New exact solutions of the Dirac equation of a charged particle propagating in a strong laser field in an underdense plasma

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1. Introduction, Volkov states, Mathieu charts in medium

2. New exact solutions of the Dirac (K-G) equation

3. Numbers, Numerical examples (Dirac, Klein-Gordon)

[4. Relevance in mechanisms of acceleration in plasma]

[1] S. V. , New exact solutions of the Dirac equation of a charged particle interacting with an electromagnetic plane wave in a medium. To be published. E-print: arxiv:1305.4370.

[2] S. V., A new class of exact solutions of the Klein-Gordon equation of a charged particle interacting with an electromagnetic plane wave in a medium. Unpublished.

Volkov states [1935]

$$[\gamma_\mu (i\partial - \varepsilon A_{rad})^\mu - \kappa]\left|\Psi\right\rangle = \varepsilon\gamma_0 V\left|\Psi\right\rangle$$

$$A_{rad}(\xi) = A_0 f(\xi)$$

$$\xi = k_\mu x^\mu = \omega(t - z/c)$$

$$\Psi_p(x) = \left[1 \pm \frac{\varepsilon(\gamma\cdot k)[\gamma\cdot A(\eta)]}{2k\cdot p}\right] u_p^{(\pm)}$$
$$\times \exp\left[\mp i[p\cdot x + \int J_p^{(\pm)}(\xi)d\xi]\right]$$

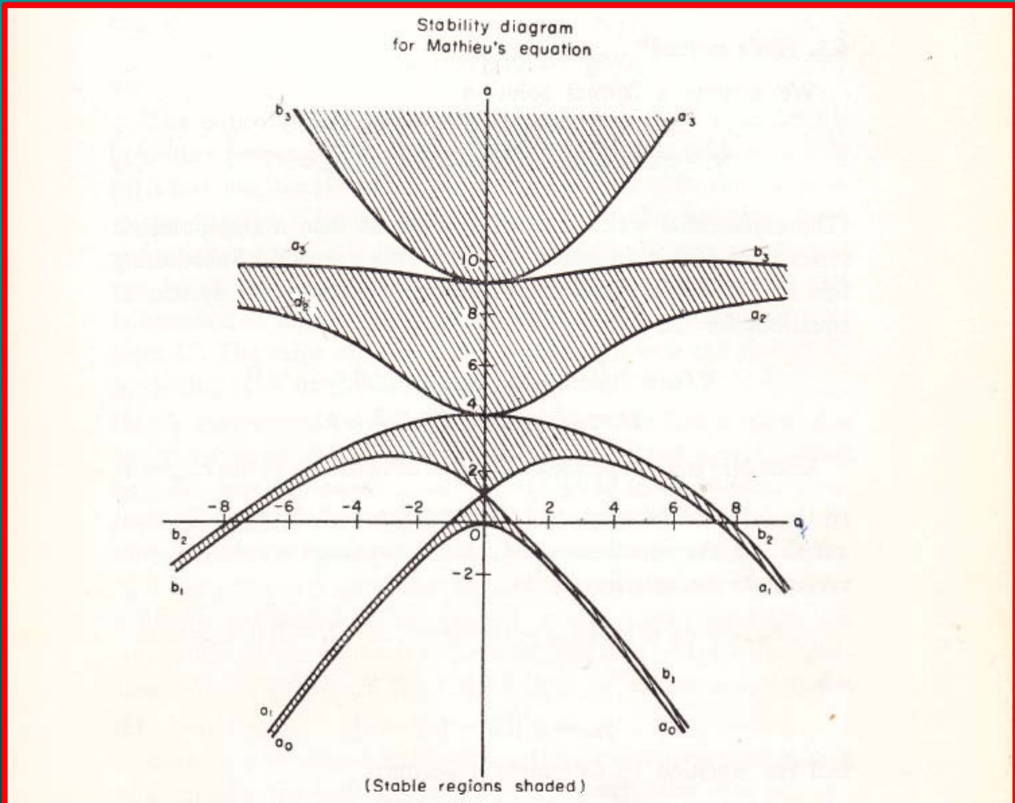
$$J_p^{(\pm)}(\xi) = (1/2k\cdot p)[\pm 2\varepsilon p\cdot A(\xi) - \varepsilon^2 A^2(\xi)$$

Wolkow D M, Über eine Klasse von Lösungen der Diracschen Gleichung. *Zeitschrift für Physik* 94, 250-260 (1935). [Application to strong-field: ~1960..]

Mathieu–type solutions $\xi = k_\mu x^\mu = \omega(t - n_m y / c)$

$$w'' + (\lambda - 2h^2 \cos 2z)w = 0$$

$$\lambda \propto k \cdot p, \quad h^2 \propto eA_0 p_\perp$$



[Figure taken from Arscott F M, ***Periodic differential equations*** (Pergamon Press, Oxford, 1964) p.123.] E.g. Nikishov & Ritus (1967), Nikishov (1970), Narozhny & Nikishov (1974), Becker (1977), ... Free Electron Laser theor.

