Interrelationship among $g-2$, EDMs and cLFV

Paride Paradisi

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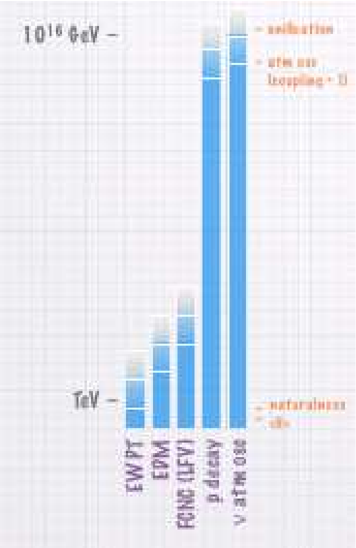
Open questions

- **The origin of flavour is still, to a large extent, a mystery. The most important open questions can be summarized as follow:**
  - **Which is the organizing principle behind the observed pattern of fermion masses and mixing angles?**
  - **Are there extra sources of flavour symmetry breaking beside the SM Yukawa couplings which are relevant at the TeV scale?**

- **Related important questions are:**
  - **Which is the role of flavor physics in the LHC era?**
  - **Do we expect to understand the (SM and NP) flavor puzzles through the synergy and interplay of flavor physics and the LHC?**

The NP "scale"

- **Gravity** $\Longrightarrow$ $\Lambda_{\text{Planck}} \sim 10^{18-19}\ \text{GeV}$
- **Neutrino masses** $\Longrightarrow$ $\Lambda_{\text{see-saw}} \lesssim 10^{15}\ \text{GeV}$
- **BAU**: evidence of CPV beyond SM
  - Electroweak Baryogenesis $\Longrightarrow$ $\Lambda_{NP} \lesssim \text{TeV}$
  - Leptogenesis $\Longrightarrow$ $\Lambda_{\text{see-saw}} \lesssim 10^{15}\ \text{GeV}$
- **Hierarchy problem**: $\Longrightarrow$ $\Lambda_{NP} \lesssim \text{TeV}$
- **Dark Matter** $\Longrightarrow$ $\Lambda_{NP} \lesssim \text{TeV}$



SM = effective theory at the EW scale

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_{d\geq 5} \frac{c_{ij}^{(d)}}{\Lambda_{NP}^{d-4}}\, O_{ij}^{(d)}$$

- $\mathcal{L}_{\text{eff}}^{d=5} = \frac{y_{\nu}^{ij}}{\Lambda_{\text{see-saw}}} L_i L_j \phi\phi$,
- $\mathcal{L}_{\text{eff}}^{d=6}$ generates FCNC operators

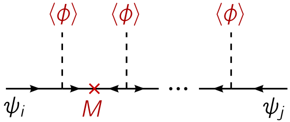
$$\textbf{BR}(\ell_i \to \ell_j \gamma) \sim \frac{1}{\Lambda_{NP}^4}$$

SM vs. NP flavor problems

- Can the SM and NP flavour problems have a common explanation?

- **Froggat-Nielsen '79: Hierarchies from SSB of a Flavour Symmetry**

$$\epsilon = \frac{\langle \phi \rangle}{M} \ll 1 \Rightarrow Y_{ij} \propto \epsilon^{(a_i + b_j)}$$



- **Flavor protection from flavor models:** [Lalak, Pokorski & Ross '10]

Operator	$U(1)$	$U(1)^2$	$SU(3)$	MFV
$(\overline{Q}_L X^Q_{LL} Q_L)_{12}$	λ	λ^5	λ^3	λ^5
$(\overline{D}_R X^D_{RR} D_R)_{12}$	λ	λ^{11}	λ^3	$(y_d y_s) \times \lambda^5$
$(\overline{Q}_L X^D_{LR} D_R)_{12}$	λ^4	λ^9	λ^3	$y_s \times \lambda^5$

- Is the this flavor protection enough?

- Is it possible to disentangle among different flavour models by means of their predicted pattern of deviation w.r.t. the SM predictions in flavour physics?

The New Physics CP problem

- **Why CP violation? Motivation:**
  - Baryogenesis requires extra sources of CPV
  - The QCD $\overline{\theta}$-term $\mathcal{L}_{CP} = \overline{\theta}\frac{\alpha_s}{8\pi}G\tilde{G}$ is a CPV source beyond the CKM
  - Most UV completion of the SM, e.g. the MSSM, have many CPV sources
  - However, TeV scale NP with $\mathcal{O}(1)$ CPV phases generally leads to EDMs many orders of magnitude above the current limits $\Rightarrow$ the New Physics CP problem.

- **How to solve the New Physics CP problem?**
  - Decoupling some NP particles in the loop generating the EDMs (e.g. hierarchical sfermions, split SUSY, 2HDM limit...)
  - Generating CPV phases radiatively $\phi^f_{CP} \sim \alpha_w/4\pi \sim 10^{-3}$
  - Generating CPV phases via small flavour mixing angles $\phi^f_{CP} \sim \delta_{fj}\delta_{fj}$ with $f = e, u, d$: maybe the suppression of FCNC processes and EDMs have a common origin?

NP search strategies

- **High-energy frontier: A unique effort to determine the NP scale**
- **High-intensity frontier (flavor physics): A collective effort to determine the flavor structure of NP**

Where to look for New Physics at the low energy?

