Superconductivity, Superfluidity and holography

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Partly based on

O. Domènech, M. Montull, A. Pomarol, A. S. and P. J. Silva, JHEP **1008** *(2010) 033 arXiv:1005.1776*

M. Montull, O. Pujolàs, A. S. and P. J. Silva, Phys. Rev. Lett. **107** *(2011) 181601 arXiv:1105.5392*

M. Montull, O. Pujolàs, A. S. and P. J. Silva, JHEP **1204** *(2012) 135 arXiv:1202.0006*

A. S., JHEP **1209** *(2012) 134 arXiv:1207.3800*

Outline

1. **Introduction**
   - Effective Field Theory Description
   - Comparison between superconductors and superfluids

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2. **Holographic model (gauge/gravity correspondence)**
   - Motivations for holographic superconductors
   - Holography at finite temperature and density and phase transitions
   - Conductivity

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   - Motivations for holographic superconductors
   - Holography at finite temperature and density and phase transitions
   - Conductivity
3. **Holographic Superfluids vs Superconductors**
   - Dynamical gauge fields in holography
   - Vortices

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Effective theories of superfluids and superconductors

along the lines of [Weinberg, 1986]

A superconductor (SC) is a material in which $U(1)_{\rm em}$ is spontaneously broken.

Simplest field content: $a_\mu \equiv (a_0, a_i)\,,\ \Phi_{\rm cl}$

For time-independent configurations and without electric fields

$$Free\ energy = F = \int d^{d-1}x\, \mathcal{L}_{\rm eff}\left(\mathcal{F}_{ij}^2, |D_i\Phi_{\rm cl}|^2, |\Phi_{\rm cl}|, \ldots\right)$$

$$\mathcal{F}_{ij} \equiv \partial_i a_j - \partial_j a_i, \quad D_i\Phi_{\rm cl} \equiv (\partial_\mu - i\,a_\mu)\Phi_{\rm cl}$$

$$J^i = -\frac{\delta F}{\delta a_i}$$

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For small enough fields we expect a Ginzburg-Landau (GL) free energy:

$$F_{\rm GL} = \int d^{d-1}x \left\{ \frac{1}{4g_0^2}\mathcal{F}_{ij}^2 + |D_i\Phi_{\rm GL}|^2 + V_{\rm GL}(|\Phi_{\rm GL}|) \right\}$$

$$\Phi_{\rm GL} = \textit{constant} \times \Phi_{\rm cl}\,, \quad V_{\rm GL} \equiv -\frac{1}{2\xi_{\rm GL}^2}|\Phi_{\rm GL}|^2 + b_{\rm GL}|\Phi_{\rm GL}|^4$$

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non-dynamical $a_i \leftrightarrow$ superfluid limit

Comparing superconductors with superfluids

	superfluids (SF)	**superconductors (SC)**
J_i	SF current density	EM current density
$\arg(\Phi_{\rm cl})$	SF velocity potential in the lab frame	condensate's phase
a_i	external velocity in the lab frame	EM vector potential

Superfluid vortices

take the vortex Ansatz: $a_\phi = a_\phi(r)\ ,\quad \Phi_{\rm cl} = e^{in\phi}\psi_{\rm cl}(r)\ ,\quad n = integer$

(r, ϕ) are the polar coordinates restricted to $0 \leq r \leq r_m$, $0 \leq \phi < 2\pi$

a_ϕ is not dynamical (it is an external angular velocity performed on the superfluid):

This is implemented by working in a *rotating frame* with a constant angular velocity $\Omega = a_\phi/r^2$. In going from the static to the rotating frame the angular velocity of the superfluid is changed accordingly: $v_\phi \to v_\phi - \Omega r^2$. Then $J_\phi \propto (v_\phi - \Omega r^2)$.

