Neutrino Masses: An Introduction

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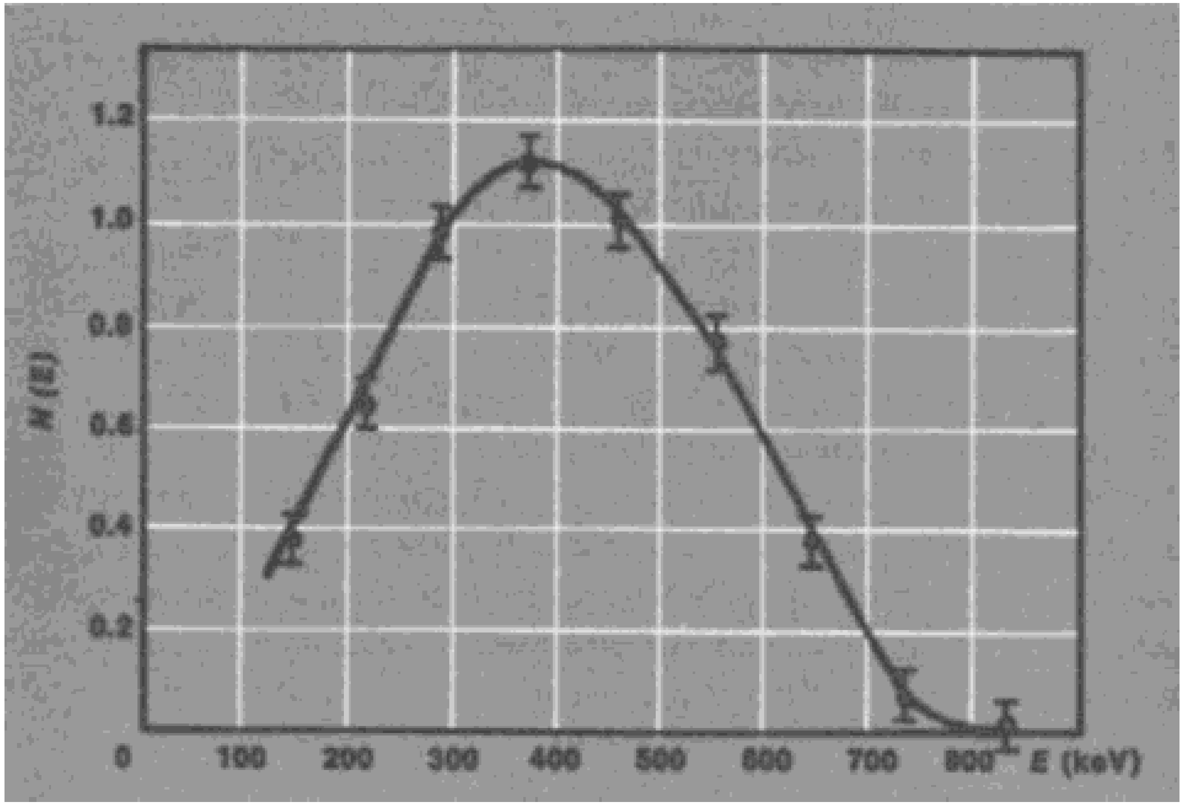
We introduce the discussion on neutrino masses, emphasizing the relevance of neutrino oscillations. We recall the basic formalism needed for the description of this phenomenon, both in vacuum and in the matter. We illustrate with various examples the importance of the oscillations in the matter to improve on the knowledge of the neutrino masses, identify the relevant quantities and touch briefly some experimental aspect.

THE FIRST STEPS

Neutrino mass affect the beta decay spectrum, e.g.

$$n \rightarrow p + e^{-} + \nu$$

especially where the electron attains its maximum energy (Fermi)

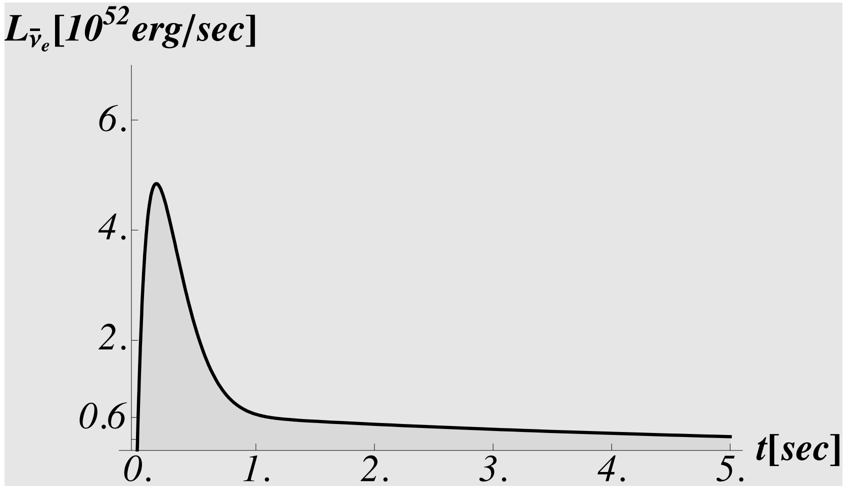


Today, $m_\nu < 2$ eV, that could improve to 0.2 eV (KATRIN)

Neutrino mass delays the slower neutrinos from a cosmic source

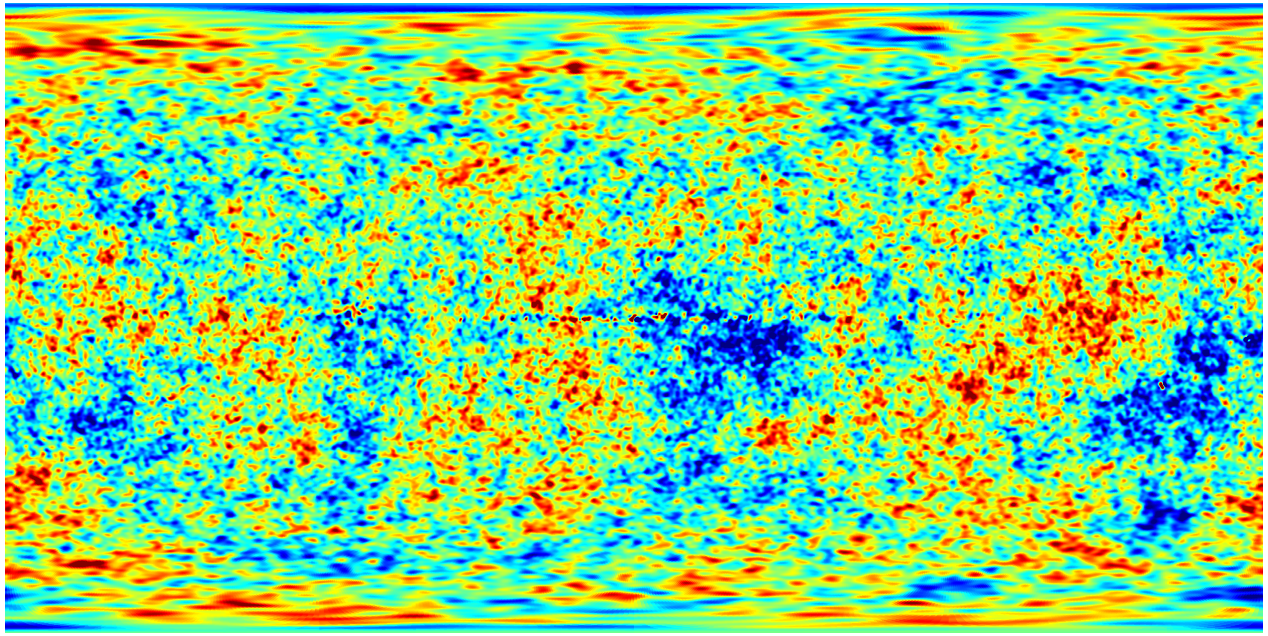
$$\delta t = \frac{D}{v} - \frac{D}{c} \approx \frac{D}{2c}\left(\frac{m_\nu c^2}{E}\right)^2$$

and therefore modifies the emission spectrum (Zatsepin).



