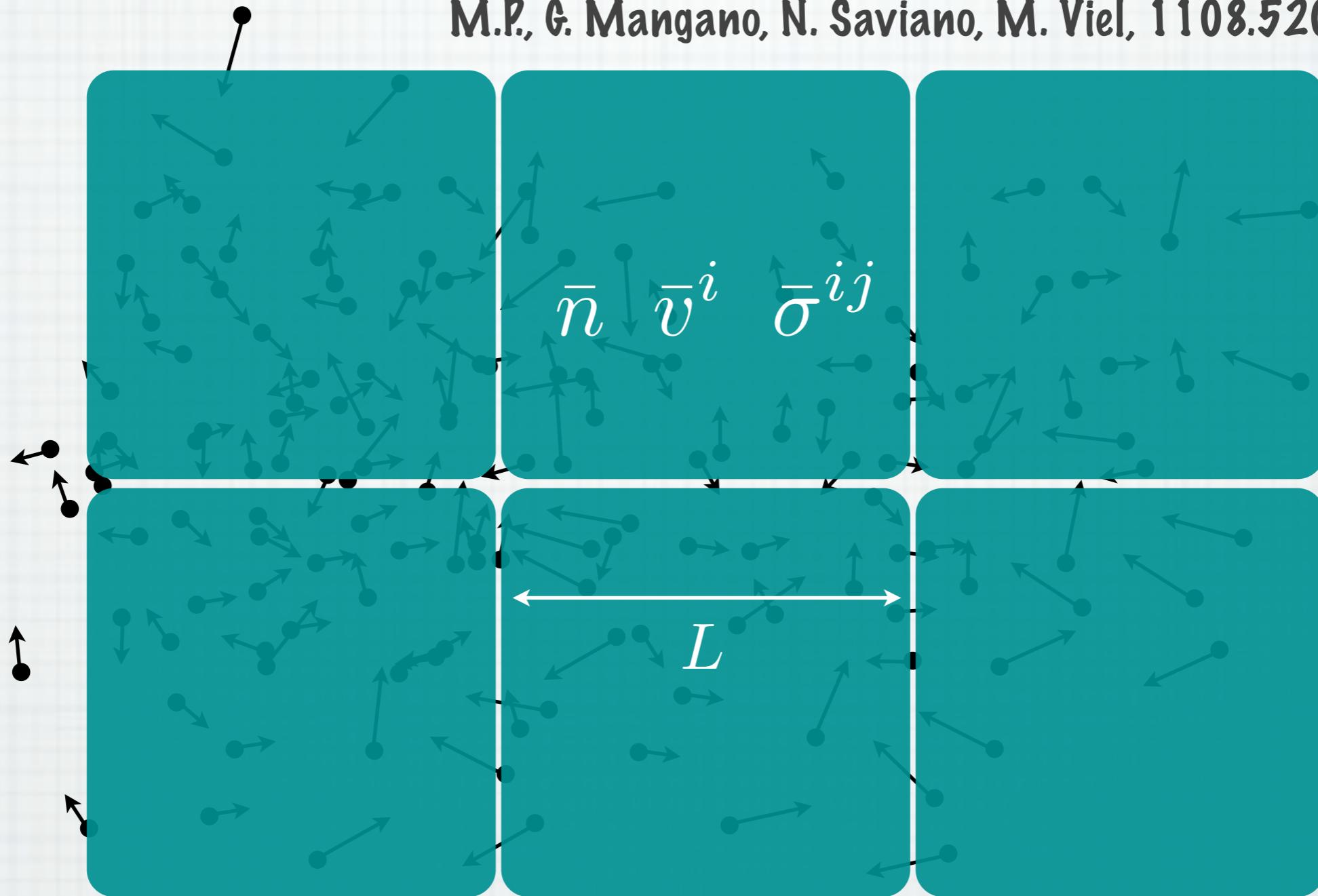
**(RESUMMED) PT IS BASED ON THE
"SINGLE STREAM APPROXIMATION"**

$$\sigma_{ij} = 0 \leftrightarrow f(\vec{x}, \vec{p}, \tau) = g(\vec{x}, \tau) \delta_D(\vec{p} - am\vec{v}(\vec{x}, \tau))$$

self-consistent, but wrong!

Rederiving the fluid equations

Buchert, Dominguez, '05, Pueblas Scoccimarro, '09, Baumann et al. '10
 M.P., G. Mangano, N. Saviano, M. Viel, 1108.5203



$$f_{mic}(x, \bar{p}, \tau) \equiv \sum_n \delta \frac{1}{V} (x - x_n(\tau)) \delta_D(p - p_n(\tau))$$

Satisfies the "Vlasov eq."

Coarse-Grained Vlasov eq.

large scales



$$\left[\frac{\partial}{\partial \tau} + \frac{p^i}{ma} \frac{\partial}{\partial x^i} - am \nabla_x^i \bar{\phi}(\mathbf{x}, \tau) \frac{\partial}{\partial p^i} \right] \bar{f}(\mathbf{x}, \mathbf{p}, \tau)$$
$$= \frac{am}{V} \int d^3y \mathcal{W} \left(\left| \frac{\mathbf{y}}{L} \right| \right) \nabla_{\mathbf{x}+\mathbf{y}}^i \delta\phi(\mathbf{x} + \mathbf{y}, \tau) \frac{\partial}{\partial p^i} \delta f(\mathbf{x} + \mathbf{y}, \mathbf{p}, \tau)$$

$$\delta\phi = \phi - \bar{\phi}$$

$$\delta f = f_{mic} - \bar{f}$$

short scales



Vlasov in the $L \rightarrow 0$ limit!

Short-distance sources

$$\frac{\partial}{\partial \tau} \bar{n}(\mathbf{x}) + \frac{\partial}{\partial x^i} (\bar{n}(\mathbf{x}) \bar{v}^i(\mathbf{x})) = 0.$$

