La Thuile, 25 February '13

# Neutrino Mixing:
# A Theoretical Overview

G. Altarelli
Universita' di Roma Tre/CERN

# Recent issues in neutrino mass and mixing

- Are sterile neutrinos coming back?

  A White Paper: K.N. Abazajian et al, ArXiv:1204.5789

- $\theta_{13}$ measured ($\sim$ 8 -10 $\sigma$ from zero, rather large: $\theta_{13} \sim 9^o$)

  T2K, MINOS, DoubleCHOOZ, Daya Bay, RENO

- Indication of $\theta_{23}$ non maximal,

  Indication of $\cos\delta_{CP} < 0$

  Related to $\theta_{13}$ large, from MINOS and T2K

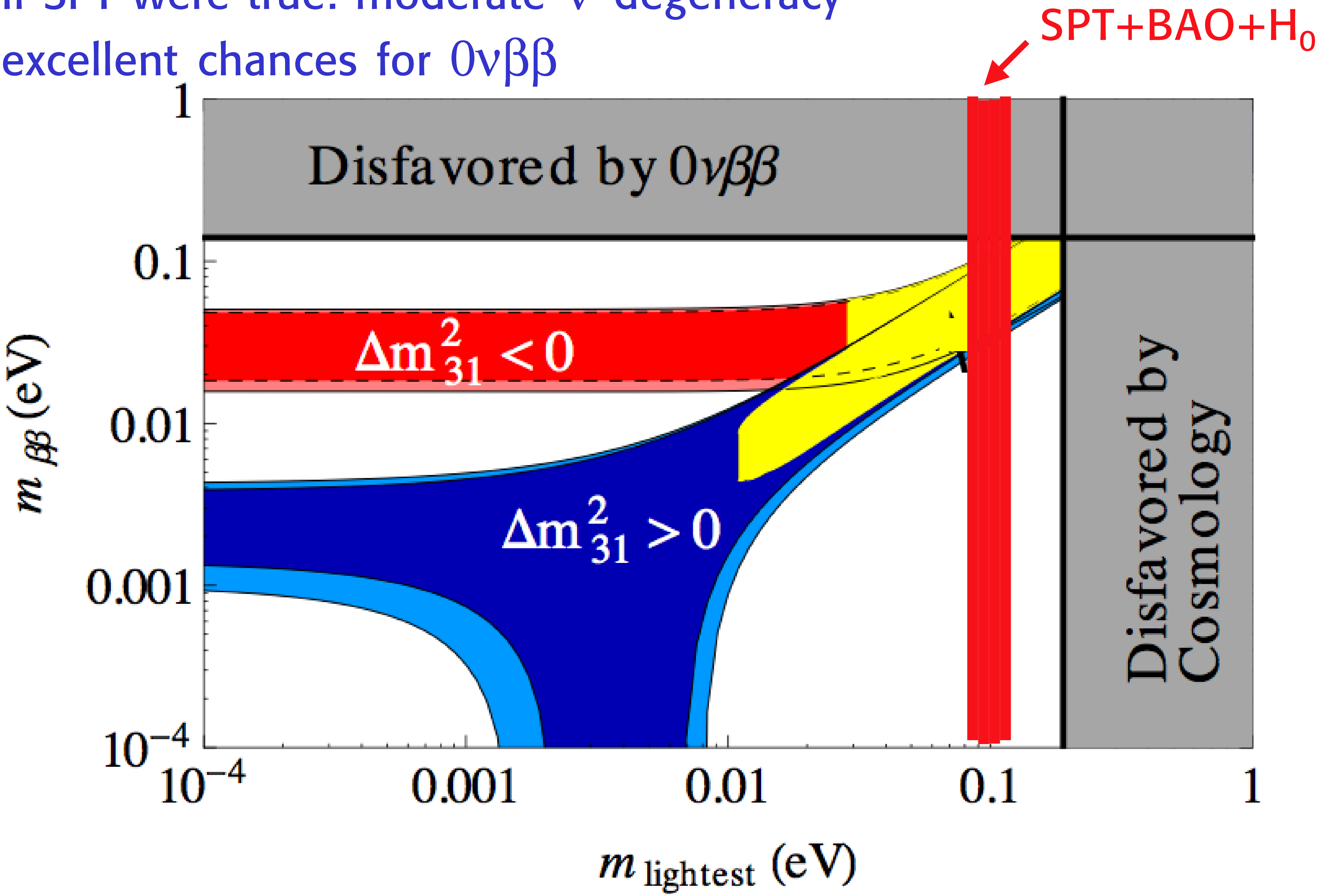
  Fogli et al '12, Forero et al '12, Gonzalez-Garcia et al '12

Also: $m_{\beta\beta} \sim < 0.14\text{-}0.38$ eV EXO'12

$\Sigma m_\nu = 0.32 \pm 0.11 \pm ?$ ← Priors

SPT + BAO + H0

If SPT were true: moderate ν degeneracy
excellent chances for 0νββ



# Sterile ν's? A number of "hints"

(they do not make an evidence but pose an important experimental problem that needs clarification)

$\bar{\nu}_\mu \to \bar{\nu}_e$ $\nu_\mu \to \nu_e$ $\bar{\nu}_\mu \to \bar{\nu}_e$

- LSND and MiniBoone (appearance)
- Reactor anomaly ( $\bar{\nu}_e$ disappearance)
- Gallium $\nu_e$ disappearance

If all true (unlikely) then need at least 2 sterile ν's

Important information also from

- Neutrino counting from cosmology

Cosmology is fully compatible with $N_{eff}$ ~3 but could accept, perhaps even favour, one sterile neutrino

The bound from nucleosynthesis is the most stringent (assuming thermal properties at decoupling)

- BBN:
  - $N_s = 0.22 \pm 0.59$ [Cyburt, Fields, Olive, Skillman, AP 23 (2005) 313, astro-ph/0408033]
  - $N_s = 0.64^{+0.40}_{-0.35}$ [Izotov, Thuan, ApJL 710 (2010) L67, arXiv:1001.4440]

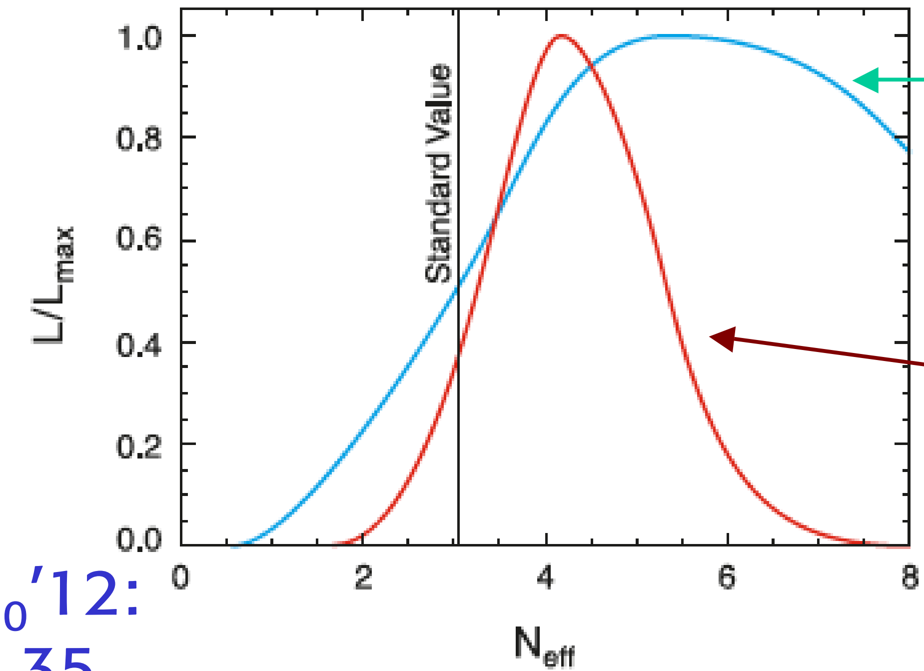
$N_s$

- BBN: $N_s < 1.2$ (95% CL) Mangano, Serpico, 1103.1261

- BBN: $N_s < 1.54$ (95% CL) [M. Pettini, et al, arXiv:0805.0594]

# From other than nucleosynthesis:

Komatsu et al




WMAP only

WMAP+BAO+$H_0$

$N_s$=1.34±0.87

well consistent with 3 ν's (prior dependence)

SPT+BAO+$H_0$'12:
$N_{eff}$= 3.71±0.35

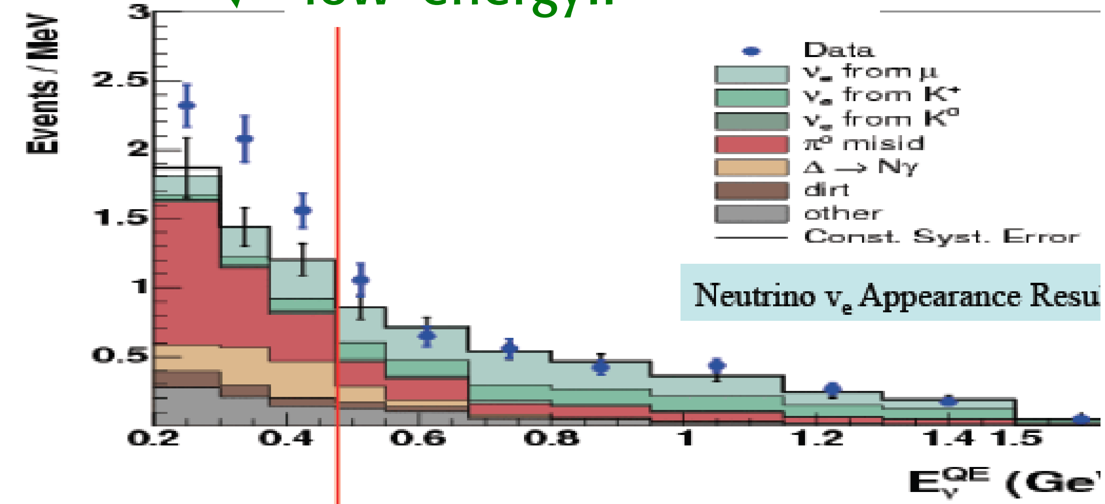
An eV scale RH ν involved in see-saw is not easy to conceive.
Most common EW scale BSM do not contain sterile neutrinos.
A sterile neutrino could be a remnant of some hidden sector or of gravity or an axino......

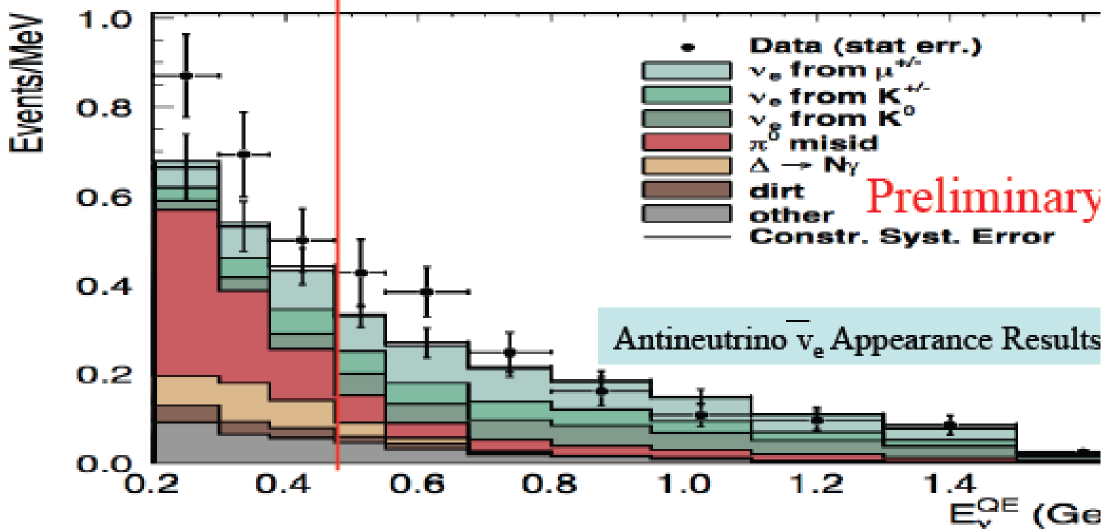
So it would be a great discovery:
An experimental clarification is needed