Today, $m_\nu < 6$ eV, it could go below 1 eV (Pagliaroli et al.)

Depending on their masses, neutrinos are radiation or matter ($T_{\text{(matt.=rad.)}} = 2.5$ eV and $T_{\text{(recomb.)}} = 0.25$ eV). The masses modify the distribution of CMB anysotropies and galaxies formation.



Today, $\sum_i m_{\nu_i} < 0.6$ eV (WMAP) that could be halved (Planck).

NEUTRINO OSCILLATIONS

The oscillations K^0–antiK^0 suggest the possibility of similar phenomena in the systems neutrino-antineutrino, neutron-antineutron, atom-antiatom etc. (1957)

The analogies between hadrons and leptons suggests that there are 2 types of hadrons and of leptons, with neutrinos possibly mixed among them (1962)

Assume a superposition of mass states

$$|\nu_e\rangle = +\cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$
$$|\nu_\mu\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle$$

A particle with given mass behaves as a De Broglie wave

$$|\nu_i, t\rangle = e^{-i\,t\,E_i\,/\,\hbar}|\nu_i\rangle \text{ with } E_i = \sqrt{(pc)^2 + (m_i\,c^2)^2}$$

Thus, the relative de-phasing of $|\nu_1, t\rangle$ e $|\nu_2, t\rangle$, implies that

$$P_{ee} \equiv |\langle\nu_e|\nu_e, t\rangle|^2 \leq 1 \text{ and } P_{e\mu} \equiv |\langle\nu_\mu|\nu_e, t\rangle|^2 \geq 0$$

$\therefore$ **If $\theta \neq 0$ and $m_1^2 \neq m_2^2$, an electronic neutrino that propagates, $|\nu_e, t\rangle$ won't be anymore an electronic neutrino $|\nu_e\rangle \equiv |\nu_e, 0\rangle$ at some $t > 0$.**

The corresponding formulae, obtained by Pontecorvo in 1967, are

$$P_{e\mu} = P_{\mu e} = \sin^2(2\theta) \times \sin^2\varphi_{12} \qquad \text{Appearance}$$
$$P_{ee} = P_{\mu\mu} = 1 - P_{e\mu} \qquad \text{Disappearance}$$

where the oscillation phase is,

$$\varphi_{12} = \frac{\left[(m_2c^2)^2 - (m_1c^2)^2\right]\ L}{4\ \hbar\ c\ E_\nu} = 1.27\ \frac{m_2^2 - m_1^2}{\text{eV}^2/c^4} \times \frac{L}{\text{km}} \times \frac{\text{GeV}}{E_\nu}$$

However, these formulae stay the same when we replace

$$\theta \to 90^\circ - \theta \quad \text{or} \quad m_1 \to m_2$$

causing ambiguity in our knowledge of neutrino masses.

APPLICATIONS

Atmospheric neutrinos

The observed atmospheric neutrino fluxes begin to disagree with the predictions when the zenith angle $\theta > 90°$. Writing,

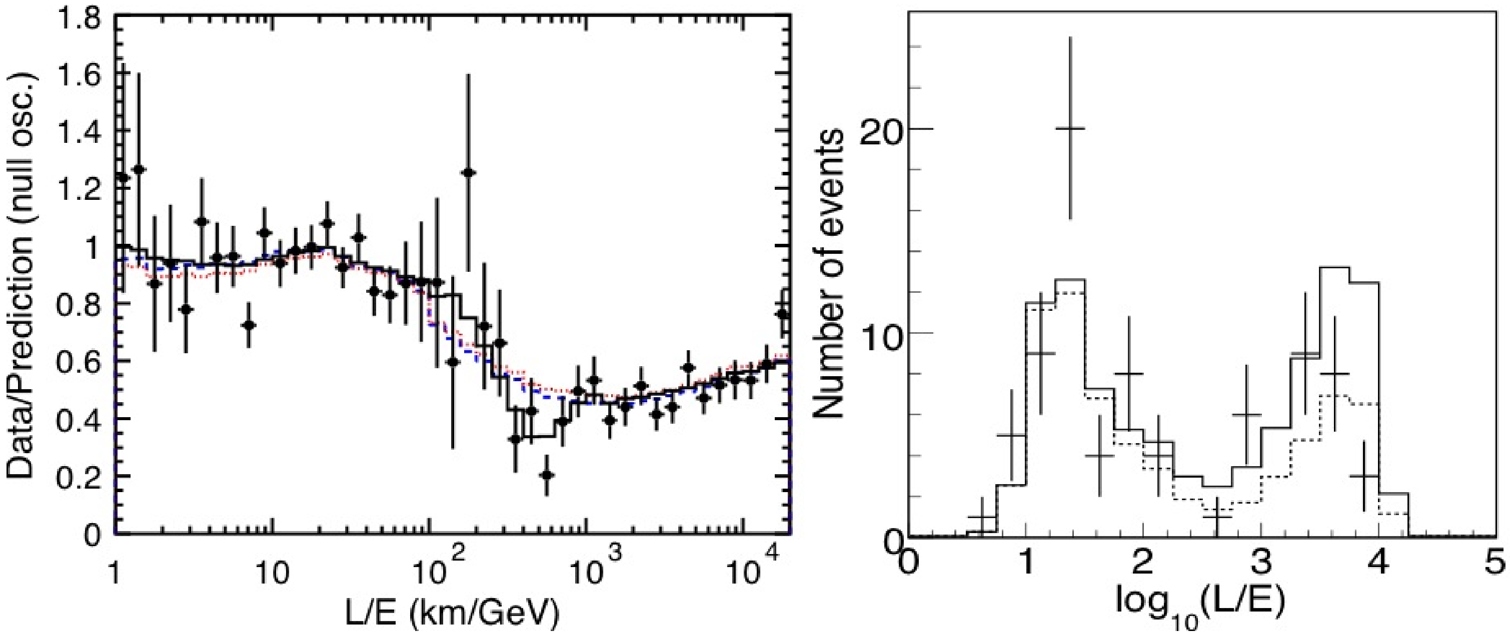
$$\varphi = \frac{\Delta m^2}{1.5 \times 10^{-3}\ \text{eV}^2} \times \frac{L}{L_0} \times \frac{E_\nu^0}{E_\nu}$$

where $L^0 = \sqrt{2\ h\ R_\oplus} \approx 500$ km (horizontal neutrinos) and $E_\nu^0 = 1$ GeV, we are not surprised that this is caused by neutrino oscillations, with

$$\boxed{\Delta m^2 = 2.4 \times 10^{-3}\ \text{eV}^2}$$

as found by Super-Kamiokande, MACRO, SOUDAN-II and as tested by K2K, MINOS, OPERA, etc.,

Remark on atmospheric neutrino "oscillations"



The "wiggles" that one could expect from the oscillations formula are not easily seen, also in dedicated L/E analyses, because the direction and the energy have to be reconstructed. The the above, by the Super-Kamiokande, have been obtained in 2004 ((left) and in 2009 (right); in the second analysis, also a proton is observed.

