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RADIATIVE LOSSES IN PLASMA-BASED ELECTRON ACCELERATORS IN ULTRAHIGH ENERGY LIMIT

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EAAC2013, 2-7 June 2013, La Biodola, Isola d'Elba, Italy

OUTLINE

- ✓ **INTRODUCTION**
- ✓ **EQUATION OF MOTION**
- ✓ **EQUATION ANALYSIS**
- ✓ **ASYMPTOTIC ACCELERATION**
- ✓ **SUMMARY AND DISCUSSION**

BETATRON OSCILLATIONS AND RADIATIVE LOSSES

$F_a = fmc\omega_p$ - accelerating force

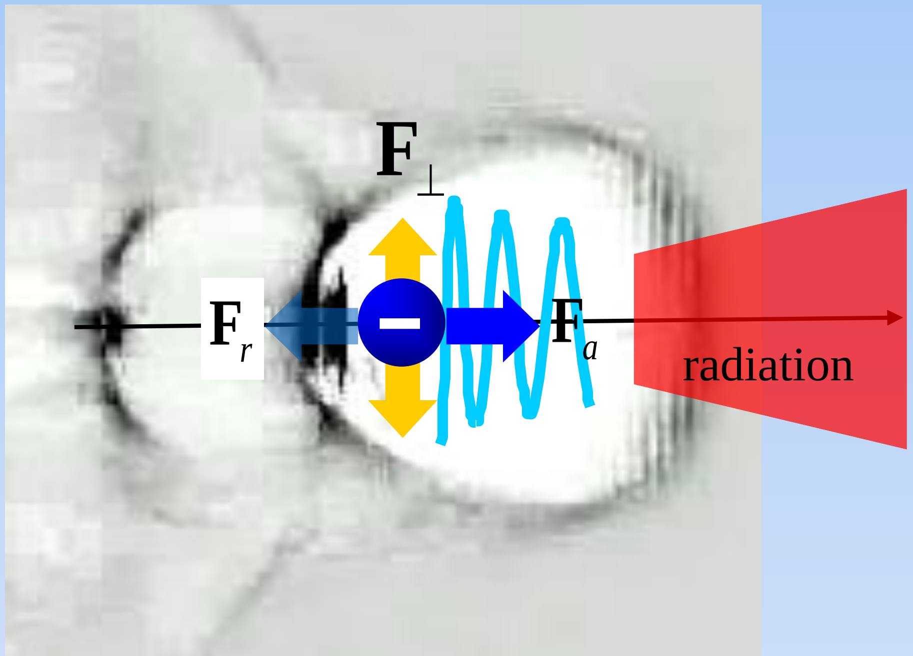
$F_\perp \approx -m\kappa^2\omega_p^2 r$ - focusing force

$F_r \approx P_{rad}/c$ - radiation damping or radiation friction force

$P_{rad} \approx 2r_e\gamma^2 F_\perp^2/(3mc)$ - radiated power

$$F_r = F_a \quad \Rightarrow \quad \boxed{\gamma_{th}^2 \approx f/(\varepsilon\kappa^4 R_\beta^2)}$$

$R_\beta = \omega_p r/c \qquad \varepsilon = 2r_e\omega_p/(3c)$



Threshold energy is about **100 GeV** for $f=0.7$, $n=10^{19}\text{cm}^{-3}$, $R_\beta=c/\omega_p$, $\kappa^2 = 0.11$.

