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# Experimental results on the thermal noise of oscillators in non equilibrium steady states

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http:www.rarenoise.lnl.infn.it

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INFN Istituto Nazionale di Fisica Nucleare

# *Non equilibrium*

*The spontaneous length fluctuations of a rod (fixed temperature), following the dissipation fluctuation theorem, have rms known and follow gaussian distributions*


….it is not known the behavior of the rod length fluctuation in case of heat flux: rms? distribution?

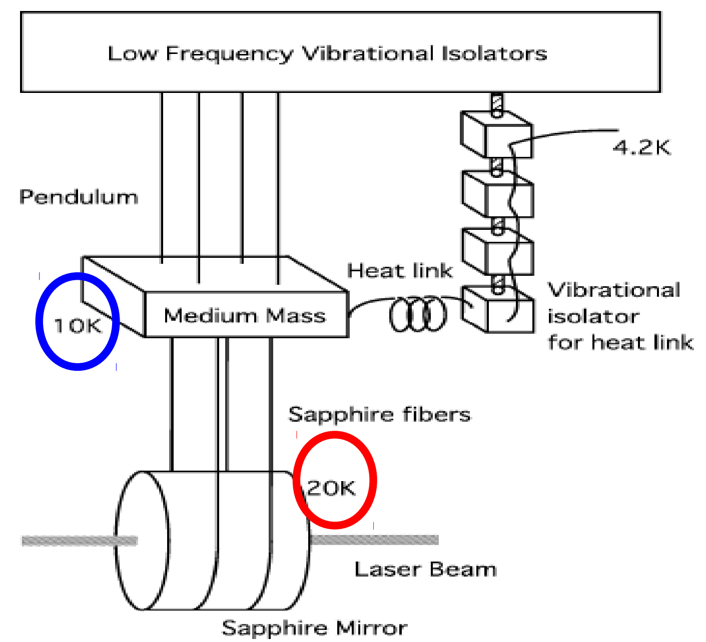

GW experiments are modeled as systems in thermodynamic equilibrium: and the thermal noise is expected have a gaussian distribution, is it correct?

approach: reproduce configurations that drive GW detectors out of equilibrium on smaller-scale

⇓

experimental and numerical studies

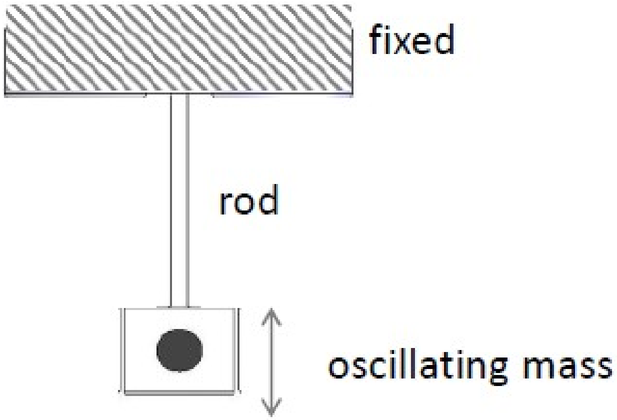
[RareNoise project]




Thermal gradient due to dissipated laser power

# RareNoise project

The aim is to observe the “thermal noise” of a mechanical oscillators, in the different condition w/o thermal gradient. Capacitive readout measures vibrations of oscillator mass.



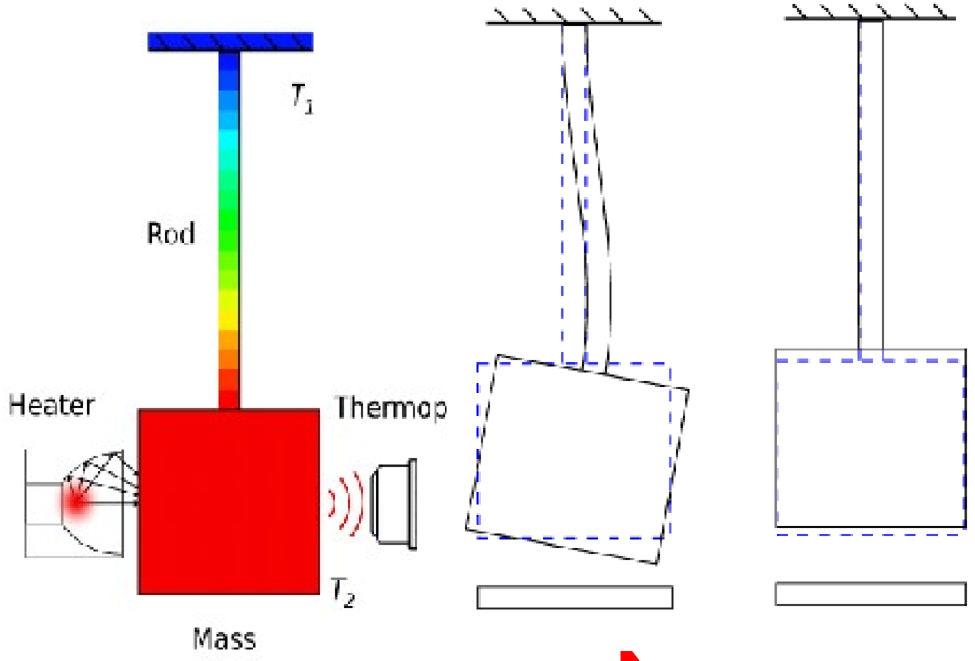

The mechanical resonators fluctuations are studied under different conditions:

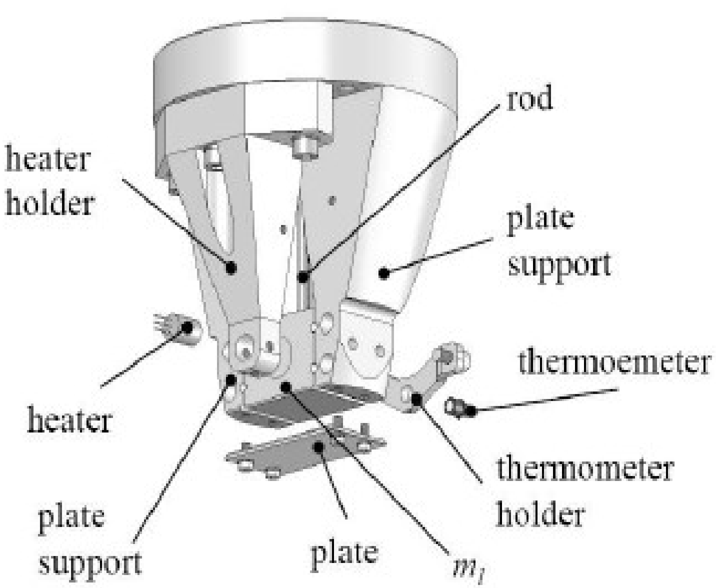
- ✗ many thermal gradients
- ✗ different materials for the ‘rod’ (aluminum, silicon)
- ✗ temperature around 300K, 77K and 4K

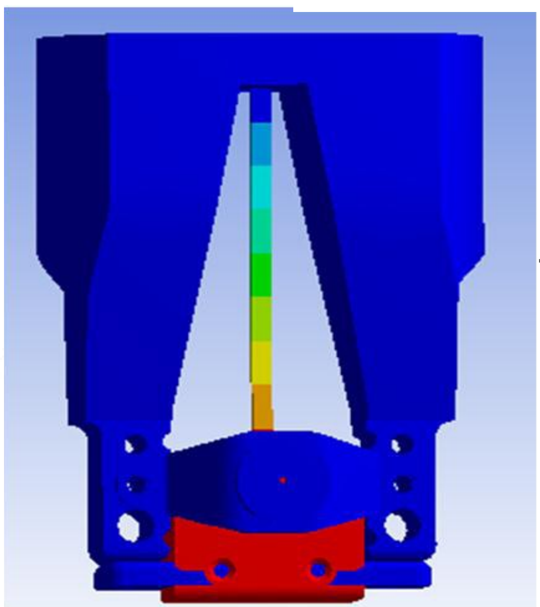
# *Experimental setup (1)*

*oscillator of Al5056:*

- ✗ *oscillator machined from a single piece of Al*
- ✗ *length 0.1 m, mass around 0.22 kg*
- ✗ *temperatures measured T1 and T2*
- ✗ *temperature T1 (top end) actively stabilized*
- ✗ *infrared heater set $\Delta T$, $\Delta T_{max}$ 15K*