- **Processes very suppressed or even forbidden in the SM**
  - FCNC processes ($\mu \to e\gamma$, $\mu \to eee$, $\mu \to e$ in N, $\tau \to \mu\gamma$, $B^0_{s,d} \to \mu^+\mu^-$...)
  - CPV effects in the electron/neutron EDMs, $d_{e,n}$...
  - FCNC & CPV in $B_{s,d}$ & D decay/mixing amplitudes
- **Processes predicted with high precision in the SM**
  - EWPO as $(g-2)_{\mu,e}$: $a_\mu^{exp} - a_\mu^{SM} \approx (3 \pm 1) \times 10^{-9}$, a discrepancy at 3σ!
  - LU in $R_M^{e/\mu} = \Gamma(M \to e\nu)/\Gamma(M \to \mu\nu)$ with $M = \pi, K$

LFV frameworks

- **Neutrino Oscillation** $\Rightarrow m_{\nu_i} \neq m_{\nu_j} \Rightarrow$ **LFV**

- **see-saw**: $m_\nu = \frac{(m_\nu^D)^2}{M_R} \sim eV,\ M_R \sim 10^{14-16} \Rightarrow m_\nu^D \sim m_{top}$

- LFV transitions like $\mu \to e\gamma$ @ 1 loop with exchange of
  - W and ν in the SM framework (GIM) with $\Lambda_{NP} \equiv M_R$

$$Br(\mu \to e\gamma) \sim \frac{m_\nu^{D\,4}}{M_R^4} \leq 10^{-50}$$

  - $\tilde{W}$ and $\tilde{\nu}$ in the MSSM framework (SUPER-GIM) with $\Lambda_{NP} \equiv \tilde{m}$

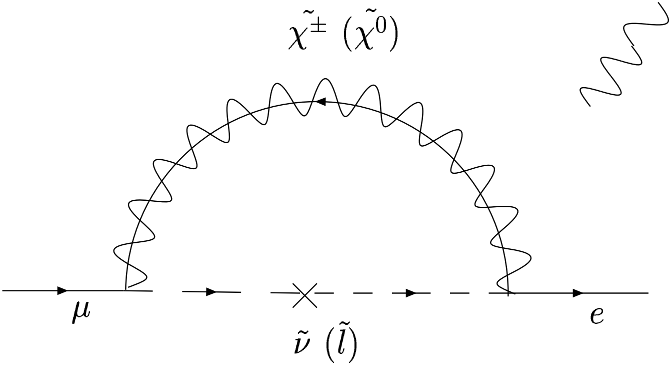
$$Br(\mu \to e\gamma) \sim \frac{m_\nu^{D\,4}}{\tilde{m}^4}$$ [Borzumati & Masiero '86]

$$\Downarrow$$

- LFV signals are undetectable (detectable) in the SM (MSSM)

LFV interactions – leptons/sleptons/gauginos

$$\mathcal{L} = \bar{\ell}_i \left(C^R_{ijA} P_R + C^L_{ijA} P_L \right) \tilde{\chi}^-_A \tilde{\nu}_j + \bar{\ell}_i \left(N^R_{ijA} P_R + N^L_{ijA} P_L \right) \tilde{\chi}^0_A \tilde{\ell}_j$$



$$\frac{BR(\ell_i \to \ell_j \gamma)}{BR(\ell_i \to \ell_j \nu_i \bar{\nu}_j)} \sim \left(\frac{m_W^4}{m_{SUSY}^4} \right) \left(\delta^{21}_{LL} \right)^2 t_\beta^2 \qquad \delta_{LL} \sim \frac{m_\nu^{D\,2}}{m_{SUSY}^2}$$

$\ell \to \ell'\gamma$: model-independent analysis

- **NP effects are encoded in the effective Lagrangian**

$$\mathcal{L} = e\frac{m_\ell}{2}\left(\bar{\ell}_R\sigma_{\mu\nu}A_{\ell\ell'}\ell'_L + \bar{\ell}'_L\sigma_{\mu\nu}A^\star_{\ell\ell'}\ell_R\right)F^{\mu\nu} \qquad \ell,\ell' = e,\mu,\tau\,,$$

$$A_{\ell\ell'} = \frac{1}{(4\pi\,\Lambda_{\rm NP})^2}\left[\left(g^L_{\ell k}\,g^{L*}_{\ell' k} + g^R_{\ell k}\,g^{R*}_{\ell' k}\right)f_1(x_k) + \frac{v}{m_\ell}\left(g^L_{\ell k}\,g^{R*}_{\ell' k}\right)f_2(x_k)\right],$$

  - **Δa_ℓ and leptonic EDMs are given by**

$$\Delta a_\ell = 2m_\ell^2\,{\rm Re}(A_{\ell\ell}), \qquad \frac{d_\ell}{e} = m_\ell\,{\rm Im}(A_{\ell\ell})\,.$$

  - **The branching ratios of $\ell \to \ell'\gamma$ are given by**

$$\frac{{\rm BR}(\ell\to\ell'\gamma)}{{\rm BR}(\ell\to\ell'\nu_\ell\bar{\nu}_{\ell'})} = \frac{48\pi^3\alpha}{G_F^2}\left(|A_{\ell\ell'}|^2 + |A_{\ell'\ell}|^2\right).$$

- **“Naive scaling”:**

$$\Delta a_{\ell_i}/\Delta a_{\ell_j} = m^2_{\ell_i}/m^2_{\ell_j}, \qquad d_{\ell_i}/d_{\ell_j} = m_{\ell_i}/m_{\ell_j}\,.$$

(for instance, if the new particles have an underlying SU(3) flavor symmetry in their mass spectrum and in their couplings to leptons, which is the case for gauge interactions).

