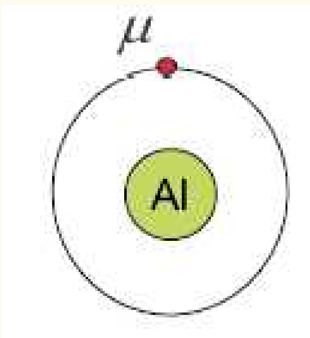
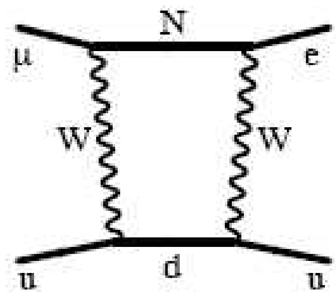
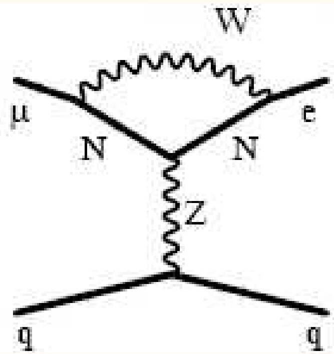
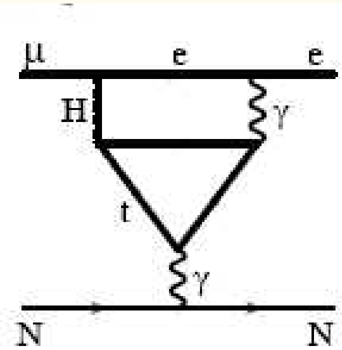
Radiative effects in exotic muon decays









1st Conference on Charged Lepton Flavour Violation

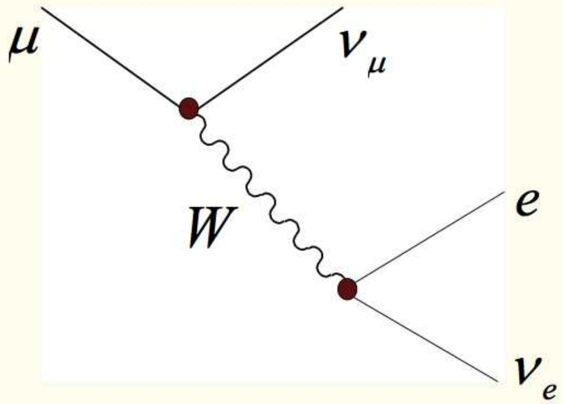
Lecce, May 2013

Andrzej Czarnecki University of Alberta

Outline

- Free muon decay
- $\mu \rightarrow e\gamma$
- $\mu \rightarrow eee$
- Muon electron conversion near nuclei
- Muon decay in orbit
- Decay $\mu \rightarrow e$+majoron

Free muon decay



A model process in particle physics
(tools for quark decays)

The first decay process known with one- and two-loop QED effects.

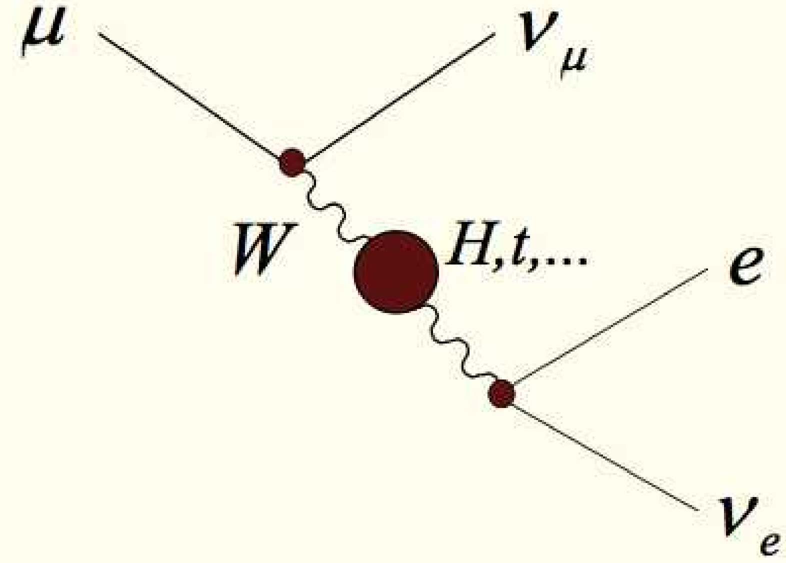
Anastasiou, Melnikov, Petriello, JHEP 0709 (2007) 014
van Ritbergen + Stuart, PRL 82 (1999) 488
Pak + Czarnecki, PRL 100 (2008) 241807

Also very thoroughly studied experimentally; most recently
* decay distributions ("Michel parameters") TWIST PRD 85 (2012) 092013
* total rate (1 ppm!) MuLan PRL 106 (2011) 041803