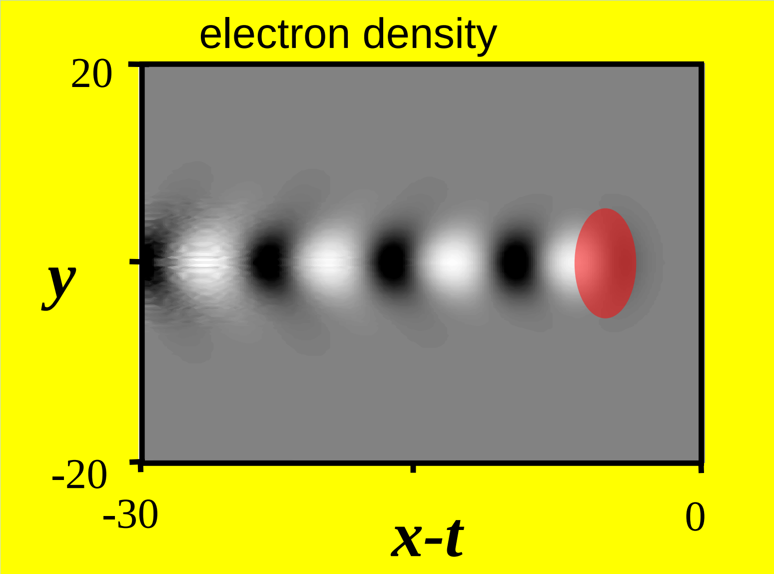
Radiative damping may be a serious limitation of electron acceleration in the high-energy regime. However, the self-consistent treatment is needed to study electron dynamics more accurately since the betatron oscillation amplitude determining radiation damping may evolve significantly during acceleration.

PLASMA-BASED LINEAR COLLIDERS

- The low density plasma is probably more profitable for high energy acceleration because of large dephasing length.
- The quasilinear regime has the advantage of symmetric accelerating properties for electrons and positrons.
- For Gaussian laser pulse, the optimal condition is $\omega_p T_L \approx 1$, $a_0 \approx \sqrt{2}$, so that

$$E_{\max} \approx 0.35 a_0^2 \frac{mc\omega_p}{e}, \quad f \approx 0.7, \quad \kappa \approx 0.11,$$

$$F_\perp \approx -m\kappa^2 \omega_p^2 r \qquad F_a = fmc\omega_p$$



K. Nakajima *et al.*, Phys. Rev. STAB **14**, 091301 (2011).
C. B. Schroeder *et al.*, Phys. Rev. STAB **13** 101301 (2010).

EQUATION OF MOTION

We start from the relativistic equation for electron motion in an electromagnetic field with the radiation reaction force in Landau-Lifshitz form

radiation reaction 4-force

$$\gamma \frac{du^i}{dt} = \frac{c r_e}{e} F^{ik} u_k + \frac{2 r_e^2}{3mc}\left(F_1^i + F_2^i + F_3^i\right),$$

$$F_1^i = \left(e/r_e\right)\frac{\partial F^{ik}}{\partial x^l} u_k u^l, \quad F_2^i = -F^{il} F_{kl} u^k, \quad F_3^i = \left(F_{kl} u^l\right)\left(F^{km} u_m\right)$$

Lorentz 4-force

The equation is derived under the assumption that the absolute value of the first term is larger than that of the second term. However, some spatial components of radiation reaction force can be larger than that of the Lorentz force. Therefore the radiative damping may dominate over acceleration.

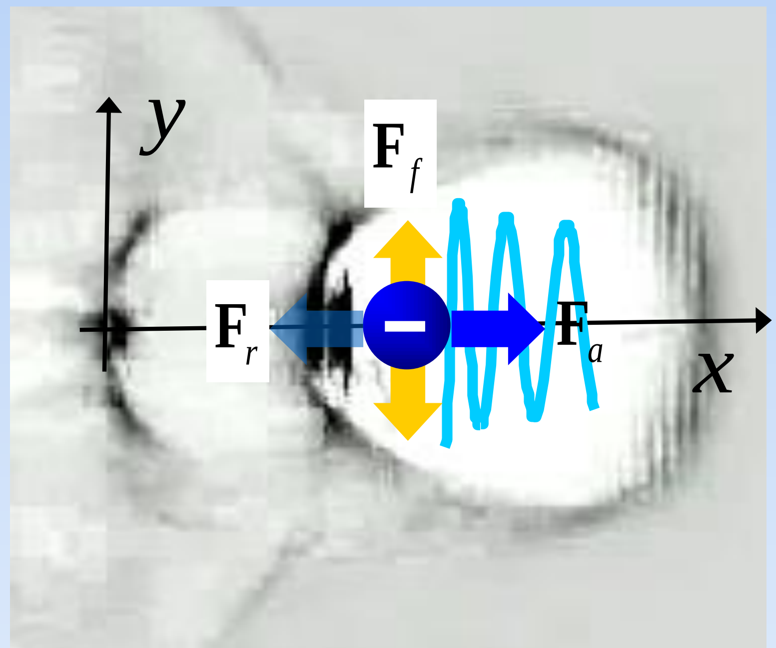
APPROXIMATIONS AND REDUCED EQUATIONS

✓ **APPROXIMATIONS**

$$\gamma >> 1, \quad F_{acc} = \text{const} = fmc\omega_p >> \frac{v_\perp}{c} F_\perp = F_\perp\left(m\kappa^2\omega_p^2 y\right) \quad \Rightarrow \quad F_3 >> F_1, F_2$$

