Hadron interactions from lattice QCD

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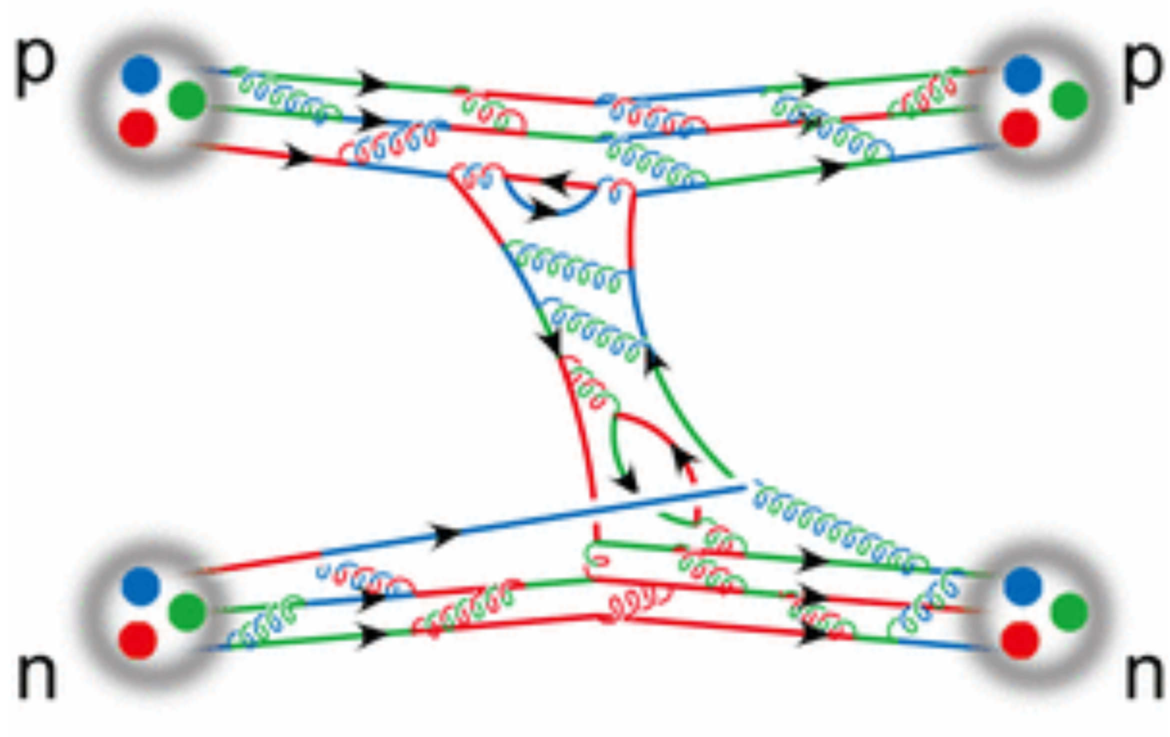
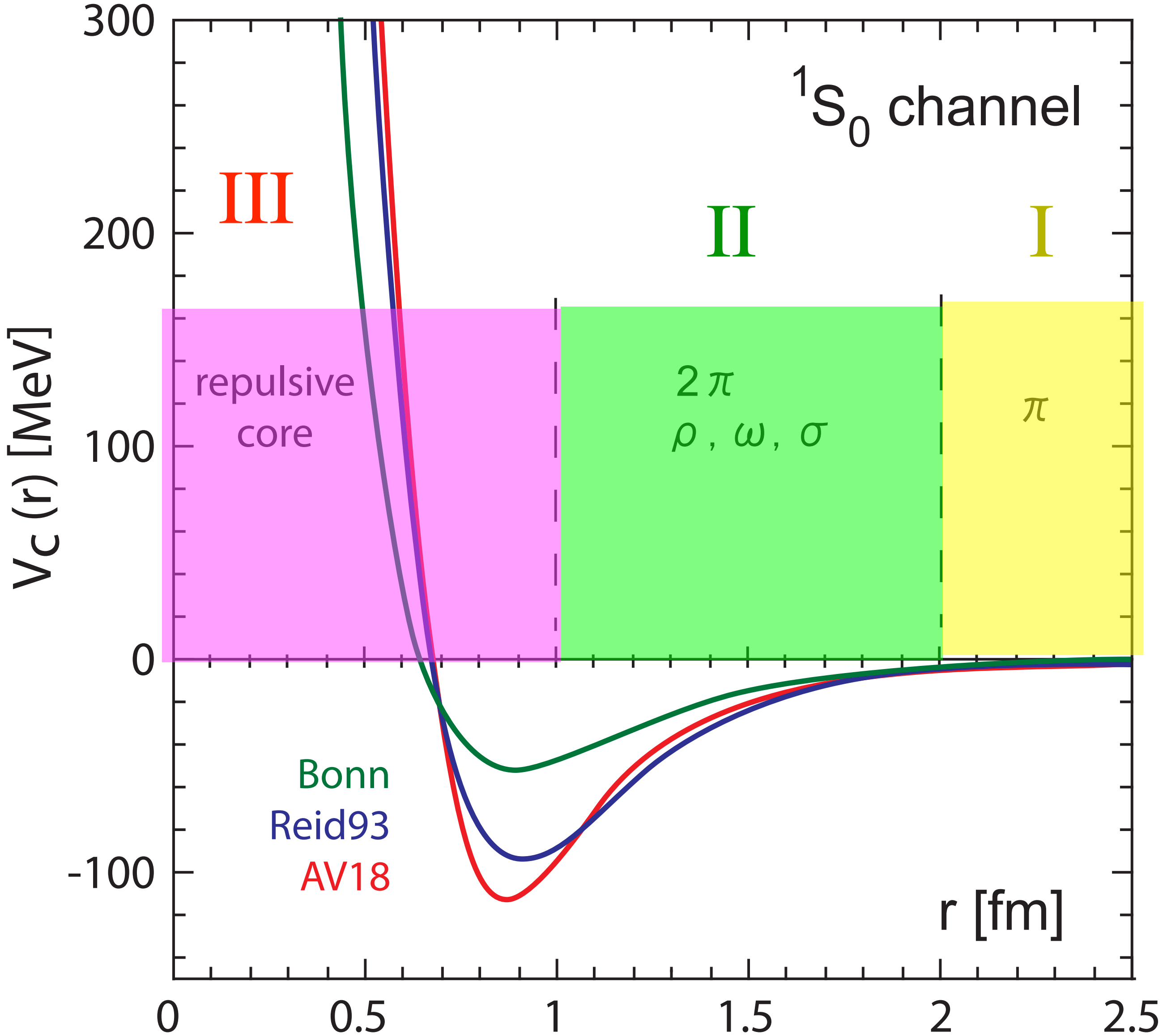


High Energy Theory Group, Sapienza University of Rome-INFN Rome
November 28, 2012

1. Introduction

How can we extract hadronic interaction from lattice QCD ?

Ex. **Phenomenological NN potential**
(~40 parameters to fit 5000 phase shift data)



I One-pion exchange

Yukawa(1935)

II Multi-pions

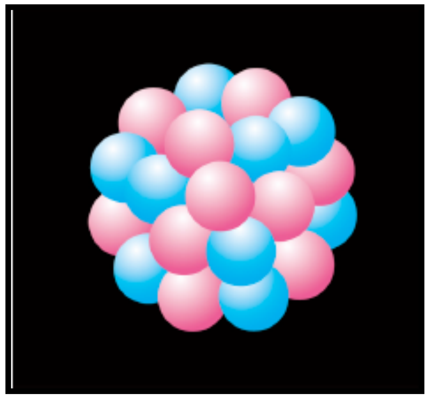
Taketani et al.(1951)

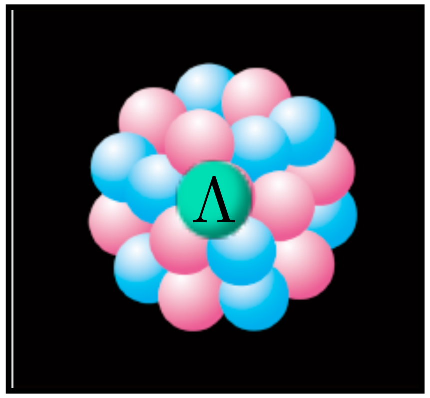
III Repulsive core

Jastrow(1951)

Nuclear force is a basis for understanding ...

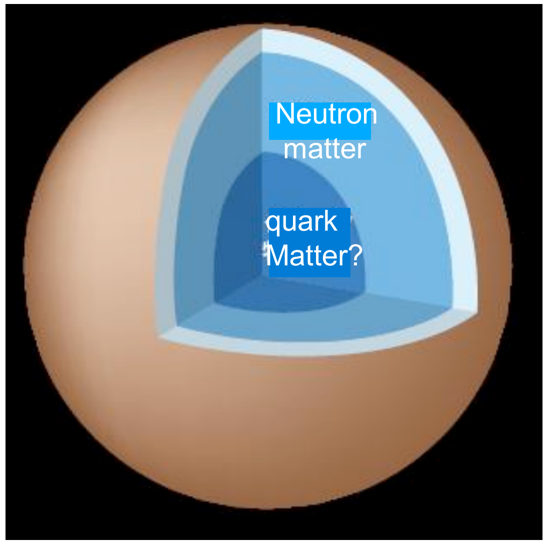
- Structure of ordinary and hyper nuclei



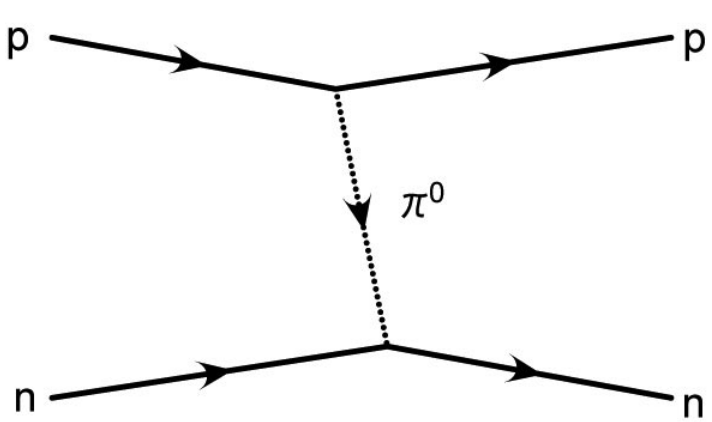


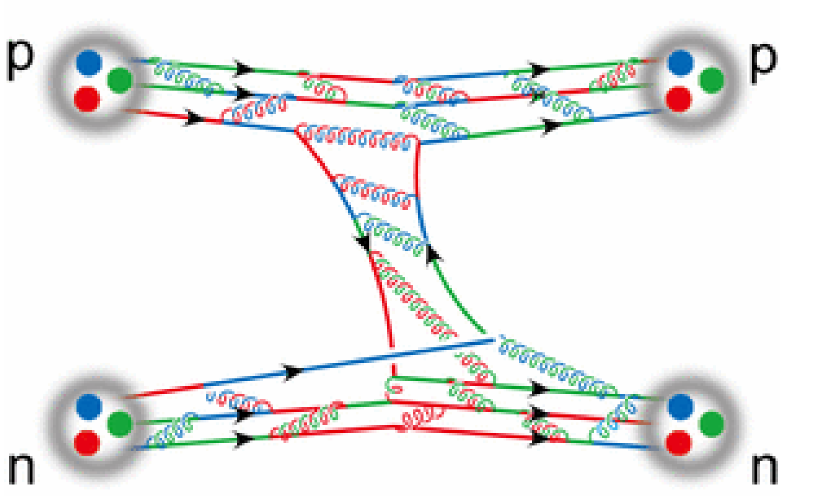
- Ignition of Type II SuperNova

- Structure of neutron star



Can we extract a nuclear force in (lattice) QCD ?





Plan of my talk

1. Introduction
2. Strategy
3. Nuclear potential
4. Hyperon Interactions
5. Some applications to Nuclear Physics
6. Conclusion

2. Strategy

3 strategies to nuclear structure from lattice QCD

Extreme: calculate nuclear structure directly from lattice QCD

Ab-Initio but (almost) impossible

difficult to extract “physics” from results

difficult to apply results to other systems

Standard: calculate NN phase shift from lattice QCD

Ab-Initio for phase shift

results can not be directly used to calculate nuclear structure

phase shift ➡ nuclear potential ➡ nuclear structure

Alternative: calculate “nuclear potential” from lattice QCD — our strategy

Ab-Initio for potential — “Physics” is clear

nuclear potential ➡ nuclear structure

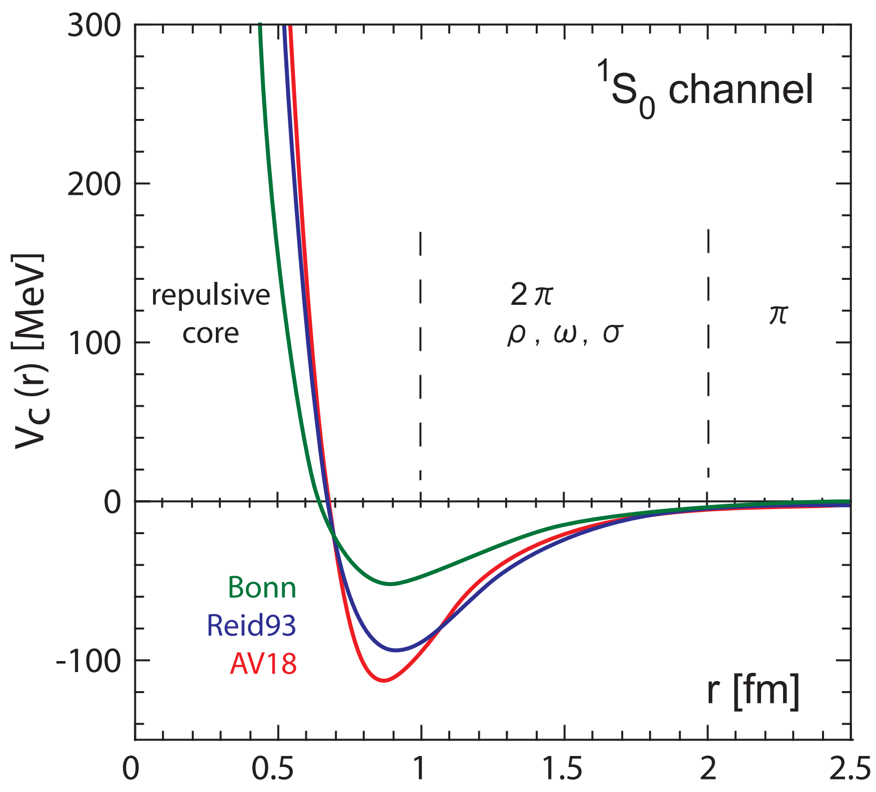
Potentials in QCD ?

What are "potentials" (quantum mechanical objects) in quantum field theories such as QCD ?

"Potentials" themselves can NOT be directly measured. cf. running coupling in QCD

scheme dependent, Unitary transformation

experimental data of scattering phase shifts → potentials, but not unique



"Potentials" are useful tools to extract observables such as scattering phase shift.

↓

One may adopt a convenient definition of potentials as long as they reproduce correct physics of QCD.

Our strategy in lattice QCD

Full details: Aoki, Hatsuda & Ishii, PTP123(2010)89.

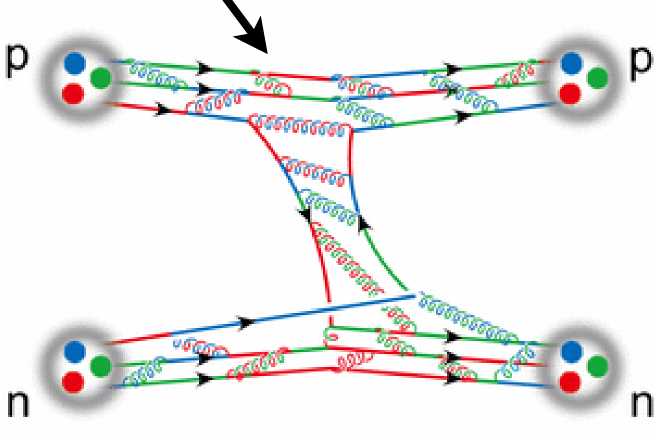
Step 1 define (Equal-time) Nambu-Bethe-Salpeter (NBS) Wave function

Spin model: Balog et al., 1999/2001

$$\varphi_{\mathbf{k}}(\mathbf{r}) = \langle 0 | N(\mathbf{x}+\mathbf{r},0) N(\mathbf{x},0) | NN, W_k \rangle$$

$W_k = 2\sqrt{\mathbf{k}^2 + m_N^2}$ energy

$N(x) = \varepsilon_{abc} q^a(x) q^b(x) q^c(x)$: local operator



Important property

partial wave $\qquad \varphi_{\mathbf{k}}^l \rightarrow A_l \dfrac{\sin(kr - l\pi/2 + \delta_l(k))}{kr} \qquad r = |\mathbf{r}| \rightarrow \infty$

Lin et al., 2001; CP-PACS, 2004/2005

$\delta_l(k)$ scattering phase shift (phase of the S-matrix by unitarity) in QCD !

How can we extract it ?

cf. Maiani-Testa theorem

cf. Luescher's finite volume method

Step 2 define non-local but energy-independent "potential" as

$$\left[\epsilon_k - H_0\right]\varphi_{\mathbf{k}}(\mathbf{x}) = \int d^3y\, U(\mathbf{x},\mathbf{y})\varphi_{\mathbf{k}}(\mathbf{y})$$

$\mu = m_N/2$

reduced mass

$$\epsilon_k = \frac{\mathbf{k}^2}{2\mu} \qquad H_0 = \frac{-\nabla^2}{2\mu}$$

non-local potential

Properties & Remarks

1. Potential itself is NOT an observable. Using this freedom, we can construct a non-local but energy-independent potential as

inner product

$$U(\mathbf{x},\mathbf{y}) = \sum_{\mathbf{k},\mathbf{k}'}^{W_k, W_{k'} \le W_{\text{th}}} \left[\epsilon_k - H_0\right]\varphi_{\mathbf{k}}(\mathbf{x})\eta^{-1}_{\mathbf{k},\mathbf{k}'}\varphi^\dagger_{\mathbf{k}'}(\mathbf{y})$$

$\eta^{-1}_{\mathbf{k},\mathbf{k}'}$: inverse of $\eta_{\mathbf{k},\mathbf{k}'} = (\varphi_{\mathbf{k}}, \varphi_{\mathbf{k}'})$

$\varphi_{\mathbf{k}}$ is linearly independent.

