LFV Decays $\tau \to \ell\ell\ell$ with polarized τ's

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Contents

Introduction

- Discovery of Neutrino Osciallations opens the road to (nontrivial) Flavour Physics of Leptons
- Due to the Quantum numbers:
  Lepton Number Violation (LNV) versus
  Lepton Flavour Violation (LFV)
- The two effects may originate from different mass scales
- What are the effects on charged lepton decays?

- Effective field theory picture:
- Standard model (without right handed ν's) is the (dim-4) starting point.
- Any new physics manifests itself as higher dimensional operators:

$$\mathcal{L} = \mathcal{L}^{SM}_{\mathrm{dim}\,4} + \mathcal{L}_{\mathrm{dim}\,5} + \mathcal{L}_{\mathrm{dim}\,6} + \cdots$$

- $\mathcal{L}_{\mathrm{dim\,n}}$ are suppressed by large mass scales

$$\mathcal{L}_{\mathrm{dim\,n}} = \frac{1}{\Lambda^{n-4}} \sum_i C_n^{(i)} O_n^{(i)}$$

$O_n^{(i)}$: Operators of dimension n,
$SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge invariant
$C_n^{(i)}$: dimensionless couplings

- The combination

$$N_i = (H^{c,\dagger} L_i) \, , \quad L_i = \begin{pmatrix} \nu_{L,i} \\ \ell_{L,i} \end{pmatrix} , \, H^c = (i\tau^2) H^* \, , H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

  has no SM Quantum numbers
- Unique dim -5 Operator:
  Generates Majorana masses for the ν's

$$\mathcal{L}_{\dim 5} = \frac{1}{\Lambda_{\rm LNV}} \sum_{ij} C_5^{ij} (\bar{L}_j H^c)^c (H^{c,\dagger} L_i)$$

- Lepton Number Violating, related to the scale $\Lambda_{\rm LNV}$
- $\Lambda_{\rm LNV}$ is known to be high, as big as the GUT scale?

Dim-6 Operators: (C)LNV

- purely leptonic

$$O_1 = (\bar{L}_i \gamma_\mu L_J)(\bar{L}_k \gamma^\mu L_l) \qquad O_2 = (\bar{L}_i \gamma_\mu \tau^a L_j)(\bar{L}_k \gamma^\mu \tau^a L_l)$$
$$O_3 = (\bar{\ell}_{R,i} \gamma_\mu \ell_{R,j})(\bar{\ell}_{R,k} \gamma^\mu \ell_{R,l}) \quad O_4 = (\bar{\ell}_{R,i} \gamma_\mu \ell_{R,j})(\bar{L}_k \gamma^\mu L_l)$$

- radiative

$$R_1 = g'(\bar{L}_i H \sigma_{\mu\nu} \ell_{R,j}) B^{\mu\nu} \qquad R_2 = g(\bar{L}_i \tau^a H \sigma_{\mu\nu} \ell_{R,j}) W^{\mu\nu,a}$$

- dim 8 leptonic:

$$P_1 = ((\bar{L}_i H)\ell_{R,j})((\bar{L}_k H)\ell_{R,l}) \quad Q_1 = ((\bar{L}_i H)\ell_{R,j})(\bar{\ell}_{R,k}(H^\dagger L_l))$$

  and alike

- *i, j, k, l are flavour indices.*

Effective Operators for τ Decays

- For LFV τ decays ($\ell, \ell', \ell'' = e, \mu$):

$$\begin{aligned}
H_{\rm eff}^{(LL)(LL)} &= \frac{g_V^{(LL)(LL)}}{\Lambda^2}(\bar\ell_L\gamma_\mu\tau_L)(\bar\ell'_L\gamma^\mu\ell''_L) \\
H_{\rm eff}^{(LL)(RR)} &= \frac{g_V^{(LL)(RR)}}{\Lambda^2}(\bar\ell_L\gamma_\mu\tau_L)(\bar\ell'_R\gamma^\mu\ell''_R) \\
H_{\rm eff}^{(RR)(LL)} &= \frac{g_V^{(RR)(LL)}}{\Lambda^2}(\bar\ell_R\gamma_\mu\tau_R)(\bar\ell'_L\gamma^\mu\ell''_L) \\
H_{\rm eff}^{(RR)(RR)} &= \frac{g_V^{(RR)(RR)}}{\Lambda^2}(\bar\ell_R\gamma_\mu\tau_R)(\bar\ell'_R\gamma^\mu\ell''_R)
\end{aligned}$$