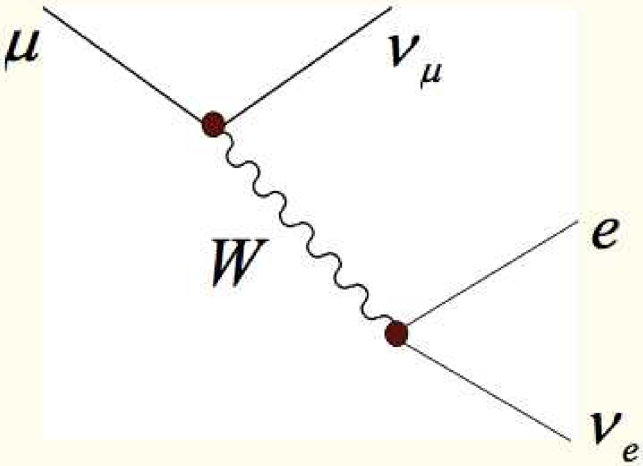
Fermi constant and tests of the SM

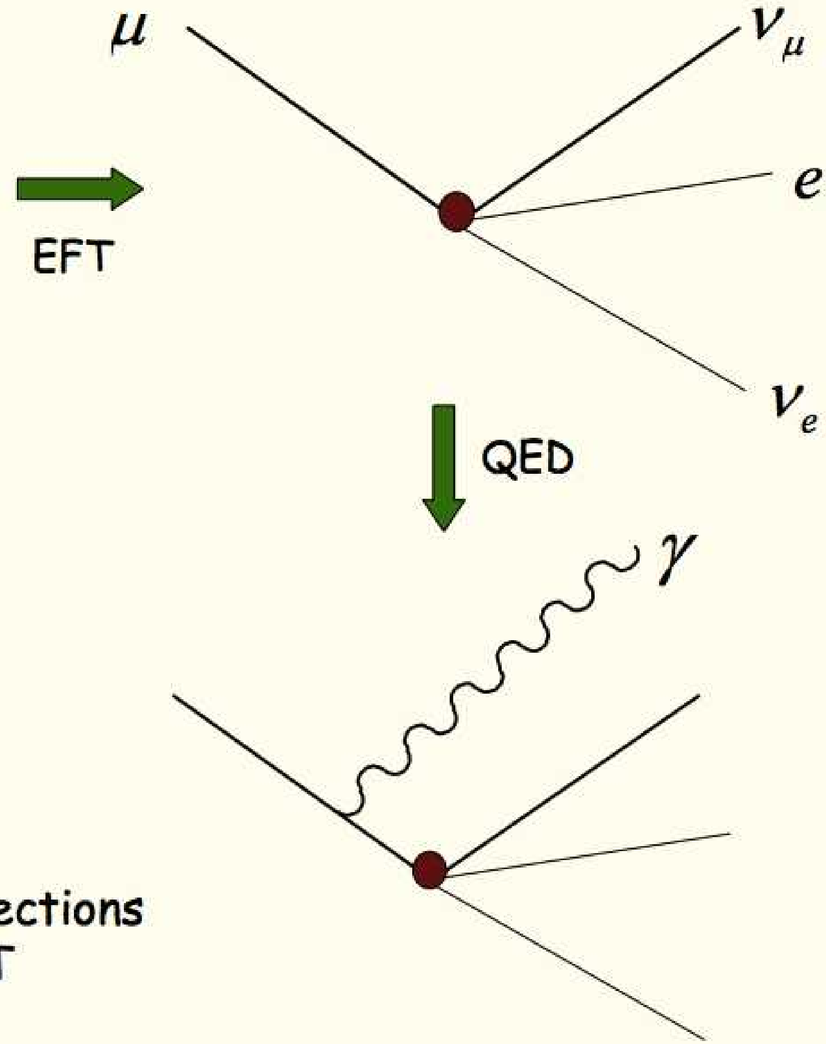
$$G_\mu \sim \frac{\alpha}{M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)}\left(1+\Delta r\right)$$

$$\Delta\alpha_{\mathrm{had}}^5 - c\,m_t^2 + c'\ln M_H^2 + \ldots$$



Determination of the Fermi constant (convention)





$$\frac{1}{\tau_\mu} = \frac{G_\mu^2 m_\mu^5}{192\pi^3}\left[1 + \Delta q\right]$$

Finite m_e and QED corrections
in the four-fermion EFT

QED radiative corrections in Fermi theory

1956: one-photon, with m_e — Behrends, Finkelstein, Sirlin

1999: two-photon, m_e=0 — van Ritbergen and Stuart

2008: two-photon, with m_e — Pak, AC

Related work:
Numerical tests of the $O(\alpha^2)$ result (not able to determine the m_e effect):
Chetyrkin, Harlander, Seidensticker, Steinhauser (1999);
Blokland, AC, Ślusarczyk, Tkachov (2004)

2005, Anastasiou, Melnikov, Petriello: $O(\alpha^2)$ electron spectrum

Muon lifetime in Fermi theory, with QED

$$\Gamma(\mu \to e\bar{\nu}\nu) = \frac{G_\mu^2 m_\mu^5}{192\pi^3}\left[X_0 + \frac{\alpha}{\pi}X_1 + \left(\frac{\alpha}{\pi}\right)^2 X_2 + \ldots\right]$$

$$X_0 = 1 - 8\rho^2 - 24\rho^4 \ln\rho + 8\rho^6 - \rho^8 \qquad \rho \equiv \frac{m_e}{m_\mu}$$

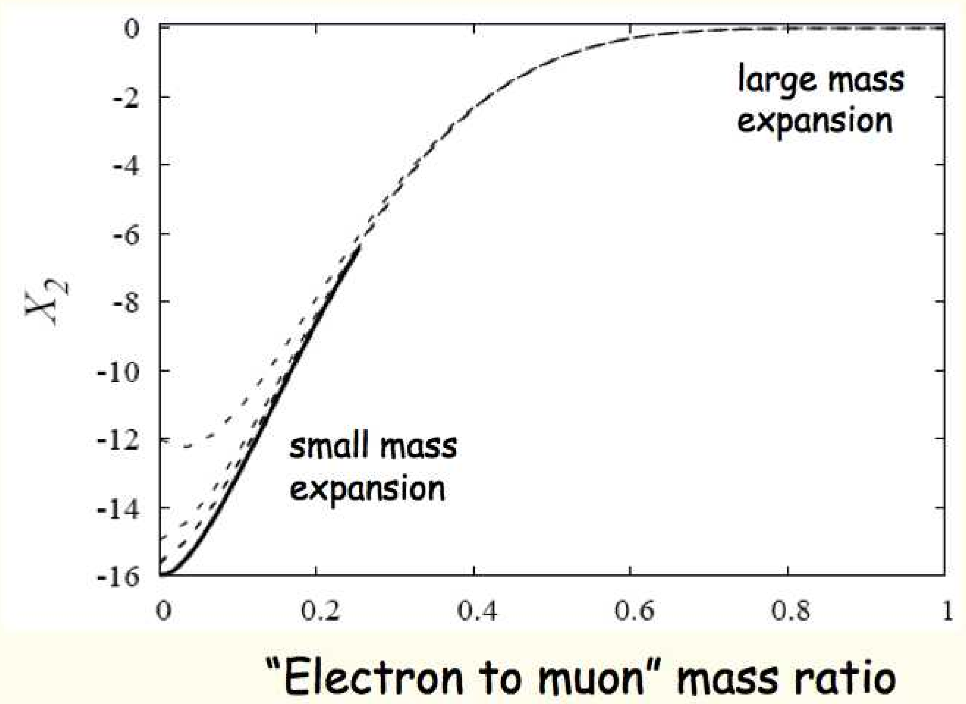
$$X_1 = \frac{25}{8} - \frac{\pi^2}{2} - (34 + 24\ln\rho)\rho^2 + 16\pi^2\rho^3 - \left(\frac{273}{2} - 36\ln\rho + 72\ln^2\rho + 8\pi^2\right)\rho^4 + \ldots.$$

$$X_2 = X_2(\rho = 0) - \frac{5}{4}\pi^2\rho + \ldots$$

Pak, AC

Can one go further: to three loops?

We have found an interesting way while checking the two-loop result: the calculation would be easier if the electron was very heavy, almost as heavy as the decaying muon.