For $^\forall W_{\mathbf{p}} < W_{\text{th}} = 2m_N + m_\pi$ (threshold energy)

$$\int d^3y\, U(\mathbf{x},\mathbf{y})\phi_{\mathbf{p}}(\mathbf{y}) = \sum_{\mathbf{k},\mathbf{k}'}\left[\epsilon_k - H_0\right]\varphi_{\mathbf{k}}(x)\eta^{-1}_{\mathbf{k},\mathbf{k}'}\eta_{\mathbf{k}',\mathbf{p}} = \left[\epsilon_p - H_0\right]\varphi_{\mathbf{p}}(x)$$

Proof of existence (cf. Density Functional Theory)

Of course, potential satisfying this is not unique. (Scheme dependence. cf. running coupling)

2. Non-relativistic approximation is NOT used. We just take the specific (equal-time) flame.

Step 3 expand the non-local potential in terms of derivative as

$$U(\mathbf{x},\mathbf{y}) = V(\mathbf{x},\nabla)\delta^3(\mathbf{x}-\mathbf{y})$$

$$V(\mathbf{x},\nabla) = V_0(r) + V_\sigma(r)(\sigma_{\mathbf{1}}\cdot\sigma_{\mathbf{2}}) + V_T(r)S_{12} + V_{\rm LS}(r)\mathbf{L}\cdot\mathbf{S} + O(\nabla^2)$$

LO LO LO NLO NNLO

spins

tensor operator $S_{12} = \frac{3}{r^2}(\sigma_{\mathbf{1}}\cdot\mathbf{x})(\sigma_{\mathbf{2}}\cdot\mathbf{x}) - (\sigma_{\mathbf{1}}\cdot\sigma_{\mathbf{2}})$

$V_A(\mathbf{x})$ local and energy independent coefficient function
(cf. Low Energy Constants(LOC) in Chiral Perturbation Theory)

Step 4 extract the local potential at LO as

$$V_{\rm LO}(\mathbf{x}) = \frac{[\epsilon_k - H_0]\varphi_{\mathbf{k}}(\mathbf{x})}{\varphi_{\mathbf{k}}(\mathbf{x})}$$

Step 5 solve the Schroedinger Eq. in the infinite volume with this potential.

→ phase shifts and binding energy below inelastic threshold

(We can calculate the phase shift at all angular momentum.)

$\delta_L(k)$ exact by construction

$\delta_L(p\neq k)$ approximated one by the derivative expansion

expansion parameter

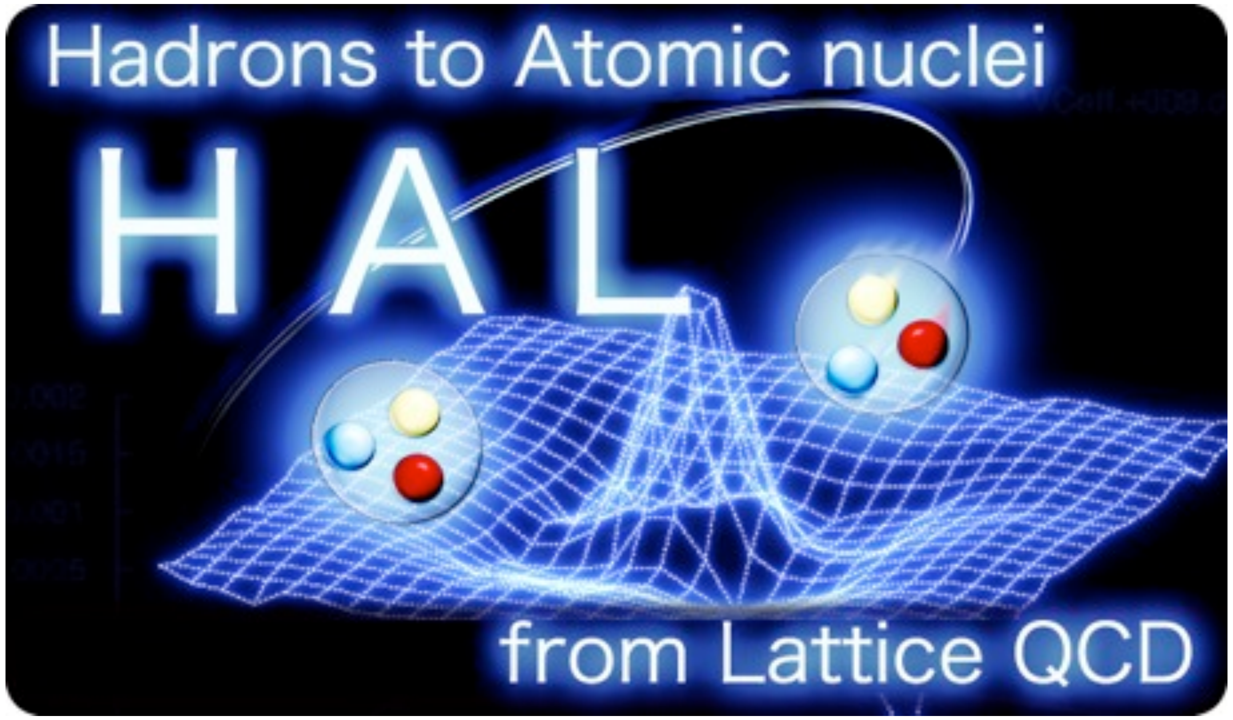
$$\frac{W_p - W_k}{W_{\rm th} - 2m_N} \simeq \frac{\Delta E_p}{m_\pi}$$

We can check a size of errors at LO of the expansion. (See later).
We can improve results by extracting higher order terms in the expansion.

This procedure gives a new method to extract phase shift from QCD.

HAL QCD method

HAL QCD Collaboration



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Keiko Murano (Riken)
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Kenji Sasaki (U. Tsukuba)
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Our strategy

Potentials from lattice QCD → Nuclear Physics with these potentials → Neutron stars Supernova explosion

3. Nuclear potential

Extraction of NBS wave function

NBS wave function

$$\varphi_{\mathbf{k}}(\mathbf{r}) = \langle 0 | N(\mathbf{x}+\mathbf{r},0) N(\mathbf{x},0) | NN, W_k \rangle$$

Potential

$$[\epsilon_k - H_0]\varphi_{\mathbf{k}}(\mathbf{x}) = \int d^3y \, U(\mathbf{x},\mathbf{y})\varphi_{\mathbf{k}}(\mathbf{y})$$

4-pt Correlation function

source for NN

$$F(\mathbf{r}, t-t_0) = \langle 0 | T\{N(\mathbf{x}+\mathbf{r},t) N(\mathbf{x},t)\} \overline{\mathcal{J}}(t_0) | 0 \rangle$$

complete set for NN

$$\begin{aligned} F(\mathbf{r}, t-t_0) &= \langle 0 | T\{N(\mathbf{x}+\mathbf{r},t) N(\mathbf{x},t)\} \sum_{n,s_1,s_2} |2N, W_n, s_1, s_2\rangle\langle 2N, W_n, s_1, s_2 | \overline{\mathcal{J}}(t_0) | 0 \rangle + \cdots \\ &= \sum_{n,s_1,s_2} A_{n,s_1,s_2} \varphi^{W_n}(\mathbf{r}) e^{-W_n(t-t_0)}, \quad A_{n,s_1,s_2} = \langle 2N, W_n, s_1, s_2 | \overline{\mathcal{J}}(0) | 0 \rangle. \end{aligned}$$

ground state saturation at large t

$$\lim_{(t-t_0)\to\infty} F(\mathbf{r}, t-t_0) = A_0 \varphi^{W_0}(\mathbf{r}) e^{-W_0(t-t_0)} + O(e^{-W_{n\neq 0}(t-t_0)})$$

NBS wave function

This is a standard method in lattice QCD and was employed for our first calculation.

Improved method

Ishii *et al.* (HALQCD), PLB712(2012) 437

normalized 4-pt Correlation function $R(\mathbf{r},t) \equiv F(\mathbf{r},t)/(e^{-m_N t})^2 = \sum_n A_n \varphi^{W_n}(\mathbf{r}) e^{-\Delta W_n t}$

$$\Delta W_n = W_n - 2m_N = \frac{\mathbf{k}_n^2}{m_N} - \frac{(\Delta W_n)^2}{4m_N}$$

$$-\frac{\partial}{\partial t} R(\mathbf{r},t) = \left\{ H_0 + U - \frac{1}{4m_N}\frac{\partial^2}{\partial t^2} \right\} R(\mathbf{r},t)$$

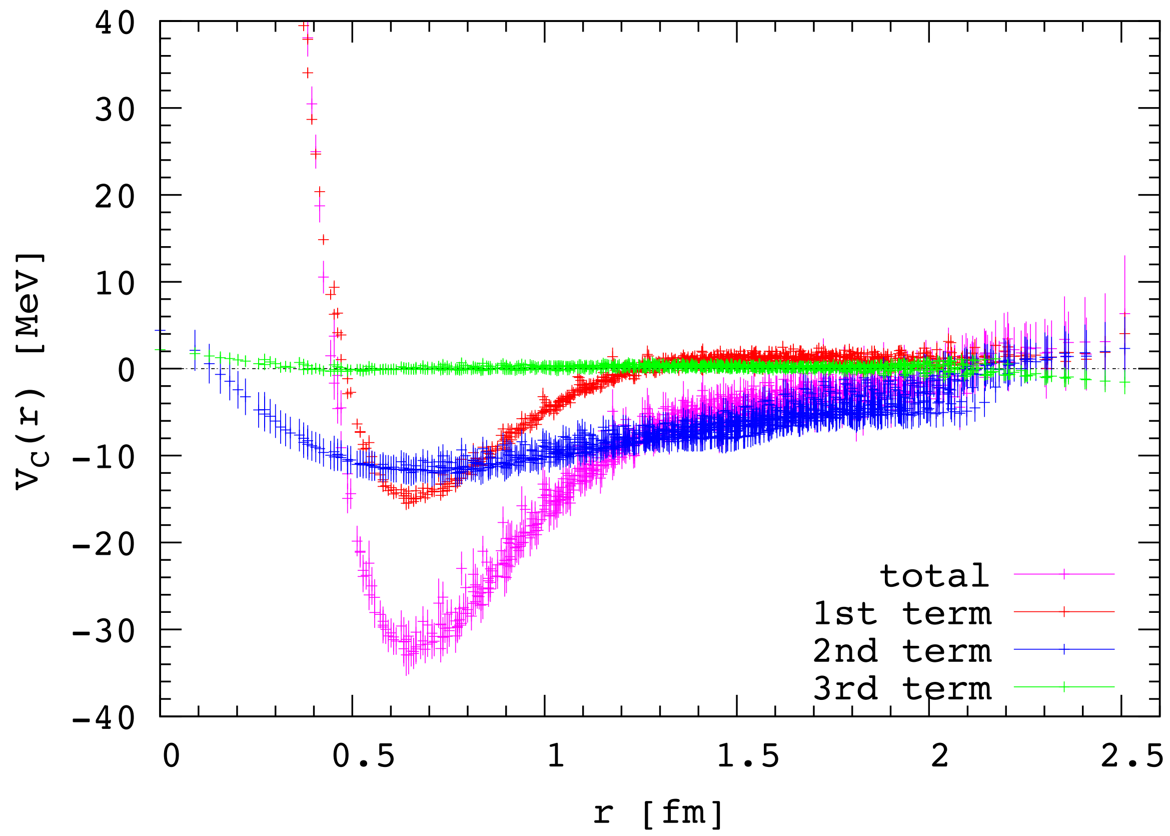
potential

Leading Order

$$\left\{ -H_0 - \frac{\partial}{\partial t} + \frac{1}{4m_N}\frac{\partial^2}{\partial t^2} \right\} R(\mathbf{r},t) = \int d^3r'\, U(\mathbf{r},\mathbf{r}') R(\mathbf{r}',t) = V_C(\mathbf{r}) R(\mathbf{r},t) + \cdots$$

1st **2nd** **3rd** **total**

3rd term(relativistic correction) is negligible.

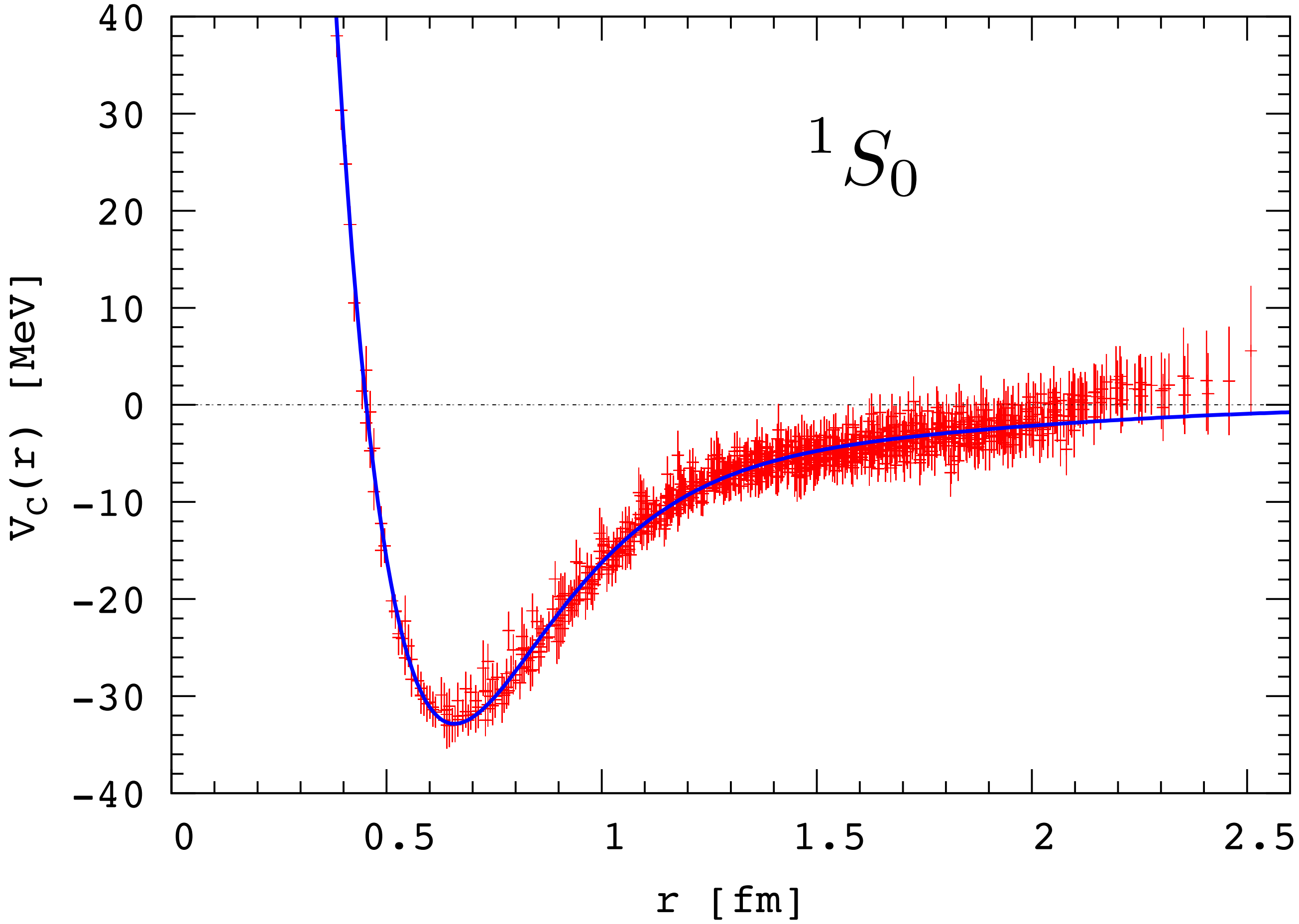


Ground state saturation is no more required ! (advantage over finite volume method.)