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Superfluids $\leftrightarrow$ superconductors in the limit in which the EM field is frozen

In the Ginzburg-Landau theory the limit is $g_0 \to 0$ while keeping the external magnetic field $B = \partial_r a_\phi/r$ constant. In this limit

$$\Omega \leftrightarrow B/2\,, \quad L_\perp \leftrightarrow 2M$$

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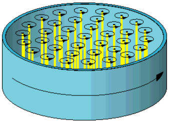
For $\quad \Omega \simeq \Omega_c$



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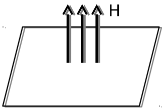


A superconducting plane probed by an external field H orthogonal to the plane

$H \neq B$ *as the magnetic field is dynamical*

Comparing superconductors with superfluids: vortices

For superconductor vortices, the dynamics of a_i is crucial

	superfluids	superconductors
field behavior	$\psi_{\rm cl} \overset{B=0}{\underset{}{\overset{\text{large } r}{\simeq}}} \psi_\infty \left(1 - n^2 \frac{\xi^2}{r^2}\right)$	$\psi_{\rm cl} \overset{\text{large } r}{\simeq} \psi_\infty + \frac{\psi_1}{\sqrt{r}} e^{-r/\xi'}$ $a_\phi \overset{\text{large } r}{\simeq} n + a_1 \sqrt{r} e^{-r/\lambda'}$
quantization of $\Phi(B)$	No	yes: $\int dr\, rB = n$
vortex energy	$F_n - F_0 \overset{\text{large } r_m}{\sim} n^2 \ln \frac{r_m}{\xi} - \frac{n}{2} B r_m^2$	finite as $r_m \to \infty$
1st critical field	$H_{c1} \overset{\text{large } r_m}{\simeq} \frac{2}{r_m^2} \ln \frac{r_m}{\xi}$	$\neq 0$ as $r_m \to \infty$
2nd critical field	$H_{c2} = \frac{1}{2\xi_{\rm GL}^2}$	$H_{c2} = \frac{1}{2\xi_{\rm GL}^2}$

Now Holography

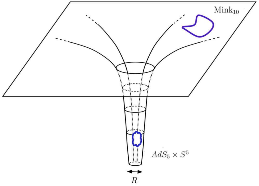
WHY?

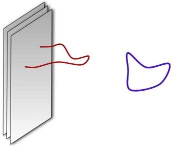
The gauge/gravity correspondence and its motivations

The goal: *describe* ***strongly*** *coupled systems by using a* ***weakly coupled*** *model with (at least) one extra dimension*

Classic example: the AdS/CFT correspondence **[Maldacena, 1997]**

Type II B string theory on $AdS_5 \times S^5$ $\leftrightarrow$ $\mathcal{N} = 4$ SYM on Minkowski





figures of **[Mateos, 2007]**

classical limit of string theory	$\leftrightarrow$	$N_c \to \infty$
classical limit and particle approximation	$\leftrightarrow$	$N_c \to \infty$, $\lambda \equiv g_{YM}^2 N_c \to \infty$ (not perturbative)

More recently: Phenomenological applications of holography to

- Condensed matter: for a review see for example **[Hartnoll, 2009]**
- To Quantum Chromodynamics **[Da Rold, Pomarol, 2005; Erlich, Katz, Son, Stephanov, 2005]**
- Strongly coupled theories beyond the Standard Model; e.g. composite Higgs models **[Agashe, Contino, Pomarol, 2004]**

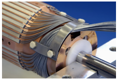
Motivations for holographic superconductors

- The most famous properties of superconductors follow from the spontaneous symmetry breaking of $U(1)_{\rm em}$ gauge invariance
- However, to understand how and when the spontaneous symmetry breaking of $U(1)_{\rm em}$ occurs one needs a microscopic theory
- BCS theory **[Bardeen, Cooper, Schrieffer, 1957]** describes "conventional superconductors" only
- There are also "unconventional superconductors"

e.g. some high-temperature superconductors (HTSC) which, unlike BCS theory, seem to involve strong coupling

important applications;
e.g. HTSC current leads

for the LHC magnets



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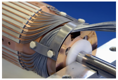