$q_\theta + q_w$

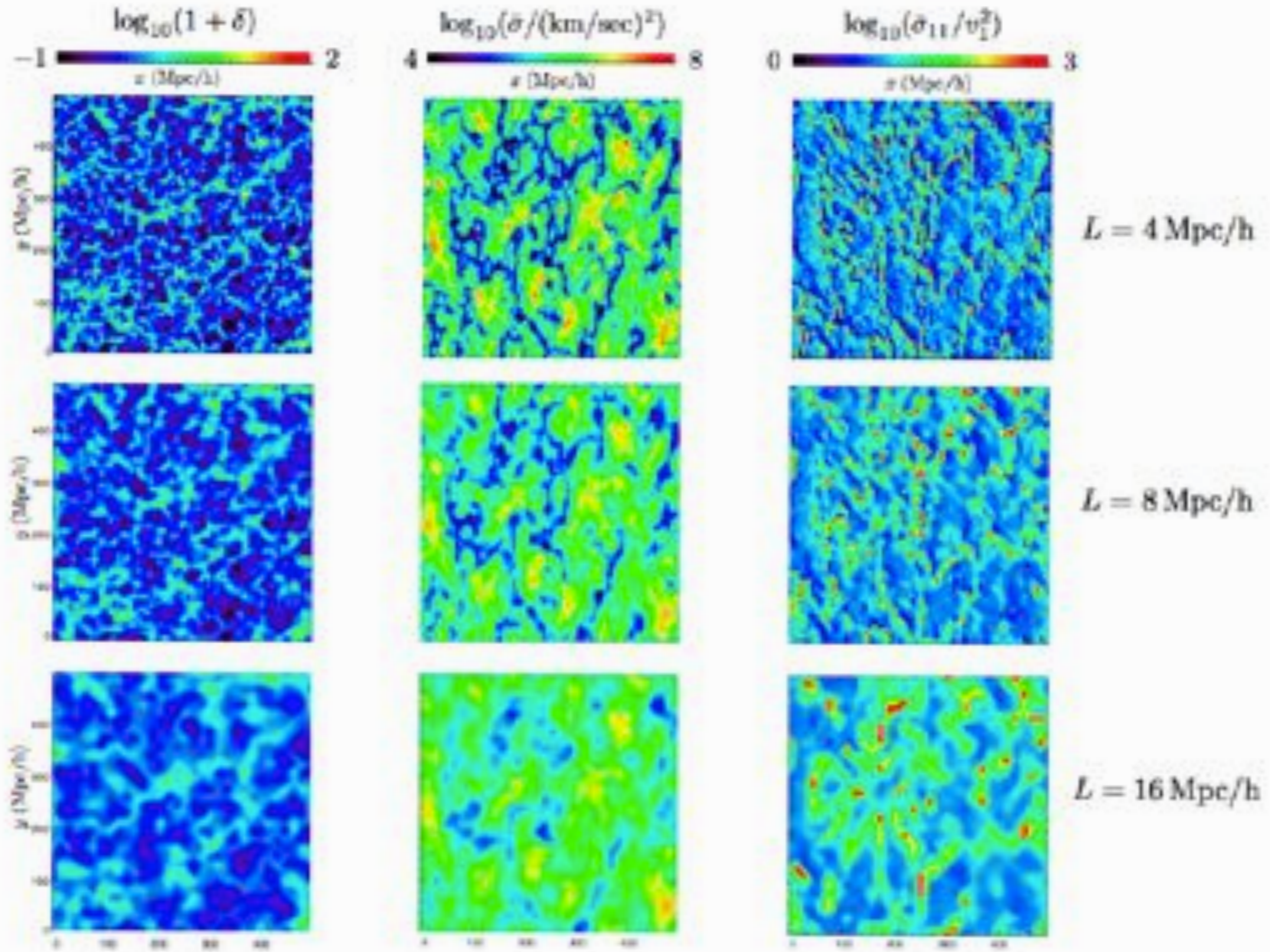
$$\begin{aligned} \frac{\partial}{\partial \tau} \bar{v}^i(\mathbf{x}) + \mathcal{H} \bar{v}^i(\mathbf{x}) + \bar{v}^k(\mathbf{x}) \frac{\partial}{\partial x^k} \bar{v}^i(\mathbf{x}) + \frac{1}{\bar{n}(\mathbf{x})} \frac{\partial}{\partial x^k} (\bar{n}(\mathbf{x}) \bar{\sigma}^{ki}(\mathbf{x})) \\ = -\nabla_x^i \bar{\phi}(\mathbf{x}) - \frac{1}{V} \int d^3 y \mathcal{W} \left(\left| \frac{\mathbf{y}}{L} \right| \right) \frac{n(\mathbf{x} + \mathbf{y})}{\bar{n}(\mathbf{x})} \nabla_{\mathbf{x} + \mathbf{y}}^i \delta \phi(\mathbf{x} + \mathbf{y}), \end{aligned}$$

Short-distance sources

$$\begin{aligned} \frac{\partial}{\partial \tau} \bar{\sigma}^{ij} + 2\mathcal{H} \bar{\sigma}^{ij} + \bar{v}^k \frac{\partial}{\partial x^k} \bar{\sigma}^{ij} + \bar{\sigma}^{ik} \frac{\partial}{\partial x^k} \bar{v}^j + \bar{\sigma}^{jk} \frac{\partial}{\partial x^k} \bar{v}^i + \frac{1}{\bar{n}} \frac{\partial}{\partial x^k} (\bar{n} \bar{\omega}^{ijk}) \\ = -\frac{1}{V} \int d^3 y \mathcal{W} \left(\left| \frac{\mathbf{y}}{L} \right| \right) \frac{n(\mathbf{x} + \mathbf{y})}{\bar{n}(\mathbf{x})} \\ \times \left[\delta v^j(\mathbf{x} + \mathbf{y}) \nabla_{\mathbf{x} + \mathbf{y}}^i + \delta v^i(\mathbf{x} + \mathbf{y}) \nabla_{\mathbf{x} + \mathbf{y}}^j \right] \delta \phi(\mathbf{x} + \mathbf{y}). \end{aligned}$$

$\bar{\sigma}^{ij}$ and all higher-order moments are dynamically generated by coarse-graining!

Coarse-Graining vs. Single-Stream



PT gets better



SSA gets worse



Well-behaved PT calls for dropping the single stream approximation

Compact form

$$\bar{\varphi}_a(\mathbf{k}, \eta) = e^{-\eta} \begin{pmatrix} \bar{\delta} \\ -\frac{\bar{\theta}}{\mathcal{H}f} \\ \frac{k^2}{\mathcal{H}^2 f^2} \bar{\sigma} \\ \frac{k^2}{\mathcal{H}^2 f^2} \bar{\Sigma} \end{pmatrix}$$

$$\bar{\sigma}(\mathbf{k}) = \bar{\sigma}^{ii}(\mathbf{k}), \quad \bar{\Sigma}(\mathbf{k}) = \frac{k^i k^j}{k^2} \bar{\sigma}^{ij}(\mathbf{k})$$

$$(\delta_{ab} \partial_\eta + \Omega_{ab}) \bar{\varphi}^b(\mathbf{k}, \eta) = e^\eta \int d^3 q_1 d^3 q_2 \delta_D(\mathbf{k} - \mathbf{q}_1 - \mathbf{q}_2) \gamma_{abc}(k, q_1, q_2) \bar{\varphi}_b(\mathbf{q}_1, \eta) \bar{\varphi}_c(\mathbf{q}_2, \eta) - h_a(\mathbf{k}, \eta)$$

(resummed) PT expansion
in γ_{abc}

$$0 \leq k \leq k_{(R)PT} \simeq \frac{2\pi}{L}$$

cosmology up to mildly non
linear scales

short-distance
sources: measure
from simulations

$$k > \frac{2\pi}{L}$$

cosmology-independent?

perturbative solution for the large scales

$$\bar{\varphi}_a^{(0)}(\mathbf{k}, \eta) = g_{ab}(\eta) \bar{\varphi}_b^{in}(\mathbf{k}) - \int_0^\eta ds g_{ab}(\eta - s) h_b(\mathbf{k}, s).$$

