Wakefield dynamics and electron acceleration in guiding structures

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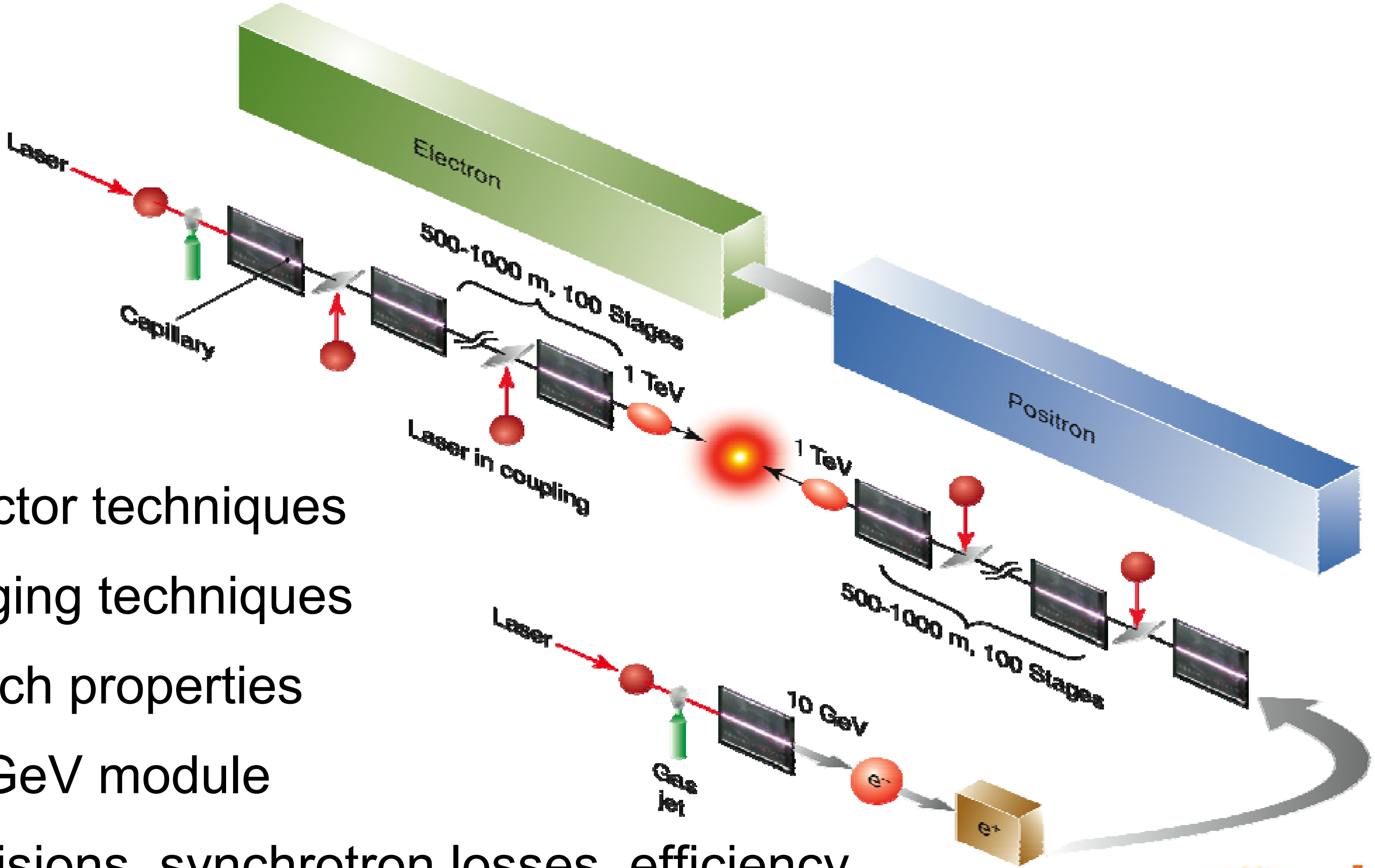
EAAC2013

1st European Advanced Accelerator Concepts Workshop
WG6 - Theory and simulations

2-7 June 2013
La Biodola, Isola d'Elba

Laser plasma accelerator based concept for a Laser Plasma Linear Collider



- Injector techniques
- Staging techniques
- Bunch properties
- 10 GeV module
- Collisions, synchrotron losses, efficiency

C.B. Schroeder et al., AAC Proceedings 2008: Leemans & Esarey, Physics Today, March 2009 ***W. Leemans***

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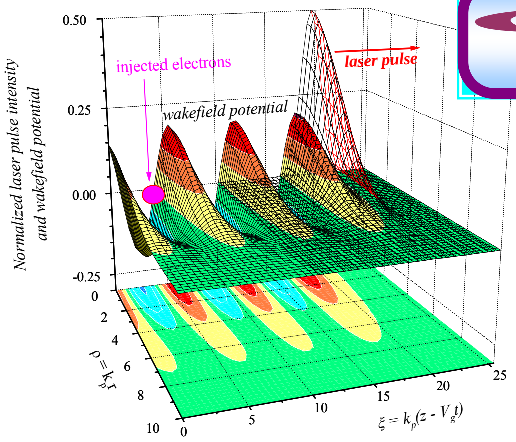
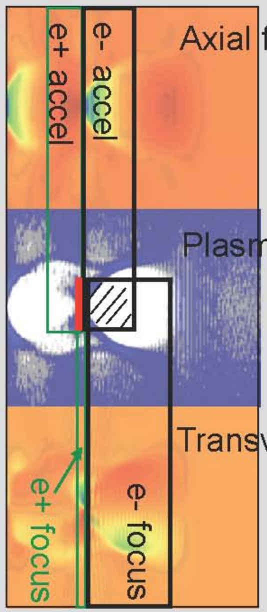
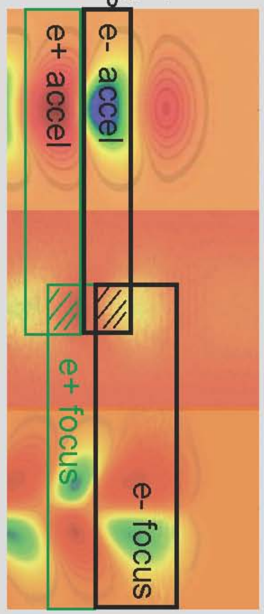
Laser-Plasma Acceleration of electrons with gradients ~ 10 GV/m



Linear & blowout regimes: e+/e- acceleration



- Blowout regime
  - high field
  - very asymmetric
    - focuses e-
    - defocuses e+

