The Problem with

Quantum Gravitation

General reference:

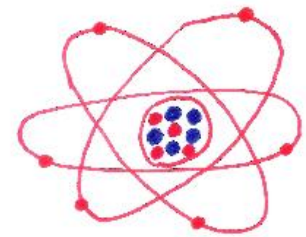
“Quantum Gravitation” (Springer Tracts in Modern Physics, 2009).

Roma I, November 2012

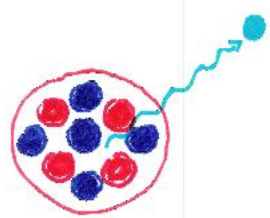
Four Fundamental Forces

Strong Force :

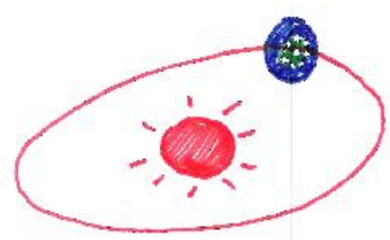
Electro-magnetism :



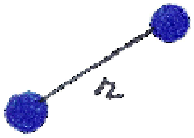
Weak Force :



Gravitational Force :



Key Facts about Gravity



two protons

By far the **Weakest** force

$$F_{Grav}/F_{E.M.} = \frac{G\,m_p^2/r^2}{e^2/r^2} = \frac{G\,m_p^2}{e^2} \sim 10^{-40}$$

… **Universal** – Couples to Mass and Energy

… **Long Range** $$F = \frac{G\,m_1\,m_2}{r^2}$$

… **Cannot be screened** (no negative gravitational "charge")

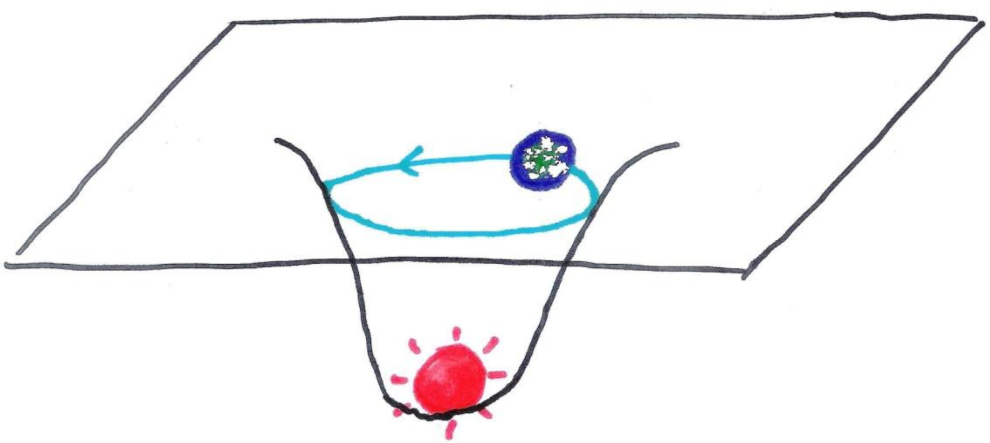
$\Rightarrow$ **Gravity affects the Structure of Matter on the Largest Scales**

Relativistic Gravity :
Einstein's General Relativity

Gravity affects the **Geometry**

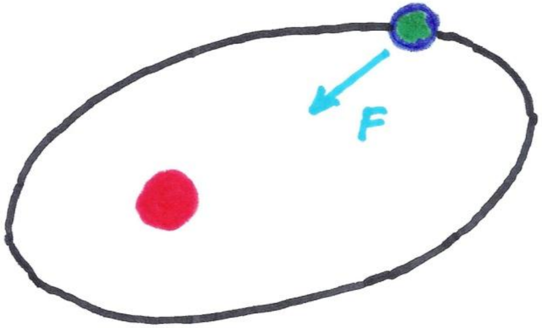
Mass-Energy curves Space-Time …

… Space-Time then tells matter how to move .

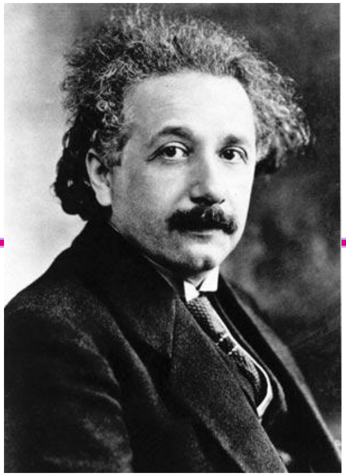


Newtonian Gravity (*1687*)

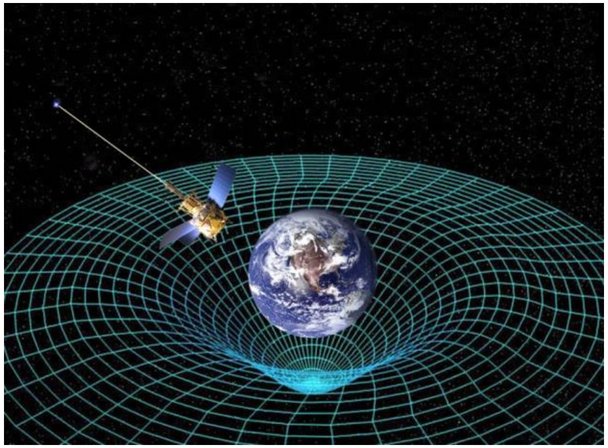
The Gravitational Force makes the Earth go around the Sun.



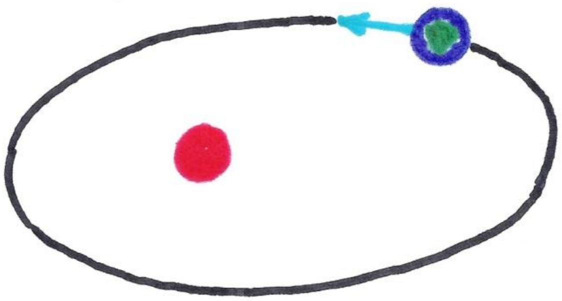
Einstein's Gravity (*1915*)



The Earth travels around the Sun by following the straightest path in Curved Space-Time.



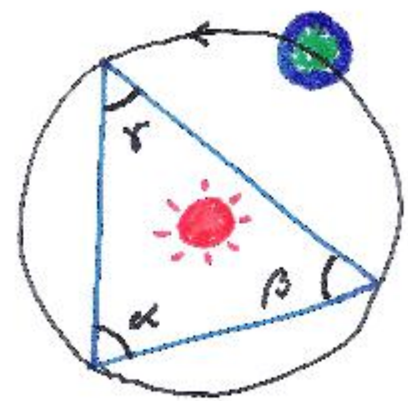
straightest path ("geodesic")



Locomotive graveyard in Uyuni, Bolivia

Curved Space-Time

Small Effect in the Solar System …



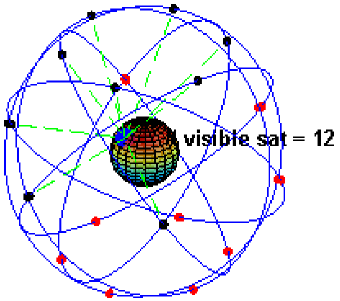
… but big for :

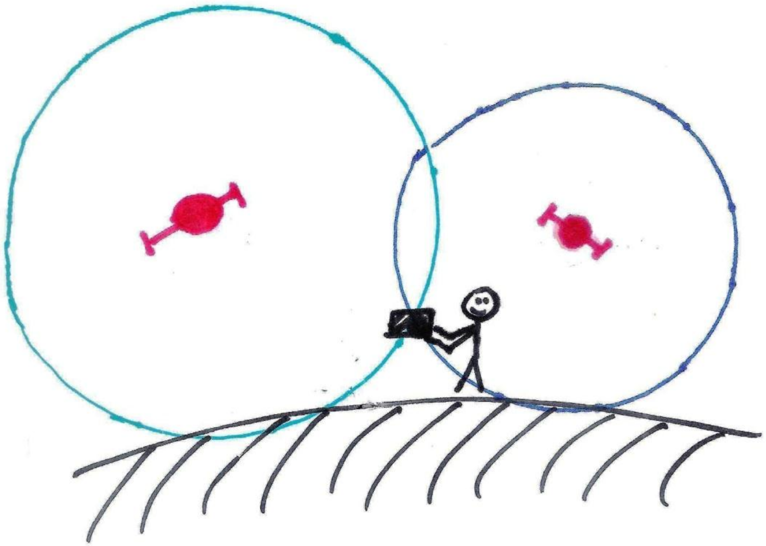
$$\alpha + \beta + \gamma \;=\; 179.9999999999...^{0}$$

- Neutron Stars
- Black Holes
- The Big Bang
- Gravitational Waves
- The Universe

GPS

A Continuously operating experiment in
Special and General Relativity





Altitude 20,000 km, orbit time 12 hrs.

Individual satellite clocks accurate to a few nanoseconds

The Large Scale Frontier …

Relativistic Gravity is important for

Massive Compact Objects

$$\frac{G\,M}{R\,c^2} \sim 1$$

Such as :

- Neutron Stars
- Black Holes
- Gravitational Waves
- The Universe (!)

Strong Gravity

The Small Scale Frontier ...

Planck Scale : $$\left(\frac{G\,\hbar}{c^3}\right)^{1/2} \sim 1.62 \times 10^{-33} cm \sim (\text{proton size}) \times 10^{-19}$$

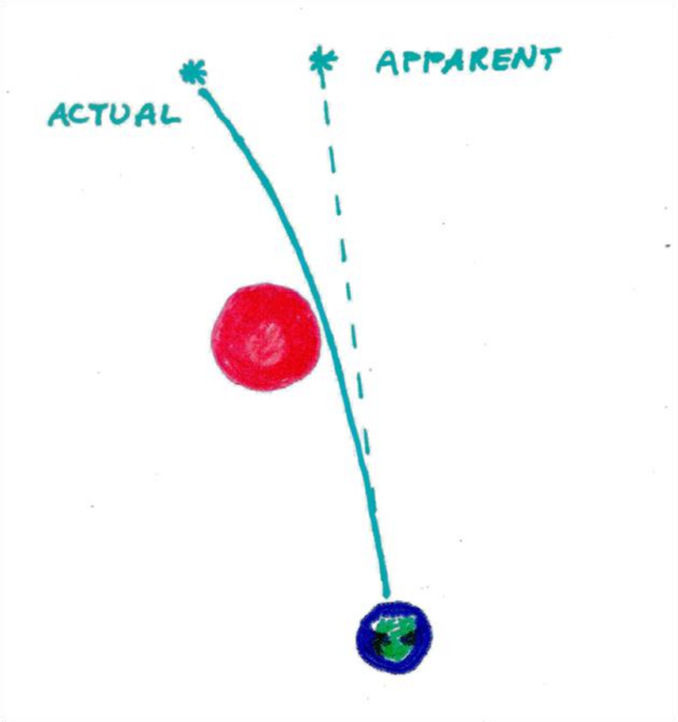
Planck Energy is about fifteen order of magnitude larger than the energy achieved at the largest accelerator $\Rightarrow$

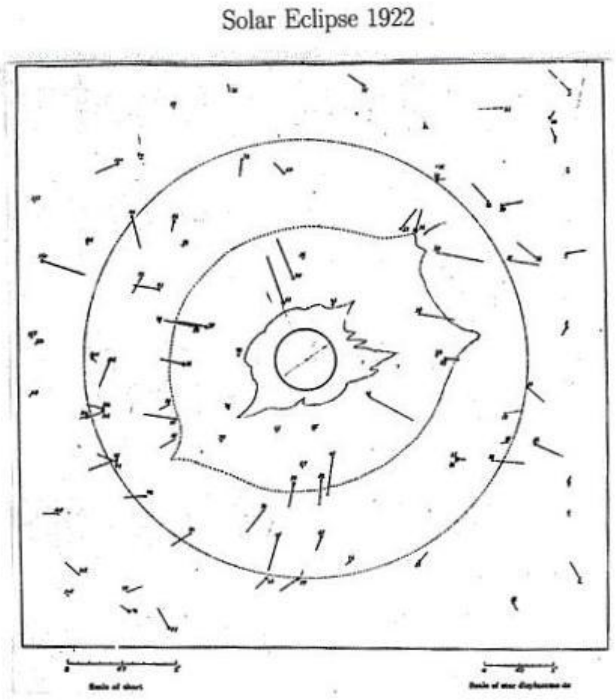
This is perhaps the scale characterizing :

- Unified Theories of Basic Forces

String Theory?