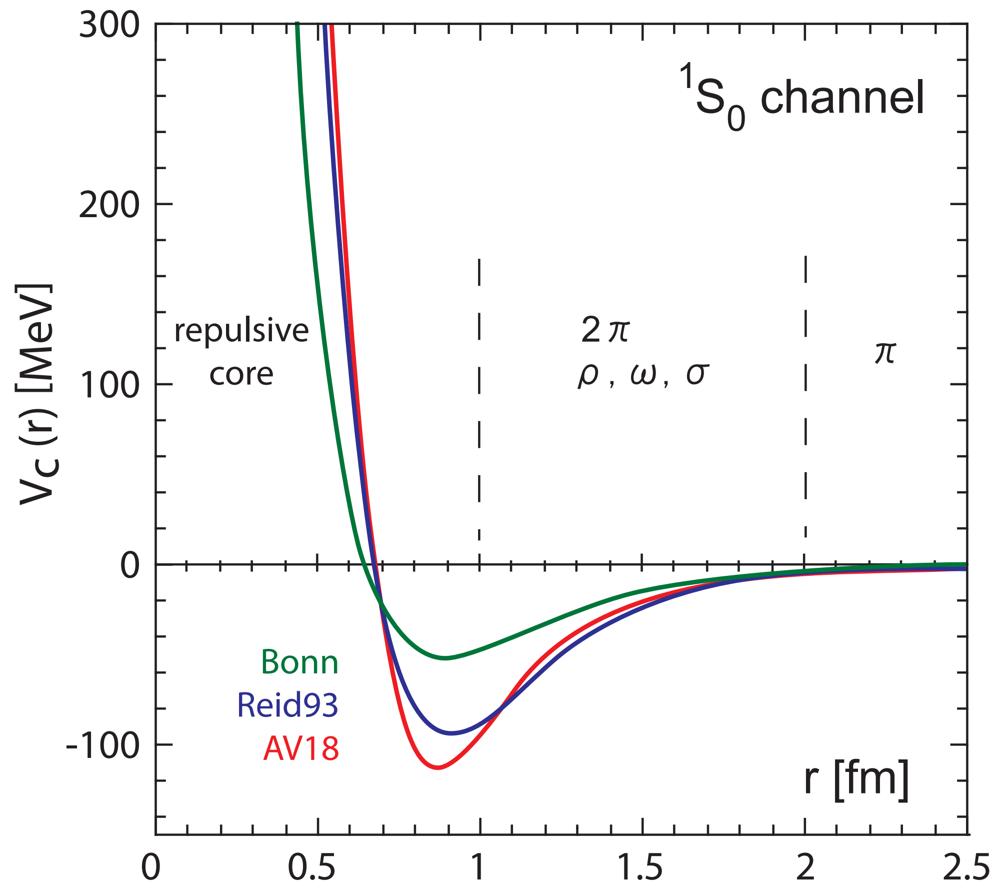
NN potential

2+1 flavor QCD, spin-singlet potential (PLB712(2012)437)

a=0.09fm, L=2.9fm $m_\pi \simeq 700$ MeV



phenomenological potential



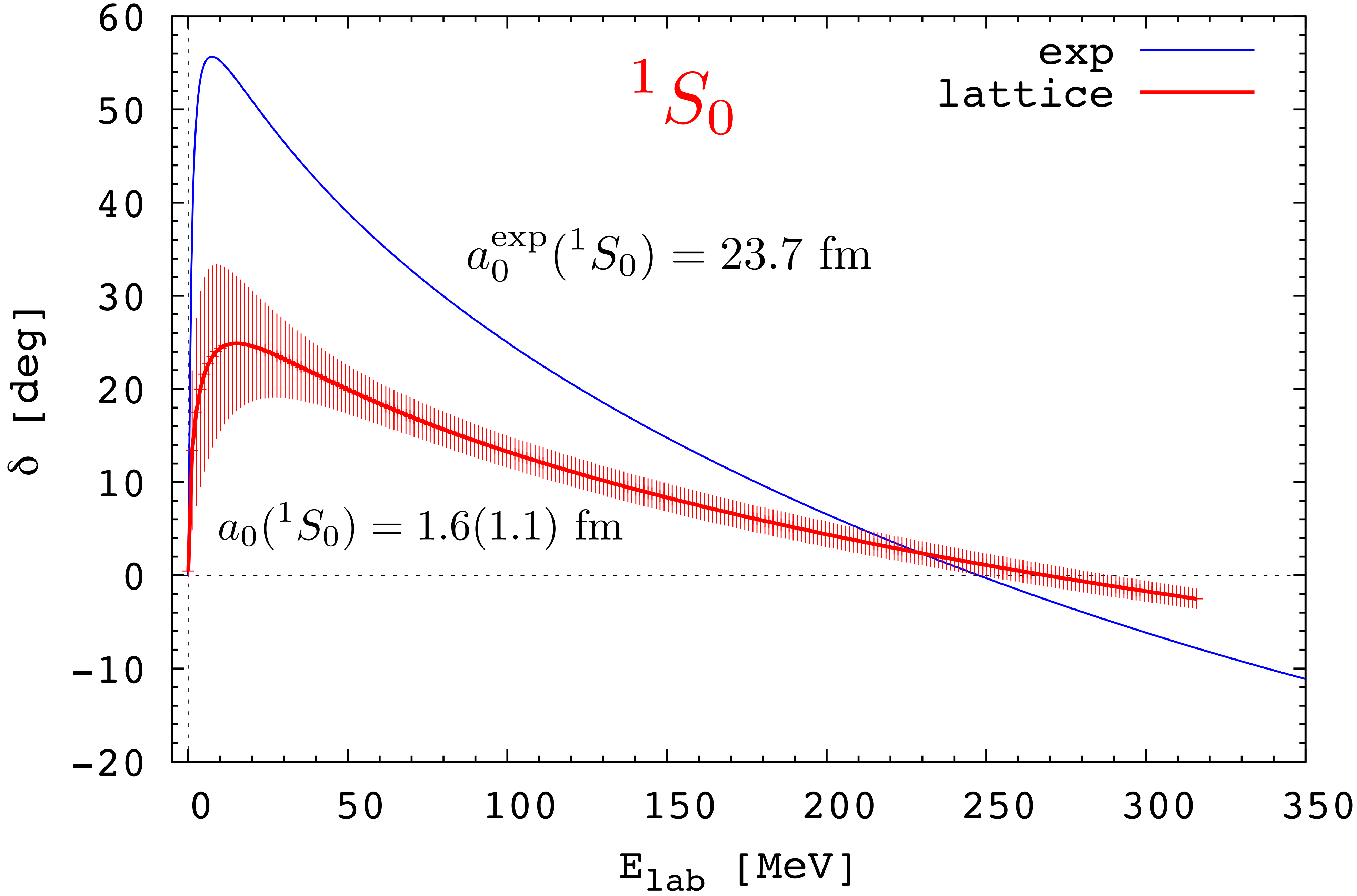
Qualitative features of NN potential are reproduced !

(1)attractions at medium and long distances
(2)repulsion at short distance(repulsive core)

1st paper(quenched QCD): Ishii-Aoki-Hatsuda, PRL90(2007)0022001

This paper has been selected as one of 21 papers in Nature Research Highlights 2007.
(One from Physics, Two from Japan, the other is on “iPS” by Sinya Yamanaka et al.)

NN potential → phase shift



It has a reasonable shape. The strength is weaker due to the heavier quark mass.

Need calculations at physical quark mass.

Convergence of velocity expansion

If the higher order terms are large, LO potentials determined from NBS wave functions at different energy become different.(cf. LOC of ChPT).

Numerical check in quenched QCD

$m_\pi \simeq 0.53$ GeV

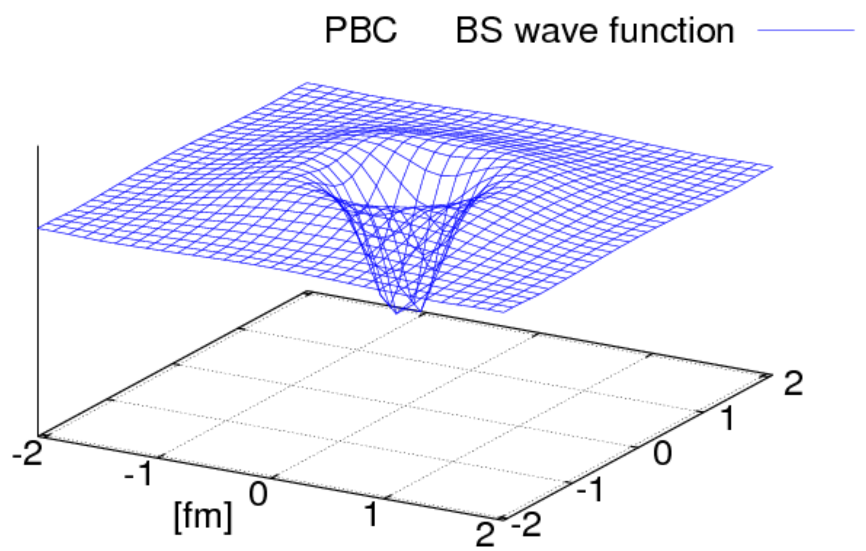
a=0.137fm, L=4.0 fm

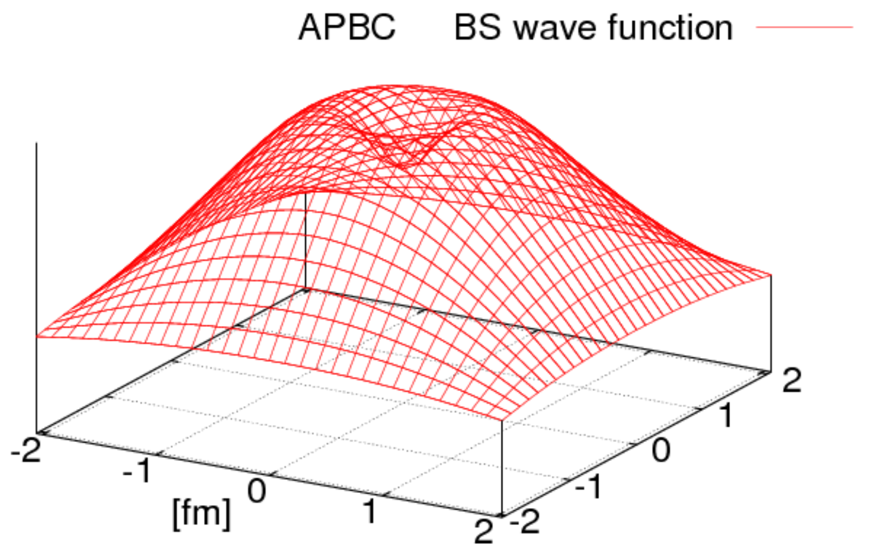
K. Murano, N. Ishii, S. Aoki, T. Hatsuda

PTP 125 (2011)1225.

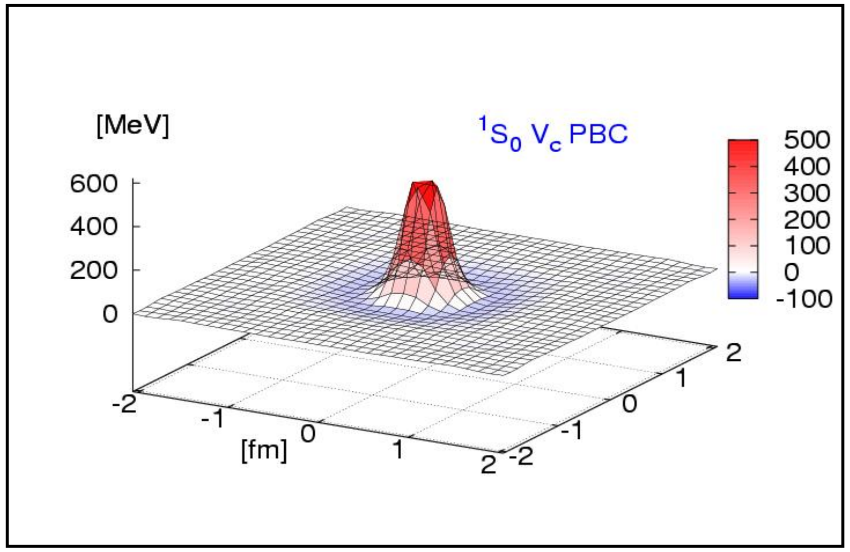
● PBC (E∼0 MeV) ● APBC (E∼46 MeV)

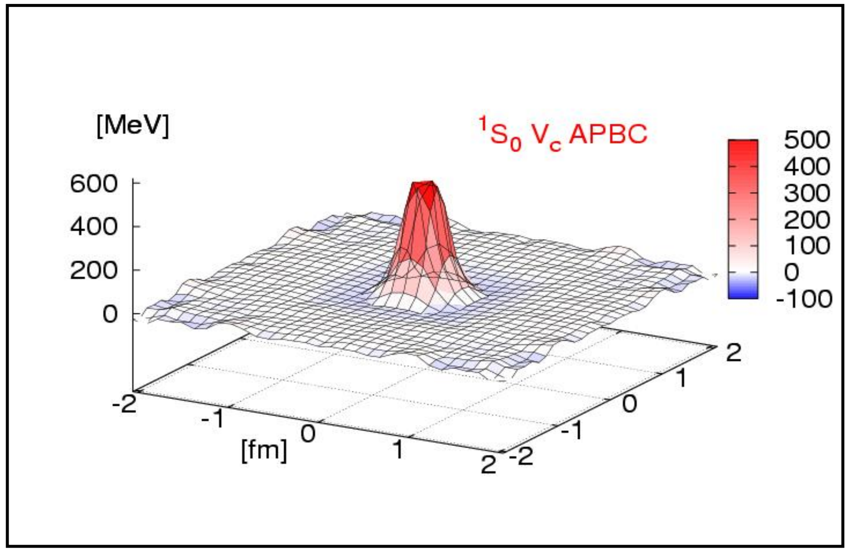
NBS wave functions

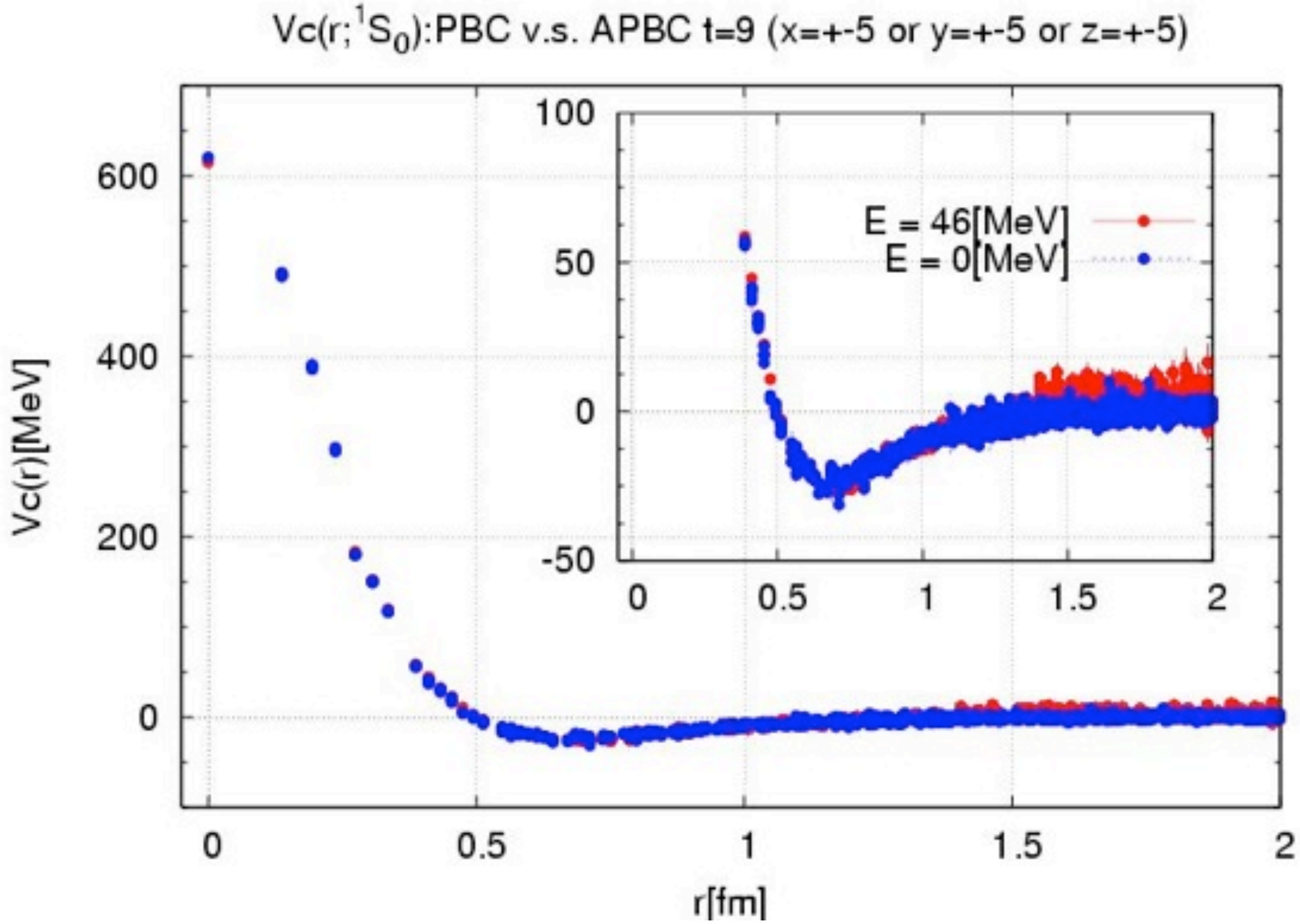


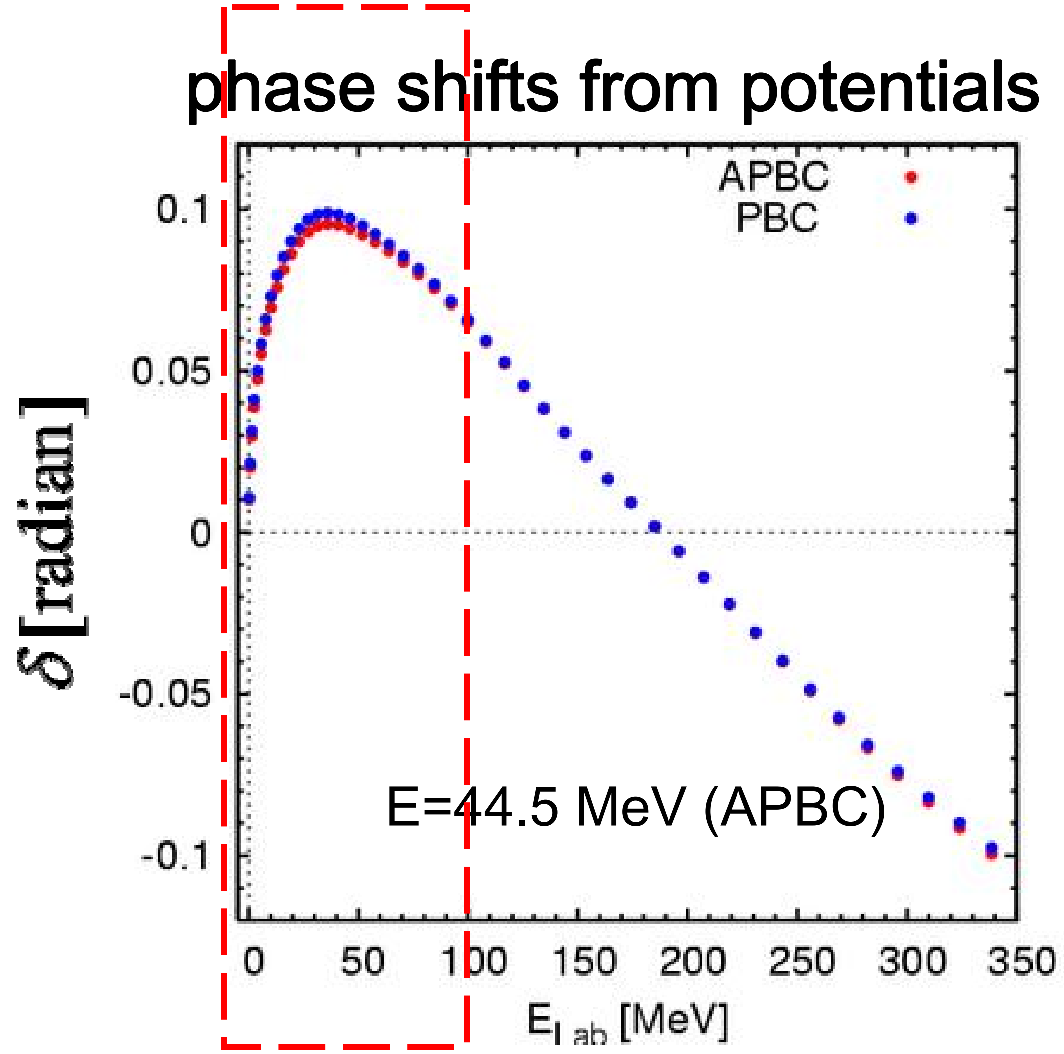


potentials









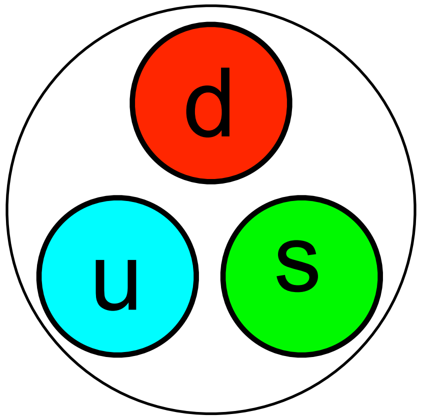
Higher order terms turn out to be very small at low energy in HAL scheme.