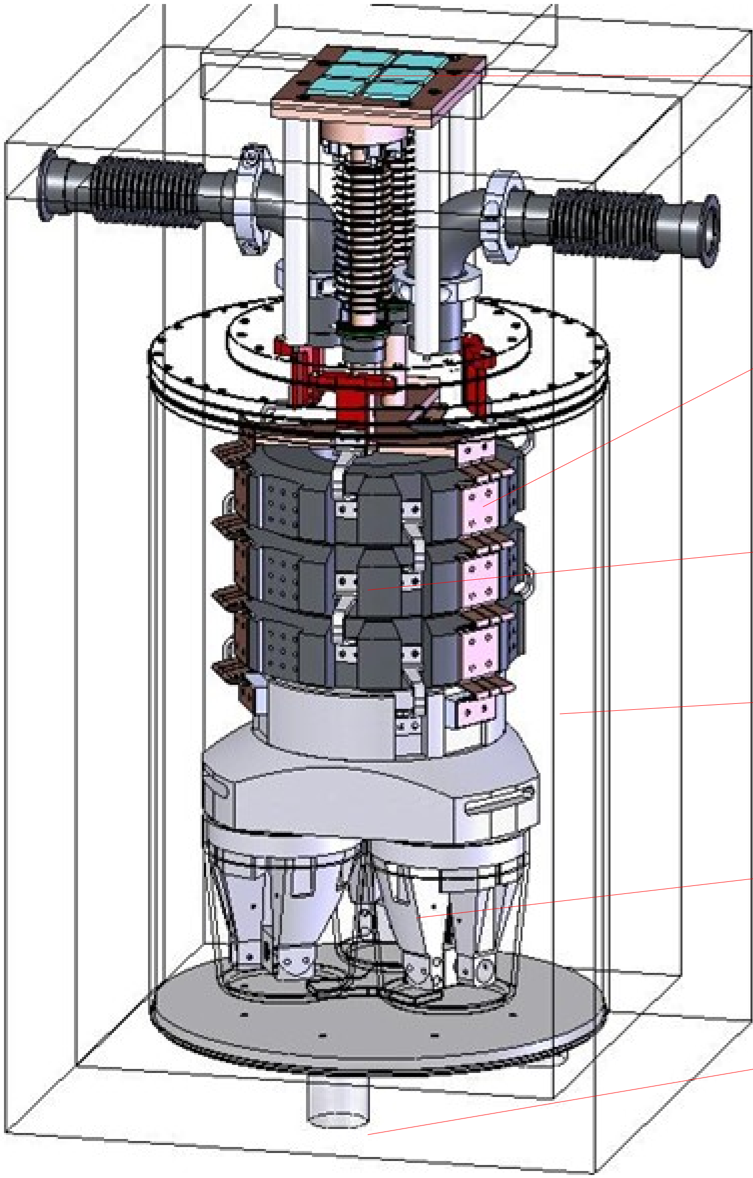


Complete dynamical and structural analysis of the part by FEM:
Transverse mode ~ 300 Hz
Longitudinal mode about ~1.4 kHz

# *Experimental setup (2)*



Peltier cells for controlling oscillator base temperature

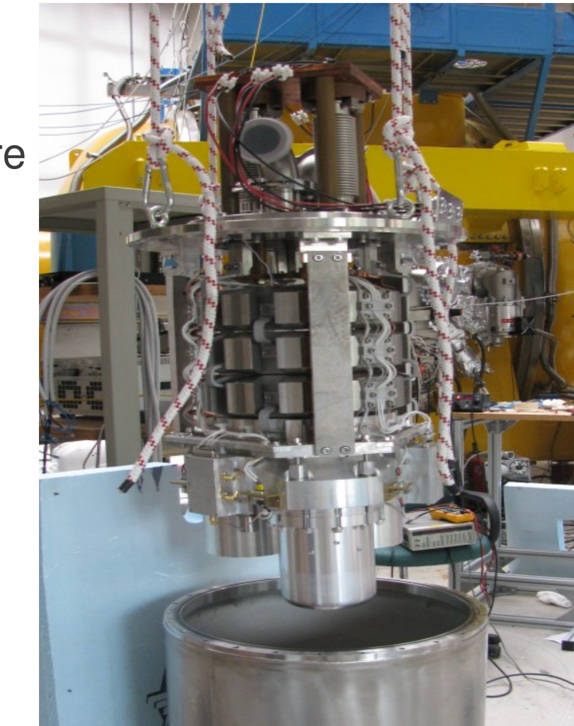
copper strips for conductive heat path

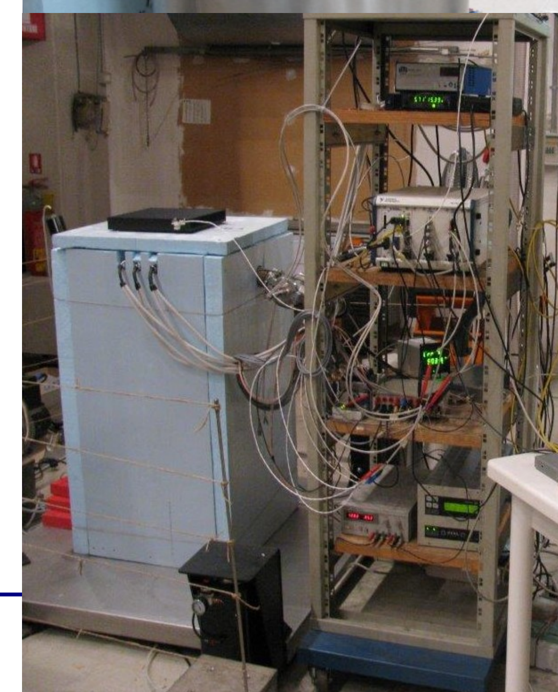
4 stages mechanical suspension

vacuum chamber

oscillator (Al5056)