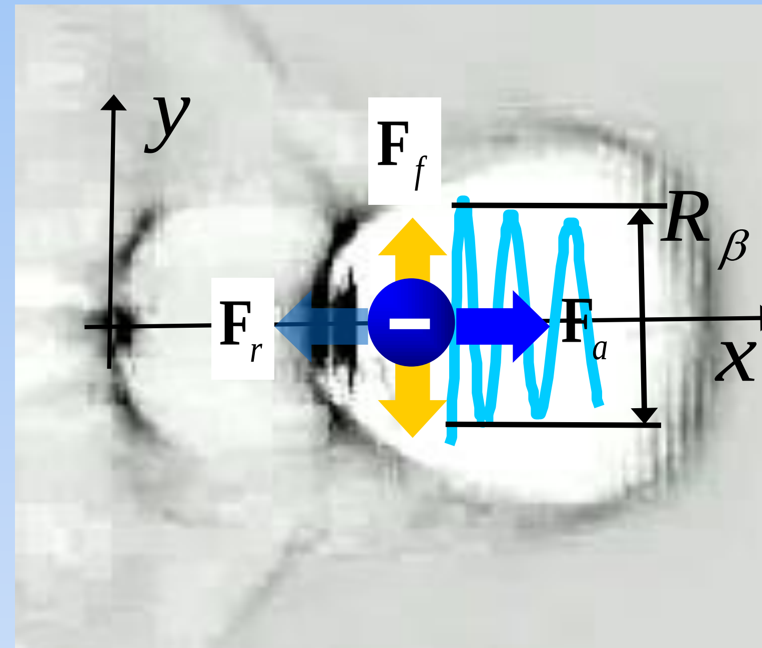
✓ **REDUCED EQUATIONS**

$$\begin{cases} \dfrac{dp_y}{dt} = -m\kappa^2\omega_p^2 y - \dfrac{2r_e}{3c}\dfrac{\kappa^4\omega_p^4 y^2}{c^2} p_y\gamma, \\ \dfrac{dy}{dt} = \dfrac{p_y}{m\gamma}, \\ \dfrac{d\gamma}{dt} = f\omega_p - \dfrac{2r_e}{3c}\dfrac{\kappa^4\omega_p^4 y^2}{c^2}\gamma^2 \end{cases}$$



EQUATION ANALYSIS

$$\begin{cases} \dfrac{dp_y}{dt} = -m\kappa^2\omega_p^2 y - \dfrac{2r_e}{3c}\dfrac{\kappa^4\omega_p^4 y^2}{c^2} p_y\gamma, \\ \dfrac{dy}{dt} = \dfrac{p_y}{m\gamma}, \\ \dfrac{d\gamma}{dt} = f\omega_p - \dfrac{2r_e}{3c}\dfrac{\kappa^4\omega_p^4 y^2}{c^2}\gamma^2 \end{cases}$$



When the force of radiative friction is disregarded, the first two equations are equivalent to the equation of linear oscillator with a slowly varying frequency.

$$y \approx C\sqrt{\omega_\beta(t)}\sin\left(\int \omega_\beta dt\right), \quad p_y \approx \frac{Cm\kappa^2\omega_p^2}{\sqrt{\omega_\beta(t)}}\cos\left(\int \omega_\beta dt\right),$$

$$\gamma \approx \gamma_0 + f\omega_p t \qquad \omega_\beta(t) \propto \gamma^{-1/2}$$

EQUATION ANALYSIS

It is convenient to introduce new variables as follows:

$$P = \frac{p_y}{mc}\varepsilon^{1/2} f^{1/2}, \quad Y = yk_p f^{3/2}\varepsilon^{1/2}, \quad T = \frac{\omega_p t\kappa^2}{f}, \quad G = \gamma\frac{\kappa^2}{f^2}$$

When the number of betatron oscillations is large, we can use the averaging method.