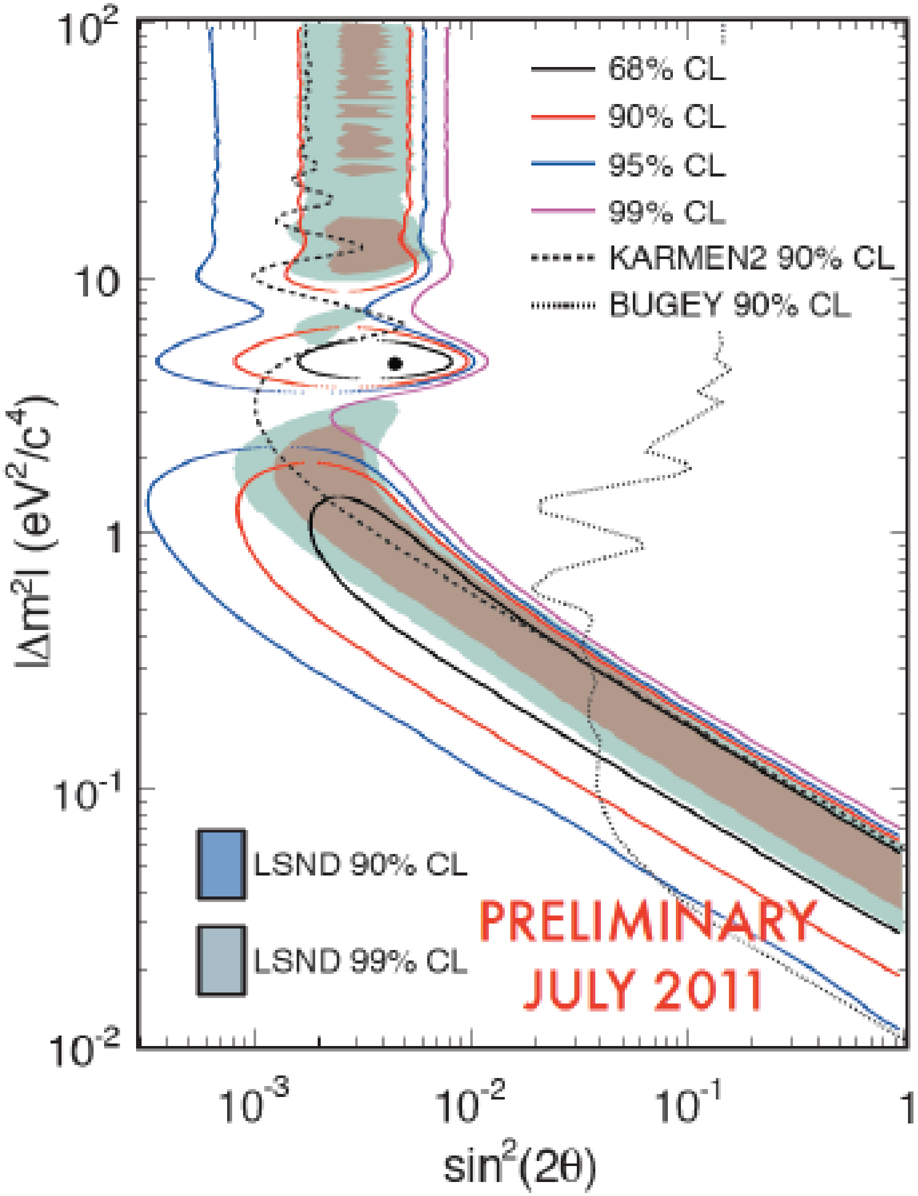
# LSND, KARMEN, MiniBooNE

MiniBooNE supports LSND in $\bar{\nu}_\mu$ but not in $\nu_\mu$ (or CP viol.?)

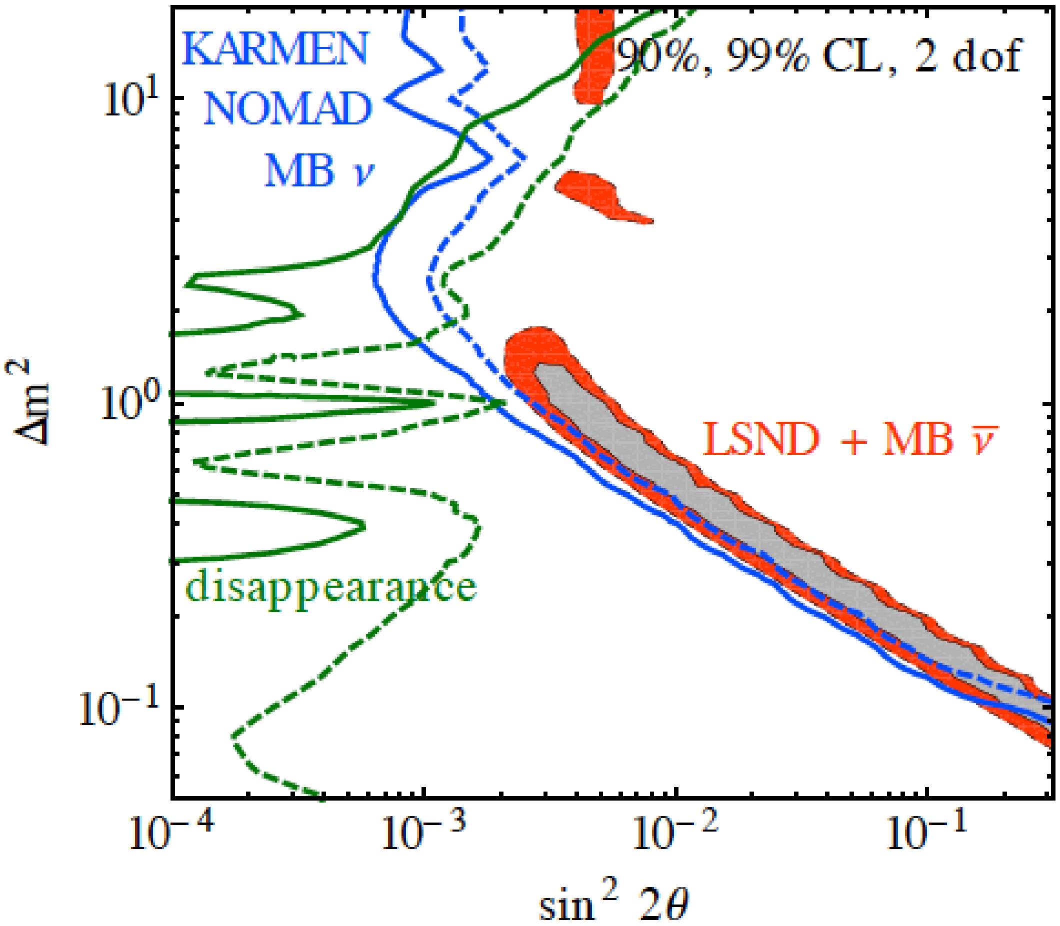
Unidentified excess at low energy!!



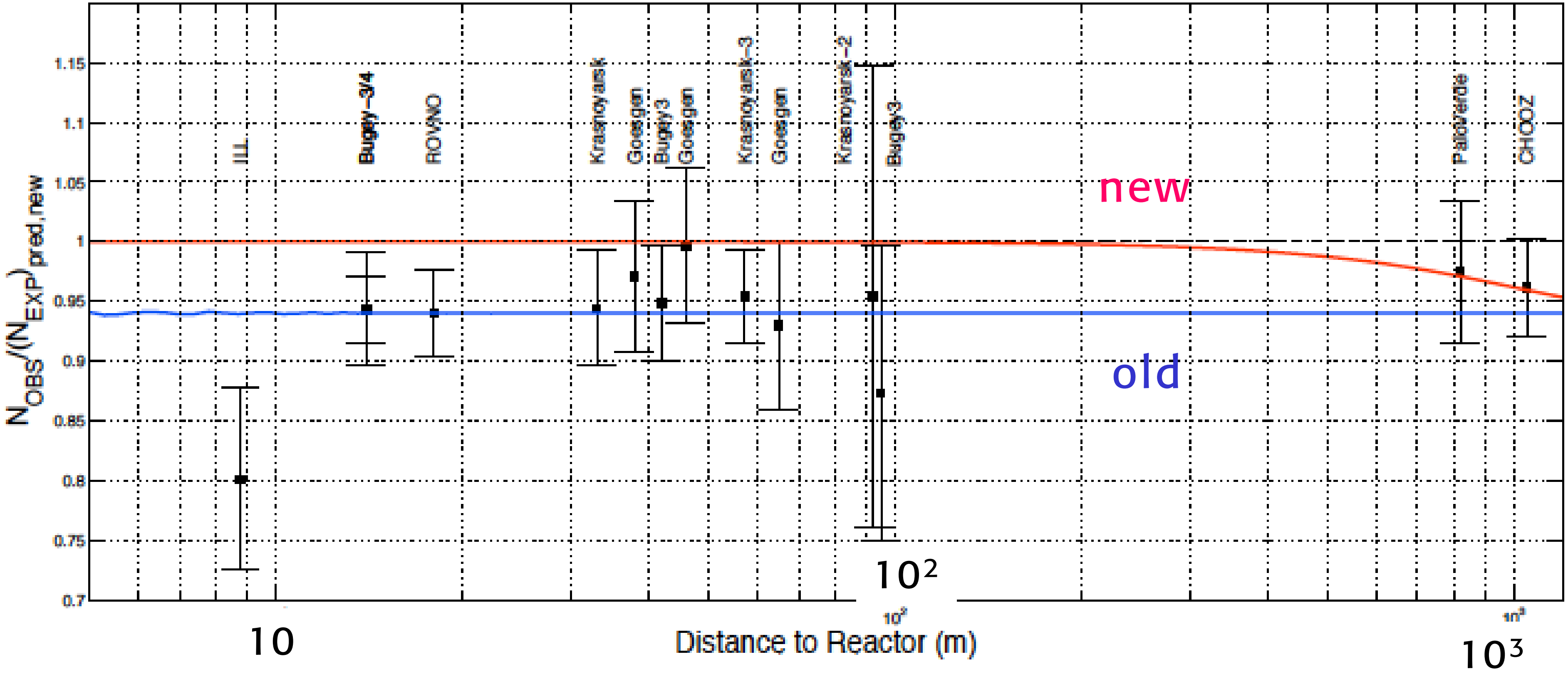
No signal in $\nu_\mu$ disappearance in accelerator experiments (CDHSW, MINOS, CCFR, MiniBooNE-SciBooNE) creates a strong tension with LSND (if no CP viol.)




For example, in 3+1 models here is the clash between appearance and disappearance

# The reactor anomaly (below 100m baseline)
# (after a revision of the theoretical flux and of crosssections)

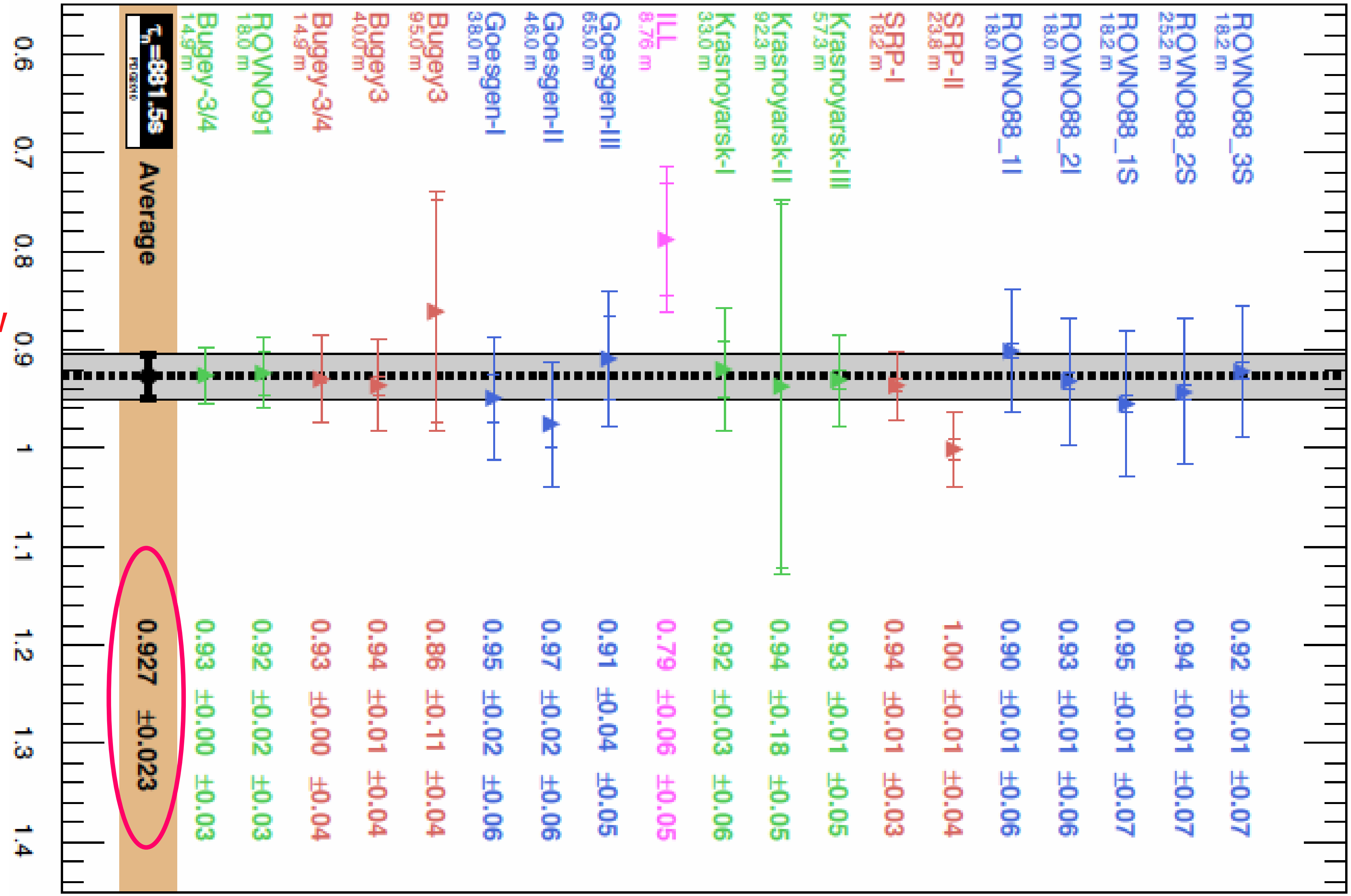
Lasserre




Systematic errors not shown in this figure (estimated in paper)!
Certainly of the same order of the shift.
They could well be larger than estimated