Need to be checked at lighter pion mass in 2+1 flavor QCD.

Note: convergence of the velocity expansion can be checked within this method.

(in contrast to convergence of ChPT, convergence of perturbative QCD)

4. Hyperon interactions



$$p = (uud), n = (udd)$$

nucleon(N)

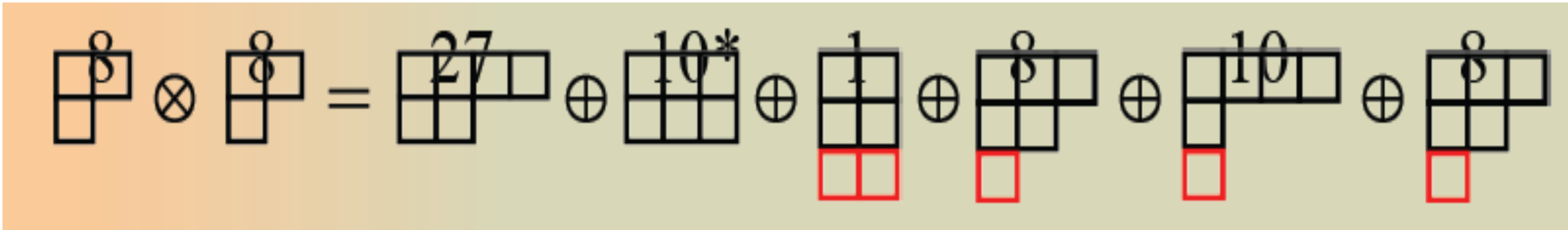
$$\Lambda = (uds)_{I=0}$$

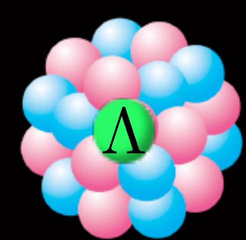
$$\Sigma^+ = (uus), \Sigma^0 = (uds)_{I=1}, \Sigma^- = (dds)$$

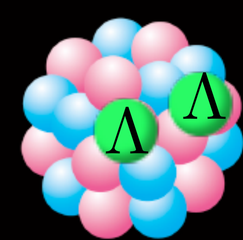
hyperon(Y)

$$\Xi^0 = (uss), \Xi^- = (dss)$$

Octet Baryon interactions







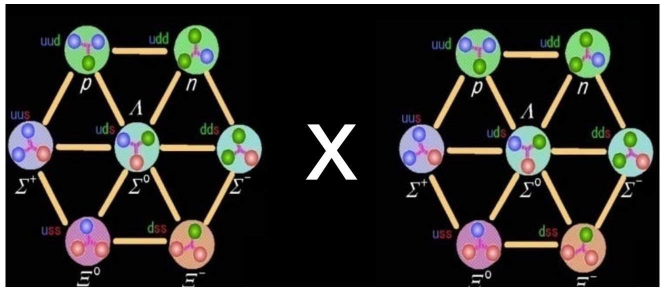
- no phase shift available for YN and YY scattering
- plenty of hyper-nucleus data will be soon available at J-PARC

- prediction from lattice QCD
- difference between NN and YN ?

Baryon Potentials in the flavor SU(3) symmetric limit

$m_u = m_d = m_s$

1. First setup to predict YN, YY interactions not accessible in exp.
2. Origin of the repulsive core (universal or not)



$$8 \times 8 = \underline{27 + 8s + 1} + \underline{10^* + 10 + 8a}$$

Symmetric Anti-symmetric

6 independent potentials in flavor-basis

$$V^{(27)}(r),\ V^{(8s)}(r),\ V^{(1)}(r) \longleftarrow {}^1S_0$$

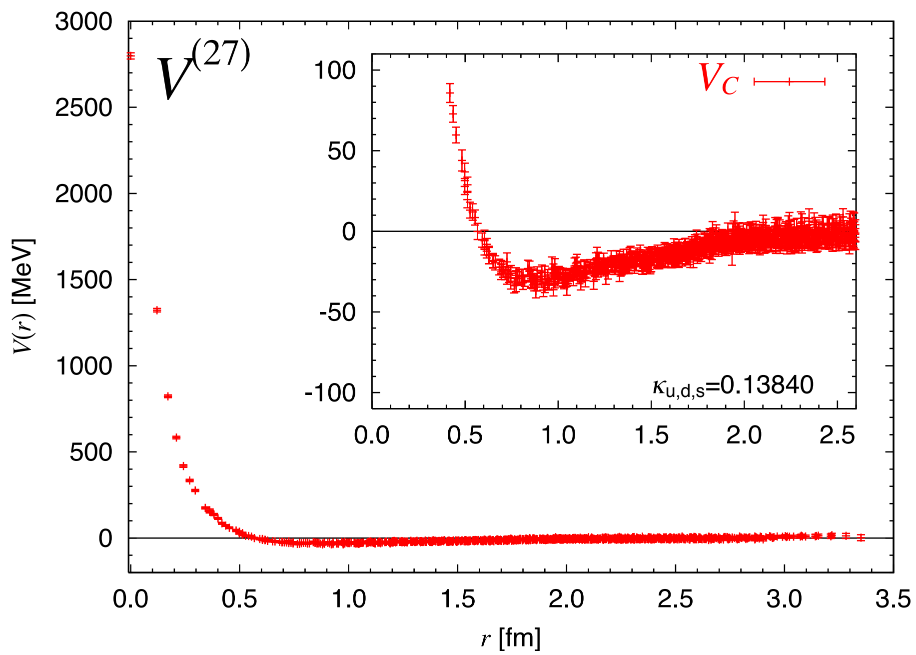
$$V^{(10^*)}(r),\ V^{(10)}(r),\ V^{(8a)}(r) \longleftarrow {}^3S_1$$

3-flavor QCD a=0.12 fm

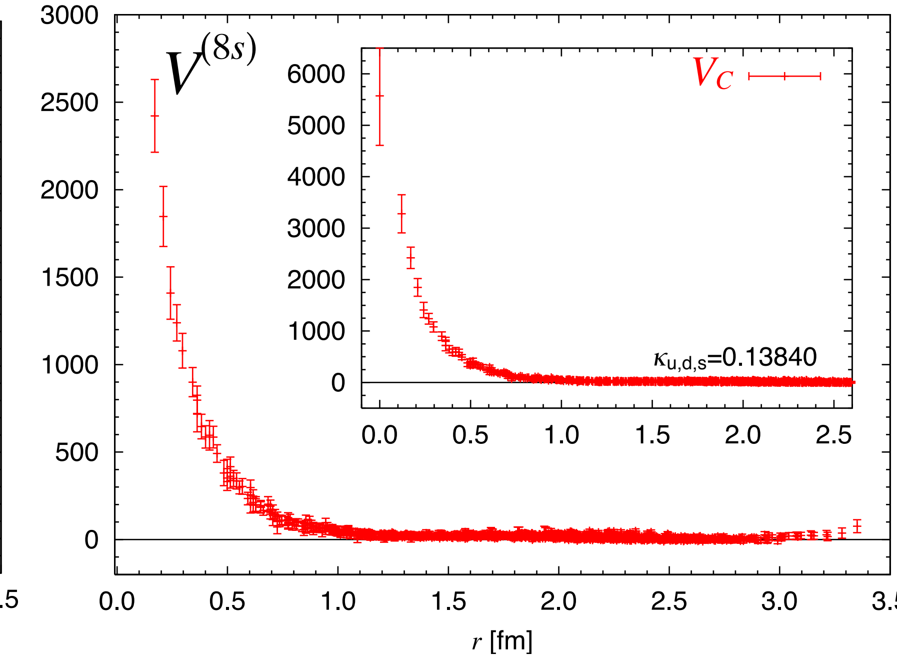
Inoue *et al.* (HAL QCD Coll.), PTP124(2010)591 L=2 fm

Inoue *et al.* (HAL QCD Coll.), NPA881(2012)28 L=2-4 fm

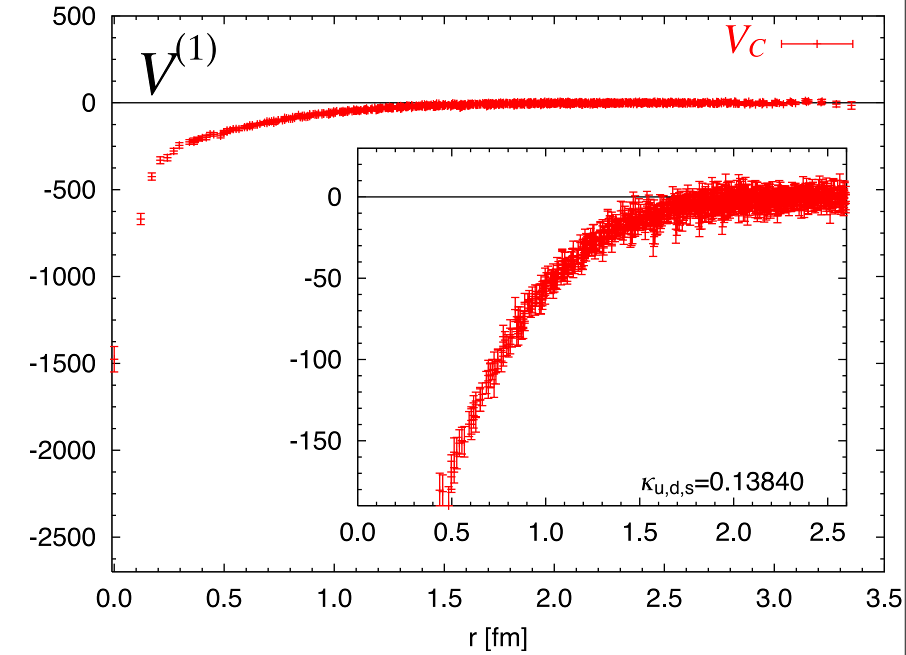
$L \simeq 4$ fm, $m_\pi \simeq 470$ MeV



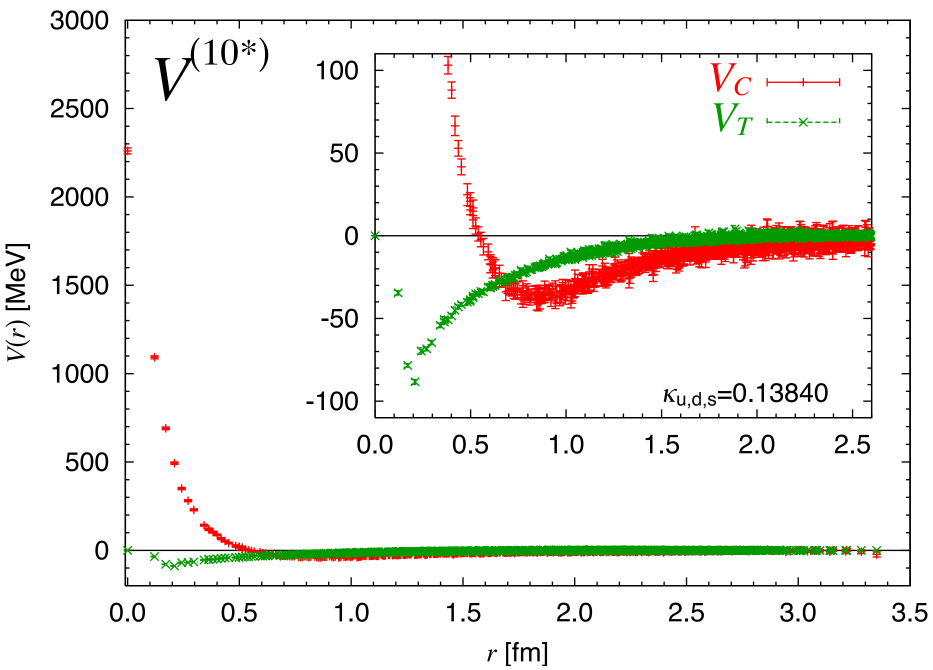
same as NN



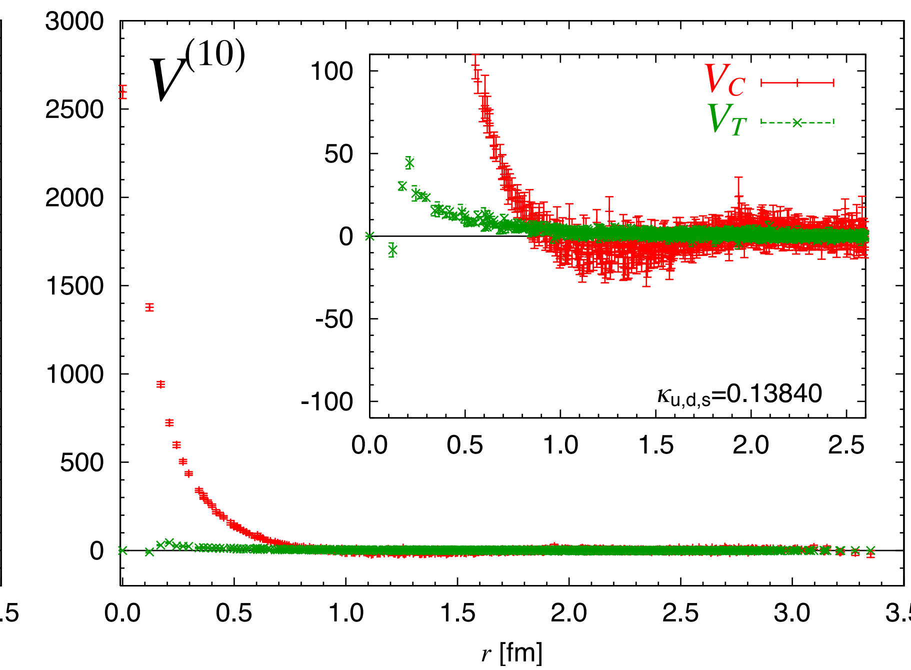
8s: strong repulsive core. repulsion only.



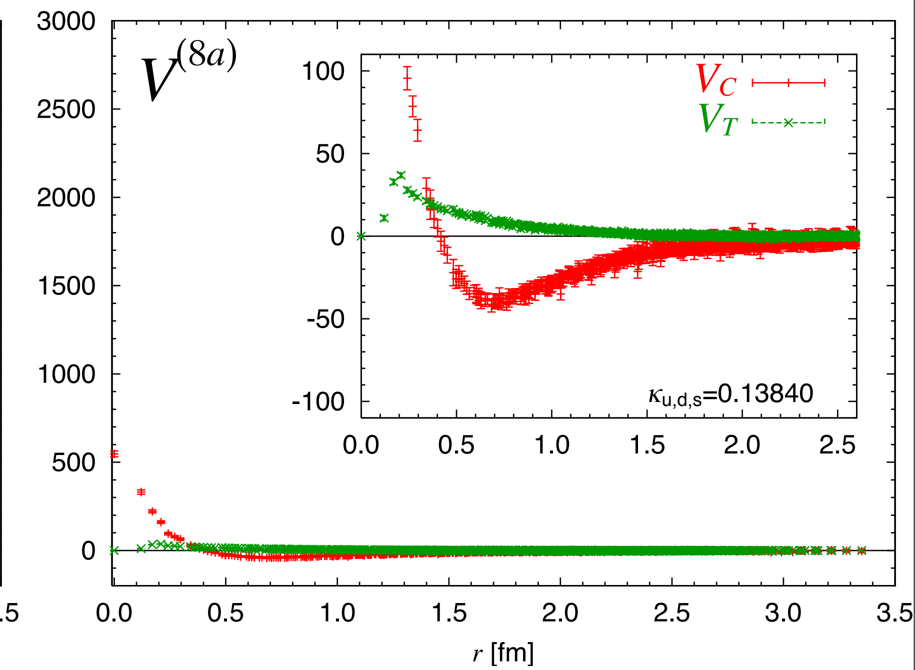
1: attractive instead of repulsive core ! attraction only .



same as NN



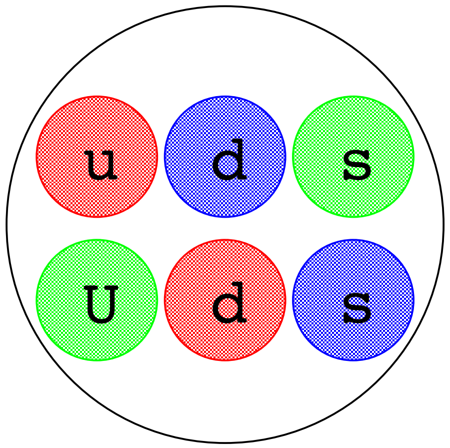
10: strong repulsive core. weak attraction.