Bending of Light by the Sun

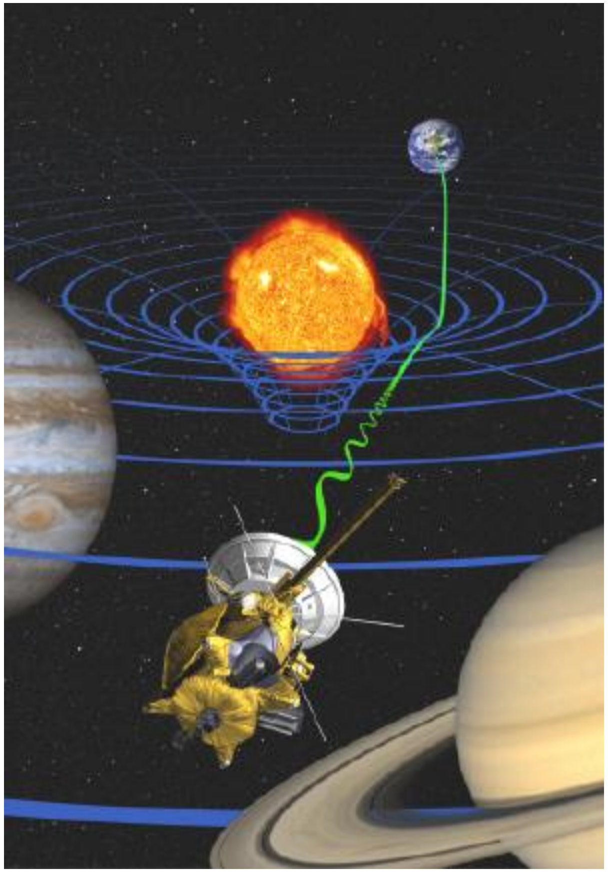




Campbell and Trumpler

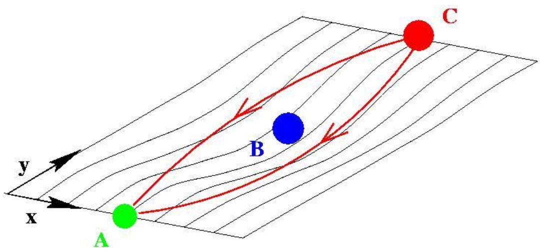
Cassini Mission (2003)

Test deflection to one part in 10^5

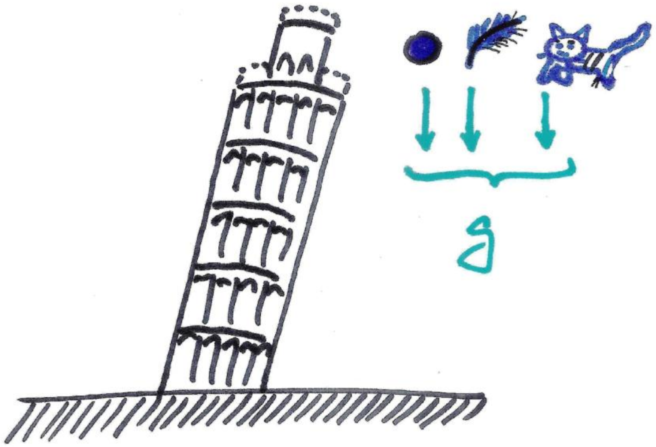


Gravitational Lensing

Galaxy Cluster 0024+1654
(HST)



Principle of Equivalence



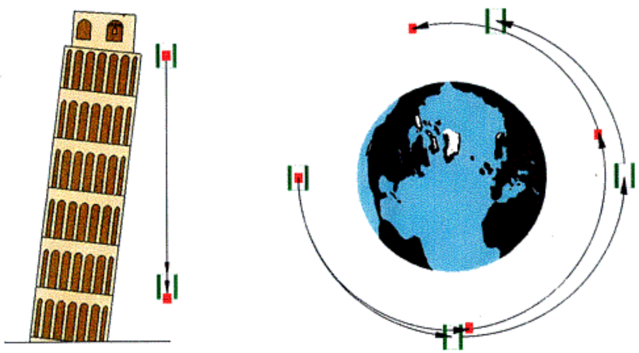
$$g's \text{ equal to } 5 \times 10^{-13}$$

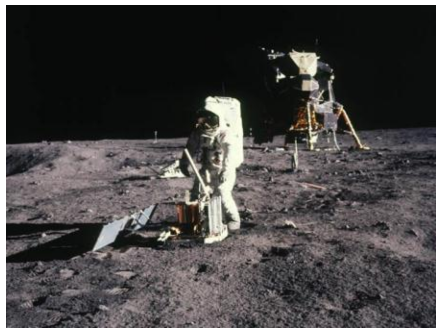
(Lunar laser ranging)

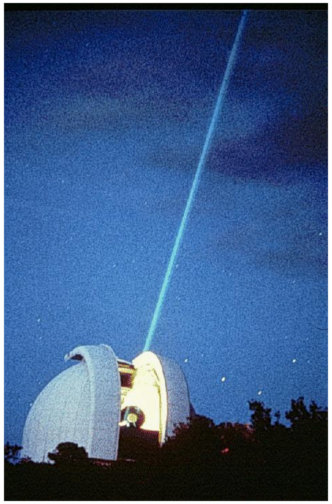
- Central to a Geometric (Gauge) Theory of Gravity

    (there is presumably only one shortest path!)
- Violations would affect of our notion of Space-Time
- … or New Forces

Tests of Equivalence Principle







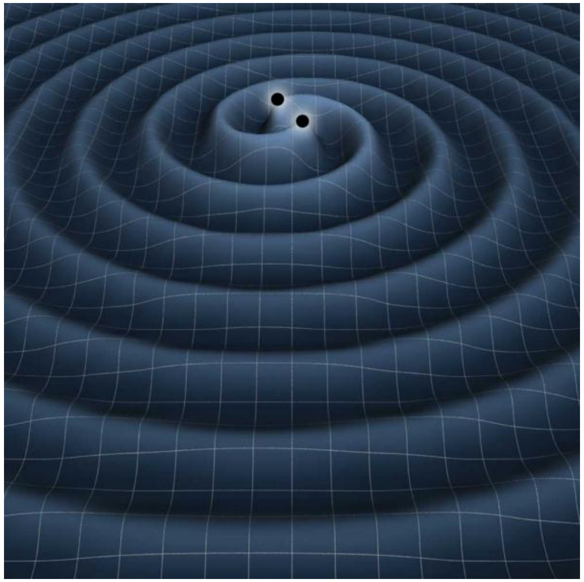
Satellites can be used to test the Equivalence Principle
(future exp.).

Lunar ranging has set stringent limits on both composition dependence, and time-dependence of G : less than 1 part in 10^{11} since 1969.

Strong Gravity: Gravitational Waves

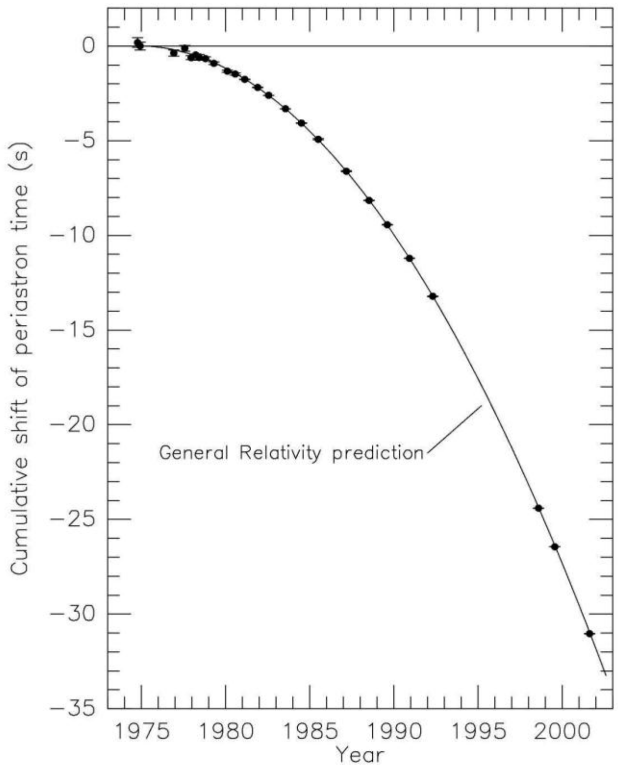
Einstein's Theory predicts

Propagating Ripples in the Fabric of Space-Time



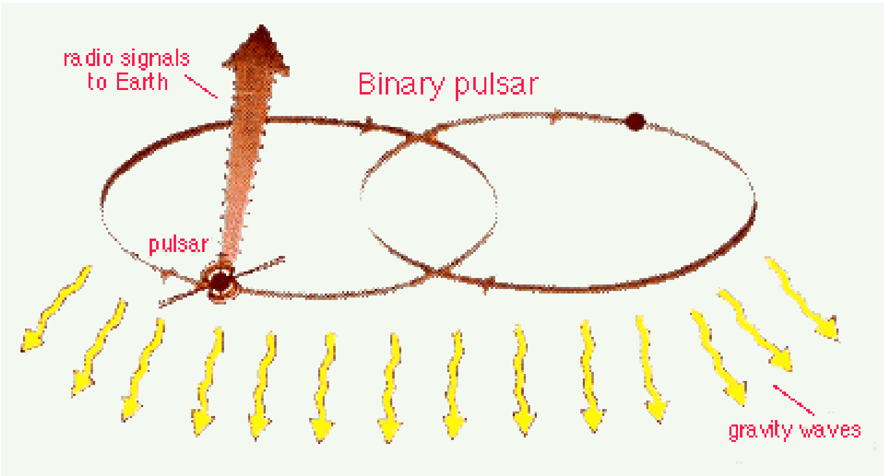
- Travels at Speed c
- Two Polarizations