[Giudice, P.P., & Passera, '12]

Model-independent predictions

- $(g-2)_\ell$ **assuming "Naive scaling"** $\Delta a_{\ell_i}/\Delta a_{\ell_j} = m^2_{\ell_i}/m^2_{\ell_j}$

$$\Delta a_e = \left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right) 0.7\times 10^{-13}\,, \qquad \Delta a_\tau = \left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right) 0.8\times 10^{-6}.$$

- **EDMs assuming "Naive scaling"** $d_{\ell_i}/d_{\ell_j} = m_{\ell_i}/m_{\ell_j}$

$$\begin{aligned} d_e &\simeq \left(\frac{\Delta a_e}{7\times 10^{-14}}\right) 10^{-24}\,\tan\phi_e\ \ e\,\mathrm{cm}\,,\\ d_\mu &\simeq \left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right) 2\times 10^{-22}\,\tan\phi_\mu\ \ e\,\mathrm{cm}\,,\\ d_\tau &\simeq \left(\frac{\Delta a_\tau}{8\times 10^{-7}}\right) 4\times 10^{-21}\,\tan\phi_\tau\ \ e\,\mathrm{cm}\,, \end{aligned}$$

- $\mathrm{BR}(\ell_i\to\ell_j\gamma)$ **vs.** $(g-2)_\mu$

$$\begin{aligned} \mathrm{BR}(\mu\to e\gamma) &\approx 3\times 10^{-13}\left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right)^2\left(\frac{\theta_{e\mu}}{10^{-5}}\right)^2\,,\\ \mathrm{BR}(\tau\to \ell\gamma) &\approx 4\times 10^{-8}\left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right)^2\left(\frac{\theta_{\ell\tau}}{10^{-2}}\right)^2\,. \end{aligned}$$

[Giudice, P.P., & Passera, '12]

A concrete SUSY scenario: “Disoriented A-terms”

- **Challenge**: Large effects for $g-2$ keeping under control $\mu \to e\gamma$ and d_e
- **“Disoriented A-terms”** [Giudice, Isidori & P.P., '12]:

$$(\delta^{ij}_{LR})_f \sim \frac{A_f \theta^f_{ij} m_{f_j}}{m_{\tilde{f}}} \qquad f = u, d, \ell \,,$$

  - Flavor and CP violation is restricted to the trilinear scalar terms.
  - Flavor bounds of the down-sector are naturally satisfied thanks to the smallness of down-type quark/lepton masses.
  - This ansatz arises in scenarios with partial compositeness where we a natural prediction is $\theta^\ell_{ij} \sim \sqrt{m_i/m_j}$ [Rattazzi et al.,'12].
- $\mu \to e\gamma$ and d_e are generated only by $U(1)$ interactions

$$A_L^{\mu e} \sim \frac{\alpha}{\cos^2\theta_W}\, \delta^{\mu e}_{LR} \,, \qquad \frac{d_e}{e} \sim \frac{\alpha}{\cos^2\theta_W}\, \mathrm{Im}\delta^{ee}_{LR} \,.$$

- $(g-2)_\mu$ is generated by $SU(2)$ interactions and is $\tan\beta$ enhanced therefore the relative enhancement w.r.t. $\mu \to e\gamma$ and d_e is $\tan\beta/\tan^2\theta_W \approx 100 \times (\tan\beta/30)$

$$\Delta a_\ell \sim \frac{\alpha}{\sin^2\theta_W}\, \tan\beta$$

A concrete SUSY scenario: "Disoriented A-terms"

- **Numerical example**: $\tilde{m} = |A_e| = 1\ \mathrm{TeV}$, $\sin\phi_{A_e}$=1, $M_2 = \mu = 2M_1 = 0.2\ \mathrm{TeV}$, and $\tan\beta = 30$ [Giudice, P.P., & Passera, '12]

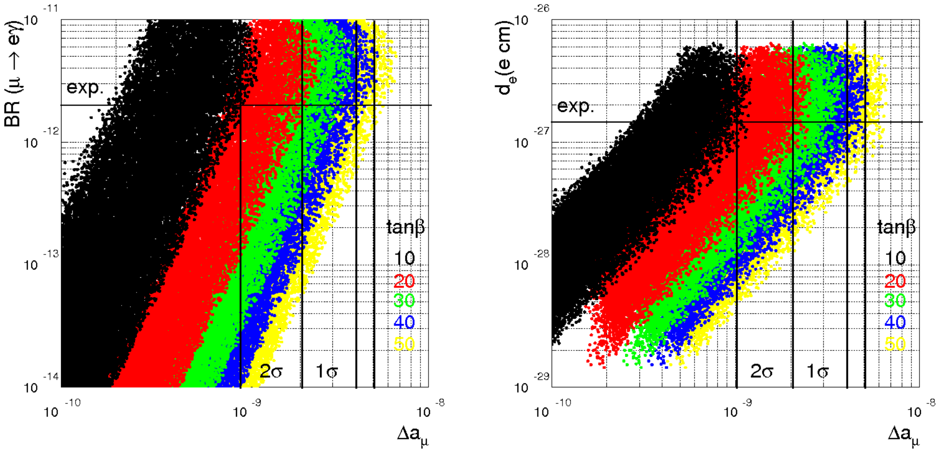
$$\mathrm{BR}(\mu \to e\gamma) \approx 6 \times 10^{-13} \left| \frac{A_\ell}{\mathrm{TeV}} \frac{\theta^\ell_{12}}{\sqrt{m_e/m_\mu}} \right|^2 \left(\frac{\mathrm{TeV}}{m_{\tilde{\ell}}}\right)^4 ,$$