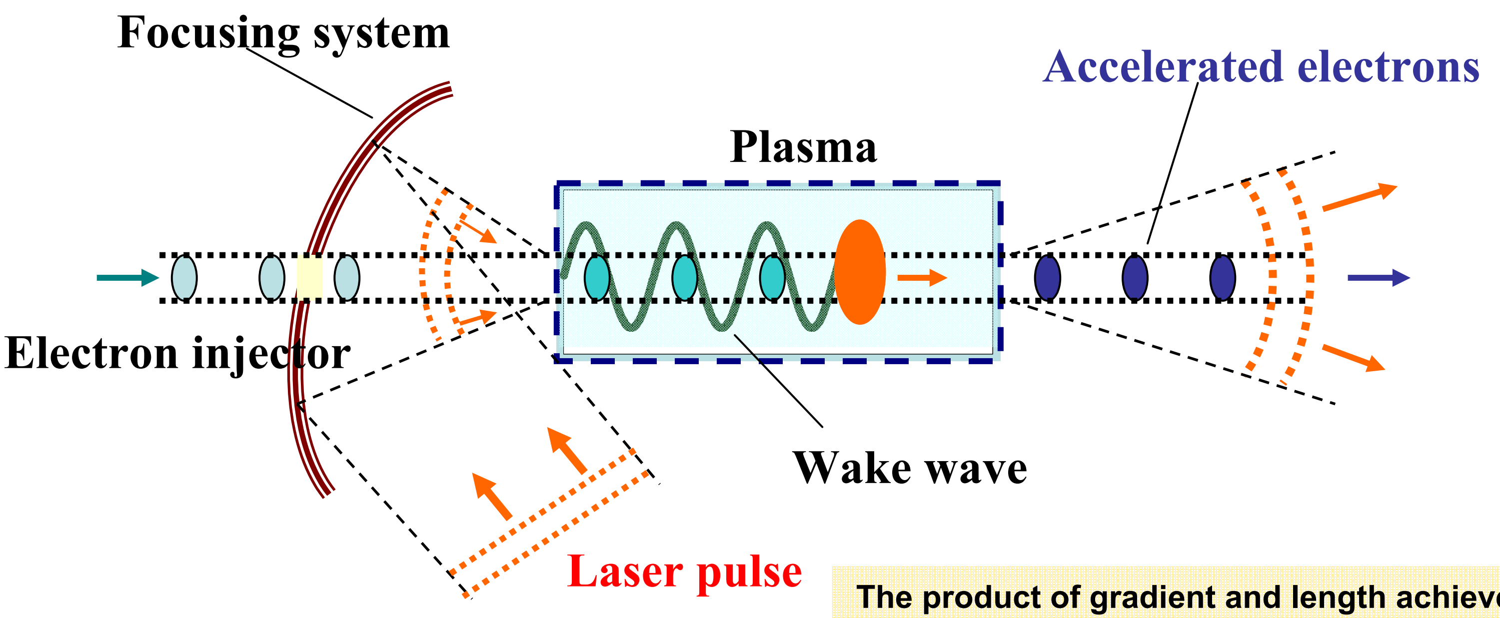


- Quasilinear
  - linear: symmetric e+/e-
  - high a0 desired for gradient
    - too high enters bubble: e+
  - a_0 ~1-2 good compromise

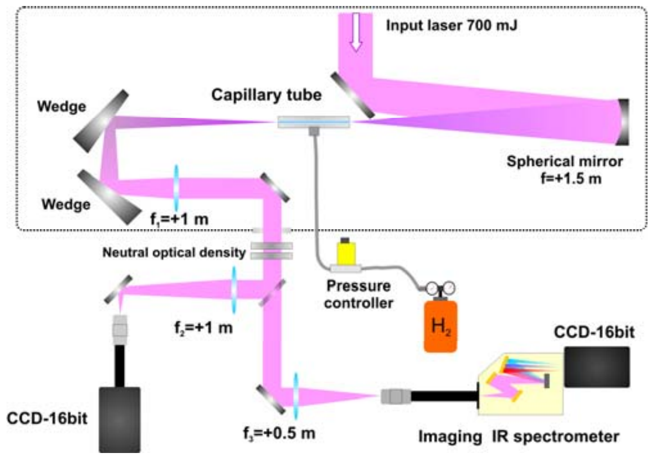
eli extreme light infrastructure

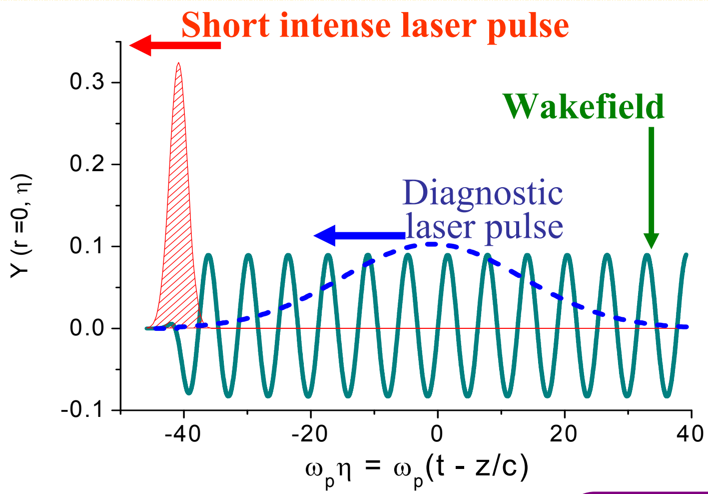
E. Esarey

Scheme of one cascade of the laser wake-field accelerator



The product of gradient and length achieved in this experiment is 0.4 GV at a pressure of 50 mbar @ 0.12 J, 51 fs





N.E. Andreev, K Cassou, F Wojda, G Genoud, M Burza, O Lundh, A Persson, B Cros, V E Fortov and C-G Wahlstrom, NJP, v. 12, 045024 (2010)

Energy spread in LWFA of short e-bunches

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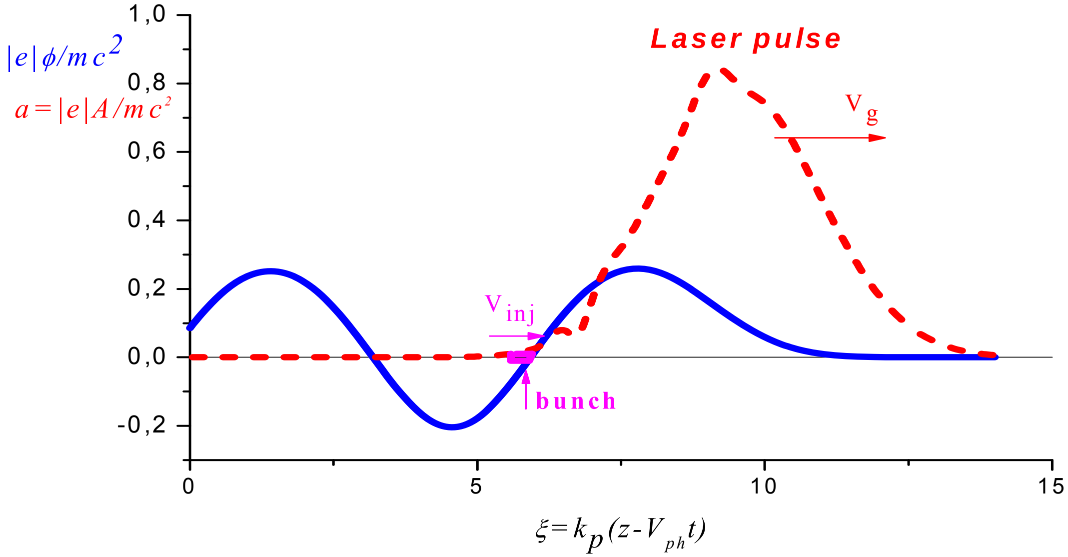
Electron bunch injection into LWFA at the maximum of accelerating field

Parameters of the laser pulse and electron bunch

$a_0 = \frac{|e| E_L}{m c \omega} = 1.0 \qquad \gamma_{ph} = \omega / \omega_p = 50 \qquad E_{inj} = 80\, mc^2 \qquad \mathbf{L_b = 0.1 k_p^{-1}}$



$$E_{\max} \approx 2mc^2 \gamma_{ph}^2 \varphi_{\max}$$

$$|\Delta E| \approx 2mc^2 \gamma_{ph}^2 k_p L_{b0} \left\{ \frac{d\phi(\xi_{inj})}{d\xi} \right\}$$

without loading effect

$$\Delta E / E_{\max} \simeq k_p L_b \simeq 10\%$$

for $L_b \approx$ 1mkm (3fs !)