First Observation of Gravitational Waves

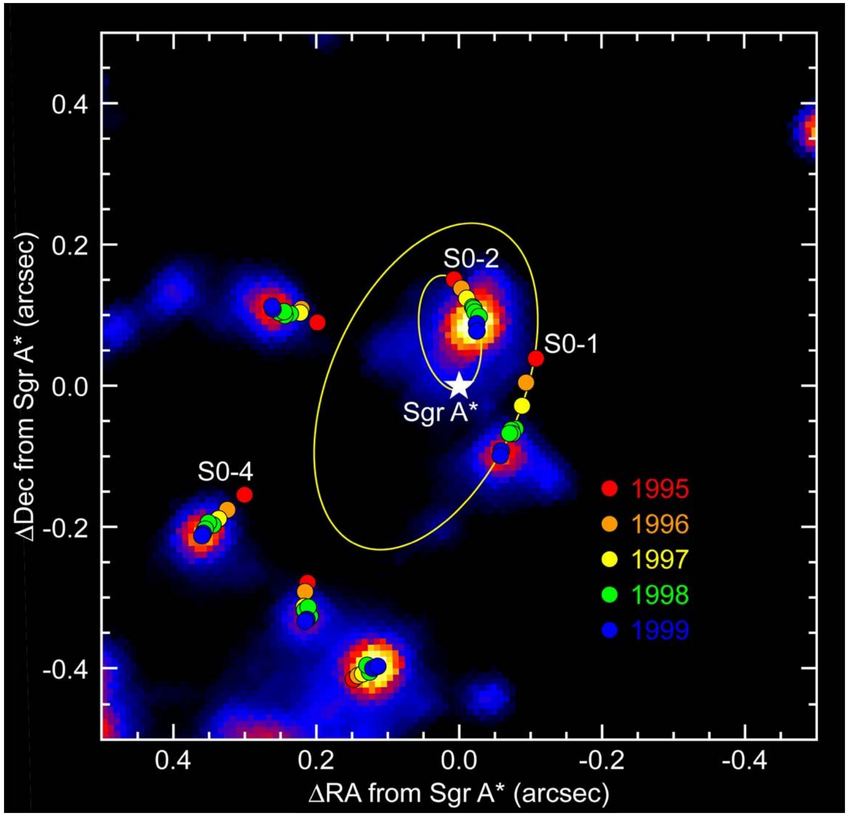


J. Taylor and J. Weisberg
agreement to 1/5 of 1%

Mass in Motion is the Source of Gravitational Waves



Black Holes : A Lot of Mass in a Very Small Region



Existence of B.H. predicted by GR

A Black Hole at the Center of our Galaxy

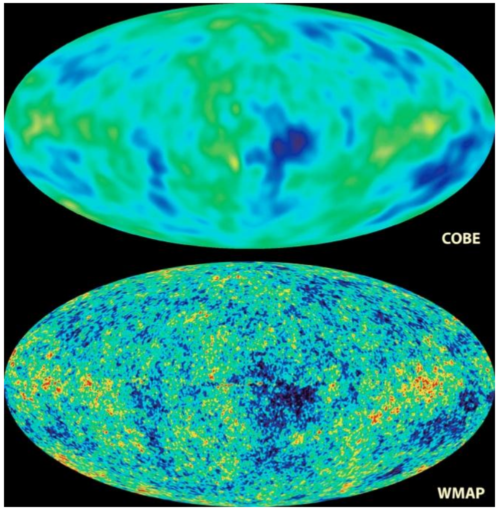
M ~ 3 x 10^6 solar masses

A. Ghez group, UCLA

Cosmology

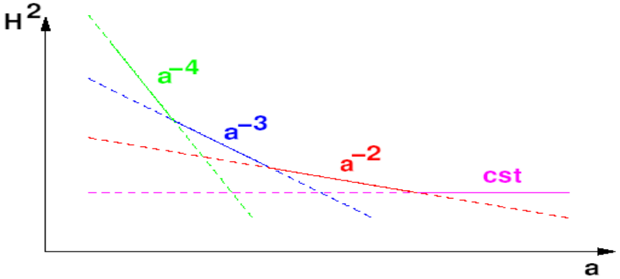
Einstein's Theory predicts the

Expansion of the Universe from a Big Bang

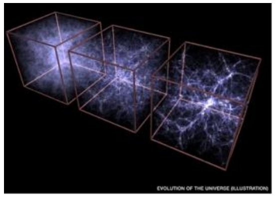


… offers a detailed description of the **Evolution of the Universe**

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi\mathcal{G}}{3c^2}\rho_{\mathrm{R}} + \frac{8\pi\mathcal{G}}{3c^2}\rho_{\mathrm{M}} - \frac{kc^2}{a^2} + \frac{\Lambda}{3}$$

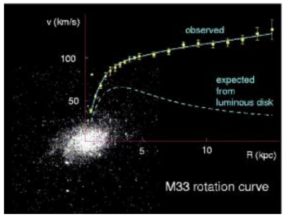


Estimated Composition of the Universe



- ~ 70% **Vacuum Energy of Unknown Origin**

  (“Dark Energy” or Cosmological Constant λ)

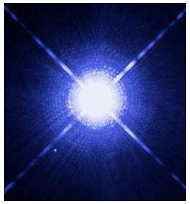


- ~ 25% **Particles of Unknown Character**

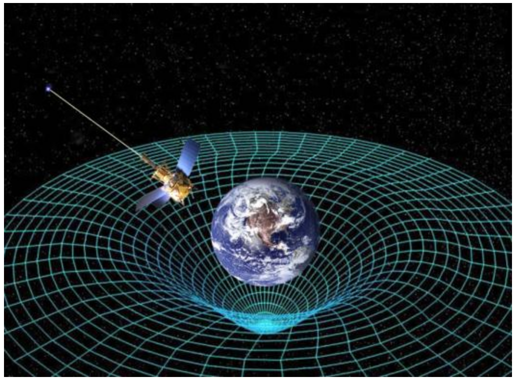
  (Dark Matter)

- ~ 5% **Familiar Particles** such as Protons, Hydrogen,

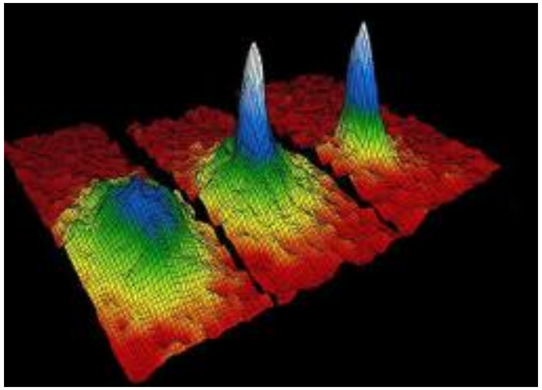
  Stars etc.



Quantum Gravitation



Curved spacetime

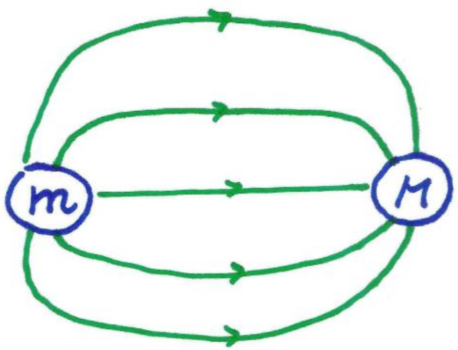


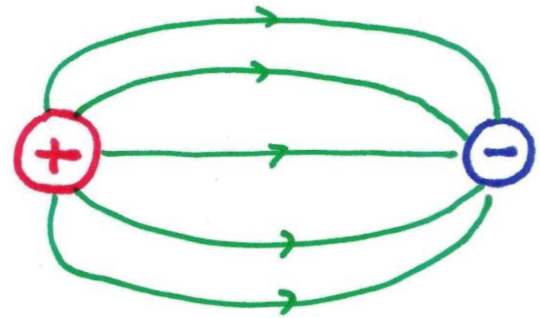
Quantum mechanics

“Space-Time Foam”

(John A. Wheeler)

Gravity *versus* Electro-Magnetism





$$\frac{1}{m}\,\phi(r) \;=\; -(G\,M)\cdot\frac{1}{r}$$

$$\phi(r) \;=\; -e^2\cdot\frac{1}{r}$$

$$[G] \sim \frac{R_0}{M_0}$$ (Feynman, 1963)

$$\frac{e^2}{\hbar c} = \frac{1}{137.}$$

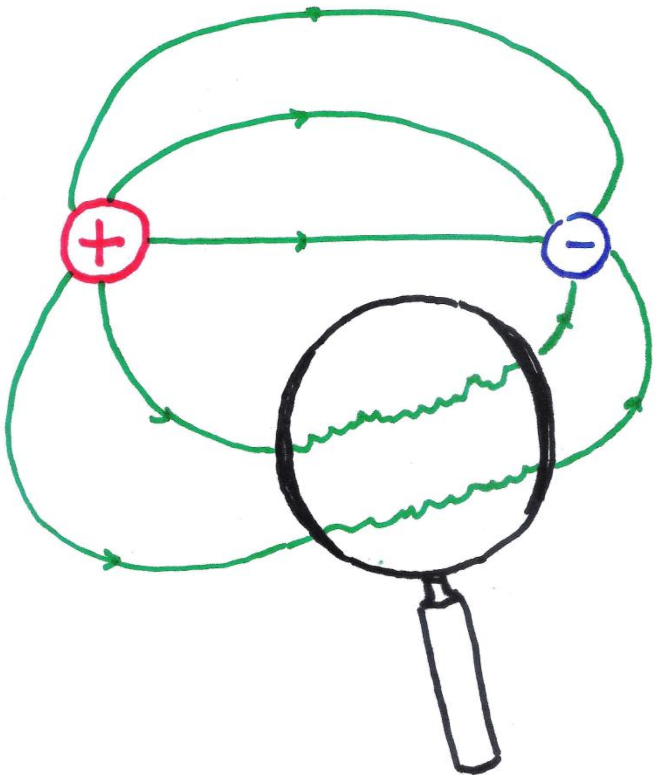
$$\begin{pmatrix} \phi & \cdots & & \\ \vdots & h_{xx} & h_{xy} & h_{xz} \\ & h_{xy} & h_{yy} & h_{yz} \\ & h_{xz} & h_{yz} & h_{zz} \end{pmatrix}$$

Grav. field is a 4 x 4 <u>matrix</u>

$$\begin{pmatrix} \phi \\ A_x \\ A_y \\ A_z \end{pmatrix}$$

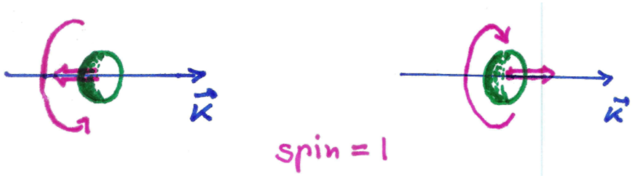
E.M. field is a <u>vector</u>

Quantum-Mechanical Fields Fluctuate

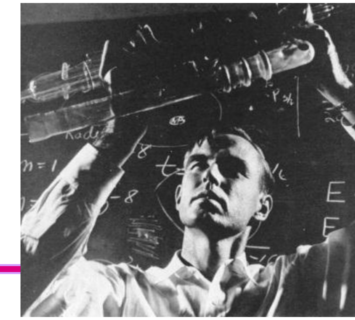


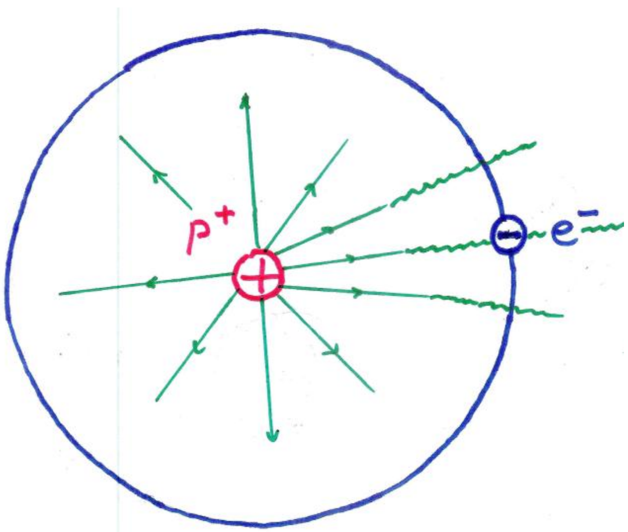
In a QM description one has microscopic field fluctuations :