New exact solutions

$$\xi = k_\mu x^\mu = \omega(t - n_m y / c)$$

$$[(i\partial - \varepsilon A)^2 - \kappa^2 - \tfrac{1}{2}\varepsilon\sigma \cdot F]\Psi = 0$$

$$\Psi_p^{(\pm)} = \sum_{s=1}^{4} \Psi_{ps}^{(\pm)} u_s$$

$$\Psi_{p1,2}^{(e)} = e^{i\overline{p}\cdot\overline{x}} e^{-(a/4)\cos\xi} g_n^k(\xi, a \,|\, +)$$

$$g'' + a\sin 2z(g' + ig) + (\eta - qa\cos 2z)g = 0$$

$$2p_x = (q+1)k_p$$

$$2(k \cdot p)^2 / k_p^4 = \eta_n^{(k)}$$

$$p_x = (n + \tfrac{1}{2})k_p$$

$$k_p \equiv k_0\sqrt{1 - n_m^2} = \omega_p / c$$

$$g_n^k(\xi, a \,|\, +) = \sum_{r=-n+1}^{n} D_r^{(k)}(a \,|\, 2n)\exp(-ir\xi)$$

New exact solutions

$$\xi = k_\mu x^\mu = \omega(t - n_m y / c)$$

$$p_x = n k_p$$

$$p_x = (n + \tfrac{1}{2}) k_p$$

$$2(k \cdot p)^2 / k_p^4 = \eta_n^{(k)}$$

$$a = 4 \frac{eF_0 \lambda_p}{\hbar \omega_0} = 2\mu_0 \frac{2mc^2}{\hbar \omega_p}$$

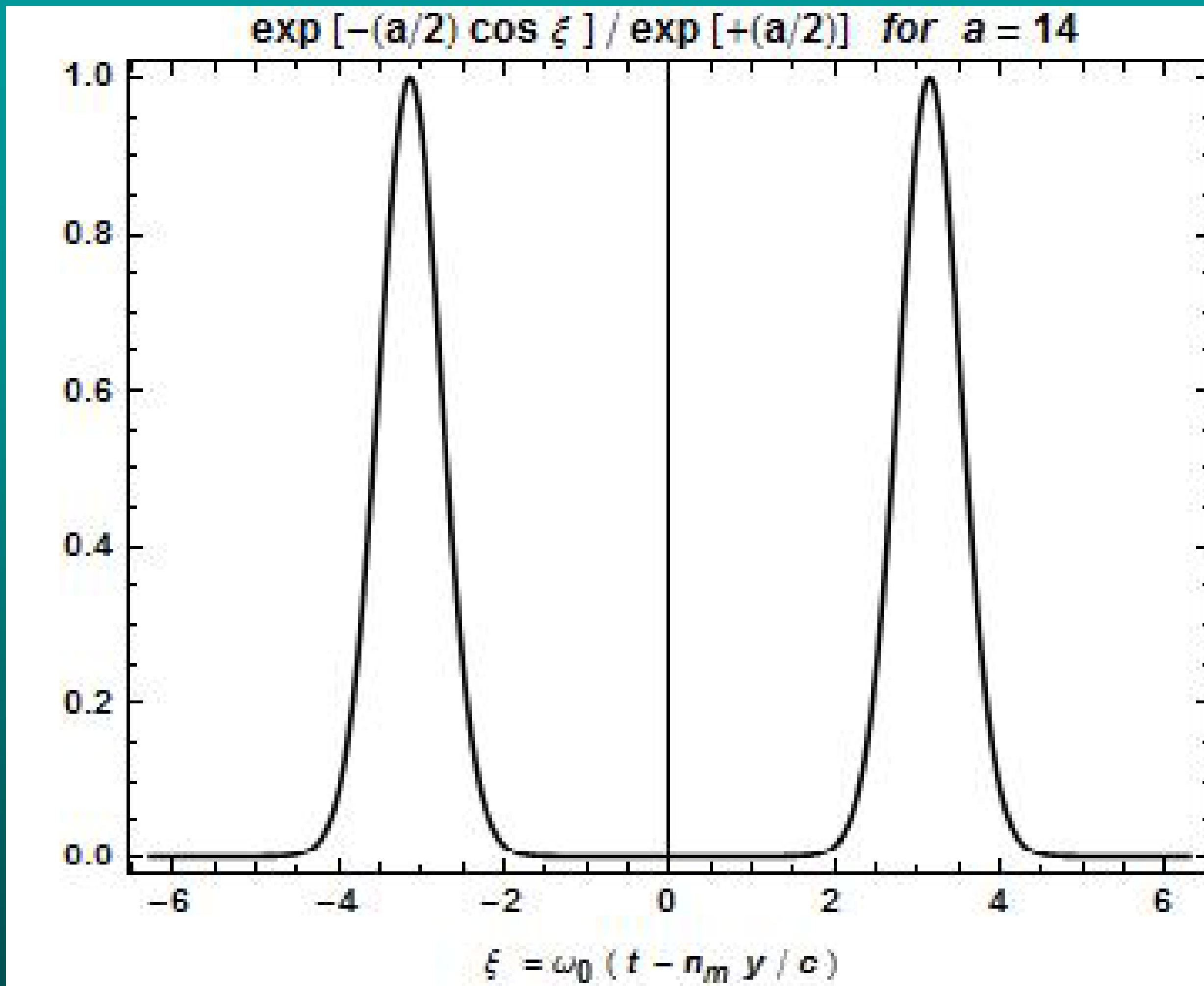
$$\mu_0 = \frac{eF_0}{mc\,\omega_0} = 8.5 \times 10^{-10} \sqrt{I_0}\,\lambda_0$$

