What is working behind the *self-modulation instability* of a charged-particle coasting beam?

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Topics discussed in WG1

- ❑ **"Self-modulation instability"**
- ❑ **Self-interaction: transverse and longitudinal**
  - ➢ **self focusing/defocusing**
  - ➢ **compression/decompression**
- ❑ **Bunch lengthening**

Possible explanation in terms of the self-consistent description provided by the Thermal Wave Model (TWM)

[*Fedele & Miele 1991, and relevant subsequent papers*]

beam/bunch density $= |\Psi(\xi,\zeta)|^2$

$U(\xi,\zeta)$ **is functional of** $|\Psi(\xi,\zeta)|^2$

TWM: example in (1+1)D:

$$i\,\epsilon_x \frac{\partial \Psi}{\partial \zeta} = \frac{\epsilon_x^2 \eta}{2} \frac{\partial^2 \Psi}{\partial \xi^2} + U(\xi,\zeta)\Psi$$

time-like var.

emittance (x dir.) *space-like var. in x dir.*

U **is provided by the set of equations governing the interaction of the beam/bunch with the surroundings:**

- ➢ **image charge and current (conven. accel.)**
- ➢ **plasma wake fields (plasma acceleration)**

Relevant example in conventional machines (longitudinal dynamics):

$$i\frac{\partial \psi}{\partial z} = \alpha\frac{\partial^2\psi}{\partial x^2} + \kappa|\psi|^2\psi + \mu\,\psi\int_{-\infty}^{x}|\psi(x',z)|^2dx'$$

long. emittance

imaginary part of coupling imped. (local effects)

real part of coupling imped. (nonlocal effects)

- ❑Coherent instability described as the *modulational instability (MI)* in optical fibers, in plasmas and BEC
- ❑Stabilizing effects due to dispersion (Landau damping)
- ❑Soliton solutions as final stage of MI (local effect)
- ❑Self-steepening and wave-breaking (nonlocal effects)

[P. Johannisson, D. Anderson, M. Lisak, M. Marklund, R. Fedele and A. Kim, *Phys. Rev. E.* **69**, 066501 (2004)]

Perspective: transfer this know how to plasma accelerators (PWF excitation):

➢*Self-consistent longitudinal dynamics of beam-plasma interaction (long beam limit)*

$$i\tilde{\varepsilon}\frac{\partial\Psi}{\partial s} = -\frac{\tilde{\varepsilon}^2}{2}\frac{\partial^2\Psi}{\partial x^2} + \frac{1}{2}K_p x^2\Psi - \frac{q^2}{e^2}\frac{\Sigma_b}{n_p\gamma^3}|\Psi|^2\Psi$$

$$\tilde{\varepsilon} \equiv \varepsilon/\gamma^2$$

plasma wave potential well (small synchrotron phase displacement)

local term

- ❑**Predicted: modulational instability & soliton formation**
- ❑**Described: synchrotron-like oscillations in the plasma wave potential well & found longitudinal envelope which includes the self-interaction**

[R. Fedele and V. G. Vaccaro , Physica Scripta. Vol. T52, 36-39, 1994]

➢*Self-consistent transverse dynamics of beam-plasma interaction (long beam limit)*

$$i\varepsilon\frac{\partial\Psi}{\partial\xi} = -\frac{\varepsilon^2}{2}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Psi}{\partial r}\right) + U_\perp^{coll}\left[|\Psi(r,\xi)|^2\right]\Psi$$

$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) - k_p^2\right]U_\perp^{coll}(r,\xi) = \frac{4\pi q^2 n_b}{m\gamma\beta^2c^2} \approx \frac{4\pi q^2 n_b}{m\gamma c^2}$$

(Zakharov-like pair of equations)

- ❑**Predicted: modulational instability in 2D (self-focusing,Weibel instability**

[R. Fedele and P.K. Shukla, *Phys. Rev. A* **44,** 4045 (1992)]

- ❑**Extension to magnetized plasmas: betatron-like oscillations & transverse envelope which includes the self-interaction and collapse & ringsolitons in local and nonlocal regimes** [Fedele et al. 2011, Tanjia et al. 2011]