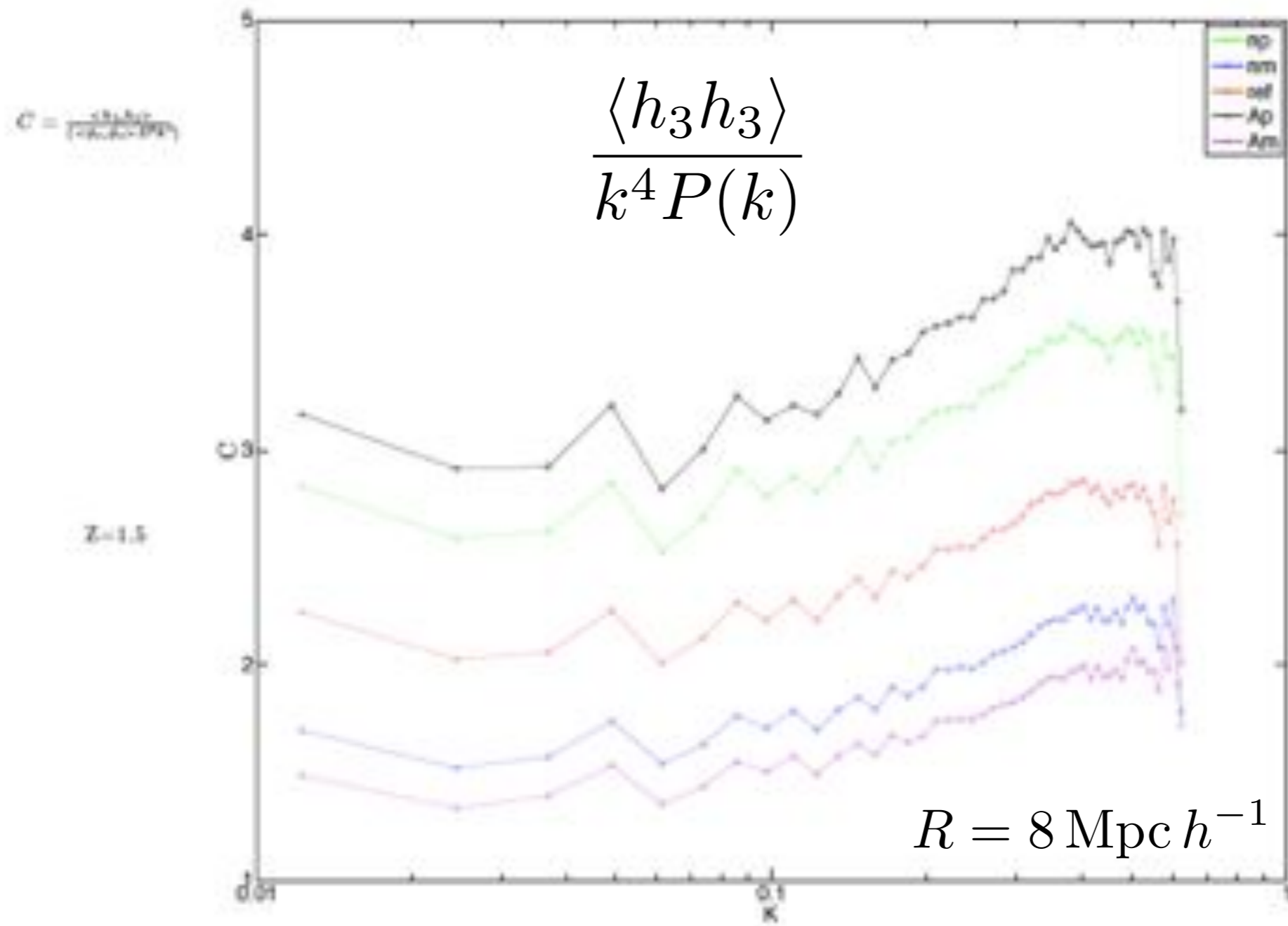
$$\bar{\varphi}_a^{(1)}(\mathbf{k}, \eta) = \int_0^\eta ds g_{ab}(\eta - s) e^s \gamma_{bcd}(k, q_1, q_2) \bar{\varphi}_c^{(0)}(\mathbf{q}_1, s) \bar{\varphi}_d^{(0)}(\mathbf{q}_2, s)$$

$$\bar{\varphi}_a^{(2)}(\mathbf{k}, \eta) = \int_0^\eta ds g_{ab}(\eta - s) e^s \gamma_{bcd}(k, q_1, q_2) \times \\ \left(\bar{\varphi}_c^{(1)}(\mathbf{q}_1, s) \bar{\varphi}_d^{(0)}(\mathbf{q}_2, s) + \bar{\varphi}_c^{(0)}(\mathbf{q}_1, s) \bar{\varphi}_d^{(1)}(\mathbf{q}_2, s) \right),$$

$\bar{\varphi}_1^{(0)} - \bar{\varphi}_2^{(0)} = \mathcal{O}(h)$ “linear” density and velocity “misaligned”

need $\langle \varphi_{a_1}^{(0)} \cdots \varphi_{a_n}^{(0)} h_{b_1} \cdots h_{b_m} \rangle$ correlators!