New reactor neutrino data

Also $\bar{\nu}_e$ from reactors begin to disagree with expectations after 1 km Double-CHOOZ, Daya Bay, RENO. Writing the phase of oscillation,

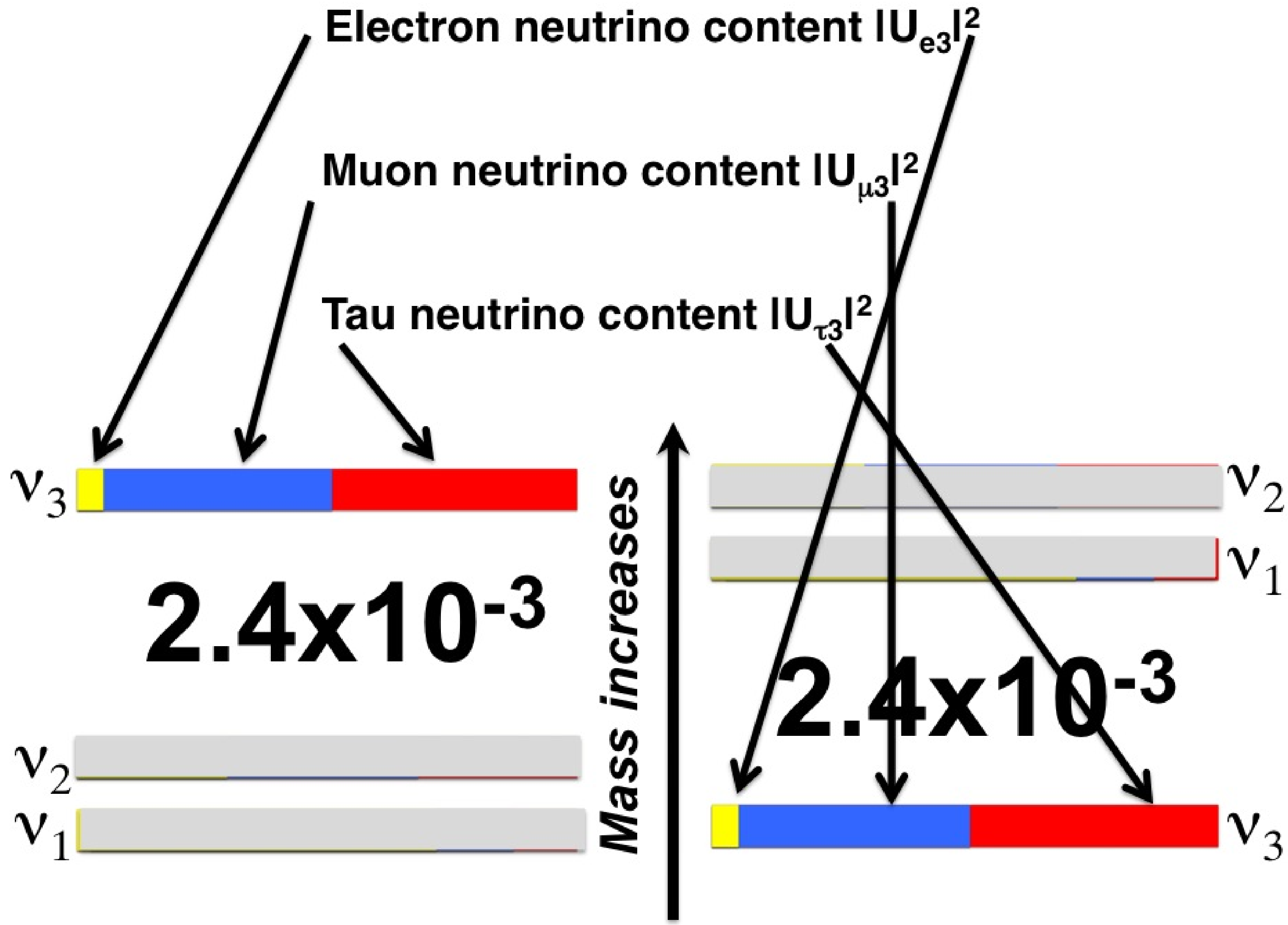
$$\varphi = \frac{\Delta m^2}{2.4 \times 10^{-3}\ \text{eV}^2} \times \frac{L}{L_0} \times \frac{E_\nu^0}{E_\nu}$$

where $L^0 = 1$ km and $E_\nu^0 = 0.003$ GeV, the same difference of mass squared, $\Delta m^2 = 2.4 \times 10^{-3}$ eV^2, gives $\varphi = 1$: just what we need.

Extending the formalism to 3 neutrinos,

$$\begin{cases} |\nu_e, t\rangle = U_{e1}|\nu_1\rangle + U_{e2}|\nu_2\rangle + U_{e3}|\nu_3\rangle e^{-i\varphi} \\ |\nu_\mu, t\rangle = U_{\mu 1}|\nu_1\rangle + U_{\mu 2}|\nu_2\rangle + U_{\mu 3}|\nu_3\rangle e^{-i\varphi} \end{cases}$$

$U_{e3}^2 \sim 2\%$ **explains reactor neutrinos,** $U_{\mu 3}^2 \sim 1/2$ **atmospheric ones.**



Electron neutrino content $|U_{e3}|^2$
Muon neutrino content $|U_{\mu3}|^2$
Tau neutrino content $|U_{\tau3}|^2$
ν_3
ν_2
ν_1
2.4x10^{-3}
Mass increases
ν_2
ν_1
2.4x10^{-3}
ν_3

KamLAND and low energy solar neutrinos

The KamLAND experiment measured reactor antineutrinos, coming from several hundred kilometers. It revealed another type of oscillation, with

$$\boxed{\Delta m^2 = 7.5 \times 10^{-5}\ \text{eV}^2}$$

and $U_{e1}^2 \sim 70\%$ (or 30% due to the ambiguity $\theta \to 90^\circ - \theta$).

Agrees with low energy neutrino measurements, since $U_{e1} \sim \cos\theta$ *and*

$$P_{ee} = 1 - \sin^2(2\theta) \times \sin^2\varphi \approx 1 - \frac{\sin^2(2\theta)}{2} \approx 0.6$$

but not with high energy solar neutrinos: Some physics is missing!