$$d_e \approx 4 \times 10^{-28}\, \mathrm{Im}\left(\frac{A_\ell\, \theta^\ell_{11}}{\mathrm{TeV}}\right) \left(\frac{\mathrm{TeV}}{m_{\tilde{\ell}}}\right)^2 e\, \mathrm{cm} ,$$

$$\Delta a_\mu \approx 1 \times 10^{-9} \left(\frac{\mathrm{TeV}}{m_{\tilde{\ell}}}\right)^2 \left(\frac{\tan\beta}{30}\right) .$$

  - Disoriented A-terms can account for $(g-2)_\mu$, satisfy the bounds on $\mu \to e\gamma$ and d_e, while giving predictions for $\mu \to e\gamma$ and d_e within experimental reach.
  - The electron $(g-2)$ follows "naive scaling".

A concrete SUSY scenario: “Disoriented A-terms”



Predictions for $\mu \to e\gamma$, Δa_μ and d_e in the disoriented A-term scenario with $\theta^{\ell}_{ij} = \sqrt{m_i/m_j}$. Left: $\mu \to e\gamma$ vs. Δa_μ. Right: d_e vs. Δa_μ [Giudice, P.P., & Passera, ’12]

Not only $\mu \to e\gamma$...

- LFV operators up to dimension-six

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda^2_{\rm LFV}} \mathcal{O}^{\rm dim-6} + \ldots .$$

$$\mathcal{O}^{\rm dim-6} \ni \quad \bar{\mu}_R \, \sigma^{\mu\nu} \, H \, e_L \, F_{\mu\nu} \, , \quad (\bar{\mu}_L \gamma^\mu e_L) \left(\bar{f}_L \gamma^\mu f_L\right) \, , \quad (\bar{\mu}_R e_L) \left(\bar{f}_R f_L\right) \, , \; f = e, u, d$$

- the dipole-operator leads to $\ell \to \ell' \gamma$ while 4-fermion operators generate processes like $\mu \to eee$ and $\mu \to e$ conversion in Nuclei.
- When the dipole-operator is dominant:

$$\frac{\mathrm{BR}(\ell_i \to \ell_j \ell_k \bar{\ell}_k)}{\mathrm{BR}(\ell_i \to \ell_j \bar{\nu}_j \nu_i)} \simeq \frac{\alpha_{el}}{3\pi} \left(\log \frac{m^2_{\ell_i}}{m^2_{\ell_k}} - 3 \right) \frac{\mathrm{BR}(\ell_i \to \ell_j \gamma)}{\mathrm{BR}(\ell_i \to \ell_j \bar{\nu}_j \nu_i)} \, ,$$

$$\mathrm{CR}(\mu \to e \text{ in N}) \simeq \alpha_{\rm em} \times \mathrm{BR}(\mu \to e\gamma) \, .$$

- $\mathrm{BR}(\mu \to e\gamma) \sim 10^{-12}$ implies $\mathrm{BR}(\mu \to eee) \leq 0.5 \times 10^{-14}$ and $\mathrm{CR}(\mu \to e \text{ in N}) \leq 0.5 \times 10^{-14}$.
- A combined analysis of $\mu \to e$ conversion on different target nuclei can discriminate among the underlying operators since the sensitivity of different processes to these operators is not the same [Okada et al. 2004].
- For three body LFV decays as $\mu \to eee$, an angular analysis of the signal would be crucial to shed light on the operator which is at work [see the talk by Mannel].

Pattern of LFV in NP models

- **Ratios like $Br(\mu \to e\gamma)/Br(\tau \to \mu\gamma)$ probe the NP flavor structure**
- **Ratios like $Br(\mu \to e\gamma)/Br(\mu \to eee)$ probe the NP operator at work**

ratio	LHT	MSSM	SM4
$\frac{Br(\mu\to eee)}{Br(\mu\to e\gamma)}$	$0.02\ldots 1$	$\sim 2\cdot 10^{-3}$	$0.06\ldots 2.2$
$\frac{Br(\tau\to eee)}{Br(\tau\to e\gamma)}$	$0.04\ldots 0.4$	$\sim 1\cdot 10^{-2}$	$0.07\ldots 2.2$
$\frac{Br(\tau\to \mu\mu\mu)}{Br(\tau\to \mu\gamma)}$	$0.04\ldots 0.4$	$\sim 2\cdot 10^{-3}$	$0.06\ldots 2.2$
$\frac{Br(\tau\to e\mu\mu)}{Br(\tau\to e\gamma)}$	$0.04\ldots 0.3$	$\sim 2\cdot 10^{-3}$	$0.03\ldots 1.3$
$\frac{Br(\tau\to \mu ee)}{Br(\tau\to \mu\gamma)}$	$0.04\ldots 0.3$	$\sim 1\cdot 10^{-2}$	$0.04\ldots 1.4$
$\frac{Br(\tau\to eee)}{Br(\tau\to e\mu\mu)}$	$0.8\ldots 2$	~ 5	$1.5\ldots 2.3$
$\frac{Br(\tau\to \mu\mu\mu)}{Br(\tau\to \mu ee)}$	$0.7\ldots 1.6$	~ 0.2	$1.4\ldots 1.7$
$\frac{\mathrm{R}(\mu\mathrm{Ti}\to e\mathrm{Ti})}{Br(\mu\to e\gamma)}$	$10^{-3}\ldots 10^{2}$	$\sim 5\cdot 10^{-3}$	$10^{-12}\ldots 26$