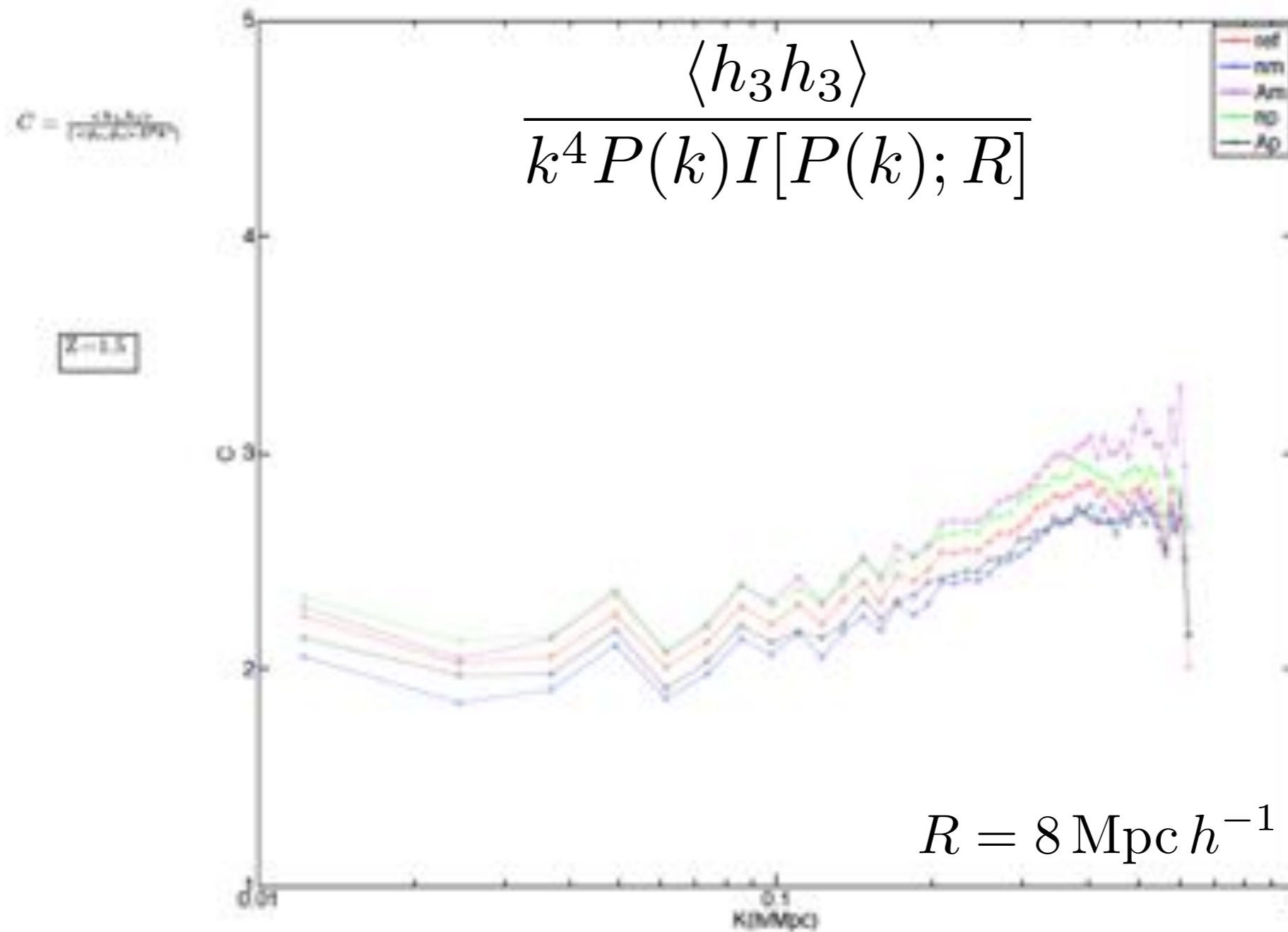
Sources: cosmology dependence



$$1.95 \cdot 10^{-9} < A_s < 3.0 \cdot 10^{-9}$$

$$0.932 < n_s < 1$$

Sources: cosmology dependence

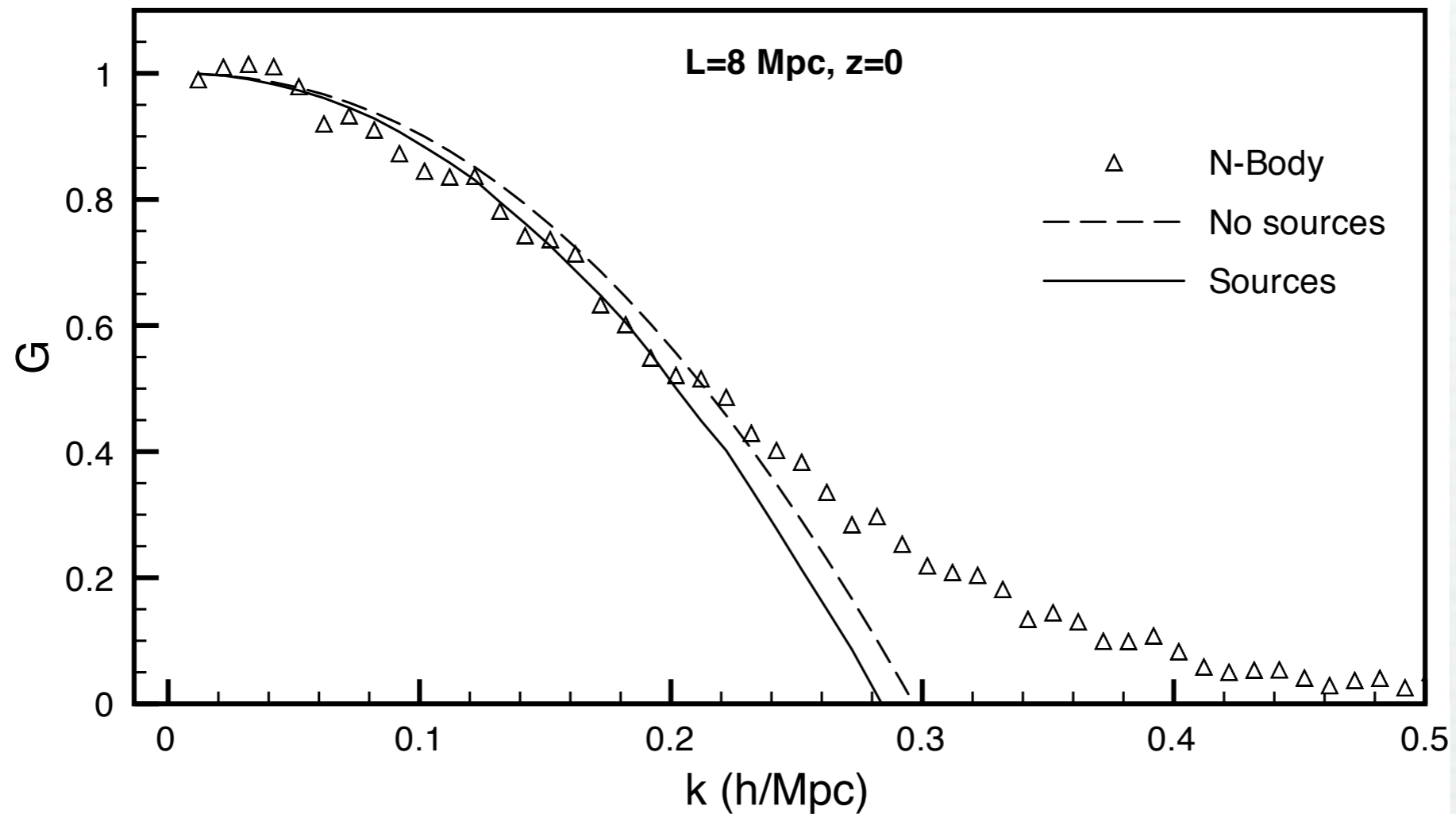


$$1.95 \cdot 10^{-9} < A_s < 3.0 \cdot 10^{-9}$$

$$0.932 < n_s < 1$$

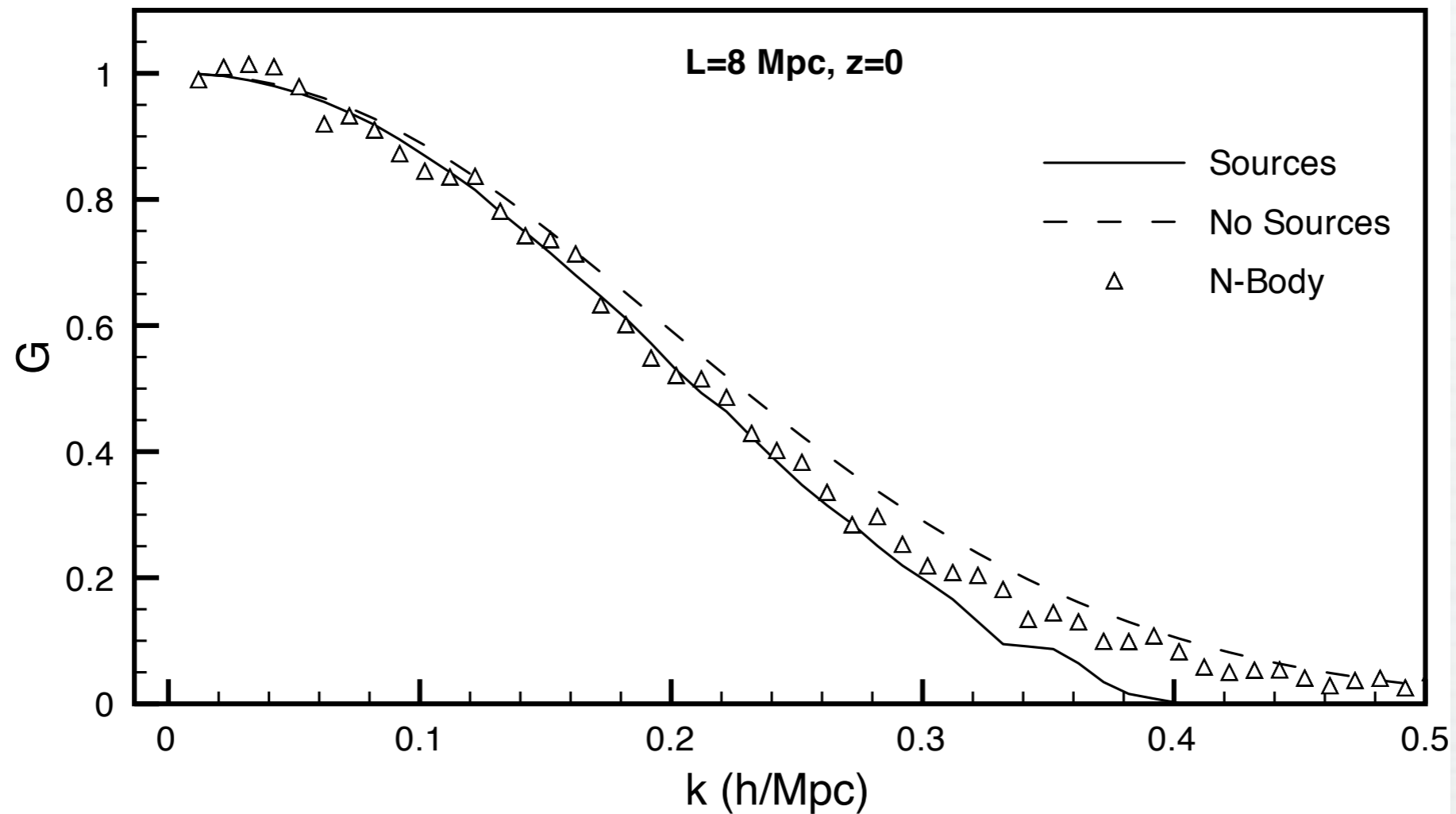