# The reactor anomaly: Meas./Exp. ~ 0.927±0.023

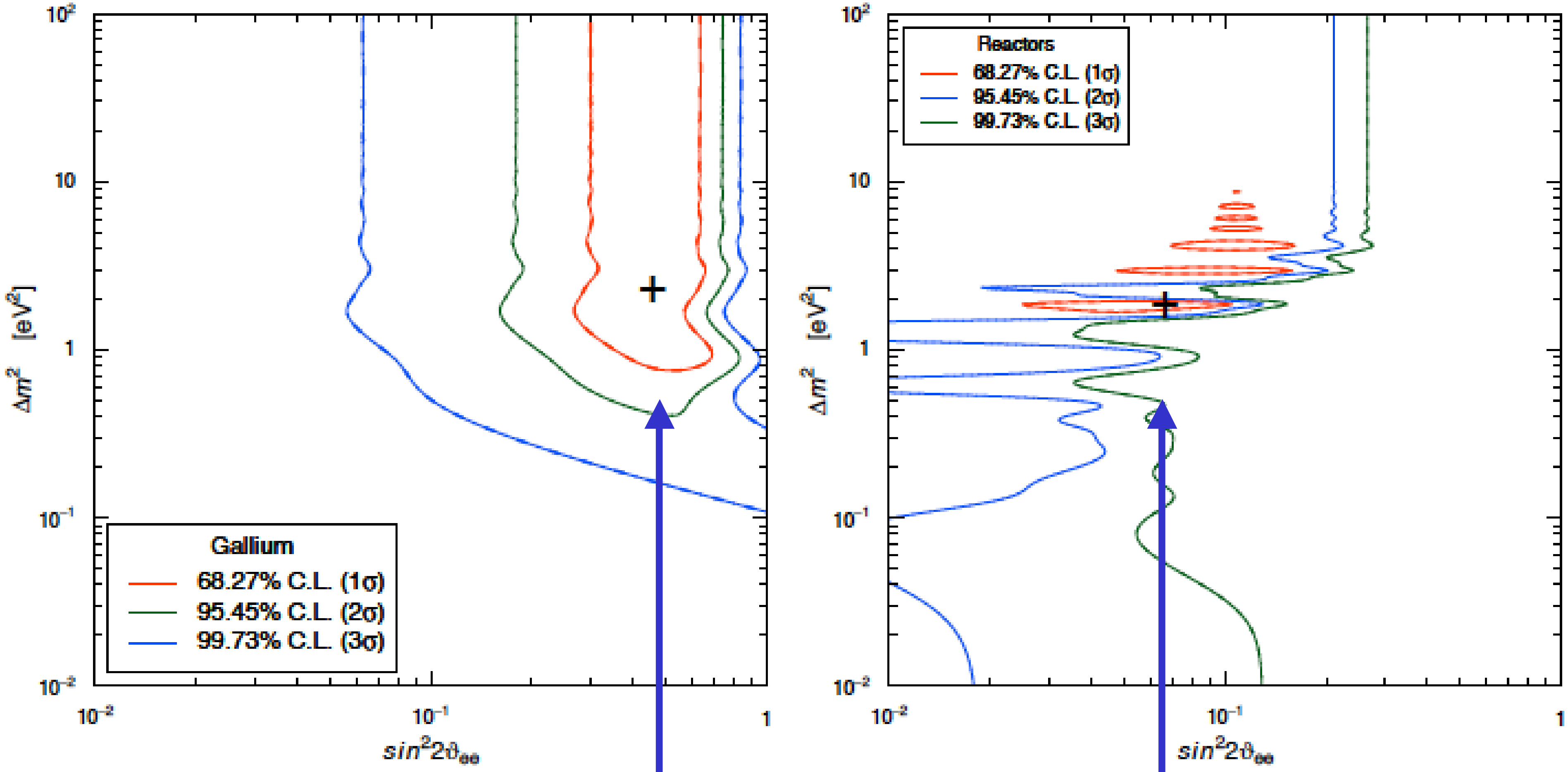
Meas./
Exp.
with new
flux



Depends on assumed cross section!

Gallium Anomaly

Reactor Anomaly




Do not really coincide!

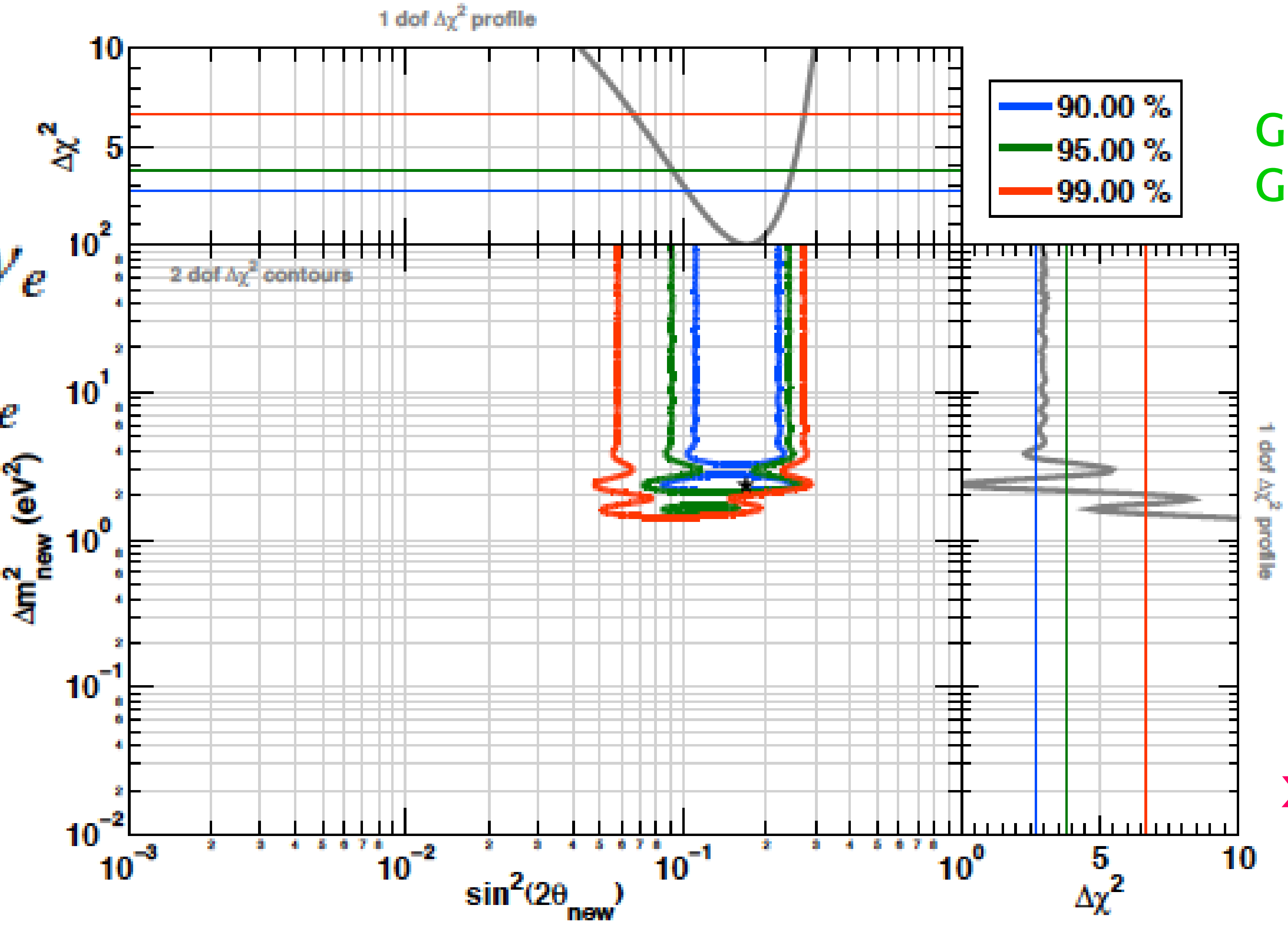
large angle

small angle

# Leaving aside LSND, Reactor and Gallium data can be fitted in 3+1

Gallium: $\nu_e$

Reactor: $\bar{\nu}_e$




Giunti, Laveder '11
Giunti et al '12

tension with cosmology

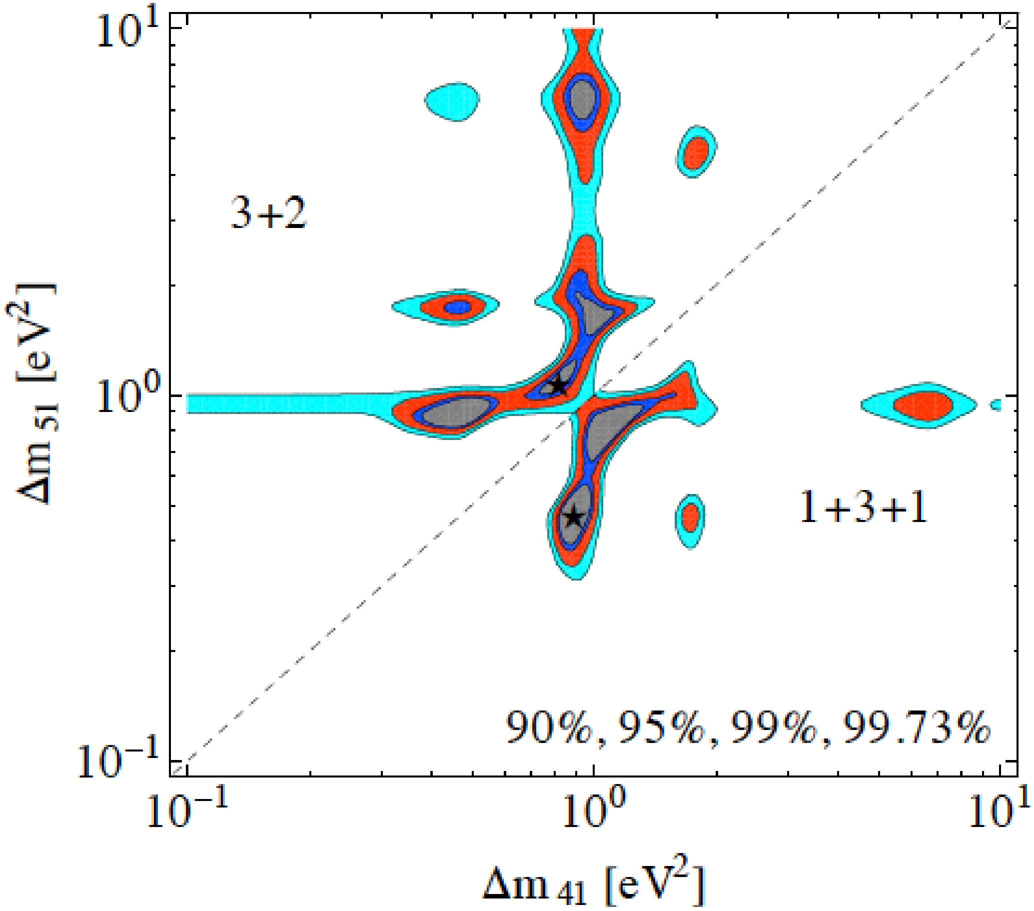
$\Sigma m_\nu <$ 0.3-0.7eV

SPT: 0.33(0.11)

Figure 60. Allowed regions in the $\sin^2(2\theta_{new})$–$\Delta m^2_{new}$ plane from the combination of reactor neutrino experiments, the Gallex and Sage calibration sources experiments, and the ILL and Bugey-3-energy spectra. The data are well fitted by the 3+1 neutrino hypothesis, while the no-oscillation hypothesis is disfavored at 99.97% C.L (3.6 $\sigma$).

# Global fit (all data)

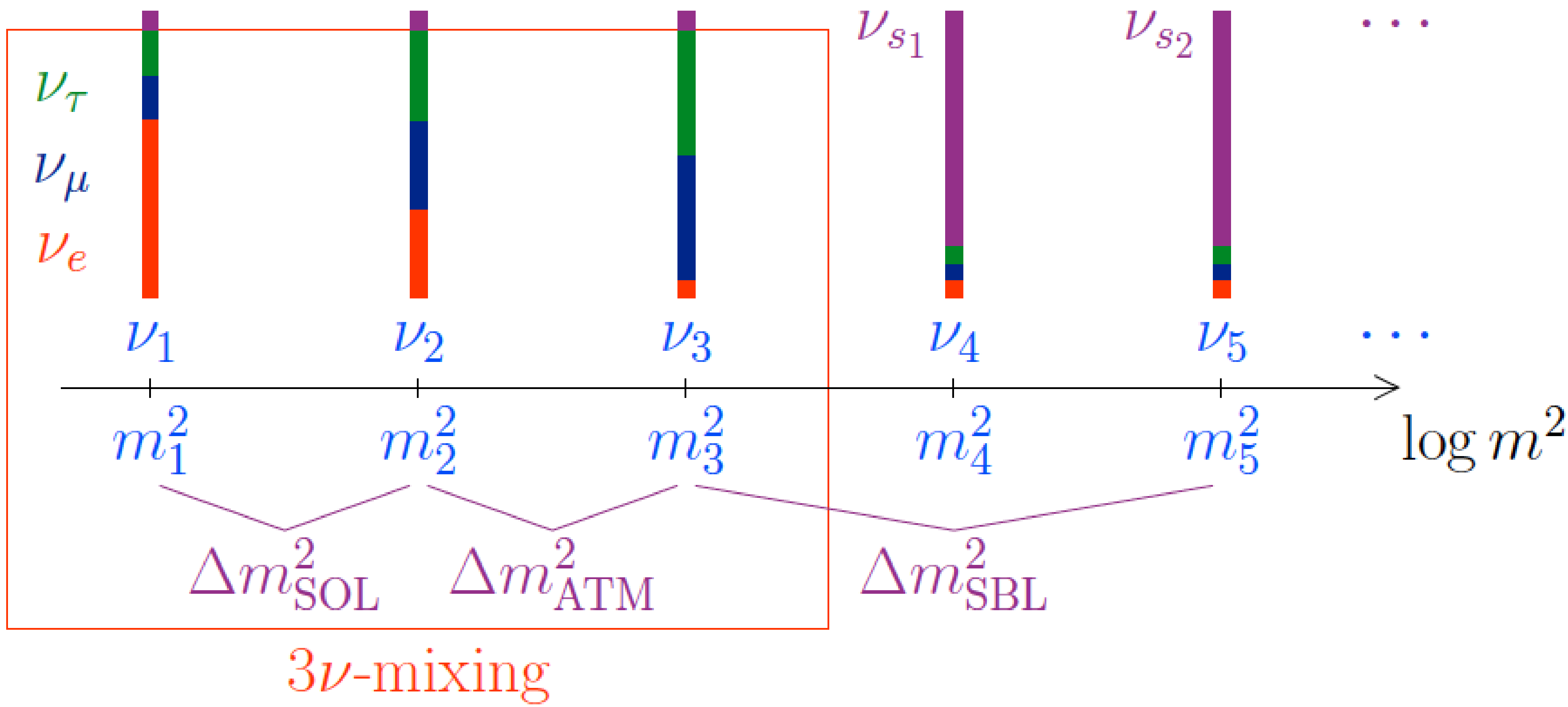
Kopp et al '12
Donini et al '12




In 3+2 or 1+3+1 the quality of the overall fit is poor: $\chi^2$/dof~5 - 6

The $\Delta m^2$ values are in tension with the cosmology mass bound $\Sigma m_\nu <$ 0.3 - 0.7 eV