[Buras et al., '07, '10]

Testing new physics with the electron $g-2$

- **Longstanding muon $g-2$ anomaly**

$$\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM} = 2.90(90)\times 10^{-9}\,, \qquad \textbf{3.5}\sigma\ \textbf{discrepancy}$$

- **NP effects are expected to be of order $a_\ell^{\rm NP}\sim a_\ell^{\rm EW}$**

$$a_\mu^{\rm EW} = \frac{m_\mu^2}{(4\pi v)^2}\left(1-\frac{4}{3}\sin^2\theta_{\rm W}+\frac{8}{3}\sin^4\theta_{\rm W}\right)\approx 2\times 10^{-9}.$$

- **Main question: which is the most convincing way to establish the origin of the a_μ discrepancy?**

- **Answer: testing new-physics effects in a_e** [Giudice, P.P, & Passera, '12]

  - a_e has never played a role in testing ideas beyond the SM. In fact, it is believed that new-physics contaminations of a_e are too small to be relevant and, with this assumption, the measurement of a_e is employed to determine the value of the fine-structure constant α.

  - The situation has now changed, thanks to advancements both on the theoretical and experimental sides.

- QED contribution [Kinoshita & Marciano, in *Quantum Electrodynamics* (1990)]

$$a_e^{\mathrm{QED}} = A_1 + A_2\left(\frac{m_e}{m_\mu}\right) + A_2\left(\frac{m_e}{m_\tau}\right) + A_3\left(\frac{m_e}{m_\mu}, \frac{m_e}{m_\tau}\right),$$

$$A_i = A_i^{(2)}(\alpha/\pi) + A_i^{(4)}(\alpha/\pi)^2 + A_i^{(6)}(\alpha/\pi)^3 + \cdots .$$

  - QED @ 1 loop [Schwinger, Phys. Rev. **73** (1948)]

$$C_1 = A_1^{(2)} = 1/2\,,$$

  - QED @ 2 loop [Sommerfield, Phys. Rev. **107** (1957); A. Petermann, Nucl. Phys. **5** (1958) 677.]

$$C_2 = A_1^{(4)} + A_2^{(4)}(m_e/m_\mu) + A_2^{(4)}(m_e/m_\tau) = -0.328\,478\,444\,002\,55\,(33).$$

  - QED @ 3 loop [Laporta & Remiddi, PLB **301** (1993), PLB **379** (1996)]

$$C_3 = 1.181\,234\,016\,816\,(11)\,, \qquad \delta a_e^{\mathrm{QED}} \sim 10^{-19}$$

  - QED @ 4 loop [Kinoshita and collaborators, PRL **99** (2007); PRD **77** (2008)]

$$C_4 = -1.9097\,(20)\,, \qquad \delta a_e^{\mathrm{QED}} \sim 5.8 \times 10^{-14}$$

  - QED @ 5 loop [Kinoshita and collaborators, 2012]

$$C_5 = 9.16\,(58) \qquad \delta a_e^{\mathrm{QED}} \sim 3.9 \times 10^{-14}$$

- **Electroweak contribution** [Czarnecki, Krause and Marciano, PRL **76** (1996)]

$$a_e^{\mathrm{EW}} = 0.3854\,(42) \times 10^{-13}.$$

- **Hadronic contribution** [Jegerlehner & and Nyffeler, Phys. Rept. **477** (2009), Nomura & Teubner, '12]

$$a_e^{\mathrm{HAD}} = 16.82\,(16) \times 10^{-13},$$

- **Standard Model prediction of a_e and value of α**

$$a_e^{\mathrm{SM}}(\alpha) = a_e^{\mathrm{QED}}(\alpha) + a_e^{\mathrm{EW}} + a_e^{\mathrm{HAD}}$$

- **Experimental situation** [Gabrielse & collaborators, PRL **100** (2008), PRL **97** (2006), PRA **83** (2011)]

$$a_e^{\mathrm{EXP}} = 115\,965\,218\,07.3\,(2.8) \times 10^{-13}$$

- **Extracting α from $a_e^{\mathrm{SM}}(\alpha) = a_e^{\mathrm{EXP}}$**

$$\alpha\,(g{-}2) = 1/137.035\,999\,174\,(34)\;[0.25\,\mathrm{ppb}],$$

This is the most precise value of α available today!

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The Standard Model prediction of the electron $g-2$

- **Second best determination of α from atomic physics**

$$\alpha\,(^{87}\mathrm{Rb}) = 1/137.035\,999\,049\,(90)\;[0.66\,\mathrm{ppb}].$$

  - $\alpha\,(^{87}\mathrm{Rb})$ is deduced from the ratios h/M_{Rb} where $M_{\mathrm{Cs,Rb}}$ is from the mass ratios $M_{\mathrm{Cs,Rb}}/m_e$ [CODATA 2010].
  - The experimental scheme combines atom interferometry with Bloch oscillation [Cladé *et al.*, PRL **96** (2006), Cadoret *et al.*, PRL **101** (2008), Bouchendira *et al.*, PRL **106** (2011)].
  - $\alpha(^{87}\mathrm{Rb})$ agrees with $\alpha(g-2)$ at the 1.3σ level, and its uncertainty $\delta\alpha(^{87}\mathrm{Rb})$ is larger than $\delta\alpha(g-2)$ just by a factor of 2.7.