8a: weak repulsive core.
strong attraction.

Flavor dependences of BB interactions become manifest in SU(3) limit !

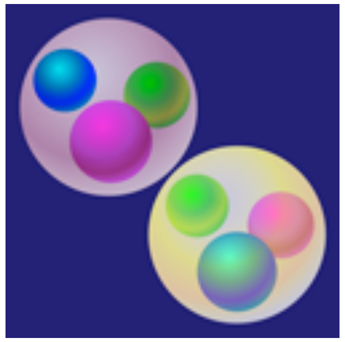
H-dibaryon:
a possible six quark state(uuddss)
predicted by the model but not observed yet.



http://physics.aps.org/synopsis-for/10.1103/PhysRevLett.106.162001

Binding baryons on the lattice

April 26, 2011



H-dibaryon in the flavor SU(3) symmetric limit

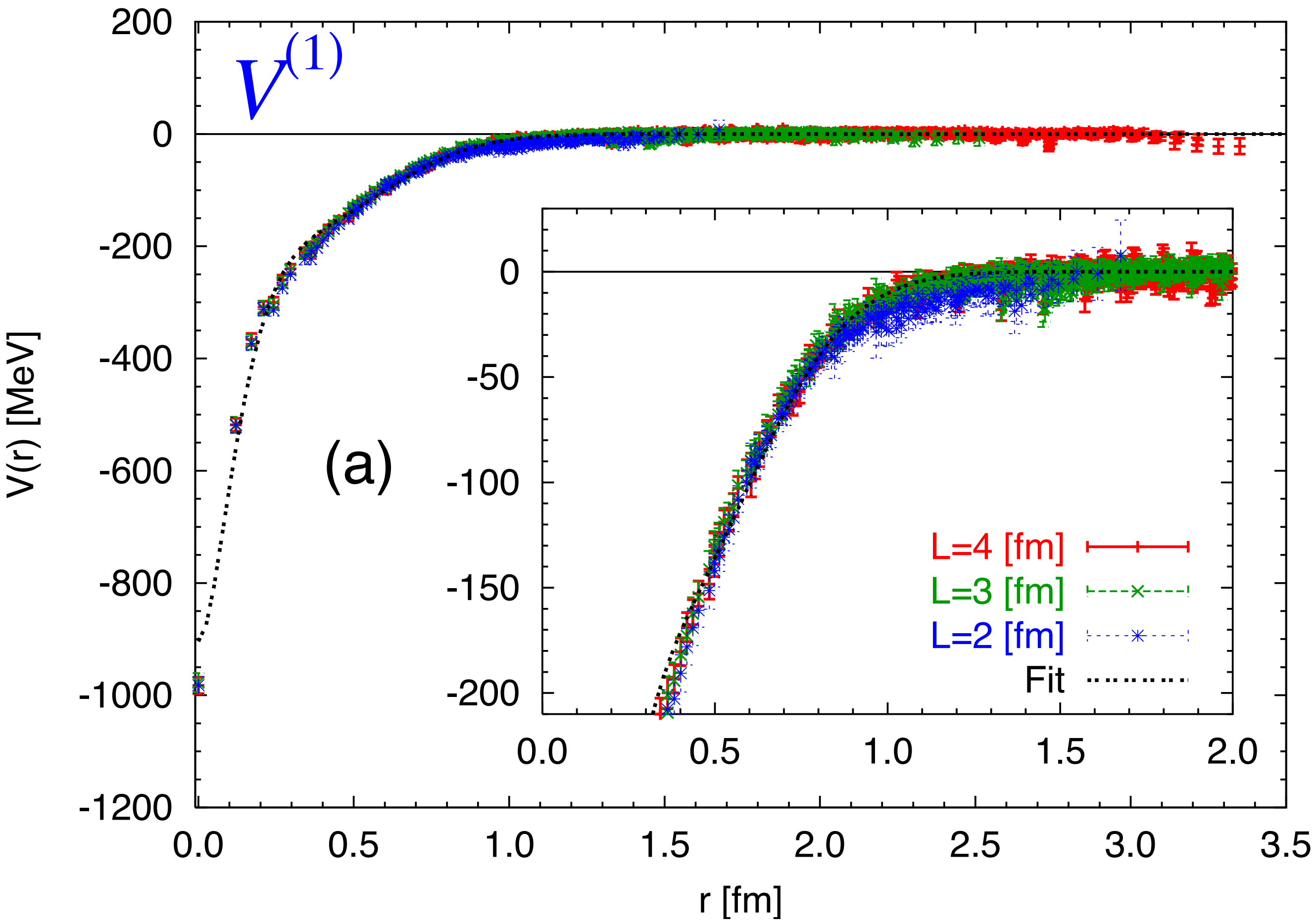
a=0.12 fm

Inoue *et al.* (HAL QCD Coll.), PRL106(2011)162002

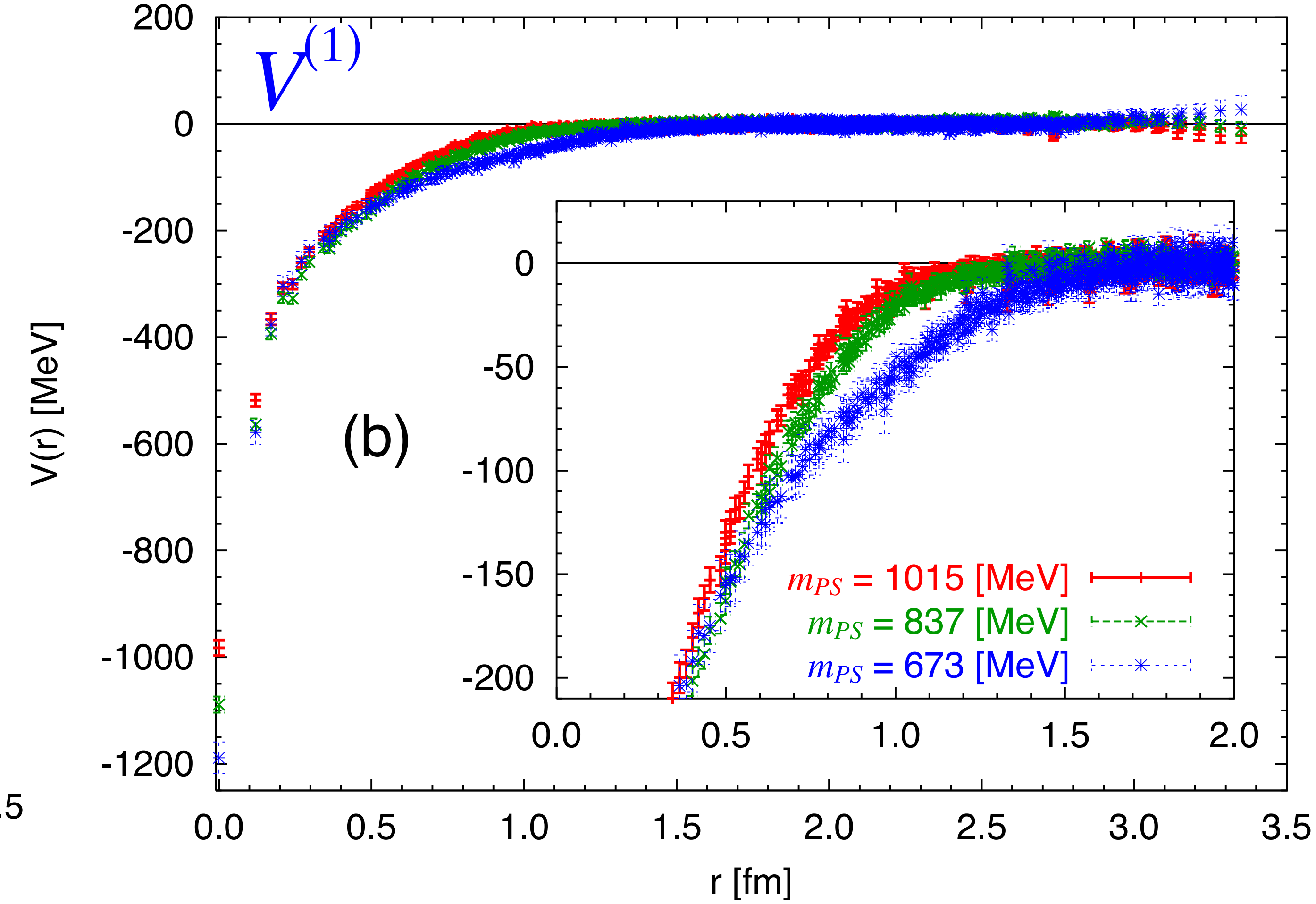
Attractive potential in the flavor singlet channel → possibility of a bound state (H-dibaryon)

$$\Lambda\Lambda - N\Xi - \Sigma\Sigma$$

volume dependence



pion mass dependence



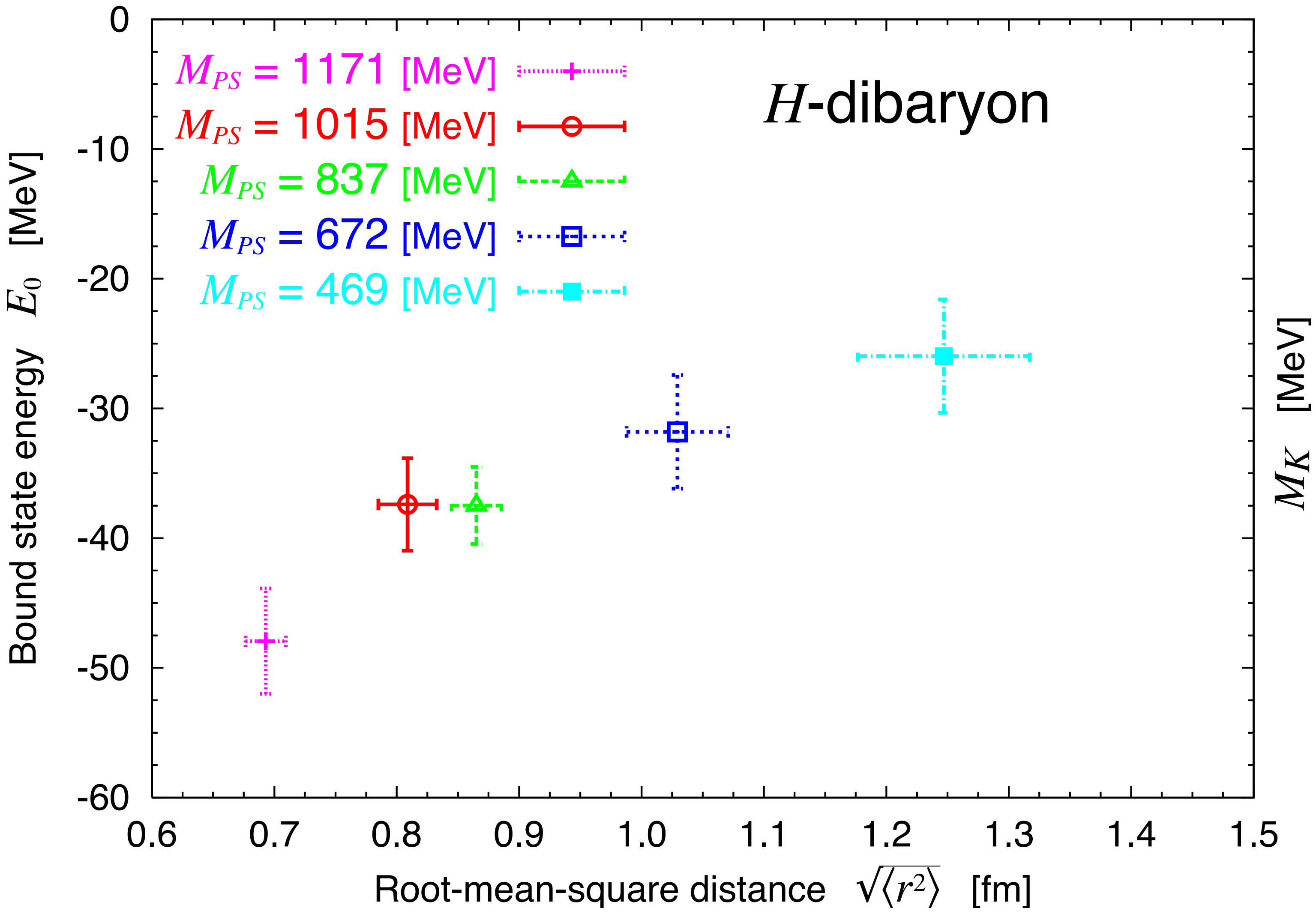
L=3 fm is enough for the potential.

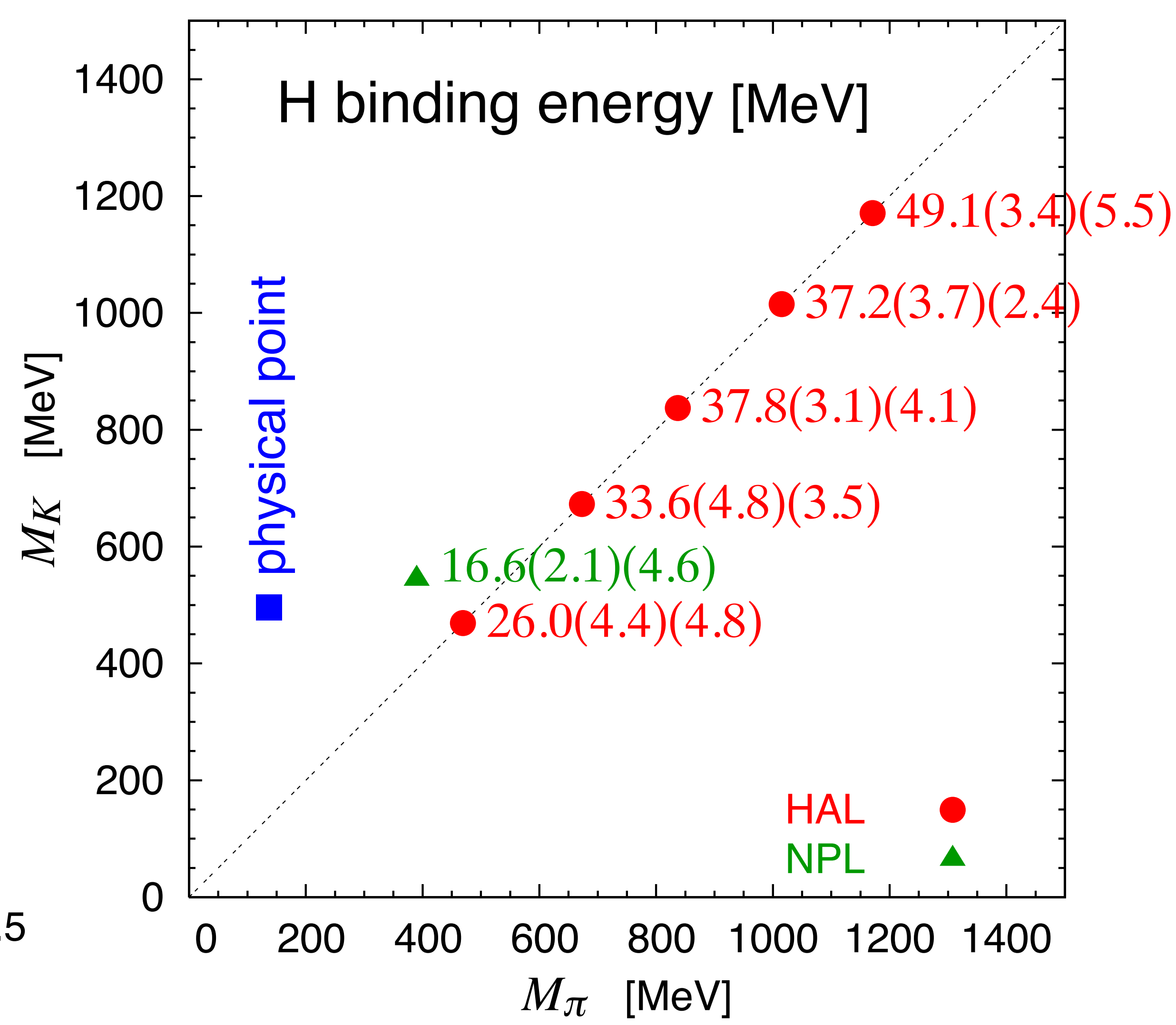
lighter the pion mass, stronger the attraction

fit potentials at L=4 fm by $V(r) = a_1 e^{-a_2 r^2} + a_3 \left(1 - e^{-a_4 r^2}\right)^2 \left(\frac{e^{-a_5 r}}{r}\right)^2$

Solve Schroedinger equation
in the infinite volume

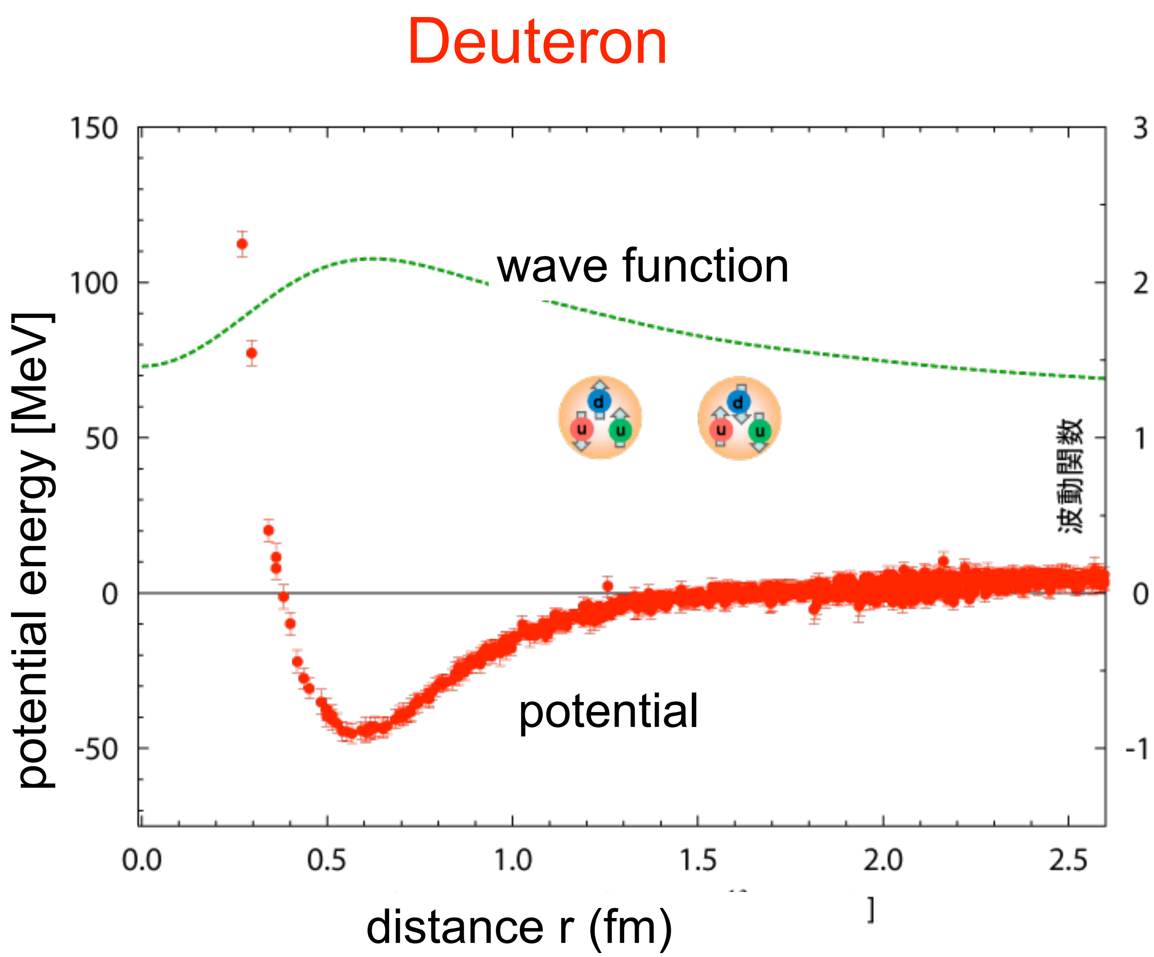
One bound state (H-dibaryon) exists.