In any case only a small leakage from active to sterile neutrinos is allowed by present data




Thus 3-ν's are still the main framework for ν mass and mixing

# Models of ν masses and mixings

$$m_\ell \to \bar{R} m_\ell L$$

$$m_\ell' = V_\ell^\dagger m_\ell U_\ell$$

$$m_\ell^{\dagger\prime} m_\ell' = U_\ell^\dagger m_\ell^\dagger m_\ell U_\ell$$

An interplay of different matrices:

$$U_{PMNS} = U_\ell^\dagger U_\nu$$

neutrino diagonalisat'n

charged lepton diagonalisat'n

$$O_5 = \ell^T \frac{\lambda^2}{M} \ell H H \to \nu_L{}^T m_\nu \nu_L$$

See-saw

$$m_\nu = m_D{}^T M^{-1} m_D$$

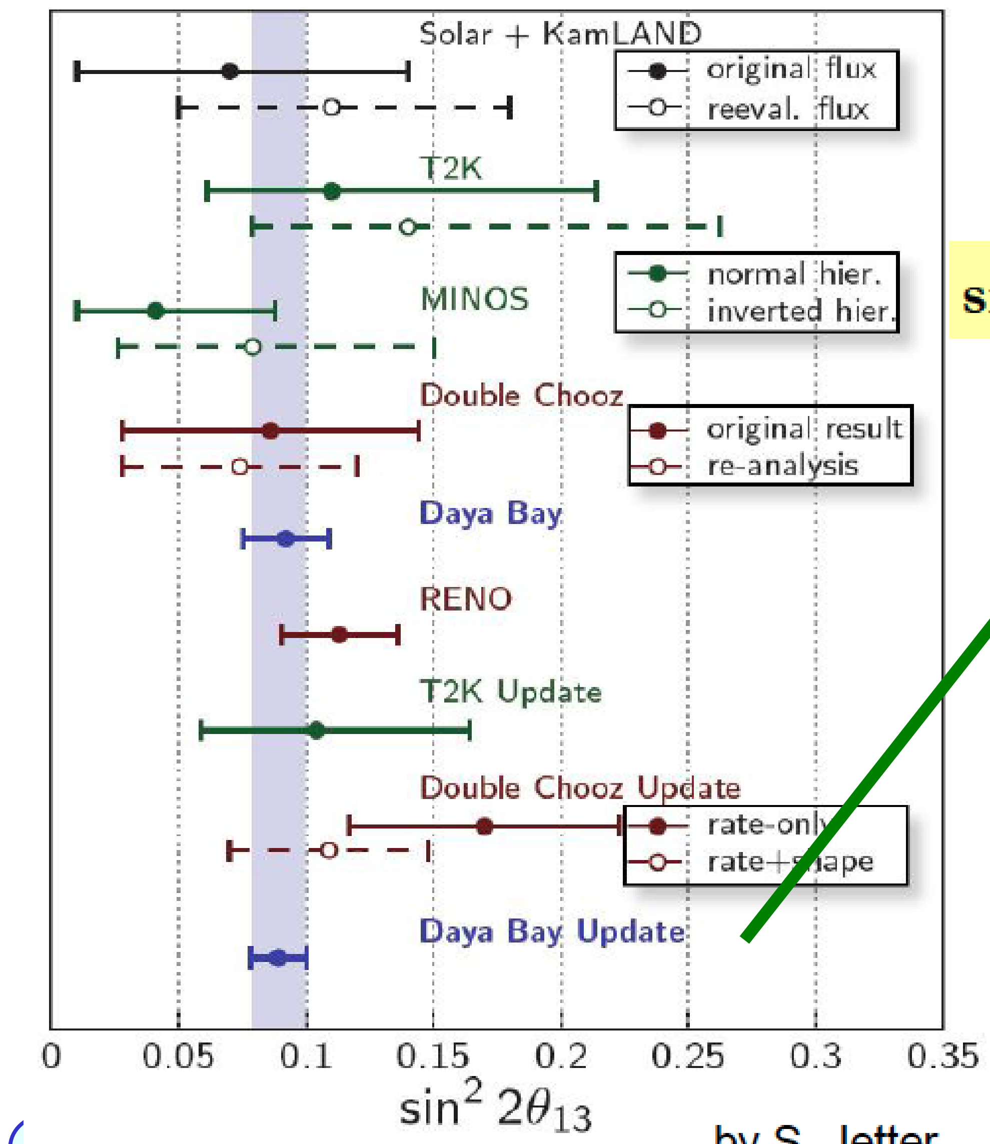
neutrino Majorana mass

neutrino Dirac mass

The large ν mixing versus the small q mixing can be due to the Majorana nature of ν's

$$m_\nu' = U_\nu^T m_\nu U_\nu$$

# Now we have a good measurement of $\theta_{13}$!!




by S. Jetter

~8σ from zero

Daya Bay

$\sin^2 2\theta_{13} = 0.089 \pm 0.010(\text{stat}) \pm 0.005(\text{syst})$

$\sin\theta_{13} = 0.15 \pm 0.01$

$\sin^2\theta_{13} = 0.023 \pm 0.003$

$\theta_{13} = 8.7^\circ \pm 0.6^\circ$

A large impact on model building!

| Parameter | Best fit | $1\sigma$ range |
|---|---|---|
| $\delta m^2/10^{-5}$ eV$^2$ (NH or IH) | 7.54 | 7.32 – 7.80 |
| $\sin^2\theta_{12}/10^{-1}$ (NH or IH) | 3.07 | 2.91 – 3.25 |
| $\Delta m^2/10^{-3}$ eV$^2$ (NH) | 2.43 | 2.33 – 2.49 |
| $\Delta m^2/10^{-3}$ eV$^2$ (IH) | 2.42 | 2.31 – 2.49 |
| $\sin^2\theta_{13}/10^{-2}$ (NH) | 2.41 | 2.16 – 2.66 |
| $\sin^2\theta_{13}/10^{-2}$ (IH) | 2.44 | 2.19 – 2.67 |
| $\sin^2\theta_{23}/10^{-1}$ (NH) | 3.86 | 3.65 – 4.10 |
| $\sin^2\theta_{23}/10^{-1}$ (IH) | 3.92 | 3.70 – 4.31 |
| $\delta/\pi$ (NH) | 1.08 | 0.77 – 1.36 |
| $\delta/\pi$ (IH) | 1.09 | 0.83 – 1.47 |

Fogli et al '12

$\theta_{23}$ non maximal

$\cos\delta < 0$ ?

| | |
|---|---|
| $\sin^2\theta_{12}$ | $0.30 \pm 0.013$ |
| $\theta_{12}/^\circ$ | $33.3 \pm 0.8$ |
| $\sin^2\theta_{23}$ | $0.41^{+0.037}_{-0.025} \oplus 0.59^{+0.021}_{-0.022}$ |
| $\theta_{23}/^\circ$ | $40.0^{+2.1}_{-1.5} \oplus 50.4^{+1.2}_{-1.3}$ |
| $\sin^2\theta_{13}$ | $0.023 \pm 0.0023$ |
| $\theta_{13}/^\circ$ | $8.6^{+0.44}_{-0.46}$ |
| $\delta_{CP}/^\circ$ | $240^{+102}_{-74}$ |
| $\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$ | $7.50 \pm 0.185$ |
| $\frac{\Delta m^2_{31}}{10^{-3}\ \text{eV}^2}$ (N) | $2.47^{+0.069}_{-0.067}$ |
| $\frac{\Delta m^2_{32}}{10^{-3}\ \text{eV}^2}$ (I) | $-2.43^{+0.042}_{-0.065}$ |

Gonzalez-Garcia et al '12

By now all mixing angles are fairly well known!

In spite of this progress viable models still span a wide range that goes from very little structure to a lot of symmetry

At one extreme are models dominated by chance
Some examples:

Anarchy
$U(1)_{\text{Froggatt-Nielsen}}$ charges
••••••

On the other hand the range for each mixing angle has narrowed and precise special patterns can be tentatively identified as starting approximations that, if significant, would lead to specified discrete symmetries:

TriBimaximal (TB), BiMaximal (BM),.......
Discrete non abelian flavour groups A4, S4, T′, $\Delta(96)$.....

$\theta_{13}$ near the previous bound and $\theta_{23}$ non maximal both go in the direction of Anarchy (a great success for Anarchy!)

Anarchy: no order for lepton mixing

In the neutrino sector no symmetry, no dynamics is needed; only chance

Hall, Murayama, Weiner '00

.....

de Gouvea, Murayama '12

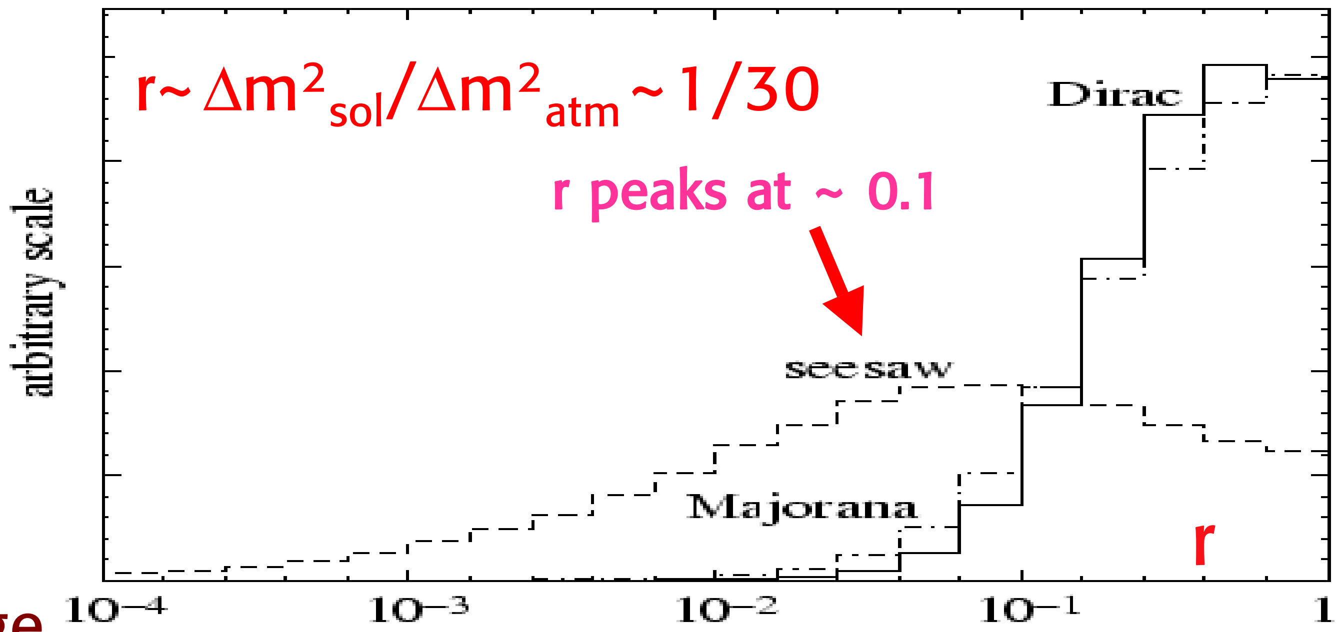
$\theta_{12}, \theta_{13}, \theta_{23}$ are just 3 random angles, the value of $r = \Delta m^2_{sun} / \Delta m^2_{atm} \sim 1/30$ is also determined by chance

# Anarchy: No structure in the neutrino sector

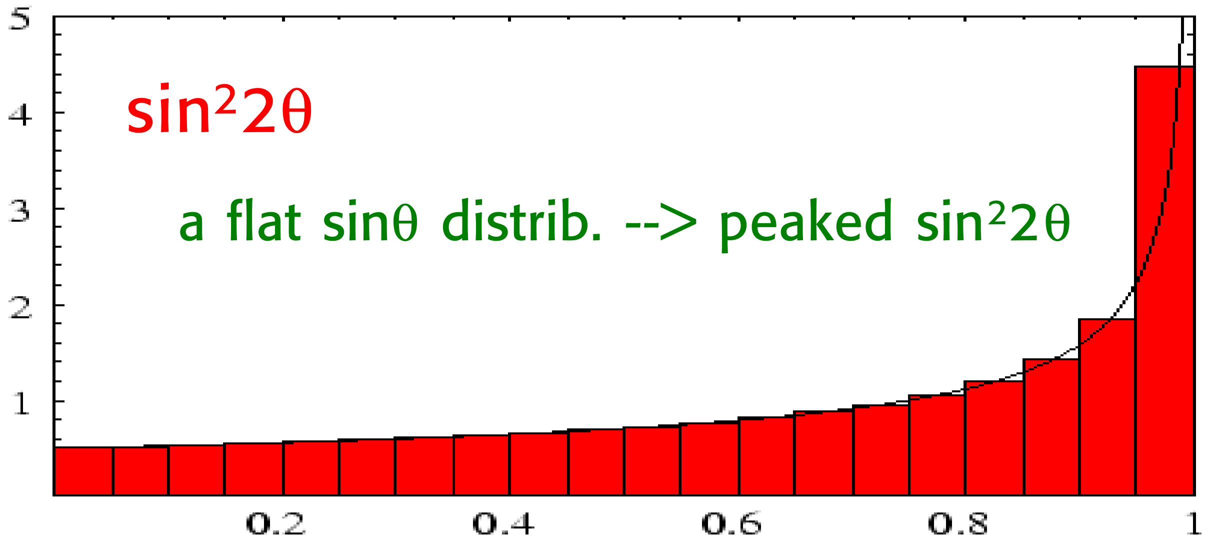
Hall, Murayama, Weiner '00

See-Saw:
$m_\nu \sim m^T M^{-1} m$
produces hierarchy
from random m, M

could fit the data on r




All mixing angles
should be not too large,
not too small




Predicts $\theta_{13}$ near old
bound and
$\theta_{23}$ sizably non maximal

successful!