Bunch loading effect: simulation and comparison with analytic predictions

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Parameters of laser pulse and electron bunch

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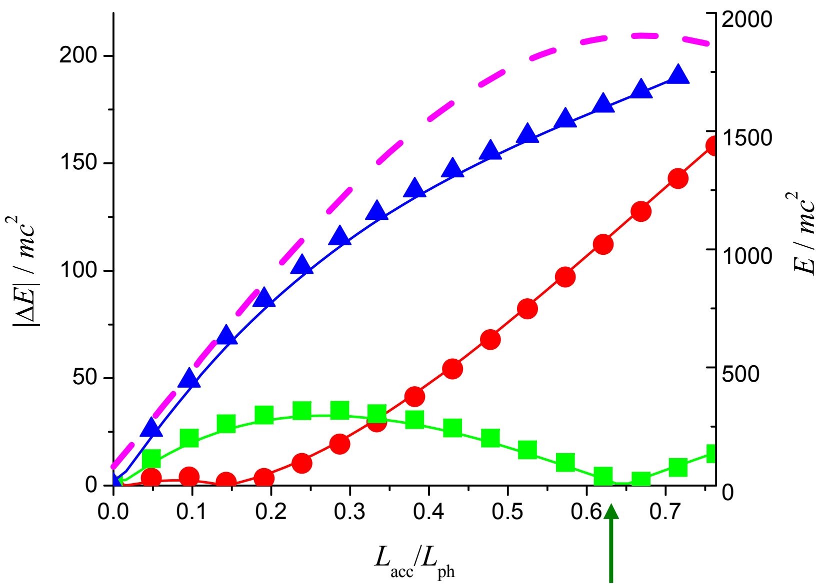

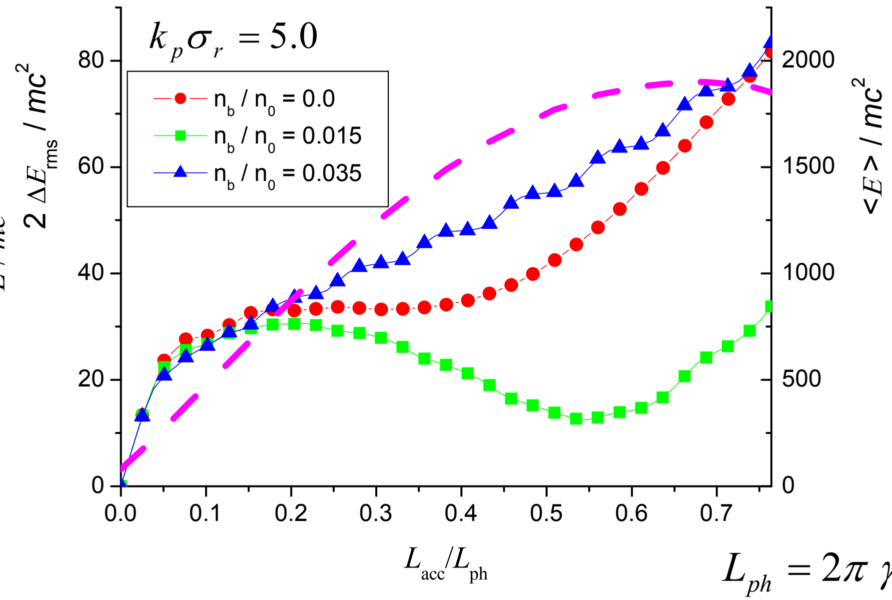

$$L_{ph} = 2\pi\gamma_{ph}^2 k_p^{-1}$$

$$|a(r_\perp, z, t)| = a(\xi)\exp\left[-\rho^2/\left(k_p r_0\right)^2\right]$$

$$r_0 = 14.14\ k_p^{-1}$$

$$|\Delta E_{min}| / mc^2 = 1.29$$

in agreement with analytical prediction

$$\frac{|\Delta E_{min}|}{mc^2} = \gamma_{ph}^2 \frac{(k_p L_b)^2}{4}\left|\frac{d^2\varphi}{d\xi^2}\right|_{max}$$

$$\tau_L = 1.1\omega_p^{-1}$$

Solid lines are analytical prediction; markers are results of numerical modeling for different bunch densities:

$n_b/n_0 =$ **0 – circles; 0.3 – triangles; 0.121 – squares.**

N.E. Andreev and S.V. Kuznetsov, IEEE Trans. on Plasma Sci., vol. 36, No.4. pp. 1765-1772, 2008

Electron Bunch Injection in Front of the Laser Pulse

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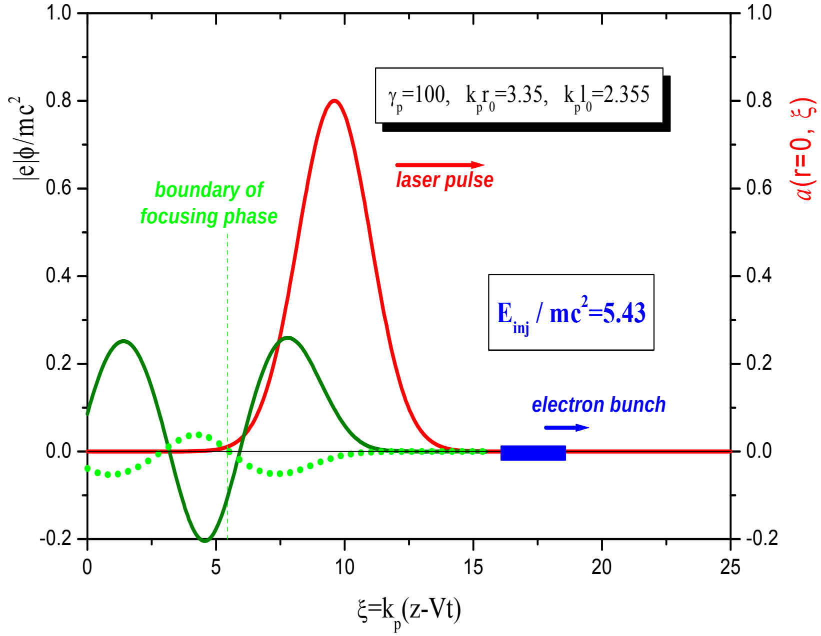
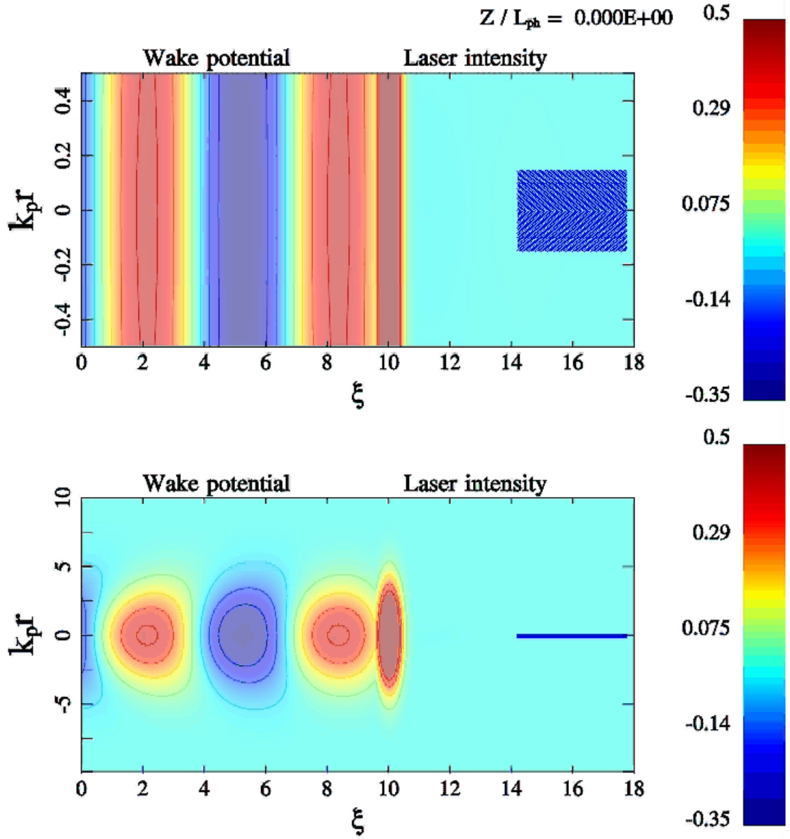