An H-dibaryon exists in the flavor SU(3) limit.
Binding energy = 25-50 MeV at this range of quark mass.
A mild quark mass dependence.

Real world ?


Deuteron
wave function
potential
potential energy [MeV]
distance r (fm)
波動関数
150
100
50
0
-50
3
2
1
0
-1
0.0
0.5
1.0
1.5
2.0
2.5
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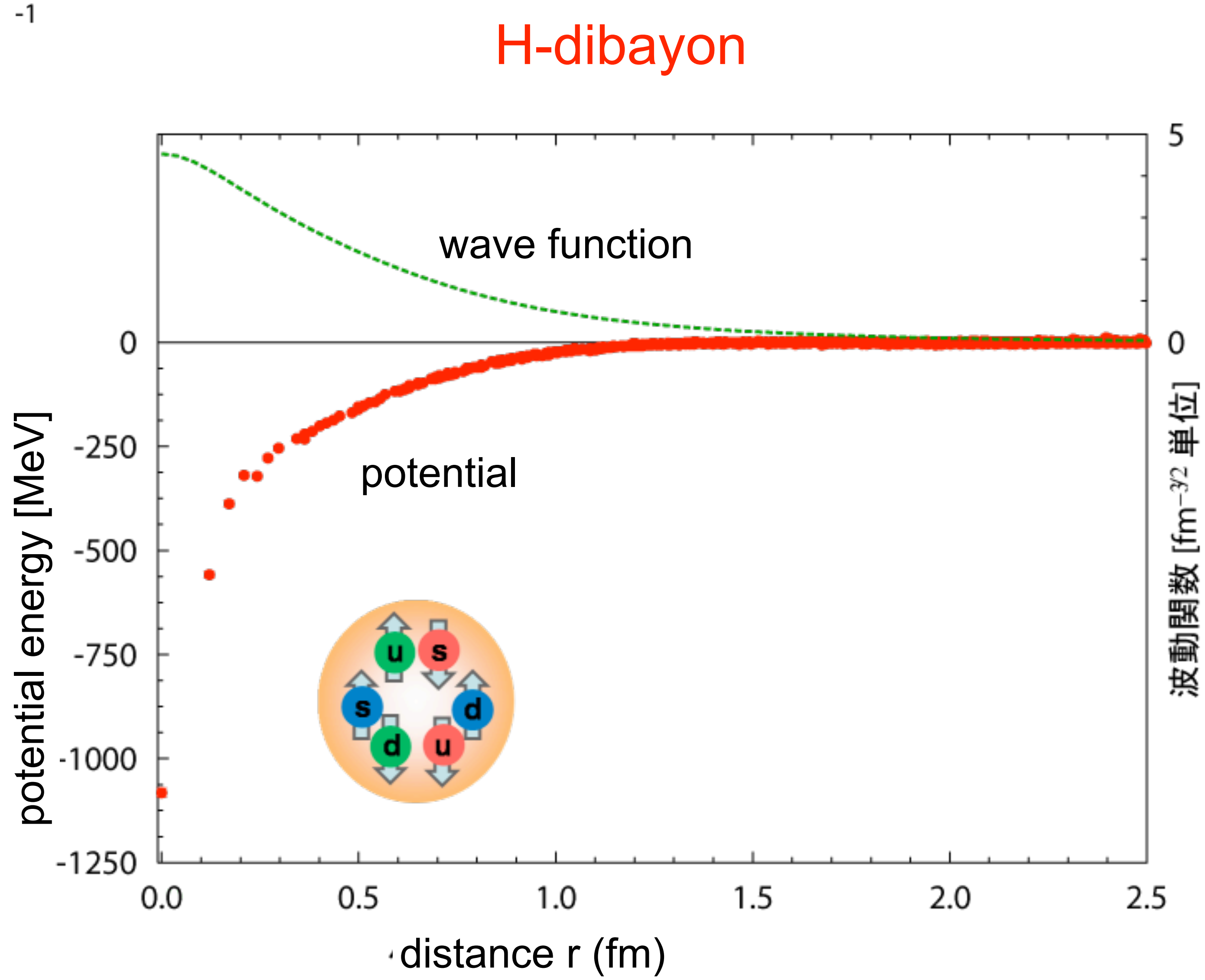
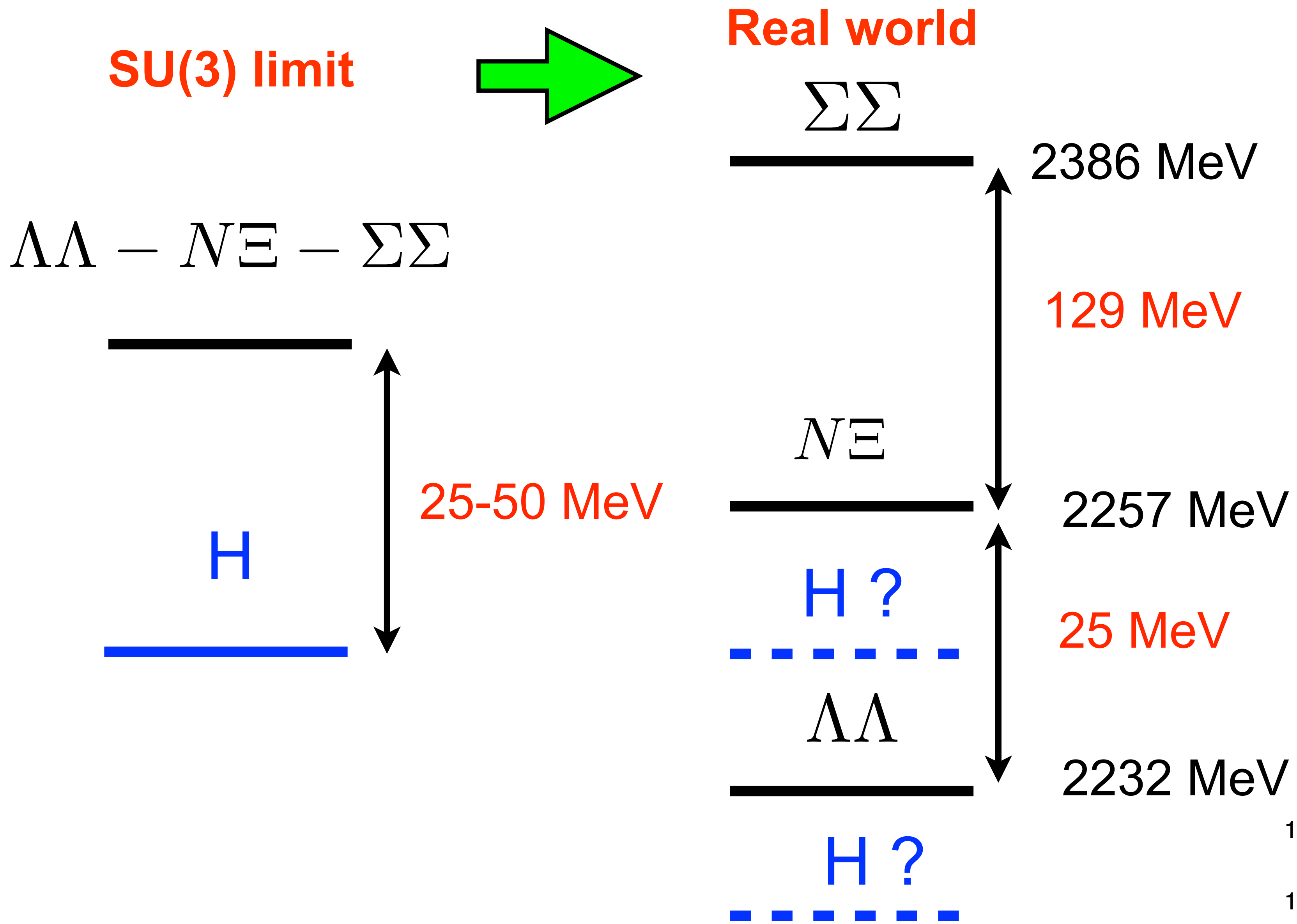
u
u

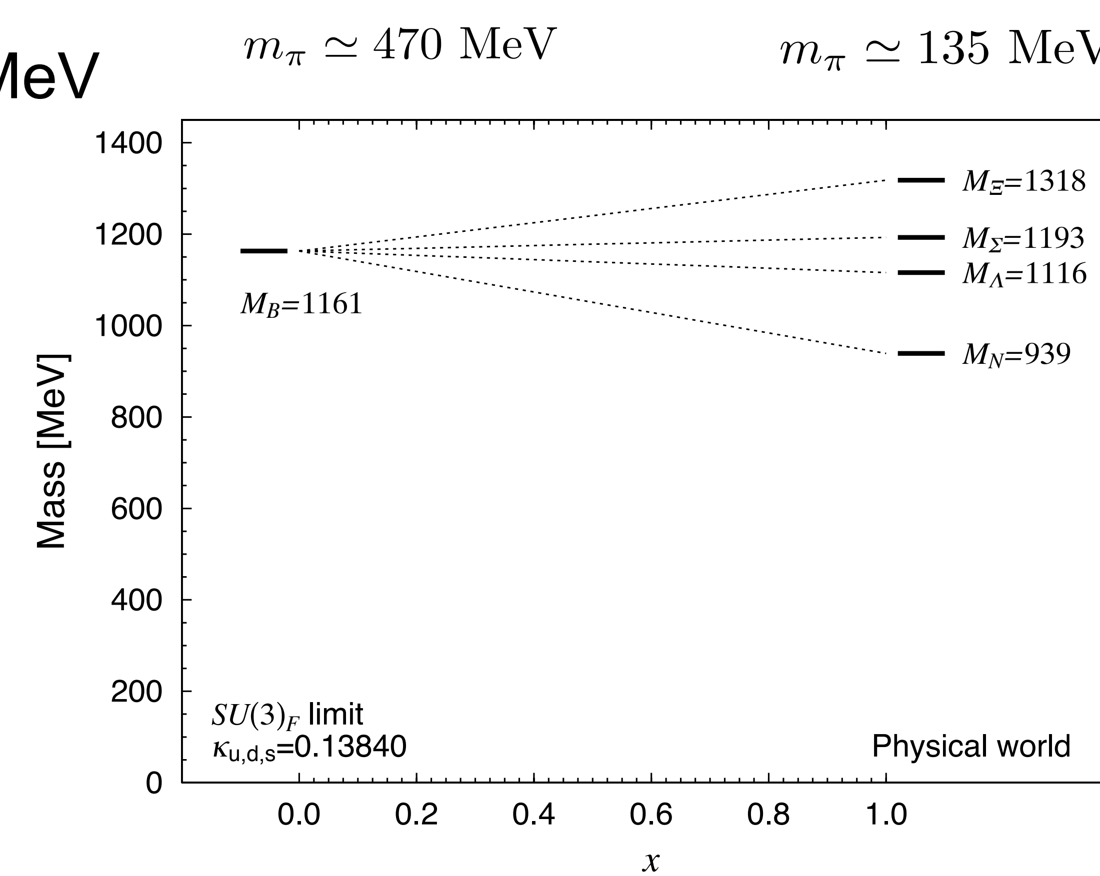

H-dibayon
wave function
potential
potential energy [MeV]
distance r (fm)
波動関数 [fm$^{-3/2}$ 単位]
0
-250
-500
-750
-1000
-1250
5
0
0.0
0.5
1.0
1.5
2.0
2.5
u
s
s
d
d
u

5. Some applications to nuclear physics

H-dibaryon with the flavor SU(3) breaking



$m_u = m_d \neq m_s$



Our approximation for SU(3) breaking

1. Linear interpolation of octet baryon masses

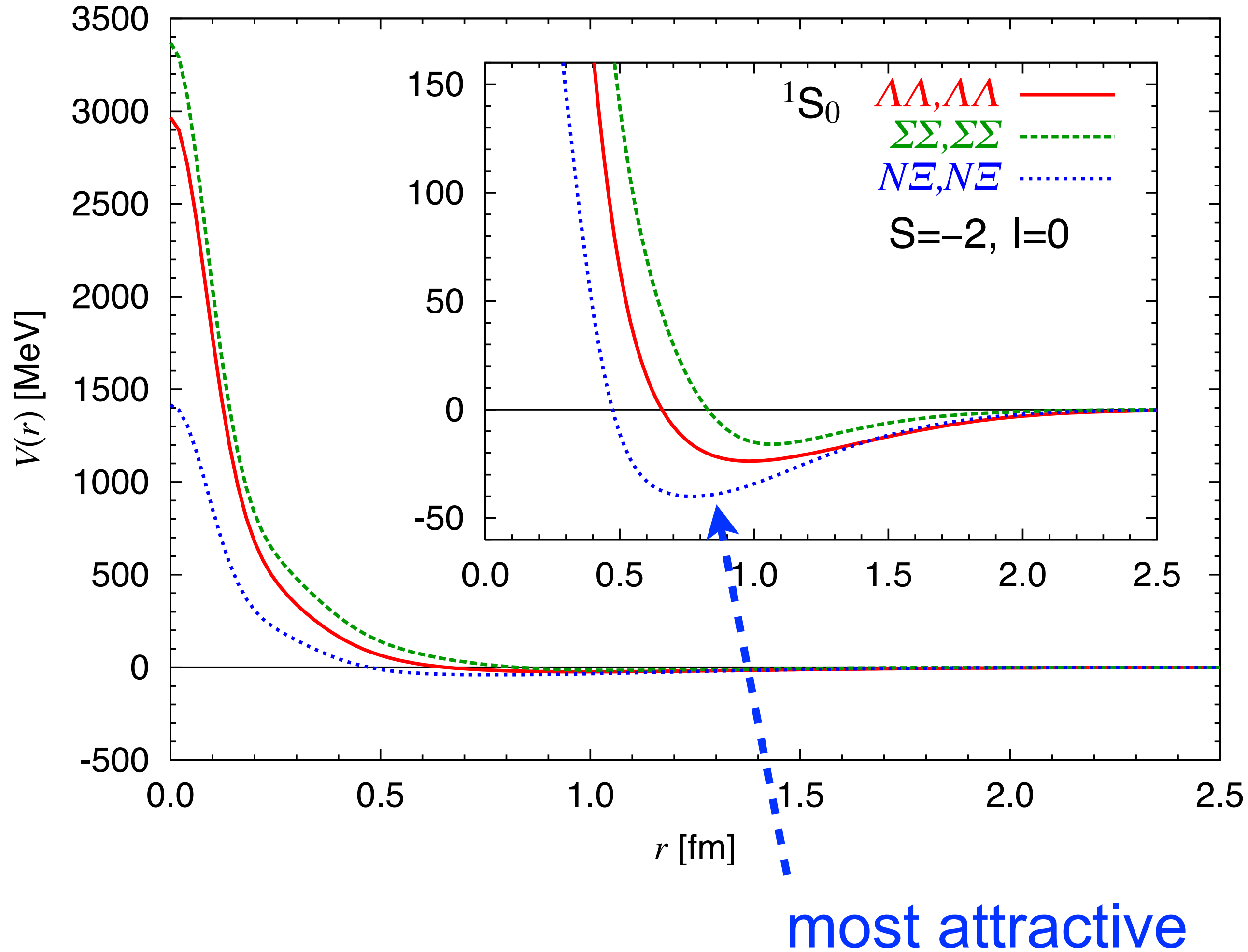
$$M_Y(x) = (1-x)M_Y^{\mathrm{SU(3)}} + xM_Y^{\mathrm{Phys}}$$