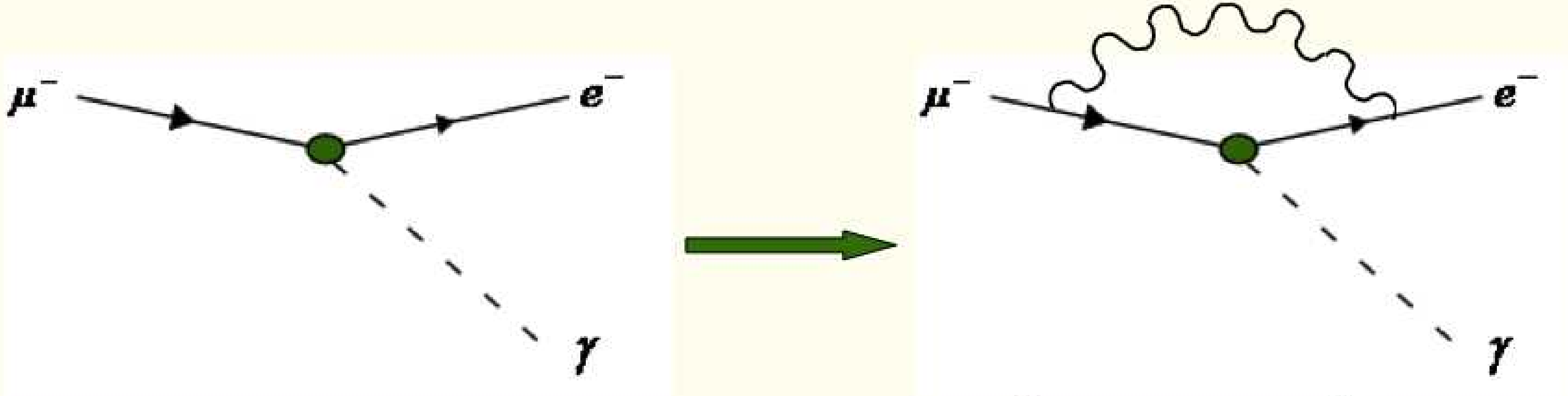


From Dowling, Piclum, AC

Note: the plot actually for QCD.
QED given by a subset of QCD results.

Lepton-flavor violating processes

QED suppression of the decay $\mu \to e\gamma$



$$\sigma_{\alpha\beta} q^{\beta} \left(E - M\gamma_5 \right) A^{\alpha} \qquad \times \left(1 - \frac{4\alpha}{\pi} \ln \frac{\Lambda}{m_\mu} \right)$$

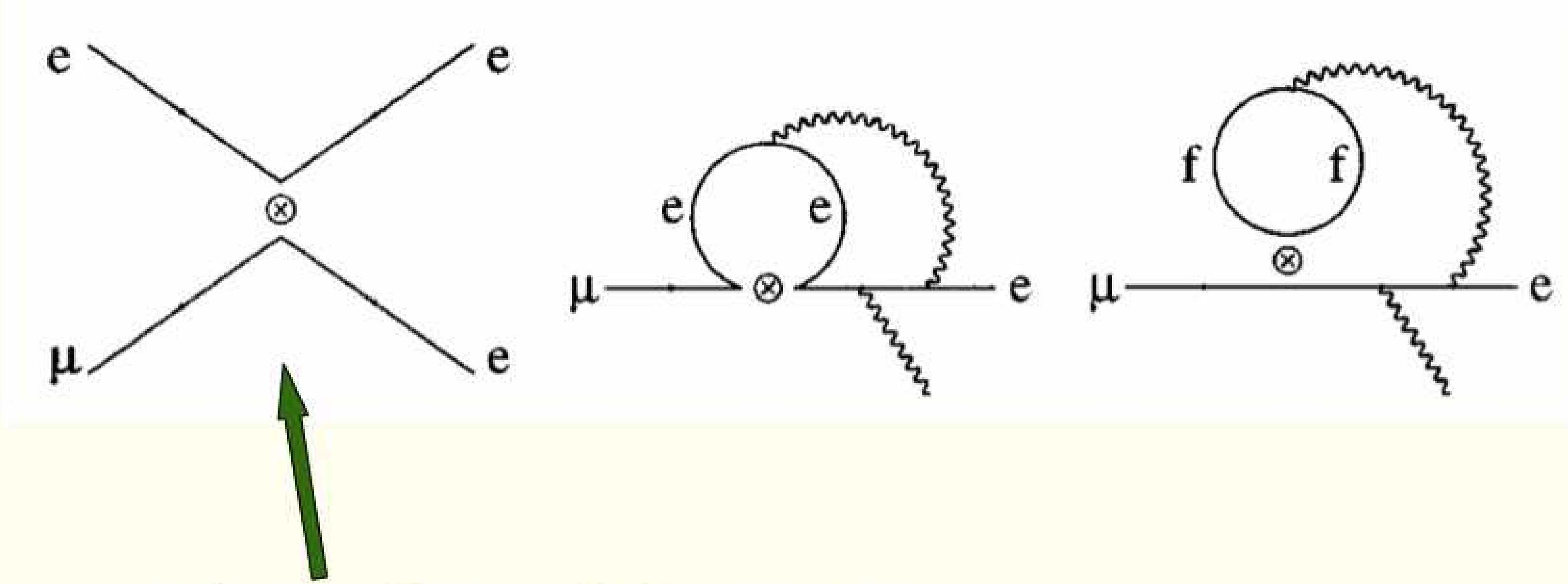
AC and Jankowski,
PRD 65, 113004 (2002)

This is the largest known QED correction to a decay rate;
15 percent for $\Lambda \sim 250$ GeV. In general, $\sim 2\ln(\Lambda/m_\mu)$ percent.

For comparison, correction for the normal muon decay is 0.4 percent.

The rate suppression $\times\left(1-\frac{8\alpha}{\pi}\ln\frac{\Lambda}{m_\mu}\right)$ **is universal (independent of the mechanism of LFV)**

This is because the non-dipole operators (four-fermion) which would have a different scaling, contribute little:



Wilson coefficient of this operator
is constrained by direct searches (SINDRUM),

$$\frac{\Gamma(\mu\to eee)}{\Gamma(\mu\to e\nu\nu)}<10^{-12}$$

Why such a large correction?

Difference between the normal muon decay and the eγ channel

$$\bar{\mu}\gamma^{\mu}Le \cdot W_{\mu}$$

dimension=4, renormalizable

$$\bar{\mu}\sigma^{\mu\nu}e \cdot F_{\mu\nu}$$

dimension=5, non-renormalizable:
large logs in the photon loops

Other processes, $\mu \to eee$ and the conversion,

have a mixture of both structures. Large logs can be present.