PT RECOVERS MOST OF THE COSMOLOGY DEPENDENCE !!

Propagator: 1-loop + sources



Mangano, Manzotti, MP, Saviano, Viel, in progress

Propagator: RPT+sources

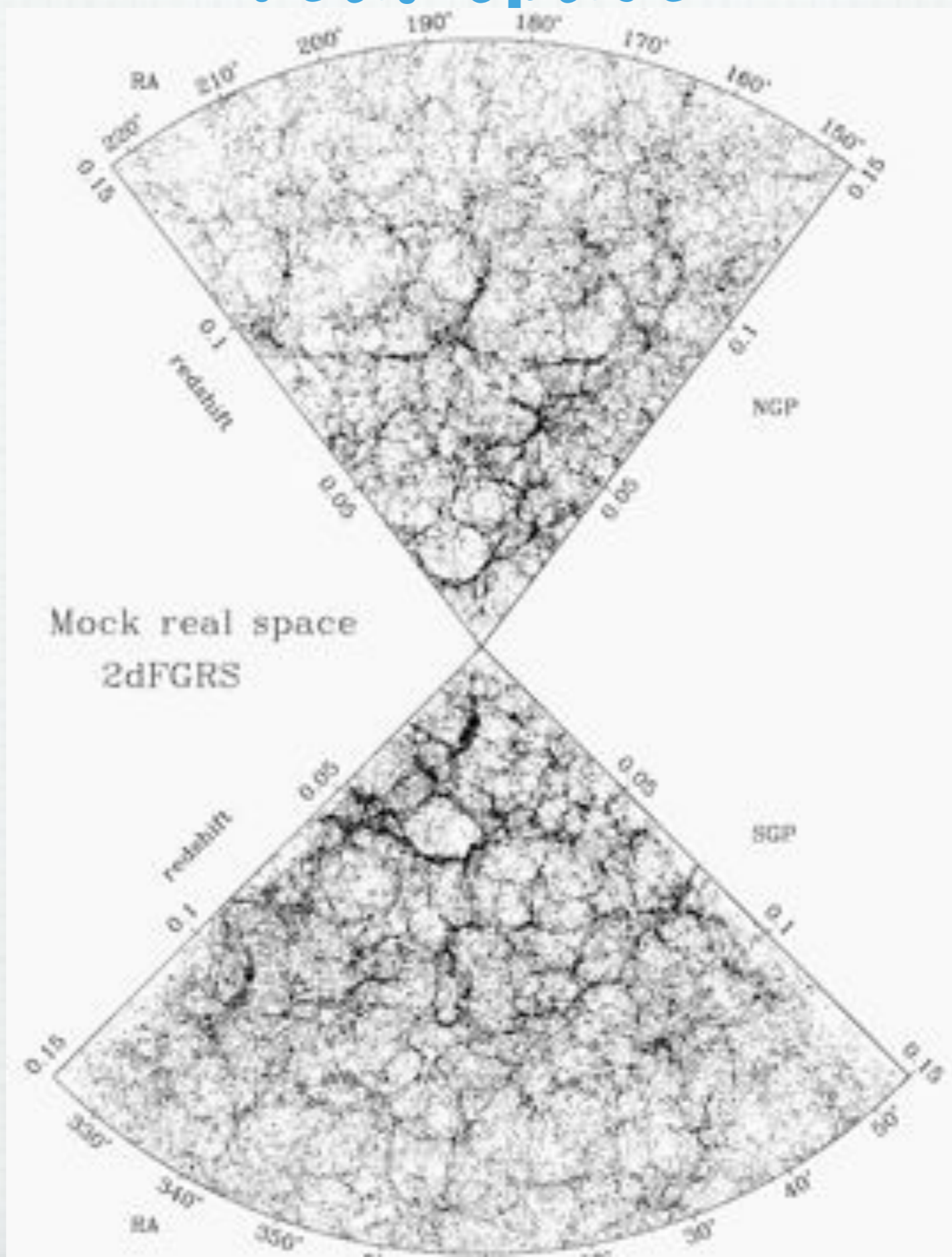


Mangano, Manzotti, MP, Saviano, Viel, in progress

PT RESUMMATIONS STILL NEEDED ON THE SMOOTH FIELDS!!