MATTER EFFECT

There is another term, that affects the propagation of neutrinos, due to

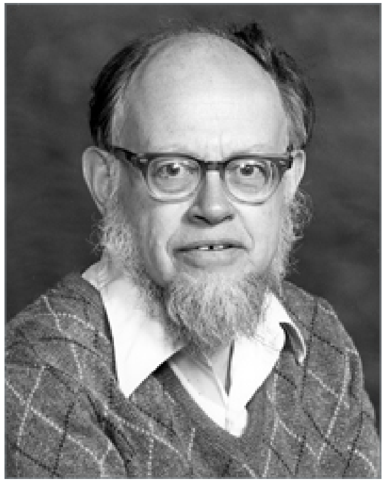
$$\mathbf{H}_{weak} = \frac{G_F}{\sqrt{2}} \int d^3x \; \overline{\nu_{\mathbf{e}}}(x)\gamma^a(1-\gamma_5)\nu_{\mathbf{e}}(x) \; \overline{\mathbf{e}}(x)\gamma_a(1-\gamma_5)\mathbf{e}(x)$$

Its matrix element between collinear neutrinos (forward scattering) gives

$$\pm\sqrt{2}\; G_F \, n_{\mathrm{e}}(x) \quad [\, + \text{ is for } \nu_e \text{ and } - \text{ for } \overline{\nu}_e \,]$$

where $n_e(x)$ is the local number density of non-relativistic electrons.

This gives a phase that affects electron neutrinos, not the other ones; recall that the vacuum oscillations are due to different phases of the mass eigenstates, $\exp(-iE_i t)$**.**




The first one who discussed the new term is Lincoln Wolfenstein, apparently triggered by a question of Emilio Zavattini. The exploration of its consequences is due to Stanislav Mikheyeev and Alexey Smirnov.

Compare the new term with the one that causes oscillations in vacuum

$$\frac{\sqrt{2}\, G_F\ n_e}{\Delta m^2/(2E_\nu)} \approx \frac{\rho}{5\ \text{gr/cc}} \times \frac{Y_e}{1/2} \times \frac{2.4\ 10^{-3}\text{eV}^2}{\Delta m^2} \times \frac{E_\nu}{6\ \text{GeV}}$$

that reads: for the bigger Δm^2 and with $E_\nu = 6$ GeV, the terms are similar in the average Earth density.

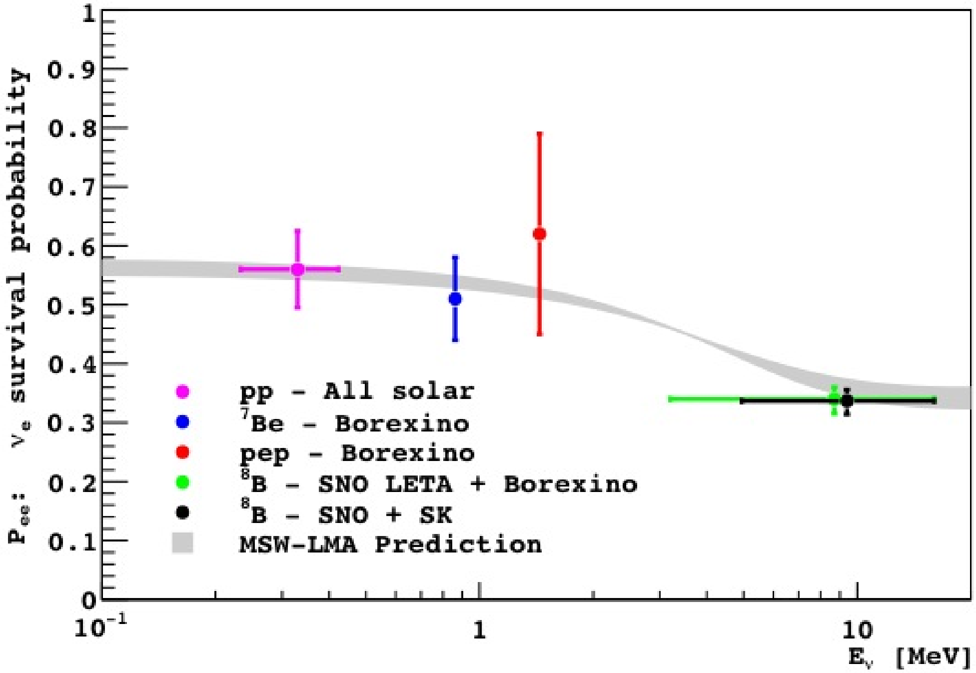
The same can be written

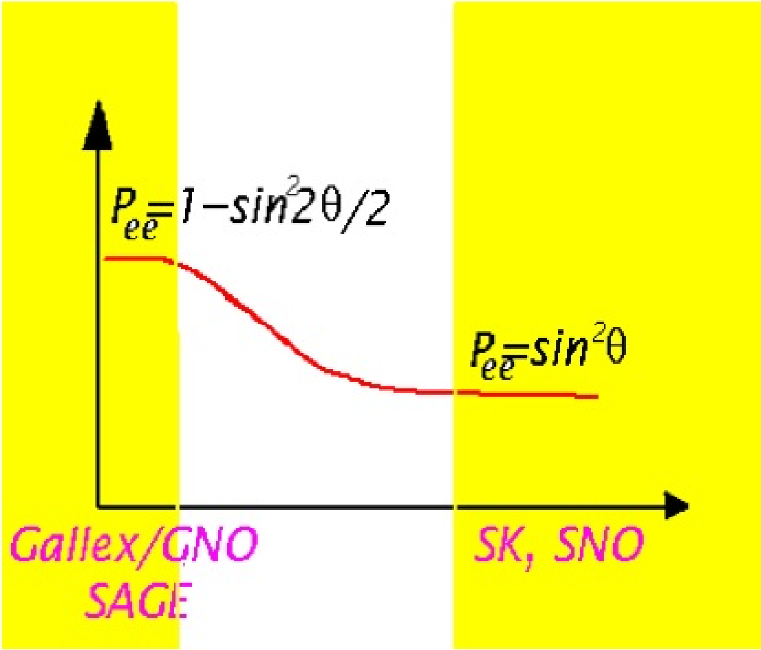
$$\frac{\sqrt{2}\, G_F\ n_e}{\Delta m^2/(2E_\nu)} \approx \frac{\rho}{100\ \text{gr/cc}} \times \frac{Y_e}{1} \times \frac{7.5\ 10^{-5}\text{eV}^2}{\Delta m^2} \times \frac{E_\nu}{5\ \text{MeV}}$$

that applies to the center of the Sun and to the smaller Δm^2, and shows that this term is relevant for high energy solar neutrinos.