- **Determination of $a_e^{\mathrm{SM}}(\alpha)$ from $\alpha(^{87}\mathrm{Rb})$**

$$a_e^{\mathrm{SM}} = 115\,965\,218\,17.9\,(0.6)(0.4)(0.2)(7.6)\times 10^{-13}.$$

  - The first (second) error is from four(five)-loop QED coefficient, the third one is $\delta a_e^{\mathrm{HAD}}$, and the last ($7.60\times 10^{-13}$) from $\delta\alpha(^{87}\mathrm{Rb})$.
  - The uncertainties of the EW and two/three-loop QED contributions are negligible.
  - $\delta a_e^{\mathrm{SM}} = 7.64\times 10^{-13}$ is about three times worse than $\delta a_e^{\mathrm{exp}}$ almost due to the uncertainty of the fine-structure constant $\alpha(^{87}\mathrm{Rb})$.

The Standard Model prediction of the electron $g-2$

- **Standard Model vs. measurement**

$$\Delta a_e = a_e^{\mathrm{EXP}} - a_e^{\mathrm{SM}} = -10.6\,(8.1) \times 10^{-13},$$

  - Beautiful test of QED at four-loop level!
  - $\delta\Delta a_e = 8.1 \times 10^{-13}$ is dominated by δa_e^{SM} through $\delta\alpha(^{87}\mathrm{Rb})$.

- **Future improvements in the determination of Δa_e**

$$\underbrace{(0.6)_{\mathrm{QED4}},\ \ (0.4)_{\mathrm{QED5}},\ \ \ (0.2)_{\mathrm{HAD}},}_{(0.7)_{\mathrm{TH}}}\ (7.6)_{\delta\alpha},\ \ (2.8)_{\delta a_e^{\mathrm{EXP}}}. \qquad (1)$$

  - The first error, 0.6×10^{-13}, stems from numerical uncertainties in the four-loop QED. It can be reduced to 0.1×10^{-13} with a large scale numerical recalculation [Kinoshita]
  - The second error, from five-loop QED term may soon drop to 0.1×10^{-13}.
  - Experimental uncertainties 2.8×10^{-13} ($\delta a_e^{\mathrm{EXP}}$) and 7.6×10^{-13} ($\delta\alpha$) dominate. We expect a reduction of the former error to a part in 10^{-13} (or better) [Gabrielse]. Work is also in progress for a significant reduction of the latter error [Nez].

- **Δa_e at the 10^{-13} (or below) is not too far! This will bring a_e to play a pivotal role in probing new physics in the leptonic sector.**

Supersymmetry and a_e

- SUSY contributions to a_ℓ comes from loops with exchange of chargino/sneutrino or neutralino/charged slepton.

- **Violations of "naive scaling"** can arise through sources of non-universalities in the slepton mass matrices in two possible ways

  - **Lepton flavor conserving (LFC) case**. The charged slepton mass matrix violates the global non-abelian flavor symmetry, but preserves $U(1)^3$. This case is characterized by non-degenerate sleptons ($m_{\tilde{e}} \neq m_{\tilde{\mu}} \neq m_{\tilde{\tau}}$) but vanishing mixing angles because of an exact alignment, which ensures that Yukawa couplings and the slepton mass matrix can be simultaneously diagonalized in the same basis.

  - **Lepton flavor violating (LFV) case**. The slepton mass matrix fully breaks flavor symmetry up to U(1) lepton number, generating mixing angles that allow for flavor transitions. Lepton flavor violating processes, such as $\mu \to e\gamma$, provide stringent constraints on this case. However, because of flavor transitions, a_e and a_μ can receive new large contributions proportional to m_τ (from a chiral flip in the internal line of the loop diagram), giving a new source of non-naive scaling.

Lepton flavor conserving case

- In the LFC case, we assume $m_{\tilde{e}} \neq m_{\tilde{\mu}} \neq m_{\tilde{\tau}}$ but flavor alignment between lepton and slepton mass matrices to avoid LFV. This is reminiscent of the alignment mechanism [Nir & Seiberg, '93], proposed to solve the supersymmetric flavor problem in the quark sector (which might arise naturally in the context of abelian flavor models).

$$\Delta a_\ell^{\rm LFC} \approx 3 \times 10^{-9} \left(\frac{m_\ell}{m_\mu}\right)^2 \left(\frac{100\ \rm GeV}{m_{\tilde{\ell}}}\right)^2 \left(\frac{\tan\beta}{3}\right) .$$