Muon-electron conversion

Muon g-2: ~3.6σ discrepancy

Encouragement for lepton flavor violation searches:

$$a_\mu^{\mathrm{NP}} \frac{e}{2m} \bar{\mu} \sigma \cdot F \mu \;\to\; \frac{e}{2m} \bar{e} \left(f_M + f_E \gamma_5 \right) \sigma \cdot F \mu$$

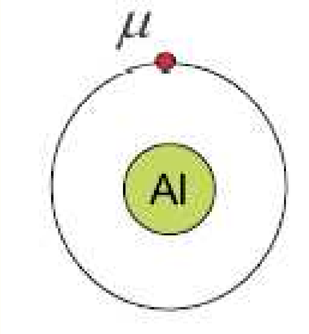
$$f_{M,E} \sim a_\mu^{\mathrm{NP}} \cdot \delta$$

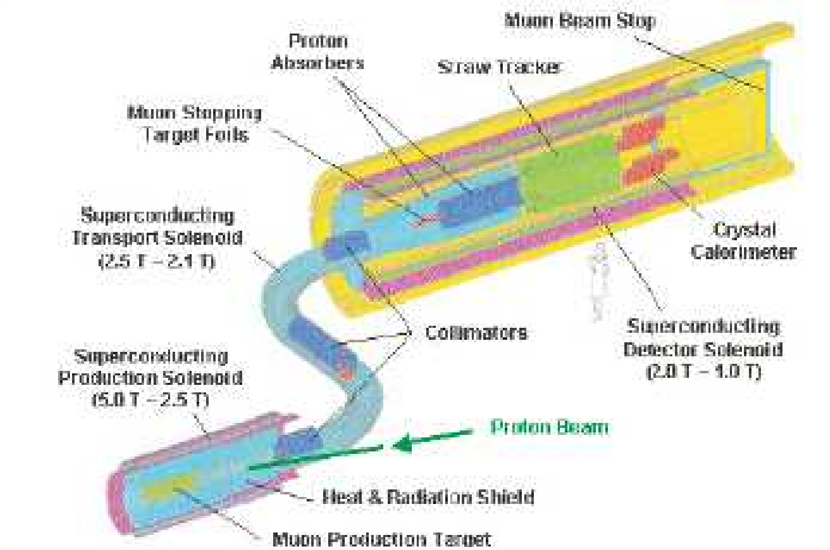
$$BR(\mu \to e\gamma) \sim 10^{-3} \delta^2$$

(Also lots of theoretical encouragement from "new physics" models.)

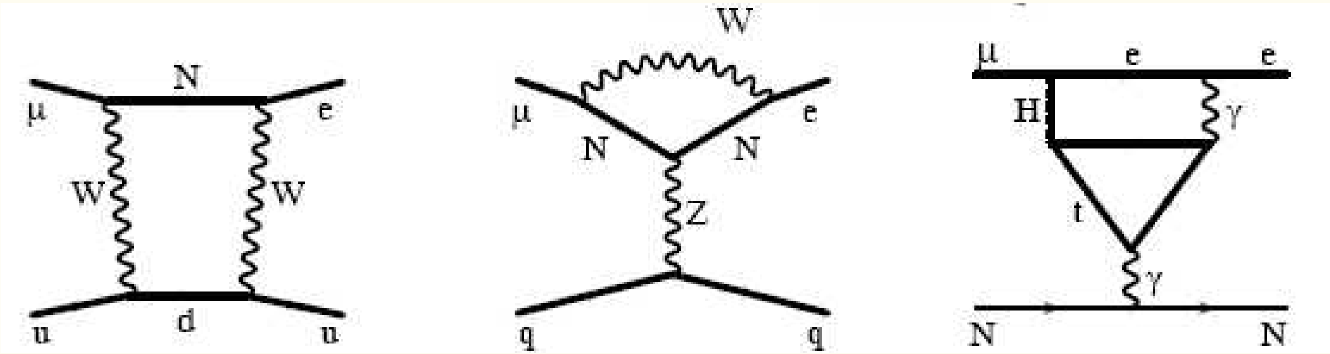
Muon-electron conversion

"The best rare process"
No accidental bkgd
(single monochromatic e^-);
10^{-17} sensitivity envisioned





Variety of mechanisms:

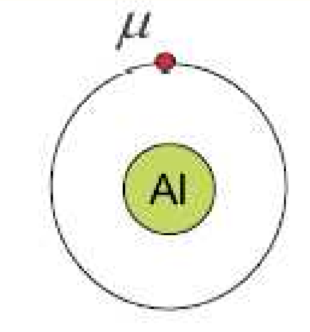


Comparison with scattering experiments

Highest luminosity in fixed-target experiments

$$\sim 10^{37\ldots38}\,/\left(\mathrm{cm}^2\cdot\mathrm{s}\right)$$

In a single muonic atom



$$= \text{density} \times \text{velocity}$$

$$= \left|\psi(0)\right|^2 \cdot Z\alpha = \frac{m_\mu^3 Z^4 \alpha^4}{\pi} \sim Z^4 \cdot 4\cdot 10^{39}\,/\left(\mathrm{cm}^2\cdot\mathrm{s}\right)$$

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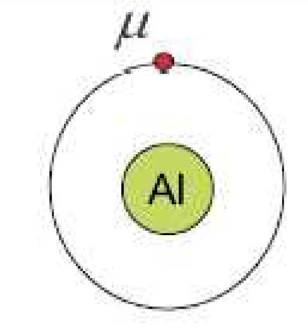

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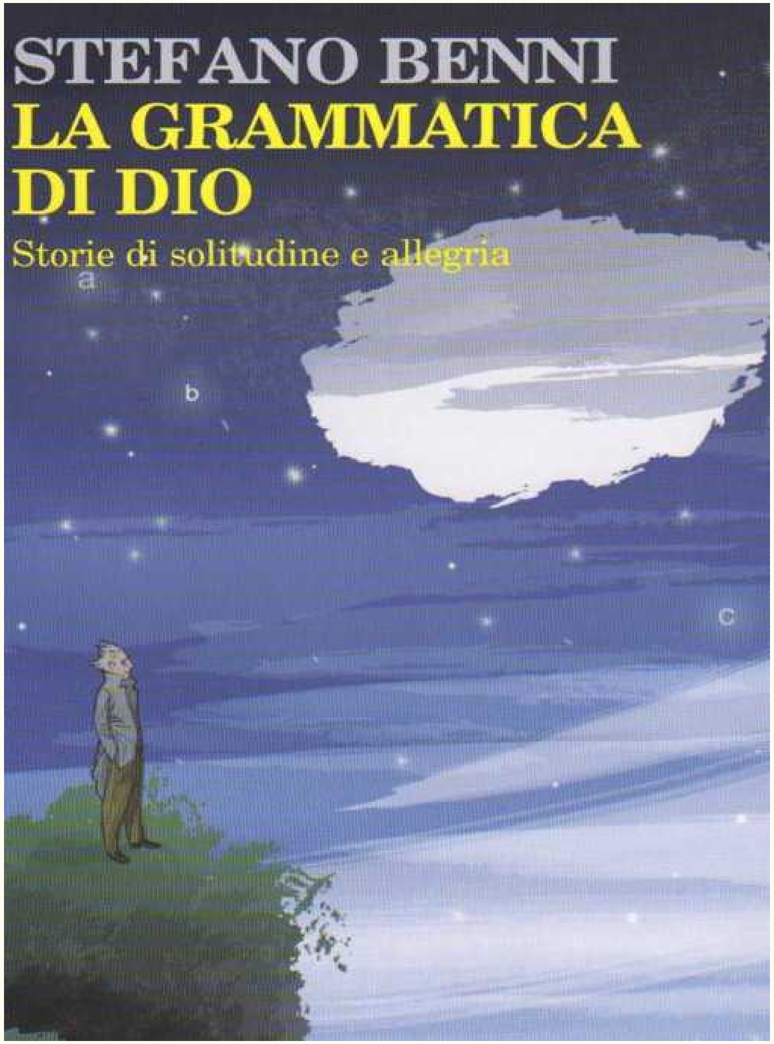
Many atoms are studied in parallel: $\sim 10^{11}$ muons stopped per second, each lives about 10^{-6} seconds: 10^5 atoms present:

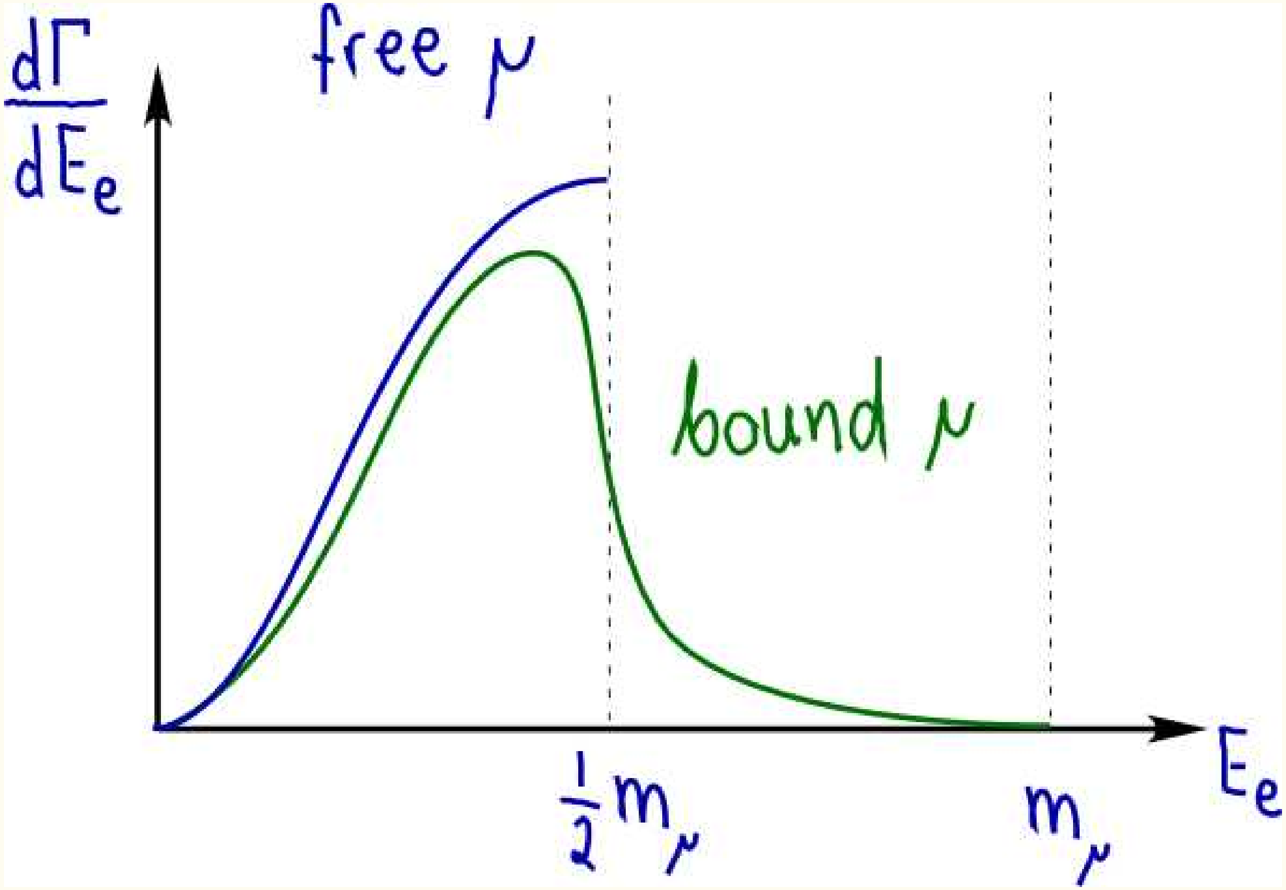
$$\sim 10^{49} / \left(\mathrm{cm}^2 \cdot \mathrm{s}\right)$$

Muon decay in orbit (DIO)

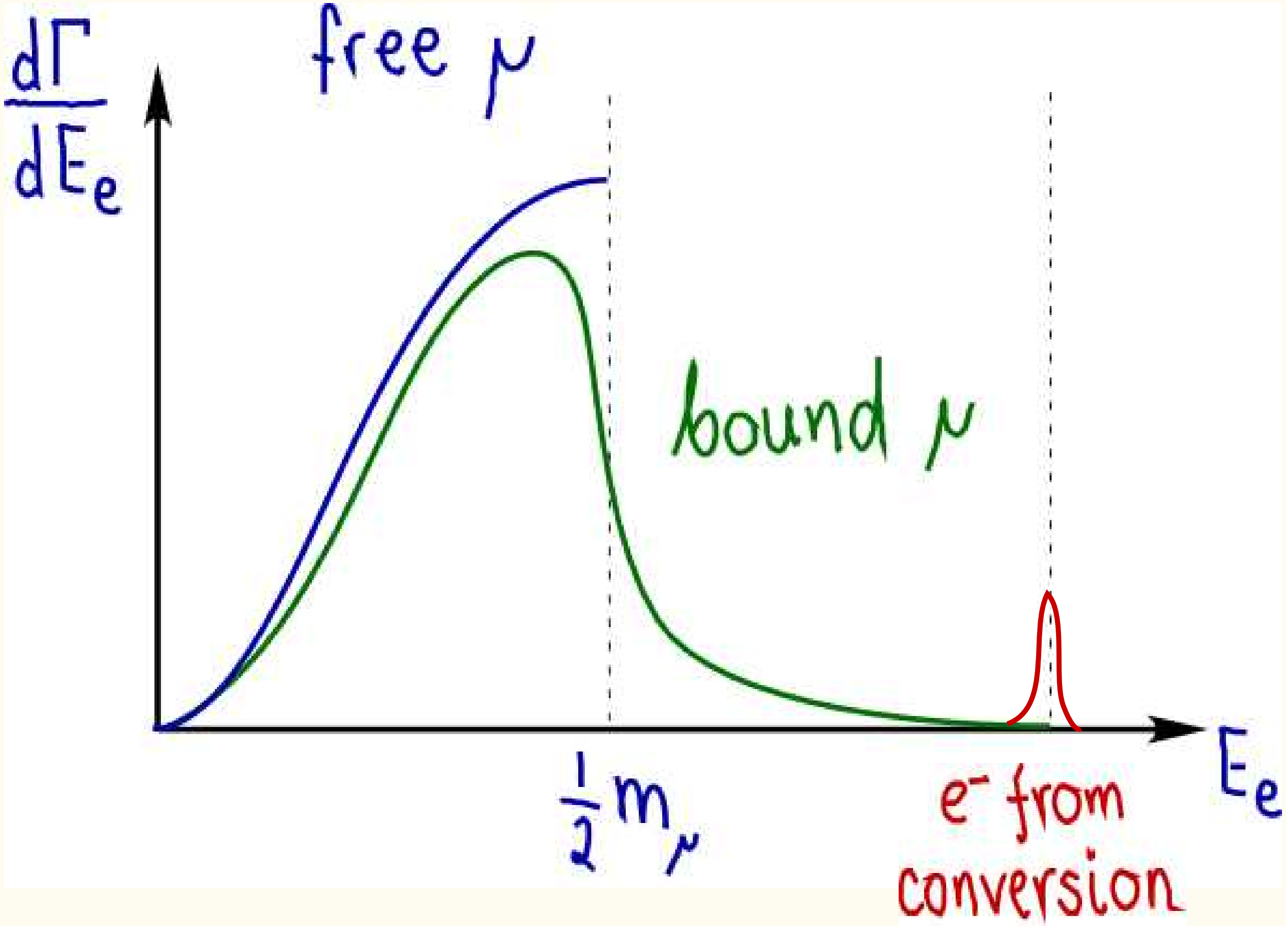
Muon decay in orbit (DIO)