For the velocity of injected electrons

$$u_{inj} = c\sqrt{1 - m^2c^4 / E_{inj}^2} < v_{ph} \quad :$$

$$\frac{\Delta E}{mc^2} = 2\gamma_{ph}^2 k_p L_b \left\{ \left(\frac{d\varphi}{d\xi_{inj}} - \frac{d\varphi}{d\xi} \right) + \frac{k_p L_b}{2} \left(\frac{d^2\varphi}{d\xi^2} - \frac{d^2\varphi}{d\xi_{inj}^2} \right) \right\}$$

$$\frac{L_b}{L_{b0}} = \frac{1-\beta}{\beta - u_{inj}/c} \qquad \beta = v_{ph}/c$$

Long low-energy electron bunch will be trapped and compressed in the wakefield

N.E Andreev., S.V. Kuznezov. Electron Bunch Compression in Laser Wakefield Acceleration.// http://www.nuclear.jp/~icfa/GIENS/workshop_papers/giens01.pdf: *Giens Workshop Proceedings, June 24 - 29, 2001*.

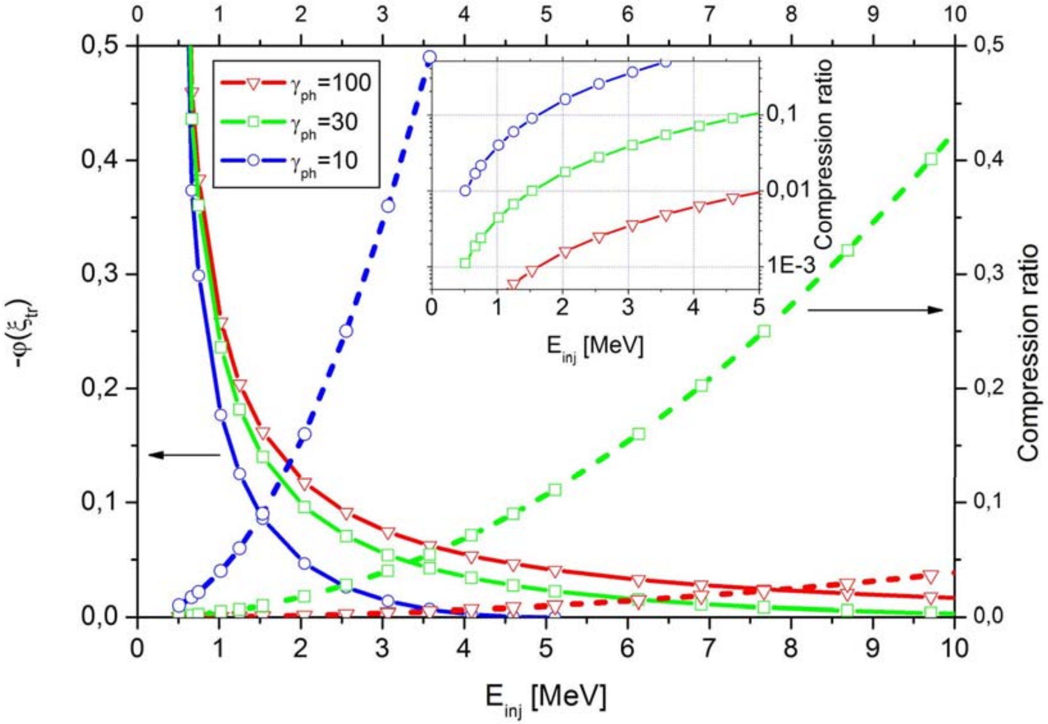
Electron Bunch Injection in Front of the Laser Pulse

trapping and compression

bunch injected in front of the laser pulse can be trapped and compressed in the wake field, if the condition

$$-\varphi(\xi_{tr}) = E_{inj}/mc^2 - \left[\left(1-\gamma_{ph}^{-2}\right)\left(E_{inj}^2/m^2c^4 - 1\right)\right]^{1/2} - 1/\gamma_{ph}$$

is fulfilled in the focusing phase of the wakefield



$$\frac{L_{b,rms}}{L_{b0}} \cong \frac{c - V_{ph}}{V_{ph} - u_{inj}} \approx \frac{E_{inj}^2}{\gamma_{ph}^2 m^2 c^4}$$

N E Andreev, S V Kuznetsov, B Cros, V E Fortov, G Maynar and P Mora "Laser wakefield acceleration of supershort electron bunches in guiding structures", Plasma Phys. Control. Fusion 53 (2011) 014001

ИТЭС
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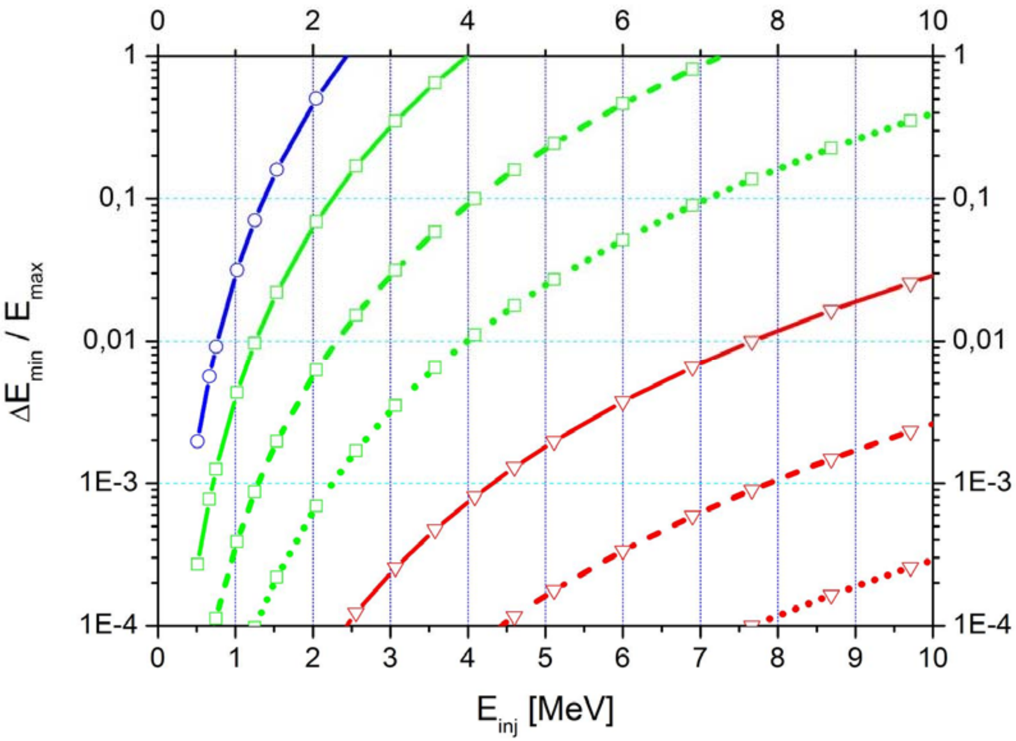
Electron Bunch Injection in Front of the Laser Pulse

energy spread at the end of acceleration

$$E_{\max} \cong 2\gamma_{ph}^2 mc^2 \varphi_{\max}$$

$$\frac{\Delta E_{\min}}{E_{\max}} \cong \frac{1}{2}(k_p L_{b,rms})^2 \cong 2\pi^2 \gamma_{ph}^{-6} \left(\frac{E_{inj}}{mc^2}\right)^4 \left(L_{b0}/\lambda_0\right)^2$$

$$\frac{\Delta E_{\min}}{mc^2} = \gamma_{ph}^2 (k_p L_{b,rms})^2 \left|\frac{d^2\varphi}{d\xi_{\max}^2}\right|$$