→ apply the gauge/gravity correspondence

The holographic model [Hartnoll, Herzog, Horowitz, 2008; Horowitz, Roberts, 2008]

$$ds^2 = \frac{L^2}{z^2}\left[-f(z)dt^2 + dx_1^2 + \ldots + dx_{d-1}^2\right] + \frac{L^2}{z^2 f(z)}dz^2\,, \quad f(z) = 1 - \left(\frac{z}{z_h}\right)^d$$

$\mathcal{O} \leftrightarrow \Psi$
$\Psi|_{z=0} = s =$ source of $\mathcal{O}$

$\hat{J}_\mu \leftrightarrow A_M$
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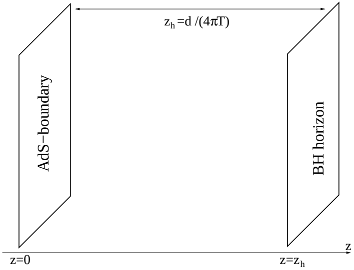
When $\Psi = 0$ the system describes a conductor (with non-zero conductivity)



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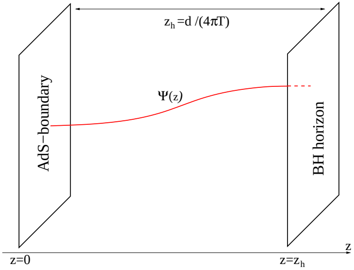
Superconducting phase $\Psi \neq 0$

no x^μ-dependence (homogeneous solutions) and $A_i = 0$

$\mu \equiv A_0|_{z=0}$

$T < T_c = 0.03(0.05)\mu \quad \text{for } d = 3(4)$



Conductivity in the unbroken phase

To compute the conductivity let us consider a small time-dependent perturbation

$$A_x(t,z) = \mathcal{A}(z) e^{i\omega(p(z)-t)}$$

The system responds creating a current which is linear in a_x: $\langle J_x \rangle = \sigma E_x$.
Using the AdS/CFT dictionary, $J_x \propto z^{3-d} \mathcal{F}_{zx}|_{z=0}$

$$g^2 \sigma = p'(0) - i \frac{\mathcal{A}'(0)}{\omega \mathcal{A}(0)}$$

Since this is a linear response problem the conductivity can be computed by solving the linearized Maxwell equation

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with appropriate boundary conditions.

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Presence of the black hole horizon $\rightarrow$ Re$[\sigma] \neq 0$
because the solution has to be ingoing in the horizon

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Now the linearized Maxwell equation is $\partial_z (f \partial_z A_x) + \omega^2 \frac{A_x}{f} - \frac{2}{z^2} \psi^2 A_x = 0$

Im$[\sigma]$ diverges like $1/\omega$ as $\omega \to 0$, corresponding through the Kramers-Kronig relation

$$\mathrm{Im}[\sigma(\omega)] = -\frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} d\omega' \frac{\mathrm{Re}[\sigma(\omega')]}{\omega' - \omega}$$

to a delta function in the real part, Re$[\sigma(\omega)] \sim \pi n_s \delta(\omega)$

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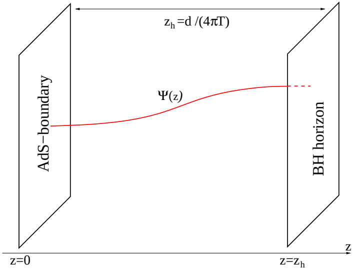

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[Albash, Johnson, 2008; Hartnoll, Herzog, Horowitz, 2008; Montull, Pomarol, Silva, 2009]

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However, that (Dirichlet) boundary condition corresponds to a superfluid

$\rightarrow$ **non-dynamical** a_i!