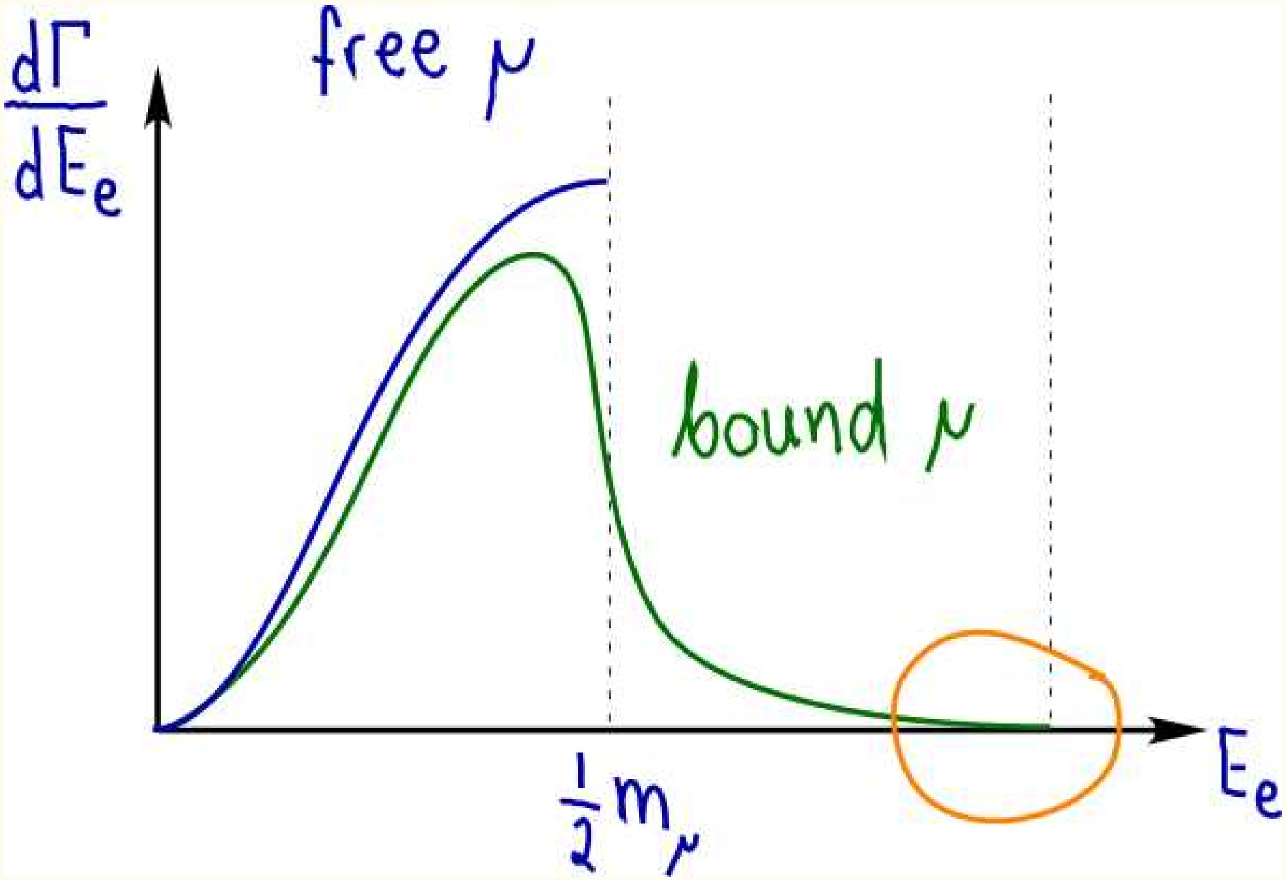
Background from the standard muon decay



Background from the standard muon decay



End point spectrum must be well understood



$$\frac{d\Gamma}{dE_e} \sim (Z\alpha)^5 (E_{max} - E)^5$$

End point spectrum

Previous studies: Shanker & Roy, Hänggi et al., Herzog & Alder

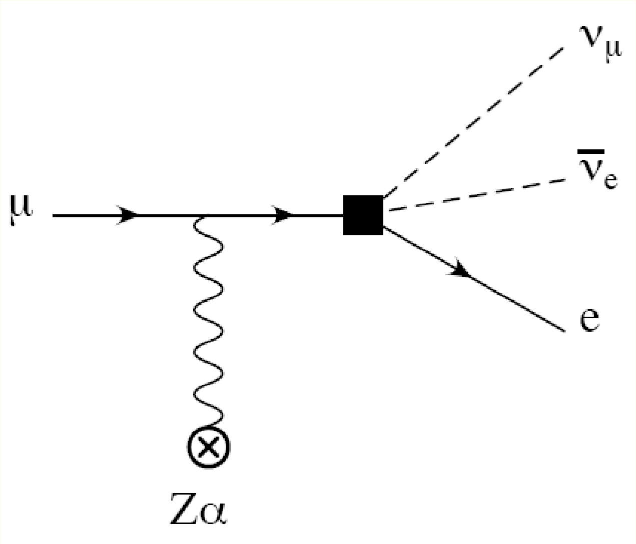
Relativistic muon wave function, nuclear size and recoil, electron final state interactions: all taken into account.