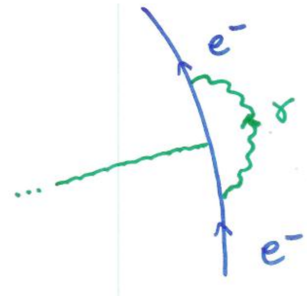
$$\Delta \mathbf{E} \cdot \Delta \mathbf{B} \geq \hbar$$



Hydrogen “Lamb Shift”

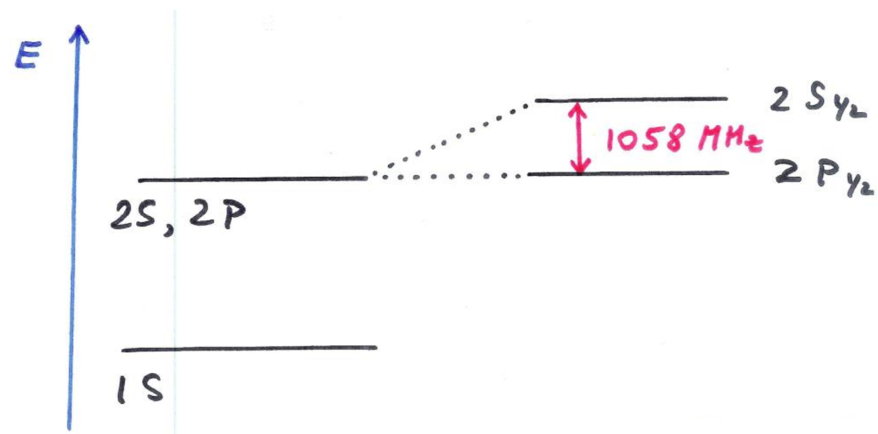






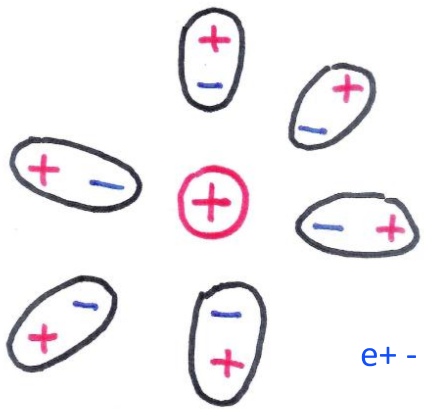
a “virtual” photon

Fluctuations in the Coulomb field affect energy levels in atoms.

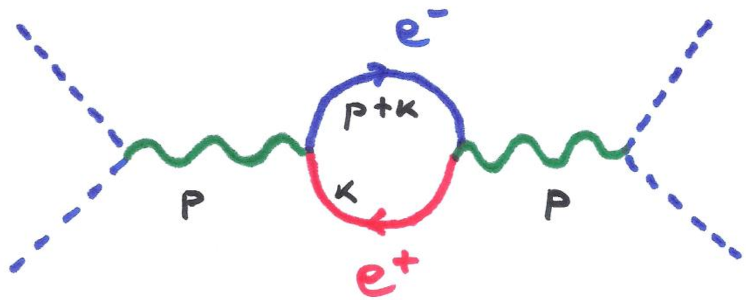


“Lamb Shift” in Hydrogen

Vacuum as a Polarizable Medium…



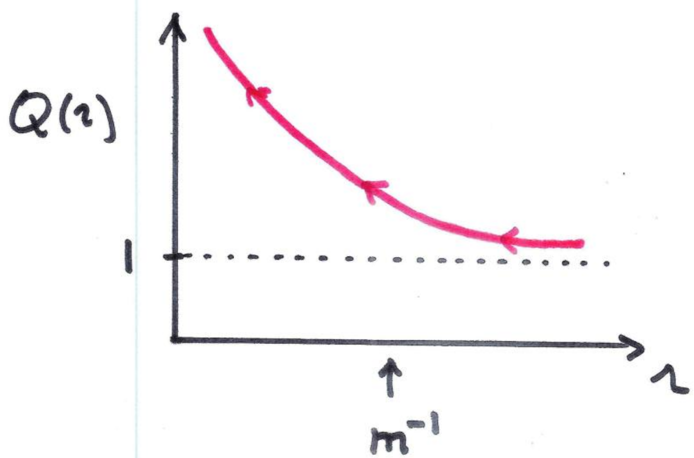
e+ - e- pairs



$$V(r) = \frac{e}{r} \to \frac{e}{r} \cdot Q(r) \qquad Q(r) \to \int_m^\Lambda d^4k \, \frac{1}{k^2} \cdot \frac{1}{(p+k)^2} \to \ln(m\,r)$$

lower cutoff at electron mass (IR cutoff) …

…leads to a "running" of α



$$1/m_e \sim 10^{-10} cm$$

$$Q(r) = 1 + \alpha \ln\left(\frac{1}{m\,r}\right) \qquad r \ll \frac{1}{m}$$

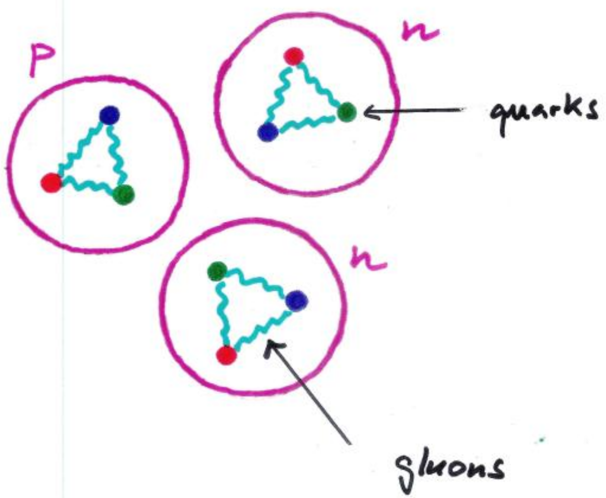
Charge Screening

Effect observed at CERN/LEP :

$$\alpha\,(\text{Hydrogen atom}) \simeq \frac{1}{137}$$

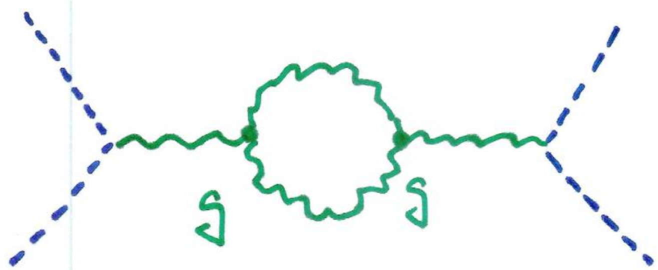
$$\alpha\left(\frac{1}{100}\text{ nucleus}\right) \simeq \frac{1}{129}$$

Strong Interactions (QCD)



8 colored Gluons bind Quarks …

Coupling $g \approx 1$

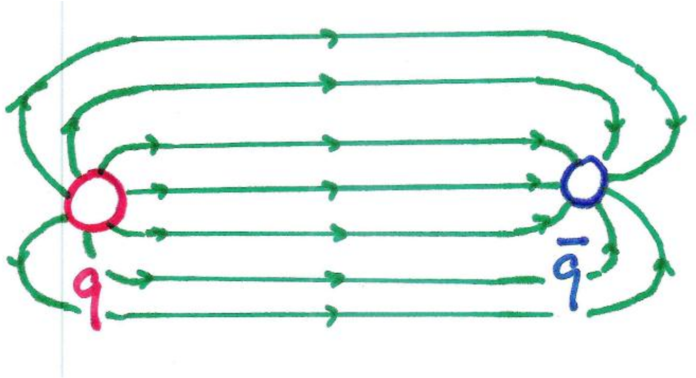


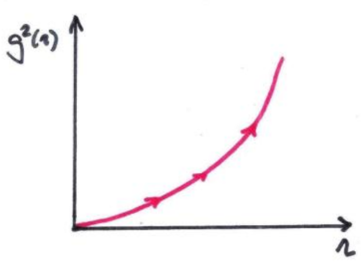
Like photons … but Gluons carry Color (charge) and interact with each other

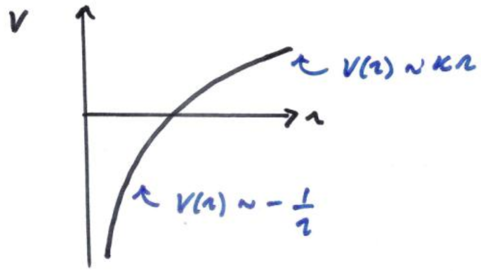
$\Rightarrow$ **Non-linear theory**

Breakdown of Perturbation Theory

A “Flux Tube” joins two quarks

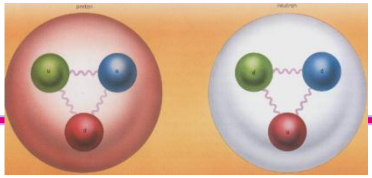






“Confinement” cannot be seen in Perturbation Theory.

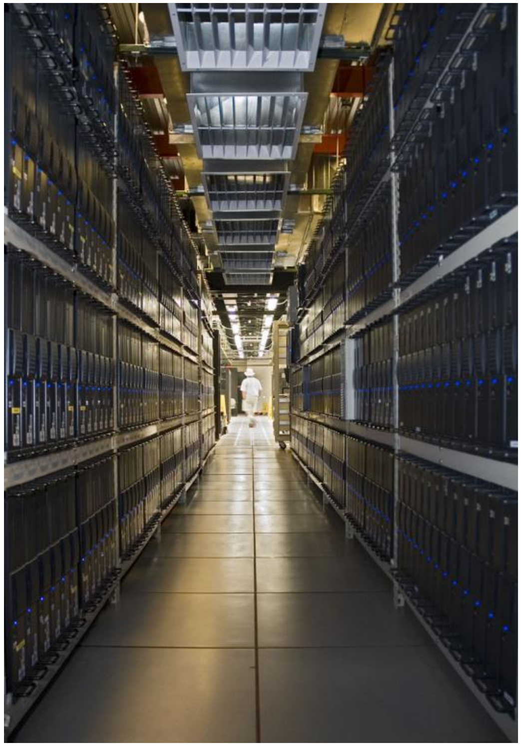
New vacuum $\Rightarrow$ **A new scale (“gluon mass”):** $\xi \simeq 10^{-13} cm$



QCD is Hard. Very Hard.

Very Serious Supercomputers.

$1M Clay Mathematics Prize.