$$k_p \equiv k_0 \sqrt{1 - n_m^2} \equiv \omega_p / c$$

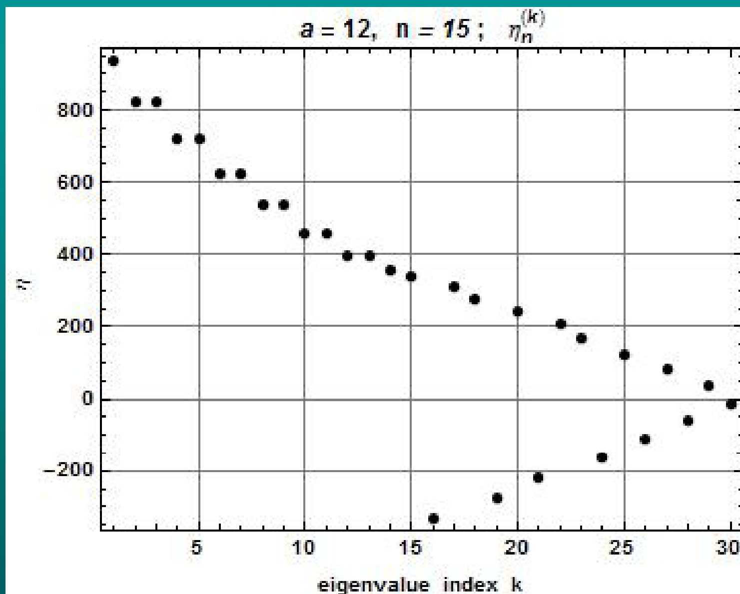
$$n_m^2(\omega_0) = 1 - \omega_p^2 / \omega_0^2$$

$$\omega = \sqrt{\omega_p^2 + (ck_y)^2}$$

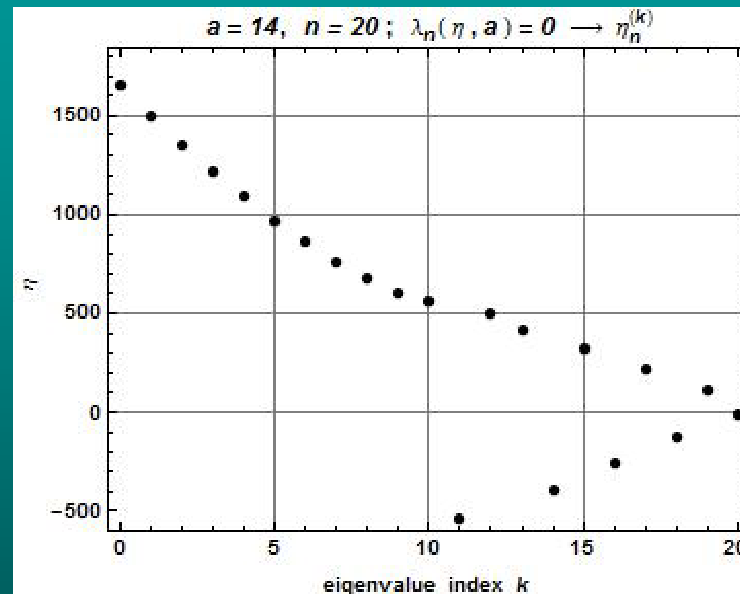
(For optical frequencies) the new parameter " a " is 6 orders of magnitude larger than the usual intensity parameter ('scaled vector potential')