passive thermal insulator box



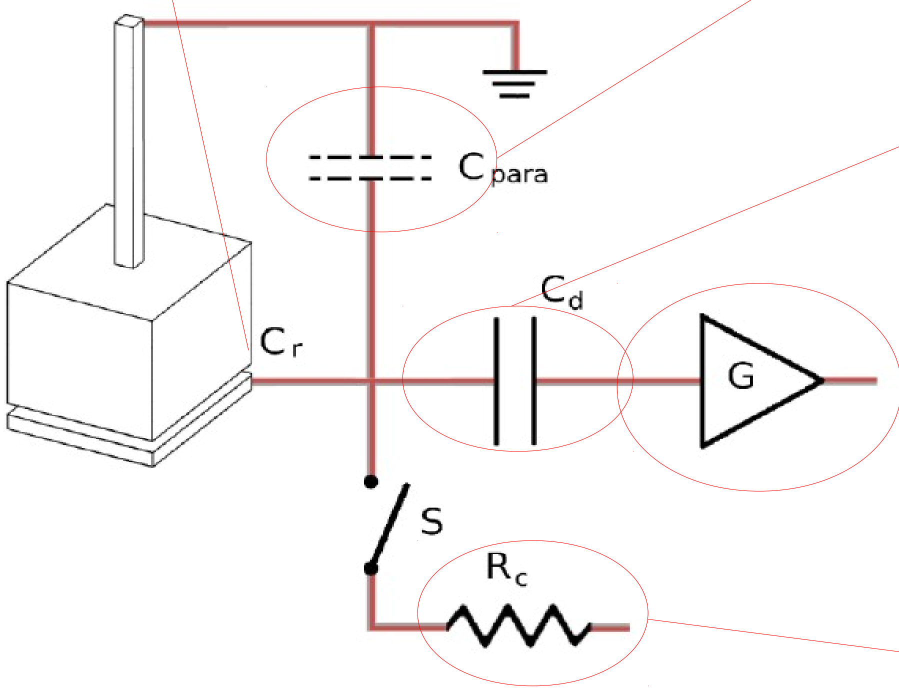


# *Capacitive readout*

Parallel plate capacitor formed by the bottom surface of the mass and a fixed plate:

$C_r \sim 160$ pF

$C_{para}$: static capacitances in parallel to $C_r$ ~160 pF




The signal from the overall parallel capacitor is sent to an amplifier G through the $C_d$ decoupling capacitor:

$C_d \sim 100$ nF

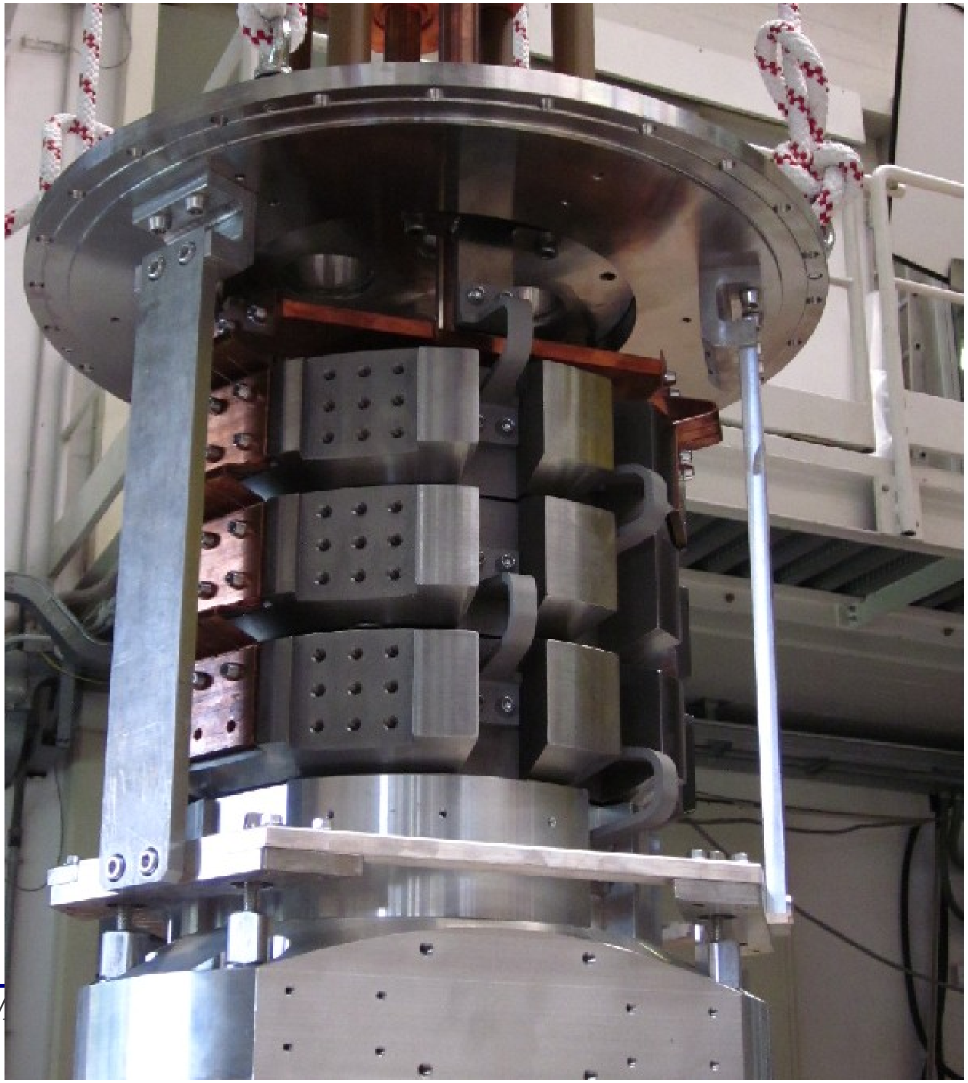
Amplifier gain G=100

Capacitor $C_r$ is biased to a constant dc voltage through a large resistor.