$\gamma_{ph} = 100$, 30, and 10 marked by triangles, squares and circles respectively, and for three initial bunch lengths $L_{b0} = 100$, 30, and 10 μm (solid, dashed and dotted lines respectively) for the laser wave length $\lambda_0 = 1$ μm

Averaged Equation for the Laser Pulse Envelope

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For the slowly varying (on the time and spatial scales ω_0^{-1} and $k_0^{-1} = c/\omega_0$) complex amplitude of the laser field E_0: $\mathbf{E} = \mathrm{Re}\{\mathbf{E}_0 \exp[-i\omega_0 t + ik_0 z]\}$

$$\left\{2ik_0\left(\frac{\partial}{\partial z} + c^{-1}\varepsilon_0 \frac{\partial}{\partial t}\right) + \Delta_\perp + k_0^2(\varepsilon_0 - 1) + \frac{\partial^2}{\partial z^2} - c^{-2}\varepsilon_0 \frac{\partial^2}{\partial t^2}\right\}\mathbf{E}_0 = $$
$$= k_0^2\left(\frac{n}{n_c \gamma} - \delta\varepsilon_a^{(NL)}\right)\mathbf{E}_0 - 4\pi i \frac{\omega_0}{c^2}\mathbf{J}^{(ion)}$$

in the dimensionless commoving with the laser pulse variables

$$\xi = k_{p0}(z - ct), \qquad \zeta = k_{p0} z, \qquad \boldsymbol{\rho} = k_{p0}\mathbf{r}_\perp$$

$$\left\{2i\frac{\partial}{\partial \zeta} + \frac{k_{p0}}{k_0}\left(\Delta_{\perp\rho} + 2\frac{\partial^2}{\partial \zeta \partial \xi}\right)\right\}\boldsymbol{a} = \frac{k_{p0}}{k_0}\left[\left(\frac{\nu}{\gamma} - \frac{3}{8}R|\boldsymbol{a}|^2\right)\boldsymbol{a} - i\mathbf{G}^{(ion)}\right]$$

where $a = eE_0/(mc\omega_0)$ is dimensionless laser envelope, $\nu = n/N_0$ **is the normalized electron density, $k_{p0} = \omega_{p0}/c$**

Computer simulation by the code LAPLAC

full scale modeling including
laser pulse dynamics, gas ionization and bunch loading

$$\left\{2ik_0\frac{\partial}{\partial z}+2\frac{\partial^2}{\partial z\,\partial\xi}+\Delta_\perp\right\}a=k_0^2\left(\frac{n}{n_c\gamma}a-iG^{(ion)}\right)$$

$$\frac{n}{\gamma}=n_0\frac{1+k_p^{-2}\Delta_\perp\Phi}{\Phi+\delta\Phi_S}$$

$$\mathbf{G}^{(ion)}=\frac{4\pi e}{m\omega_{p0}^2c}\mathbf{J}^{(ion)}=\frac{k_{p0}}{k_0}\left[\frac{2\boldsymbol{a}}{|\boldsymbol{a}|^2}\sum_{k=0}^{Z_n-1}S_0^{(k)}\frac{U_k}{mc^2}-\frac{\boldsymbol{a}^*}{4}S_2\right]$$

$$S_0=\frac{\Gamma_0}{\mathrm{N}_0\omega_{p0}}=\frac{n_a}{\mathrm{N}_0\omega_{p0}}\sum_{k=0}^{Z_n-1}\overline{W}_kD_k\equiv\sum_{k=0}^{Z_n-1}S_0^{(k)},\qquad S_2=\frac{\Gamma_2}{\mathrm{N}_0\omega_{p0}}\cong 2\mu S_0$$

N.E. Andreev and S.V. Kuznetsov, IEEE Trans. on Plasma Sci., vol. 36, No.4. pp. 1765-1772, 2008

N.E. Andreev, Y. Nishida, and N. Yugami. Physical Review E, vol. 65, pp. 056407-1 – 056407-10, 2002

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Nonlinear plasma response – Effective Potential

The nonlinear relativistic plasma response can be expressed through a single scalar function (potential) Φ :

$$\frac{\nu}{\gamma} = \frac{\nu_0 + \Delta_\perp \Phi}{\Phi + \delta\Phi_S} \qquad \delta\Phi_S = -\frac{1}{\nu_0}\int_{+\infty}^{\xi} \frac{\partial \nu_0}{\partial \xi'}\left(\Phi - 1 + \frac{|\boldsymbol{a}|^2}{4} - \frac{\mu}{4}\mathrm{Re}\left(\boldsymbol{a}^* \cdot \boldsymbol{a}^*\right)\right) d\xi'$$

$$\Phi = \gamma - q_z + \int_{+\infty}^{\xi} \frac{S_0}{\nu}\left(q_z - \frac{|\boldsymbol{a}|^2}{2} - \frac{\mu}{4}\mathrm{Re}\left(\boldsymbol{a}^* \cdot \boldsymbol{a}^*\right)\right) d\xi' \equiv \gamma - q_z - \delta\Phi_S$$

The electric and magnetic fields in plasma can be also expressed through the potential Φ:

$$E_z = \frac{\partial \Phi}{\partial \xi}, \qquad E_r - B_\varphi = \frac{\partial \Phi}{\partial \rho},$$

For a wide (in comparison with the plasma skin depth $1/k_p$) laser pulse
the equation for the potential can be linearized with respect to the small parameter $|\Phi - 1|/(k_p L_\perp)^2$

$$\left\{(\Delta_\perp - \nu_0)\frac{\partial^2}{\partial \xi^2} - \frac{\partial \ln \nu_0}{\partial \rho}\frac{\partial^3}{\partial \rho \partial \xi^2} + \nu_0 \Delta_\perp\right\}\Phi - \frac{\nu_0^2}{2}\left[1 - \frac{1 + |\boldsymbol{a}|^2}{\left(\Phi + \delta\Phi_S\right)^2}\right] = \nu_0\left[\Delta_\perp \frac{|\boldsymbol{a}|^2}{4} - \boldsymbol{N_b}(\xi, \rho, z)\right]$$