'Void regions' in the centre of the cycle

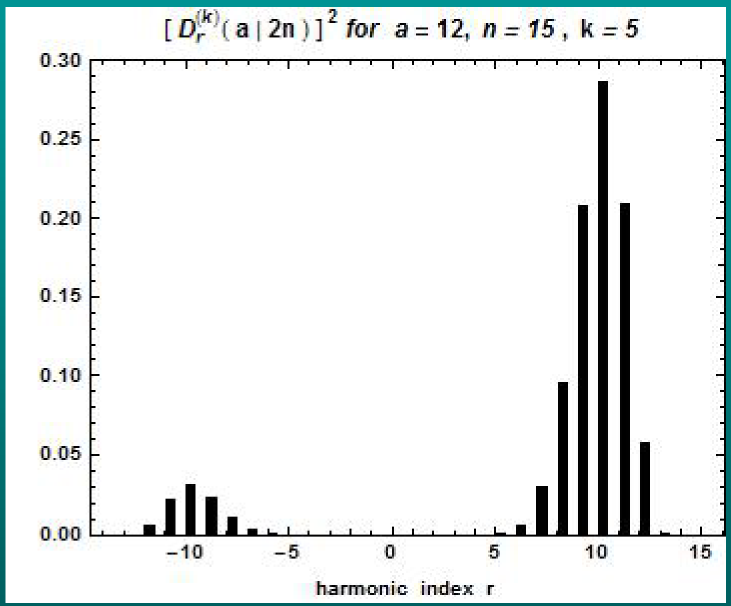


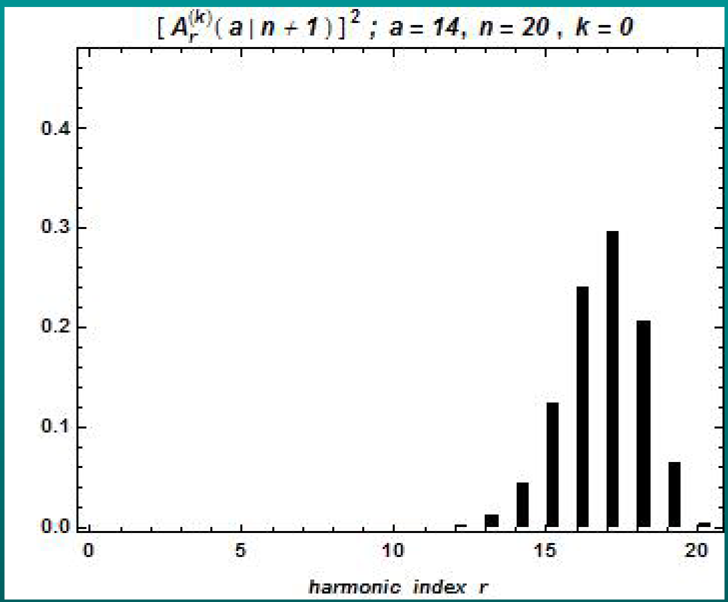
Dirac



Klein-Gordon

'Hyperfine splitting' of the longitudinal momentum spectrum

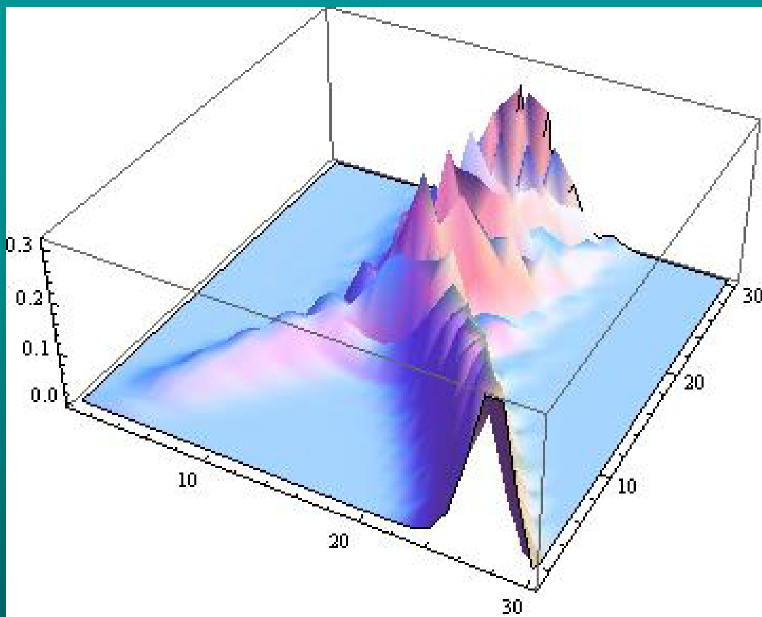


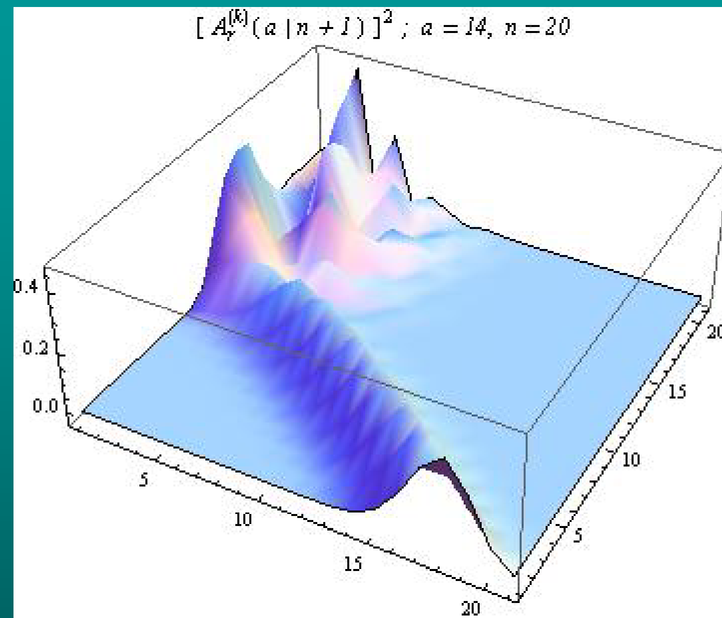


Dirac

Klein-Gordon

Double peak structure. Single peak structure. Oscillatory spectrum.