- In addition we have the radiative operators

$$H_{\rm rad}^{\rm (LR)} = \frac{g_{\rm rad}^{(LR)} v}{\Lambda^2} (\bar{\ell}_L \sigma_{\mu\nu} \tau_R) F^{\mu\nu}$$
$$H_{\rm rad}^{\rm (RL)} = \frac{g_{\rm rad}^{(LR)} v}{\Lambda^2} (\bar{\ell}_R \sigma_{\mu\nu} \tau_L) F^{\mu\nu}$$

- ... and the helicity changing operators:

$$Q_{\rm eff,S}^{(LR)(LR)} = \frac{g_S^{(LR)(LR)}}{\Lambda^2} (\bar{\ell}_L \tau_R)(\bar{\ell}'_L \ell''_R)$$
$$Q_{\rm eff,T}^{(LR)(LR)} = \frac{g_T^{(LR)(LR)}}{\Lambda^2} (\bar{\ell}_L \sigma_{\mu\nu} \tau_R)(\bar{\ell}'_L \sigma^{\mu\nu} \ell''_R)$$
$$\cdots$$

Coupling Constants

- Coupling constants C_{ijkl} (and thus the g) depend on the specific model.
- ... see talks on specific models with CLVF
- Alternatively: Make us of leptonic minimal flavor violation (LMFV)

Cirigliano, Grinstein 2007; Dassinger, Feldmann, M., Turczyk 2007

Leptonic Minimal Flavour Violation

- In case of vanishing lepton and neutrino masses: Leptons have a $SU(3)_{L_L} \times SU(3)_{\ell_R}$ flavour symmetry
- Charged Lepton masses and Neutrino masses induce an explicit breaking:
  - Charged Lepton Mass Matrices:

$$\lambda = \frac{1}{v} \mathrm{diag}(m_e, m_\mu, m_\tau)$$

  - Majorana Mass Term from the dim-5 Operator: C_5 (3×3 Matrix)
- Spurion Analysis:

$$\lambda \sim (\bar{3}, 3) \qquad C_5 \sim (\bar{6}, 1)$$

- Two spurion insertions:

$$C_5^\dagger \times C_5 \sim \bar{6} \times 6 = 1 + 8 + 27$$

- The octet part is

$$C_5^\dagger C_5 = \frac{\Lambda^2}{v^4} U_{PMNS} \Delta m_\nu^2 U_{PMNS}^\dagger$$

- The (order of magnitude of the) couplings can be expressed in terms of PMNS matrix elements and the mass-squared differences of the neutrinos.
- Consequences for the g's

$$g_V^{(RR)(RR)},\ g_V^{(RR)(LL)},\ g_{\rm rad}^{(RL)} \sim 0$$

due to small lepton masses.

Unpolarized Results

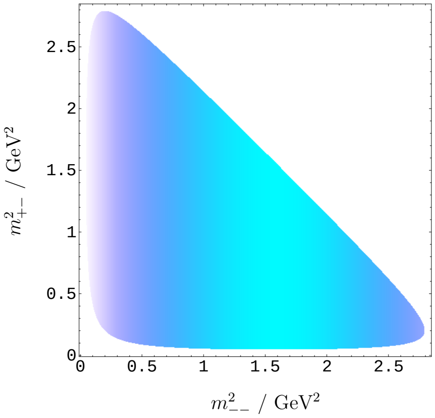
- Focus first on the decay $\tau \to 3\mu$:
- Study the Dalitz distribution in

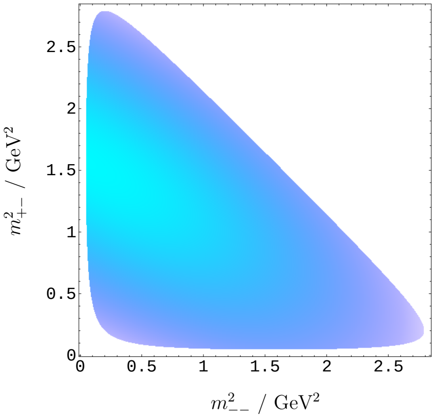
$$m^2_{--} = (p_{\mu^-} + p'_{\mu^-})^2 \quad \text{and} \quad m^2_{+-} = (p'_{\mu^-} + p_{\mu^+})^2$$

- As an example: (LL)(LL) versus (LL)(RR)

$$\frac{d\Gamma^{(LL)(LL)}}{dm^2_{--}\, dm^2_{+-}} \propto (2m^2_\mu - m^2_{--})(m^2_\mu - m^2_{--} + m^2_\tau)$$

$$\frac{d\Gamma^{(LL)(RR)}}{dm^2_{--}\, dm^2_{+-}} \propto f(m^2_{--}, m^2_{+-})$$