To do this let us introduce a new variable, $U\exp\left(i\int G^{-1/2}dT\right) = Y - iG^{-1/2}P$,

where $2S = |U|^2 = Y^2 + P^2/G = R_\beta^2 f^3\varepsilon \approx 2\langle Y^2\rangle$

Averaging over the fast time related to the betatron oscillations yields the averaged equations:

$$\begin{cases} \dfrac{dS}{dT} = -\dfrac{1}{2}\dfrac{S}{G} - \dfrac{1}{4}GS^2, \\ \dfrac{dG}{dT} = 1 - SG^2 \end{cases}$$

EQUATION ANALYSIS

1) At the absence of the accelerating force (*f = 0*)

$$\gamma = \gamma_0 \left(1 + \frac{5\,\varepsilon\, R_{\beta,0}^2\, \gamma_0}{16} \omega_p t \right)^{-4/5},$$

I.Yu. Kostyukov, E.N. Nerush, and A.M. Pukhov, JETP 103, 800 (2006).

2) At the absence of the radiation reaction

$$y \approx C\sqrt{\omega_\beta(t)}\,\sin\left(\int \omega_\beta dt\right), \quad p_y \approx \frac{C m \kappa^2 \omega_p^2}{\sqrt{\omega_\beta(t)}} \cos\left(\int \omega_\beta dt\right), \quad \gamma \approx \gamma_0 + f\omega_p t$$

3) If the radiation reaction force is much weaker than the accelerating one, then to the first order in the radiation reaction force the normalized electron energy is

$$G = G_0 + T - \frac{2}{5}\left[1 - (G_0 + T)^{5/2}\right],$$

P. Michel, C. B. Schroeder, B. A. Shadwick, E. Esarey, W. P. Leemans, Physical Review E 74, 026501 (2006).

EQUATION ANALYSIS

$$\frac{dS}{dT} = -\frac{1}{2}\frac{S}{G} - \frac{1}{4}GS^2, \quad \frac{dG}{dT} = 1 - SG^2$$

CONSERVATION OF INTEGRAL

$$I = \frac{1 - 3SG^2/2}{S^{9/4}\left(SG^2\right)^{3/4}} = \text{const}.$$

NEW VARIABLES

$$G_{tr} = T_{tr} = I^{2/9}, \quad S_{tr} = I^{-1/9} \qquad \tau = T/T_{tr}, \qquad g = G/G_{tr}, \qquad s = \left(S/S_{tr}\right)\left(G/G_{tr}\right)^{-1/4}$$

EXACT SOLUTION

$$\frac{ds}{d\tau} = -\frac{3}{4}s^{13/9}\left(s^2 + \frac{3}{2}\right)^{4/9}$$

$$\varphi(s) - \varphi(s_0) = -\tau, \quad \varphi(x) = 2^{4/9}\left(3 + 2s^2\right)^{5/9} s^{-4/9} - 2^{13/9}3^{5/9}s^{14/9}\,{}_2F_1\left(\frac{7}{9}, \frac{4}{9}; \frac{16}{9}; -\frac{2s^2}{3}\right)$$