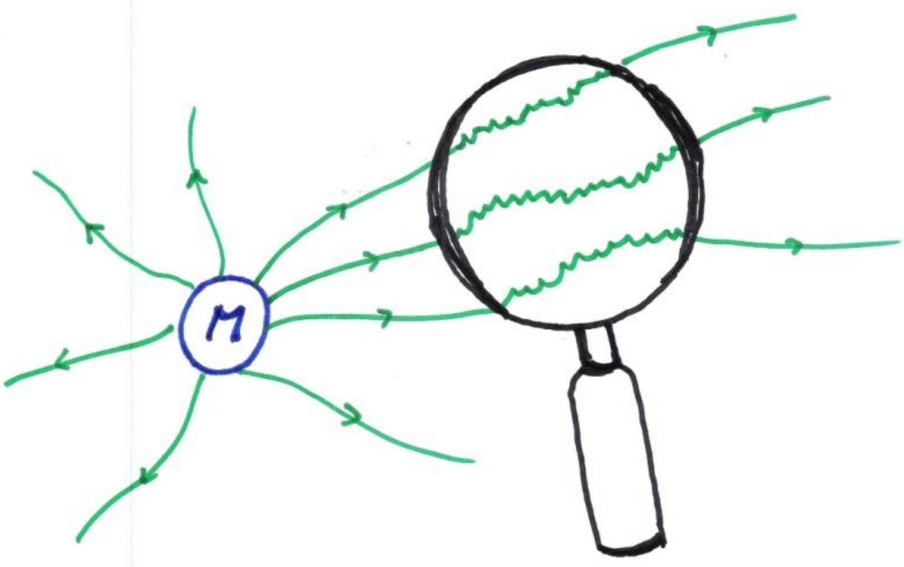


Fermilab Cluster

Quantum Gravitation

Qualitative Features:

- Quantum Fluctuations of the Gravitational Field, on <u>all</u> distance scales



- Spin-Two, Massless Graviton

$$[h,\, \dot{h}'] \;=\; i\hbar\, \delta^{(3)}$$

Gravity : A Beautiful Simple Theory

$$\boxed{L_{\text{Einstein}} = \frac{1}{G}\sqrt{g}\,R + \text{matter}}$$

curvature of spacetime

Feynman proved in 1963* :

Theory of a massless spin-two particle leads to Einstein's General Relativity.

$\Rightarrow$ Unique Theory of *m=0 s=2 (q-mechanical)* particle.

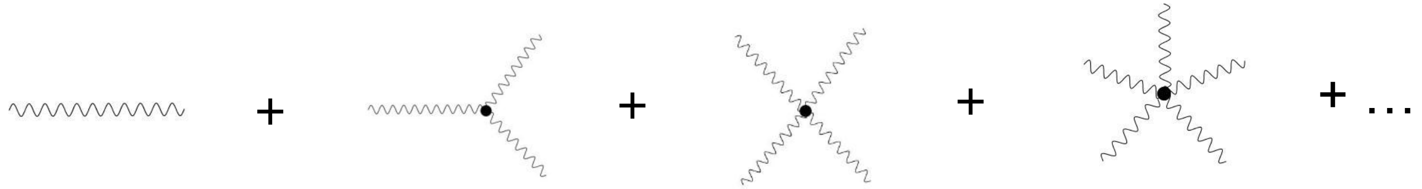
* R.P. Feynman "Lectures on Gravitation", ed. by F. Morinigo & F. Wagner, Caltech 1963.

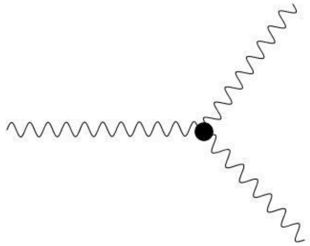
Feynman Diagrams

Infinitely many graviton interaction terms in *L :*

$$L_{\text{Einstein}} = \left(1 + G\,h + G^2\,h^2 + \ldots\right) \cdot \partial h \cdot \partial h$$

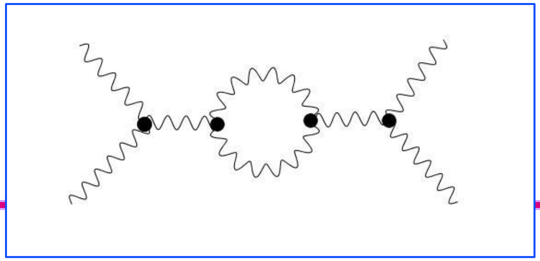
Gravitons self-interact, or ''Gravitate" :

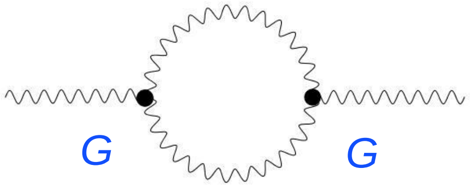




$$V(k) \sim h \cdot \partial h \cdot \partial h \sim k^2$$

“One Graviton Loop”



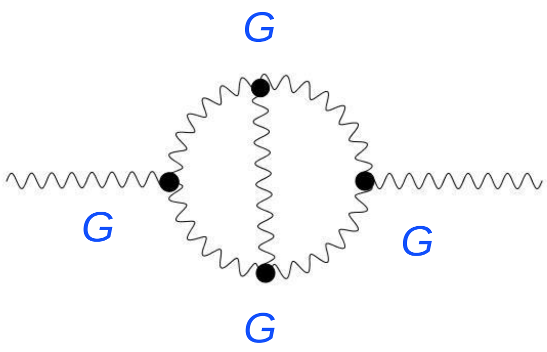


$$\sim\ G^2 \int d^4k \, \frac{1}{k^2} \cdot V(p+k) \cdot \frac{1}{(p+k)^2} \cdot V(p+k)$$

$$\sim\ G^2 \int^{\infty} \frac{d^4k}{k^4} \cdot k^2 \cdot k^2 \ \sim\ G^2 \cdot \infty^4$$

One loop diagram quartically divergent in d = 4.

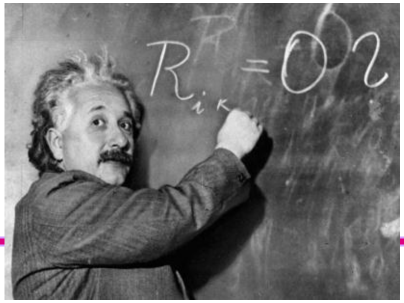
“Two Graviton Loops” : worse



$$\sim\ G^4 \int \frac{(d^4k)^2}{(k^2)^5} \cdot \left(k^2\right)^4 \ \sim\ G^4\,\infty^6$$

Perturbation theory badly divergent … wrong ground state ?

The Cosmological Constant



Add to the action λ x (Volume of Space-Time), or :

$$L = \left(1 + G\,h + G^2\,h^2 + \ldots\right) \cdot \partial h \cdot \partial h + \lambda \left(1 + G\,h + G^2\,h^2 + \ldots\right)$$

A new length scale $m \sim \sqrt{\lambda}$ $\xi \sim 1/\sqrt{\lambda}$

… λ acts like a mass (IR cutoff).

Einstein thought this was his “Biggest Blunder”(1917, 1922).

Still A Beautiful Simple Theory…

$$L = \frac{1}{G}\sqrt{g}\,R + \lambda\sqrt{g} + \text{matter}$$

Curvature

Volume

Many Paths Open

- Denial : Gravity should *not* be quantized, only matter fields.

  … goes against rules of QM & QFT.

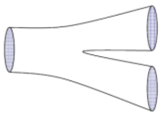
- Keep gravity, but resort to "*other*" methods:

  New RG fixed points, Lattice Gravity, Loop QG, …

- Add more fields so as to reduce (or *eliminate*) divergences:

  N=8 SuperGravity $\longrightarrow$ 70 massless scalars…

- Embed gravity in a larger non-local (finite) theory : SuperStrings in d=10 … Gravity emerges as an effective theory.



Super-Symmetric "*N=8*" Gravity

= finite

graviton loop　　　gravitino loop

$\cancel{N=1}$

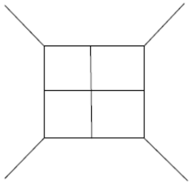
$\cancel{N=2}$

$\cancel{N=4}$

$$\left.\begin{array}{l} 1 \times \; S = 2 \\ 8 \times \; S = 3/2 \\ 28 \times \; S = 1 \\ 56 \times \; S = 1/2 \\ 70 \times \; S = 0 \end{array}\right\} \text{all } m = 0$$

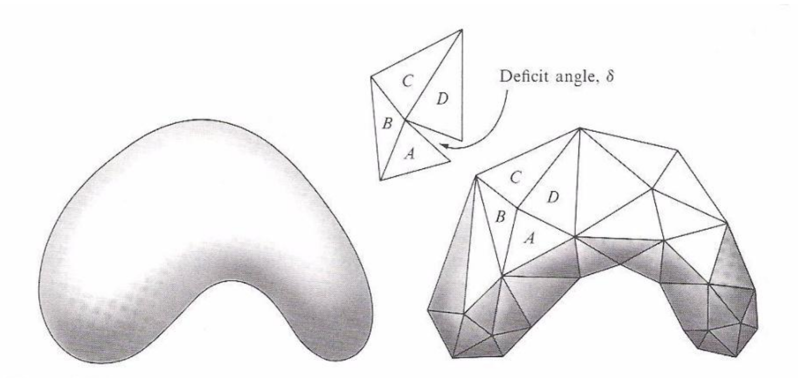
Fine tune extra particles till you get a finite result … *N=8* finite to 4 loops ?

One scalar might have been seen …



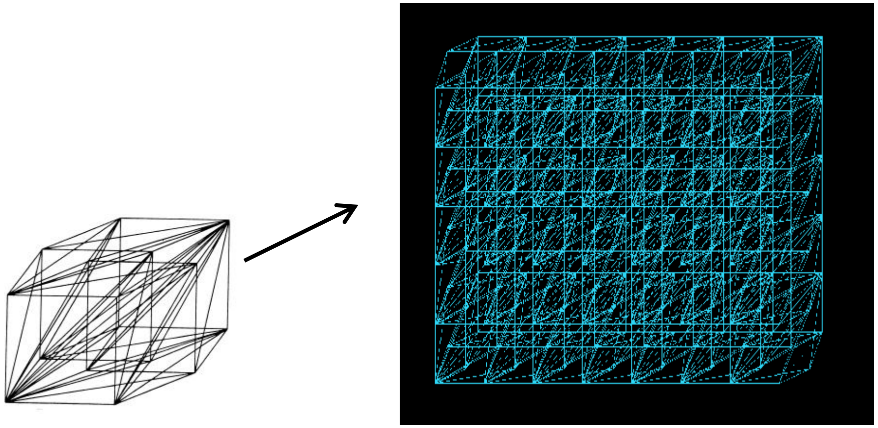
Another Scenario :
Gravity on a Lattice

T. Regge and J.A.Wheeler



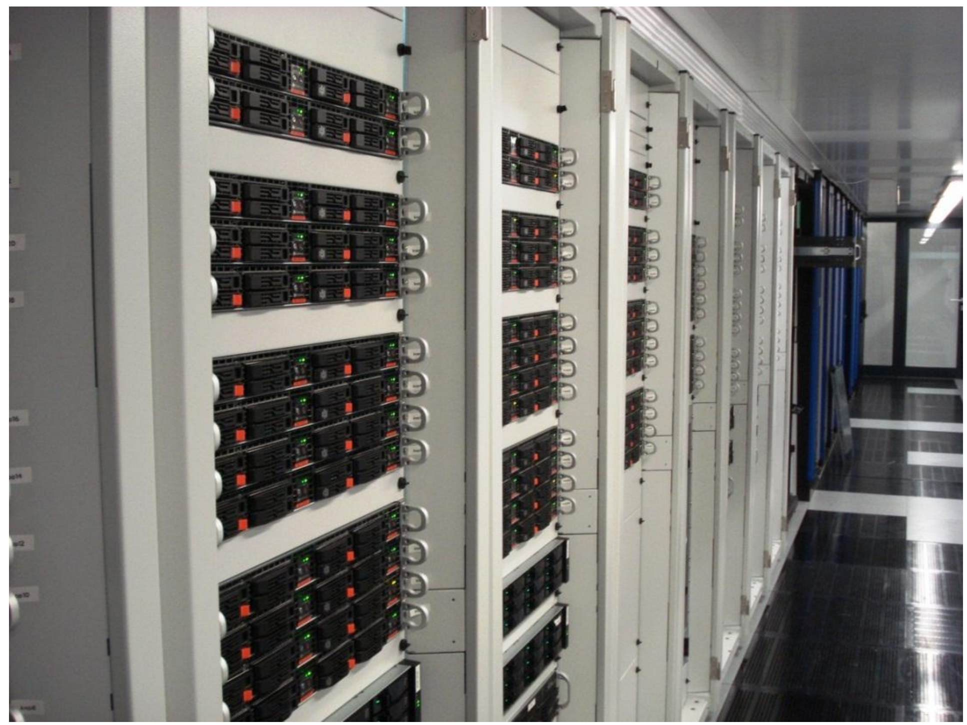
MTW ch. 42

- Discretize Einstein's General Relativity by *triangulating* the space-time manifold.
- Only lattice theory of gravity known to have correct continuum limit (<u>unique</u>).
- Put within the reach of computation problems which in practical terms are beyond the power of analytical methods.
- Exact in the limit lattice spacing $\to 0$.