Redshift-space distortions

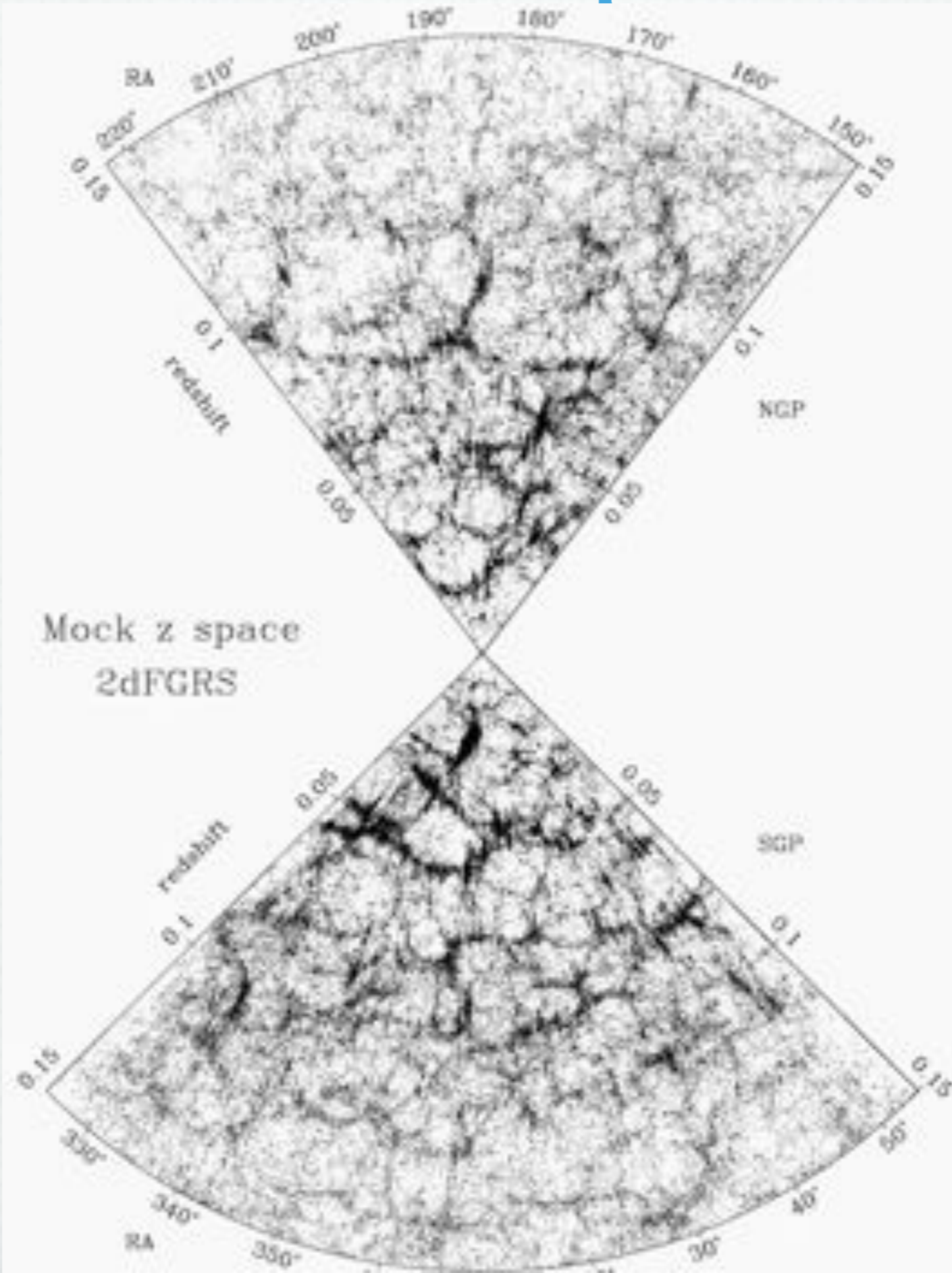
real-space



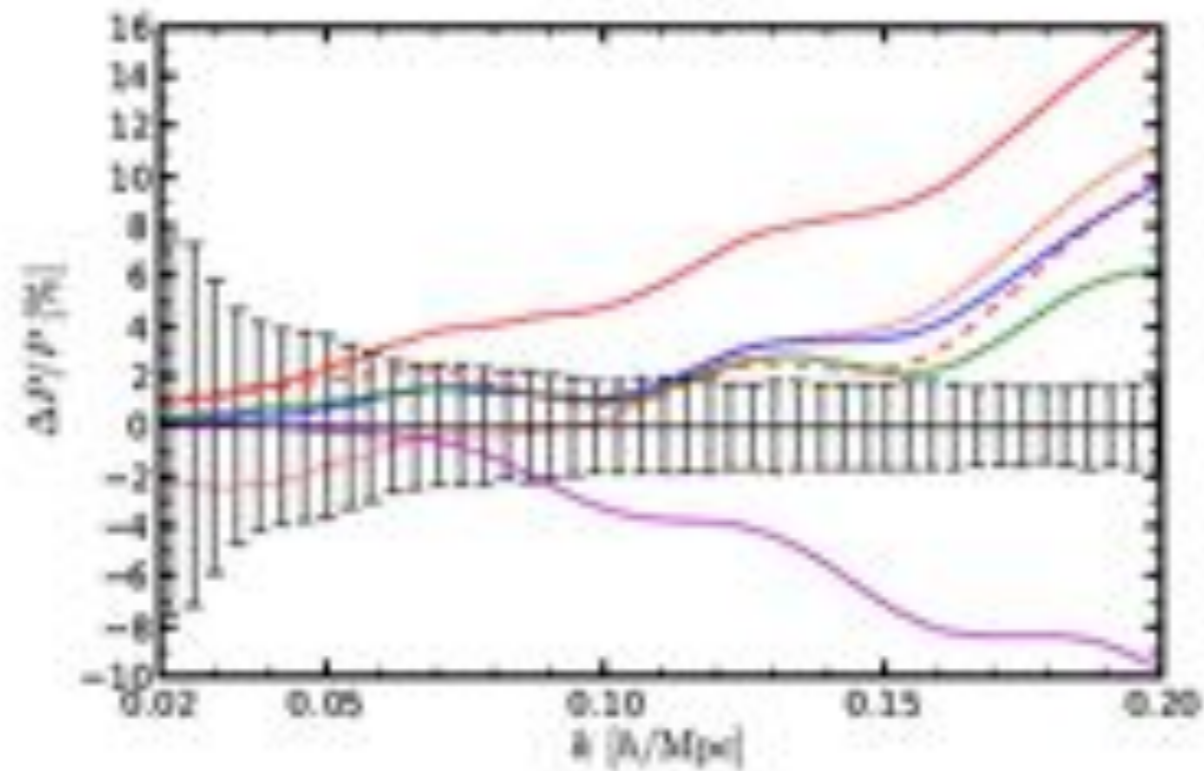
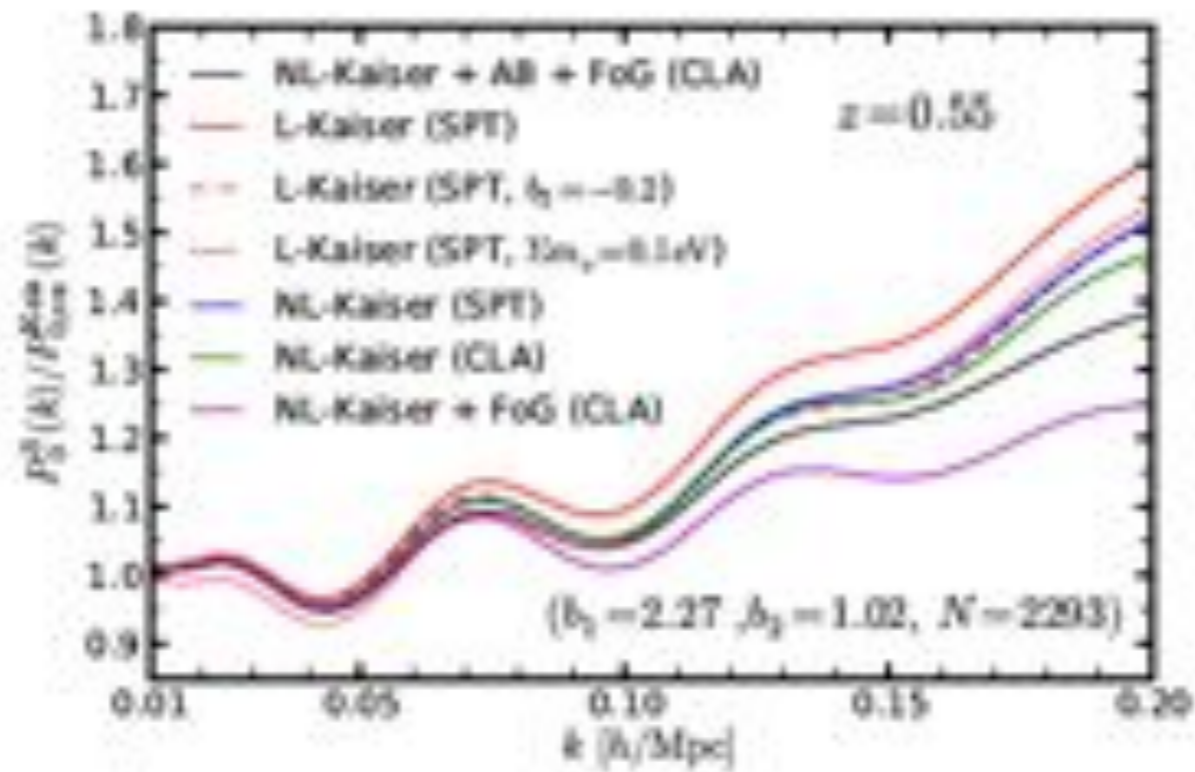
redshift-space

large scale:
Kaiser

small scale:
FoG



RSD modeling



Zhao et al
1211.3741

IR-UV mixing in redshift space

microscopic level:

$$\vec{x}_n \rightarrow \vec{s}_n = \vec{x}_n + \frac{p_n^z}{a\mathcal{H}m} \hat{z} \quad (\text{plane parallel approx.})$$

$$\delta_D(\vec{k}) + \delta_s(\vec{k}) = \int \frac{d^3\vec{x}}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} [1 + \delta(\vec{x})] \exp [ik_z v_z(\vec{x})/\mathcal{H}]$$