Anarchy and its variants can be embedded in a simple GUT context based on

$SU(5) \times U(1)_{flavour}$

Froggatt Nielsen '79

Offers a simple description of hierarchies for quarks and leptons, but only orders of magnitude are predicted (large number of undetermined o(1) parameters $c_{ab}$)

The typical order parameter is $o(\lambda_C)$ and the entries of mass matrices are suppressed by $m_{ab} \sim c_{ab} (\lambda_C)^{nab}$

The exponents $n_{ab}$ are fixed by the charge imbalance

Anarchy can be realised in SU(5) by putting all the flavour structure in T ~ 10 and not in $F^{bar}$ ~ $5^{bar}$

$m_u \sim 10 . 10$ strong hierarchy $m_u : m_c : m_t$

$m_d \sim 5^{bar} . 10 \sim m_e^T$ milder hierarchy $m_d : m_s : m_b$

or $m_e : m_\mu : m_\tau$

Experiment supports that down quark & charged lepton hierarchy is roughly the square root of up quark hierarchy

$m_\nu \sim \nu_L^T m_\nu \nu_L \sim 5^{barT} . 5^{bar}$ or for see saw $(5^{bar}.1)^T$ (1.1) $(1.5^{bar})$

For example, for the simplest flavour group, $U(1)_F$

1st fam. 2nd 3rd

Anarchy

$$\begin{cases} T : & (3, 2, 0) \\ F^{bar} : & (0, 0, 0) \\ 1 : & (0, 0, 0) \end{cases}$$

# A milder ansatz - $\mu-\tau$ anarchy: no structure only in 23

Consider a matrix like $m_\nu \sim L^T L \sim \begin{pmatrix} \varepsilon^4 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon^2 & 1 & 1 \\ \varepsilon^2 & 1 & 1 \end{pmatrix}$ Note: $\theta_{13} \sim \varepsilon^2$, $\theta_{23} \sim 1$

$q(5^{bar}) \sim (2, 0, 0)$

with coeff.s of o(1) and det23~o(1)

["semianarchy", while $\varepsilon \sim 1$ corresponds to anarchy]

After 23 and 13 rotations $m_\nu \sim \begin{pmatrix} \varepsilon^4 & \varepsilon^2 & 0 \\ \varepsilon^2 & \eta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$r = \Delta m^2_{sun} / \Delta m^2_{atm}$

Normally two masses are of o(1) or r ~1 and $\theta_{12} \sim \varepsilon^2$

But if, accidentally, $\eta \sim \varepsilon^2$, then r is small and $\theta_{12}$ is large.

The advantage over anarchy is that $\theta_{13}$ is naturally small and a single accident is needed to get both $\theta_{12}$ large and r small

Ramond et al, ........

recently reanalysed by Buchmuller et al, '11

# SU(5)xU(1)

One can try different charge assignments

1st fam. 2nd 3rd

$\Psi_{10}$: (5, 3, 0)
$\Psi_5$: (2, 0, 0)
$\Psi_1$: (1,-1, 0)

Equal 2,3 ch. for lopsided

Recall: $m_u \sim$ 10 10
$m_d = m_e^T \sim$ $\bar{5}$ 10
$m_{\nu D} \sim$ $\bar{5}$ 1; $M_{RR} \sim$ 1 1

| Model | $\Psi_{10}$ | $\Psi_5$ | $\Psi_1$ |
|---|---|---|---|
| Anarchy ($A$) | (3,2,0) | (0,0,0) | (0,0,0) |
| Semianarchy $\mu\tau$-Anarchy ($A_{\mu\tau}$) | (3,2,0) | (1,0,0) | (2,1,0) |
| Pseudo $\mu\tau$-Anarchy ($PA_{\mu\tau}$) | (5,3,0) | (2,0,0) | (1,-1,0) |
| Hierarchy ($H$) new | (5,3,0) | (2,1,0) | (2,1,0) |

No structure for leptons → Anarchy ($A$)

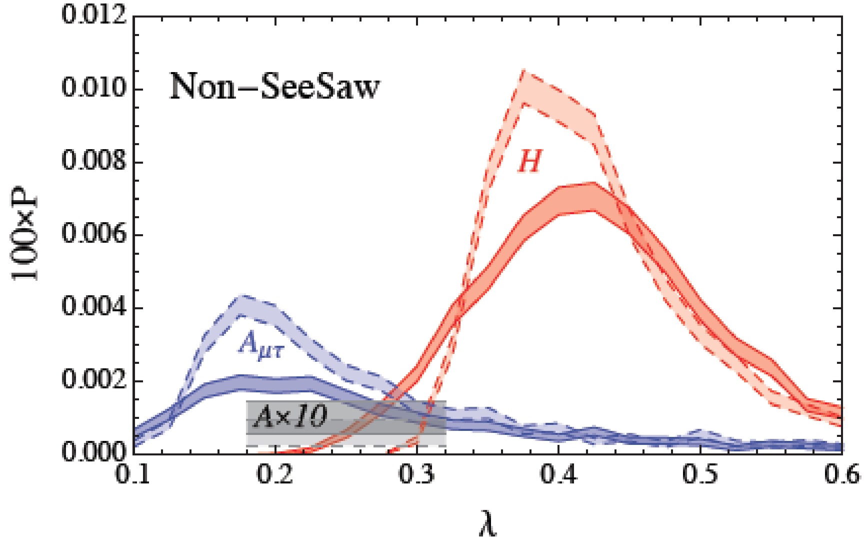
No automatic det23 = 0 → $\mu\tau$-Anarchy ($A_{\mu\tau}$) — all charges non negative

Automatic det23 = 0 → Pseudo $\mu\tau$-Anarchy ($PA_{\mu\tau}$) — charges of both signs

here r, $\theta_{23}$ are suppressed (Hierarchy)

With suitable charge assignments many relevant patterns can be obtained

If we embed anarchy in GUT's and explain quark hierarchies in terms of FN charges, then more effective variants of anarchy can be built, where chance is somewhat mitigated



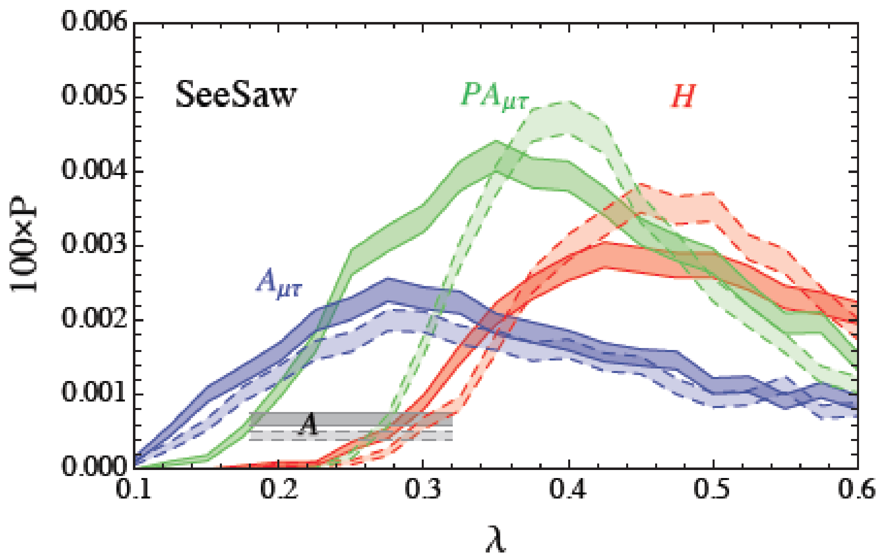

Anarchy (A): both r and $\theta_{13}$ small by accident

$\mu\tau$-anarchy ($A_{\mu\tau}$): only r small by accident

H, $PA_{\mu\tau}$ : no accidents

extraction range:
solid [0.5-2.0] dashed [0.8-1.2]

GA, Feruglio, Masina '02,'06
GA, Feruglio, Masina, Merlo '12




Optimal values of $\lambda \sim o(\lambda_C)$

$A_{\mu\tau}$: $\lambda$ ~ 0.2 (non SS), 0.3 (SS)

$PA_{\mu\tau}$: $\lambda$ ~ 0.35-0.4

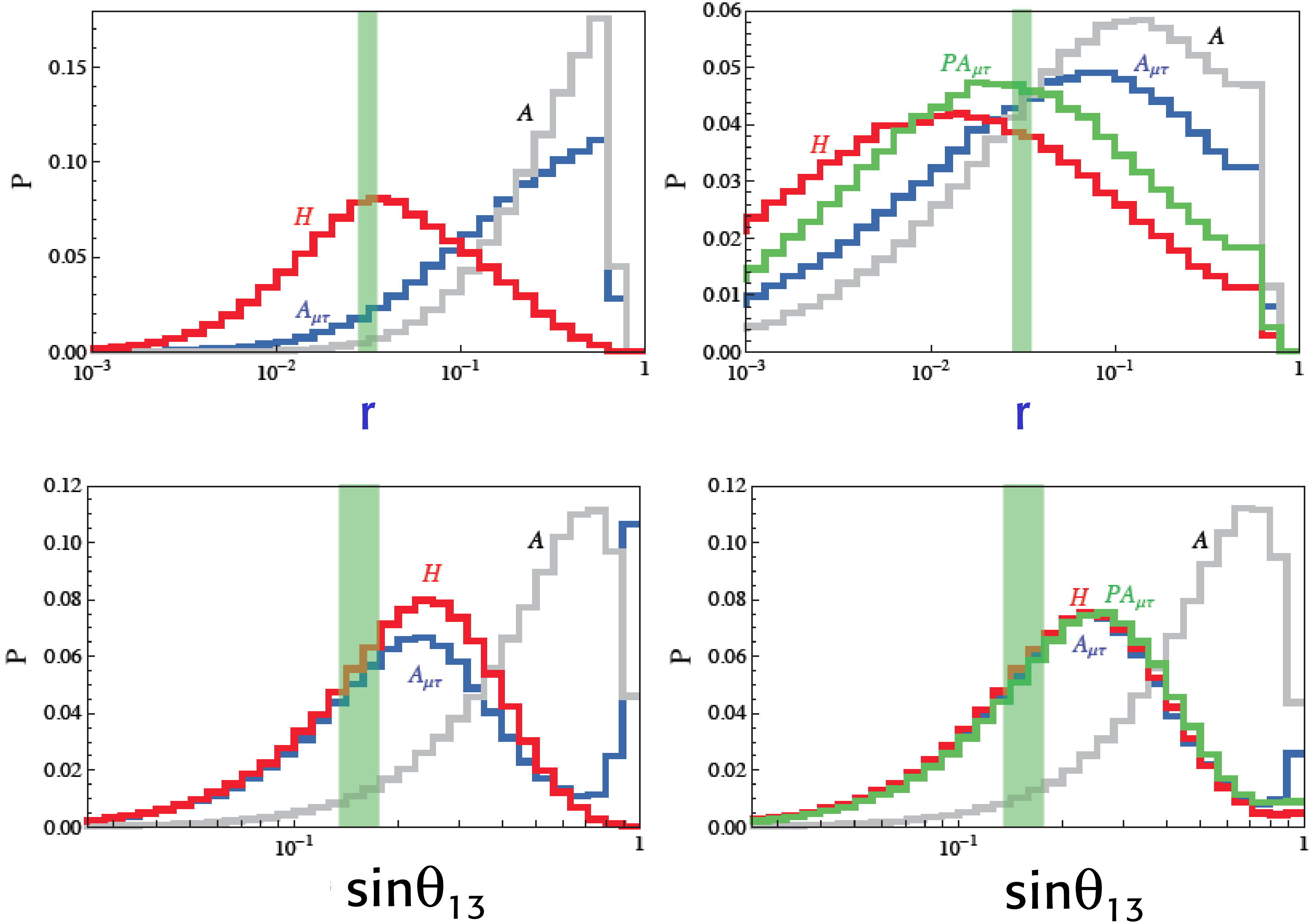
H: $\lambda$ ~ 0.4 (non SS), 0.45 (SS)

## no see-saw

$$O_5 = \ell^T \frac{\lambda^2}{M} \ell HH \to \nu_L{}^T m_\nu \nu_L$$

## when all charges are positive see-saw only affects r

## see-saw



## From Anarchy to more symmetry

Larger than U(1) continuous symmetries:

e.g $U(3)_l x U(3)_e$ ----> $U(2)_l x U(2)_e$

Blankenburg, Isidori, Jones-Perez '12

## At the other extreme from Anarchy

models with a maximum of order:
based on non abelian discrete flavour groups

(reviews: G.A., Feruglio, Rev.Mod.Phys. 82 (2010) 2701;
G.A., Feruglio, Merlo'12 ;
King, Luhn'13 )

A number of "coincidences" could be hints pointing to the underlying dynamics

## An incomplete list of recent papers on discrete groups and large $\theta_{13}$


S. King, Phys.Lett. **B718**, 136 (2012), 1205.0506.

S. Antusch, C. Gross, V. Maurer, and C. Sluka, Nucl.Phys. **B866**, 255 (2013), 1205.1051.

I. de Medeiros Varzielas, JHEP **1201**, 097 (2012), 1111.3952. +D. Pidt 1211.5370

S. King, Phys.Lett. **B675**, 347 (2009), 0903.3199. +et al **1301.7065**

G. Altarelli, F. Feruglio, and L. Merlo (2012), 1205.5133.

I. de Medeiros Varzielas and G. G. Ross, arXiv:1203.6636

A. Meroni, S. T. Petcov and M. Spinrath, arXiv:1205.5241.

J. Barry and W. Rodejohann, Phys.Rev. **D81**, 093002 (2010), 1003.2385.

Y. Shimizu, M. Tanimoto, and A. Watanabe, Prog.Theor.Phys. **126**, 81 (2011), 1105.2929.

Y. Ahn and S. K. Kang, Phys.Rev. **D86**, 093003 (2012), 1203.4185.

M.-C. Chen, J. Huang, J.-M. O'Bryan, A. M. Wijangco, and F. Yu (2012), 1210.6982.