Numerical Quantum Gravity

Intel Xeon Cluster

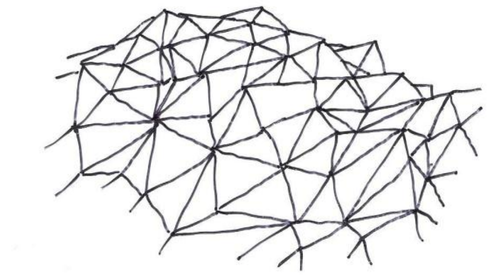


Machine room in MPI-AEI basement

Phases of Quantum Gravity

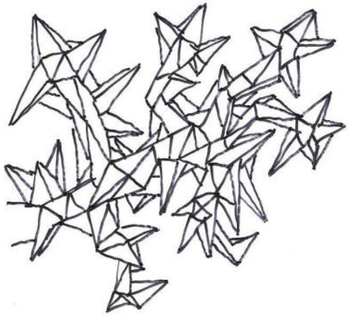
Quantum Gravity in D=4 has two phases (PLB 1984)…

$G > G_c$ Smooth phase: R ≈ 0



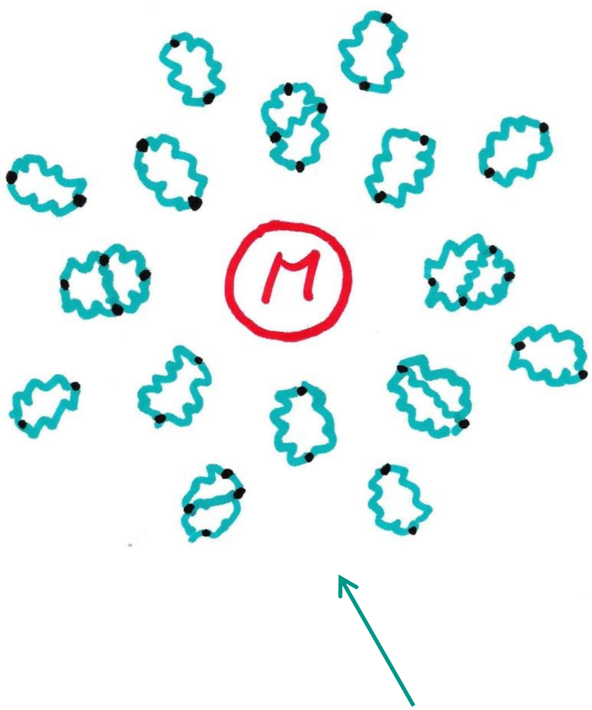
Physical

$G < G_c$ Unphysical (branched polymer-like, d ≈ 2)



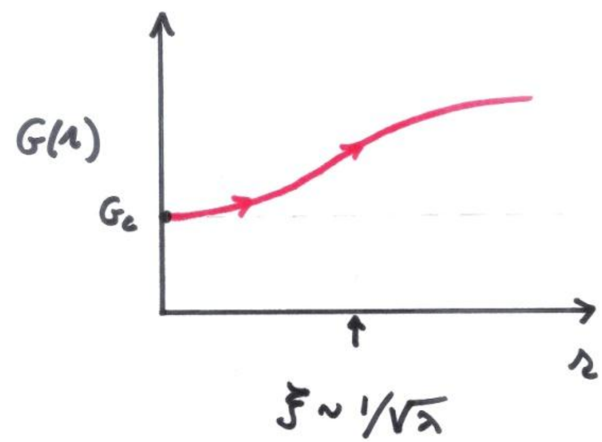
Unphysical

Newton's "Constant" no longer Constant



virtual graviton cloud

Gravitational Field Fluctuations give *anti-screening,* and a slow "running" of G(r)



infrared cutoff

Running of Newton's G(□)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \lambda g_{\mu\nu} = 8\pi G(\Box) T_{\mu\nu}$$

ν = 1/3

$$G(\Box) = G_0 \left[1 + c_0 \left(\frac{1}{\xi^2 \Box} \right)^{1/2\nu} + \ldots \right]$$

New RG invariant scale of gravity $\xi \sim 1/\sqrt{\lambda}$ (infrared cutoff)

⇒ Expect deviations from GR on largest scales.

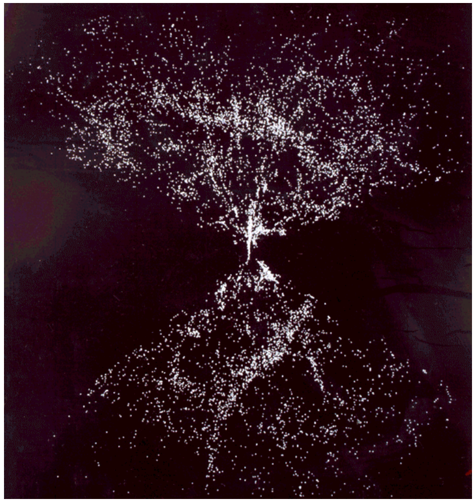
An example : *Matter density perturbations*

[with R. Wiliams PRD 2006]

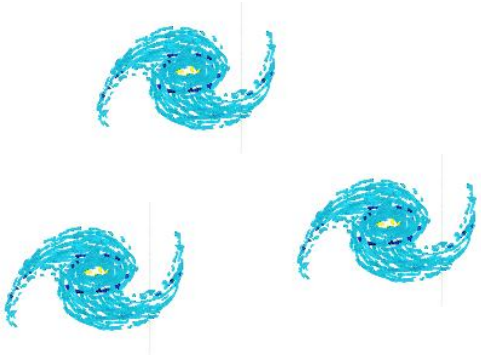
Matter Density Perturbations

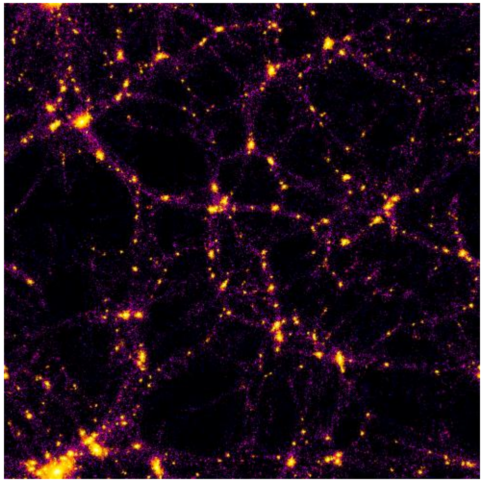
Visualizing Density Perturbations
on very large scales …

Evolution predicted by GR



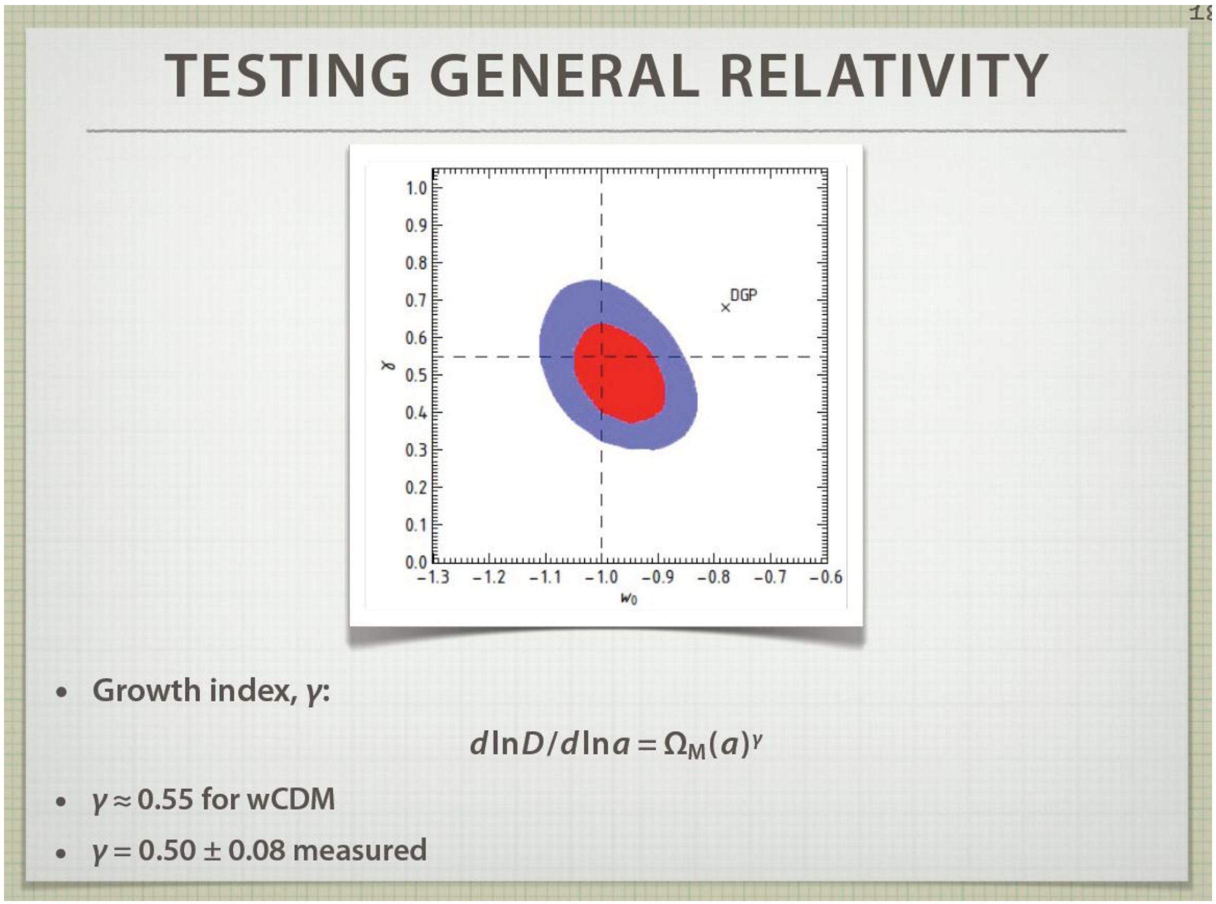
Observed (CfA)