Dynamical a_μ in holography

- impose a **dynamical equation for a_μ**

$$J^\mu + \frac{1}{g_b^2}\partial_\nu \mathcal{F}^{\nu\mu} + J^\mu_{ext} = 0$$

  Here, for generality, we have added a kinetic term for a_μ and a background external current J^μ_{ext}
- Then we must add to S the following term

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- by using $J_\mu = \frac{L^{d-3}}{g^2} z^{3-d} \mathcal{F}_{z\mu}|_{z=0}$

$$\frac{L^{d-3}}{g^2} z^{3-d} \mathcal{F}_z{}^\mu \Big|_{z=0} + \frac{1}{g_b^2}\partial_\nu \mathcal{F}^{\nu\mu}\Big|_{z=0} + J^\mu_{ext} = 0$$

This is an AdS-boundary condition of the Neumann type

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$d = 3 + 1$ case

J_μ is logarithmically divergent:

$$\frac{1}{z} \mathcal{F}_{z\mu}\Big|_{z=0} = -\partial^\nu \mathcal{F}_{\nu\mu} \ln z\Big|_{z=0} + \dots$$

We can absorb the divergence in $\frac{1}{g_b^2} \partial_\nu \mathcal{F}^{\nu\mu}\Big|_{z=0}$ to define a renormalized electric charge g_0 in the normal phase ($\Phi_{\rm cl} = 0$):

$$\frac{1}{g_0^2} = \frac{1}{g_b^2} - \frac{L}{g^2} \ln z|_{z=0} + \textit{finite terms}$$

a_μ breaks conformal invariance
(the same is true for any $d > 4$)

Dynamical a_μ in holography

$$\frac{L^{d-3}}{g^2} z^{3-d} \mathcal{F}_z{}^{\mu}\Big|_{z=0} + \frac{1}{g_b^2}\partial_\nu \mathcal{F}^{\nu\mu}\Big|_{z=0} + J^{\mu}_{ext} = 0$$

<u>$d = 3+1$ **case**</u>

J_μ is logarithmically divergent:

$$\frac{1}{z}\mathcal{F}_{z\mu}\Big|_{z=0} = -\partial^\nu \mathcal{F}_{\nu\mu} \ln z\Big|_{z=0} + \dots$$

We can absorb the divergence in $\frac{1}{g_b^2}\partial_\nu \mathcal{F}^{\nu\mu}\Big|_{z=0}$ to define a renormalized electric charge g_0 in the normal phase ($\Phi_{\rm cl} = 0$):

$$\frac{1}{g_0^2} = \frac{1}{g_b^2} - \frac{L}{g^2} \ln z|_{z=0} + \textit{finite terms}$$

a_μ **breaks conformal invariance**
(the same is true for any $d > 4$)

<u>$d = 2+1$ **case**</u>

no divergence $\Rightarrow$
we can take $g_b \to \infty$
so $\frac{1}{g_b^2}\partial_\nu \mathcal{F}^{\nu\mu}\Big|_{z=0} \to 0$

In this case a_μ *does not* break conformal invariance and can be considered as an emerging phenomenon: its kinetic term is induced by the dynamics
see also **[Witten, 2003]**

Vortex solutions in holographic superfluids

Vortex ansatz: $\Psi = \psi(z,r)e^{in\phi}\ , \quad A_0 = A_0(z,r)\ , \quad A_\phi = A_\phi(z,r)\,,$

AdS-boundary conditions: $s = 0\ , \quad \mu = constant\ ,$

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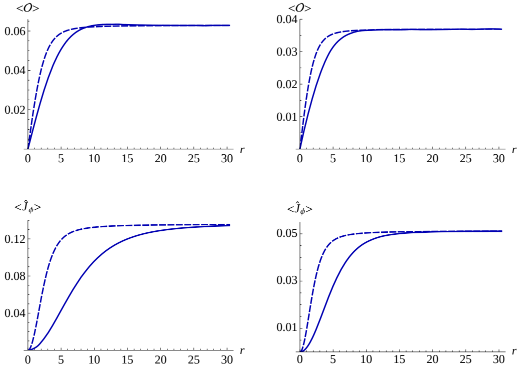
Plots: for $n = 1$, $T/T_c = 0.3$ and $B = 0$

- solid lines: holographic profiles for $d = 2+1$ (left) and $d = 3+1$ (right)
- dashed lines: corresponding profiles in the GL model



Determination of GL parameters:

- $\xi_{\rm GL}^2 = \frac{1}{2B_{c2}}$
- the matching at large r then gives b_{GL}.