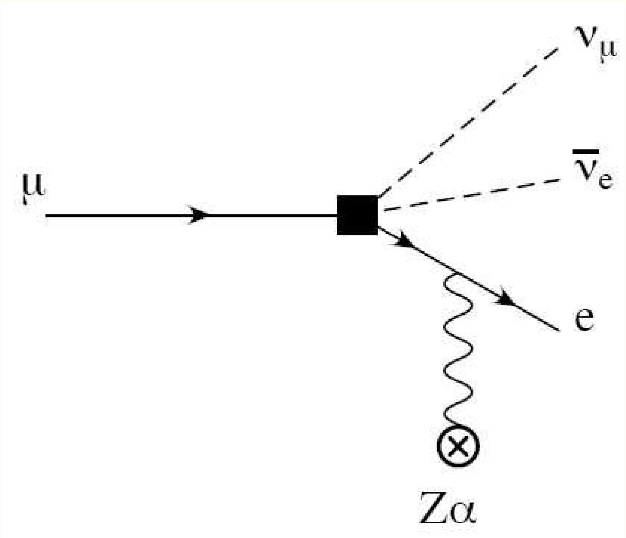
$$N(E_e)dE_e \simeq 0.4 \cdot 10^{-21}\left(1-\frac{E_e}{E_{\max}}\right)^5 dE_e$$

New evaluation: AC, X. Garcia i Tormo, W. J. Marciano PRD84,013006,2011

Planned energy resolution in Mu2e: ~250 keV → 0.22 background events.

How can the electron get muon's whole energy?





Neutrinos get no energy;
The nucleus balances electron's momentum, takes no energy.
Near the end point:

$$\frac{d\Gamma}{dE_e} \sim |\psi(0)|^2 (Z\alpha)^2 \frac{d^3\nu_e}{\nu_e}\frac{d^3\nu_\mu}{\nu_\mu}\delta(E_{\max} - E_e - \nu_e - \nu_\mu)\,\mathrm{Tr}\ldots \not{\nu}_e \ldots \not{\nu}_\mu$$

$$\sim (Z\alpha)^5 (E_{\max} - E_e)^5$$

μ-e conversion may be caused by a majoron

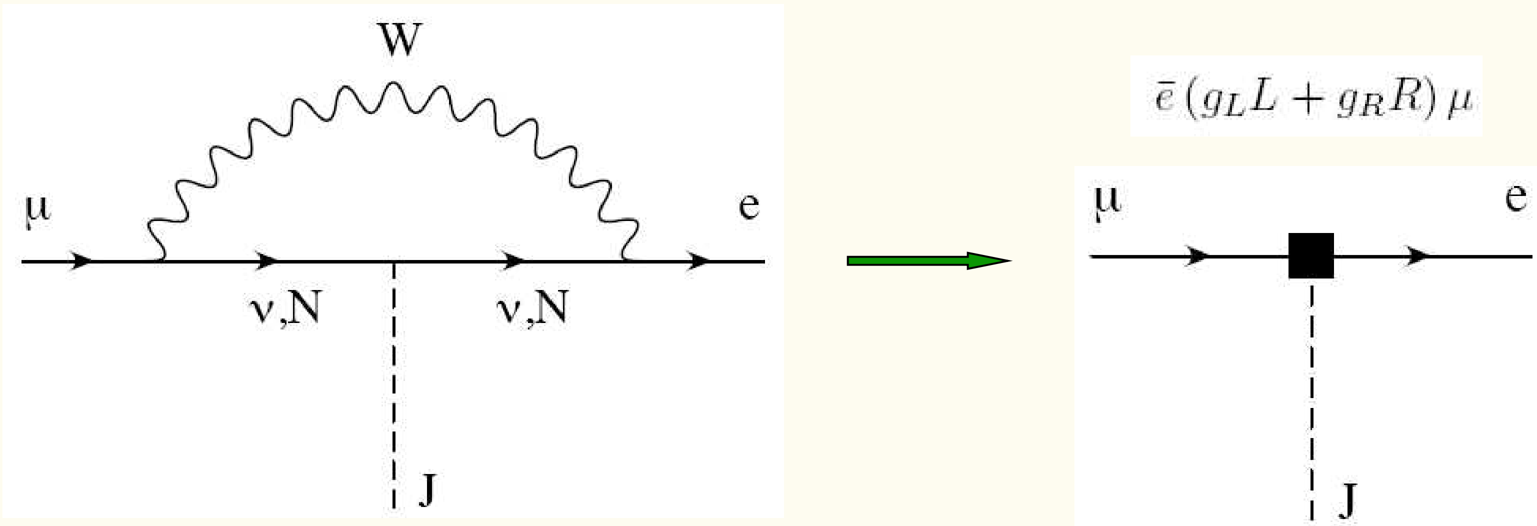
What is the majoron?

If neutrinos have Majorana masses: lepton number is not conserved.

How can lepton conservation be broken?

* explicitly by the Majorana mass term;
* spontaneously, locally; or
* spontaneously, globally → Goldstone boson.

Majoron can violate lepton flavor number



$$\bar{e}\,(g_L L + g_R R)\,\mu$$

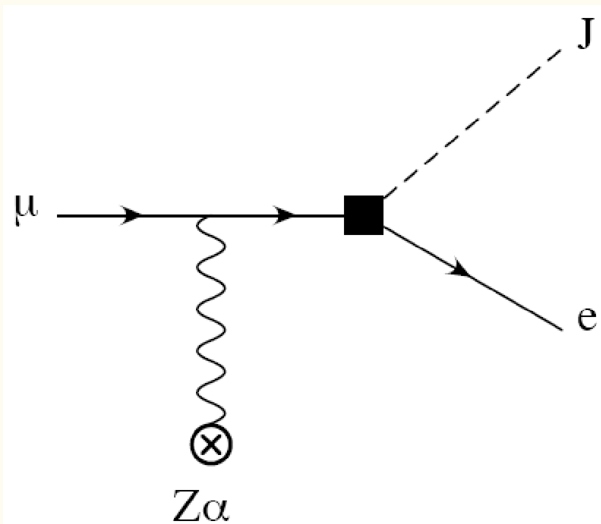
The resulting extra muon decay rate:

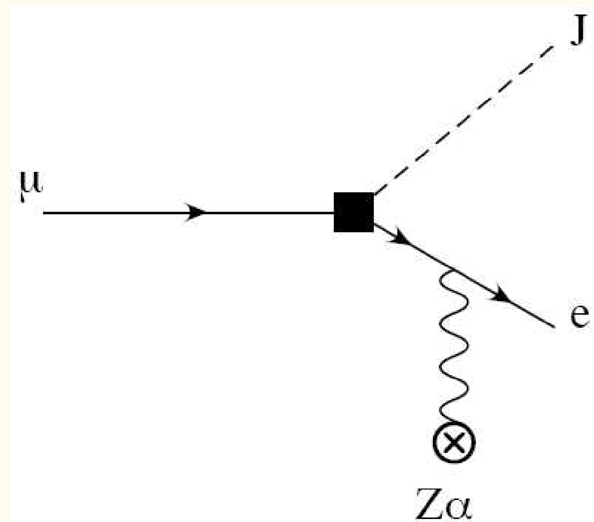
$$\Gamma\,(\mu \to eJ) = \frac{m_\mu}{32\pi}\,\left(g_L^2 + g_R^2\right)$$

What is the electron spectrum in $\mu \rightarrow e+J$?