Basic Model Equations: electron beam dynamics

$$\frac{dP_z}{d\tau} = \frac{\partial}{\partial \eta}\Phi + F_z$$

$$\frac{d\mathbf{P}_r}{d\tau} = \frac{\partial}{\partial \boldsymbol{\rho}}\Phi + \mathbf{F}_r$$

$$\frac{d\eta}{d\tau} = P_z / \gamma - 1$$

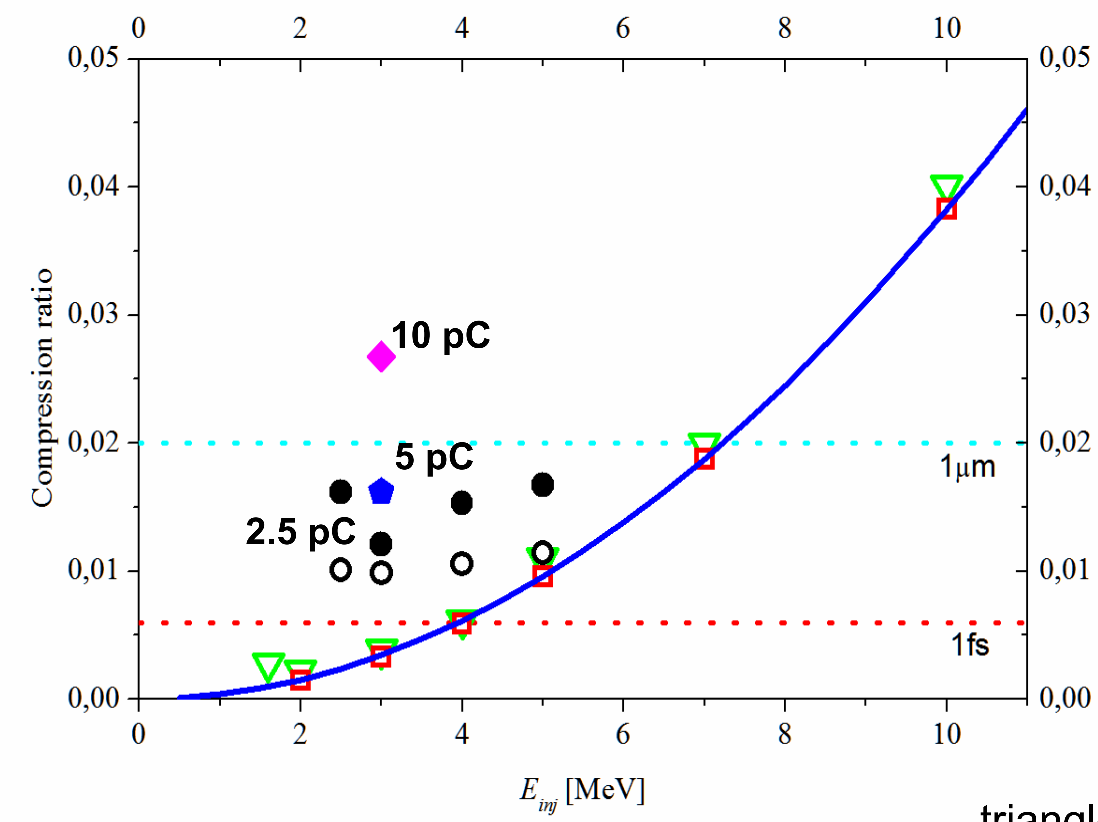
$$\frac{d\boldsymbol{\rho}}{d\tau} = \mathbf{P}_r / \gamma$$

$$F_z = -\frac{1}{\gamma}\frac{\partial}{\partial \eta}|\boldsymbol{a}|^2 / 4$$

$$\mathbf{F}_r = -\frac{1}{\gamma}\frac{\partial}{\partial \boldsymbol{\rho}}|\boldsymbol{a}|^2 / 4$$

Restrictions on the e-bunch compression

Initial emmitance and loading effect



$\gamma_{ph} = 100$

L_{b0}=47μm
R_{b0}=33μm

$\varepsilon_{n0} = 1\,\mathrm{mm}\times\mathrm{mrad}$

open symbols represent 3-D modelling results for the prescribed laser pulse

triangles and squares without loading effect, R_{b0}=2 μm; squares—without influence of ponderomotive force.

N.E. Andreev, V.E. Baranov, B. Cros, V.E. Fortov, S.V. Kuznetsov, G. Maynard, P. Mora, NIM A 653, 66–71 (2011)
N E Andreev, V.E. Baranov, B Cros, G Maynar, P Mora and M. Veysman., J. Plasma Physics (2013), vol. 79, p. 143–152

Computer simulation by the code LAPLAC

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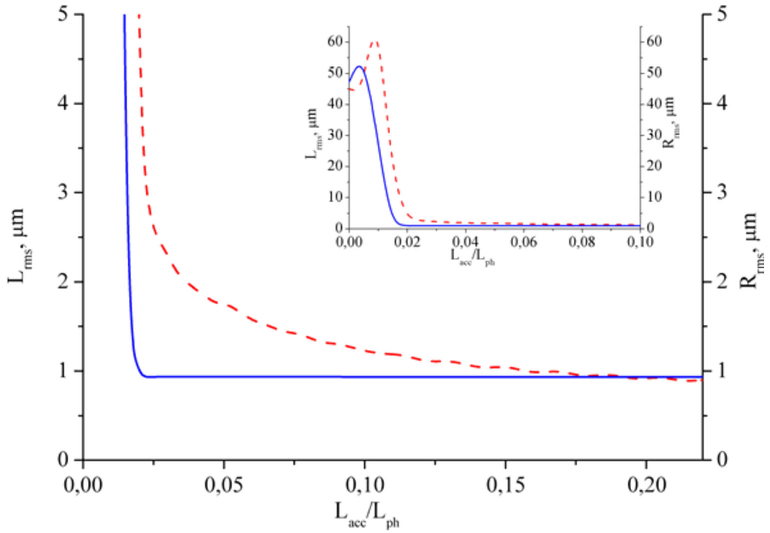
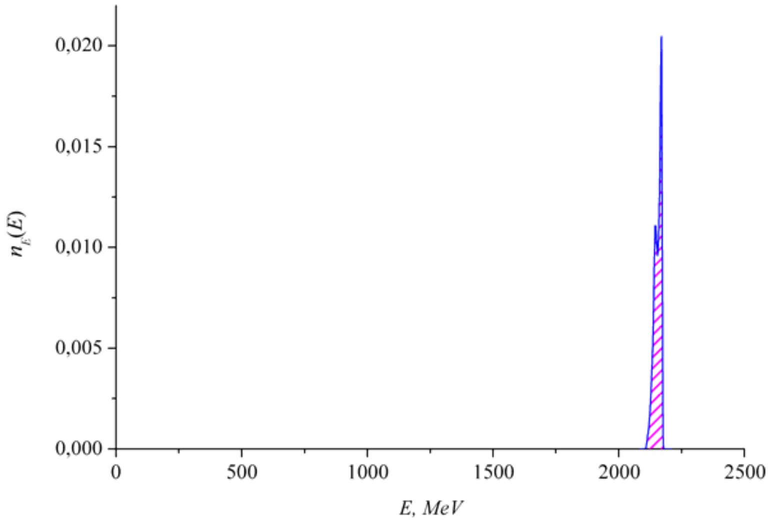
accelerated electron bunch externally injected in front of the laser pulse

the bunch has acquired an energy of **2.2 GeV** with a narrow energy spectrum and low emittance 5.4 mm × mrad





The total trapped and accelerated number of particles in the bunch is about 25% of the injected electrons = 2.5 pC

$E_{inj} = 3$ MeV $L_{b0} = 2\sigma_z = 47\,\mu\text{m}$ $r_0 = 37\,\mu\text{m}$ $I_L = 2.7\times10^{18}\,\text{W/cm}^2$ $P_L / P_{cr} = 0.35$

Q_b=10 pC R_{b0}=45μm $\tau_{\text{FWHM}} = 31$ fs   laser energy 2.25 J $n_0(0) = 1.1\times10^{17}\,\text{cm}^{-3}$