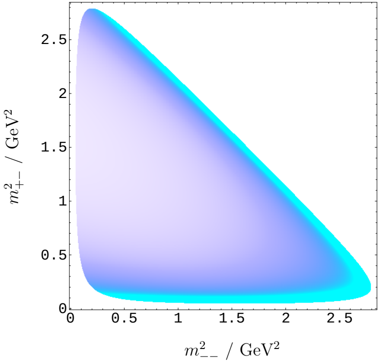
(LL)(LL)
= (RR)(RR)

(LL)(RR)

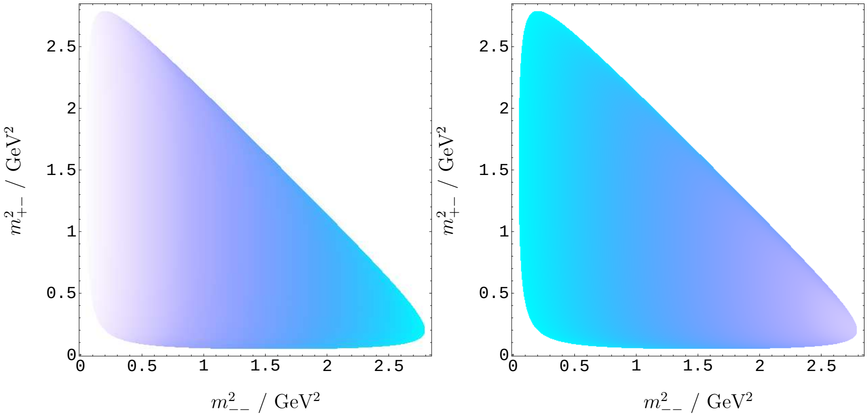
Radiative Contributions

- Aside from the purely leptonic operators we also have

$$\tau \to \mu\gamma \to \mu\ \mu^+\mu^-$$



... and the corresponding interference terms



(LL)(LL) $\times$ rad (LL)(RR) $\times$ rad

Sign of the terms depends on the model!

Typical Model predictions

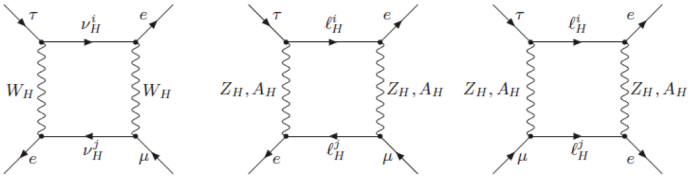
ratio	LHT	MSSM (dipole)	MSSM (Higgs)	4G
$\frac{\mathcal{B}(\mu^- \to e^- e^+ e^-)}{\mathcal{B}(\mu \to e\gamma)}$	$0.02 \ldots 1$	$\sim 6 \cdot 10^{-3}$	$\sim 6 \cdot 10^{-3}$	$0.06 \ldots 2.2$
$\frac{\mathcal{B}(\tau^- \to e^- e^+ e^-)}{\mathcal{B}(\tau \to e\gamma)}$	$0.04 \ldots 0.4$	$\sim 1 \cdot 10^{-2}$	$\sim 1 \cdot 10^{-2}$	$0.07 \ldots 2.2$
$\frac{\mathcal{B}(\tau^- \to \mu^- \mu^+ \mu^-)}{\mathcal{B}(\tau \to \mu\gamma)}$	$0.04 \ldots 0.4$	$\sim 2 \cdot 10^{-3}$	$0.06 \ldots 0.1$	$0.06 \ldots 2.2$
$\frac{\mathcal{B}(\tau^- \to e^- \mu^+ \mu^-)}{\mathcal{B}(\tau \to e\gamma)}$	$0.04 \ldots 0.3$	$\sim 2 \cdot 10^{-3}$	$0.02 \ldots 0.04$	$0.03 \ldots 1.3$
$\frac{\mathcal{B}(\tau^- \to \mu^- e^+ e^-)}{\mathcal{B}(\tau \to \mu\gamma)}$	$0.04 \ldots 0.3$	$\sim 1 \cdot 10^{-2}$	$\sim 1 \cdot 10^{-2}$	$0.04 \ldots 1.4$
$\frac{\mathcal{B}(\tau^- \to e^- e^+ e^-)}{\mathcal{B}(\tau^- \to e^- \mu^+ \mu^-)}$	$0.8 \ldots 2$	~ 5	$0.3 \ldots 0.5$	$1.5 \ldots 2.3$
$\frac{\mathcal{B}(\tau^- \to \mu^- \mu^+ \mu^-)}{\mathcal{B}(\tau^- \to \mu^- e^+ e^-)}$	$0.7 \ldots 1.6$	~ 0.2	$5 \ldots 10$	$1.4 \ldots 1.7$
$\frac{\mathrm{R}(\mu \mathrm{Ti} \to e \mathrm{Ti})}{\mathcal{B}(\mu \to e\gamma)}$	$10^{-3} \ldots 10^{2}$	$\sim 5 \cdot 10^{-3}$	$0.08 \ldots 0.15$	$10^{-12} \ldots 26$