S. F. King, C. Luhn and A. J. Stuart, Nucl. Phys. B **867** (2013) 203 [arXiv:1207.5741].

C. Hagedorn, S. F. King and C. Luhn, Phys. Lett. B **717** (2012) 207 [arXiv:1205.3114]

N. Memenga et al, [arxiv:1301.2963]

# TB Mixing

TB mixing is close to the data:
$\theta_{12}, \theta_{23}$ agree within ~ $2\sigma$
$\theta_{13}$ is the smallest angle

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

At $1\sigma$: Fogli et al '12

$\sin^2\theta_{12} = 1/3$ : 0.291- 0.325
$\sin^2\theta_{23} = 1/2$ : 0.36 - 0.41
$\sin\theta_{13} = 0$ : 0.14 - 0.16

A coincidence or a hint?

Called:
Tri-Bimaximal mixing

Harrison, Perkins, Scott '02

$$\nu_3 = \frac{1}{\sqrt{2}}(-\nu_\mu + \nu_\tau)$$

$$\nu_2 = \frac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)$$

⊕ $\theta_{13}$ largish and $\theta_{23}$ non maximal tend to move away from TB

## LQC: Lepton Quark Complementarity

$\theta_{12} + \theta_C = (46.4 \pm 0.8)^o \sim \pi/4$ ← Gonzalez-Garcia et al '12

Suggests Bimaximal mixing corrected by diagonalisation of charged leptons

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

A coincidence or a hint?

## Golden Ratio

$$\sin^2\theta_{12} = \frac{1}{\sqrt{5}\phi} = \frac{2}{5+\sqrt{5}} \approx 0.276$$

$$U_{GR} = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ \frac{\sin\theta_{12}}{\sqrt{2}} & -\frac{\cos\theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin\theta_{12}}{\sqrt{2}} & -\frac{\cos\theta_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

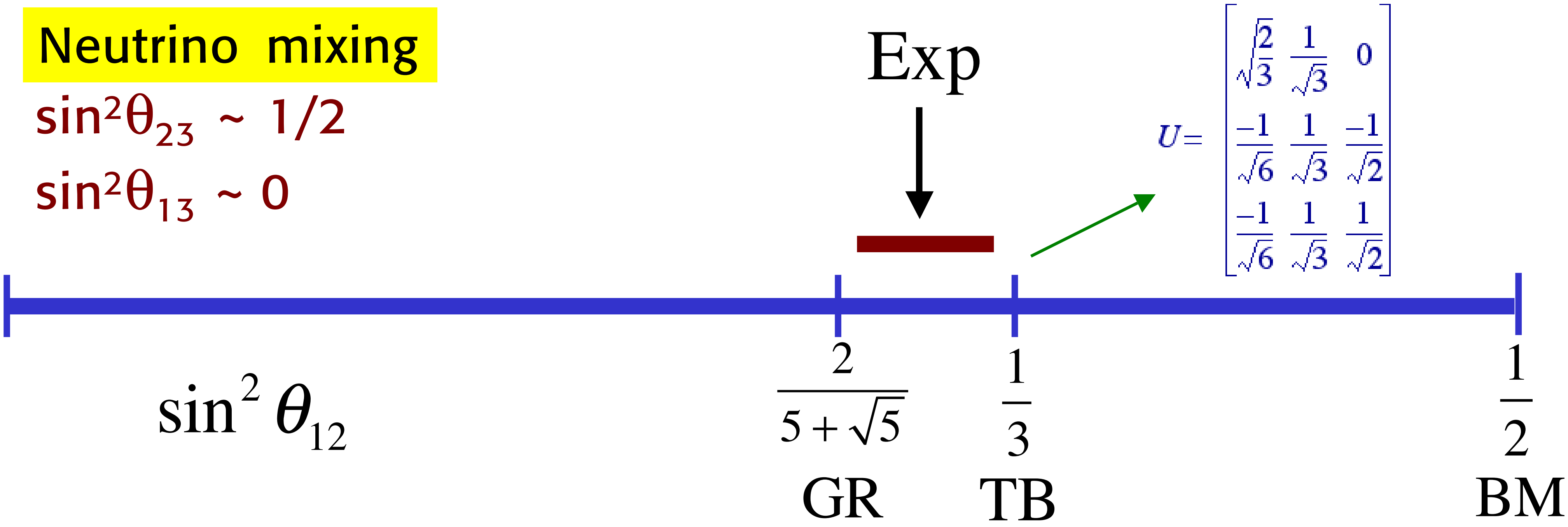
A coincidence or a hint?

Cannot all be true hints, perhaps none

## Neutrino mixing

$\sin^2\theta_{23} \sim 1/2$

$\sin^2\theta_{13} \sim 0$




TB: Group A4, S4..... A vast literature (Ma, Rajasekaran '01.....)

GR: Golden Ratio - Group A5 Feruglio, Paris '11; G. J. Jing et al '11 Cooper et al '12

BM: Group S4 GA, Feruglio, Merlo '09

TB Mixing naturally leads to discrete flavour groups (similarly for GR, BM....)

TB Mixing: $$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

This is a particular rotation matrix with specified fixed angles

# Why and how discrete groups, in particular A4, work?

TB mixing corresponds to m in the basis where charged leptons are diagonal

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

Crucial point 1:
m is the most general matrix invariant under $SmS = m$ and $A_{23} m A_{23} = m$ with:

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

$$A_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

2-3 symmetry

$S^2 = A_{23}^2 = 1$

Crucial point 2:

Charged lepton masses:
a generic diagonal matrix
is defined by invariance under T
(or $\eta$T with $\eta$ a phase):

$$m_l = v_T \frac{v_d}{\Lambda} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

$$m_l^+ m_l = T^+ m_l^+ m_l T$$

a possible T is

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

$\omega^3=1$ --> $T^3=1$

An essential observation is that

S, T and $A_{23}$ are all contained in S4

$S^4=T^3=(ST^2)^2=1$ define S4

Thus S4 is the reference group for TB mixing

A4 is a subgroup of S4

Lam

## A4: a vast literature (Ma, Rajasekaran '01.....)

A4 is the discrete group of even perm's of 4 objects.
(the inv. group of a tetrahedron). It has 4!/2 = 12 elements.

$S^2=T^3=(ST)^3=1$ define A4

A4 has 4 inequivalent irreducible representations:
a triplet and 3 different singlets

3, 1, 1′, 1″ (promising for 3 generations!)

Ch. leptons $l \sim 3$ $e^c, \mu^c, \tau^c \sim 1, 1'', 1'$

Invariance under S and T is automatic in A4 while
$A_{23}$ is not contained in A4 (2<->3 exchange is an odd perm.)

But 2-3 symmetry happens in A4 if 1′ and 1″ symm. breaking
flavons are absent or have equal VEV's [2 of S4 = 1′ + 1″ of A4].

# Crucial point 3: A4 must be broken: the alignment

Before SSB the model is invariant under the flavour group A4

There are flavons $\phi_T$, $\phi_S$ , $\xi$... with VEV's that break A4:

$\phi_T$ breaks A4 down to $G_T$, the subgroup generated by
1, T, $T^2$, in the charged lepton sector

$\phi_S$ , $\xi$ break A4 down to $G_S$, the subgroup generated by
1, S, in the neutrino sector

$$\langle \varphi_T \rangle = (v_T, 0, 0)$$
$$\langle \varphi_S \rangle = (v_S, v_S, v_S)$$
$$\langle \xi \rangle = u \quad , \quad \langle \tilde{\xi} \rangle = 0$$

$\phi_T$, $\phi_S$ ~ 3
$\xi$ ~ 1

The 2-3 symmetry occurs in A4 if 1′ and 1″ flavons are absent

This aligment along subgroups of A4 naturally occurs in the good A4 models

# At LO TB mixing is exact

When NLO corrections are included from operators of higher dimension in the superpotential each mixing angle generically receives corrections of the same order $\delta\theta_{ij} \sim o(VEV/\Lambda) \sim o(\xi)$

Typical predicted pattern

$$\sin^2\theta_{12} = \frac{1}{3} + o(\xi)$$ ← ~ -0.03

$$\sin^2\theta_{23} = \frac{1}{2} + o(\xi)$$ ← ~ - 0.1

$$\sin\theta_{13} = o(\xi)$$ ← ~ 0.15

exp values of $o(\xi)$

As the maximum allowed corrections to $\theta_{12}$ are numerically $o(\lambda_C^2)$, one typically expected $\theta_{13} \sim o(\lambda_C^2)$

This generic prediction can be altered in special versions e.g. Lin '09 discussed a A4 model where $\theta_{13} \sim o(\lambda_C)$

We now compare

"Typical" A4 models

"Special" A4 models → with extra symmetry to separate $\theta_{13}$ and $\theta_{23}$ from $\theta_{12}$ up to NNLO

Bimaximal models

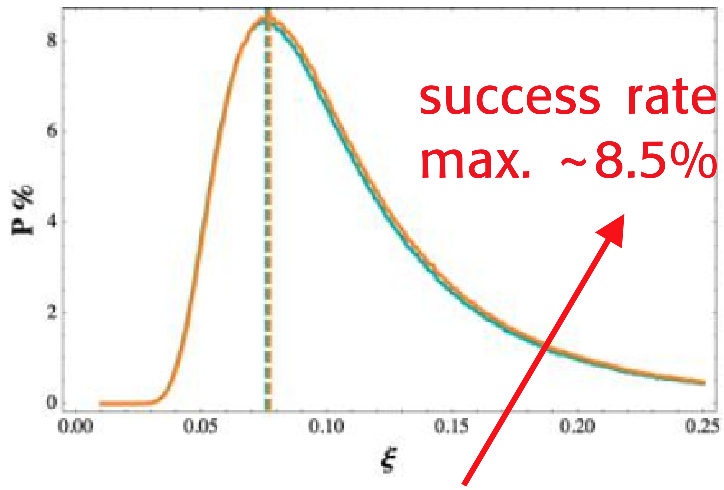
At LO the mixing angles are fixed at either TB or BM

Higher order operators lead to departures of o(VEV/$\Lambda$).
But the coeffs of these operators are not fixed.