Simulation (MPI Garching)

Measured growth parameter γ



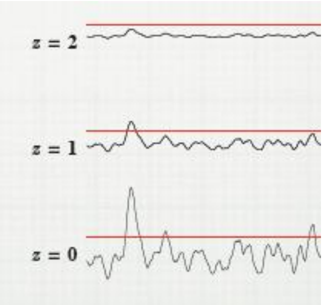
Peebles index

(Alexei Vikhlin, CfA)

Density Perturbations with G(□)

Standard GR result for density perturbations :

$$\ddot{\delta}(t) + 2\,\frac{\dot{a}}{a}\,\dot{\delta}(t) - 4\pi\,G(t)\,\bar{\rho}(t)\,\delta(t) = 0$$



GR solution, in matter-dominated era : $\delta_{\mathbf{q}}(t) = \delta_{\mathbf{q}}(t_0)\left(\frac{t}{t_0}\right)^{2/3}$

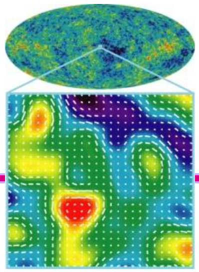
If Gravity is modified, then Peebles exponents will change:

$$\boxed{\gamma = 0.556 - 106.4\,c_t + O(c_t^2)}$$

$c_t \lesssim 5\times 10^{-4}$

[with R. Toriumi PRD 2011]

Gravitational “Slip” with $G(\Box)$



In the conformal Newtonian gauge

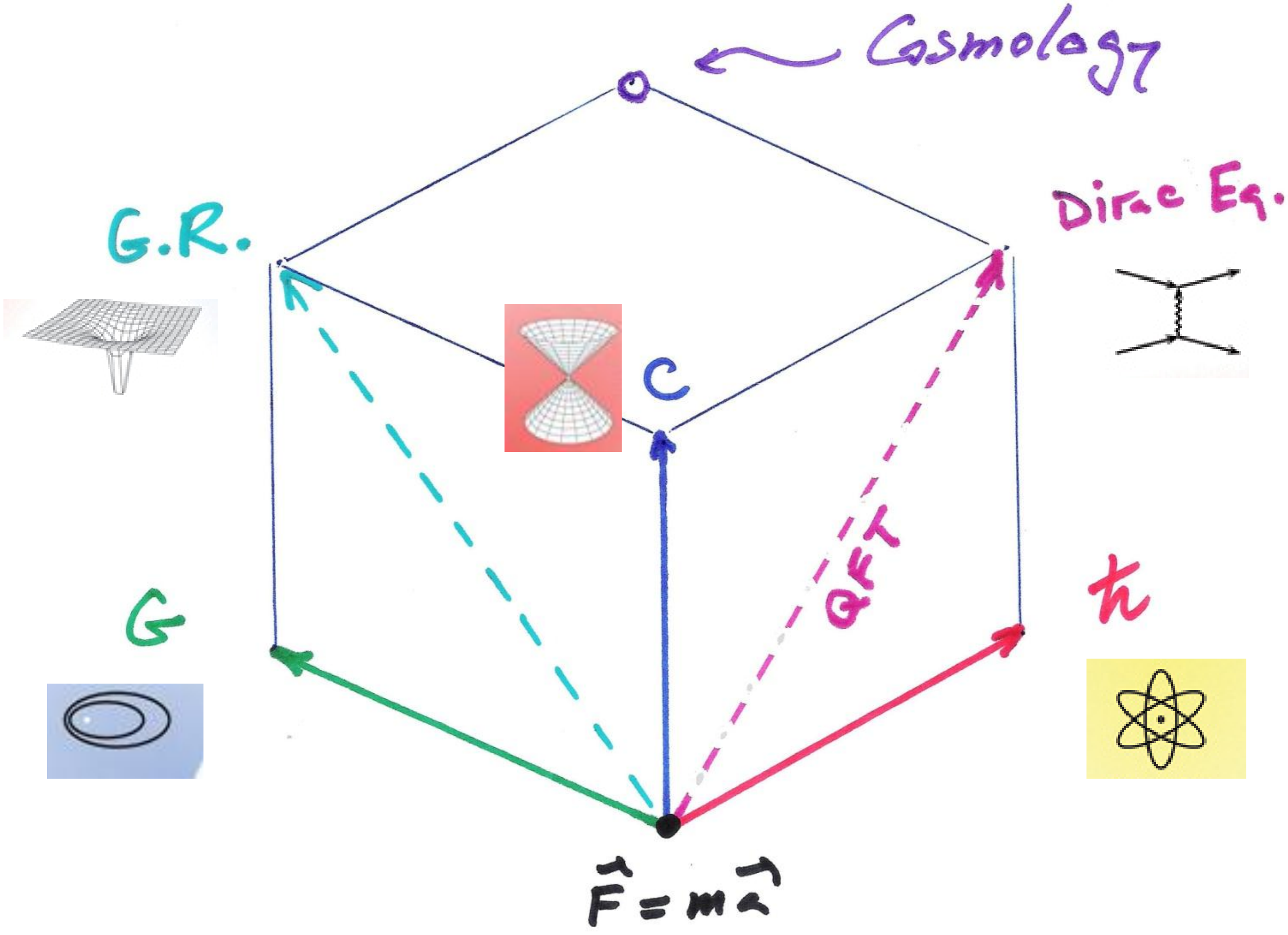
$$ds^2 = a^2(\tau)\left\{-(1+2\psi)d\tau^2 + (1-2\phi)dx^i dx_i\right\}$$

In classical GR : $\eta = \psi/\phi - 1 = 0$. Good test for deviations …

$$\eta(z) = -0.766\, c_t - 4.109\, c_t\, z + 12.188\, c_t\, z^2 + \cdots$$

CMB measurements give values around 0.09 ± 0.7

[with R. Toriumi PRD 2011]



Cosmology
G.R.
Dirac Eq.
c
QFT
G
ħ
F = ma

The End

Locomotive graveyard in Uyuni, Bolivia

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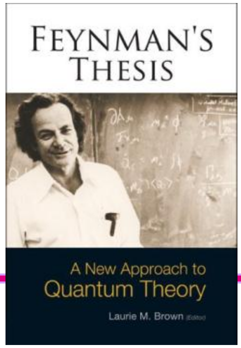
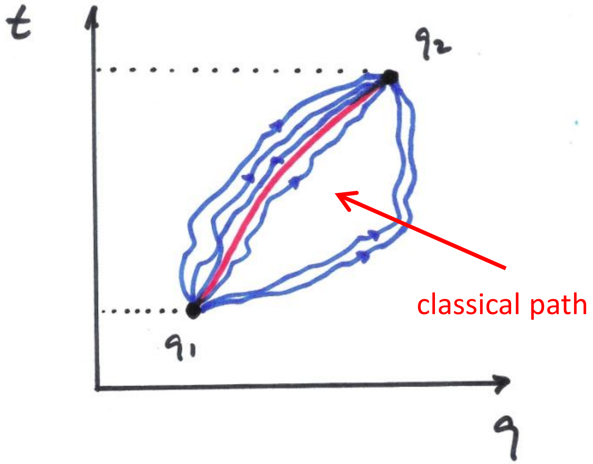
Feynman Path Integral



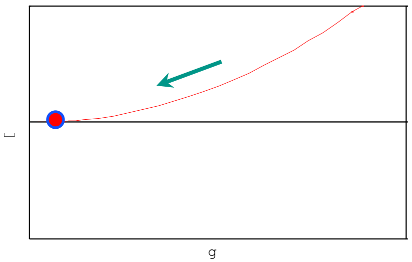
Reformulate QM amplitudes in terms of :

Discrete *Sum over Paths on a Space-Time Lattice*

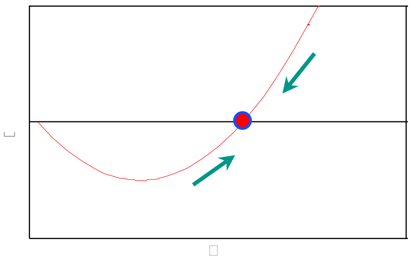


$$\boxed{Amp(A \to B) = \sum_{all\ paths} e^{\frac{i}{\hbar} S(path)}}$$

RG Running Scenarios

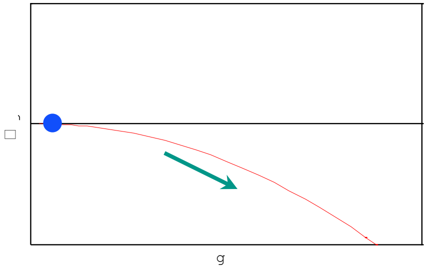


“Triviality” of lambda phi 4

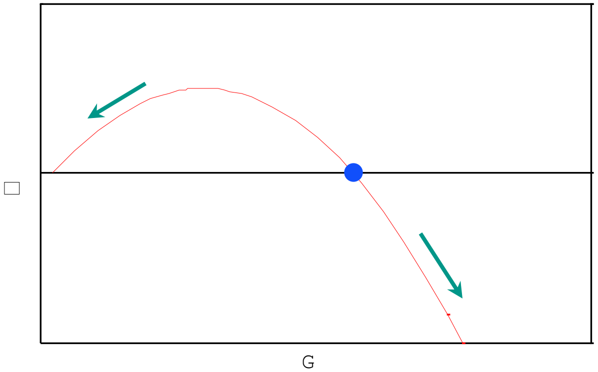


Wilson-Fisher FP in d<4

Nontrivial UV fixed point



Asymptotic freedom of YM



Ising model, σ-model, Gravity (2+ε, lattice)

Callan-Symanzik. beta functions,

$$\boxed{\mu \frac{\partial}{\partial \mu} G(\mu) = \beta(G(\mu))}$$

Gravity in in 2+ε Dimensions

Wilson 1973
Weinberg 1977 …
Kawai, Ninomiya 1995
Kitazawa, Aida 1998

G is dim-less, theory is perturbatively renormalizable

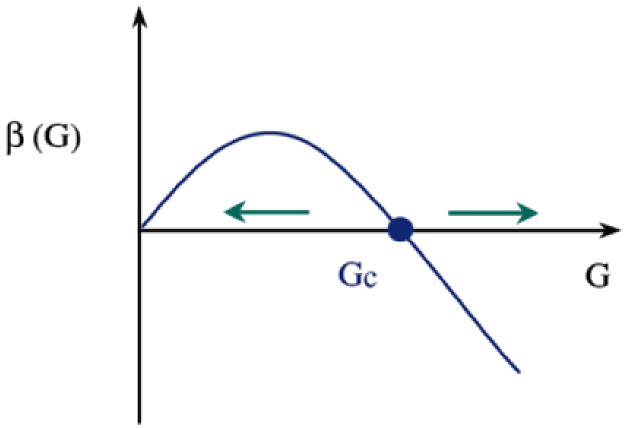
$$\mu \frac{\partial}{\partial \mu} G(\mu) \;=\; \beta(G(\mu))$$

$$\beta(G) \;=\; (d-2)\, G \;-\; \frac{2}{3}\,(25-n_s)\, G^2 \;-\; \frac{20}{3}\,(25-n_s)\, G^3 \;+\; \ldots$$