2. Potentials in particle basis in SU(3) limit

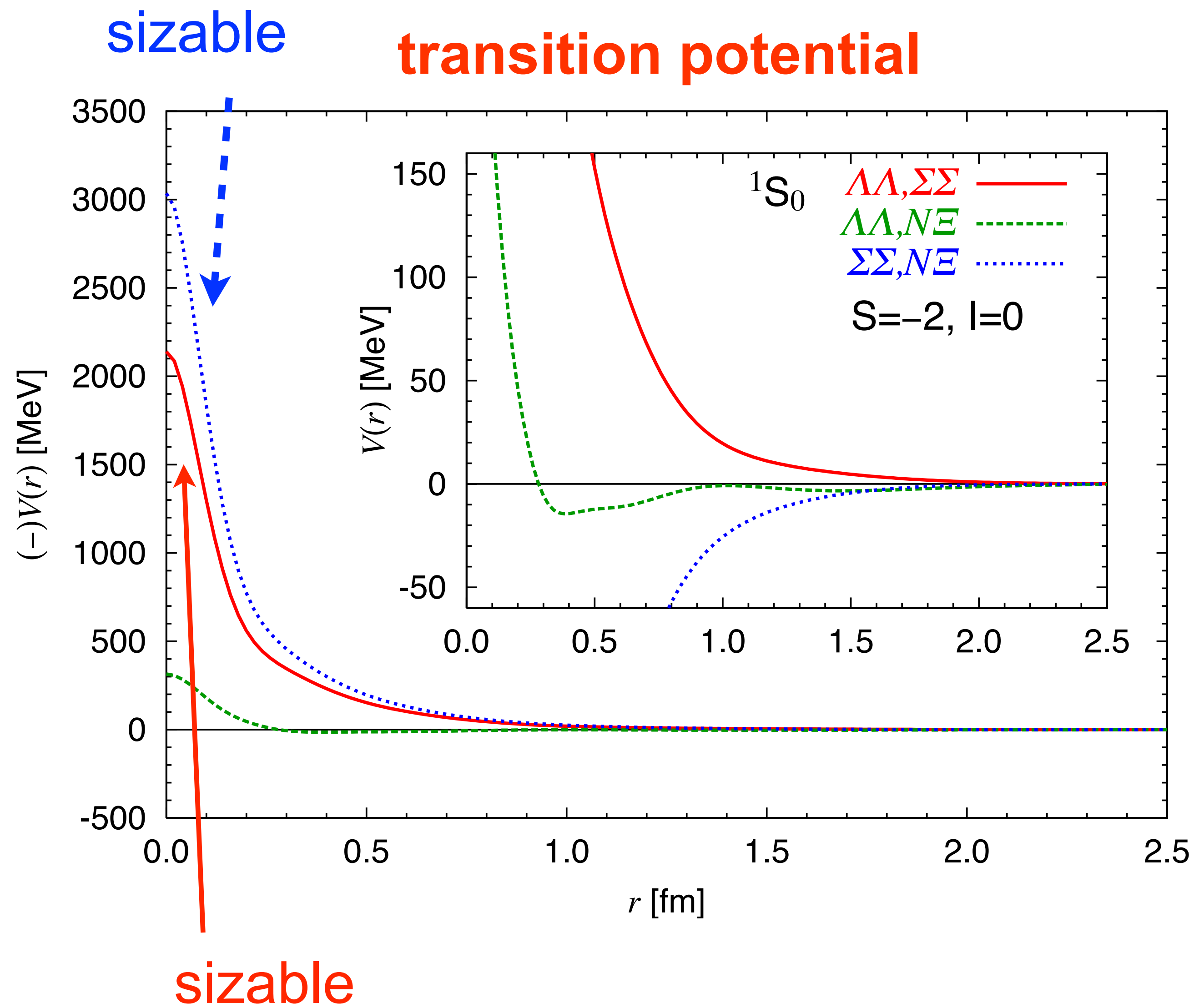
Potentials in particle basis in SU(3) limit (S=-2, I=0)

$m_\pi \simeq 470$ MeV

diagonal potential



transition potential



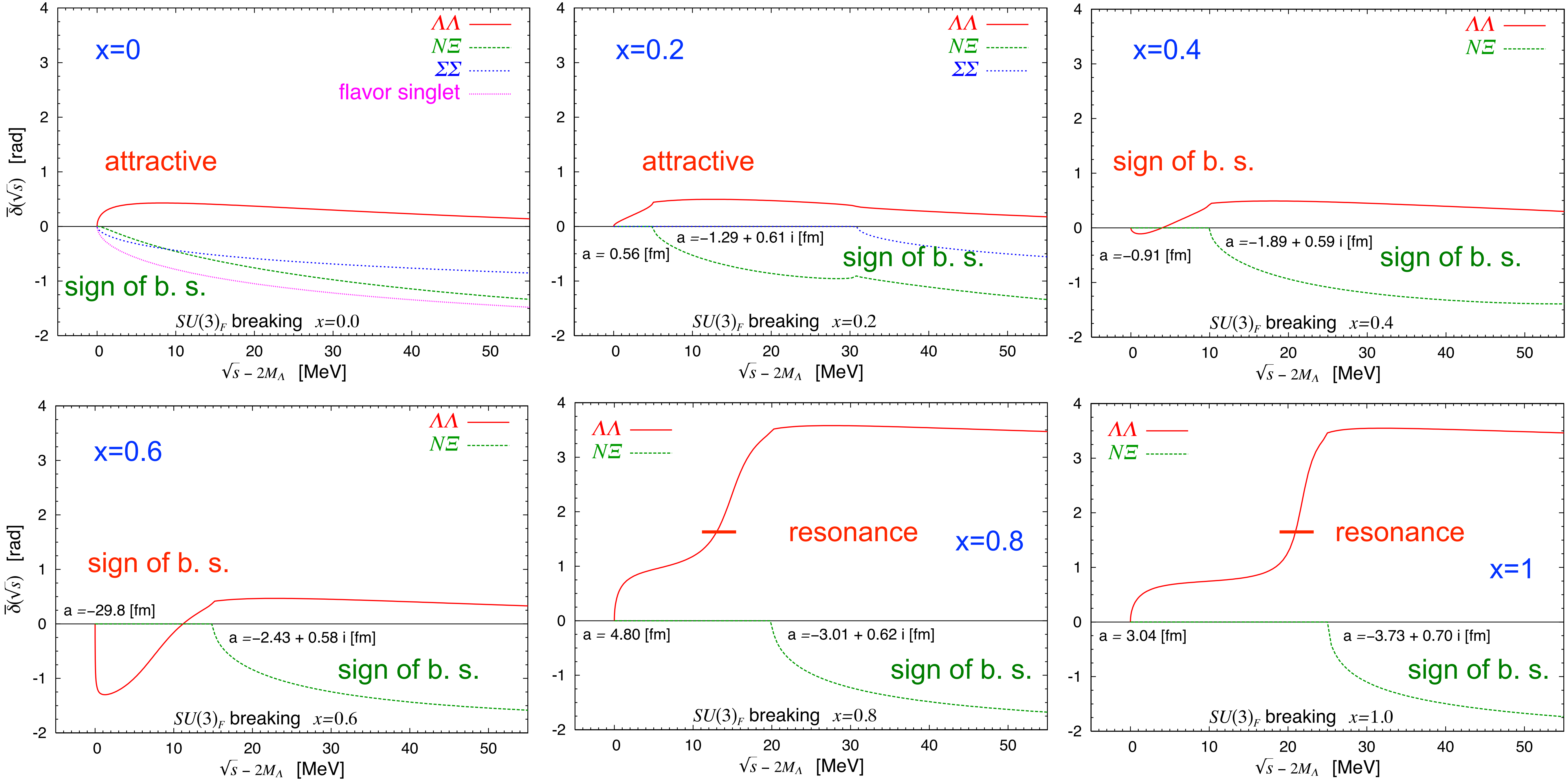
This part needs to be improved.

The direct calculation of potentials in 2+1 flavor QCD is in progress.

K. Sasaki *et al.* (HAL QCD Coll.), Lat 2012

Phase shift

Inoue *et al.* (HAL QCD Coll.), arXiv:1112.5926[hep-lat]



H-dibaryon seems to become resonance at physical point.

H couples most strongly $N\Xi$.

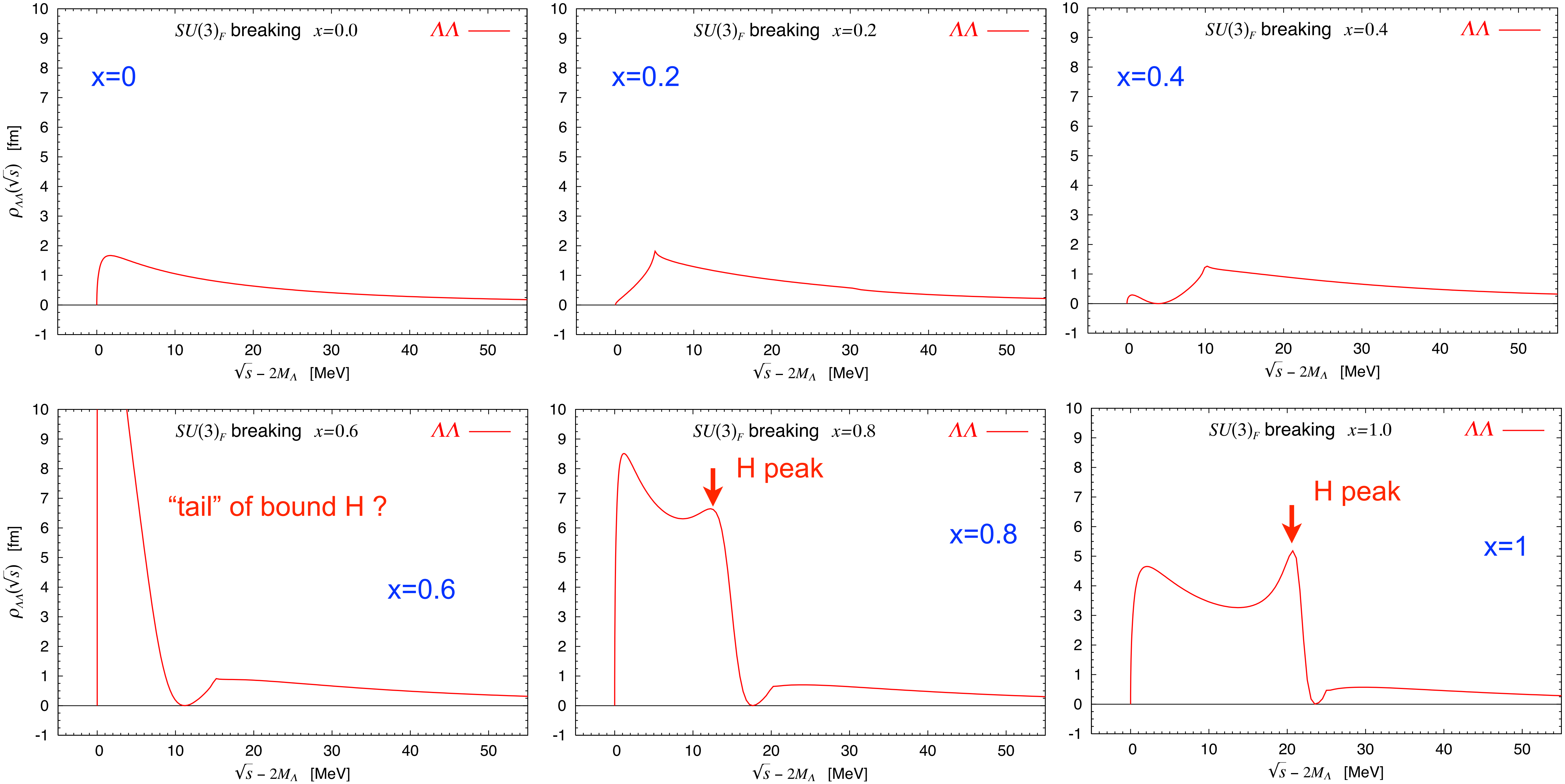
$\Lambda\Lambda$ interaction is attractive.

H has a sizable coupling to $\Lambda\Lambda$ near and above the threshold.

Invariant mass spectrum

$\Lambda\Lambda \to \Lambda\Lambda$

Inoue *et al.* (HAL QCD Coll.), arXiv:1112.5926[hep-lat]



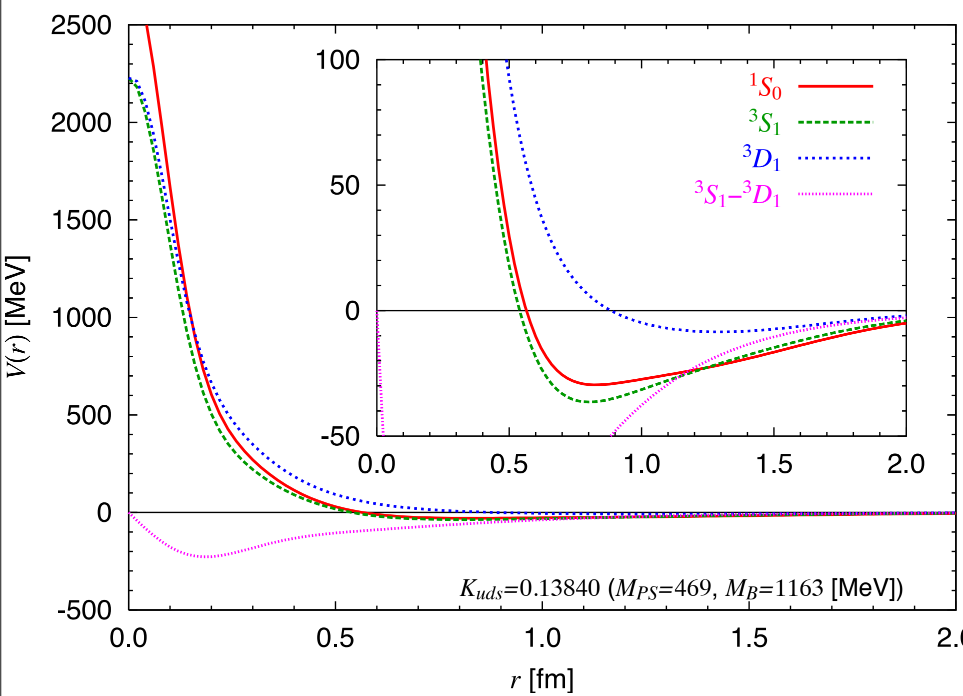
A peak of the resonance H might be observed in experiments !?

Equation of State for nuclear/neutron matter

EoS of nuclear matter

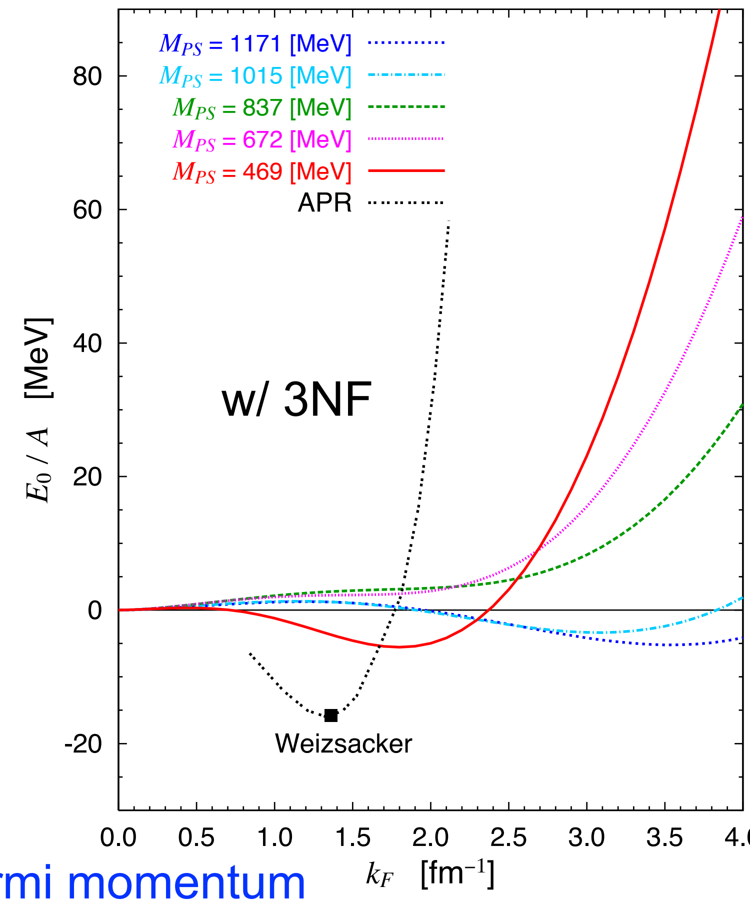
Inoue *et al.* (HAL QCD Coll.), in preparation

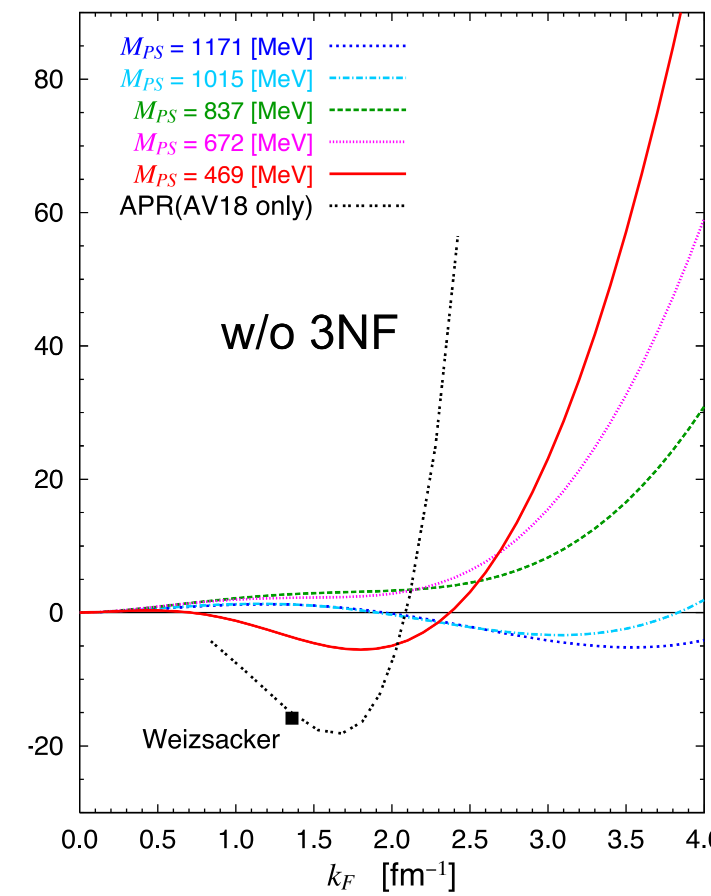
NN potentials $m_\pi = 470\ \mathrm{MeV}$