GUT versions of these models exist but from the quark sector no additional hint for discrete groups is found

# In a typical A4 model

GA, Feruglio, Merlo, Stamou '12




Optimal value $\xi = 0.076$

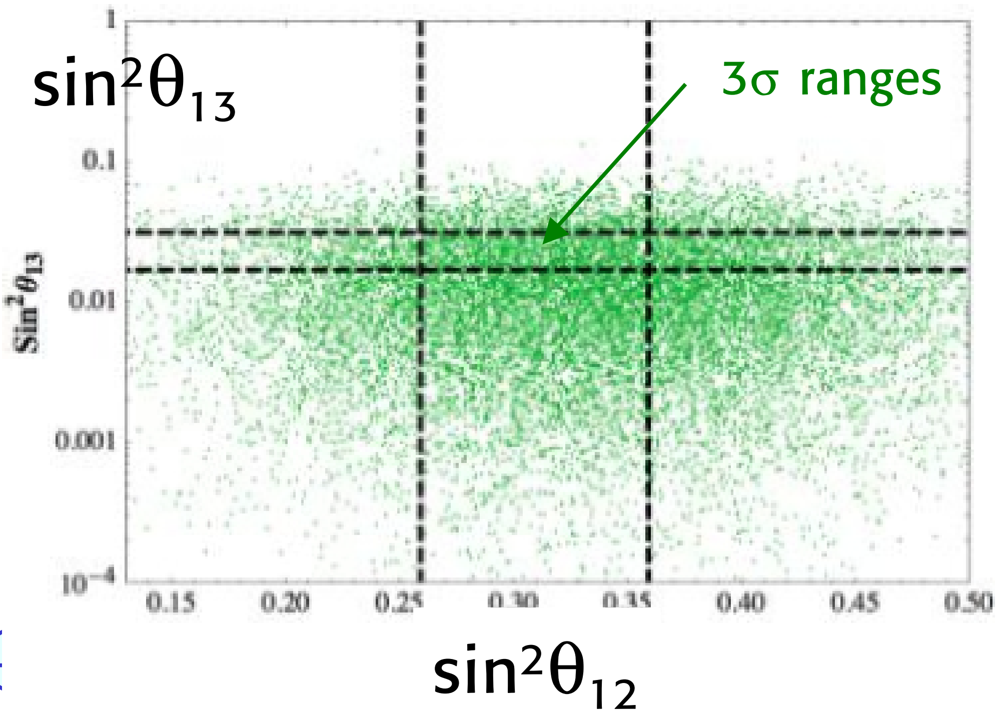
$c^e$: ch. lept. $c^\nu$: neutrinos

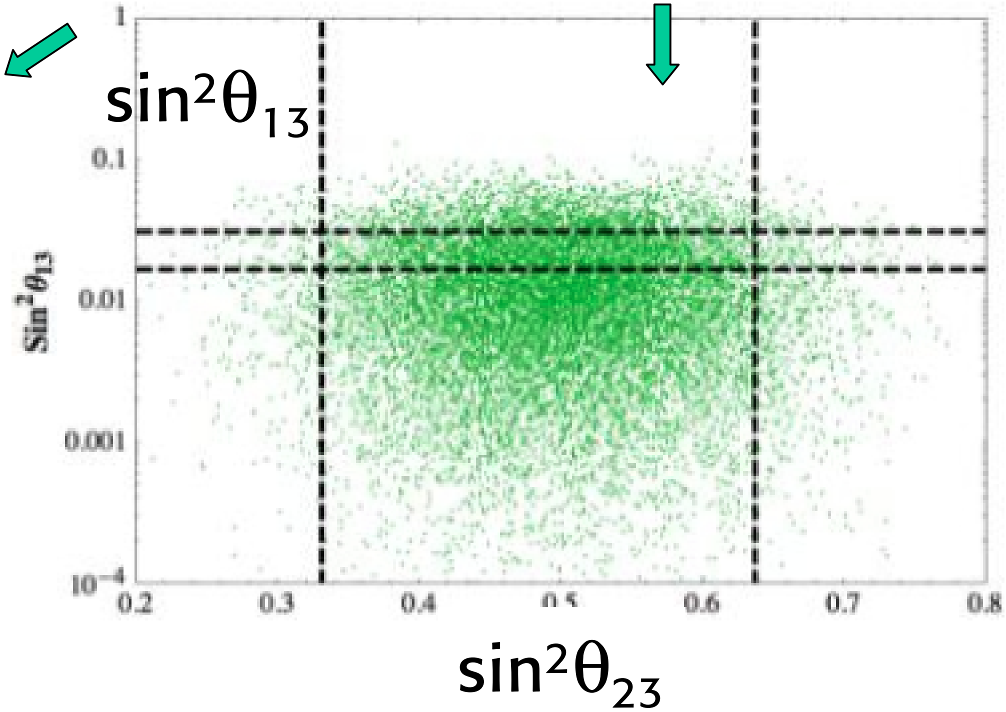
$$\sin^2\theta_{23} = \frac{1}{2} + \mathcal{R}e(c^e_{23})\,\xi + \frac{1}{\sqrt{3}}\left(\mathcal{R}e(c^\nu_{13}) - \sqrt{2}\,\mathcal{R}e(c^\nu_{23})\right)\xi$$

$$\sin^2\theta_{12} = \frac{1}{3} - \frac{2}{3}\mathcal{R}e(c^e_{12} + c^e_{13})\,\xi + \frac{2\sqrt{2}}{3}\mathcal{R}e(c^\nu_{12})\,\xi$$

$$\sin\theta_{13} = \frac{1}{6}\left|3\sqrt{2}\,(c^e_{12} - c^e_{13}) + 2\sqrt{3}\left(\sqrt{2}\,c^\nu_{13} + c^\nu_{23}\right)\right|\xi\,.$$

$c^a_{ij}$: random complex with abs. value gaussian around 1 with variance 0.5



In the Lin version of A4 (Lin '12)
ch. leptons ($\xi$) and $\nu$'s ($\xi'$)
are kept separate also at NLO.

$$\sin\theta_{13} = \left|\sqrt{\frac{2}{3}}\,\xi' + \frac{c^e_{12} - c^e_{13}}{\sqrt{2}}\,\xi\right|$$

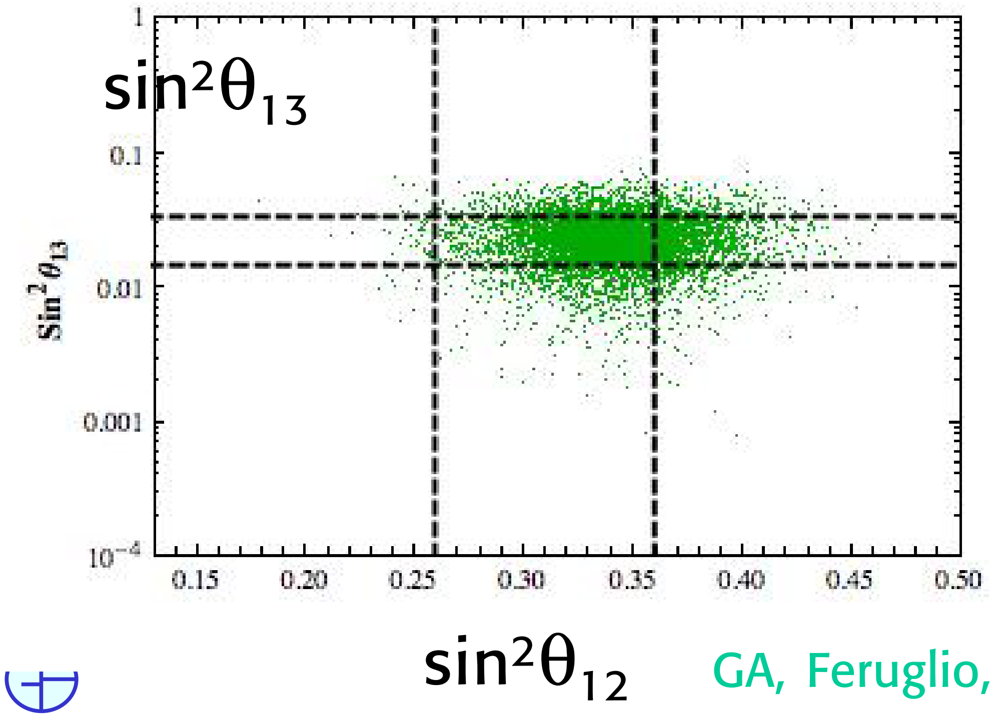
$$\sin^2\theta_{12} = \frac{1}{3 - 2\,|\xi'|^2} - \frac{2}{3}\,\mathcal{R}e(c^e_{12} + c^e_{13})\,\xi$$

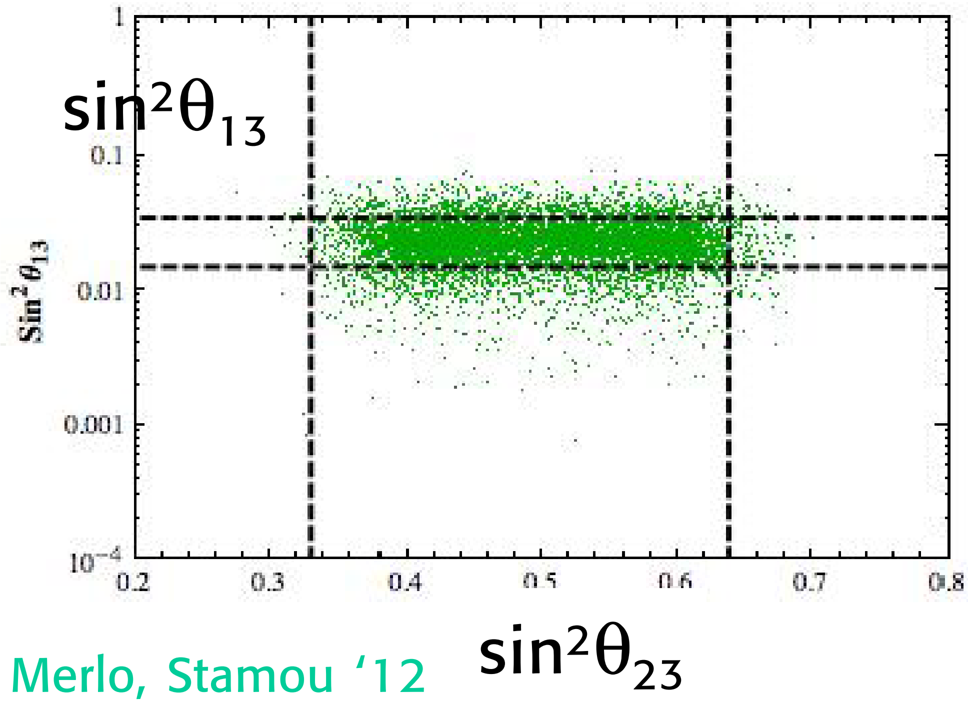
$$\sin^2\theta_{23} = \frac{1}{2} + \frac{1}{\sqrt{3}}\,|\xi'|\,\cos\delta + \mathcal{R}e(c^e_{23})\,\xi$$

Thus a separate minimisation allows for different scales

$|\xi'| \sim 0.184$ and $\xi \sim 0.005$-$0.06$

Less fine tuning
Larger success rate ~55%



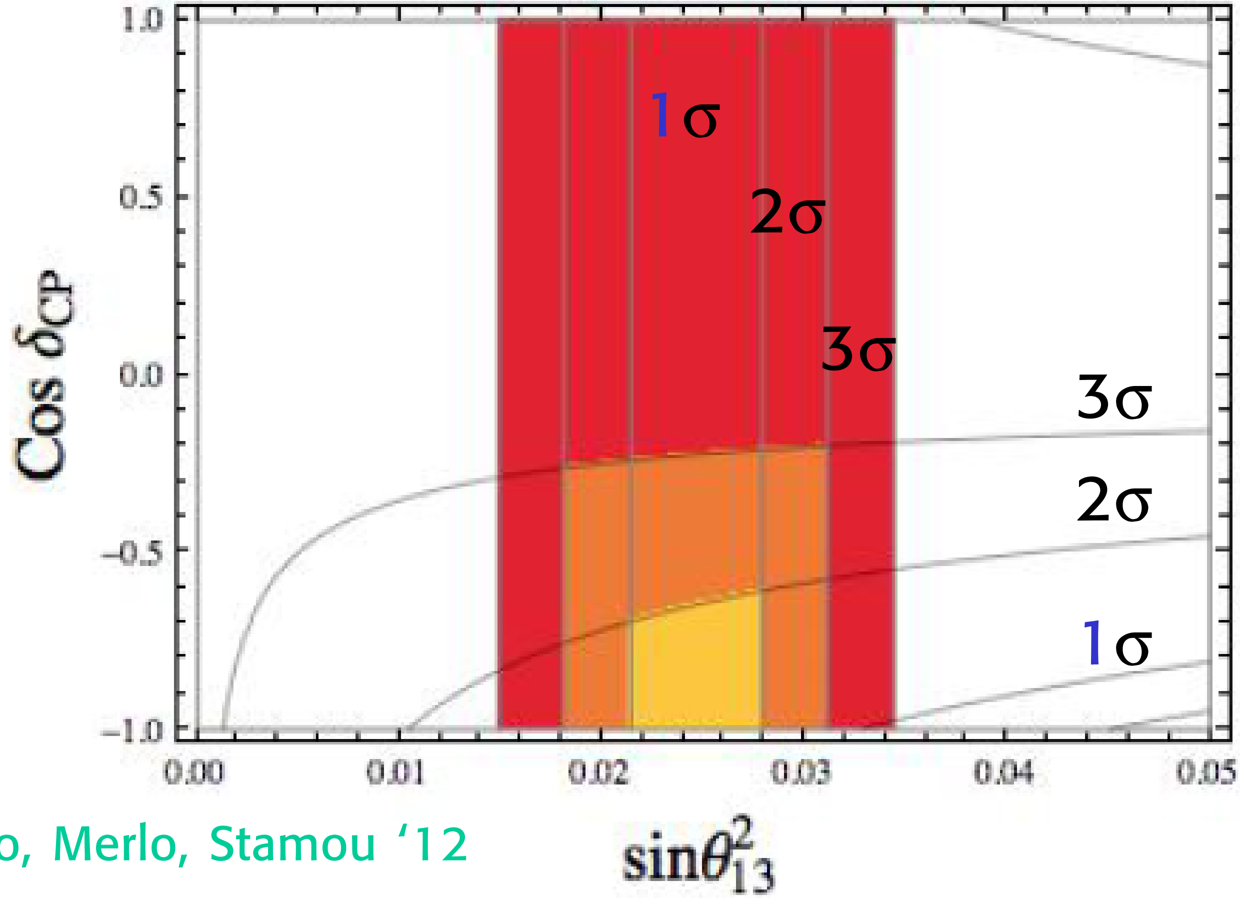




GA, Feruglio, Merlo, Stamou '12

In Lin model by neglecting the small corrections proportional to $\xi$ a sum rule is obtained:

$$\sin^2\theta_{23} = \frac{1}{2} + \frac{1}{\sqrt{2}} \sin\theta_{13} \cos\delta_{CP}$$

which is in agreement with exp. indication of $\cos\delta_{CP} < 0$




GA, Feruglio, Merlo, Stamou '12

# Bimaximal Mixing

Taking the "complementarity" relation seriously:

$$\theta_{12} + \theta_C = (46.4 \pm 0.8)^o \sim \pi/4$$

Raidal'04

leads to consider models that give $\theta_{12} = \pi/4$ but for corrections from the diag'tion of charged leptons

$$U_{PMNS} = U_\ell^\dagger U_\nu$$

Recall:

$$\lambda_C \approx 0.22 \text{ or } \sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$$