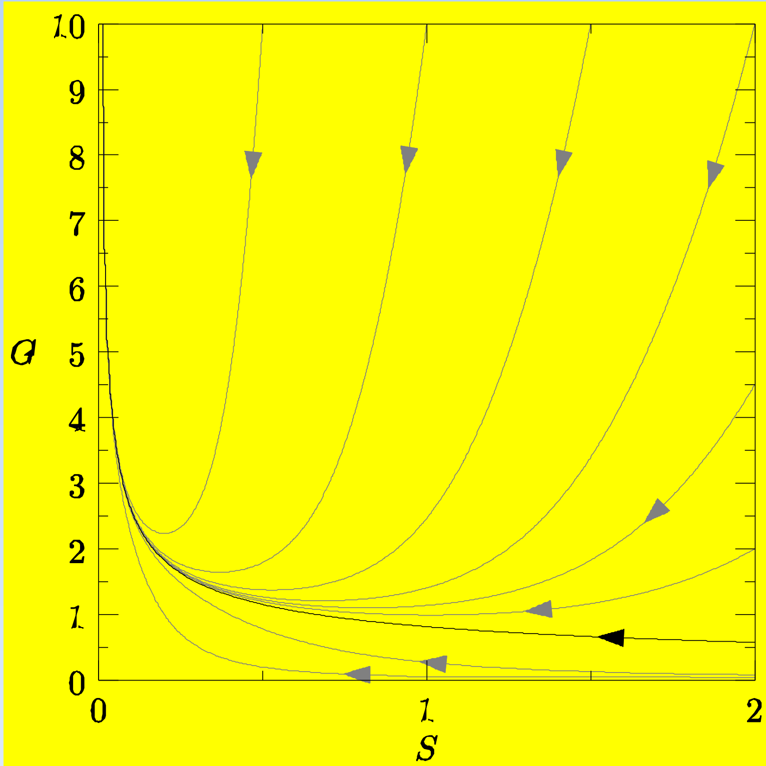
I.Yu. Kostyukov, E.N. Nerush, A.G. Litvak, PR STAB 15, 111001 (2012).

EQUATION ANALYSIS

$$\begin{cases} \dfrac{dS}{dT} = -\dfrac{1}{2}\dfrac{S}{G} - \dfrac{1}{4}GS^2, \\ \dfrac{dG}{dT} = 1 - SG^2 \end{cases}$$

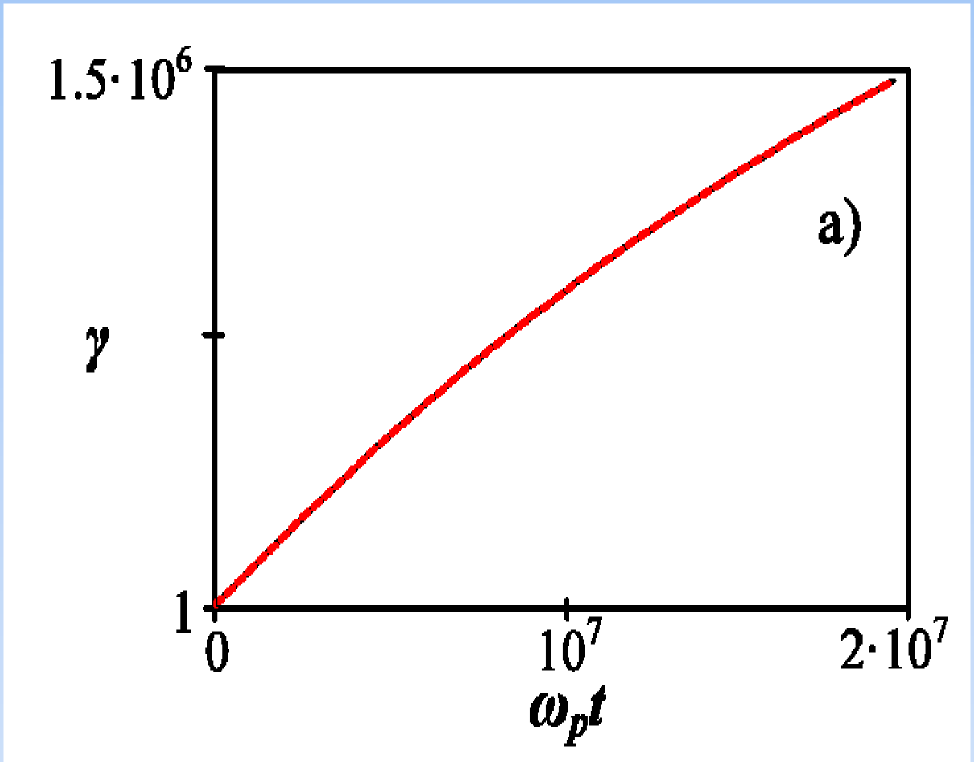
As $G > 0$ and $S > 0$ then $dS/dt < 0$ and the amplitude of the betatron oscillations always decreases with time. This means that for arbitrary electron energy the betatron oscillation amplitude will be small enough at a certain instance of time to be radiation reaction force less than the accelerating force.