APPLICATIONS

Solar neutrino oscillations

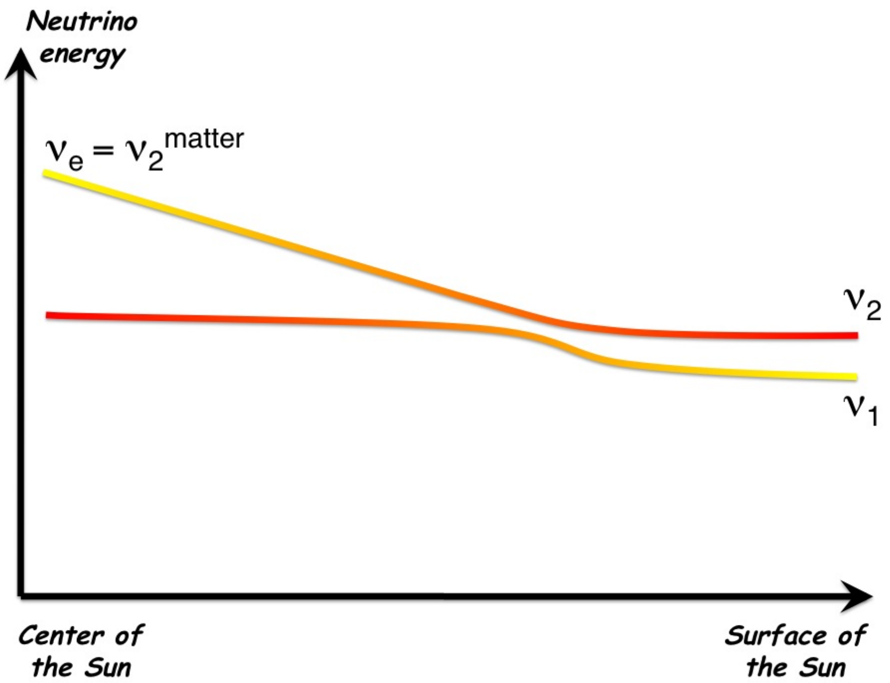




Variation of P_{ee} with the neutrino energy, as compiled by Borexino 2012 (left), and the explanation, from La Thuile 2003 (right): at low/high energy, we have oscillations in vacuum/matter.

Solar neutrinos [contd.]

The transformation among neutrino types can be described by a Hamiltonian of propagation, $\partial_t \psi = -i[H_{vac.}(t) + H_{matt.}]\psi$, that includes the matter term and the vacuum terms. For the conditions in the Sun, there is a simple solution.



Due to the matter term, the high energy, electronic solar neutrinos are produced as the heavier state. They remain always such, and exit the Sun as ν_2. Since $\nu_e = \cos\theta\nu_1 + \sin\theta\nu_2$, the overlap is just $P_{ee} = \sin^2\theta = 0.3$.

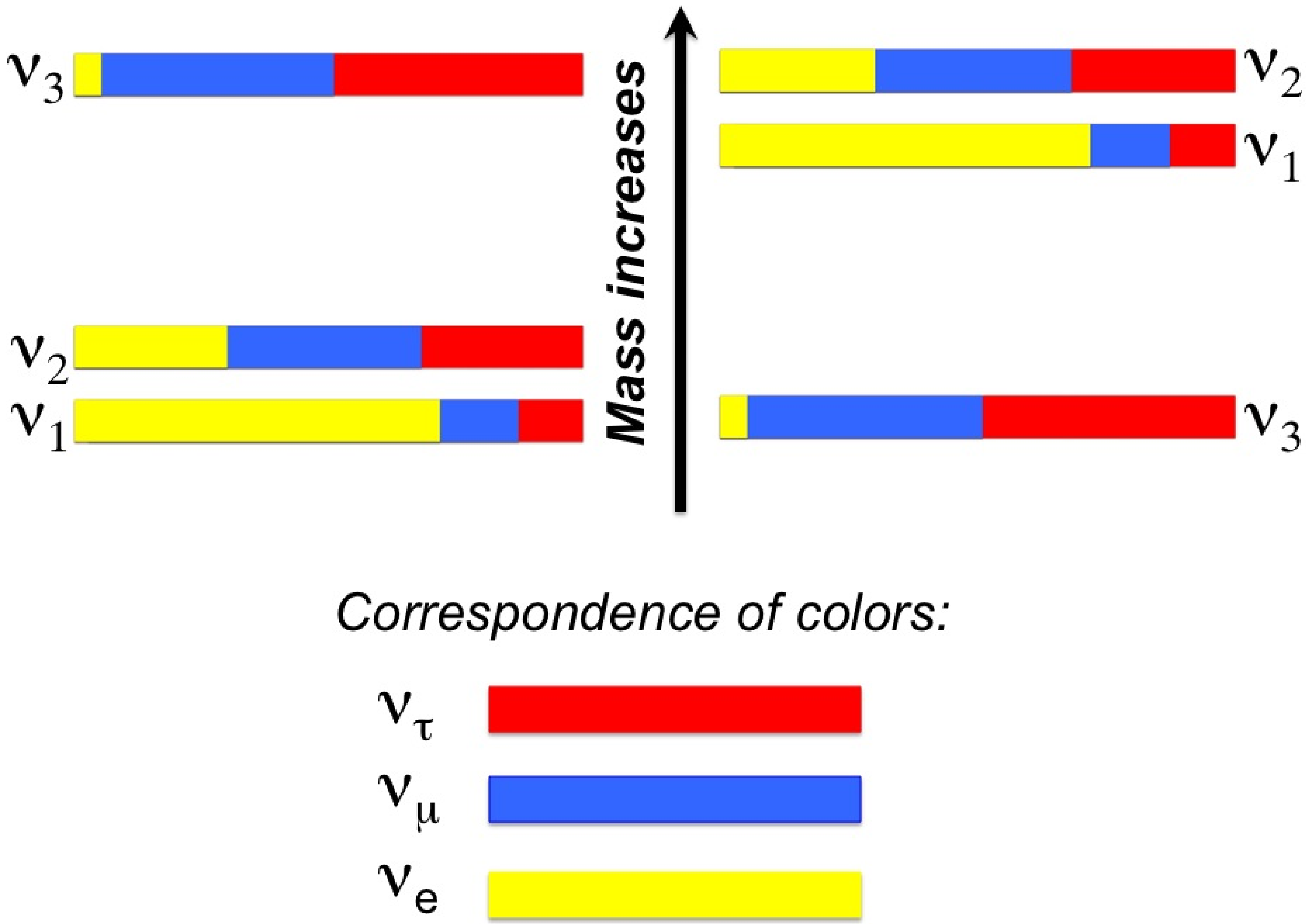
Constant matter density (the simplest case)

The 2 flavor expression for P_{ee} is the same as the vacuum one for constant density, after suitable replacements of the Δm^2 and of the mixing angle, e.g.,

$$\tan 2\theta_{matter} = \frac{\sin 2\theta}{\cos 2\theta - \xi} \text{ where } \xi = \pm \frac{\sqrt{2} G_F n_e}{\Delta m^2/(2E_\nu)}$$

Note that this expression is not anymore symmetric under the replacement $\theta \to 90° - \theta$; e.g., if $\theta < 45°$, $\theta_{matter} > \theta$ for neutrinos. Thus, the observation of matter effect reduces the ambiguity.

This is in essence the reason why we know, from solar neutrinos, that $U_{e1}^2 \approx 70\%$ rather than $U_{e1}^2 \approx 30\%$: the electron neutrino is mostly in ν_1.



ν_3
ν_2
ν_1
Mass increases
ν_2
ν_1
ν_3
Correspondence of colors:
ν_τ
ν_μ
ν_e

The last ingredient is the CP violating phase in the mixing matrix, that plays a similar role to the analogous term in the Cabibbo-Kobayashi-Maskawa matrix.