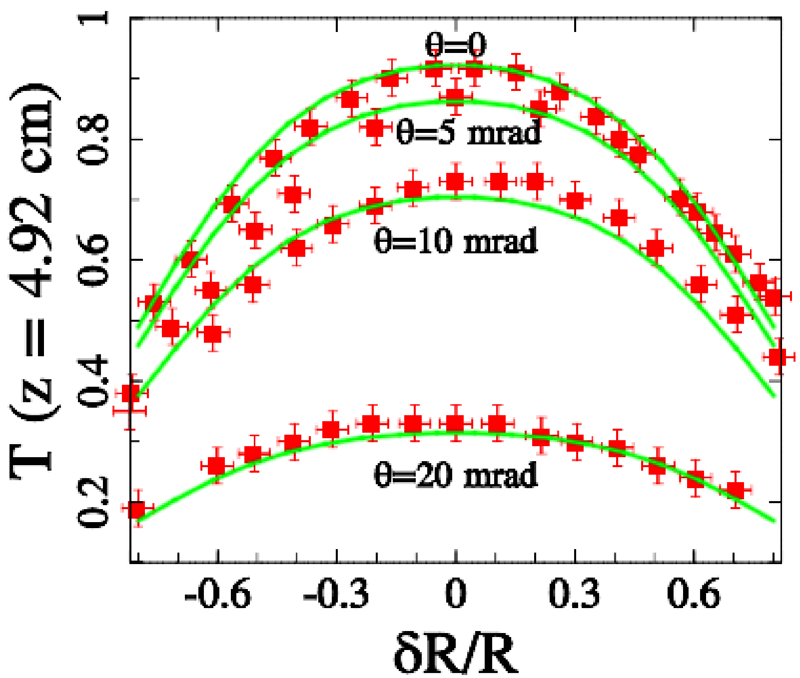
$\varepsilon_{N,r} = 4 r_{rms}\sigma_{P_r/mc} = 1\,\text{mm mrad}$

$L_b \approx R_b \approx 0.9$ μm

ΔE/E ≈ 1%

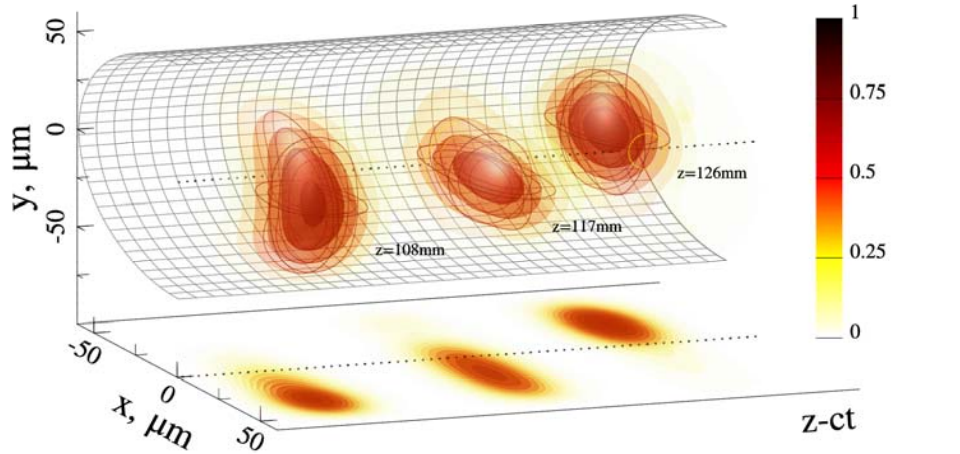
N.E. Andreev, V.E. Baranov, B. Cros, V.E. Fortov, S.V. Kuznetsov, G. Maynard, P. Mora, NIM A (2011)

Laser wakefield generation in capillary at broken symmetry



Silicon capillary, $R_{cap} = 51\mu m$,
$r_0 = 32\mu m$, $\lambda_L = 0.63\mu m$,
circular polarization

linearly-polarized laser pulse with r_0 = 40μm,
FWHM duration 135 fs, R_{cap} = 60μm
The angle between laser and capillary axis $\theta = 6\,\mathrm{mrad}$



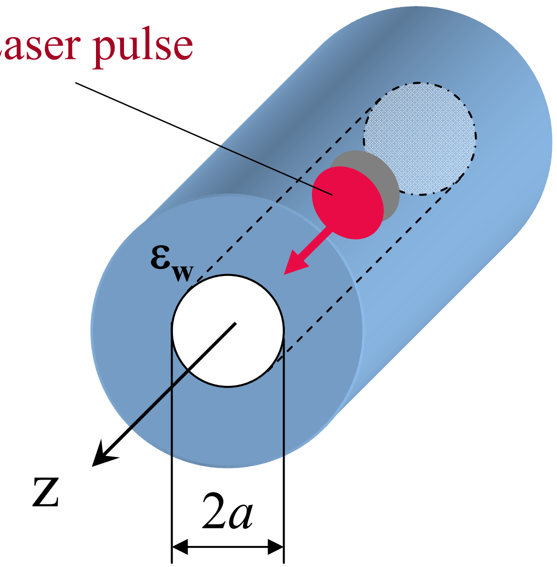
For regular wakefield generation and effective electron acceleration the angle between laser and capillary axis has to be less than 1 mrad

Veysman, N. E. Andreev, G. Maynard, and B. Cros, PHYSICAL REVIEW E 86, 066411 (2012)
M.Veysman, N. E. Andreev, K. Cassou, Y. Ayoul, G. Maynard, and B. Cros, J. Opt. Soc. Am. B, V. 27, No. 7 (2010)
N E Andreev, V.E. Baranov, B Cros, G Maynar, P Mora and M. Veysman J. Plasma Physics (2013), vol. 79, pp. 143–152

Propagation of short intense laser pulses in capillaries

➢ *equations for angular harmonics of field components*

$$\widetilde{\boldsymbol{E}}(\widetilde{\boldsymbol{B}}) = \frac{1}{2}e^{-i\omega_0 t + ik_0 z}\sum_{l=-L}^{L} e^{il\varphi}\boldsymbol{E}_l(\boldsymbol{B}_l) + \text{c.c.},$$



a) Inside capillary interior

$$\left[\Delta_\rho - \frac{l^2}{\rho^2} + 2i\frac{\partial}{\partial\zeta} + 2\frac{\partial^2}{\partial\xi\partial\zeta}\right]\begin{Bmatrix} E_z \\ B_z \end{Bmatrix}_l = 0,$$

$$2i\frac{\partial}{\partial\zeta}\begin{Bmatrix} E_\varphi \\ B_\varphi \end{Bmatrix}_l = i\begin{Bmatrix} -\partial_\rho B_z \\ \varepsilon\partial_\rho E_z \end{Bmatrix}_l - \frac{l}{\rho}\begin{Bmatrix} E_z \\ B_z \end{Bmatrix}_l,$$

b) Inside capillary walls:

$$\left[\frac{\partial^2}{\partial\rho^2} + \varepsilon - 1\right]\begin{Bmatrix} E_z \\ B_z \end{Bmatrix}_l = 0; \qquad [\varepsilon - 1]\begin{Bmatrix} E_\varphi \\ B_\varphi \end{Bmatrix}_l = i\begin{Bmatrix} -\partial_\rho B_z \\ \varepsilon\partial_\rho E_z \end{Bmatrix}_l.$$

c) Radial components:

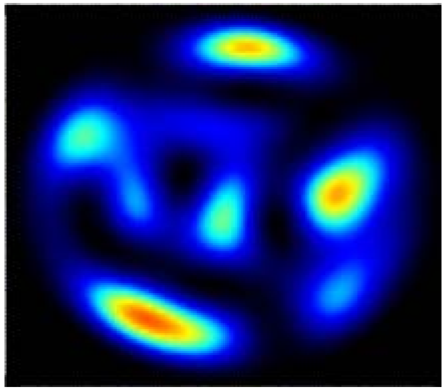
$$\begin{Bmatrix} E_r \\ B_r \end{Bmatrix}_l = \begin{Bmatrix} \varepsilon^{-1}B_\varphi \\ -E_\varphi \end{Bmatrix}_l - \frac{l}{\rho}\begin{Bmatrix} \varepsilon^{-1}B_z \\ -E_z \end{Bmatrix}_l.$$