Dirac

Klein-Gordon

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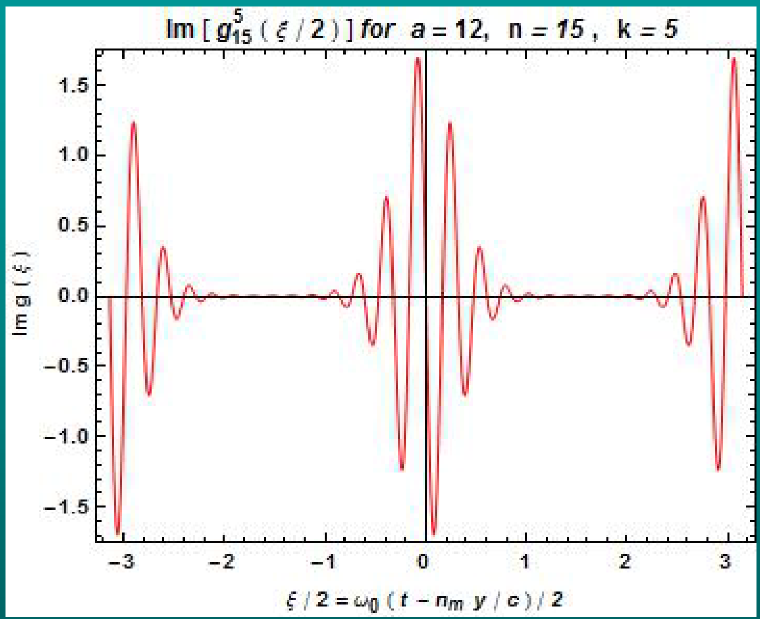
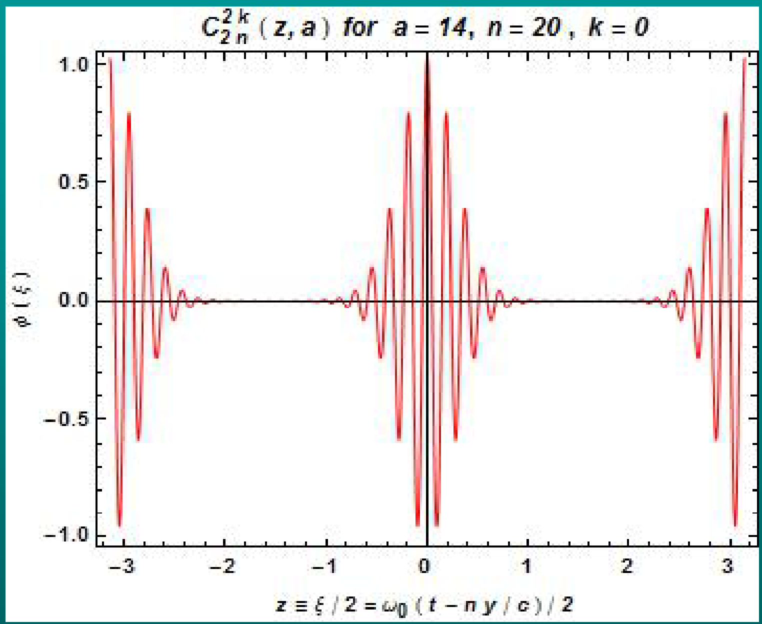
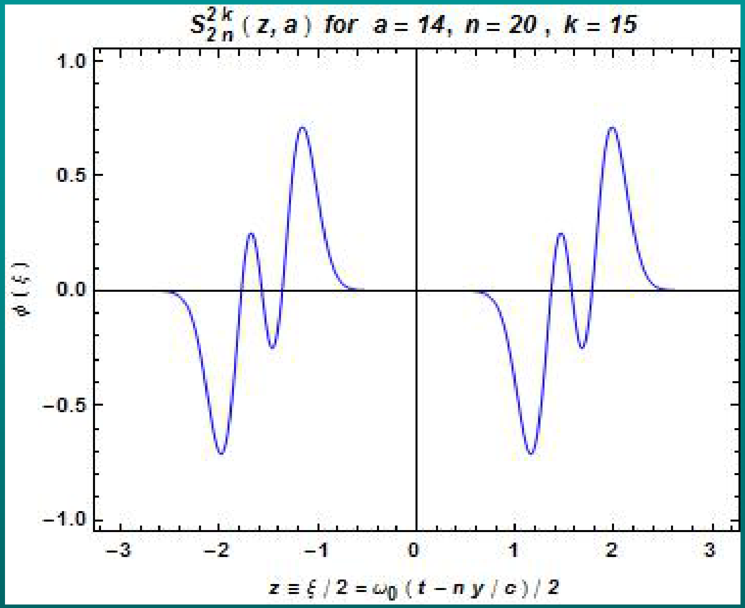


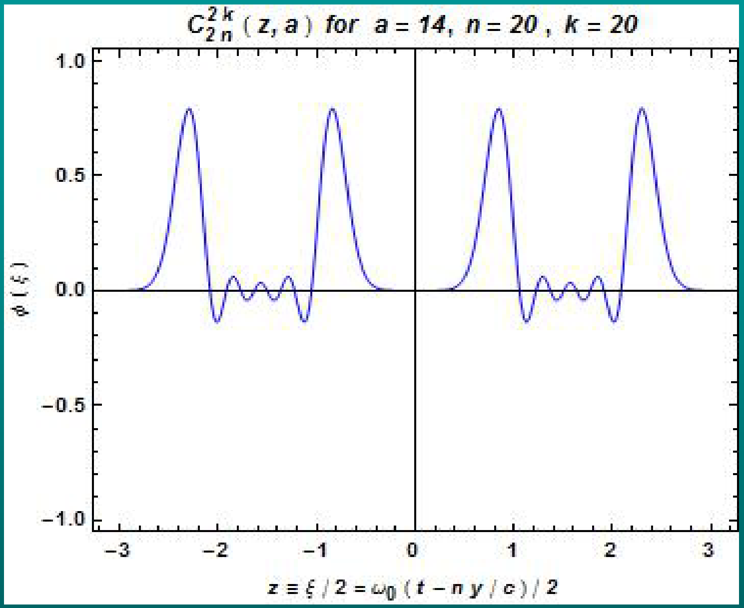


Dirac

Klein-Gordon

Wave functions





Klein-Gordon. K = 15.

Klein-Gordon. k = 20.

Wave functions with negative eigenvalues (imaginary longitudinal momentum)

Summary

1. Discrete transverse and longitudinal momenta

2. Basically–periodic motion, ‘void regions’

3. Large new parameter

$$a = 4\frac{eF_0\lambda_p}{\hbar\omega_0} = 2\mu_0\,\frac{2mc^2}{\hbar\omega_p}$$

4. Half-integer harmonics

Acknowledgment.
This work has been supported by the Hungarian Scientific Research Foundation OTKA, Grant No. K 104260.