# *Mechanical suspension*

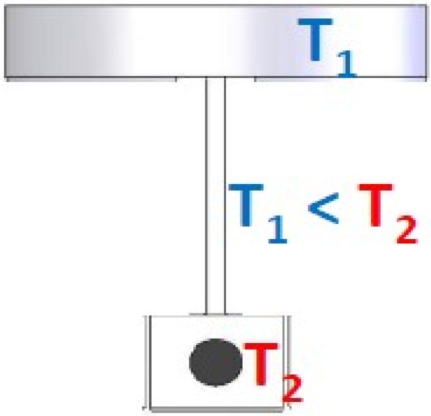
To reduce mechanical noise in the setup active and passive filters have been used:

- ✗ active filter is provided by an air suspended platform which supports all the experimental apparatus.
- ✗ passive suspension consists of a cascade of 4 mechanical filters effective in all directions and housed inside the vacuum chamber.



Overall we estimate to achieve vibration isolation of more than 200dB at 1.5kHz in all spatial directions.

# *Temperature control*




The temperature T1 of rod top end is stabilized with feedback loop. Temperature T1 can be set within a 20 K interval around room temperature.

The oscillating mass is radiatively, raising T2 up to a thermal difference T2-T1 of 15 K (flowing about 1W power). T2 is measured using contact-less thermopiles.

$T_{avg}$ = (T1+T2)/2 for longitudinal mode

Non Equilibrium Steady State (NESS) condition: T2>T1

The steady state condition is:

$\Downarrow$

Temperature stability of < 9 μK/s

$$\sqrt{\left(\frac{dT_1}{T_1 dt}\right)^2 + \left(\frac{dT_2}{T_2 dt}\right)^2} < 6 * 10^{-8} s^{-1}$$

# *Data acquisition*

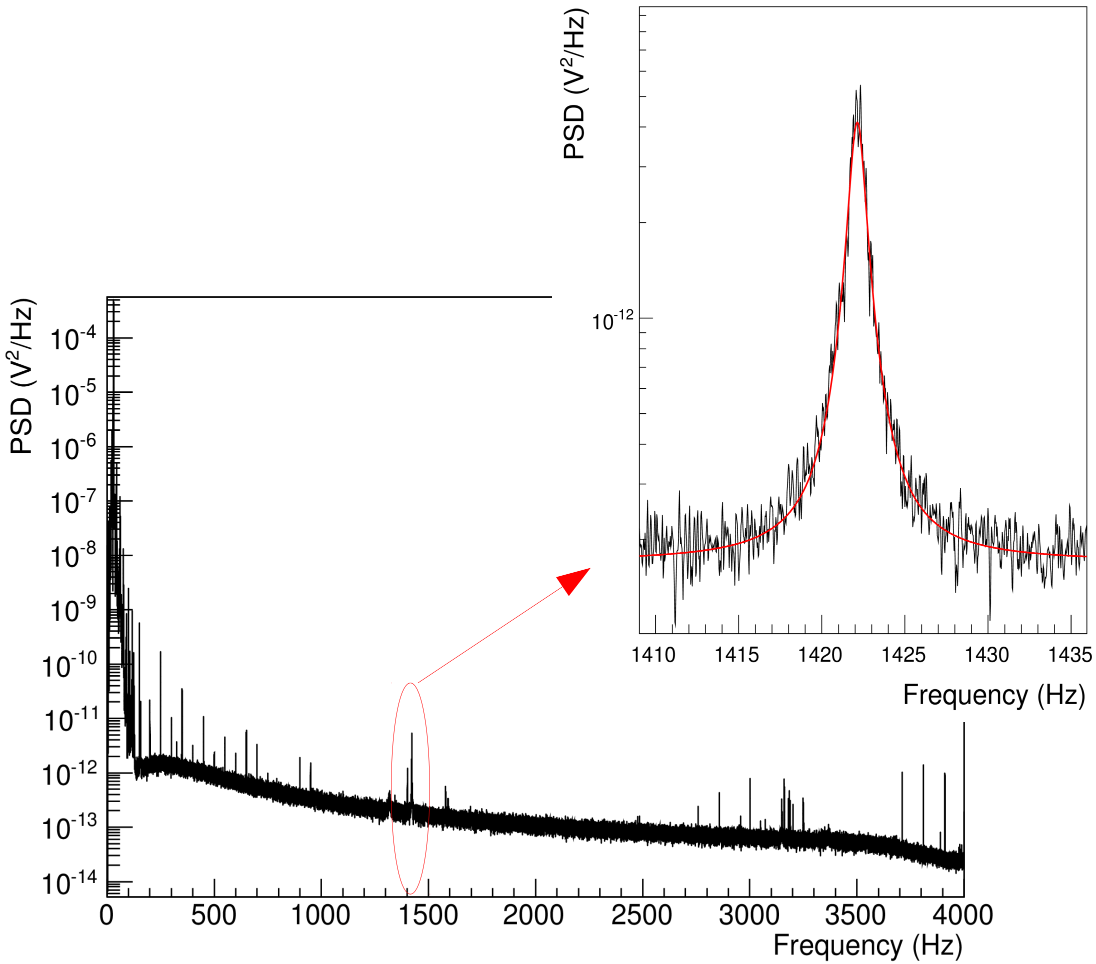
Data taking: November 2011 - May 2012:
✗ oscillator amplifier output →sampling frequency 8kHz
✗ auxiliary channel: temperatures, voltage, time: 27 channels →sampling frequency 0.1Hz