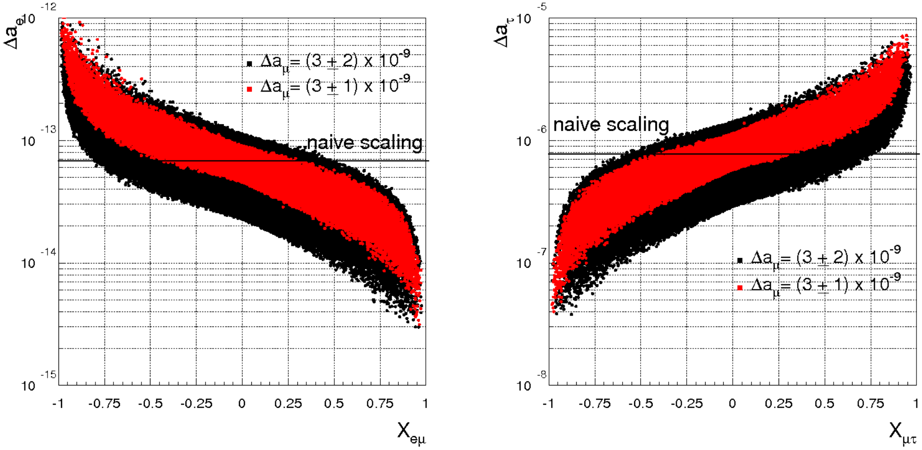
- Assuming that sleptons are the heaviest particles running in the loop

$$\Delta a_e \approx \Delta a_\mu \frac{m_e^2}{m_\mu^2} \frac{m_{\tilde{\mu}}^2}{m_{\tilde{e}}^2} \approx \frac{m_{\tilde{\mu}}^2}{m_{\tilde{e}}^2} \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 10^{-13} ,$$

$$\Delta a_\tau \approx \Delta a_\mu \frac{m_\tau^2}{m_\mu^2} \frac{m_{\tilde{\mu}}^2}{m_{\tilde{\tau}}^2} \approx \frac{m_{\tilde{\mu}}^2}{m_{\tilde{\tau}}^2} \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 10^{-6} .$$

- For values of Δa_μ explaining the muon $g-2$, non-degenerate sleptons at the level $m_{\tilde{\mu}} \approx 3\, m_{\tilde{e}}$ lead to $\Delta a_e \approx 10^{-12}$, which is at the limit of present experimental sensitivity.

Lepton flavor conserving case



Left: Δa_e as a function of $X_{e\mu} = (m^2_{\tilde{e}} - m^2_{\tilde{\mu}})/(m^2_{\tilde{e}} + m^2_{\tilde{\mu}})$. Right: Δa_τ as a function of $X_{\mu\tau} = (m^2_{\tilde{\mu}} - m^2_{\tilde{\tau}})/(m^2_{\tilde{\mu}} + m^2_{\tilde{\tau}})$. Black points satisfy the condition $1 \leq \Delta a_\mu \times 10^9 \leq 5$, while red points correspond to $2 \leq \Delta a_\mu \times 10^9 \leq 4$.

Correlation between a_e and violation of lepton universality in LFC

- In SUSY, “naive scaling” violations for $(g-2)_\ell$ can arise through sources of non-universalities in the slepton masses.
- In turn, these non-universalities will induce violations of lepton flavor universality in $P \to \ell\nu$, $\tau \to P\nu$ (where $P = \pi, K$), $\ell_i \to \ell_j \bar{\nu}\nu$, $Z \to \ell\ell$ and $W \to \ell\nu$ through loop effects.
- LFU has been tested at the 0.1% level so far.
- It is interesting to study the **correlation** between such **LFU** and departures from **“naive scaling”** for Δa_ℓ.
- Taking for example the process $P \to \ell\nu$, we can define the quantity

$$\frac{(R_P^{e/\mu})_{\rm EXP}}{(R_P^{e/\mu})_{\rm SM}} = 1 + \Delta r_P^{e/\mu} \; .$$

  - $R_P^{e/\mu} = \Gamma(P \to e\nu)/\Gamma(P \to \mu\nu)$
  - $\Delta r_P^{e/\mu} \neq 0$ signals the presence of new physics violating LFU.

Correlation between a_e and violation of lepton universality in LFC

- In SUSY, in the absence of LFV sources, $\Delta r_P^{e/\mu}$ is induced at the loop level through sparticle exchange. The parametrical structure of $\Delta r_{\mathrm{P}}^{e/\mu}$ is

$$\Delta r_{\mathrm{P}}^{e/\mu} \sim \frac{\alpha}{4\pi}\left(\frac{m_{\tilde{e}}^2 - m_{\tilde{\mu}}^2}{m_{\tilde{e}}^2 + m_{\tilde{\mu}}^2}\right)\frac{v^2}{\min(m_{\tilde{e},\tilde{\mu}}^2)}\,,$$

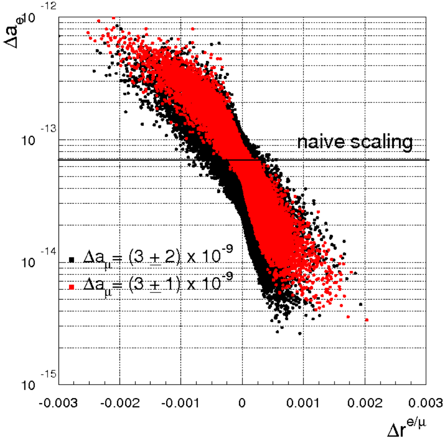
- The term $v^2/\min(m_{\tilde{e},\tilde{\mu}}^2)$ stems from SU(2) breaking effects which arise from 1) left-right soft breaking terms, 2) mixing terms in the chargino/neutralino mass matrices, or 3) D-terms.