Appendices

Large electric field strengths

$$[F_0 /(V / cm)] = 27.46 \times [I_0 /(W / cm^2)]^{1/2}$$

'LASER ACCELERATION OF PARTICLES' [Los Alamos, NM, USA, 1982]

AIP Conference Proceedings
Series Editor: Hugh C. Wolfe
Number 91

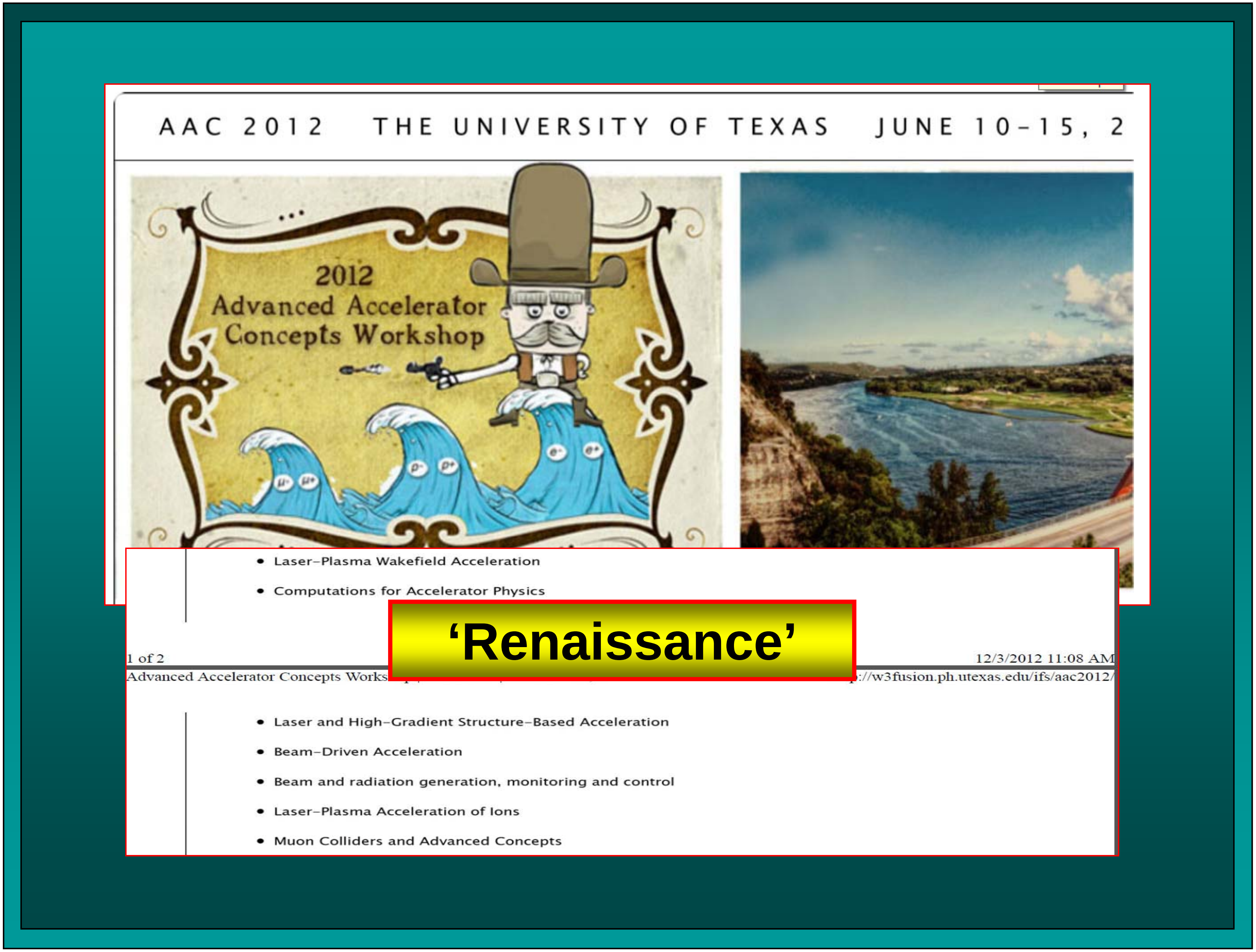
Laser Acceleration of Particles
(Los Alamos, 1982)

Edited by
Paul J. Channell
Los Alamos National Laboratory

...eration schemes.

Some mechanisms of acceleration viewed as potential candidates high-gradient accelerators are the following (not in order of ...ority):

1. Grating Accelerator - a special case of a conventional accelerating waveguide in which the waveguide structure is open sided, for example, a diffraction grating in the presence of a strong optical field.
2. High Gradient Structures and Power Sources at Wavelengths Near 1 cm - a scaledown of conventional waveguide structures to millimeter size with emphasis on avoidance of breakdown problems.
3. Inverse Cherenkov Accelerator - relies on index of refraction of background medium to provide phase matching between optical field and electron. A novel geometry has been proposed to alleviate gas-breakdown limitations.
4. Beat-Wave Accelerator - acceleration is achieved by the fields produced as a result of charge separation generated by a traveling plasma wave. The plasma wave, in turn, is set up by a laser pulse whose amplitude is modulated so that the beat frequency matches the plasma frequency.
5. Inverse Free Electron Laser Accelerator - uses the energy exchange occurring when an undulating electron in a periodic magnetic field has a transverse velocity in phase with the transverse E-field of an intense optical beam.
6. Two-Wave Accelerator - similar to (5) except that the magnetic wiggler is replaced by a slow microwave field whose phase velocity relative to the electron beam may be adjusted by changing the relative angle.