Free muon: monoenergetic electron, $E_e = m_\mu/2$

Muon bound in an atom: spread out up to m_μ





$$\frac{d\Gamma}{dE_e} \sim |\psi(0)|^2 (Z\alpha)^2 \frac{d^3 J}{J} \delta\left(E_{\max} - E_e - J\right) |\mathcal{M}|^2$$

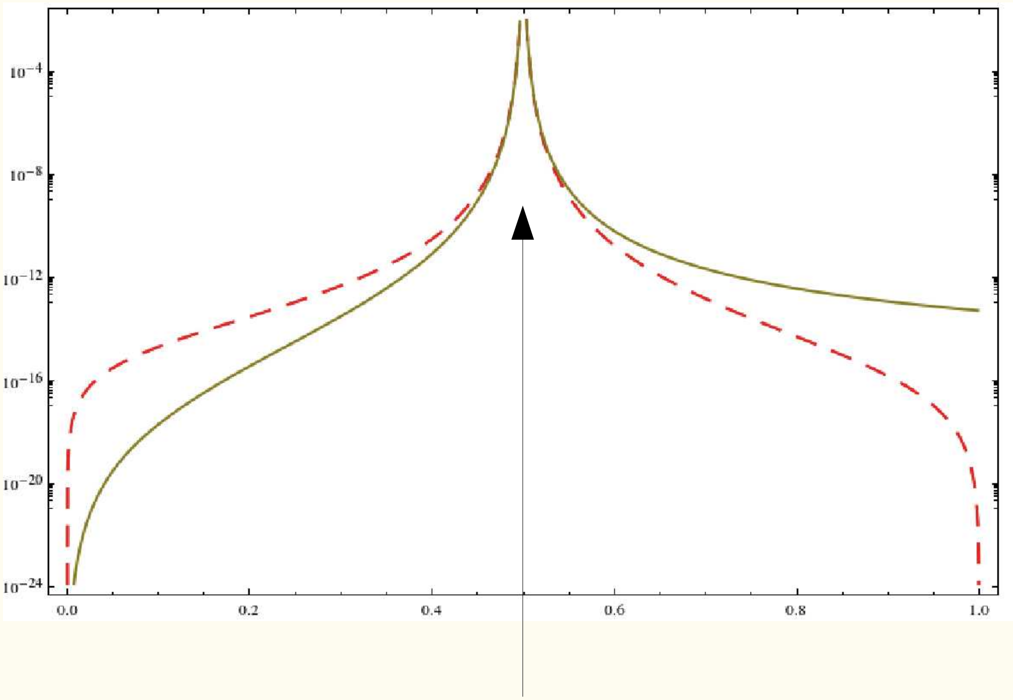
$$\sim (Z\alpha)^5 \left(E_{\max} - E_e\right)^3$$

Vanishes at end point

With D. Bryman, M. Dowling, X. Garcia i Tormo

Results: electron spectrum in $\mu \rightarrow e+J$

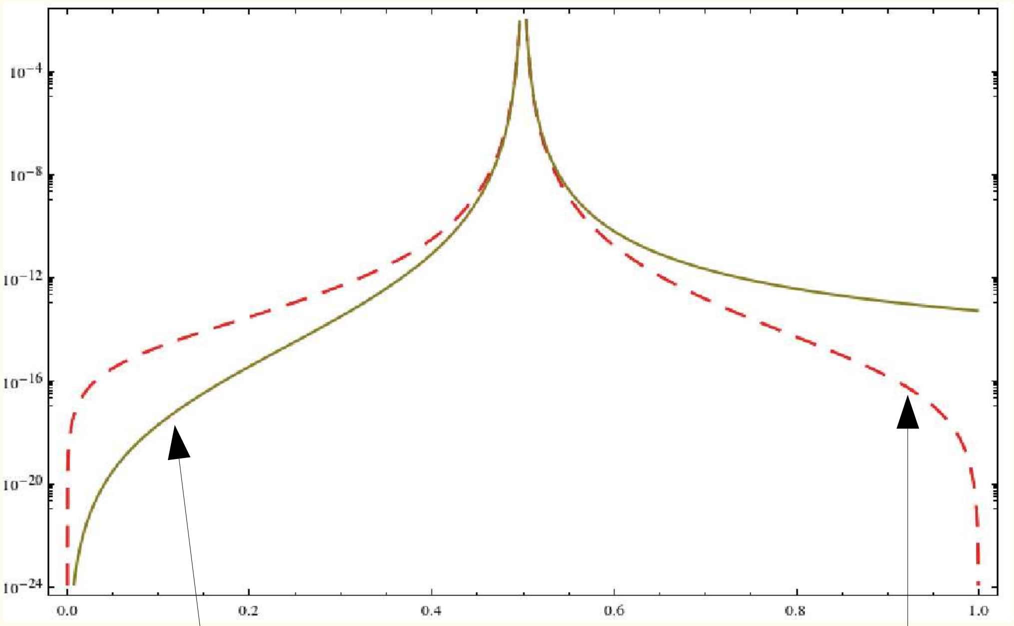
Robert Szafron

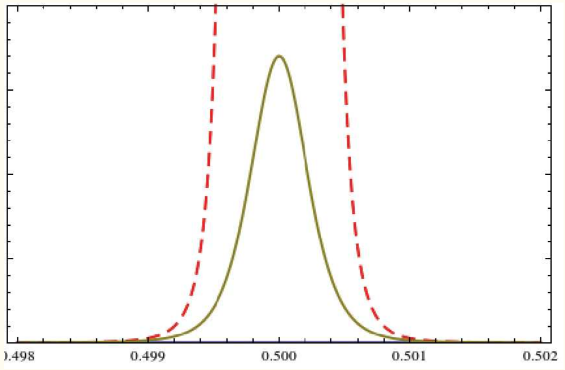


without binding effects,
the electron spectrum is
monochromatic,
concentrated here
at half muon mass

Results: electron spectrum in $\mu \to e+J$

Robert Szafron





smearing due to
muon's motion.
Dominates in the center.

expansion
in Z*alpha
Correct far
from the center

Summary

We have determined spectra of daughter electrons in decays of bound muons.

Simple interpretation of the high-energy tail: hard photon exchange with the nucleus.

The signal of possible decays into majorons is enhanced by two powers of (E_{max} - E_e) but not by four powers.

Ongoing work:

* radiative background for $\mu \to e\gamma$ (with Yi Liang and K. Melnikov); and

* better understanding of the muon decay in orbit: an effective theory approach to various regions of the spectrum (with R. Szafron).