(pure gravity : $n_s = 0$)

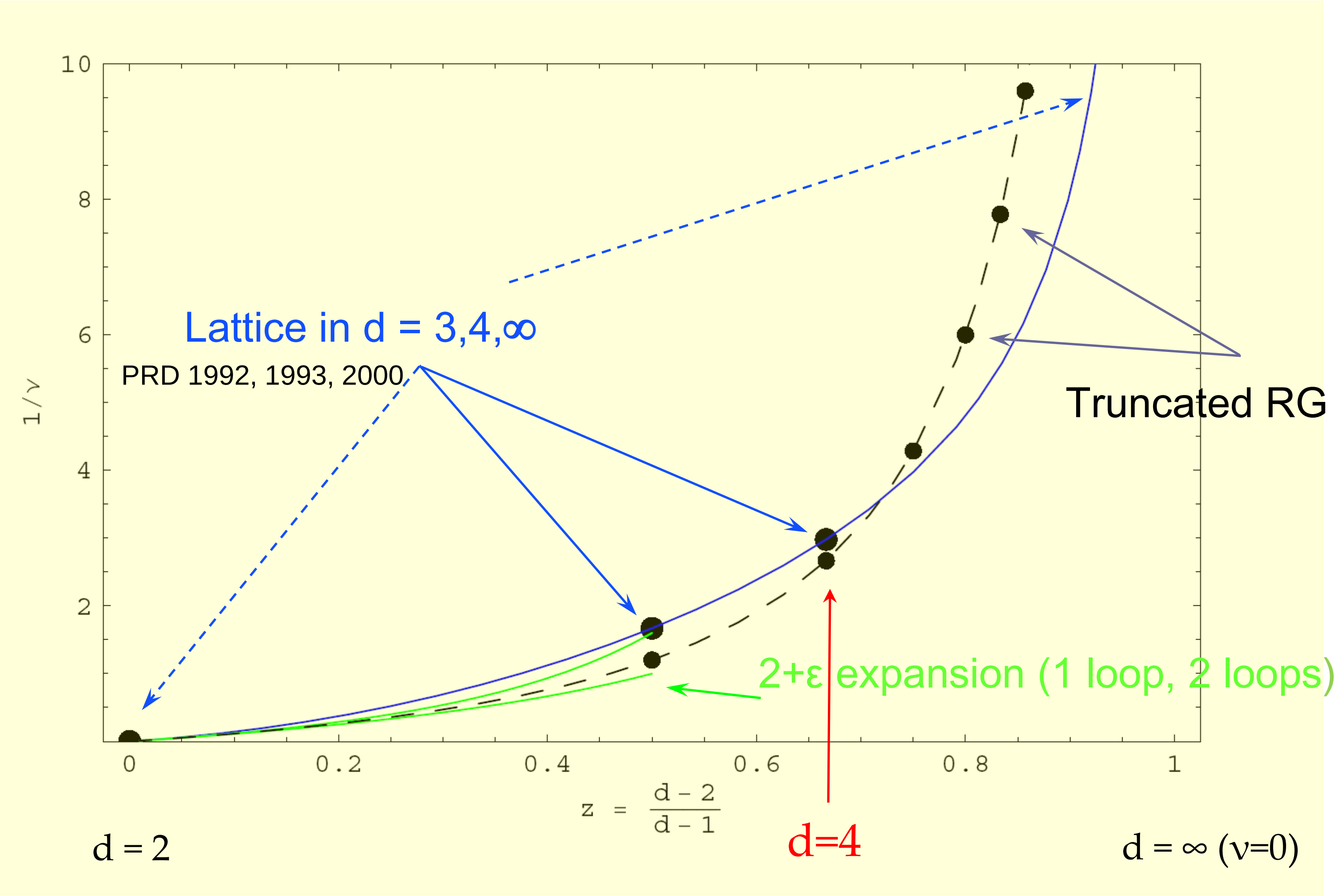
Non-trivial UV fixed point :

$$\begin{cases} G_c \;=\; \dfrac{3}{2\,(25-n_s)}\,(d-2) - \dfrac{45}{2(25-n_s)^2}\,(d-2)^2 + \ldots \\ \boxed{\nu^{-1}} \;=\; -\beta'(G_c) \;=\; (d-2) \;+\; \dfrac{15}{25-n_s}\,(d-2)^2 \;+\; \ldots \end{cases}$$



Two phases of Quantum Gravity

Gravitational exponent ν compared



Determination of Scaling Exponents

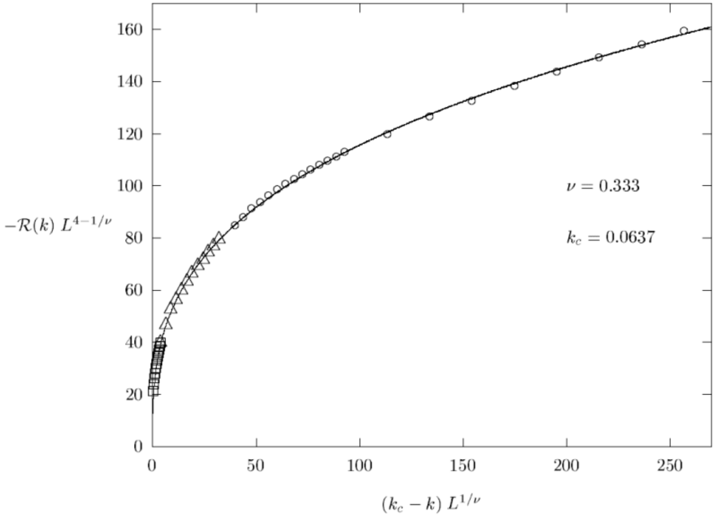
$$\mathscr{R}(k) \sim \frac{< \int dx \sqrt{g} R(x) >}{< \int dx \sqrt{g} >} \sim \frac{1}{V} \frac{\partial}{\partial k} \ln Z \underset{k \to k_c}{\sim} -A_{\mathcal{R}} (k_c - k)^{\delta} \qquad \nu = \frac{1+\delta}{d}$$

$$\chi_{\mathscr{R}}(k) \sim \frac{< (\int dx \sqrt{g} R)^2 > - < \int dx \sqrt{g} R >^2}{< \int dx \sqrt{g} >} \sim \frac{1}{V} \frac{\partial^2}{\partial k^2} \ln Z \underset{k \to k_c}{\sim} -A_{\mathcal{R}} (k_c - k)^{-(1-\delta)}$$

Scaling assumption:

$$F_{sing}(G) \sim \xi^{-d}$$

$$\xi(k) \equiv m(k)^{-1} \underset{k \to k_c}{\sim} A_{\xi} (k_c - k)^{-\nu}$$



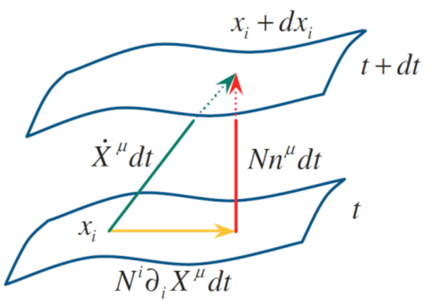
Find value for ν close to $1/3$:

$$k_c = 0.0636(11) \qquad \nu = 0.335(9)$$

$$\boxed{\nu \approx 1/3}$$

[Phys Rev D 2000, 2012]

Canonical Quantization



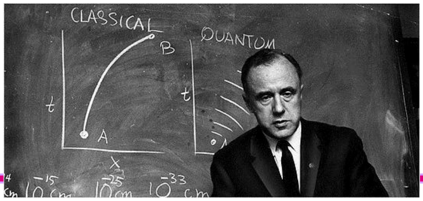
- Split space-time into **space** and **time** (3+1)

- Evolve spatial geometry forward in time according to Einstein's equations

$$g_{ij}(t, \mathbf{x}) \quad \text{and} \quad \Pi^{ij}(t, \mathbf{x}) = \frac{\delta \mathcal{S}_{\text{Einstein}}}{\delta \dot{g}_{ij}(t, \mathbf{x})}$$

- Dynamics determined by constraints (via lapse and shift)

Wheeler-DeWitt Equation



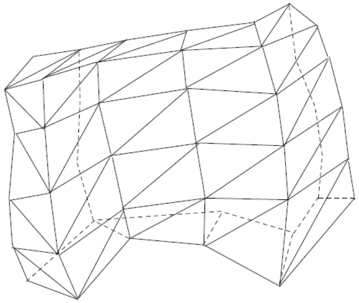
- Position rep. → Functional Schrödinger equation :

$$\hat{g}_{ij}(\mathbf{x}) \;\rightarrow\; g_{ij}(\mathbf{x}) \qquad \hat{\pi}^{ij}(\mathbf{x}) \;\rightarrow\; -i\hbar \cdot 16\pi G \cdot \frac{\delta}{\delta g_{ij}(\mathbf{x})}$$

$$\boxed{\left\{ -(16\pi G)^2\, G_{ij,kl}(\mathbf{x}) \frac{\delta^2}{\delta g_{ij}(\mathbf{x})\, \delta g_{kl}(\mathbf{x})} \;-\; \sqrt{g(\mathbf{x})}\; \left({}^3R(\mathbf{x}) \;-\; 2\lambda \right) \right\} \Psi[g_{ij}(\mathbf{x})] \;=\; 0}$$

- Discretize and solve :

$$\left\{ -(16\pi G)^2 \sum_{i,j\subset\sigma} G_{ij}\,(\sigma)\, \frac{\partial^2}{\partial l_i^2\, \partial l_j^2} \;-\; 2\, n_{\sigma h} \sum_{h\subset\sigma} l_h\, \delta_h \;+\; 2\lambda\, V_\sigma \right\} \Psi[\, l^2\,] \;=\; 0$$



…like Hamiltonian lattice gauge theory (K-S)

[Phys Rev D 2011, 2012]

“Loop” Quantum Gravity

- Canonical variables are *holonomies* along link *e*

$$h_e[A] = \mathcal{P}\exp\int_e A$$

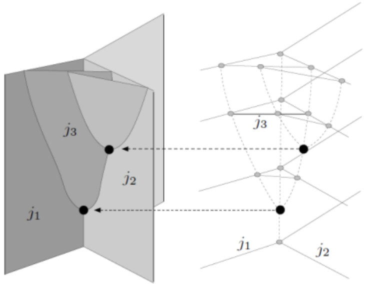
- Conjugate variables : flux through area element *S*

$$F_S^a[E] := \int_S dF^a = \int_S \epsilon_{mnp} {E_a}^m dx^n \wedge dx^p$$

- Construct wave functionals, defined on spin networks

$$\Psi_{\{\Gamma,C\}}[A] = f_C\Big(h_{e_1}[A], \ldots, h_{e_n}[A]\Big)$$

“Spin Foam” Models



- Dynamics defined via a generalized *spin state sum* model

$$Z_\phi = \sum_{\text{spins } \{j\}} \prod_{f,e,v} A_f(\{j\}) \; A_e(\{j\}) \; A_v(\{j\})$$

A’s = amplitudes for faces *f*, edges *e* and vertices *v*

- A number of very basic issues :

Riemannian SO(4) or Lorentzian SO(3,1) ?

Weights are exp(i S) - but with S Euclidean ?

Diffeomorphisms (edges are half-integer valued) ? Classical limit = GR ?

Renormalization ?