AAC 2012 THE UNIVERSITY OF TEXAS JUNE 10–15, 2
2012
Advanced Accelerator
Concepts Workshop
• Laser–Plasma Wakefield Acceleration
• Computations for Accelerator Physics
‘Renaissance’
1 of 2
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Advanced Accelerator Concepts Works
://w3fusion.ph.utexas.edu/ifs/aac2012/
• Laser and High–Gradient Structure–Based Acceleration
• Beam–Driven Acceleration
• Beam and radiation generation, monitoring and control
• Laser–Plasma Acceleration of Ions
• Muon Colliders and Advanced Concepts

PWFA Proposal

J. Plasma Physics (2012), *vol.* 78, *part* 4, *pp.* 347–353.
doi:10.1017/S0022377812000086

A proposed demonstration of an experiment of proton-driven plasma wakefield acceleration based on CERN SPS

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Abstract. The proton bunch-driven plasma wakefield acceleration (PWFA) has been proposed as an approach to accelerate an electron beam to the TeV energy regime in a single plasma section. An experimental program has been recently proposed to demonstrate the capability of proton-driven PWFA by using existing proton beams from the European Organization for Nuclear Research (CERN) accelerator complex. At present, a spare Super Proton Synchrotron (SPS) tunnel, having a length of 600 m, could be used for this purpose. The layout of the experiment is introduced. Particle-in-cell simulation results based on realistic SPS beam parameters are presented. Simulations show that working in a self-modulation regime, the wakefield driven by an SPS beam can accelerate an externally injected $\sim$10 MeV electrons to $\sim$2 GeV in a 10-m plasma, with a plasma density of $7 \times 10^{14}\,\mathrm{cm}^{-3}$.

Rubidium [instead of Lithium] for plasma generation by photoionization

Photo-Ionized Lithium Source for Plasma Accelerator Applications

P. Muggli, K. A. Marsh, S. Wang, C. E. Clayton, S. Lee, T. C. Katsouleas, and C. Joshi

***Abstract*— A photo-ionized lithium source is developed for plasma acceleration applications. A homogeneous column of lithium neutral vapor with a density of 2×10^{15} cm^{-3} is confined by helium gas in a heat-pipe oven. A UV laser pulse ionizes the vapor. In this device, the length of the neutral vapor and plasma column is 25 cm. The plasma density was measured by laser interferometry in the visible on the lithium neutrals and by CO_2 laser interferometry on the plasma electrons. The maximum measured plasma density was 2.9×10^{14} cm^{-3}, limited by the available UV fluence (≈83 mJ/cm^2), corresponding to a 15% ionization fraction. After ionization, the plasma density decreases by a factor of two in about 12 μs. These results show that such a plasma source is scaleable to lengths of the order of 1 m and should satisfy all the requirements for demonstrating the acceleration of electrons by 1 GeV in a 1-GeV/m amplitude plasma wake.**

***Index Terms*—Alkali plasma, electron acceleration, heat-pipe oven, plasma accelerator, wakefield.**

times the accelerating length

$$W[\text{MeV}] = 96\varepsilon L[\text{cm}]\sqrt{n_e[\text{cm}^{-3}]/10^{16}} \qquad (1)$$

where $\varepsilon = \delta n_e/n_e$ is the wake amplitude ($\varepsilon \leq 1$, typically $0.3 < \varepsilon < 0.5$), and L is the shorter of the length over which the wake is excited or the dephasing length, i.e., the length in the laboratory frame in which the accelerating electrons slips by 90° in phase with respect to the plasma wave. Future experiments will aim at demonstrating energy gains of the order of 1 GeV. This can be achieved in an $n_e \approx 10^{18}$ cm^{-3} plasma with a length $L \approx 1$ cm, in an $n_e \approx 10^{16}$ cm^{-3} plasma with $L \approx 10$ cm, or in an $n_e \approx 10^{14}$ cm^{-3} plasma with a length $L \approx 1$ m. Although in the latter case the gradient is not as high as already achieved in other plasma-based acceleration experiments, the 1-GeV/m gradient is much larger than that