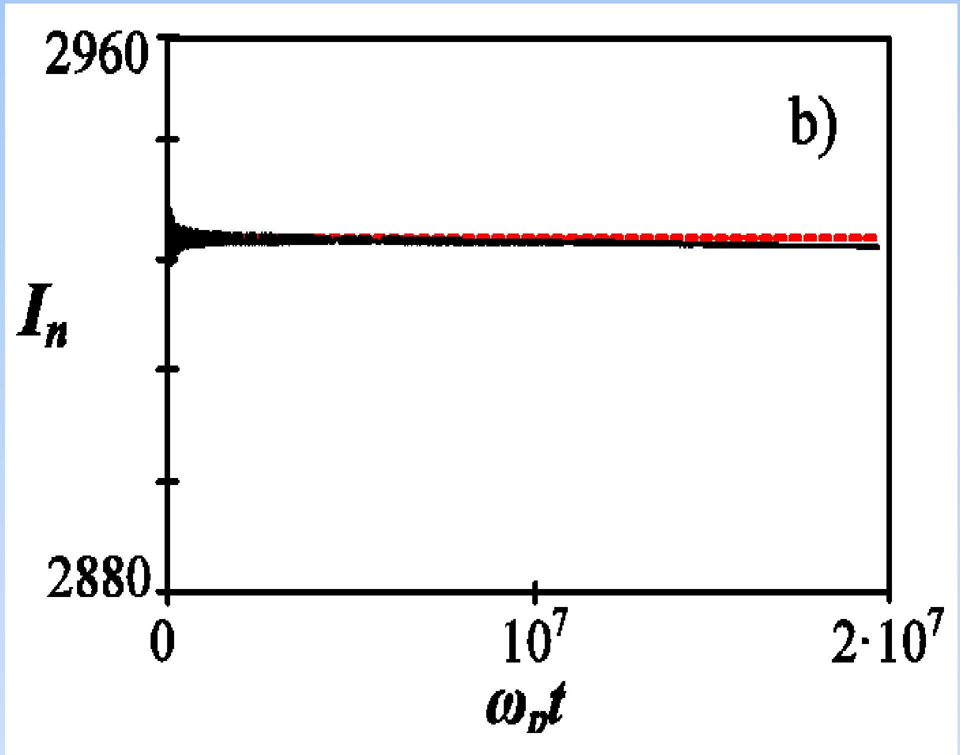
$$I = \frac{1 - 3SG^2/2}{S^{9/4}\left(SG^2\right)^{3/4}} = \text{const}.$$



If initially the accelerating force is stronger than the radiation reaction force ($SG^2 < 1$) then the electron energy monotonically increases with time. Otherwise ($SG^2 > 1$) the electron energy decays up to the time instance when ($SG^2 = 1$) that corresponds to $F_{acc} = F_{rrf}$ and then it monotonically increases with time.