M. Veysman, N. E. Andreev, K. Cassou, Y. Ayoul, G. Maynard,B. Cros, J. Opt. Soc. Am. B, V. 27, No. 7 (2010)

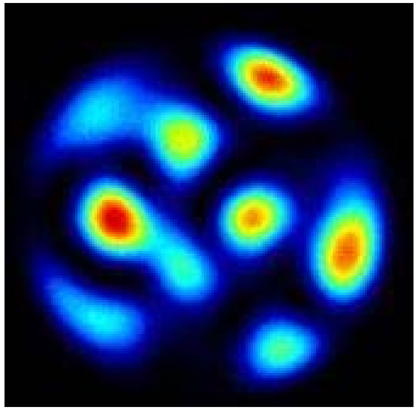
N E Andreev, V.E. Baranov, B Cros, G Maynar, P Mora and M. Veysman J. Plasma Physics (2013), vol. 79, part 2, pp. 143–152

Experimental fluence distributions confirm the modelling results

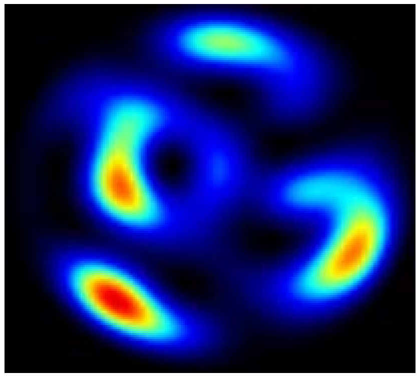
Theory, z=49.5 mm



experiment, z=49.5mm

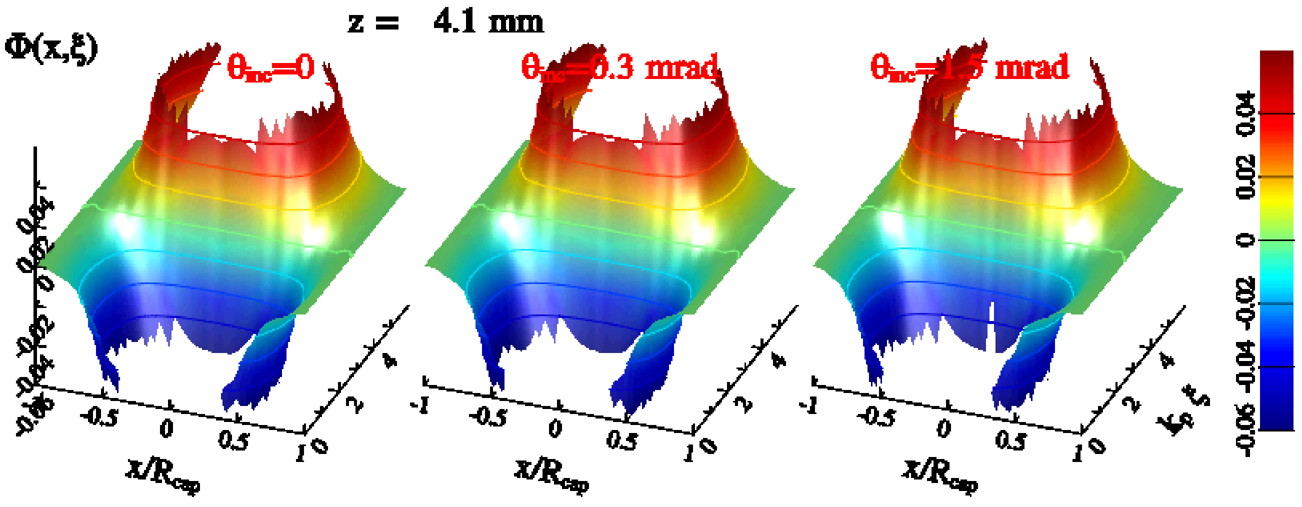


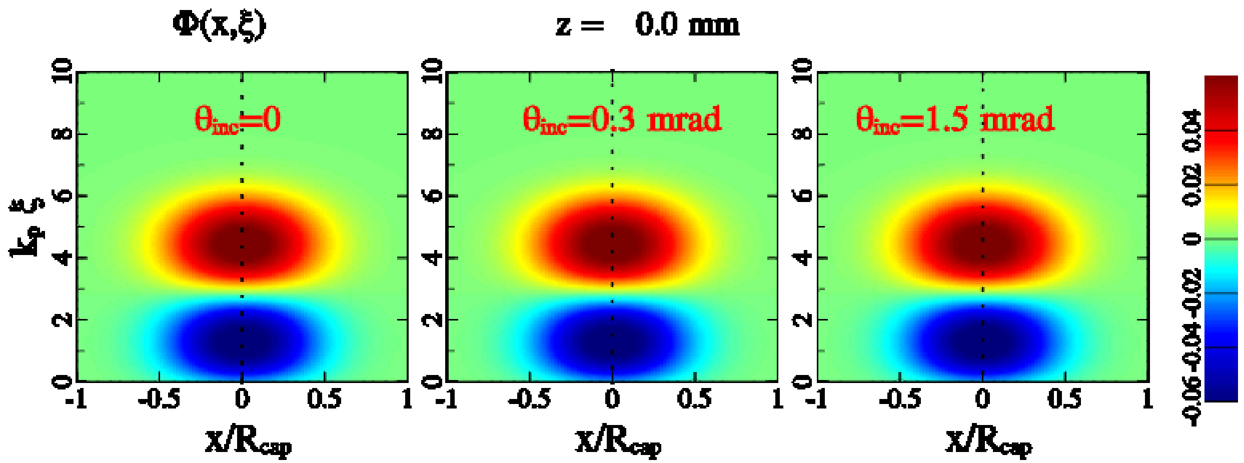
Theory, z=48.5 mm



M. Veysman, N.E. Andreev, K. Cassou, Y. Ayoul, G. Maynard, and B. Cros, JOSA B (2010)

Wakefield generation in capillary at broken symmetry





Conclusions

- The control of the wakefield phase velocity (nonlinear laser dynamics) is necessary for an effective electron bunch compression
- The transverse focusing of the bunch (lens effect), while it propagates in plasma before the laser pulse overtakes the bunch, is important for the decrease of the final bunch emittance
- The effective longitudinal bunch compression in this scheme of injection (to μm and sub-μm sizes) leads to a small relative energy spread (of order 1%) at the end of the acceleration stage
- Loading effect can be controlled and used to optimize electron bunch parameters for low energy spread

  (but it limits the bunch charge!)
- Broken symmetry of the laser pulse entrance to the waveguide will prevent regular acceleration for an angle of incidence > 1 mrad

With thanks for collaboration to

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V.E. Fortov - *Joint Institute for High Temperatures RAS, Russia*
S.V. Kuznetsov - *Joint Institute for High Temperatures RAS, Russia*
W. Leemans - *Lawrence Berkeley National Laboratory, USA*
G. Maynard - *CNRS-Universitè Paris XI, France*
P. Mora - *CPHT, CNRS- Ecole Polytechnique, France*
M.E. Veisman - *Joint Institute for High Temperatures RAS, Russia*
C-G Wahlström - *Department of Physics, Lund University, Swede*
F. Wojda - *CNRS-Universitè Paris XI, France*

Thank You

for your attention!