A common (PDG) parameterization is

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - s_{13}c_{12}s_{23}e^{i\delta} & c_{12}c_{23} - s_{13}s_{12}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - s_{13}c_{12}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{13}s_{12}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. Setting $\ell = e, \mu, \tau$ and $i = 1, 2, 3$ we have

$$|\nu_\ell\rangle = U^*_{\ell i}|\nu_i\rangle \ , \ |\bar{\nu}_\ell\rangle = U_{\ell i}|\bar{\nu}_i\rangle$$

θ_{13} is the angle newly seen in the reactor, θ_{12} is the solar/KamLAND mixing angle, θ_{23} is the atmospheric/longbaseline mixing, whereas the CP phase δ is unknown.

TABLE I: Results of the global 3ν oscillation analysis, in terms of best-fit values and allowed 1, 2 and 3σ ranges for the 3ν mass-mixing parameters. We remind that Δm^2 is defined herein as $m_3^2 - (m_1^2 + m_2^2)/2$, with $+\Delta m^2$ for NH and $-\Delta m^2$ for IH.

Parameter	Best fit	1σ range	2σ range	3σ range
$\delta m^2/10^{-5}$ eV^2 (NH or IH)	7.54	7.32 – 7.80	7.15 – 8.00	6.99 – 8.18
$\sin^2\theta_{12}/10^{-1}$ (NH or IH)	3.07	2.91 – 3.25	2.75 – 3.42	2.59 – 3.59
$\Delta m^2/10^{-3}$ eV^2 (NH)	2.43	2.33 – 2.49	2.27 – 2.55	2.19 – 2.62
$\Delta m^2/10^{-3}$ eV^2 (IH)	2.42	2.31 – 2.49	2.26 – 2.53	2.17 – 2.61
$\sin^2\theta_{13}/10^{-2}$ (NH)	2.41	2.16 – 2.66	1.93 – 2.90	1.69 – 3.13
$\sin^2\theta_{13}/10^{-2}$ (IH)	2.44	2.19 – 2.67	1.94 – 2.91	1.71 – 3.15
$\sin^2\theta_{23}/10^{-1}$ (NH)	3.86	3.65 – 4.10	3.48 – 4.48	3.31 – 6.37
$\sin^2\theta_{23}/10^{-1}$ (IH)	3.92	3.70 – 4.31	3.53 – 4.84 $\oplus$ 5.43 – 6.41	3.35 – 6.63
δ/π (NH)	1.08	0.77 – 1.36	—	—
δ/π (IH)	1.09	0.83 – 1.47	—	—

Professional, global neutrino data analyses are performed by the Bari team. A useful convention (=definition) of the two differences of mass squared is suggested. There is no hint on mass hierarchy in the present data. (Fogli, Lisi, et al. 2012).

WHAT NEXT?

The probability of oscillations from cosmic distances is reliably predicted (see Aharonian and FV, 2012) but irrelevant to mass hierarchy and CP studies.

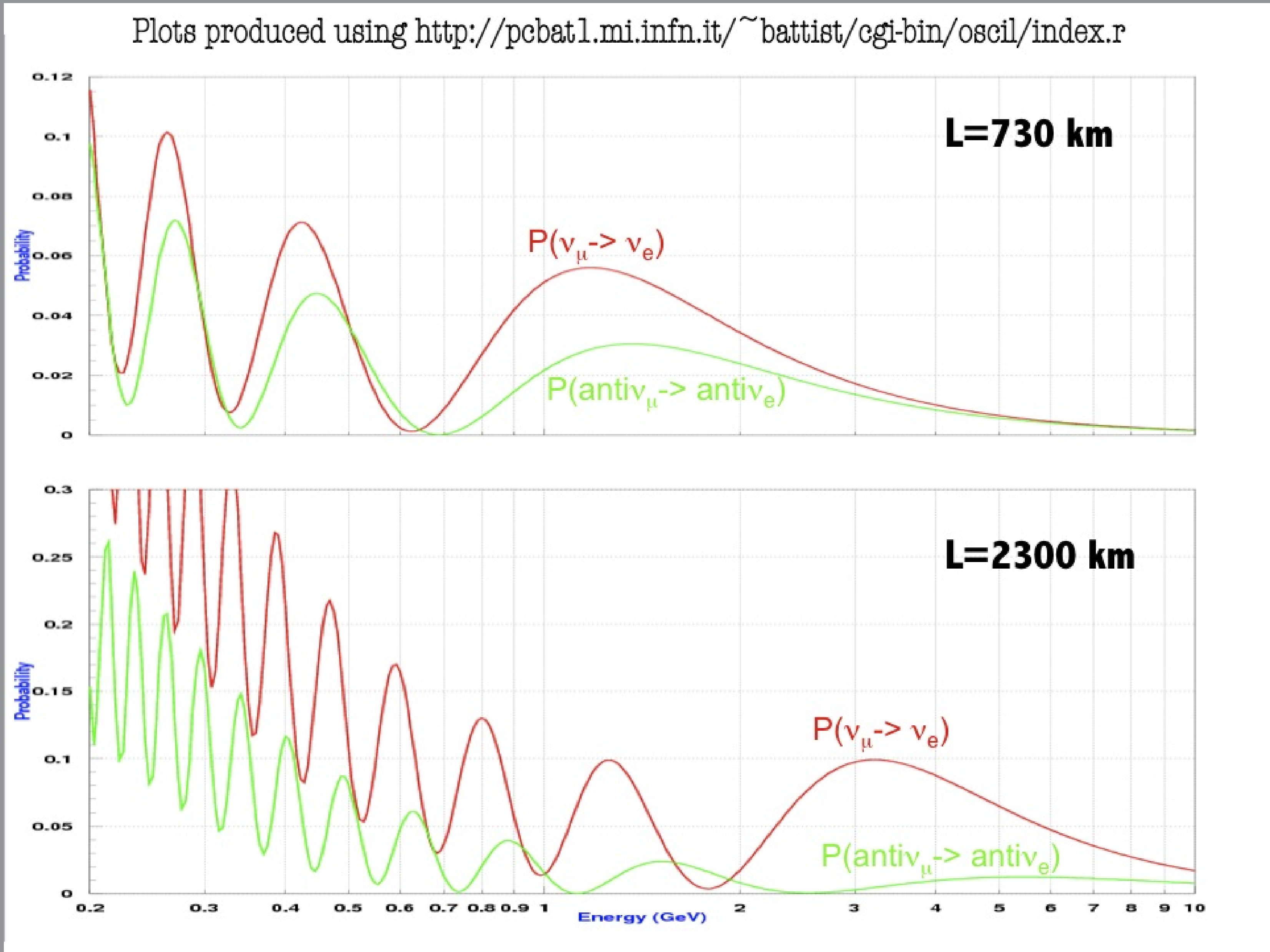
In the case of inverted hierarchy, we have lower bounds on neutrino mass entering the neutrinoless double beta decay rate

$$m_{ee} \equiv \left| \sum_{i=1}^{3} U_{ei}^2 \; m_i \right| > 18.2 \pm 3.2 \text{ meV}$$