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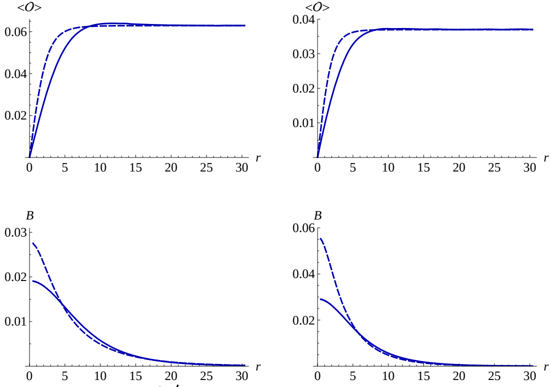
$g_b/g \to \infty$ for $d = 2+1$, while, for $d = 3+1$, we have taken g_b to satisfies $g_0^{-2}(T = T_c) \simeq 1.7L/g^2$



Determination of GL parameters:

- $\xi_{\rm GL}^2 = \frac{1}{2H_{c2}}$,
- the matching at large r gives b_{GL} and g_0 in the GL free energy.

We observed $a_\phi \simeq n + a_1\sqrt{r}e^{-r/\lambda'}$, for large r.

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Plots:

- left: $d = 2 + 1$
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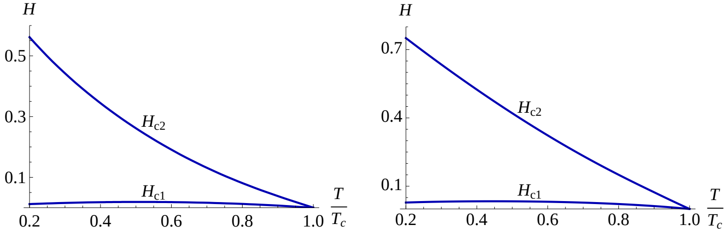

$H_{c1} < H_{c2}$ for every T, so **the holographic superconductors are of Type II**

Interestingly, HTSC are also of Type II

Holographic insulator/superconductor transition

- The model above realizes a conductor/superconductor transition
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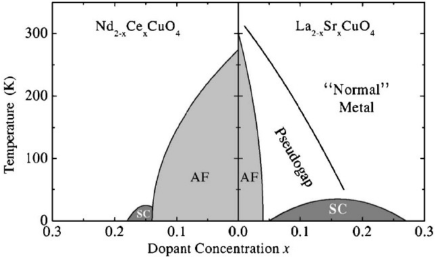

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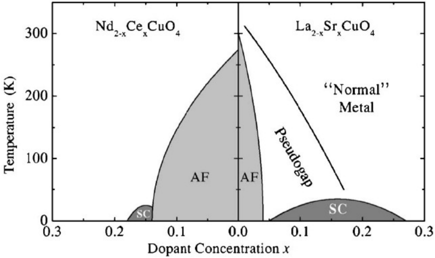

Holography also overcomes the challenge to describe *insulating* materials that display superconductivity at low enough temperatures **[Nishioka, Ryu, Takayanagi, 2009; Salvio 2012]**

In holography we compactify a spatial dimension: $\chi \sim \chi + 2\pi R$

We have two static metrics with symmetry IO$(d-1) \times U(1)$ or Poincaré$(d-2,1) \times U(1)$