PWFA Proposal

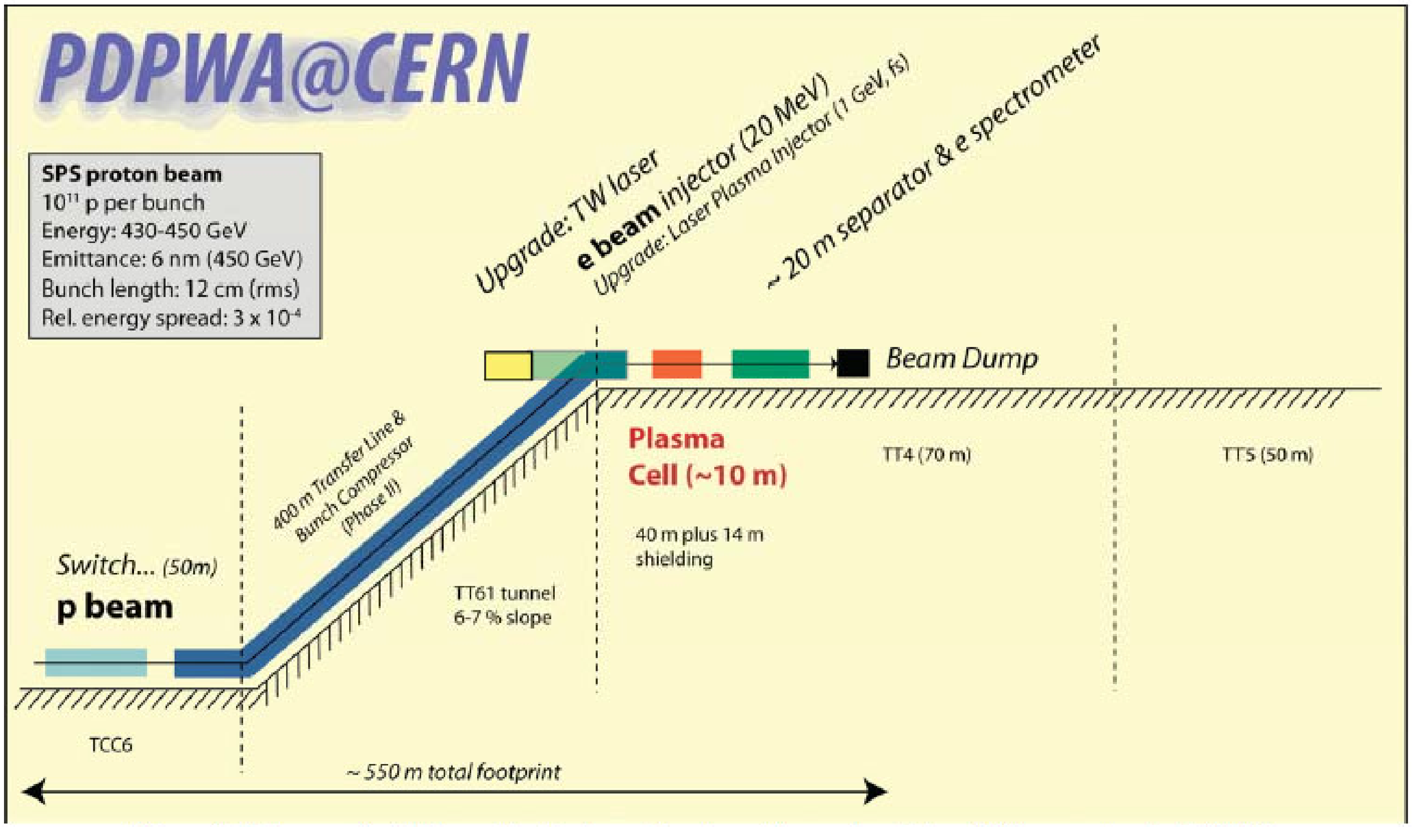

Figure 2. (Colour online) Schematic of a beam-line layout for proton-driven PWA experiment at CERN.

Diffuse scattering, Radiation diffusion

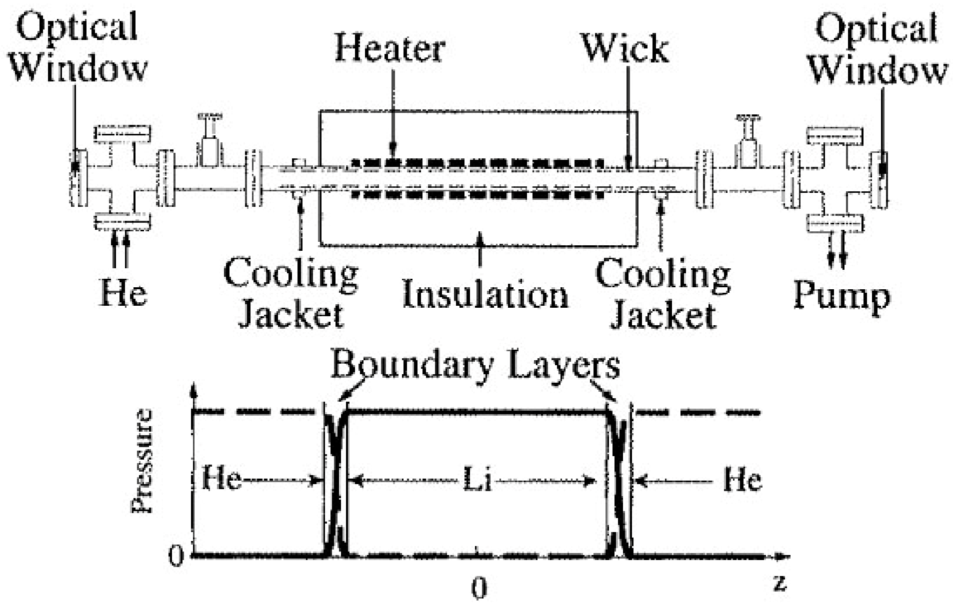

Fig. 3. Schematic of the heat-pipe oven (not to scale) and expected pressure profiles for the buffer gas (He) and the Li vapor (assuming no significant flow in the oven). The length of the Li column and width of the boundary layers are arbitrary on this figure.

P. Muggli, K. A. Marsh, S. Wang, C. E. Clayton, S. Lee, T. C. Katsuleas and C. Joshi, Photo-ionized Lithium source for plasma accelerator applications. IEEE Transactions on Plasma Science, Vol. 27, No. 3, June 1999. pp 791-799.

Excitation by diffuse light; homogenity in ‘equilibrium’