A few remarks on other $\tau \to 3\ell$ decays

- Including also electrons we can have modes with no "radiative pollution":

$$\tau^- \to e^- e^- \mu^+ \qquad \tau^- \to \mu^- \mu^- e^+$$

- There are models which have such "doubly lepton number violating" decays eg. Little Higgs with T-Parity



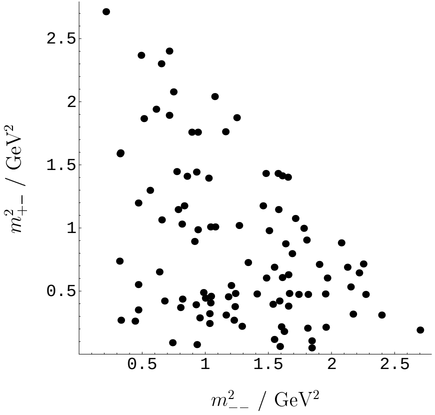
Reality

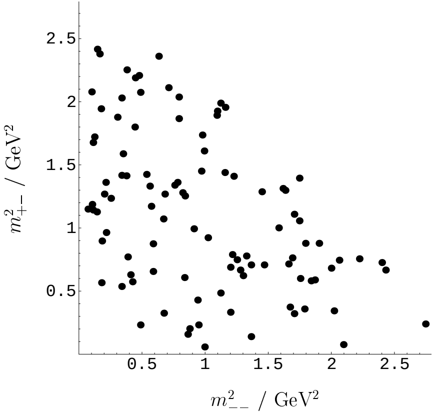
- CLNV processes appear in the SM, augmented by $\mathcal{L}_{\text{dim }5}$ for neutrino masses.

$$\mathrm{Br}(\tau \to \mu\gamma) \sim 10^{-54} \quad \mathrm{Br}(\tau \to 3\mu) \sim 10^{-56}$$

- A huge enhancement factor is needed to arrive at a measurable Br.
- Current limits [PDG]: $\mathrm{Br}(\tau \to 3\ell) \sim 10^{-8}$
- There are models with Br's not far below this
- Assume that this is true and we can collect $\mathcal{O}$ (100) events.

- Simulation of the Dalitz plots with 100 Events





Benefits from Polarization

- Disentangle the helicity structure of the NP operators
- Obtain a suppression of backgrounds

These points deserve a detailed analysis which has (to the best of my knowledge) not yet been done

Some well known facts on polarization

- Consider a τ-Charm factory
  = High Lumi e^+ e^- collider at the τ threshold
- (Longitudinally) polarized e^+ e^- beams
  with a sizable polarization (90%)
- At threshold: τ pairs are produced in s waves:
  Highly polarized τ's, polarization in beam direction
- For electrons at these energies: helicity = chirality
- For positrons at these energies: helicity = - chirality
- e^+ e^- cross sections: $\sigma(++) = 0 = \sigma(--)$

- Unpolarized beams:

$$\sigma = \frac{1}{4}[\sigma(+-) + \sigma(-+)]$$

- polarization of the e^- beam:

$$\sigma = \frac{1}{4}[\sigma(+-) + \sigma(-+)] + \frac{w_-}{4}[\sigma(+-) - \sigma(-+)]$$

- Both beams polarized

$$\sigma = \frac{1 + w_+ w_-}{4}[\sigma(+-) + \sigma(-+)] + \frac{w_+ + w_-}{4}[\sigma(+-) - \sigma(-+)]$$

with $w_+ = -\frac{N_+^+ - N_+^-}{N_+^+ + N_+^-}$ and $w_- = \frac{N_-^+ - N_-^-}{N_-^+ + N_-^-}$

- Positron polarization does not yield new information
- However, positron polarization enhances the "effective" polarization