Black Hole (deconfined) phase: a conductor

$$ds^2 = \frac{L^2}{z^2}\left[-f(z)dt^2 + d\chi^2 + dy_{d-2}^2 + \frac{dz^2}{f(z)}\right]$$

$f(z) = 1 - (z/z_h)^d$, $z_h = d/4\pi T$, **Favorable for $R > 1/2\pi T$** (at $\mu = 0$)

"Soliton" (confined) phase [Witten, 1998; Horowitz, Myers, 1998]: an insulator

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- the transition between them occurs at μ and/or T around $1/R$ (known as a Hawking-Page transition (1983))
- both phases exhibit SC bahavior: below $T \sim 1/R$ and increasing μ, one finds first a Soliton SC state and then (for $\mu \gtrsim 1/R$) a Black Hole SC

For the soliton (with no metric backreaction) $R_c \simeq \dfrac{1.81\ (1.70)}{\mu}$, for $d = 2{+}1\ (3{+}1)$

Conductivity in the confined phase: insulator

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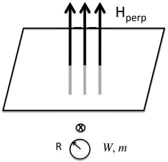
- Fluid mechanical interpretation of an insulator: a solid

Magnetic fields in the presence of a compact space-dimension

In principle, there are *two ways* to turn on an external magnetic field *H*:



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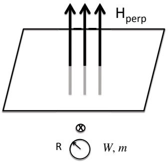

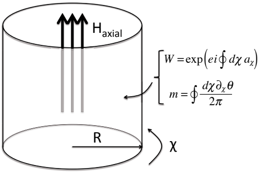

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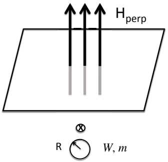

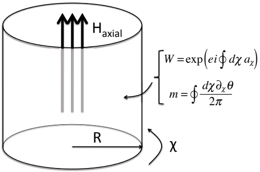

We focus on the possibility on the left because here we interpret the compact extra dimension only as a tool to have a gapped system.

For an analysis of the second possibility see **[Montull, Pujolàs, Salvio, Silva, 2011, 2012]**

$H = H_{\rm perp}$ in the holographic insulator/superconductor transition

Vortex ansatz (for $d \geq 4$): $\Psi = \psi(z,r)e^{in\phi}$, $A_0 = A_0(z,r)$, $A_\phi = A_\phi(z,r)$,

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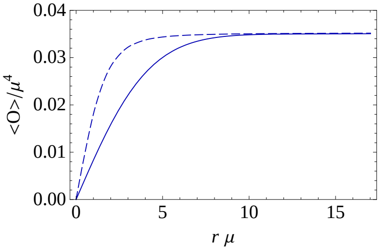

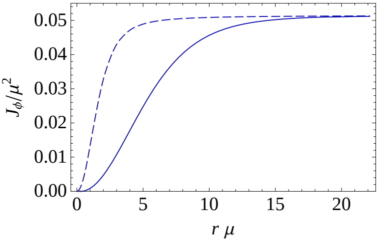

Plots: $n = 1$, $R/R_c = 5$ and $B = 0$

- Solid lines: holographic model for $d = 3 + 1$
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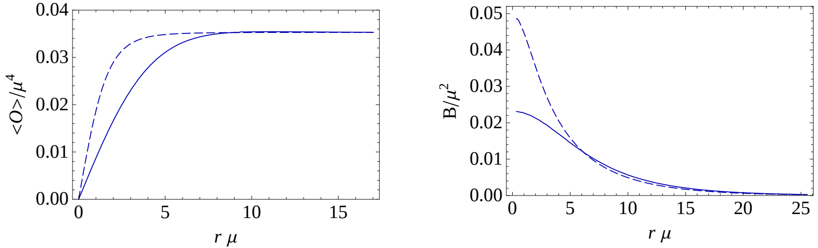
The gauge field is emergent for $R \to 0$



Plots: The modulus of $\langle \mathcal{O} \rangle$ (up to L^{d-3}/g^2) and B versus r for $n = 1$