Different data taking condition:
✔electric field applied (dc bias)
✔temperature w/o gradient

average PSD

*Longitudinal mode: 50 non overlapping buffer, frequency resolution 0.03Hz, time average 27 min*

*Transversal mode: 25 non overlapping buffer, frequency resolution 0.0076 Hz, time average 54 min*



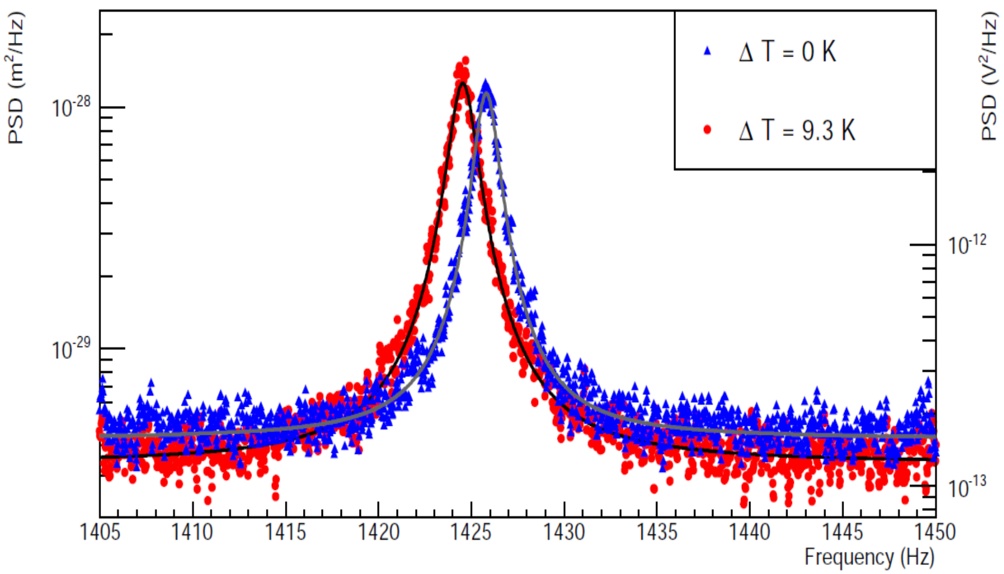
# *Spectral analysis of longitudinal mode*

The output signal can be converted into longitudinal vibrations of the mass.

*The resonant peak in the PSD is fitted by a Lorentzian curve plus a constant (accounting for the electronic noise)*

$$y(f)=\frac{p_1 p_2}{(f-p_0)^2+p_1^2}+p3$$

$p_0$ resonant frequency, $p_1$ FWHM,
$p_2$ Lorentzian peak area, $p_3$ noise level




PSD in equilibrium (blue triangles) and in NESS (red circles). Estimated:
$T_{eff}$ EQ = 319 ± 5(stat) ± 18(syst) K
$T_{eff}$ NESS = 402 ± 6(stat) ± 18(syst) K

the area of the fitting curve becomes an estimate of the mean square vibration of the oscillator: $x(t)^2$. At thermodynamic equilibrium, following the equipartition law:

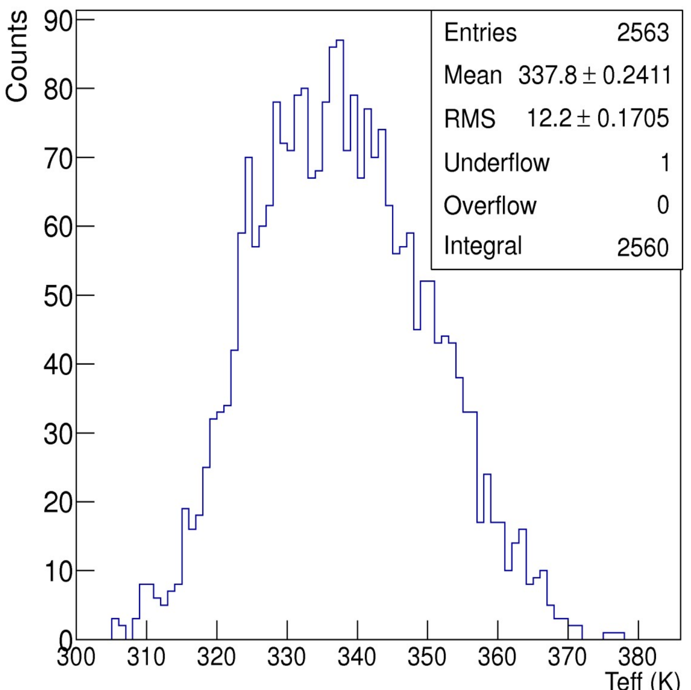
$$\langle x_l(t)^2 \rangle = \frac{k_b T}{m_l \omega_l^2}$$