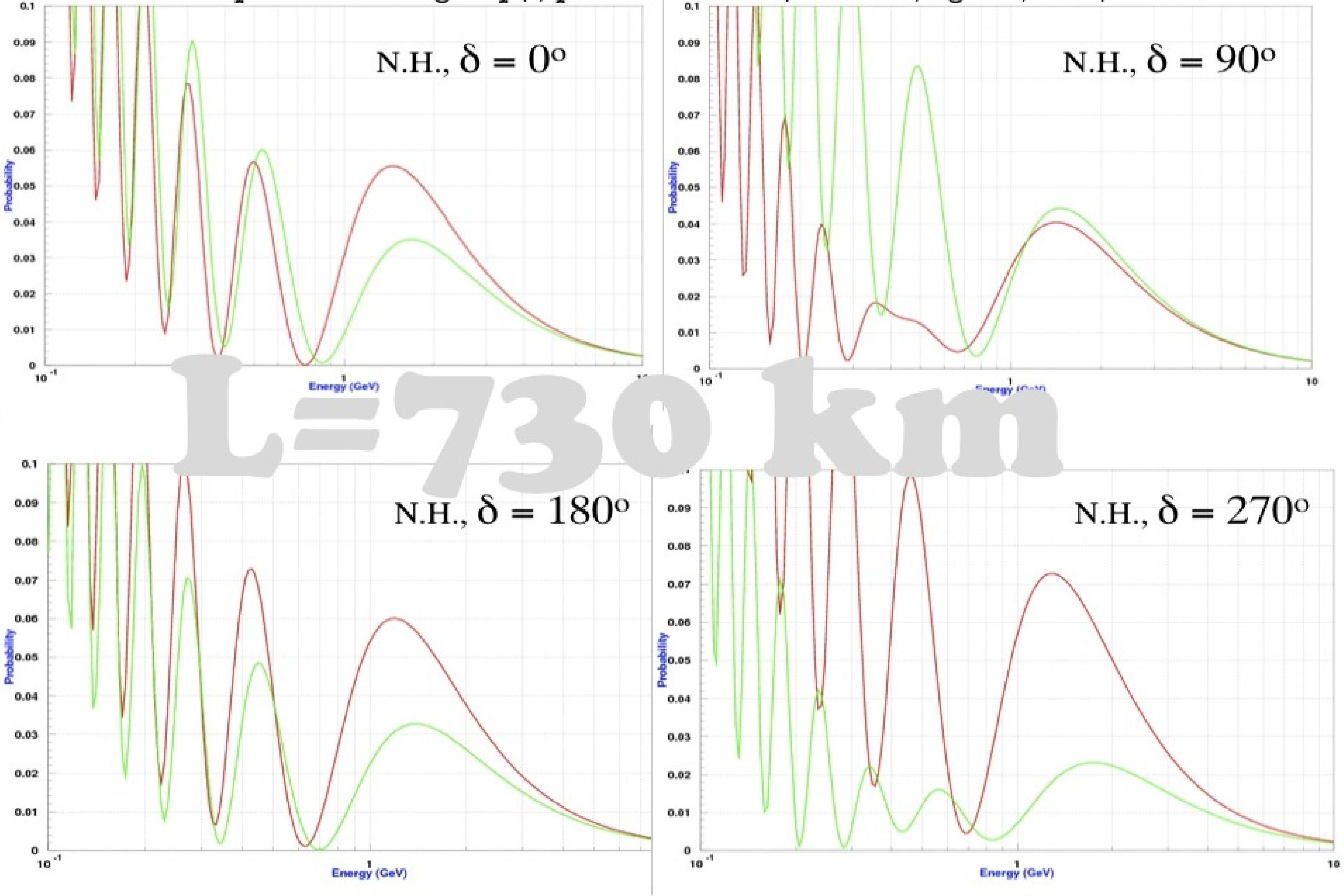
and on the mass that can be probed in cosmology

$$m_{cosm} \equiv \sum_{i=1}^{3} m_i > 98.4 \pm 3.0 \text{ meV}$$

Finally, we offer some remarks on oscillation studies with long-baseline experiments (at a fixed and known distance) and with atmospheric neutrinos passing through the Earth (various zenith angles, various energies).

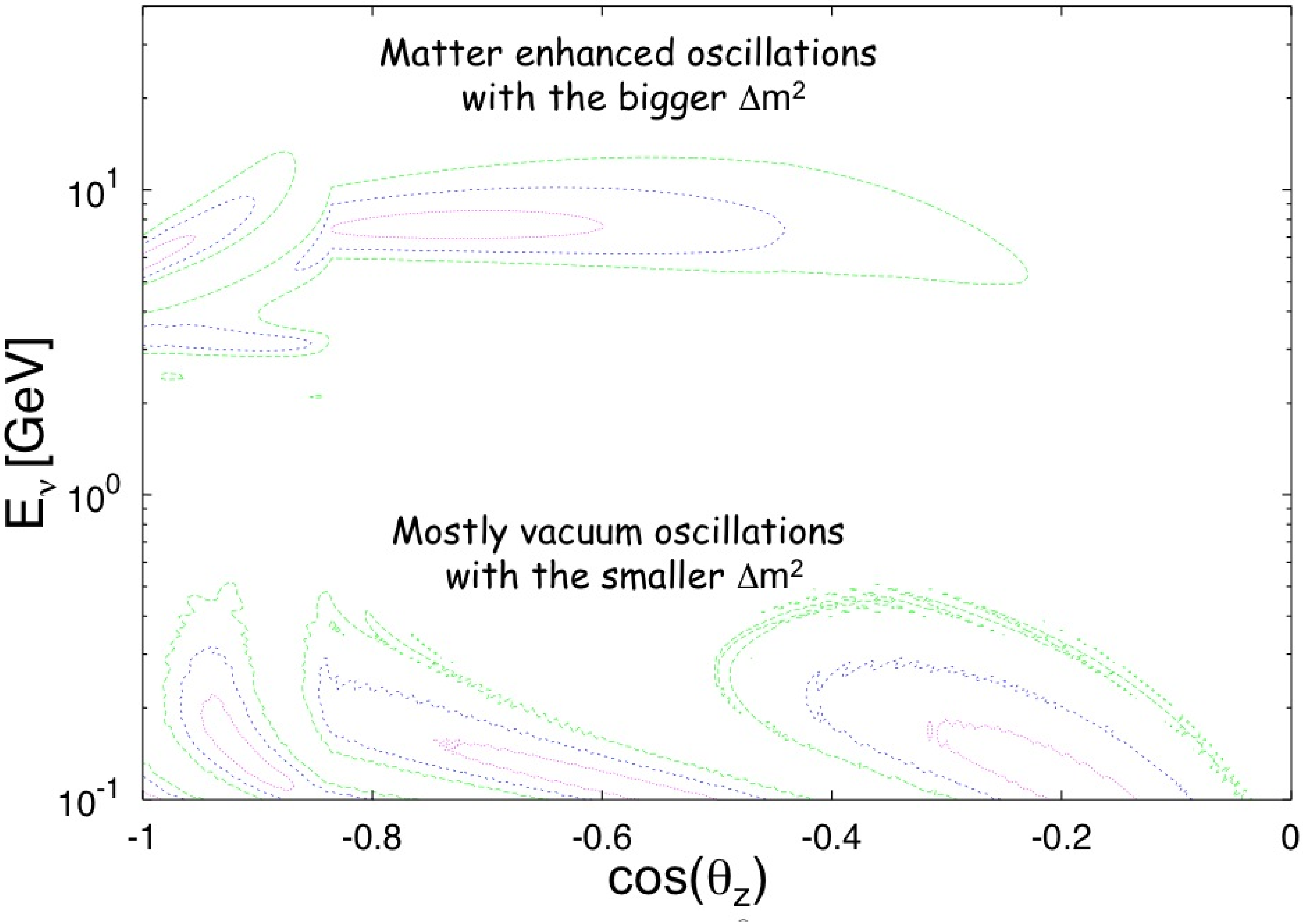

Plots produced using http://pcbat1.mi.infn.it/~battist/cgi-bin/oscil/index.r
L=730 km
P(ν_μ-> ν_e)
P(antiν_μ-> antiν_e)
Probability
0.12
0.1
0.08
0.06
0.04
0.02
0
L=2300 km
P(ν_μ-> ν_e)
P(antiν_μ-> antiν_e)
Probability
0.3
0.25
0.2
0.15
0.1
0.05
0
0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 2 3 4 5 6 7 8 9 10
Energy (GeV)