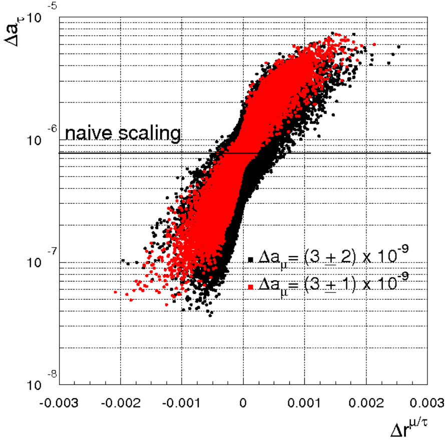
- **"naive scaling"** violations for Δa_ℓ

$$\Delta a_e \approx \Delta a_\mu \, \frac{m_e^2}{m_\mu^2}\frac{m_{\tilde{\mu}}^2}{m_{\tilde{e}}^2} \approx \frac{m_{\tilde{\mu}}^2}{m_{\tilde{e}}^2}\left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right)10^{-13}\,,$$

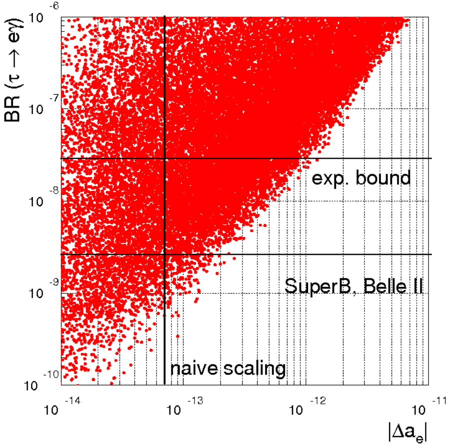
- For values of $\Delta a_\mu \sim few \times 10^{-9}$ (explaining the muon $g-2$ anomaly), non-degenerate sleptons at the level $m_{\tilde{\mu}} \approx 3\, m_{\tilde{e}}$ lead to $\Delta a_e \approx 10^{-12}$, ($\Delta a_e \approx 10^{-13}$ in "naive scaling") and $\Delta r_{\mathrm{P}}^{e/\mu} \approx 10^{-3}$.

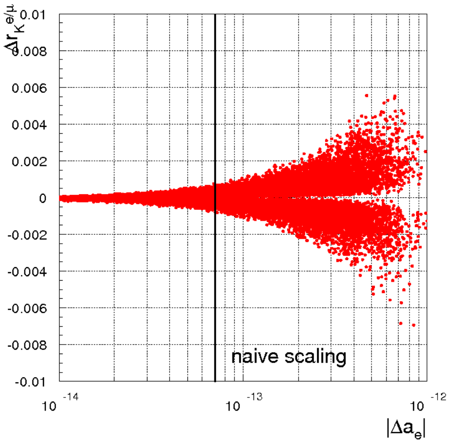
Lepton flavor conserving case





Left: $\Delta r_{\rm P}^{e/\mu}$ vs. Δa_e, where $\Delta r_{\rm P}^{e/\mu}$ measures violations of lepton universality in $\Gamma(P \to e\nu)/\Gamma(P \to \mu\nu)$ with $P = K, \pi$. Right: $\Delta r_{\rm P}^{\mu/\tau}$ vs. Δa_τ where $\Delta r_{\rm P}^{\mu/\tau}$ measures violations of lepton universality in $\Gamma(P \to \mu\nu)/\Gamma(\tau \to P\nu)$.





Left: $\mathrm{BR}(\tau \to e\gamma)$ vs. $|\Delta a_e|$. Right: $\Delta r_K^{e/\mu}$ vs. $|\Delta a_e|$. The vertical line corresponds to the prediction for Δa_e assuming NS, setting Δa_μ equal to its central value $\Delta a_\mu = 3 \times 10^{-9}$.

Conclusions and future prospects

- **Important questions in view of ongoing/future experiments are**:
  - What are the expected deviations from the SM predictions induced by TeV NP?
  - Which observables are not limited by theoretical uncertainties?
  - In which case we can expect a substantial improvement on the experimental side?
  - What will the measurements teach us if deviations from the SM are [not] seen?

- **(Personal) answers**:
  - The expected deviations from the SM predictions induced by NP at the TeV scale with generic flavor structure are already ruled out by many orders of magnitudes.
  - On general grounds, we can expect any size of deviation below the current bounds.
  - cLFV processes, leptonic EDMs and LFU observables do not suffer from theoretical limitations (clean th. observables).
  - On the experimental side there are still excellent prospects of improvements in several clean channels especially in the leptonic sector: $\mu \to e\gamma$, $\mu N \to eN$, $\mu \to eee$, τ-LFV, EDMs and leptonic $(g-2)$.
  - The the origin of the $(g-2)_\mu$ discrepancy can be understood testing new-physics effects in the electron $(g-2)_e$. This would require improved measurements of $(g-2)_e$ and more refined determinations of α in atomic-physics experiments.

Conclusions

The origin of flavour is still, to a large extent, a mystery. The most important open questions can be summarized as follow:

- **Which is the organizing principle behind the observed pattern of fermion masses and mixing angles?**

- **Are there extra sources of flavour symmetry breaking beside the SM Yukawa couplings which are relevant at the TeV scale?**

Irrespectively of whether the LHC will discover or not new particles, flavor physics in the leptonic sector (especially cLFV, leptonic $g-2$ and EDMs) will teach us a lot...