BHF

Energy density of Nuclear matter

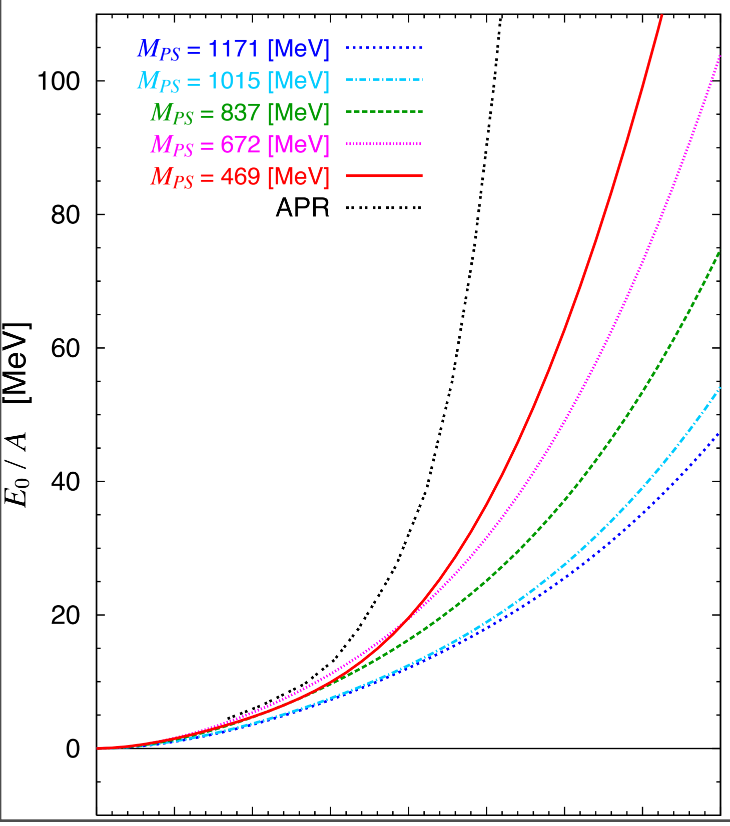




Fermi momentum

A. Akmal, V.R. Pandharipande, G.G. Ravenhall, Phys. Rev. C58 1804 (1998)

Neutron matter

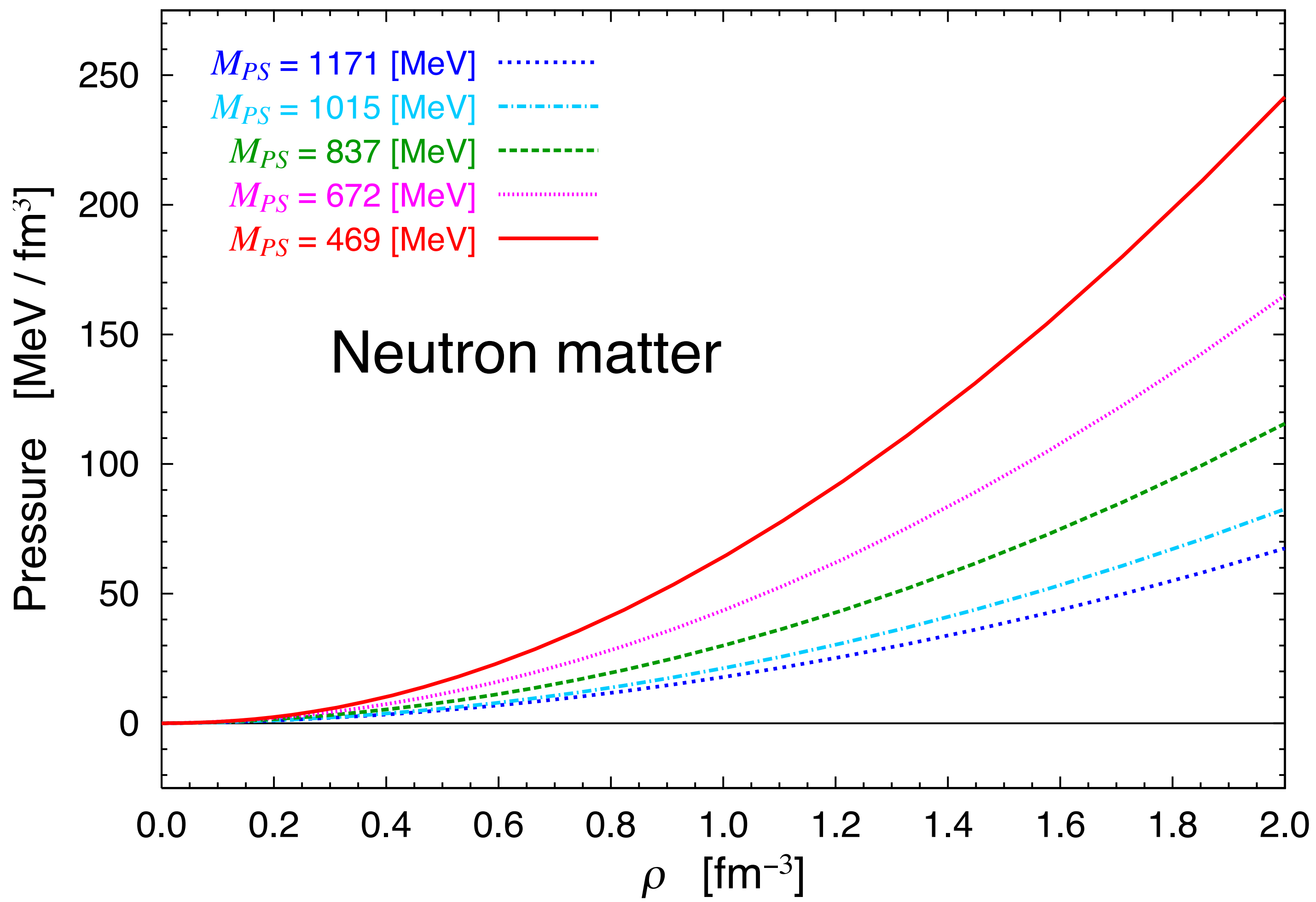


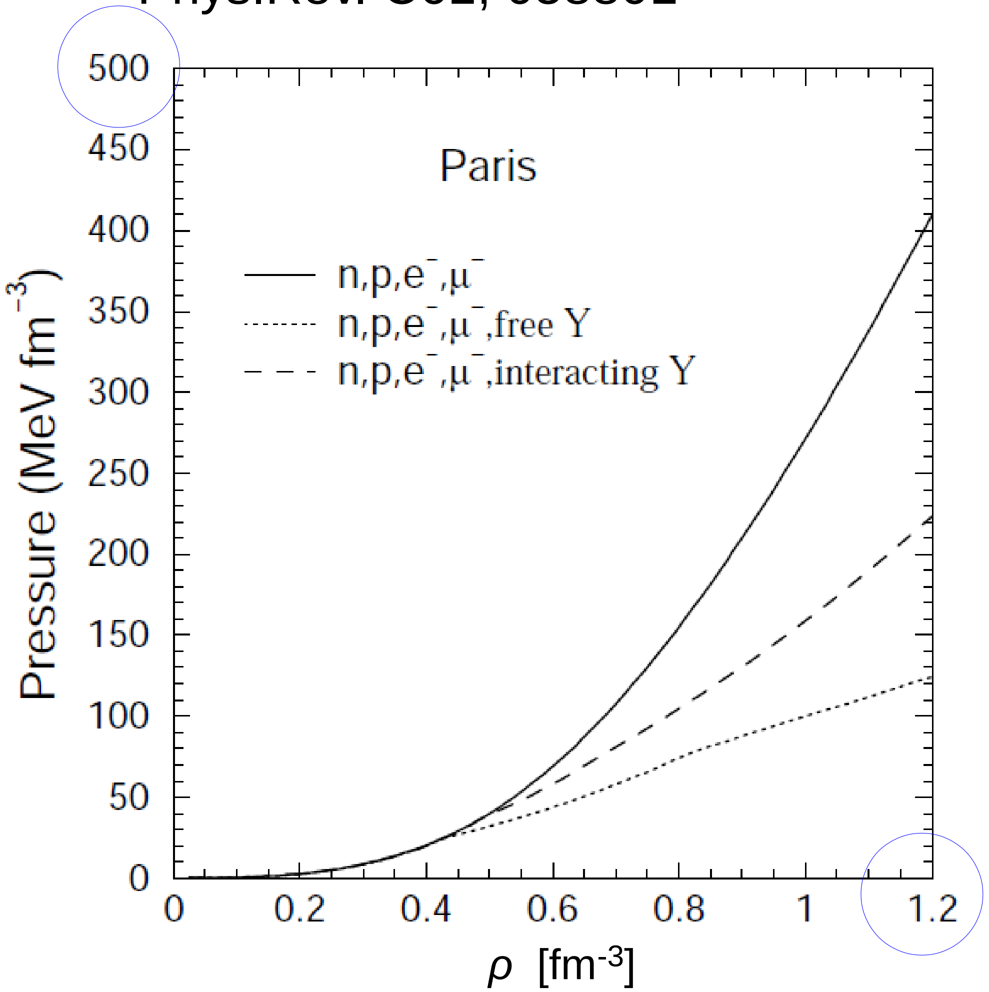
Nuclear matter shows the saturation at the lightest pion mass, but the saturation point deviates from the empirical one obtained by Weizsacker mass formula.

No saturation for Neutron matter.

Pressure of Neutron matter

M. Baldo, F. Burgio, H.-J.Schulze, Phys.Rev. C61, 058801



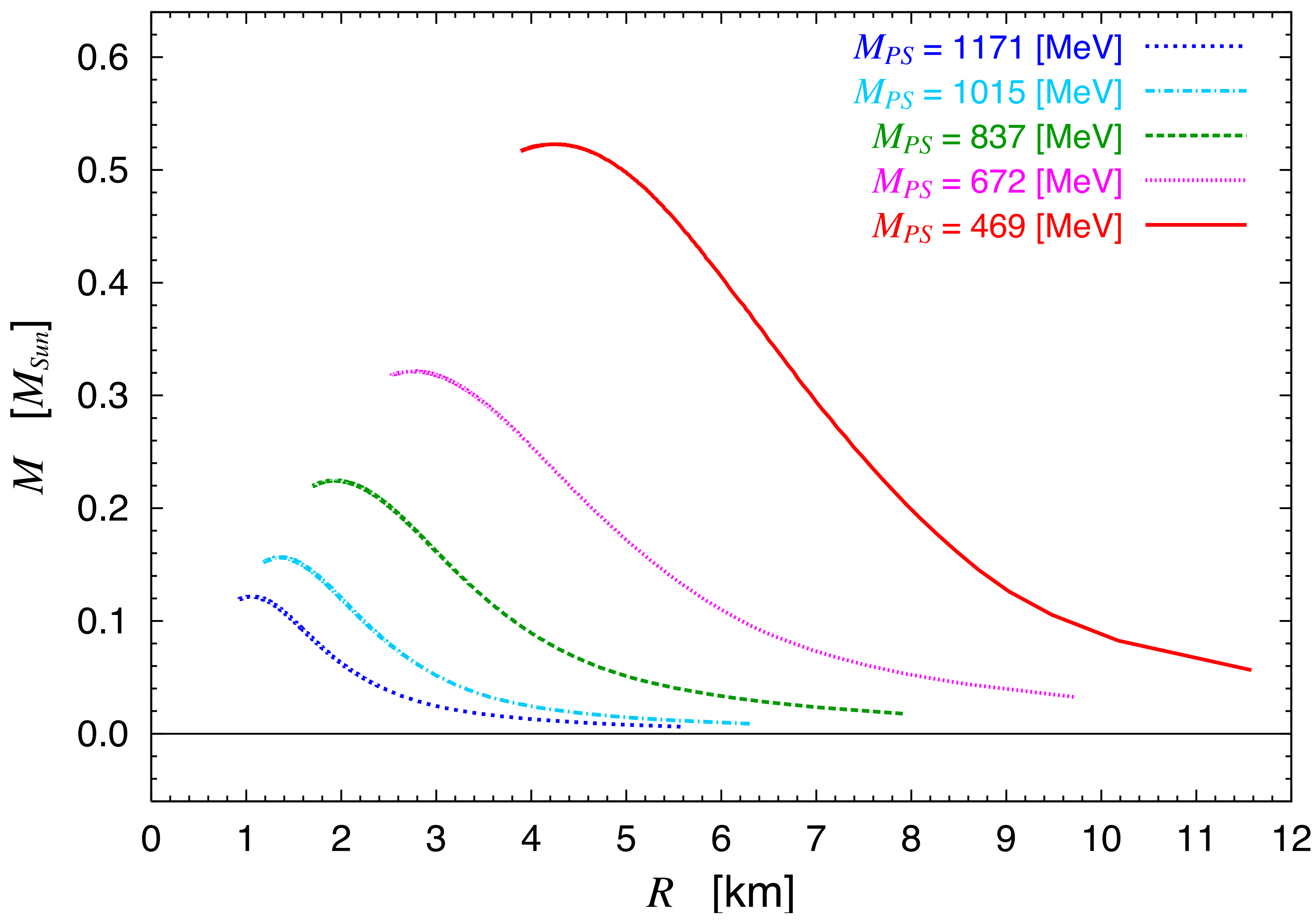


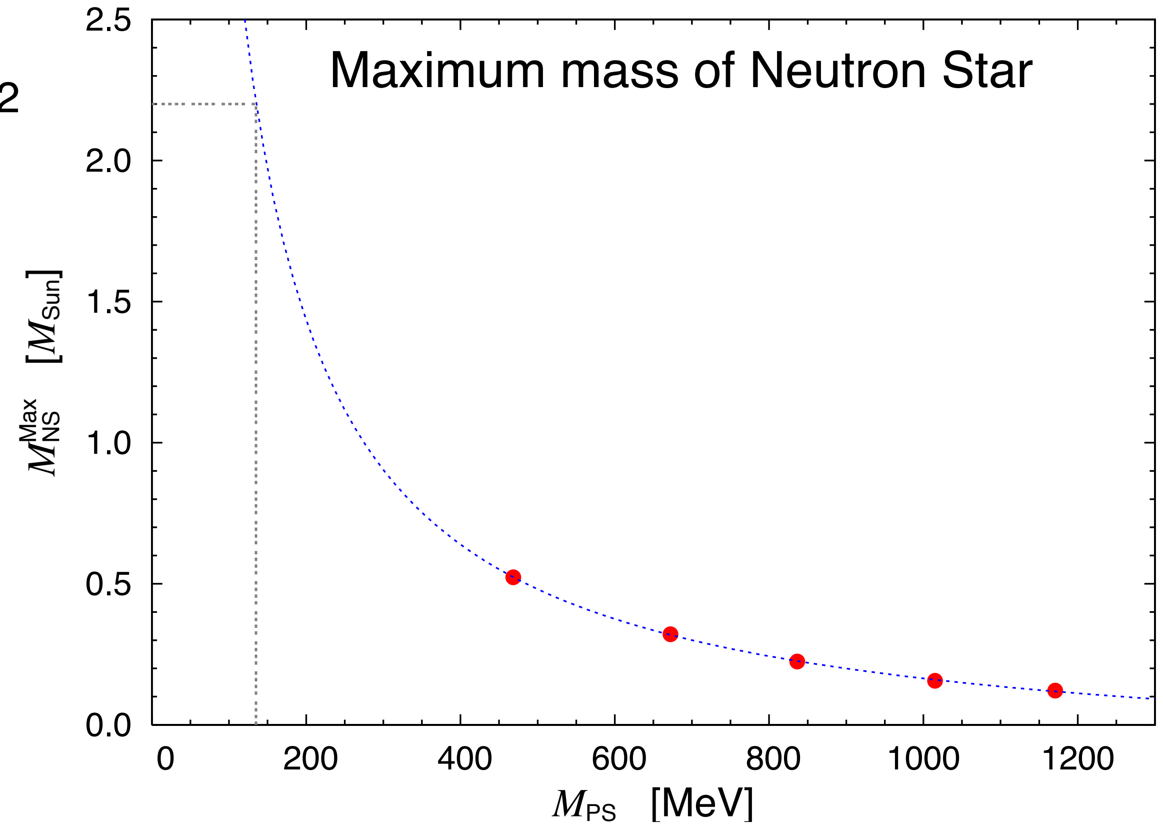
pressure $$P = \rho^2 \frac{d(E_0/A)}{d\rho} = \frac{\gamma k_F^4}{18\pi^2} \frac{d(E_0/A)}{dk_F}$$

density $$\rho = \frac{\gamma k_F^3}{6\pi^2}$$

Our Neutron matter becomes harder as the pion mass decreases, but it is still softer than phenomenological models.

Neutron star M-R relation





Maximum mass of Neutron vs. pion mass

6. Conclusion

- HAL QCD scheme is shown to be a promising method to extract hadronic interactions in lattice QCD.
  - ground state saturation is not required.
  - Calculate potential (matrix) in lattice QCD on a finite box.
  - Calculate phase shift by solving (coupled channel) Shroedinger equation in infinite volume.
  - bound/resonance/scattering
- Future directions
  - calculations at the physical pion mass on “K-computer”
  - hyperon interactions with the SU(3) breaking
  - Baryon-Meson, Meson-Meson
  - Exotic other than H such as penta-quark, X, Y etc.
  - 3 Nucleon forces
  - Other applications ? (weak interaction ?)

Thank you !