NUMERICAL SOLUTION



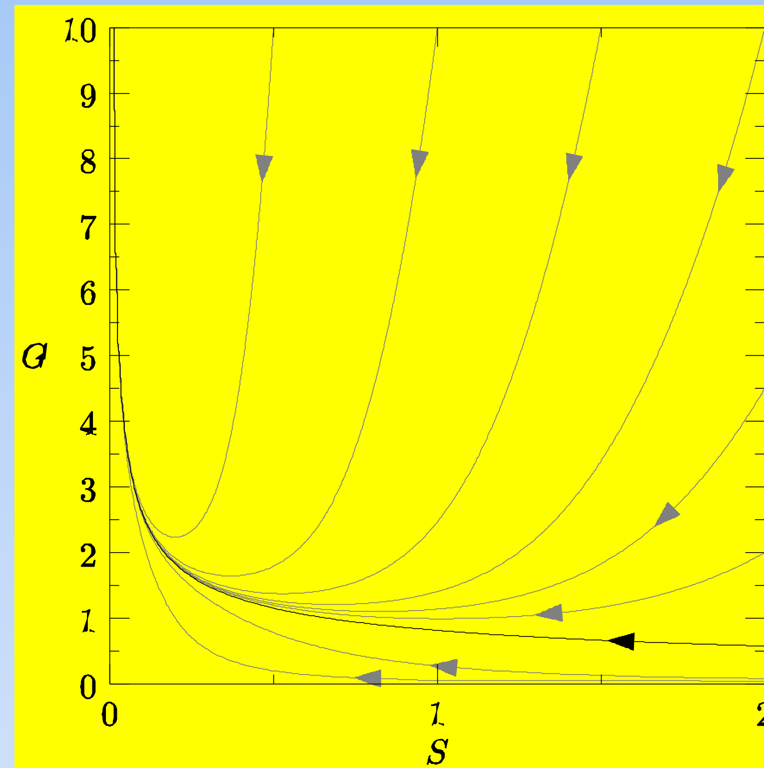


The dependence of $\gamma(t)$ and $I_n(t)$ calculated by solving the exact equation of motion with radiation reaction force (black solid lines) and by solving the reduced equations (red dashed lines) for parameters $\kappa^2 = 0.5$, $f = 0.1$, $n = 10^{15}\,\text{cm}^{-3}$ and for initial conditions

$\gamma_0 = 2000$, $R_{\beta,0} = 0.8c/\omega_p$, $p_{y,0} = 0$.

ASYMPTOTIC ACCELERATION

All electron trajectories merge in the limit $t \to \infty$ so that $G \to \infty$ and $S \to 0$. This is asymptotic regime of electron acceleration.



In this limit

$$I \to 0, \quad S \approx \frac{2}{3} G^{-2}, \quad G \propto \frac{1}{3} T.$$

The characteristic time of transition to asymptotic acceleration is $T_{tr} = I^{2/9}$.

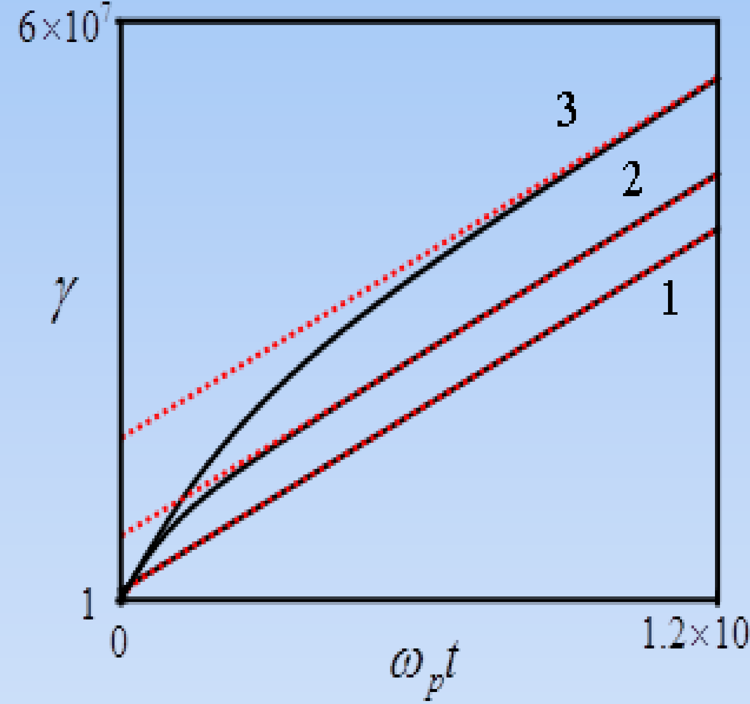
Asymptotic acceleration regime

$$T >> T_{tr} \qquad G = \frac{\delta}{3} G_{tr} + \frac{1}{3} T, \;\; S = \frac{2}{3} G^{-2}$$

$$\delta = -\frac{27\Gamma\left(-\frac{1}{3}\right)\Gamma\left(\frac{16}{9}\right)}{28\Gamma\left(\frac{4}{9}\right)} \approx 1.85$$

ASYMPTOTIC ACCELERATION

$$T >> T_{tr} \qquad G = \frac{\delta}{3} G_{tr} + \frac{1}{3} T, \;\; S = \frac{2}{3} G^{-2}$$