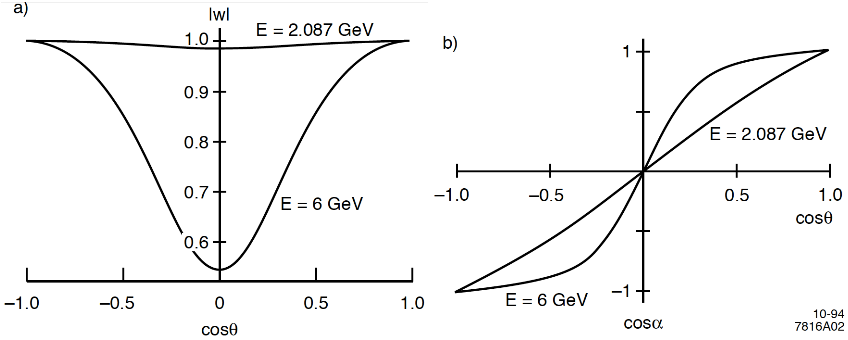
$$w_- \to w_{\rm eff} = \frac{w_+ + w_-}{1 + w_+ w_-}$$

- for $w_- = w_+ = 0.9$ we have $w_{\rm eff} = 0.994$.
- At threshold: Both τ's are in an s wave: Spins aligned with the beam axis!

τ polarization

- $\vec{s}_\pm$ (unit length) spin vectors of $\tau_\pm$
- Polarization of $\tau_\pm$

$$w_\pm = \frac{[N_\pm \text{ with } +\vec{s}_\pm] - [N_\pm \text{ with } -\vec{s}_\pm]}{[N_\pm \text{ with } +\vec{s}_\pm] + [N_\pm \text{ with } -\vec{s}_\pm]}$$



$\tau \to 3\mu$ with polarized τ's

- A lot of work has been dedicated to CP studies in polarized τ decays
- Here one studies “bread and butter” decays, looking at specific observables
- $\to$ A lot of data, allowing detailed angular analyses.
- However, in CLFV decays we do not have large data samples
  $\to$ a different strategy

- Energy distribution of the μ^+ in $\tau \to \mu^-\mu^-\mu^+$

$$d\Gamma = f(x) \pm g(x)\, \vec{s} \cdot \vec{e}_+$$

- $x = 2E_+/m_\tau$: rescaled energy variable
  $\vec{s}$: Spin of the τ in its rest frame
  $\vec{e}_+$: Flight direction of the μ^+ in the rest frame of τ
- "Forward backward asymmetry" $a_{\rm FB}$ ($\vec{s} \cdot \vec{e}_+ = \cos\theta$)

$$a_{\rm FB} = \frac{1}{d\Gamma}\left[\int_{\theta=0}^{\theta=\pi/2} d\Gamma d\Omega_\theta - \int_{\theta=\pi/2}^{\theta=\pi} d\Gamma d\Omega_\theta\right] = \frac{g(x)}{f(x)}$$

- (LL)(LL) contribution ($m_\mu = 0$)

$$d\Gamma \propto x(1-x)\left[1 \mp \vec{s}\cdot\vec{e}_+\right]$$
$$a_{\rm FB} = 1$$

- (LL)(RR) contribution ($m_\mu = 0$)

$$d\Gamma \propto x(3-2x) \mp x(2x-1)\vec{s}\cdot\vec{e}_+$$
$$a_{\rm FB} = \frac{2x-1}{3-2x}$$

- (RR)(RR) contribution ($m_\mu = 0$)

$$d\Gamma \propto x(1-x)\left[1 \pm \vec{s}\cdot\vec{e}_+\right]$$
$$a_{\rm FB} = -1$$

- Likewise (RR)(LL):
  Sign change in the spin-dependent term.

- (LR)(LR) scalar contribution ($m_\mu = 0$)

$$d\Gamma \propto x(1-x)\left[1 \pm \vec{s}\cdot\vec{e}_+\right]$$
$$a_{\rm FB} = 1$$

- Sign flip of the spin dependent piece for (RL)(RL)
- Clean handle to disentangle helicities
- Integration over energy
- A serious study should include the μ mass.

Muon polarization

- Additional information is contained in the polarization of the muon
- Needs a analysis if the μ decay products.
- $\rightarrow$ Needs a sufficient data sample

Outlook

- Polarization of initial lepton is very useful
- ... both beams, if possible
- However, not for the discovery of the decay modes,
- but for the identification of the underlying interaction.
- Role of polarization in background suppression ...

All these points deserve a more detailed study!