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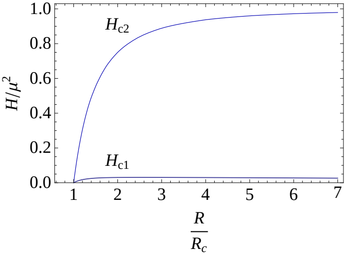

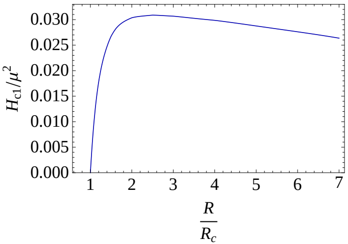

Plots: H_{c1} and H_{c2} versus R from holography for $d = 3+1$ and g_b chosen to satisfy $g_0^{-1}(R = R_c) \simeq 1.7L/g^2$

$H_{c1} < H_{c2}$ for every R, **so also in this phase the holographic superconductor is of Type II, like the high-temperature superconductors**

Dilaton-Gravity

An (approximate) insulating normal phase can also be obtained with a dilaton

Other reasons for dilatonic extensions are

- Charged dilaton black holes have more physical low-temperature behavior
  [Charmousis, Gouteraux, Kim , Kiritsis, Meyer, 2010]
- Dilatons typically emerge in low-energy effective descpritions of string theories

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The gravity action:

$$S_{\rm gravity} = \frac{1}{16\pi G_N}\int d^4x\,\sqrt{-g}\left[\mathcal{R} - (\partial_\alpha\phi)^2 - V(\phi)\right]$$

The most general static asymptotically AdS planar black hole with two-dimensional rotation and translation invariance has recently been derived **[anabalón, 2012]**. *This allows us to extend the previous analysis to general dilaton-gravity model in the limit $G_N \to 0$.*

In particular we have

$$\phi(z) = \sqrt{\frac{\nu^2-1}{2}}\,\ln(1+z/L)$$

Holographic model for superfluid phase transitions

$$S = S_{\rm gravity} + \int d^4x \sqrt{-g} \left\{ -\frac{Z_A(\phi)}{4g^2} \mathcal{F}^2_{\alpha\beta} - \frac{Z_\psi(\phi)}{L^2 g^2} |D_\alpha \Psi|^2 \right\}$$

the dilaton couples to A_α and Ψ through two *generic* functions $Z_A(\phi)$ and $Z_\psi(\phi)$

There are no special requirements for the Zs at this level, besides the fact that they should be regular and nonvanishing for any ϕ in order for the semiclassical approximation to be valid

Again one can show that

- There is a superfluid phase transition at small enough T and big enough μ
- There are vortex solutions both in the superfluid and in the superconductor case

Conductivity

We can study the conductivity using the same approach we used without the dilaton

One can show

$$\lim_{\omega \to 0} \text{Re}[\sigma] = \frac{1}{g^2} Z_A|_{z=z_h}$$

$\to$ the DC conductivity can be suppressed or enhanced depending on $Z_A(\phi)$

Example: $Z_A(\phi) = e^{\gamma\phi}$, the bigger γ the bigger the DC conductivity, *while a large negative value of γ corresponds to an approximate insulating behavior*.
This effect is even stronger at low temperatures

$$\lim_{\omega \to 0} \text{Re}[\sigma] \sim T^{-\gamma\sqrt{(\nu^2-1)/2}}$$

For a $Z_A(\phi)$ such that the DC conductivity is small for $\phi \neq 0$ the superfluid phase transition is a conductor/superconductor transition at high T and a (non-ideal)insulator/superconductor one at low T

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- *Effective field theory description and comparison between superfluids and superconductors*

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Outlook

- *Applications of the method to introduce a dynamical gauge field in holography to color superconductivity*
- *Extension of the insulator/superconductor models to describe a bigger portion of (or even the complete) phase diagram of cuprate high-temperature superconductors*