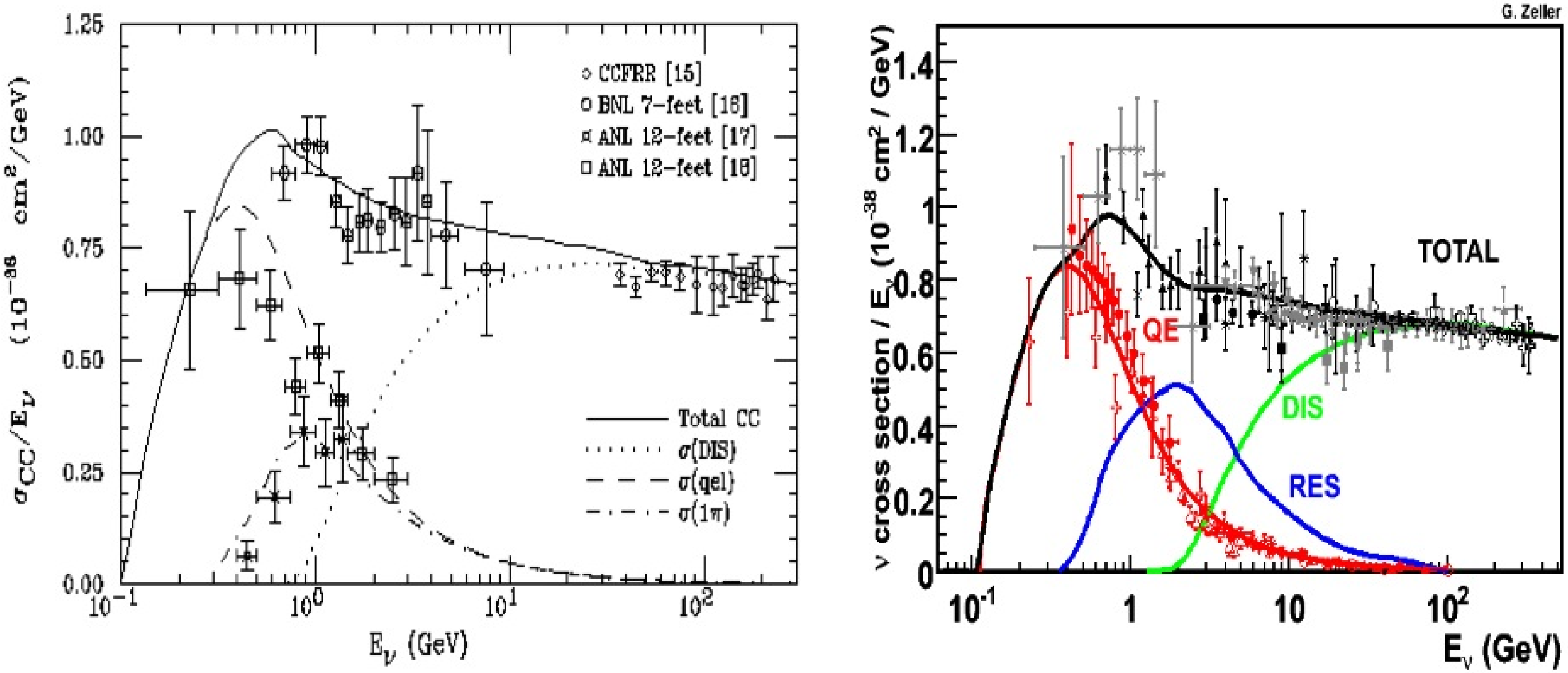
Plots produced using http://pcbat1.mi.infn.it/~battist/cgi-bin/oscil/index.r



Red curves, $P(\nu_\mu \to \nu_e)$; green curves, $P(\text{anti}\nu_\mu \to \text{anti}\nu_e)$

P_{ee} in the Earth = 0.3, 0.5, 0.7 (La Thuile 2003)





A comparison of two calculations of the inclusive neutrino cross section at GeV energies, summing the quasi-elastic, the delta resonance, and the deep inelastic contributions. Left, Lipari, Lusignoli, Sartogo 1995; right, Zeller 2012. The consistence is remarkable, except on the delta contribution, that has an important uncertainty.

Competing experimental projects

T2K (Japan). J-PARC neutrino beam, $L = 295$ km and $E_\nu = 0.4 - 0.8$ GeV. Now the off-axis detectors is Super-Kamiokande (22.5 kt), possibly followed by Hyper-Kamiokande (0.560Mt). Could probe the hierarchy. [Mezzetto is in T2K]

NOνA (USA). NuMI beam, $L = 730$ km, $E_\nu = 1.5 - 2.5$ GeV. 14 kton scintillator. More sensitivity to matter effect.

INO (India). 50 kton of magnetized iron, uses atmospheric neutrinos. Expect $\sim 230\nu + \bar{\nu}$ events per year above 2 GeV without oscillations. Estimate a sensitivity weaker than Monolith (Tabarelli de Fatis 2002) this needs clarification.

Daya Bay 2 (China). L=60 km, $E_\nu \sim 3$ MeV. Aims at seeing very small differences due to mass hierarchy; should meet tight experimental requirements.

For more discussion of the current experimental plans and projects, see the νTURN workshop at Gran Sasso at `http://nuturn2012.lngs.infn.it/`

Other light neutrinos?

No clear evidence, as argued in Cirelli et al., 2004 and as discussed in the "Workshop on Beyond Three Family Neutrino Oscillations" at Gran Sasso, 2011.

There are various hints (even if the agreement among them is not good and they are not easily reconciled with the cosmological bound on neutrino masses).

Various experimental proposals to proceed, using EC capture sources, reactors neutrinos, pion-at-rest beams, accelerators.

For a more complete discussion ask again next speaker, who authored this year a review on sterile neutrinos and participates in a new proposal for sterile neutrino search at CERN.

SUMMARY

⋆ The evidences for massive neutrinos are overwhelming and come from neutrino oscillations, a phenomenon on solid experimental basis.

⋆ The underlying theory is well understood and efficient software is available to describe oscillations through the Earth.

⋆ Open issues: the type of mass hierarchy; the parameter of CP violation; the size of absolute neutrino mass; the nature of neutrino masses.

⋆ All these steps are considered rather demanding. However, I would be not surprised if, as in the past, we will learn from global analyses and the right way to go will become more clear as we will proceed further.

Thanks for the attention!