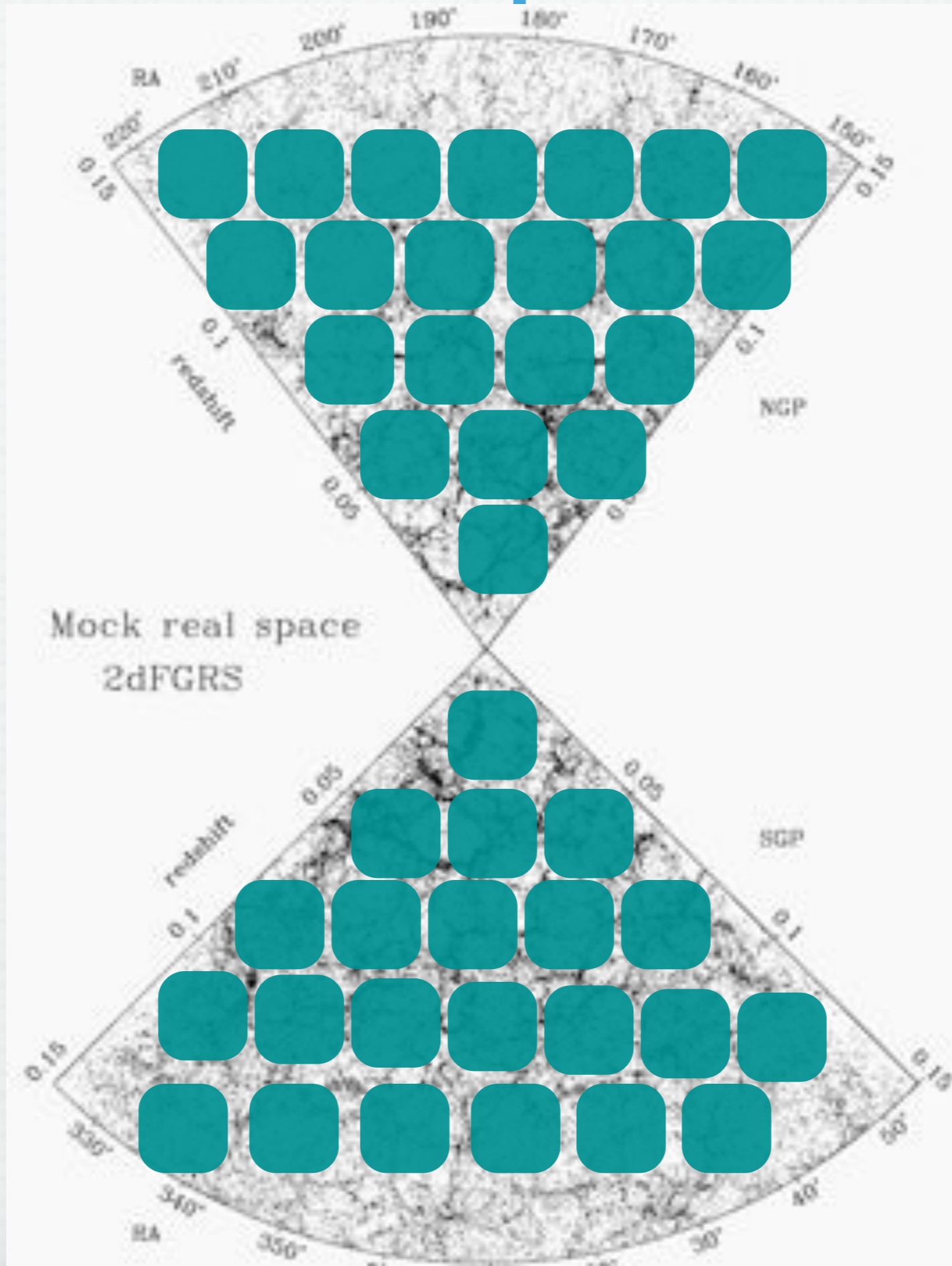
see Scoccimarro '04,

$\langle \delta_s(\vec{x}) \delta_s(\vec{y}) \rangle$ gets contributions from terms like $\langle \delta(\vec{x}) \delta(\vec{y}) v_z^2(\vec{y}) \rangle \sim \langle \delta(\vec{x}) \delta(\vec{y}) \rangle \langle v_z^2 \rangle$

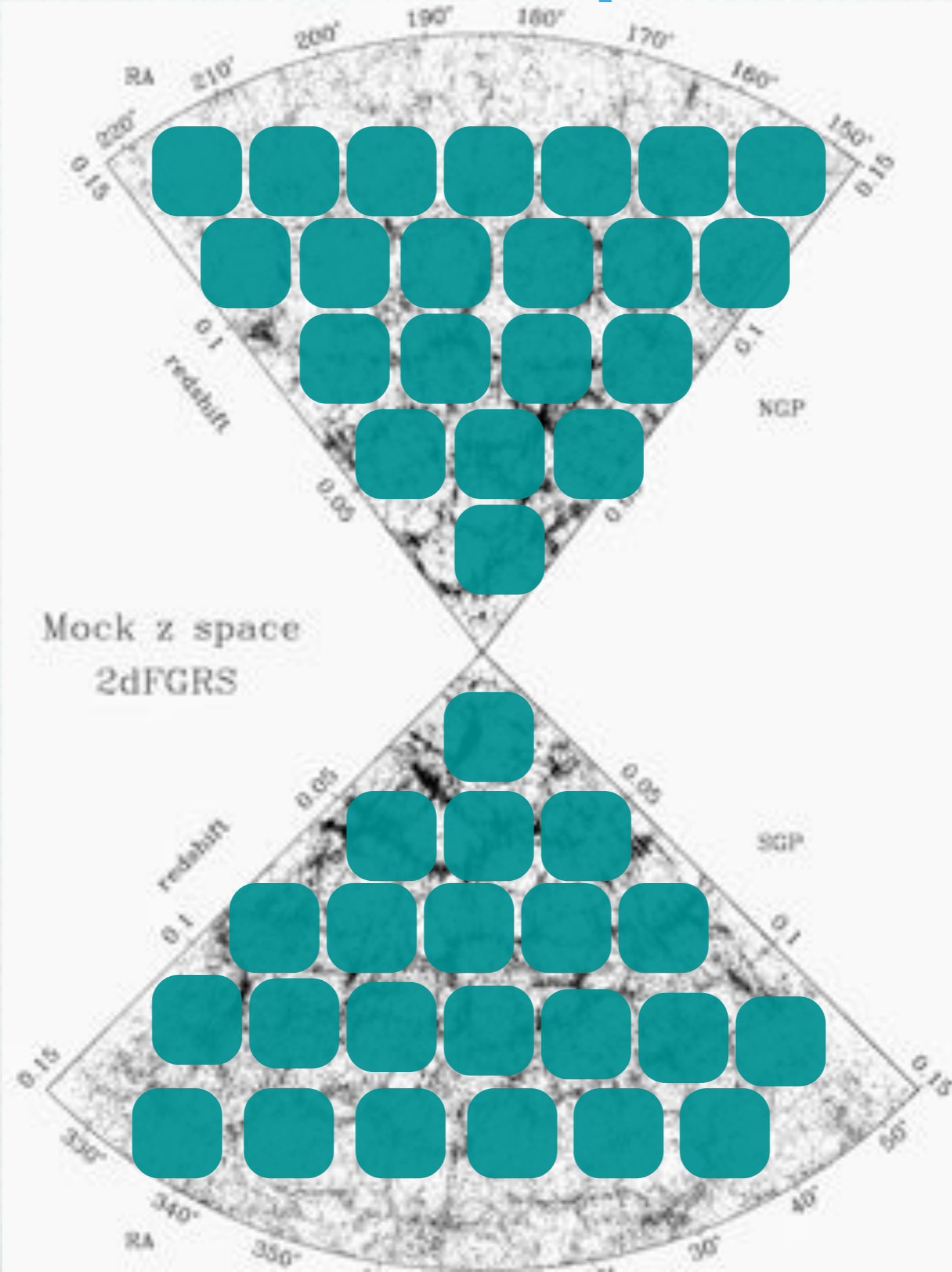
Large scales feel short ones!!

Problems for PT even at very large scales:
Desjacques, Baldauf, McDonald, Okumura, Seljak, Vlah, '11-'12

real-space



redshift-space



Integrate
out the FoG!

Coarse-grained redshift space

smoothed fields:

$$\delta_D(\vec{k}) + \bar{\delta}_s(\vec{k}) = \int \frac{d^3 \vec{x}}{(2\pi)^3} e^{i\vec{k} \cdot \vec{x}} [1 + \bar{\delta}(\vec{x})] \exp \left[ik_z \bar{v}_z(\vec{x}) / \mathcal{H} - \bar{\sigma}_{zz}(\vec{x}) k_z^2 / (2\mathcal{H}^2) + \dots \right]$$

better behavior of the expanded exponential

$$\bar{\delta}(\vec{k}) \neq -\frac{\bar{\theta}}{f\mathcal{H}}$$

“tree-level” corrections to Kayser’s formula

$$e^{-\bar{\sigma}_{zz}(\vec{x}) k_z^2 / (2\mathcal{H}^2)}$$

FoG resummation!

Summary

- * Future surveys demand a lot of effort on non-linear effects: real space, redshift space, bias;
- * Improved PT methods OK for BAOs and beyond (up to $O(1 h/\text{Mpc})$);
- * Agreement between independent approaches;
- * Good: speed, flexibility;
- * Common limit: the single stream approximation;
- * PT and N-body are complementary tools: let's exploit it! Coarse-grained PT