Normally one obtains $\theta_{12} + o(\theta_C) \sim \pi/4$ "weak compl." rather than $\theta_{12} + \theta_C \sim \pi/4$

$s^e_{23}$ is negligible

$$U_e = \begin{pmatrix} 1 & c^e_{12}\,\xi & c^e_{13}\,\xi \\ -c^{e*}_{12}\,\xi & 1 & 0 \\ -c^{e*}_{13}\,\xi & 0 & 1 \end{pmatrix}$$

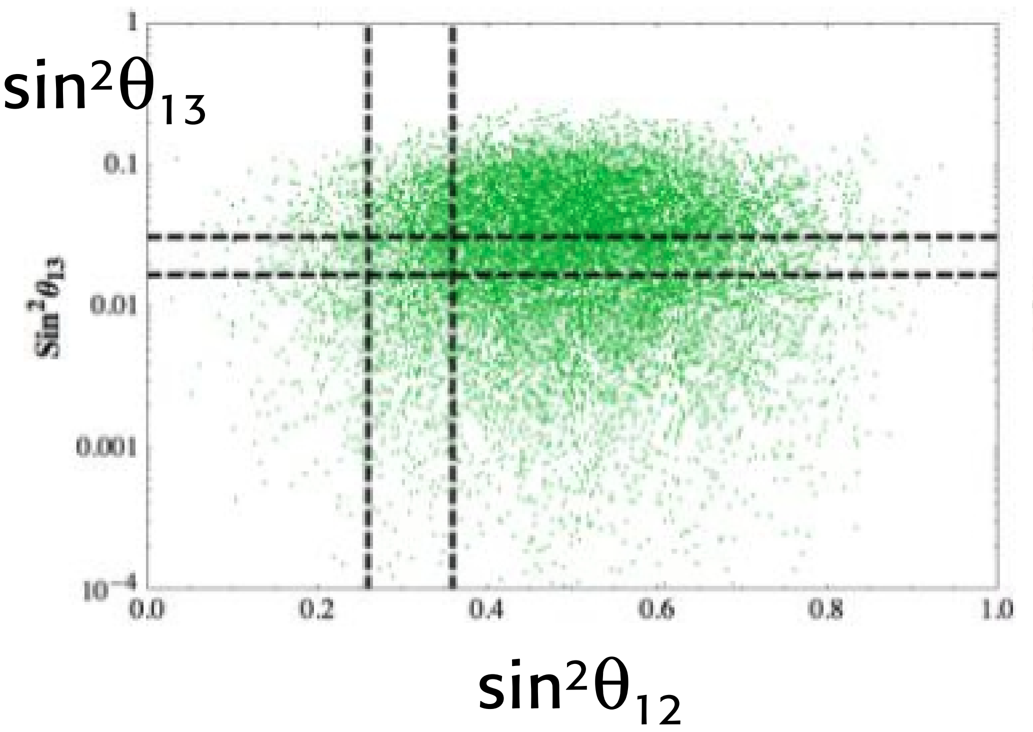
$$\delta_{CP} = \pi + \arg\left(c^e_{12} - c^e_{13}\right)$$

$$\sin\theta_{13} = \frac{1}{\sqrt{2}}\,|c^e_{12} - c^e_{13}|\,\xi$$

$$\sin^2\theta_{12} = \frac{1}{2} - \frac{1}{\sqrt{2}}\mathcal{R}e(c^e_{12} + c^e_{13})\,\xi$$

$$\sin^2\theta_{23} = \frac{1}{2}\,.$$

$\xi \sim\ 0.172$




In a random generation of coefficients the success rate is small (2.6%).
The main problem here is to get $\sin^2\theta_{12}$ right by chance

GA, Feruglio, Merlo, Stamou '12
GA, Feruglio, Merlo '09
D. Meloni '11

$$\delta_{CP} = \pi + \arg\left(c^e_{12} - c^e_{13}\right)$$

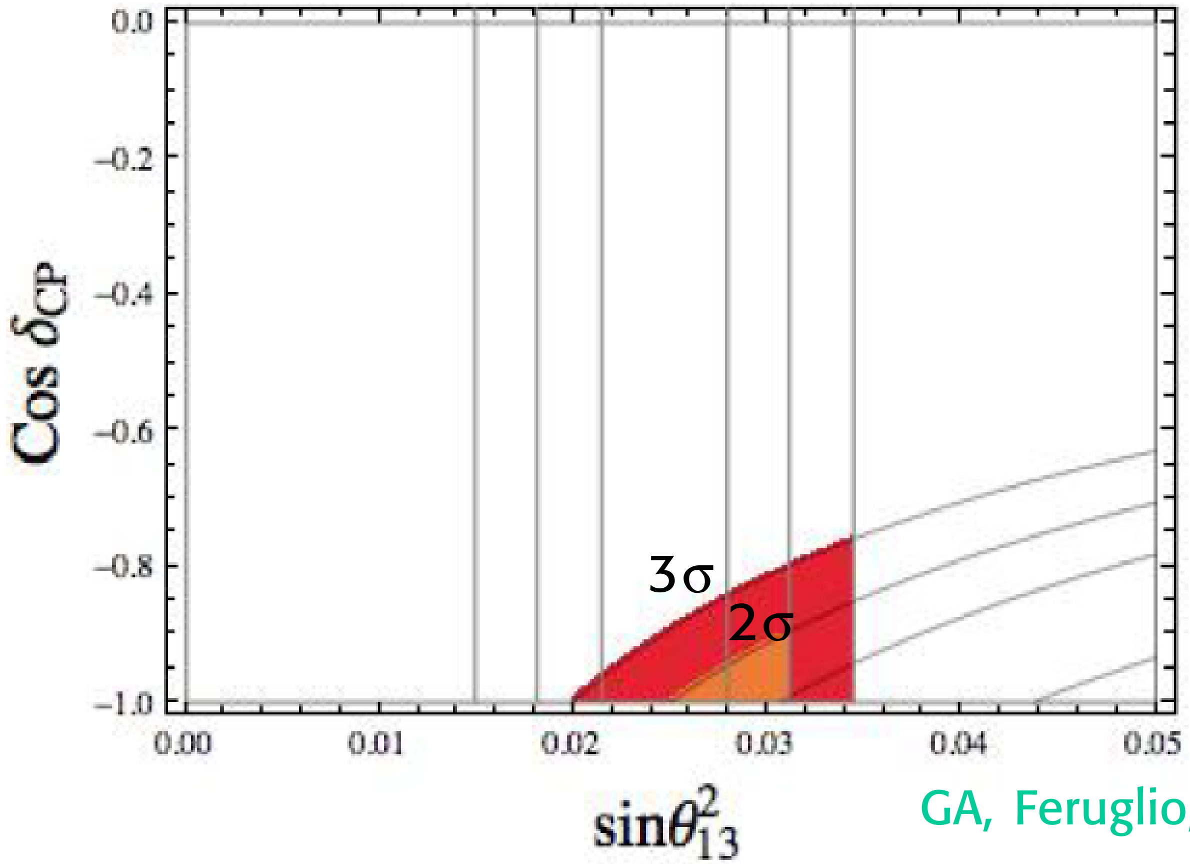
$$\sin\theta_{13} = \frac{1}{\sqrt{2}}\,|c^e_{12} - c^e_{13}|\,\xi$$

$$\sin^2\theta_{12} = \frac{1}{2} - \frac{1}{\sqrt{2}}\mathcal{R}e(c^e_{12} + c^e_{13})\,\xi$$

$$\sin^2\theta_{23} = \frac{1}{2}\,.$$

For dominance of a single $c^e$, e.g. $c^e{}_{13}$=0 we have a sum rule

$$\sin^2\theta_{12} = \frac{1}{2} + \sin\theta_{13}\,\cos\delta_{CP}$$




Then $\cos\delta_{CP} \sim -1$ is predicted

GA, Feruglio, Merlo, Stamou '12

## Constraints from lepton flavour violation (LFV)

These SUSY GUT models with A4 or S4 flavour symmetry imply LFV, thru non diagonal lepton mass terms

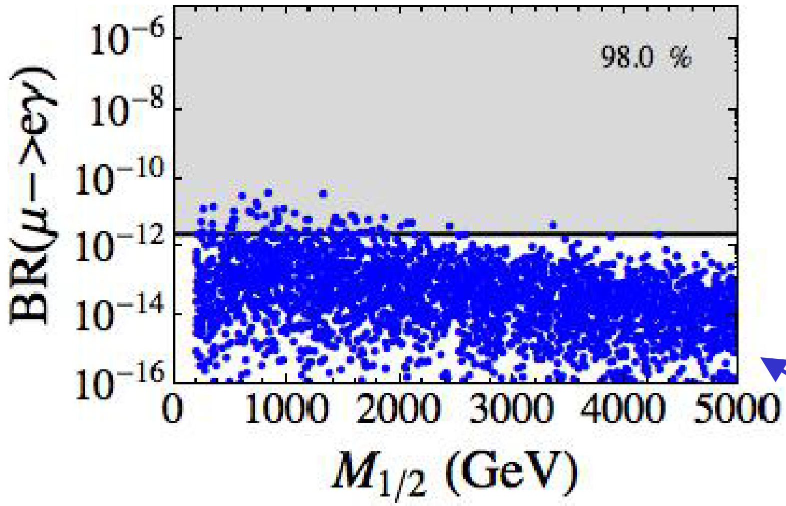
Existing bounds on LFV, e.g. from $\mu \to e\,\gamma$, $\tau \to \mu\,\gamma$, lead to constraints that are particularly strong for the S4 model of Bi-mixing with (large) corrections from charged leptons

The MEG recent bound on $Br(\mu \to e\,\gamma) < 2.4\ 10^{-12}$

poses a serious constraint on SUSY models with non diagonal mass matrices at the GUT scale

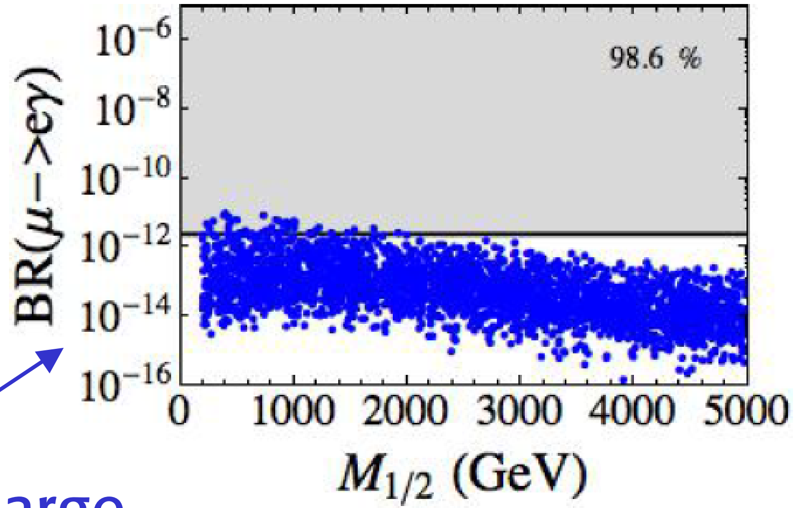
# Br($\mu$ -> e $\gamma$) < 2.4 $10^{-12}$: a serious constraint

**CMSSM**

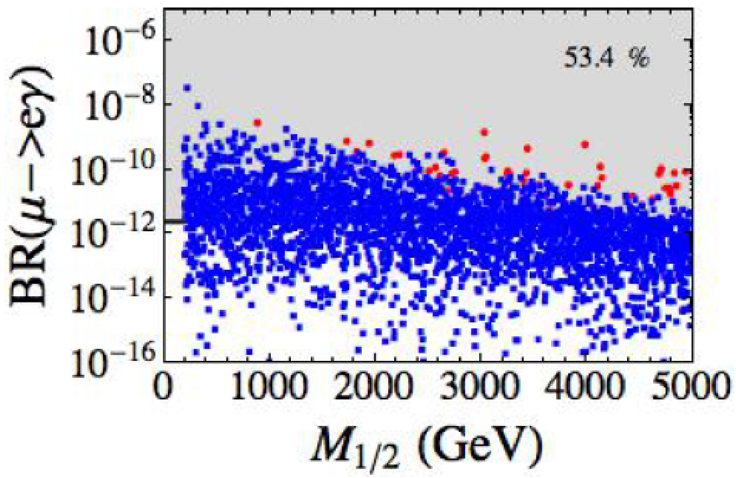



Typical A4, $\xi$ = 0.076

$m_0$ ~ 5 TeV large
tan$\beta$ ~ 2




Lin-type A4, $\xi'$ = 0.184
[main effect o($\xi'^2$)]




S4, $\xi$ = 0.172

S4 is disadvantaged as large off diagonal ch. lepton mass terms are needed (of o($\lambda_C$))

Needs either $m_0$ or $M_{1/2}$ heavy

GA, Feruglio, Merlo, Stamou '12

## Conclusion

Data on mixing angles are much better now but models of neutrino mixing still span a wide range from anarchy to discrete flavour groups

In the near future it will not be easy to decide from the data which ideas are right

So far no real illumination came from leptons to be combined with the quark sector for a more complete theory of flavour

Future: Normal vs Inverse hierarchy, phase $\delta, 0\nu\beta\beta, \Sigma m_\nu$....