In asymptotic acceleration regime the radiation reaction force is equal to two-thirds of the accelerating force:

$$F_{rrf} = \frac{2}{3} F_{acc}$$

The electron energy increases linearly with time while the betatron amplitude and the normalized energy spread are reversely proportional to the time.

$$\langle G \rangle \propto T, \qquad R_\beta \propto \sqrt{S} \propto \frac{1}{T}, \qquad \frac{\sigma_G}{\langle G \rangle} = \frac{\sqrt{\langle G^2 \rangle - \langle G \rangle^2}}{\langle G \rangle} \propto \frac{1}{T}$$

DISCUSSION: VALIDITY CONDITIONS

1) Validity of the averaged equations

$$T >> T_{tr} \qquad S \approx \frac{2}{3} G^{-2}, \qquad G \propto \frac{1}{3} T$$

$$\kappa \sim f \sim 1$$

$$\Rightarrow \quad F_2/F_3 \sim f\varepsilon << 1$$

$$F_1/F_3 \sim (3/4)\kappa^2 f \gamma^{-1/2} \varepsilon^{1/2} << 1$$

2) Justification of classical approach

$$\chi = \frac{\left[(mc\gamma\mathbf{E} + \mathbf{p} \times \mathbf{H})^2 - (\mathbf{p} \cdot \mathbf{E})^2 \right]^{1/2}}{mcE_{cr}} \approx \frac{\gamma F_{\perp}}{eE_{cr}}$$

$$T >> T_{tr} \qquad S \approx \frac{2}{3} G^{-2}, \qquad G \propto \frac{1}{3} T$$

$$\Rightarrow \quad \chi \approx \sqrt{\frac{2f}{\alpha} \frac{\hbar \omega_p}{mc^2}} << 1$$

3) Coulomb effect

$$r \propto \sqrt{S} \approx G^{-1} \quad \Rightarrow \quad F_{\perp} \approx \kappa^2 mc\omega_p r \propto \gamma^{-1}, \qquad F_{\perp,Coulomb} \propto \frac{Q}{r\gamma^2} \propto \gamma^{-1}$$

DISCUSSION: ESTIMATES

The distance passed by the electron before reaching *asymptotic acceleration regime* is

$$l_{tr}\frac{\omega_p}{c} \approx \frac{f}{\kappa^2} T_{tr} \approx 1.6\left(\varepsilon^2 \gamma_0 R_{\beta,0}^4 f \kappa^8\right)^{-1/3}.$$

For the initial parameters $n = 10^{18}\,\mathrm{cm}^{-3}$, $R_\beta = c/\omega_p$, $\gamma_0\, mc^2 = 1$ GeV, $f = 0.7$, $\kappa = 0.11$ **the electron comes into *asymptotic acceleration regime* after passing** 7800 **laser-driven acceleration stages with total distance** $l_{tr} = 73$ m**, achieving the energy** $\gamma mc^2 = 5$ TeV **and** $R_\beta = 0.008\, c/\omega_p$**, where the stage distance is chosen to be equal to the half dephasing length and the distance between the acceleration stages is neglected. For the rarefied plasma** $n = 10^{15}\,\mathrm{cm}^{-3}$ ***asymptotic acceleration regime* is achieved within** 78 **stages with** $l_{tr} = 23$ km**,** $\gamma mc^2 = 48$ TeV **and** $R_\beta = 0.005\, c/\omega_p$**.**

***Asymptotic acceleration regime* may be achieved within a few acceleration stages in the proton-driven acceleration schemes because of the very large dephasing length.**

CONCLUSIONS

1) Electron acceleration is not limited by the radiative damping in plasma-based accelerators. Even if the radiation reaction force is stronger than the accelerating force at the beginning, then acceleration eventually succeeds deceleration with time.

2) The damping of the betatron oscillations leads to the transition to the self-similar asymptotic acceleration regime in the infinite-time limit when the radiation reaction force becomes equal to 2/3 of the accelerating force.

3) The relative energy spread induced by the radiative damping in the accelerated electron bunch decreases with time in this regime.

4) The obtained results can be also applied to any other accelerating systems with the linear focusing forces.

THANK YOU FOR YOUR ATTENTION!