# Data selection

- ✗ Relative error in the estimate of the resonant frequency: $\sigma_{p0}/p_0 < 10^{-5}$ (longitudinal mode) and $\sigma_{p0}/p_0 < 2\times10^{-5}$ (transverse mode).
- ✗ Relative error in the estimate of the Lorentzian curve area: $\sigma_{p2}/p_2 < 0.5$
- ✗ Area of the payload A limited
- ✗ steady state: limit to the maximum total derivative of T1 and T2 during the averaging time of the PSD. For T=300K this corresponds to maximum time derivative of 9 μK/s

The total data considered, after cuts, correspond to a 85 days for the longitudinal mode (4412) and 61 days for the transversal mode (1599).

The analysis takes into account the discharge of the capacitor readout.




$T_{eff}$ EQ distribution (linear dependence with thermodynamic temperature not visible due to too limited range variation):

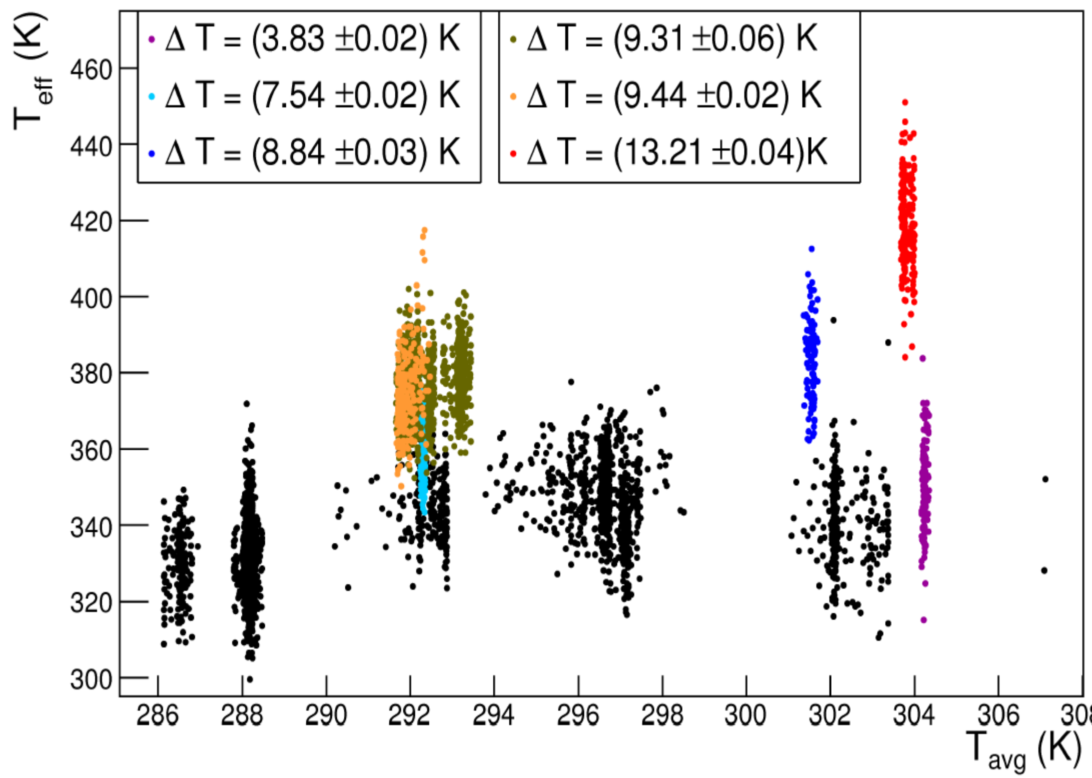
$$T_{eff}EQ = 338 \pm 12(stat) \pm 18(syst)\ K$$

# *Effective temperature*

<u>Longitudinal mode</u>

Effective temperature of the first longitudinal acoustic mode against mode average temperature:

- ✗ black point: equilibrium
- ✗ colored point: NESS with different gradient as shown




At the thermodynamic equilibrium, due to equipartition:

*$T_{eff}$ it is a good "thermometer" of the thermodynamic temperature*

In NESS $T_{eff}$ depends not only on $T_{avg}$ but also on $\Delta T$

⇓

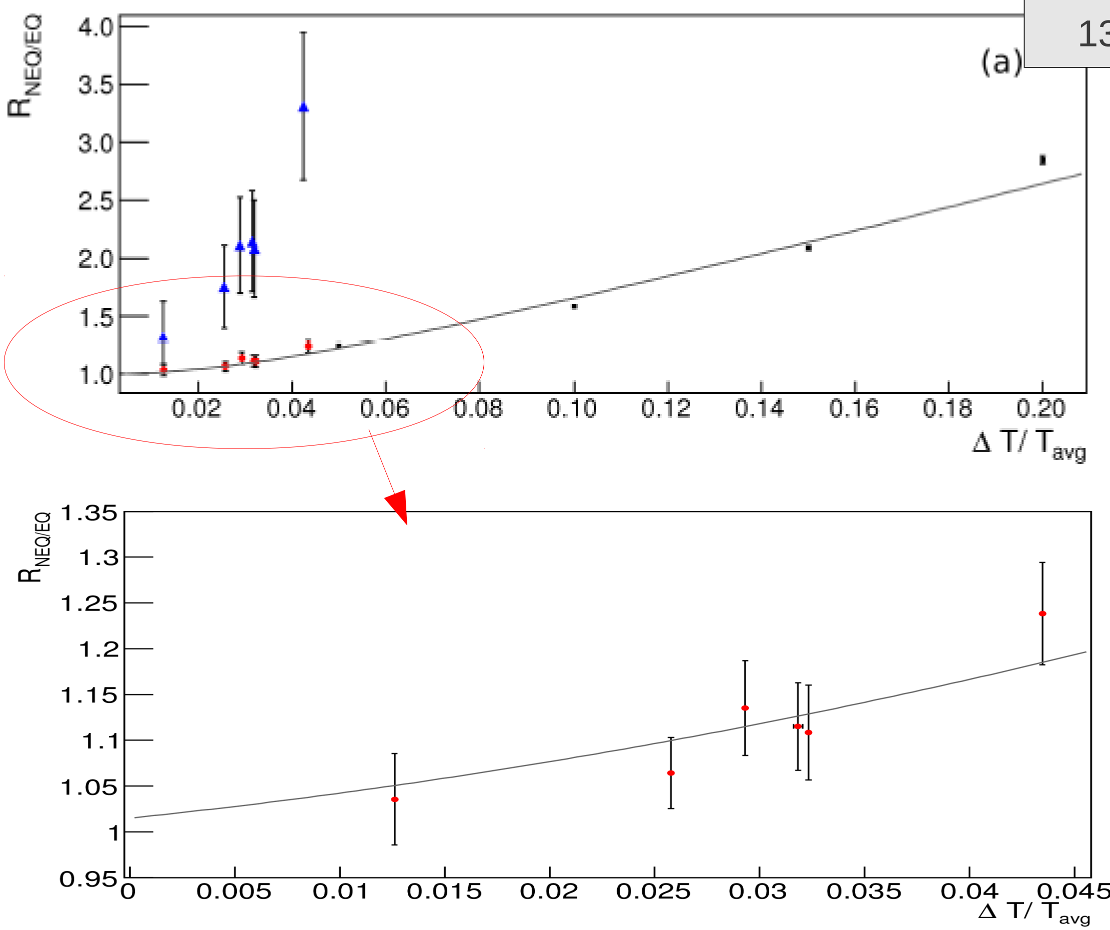
*$T_{eff}$ is not longer a valid measure of thermodynamic temperature*

# Equilibrium vs NESS

*ratio $R_{NEQ/EQ}$ of the effective temperatures in the NESS over equilibrium*

$$R_{NEQ/EQ} = \frac{T_{eff}\,NESS}{T_{eff}\,EQ}$$

| $\Delta T$ | $R_{NEQ}/R_{EQ}$ |
|---|---|
| 3.83±0.02(stat)±0.2(syst) | 1.04±0.050(stat)±0.013(syst) |
| 7.54±0.02(stat)±0.2(syst) | 1.06±0.039(stat)±0.015(syst) |
| 8.84±0.03(stat)±0.2(syst) | 1.13±0.052(stat)±0.017(syst) |
| 9.31±0.06(stat)±0.2(syst) | 1.12±0.048(stat)±0.018(syst) |
| 9.44±0.02(stat)±0.2(syst) | 1.11±0.051(stat)±0.018(syst) |
| 13.21±0.04(stat)±0.2(syst) | 1.24±0.055(stat)±0.025(syst) |

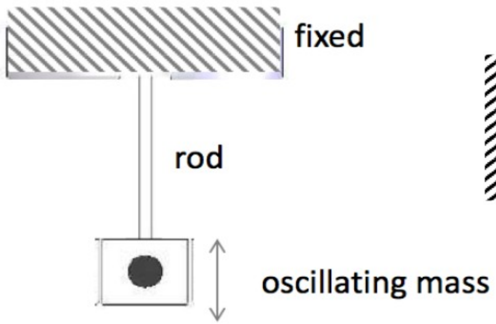


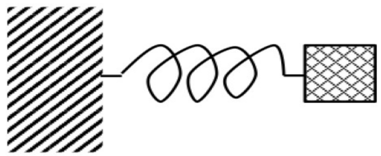

*Red point: longitudinal mode*
*Blue point  transverse mode*
*Gray line: best fit of the numerical point*

*Error bars include statistical uncertainty, the systematic errors are estimated of*
*± 0.01- 0.03 along the vertical axis*
*$\pm 7 \times 10^{-4}$ along the horizontal axis*

# Numerical simulation molecular dynamic

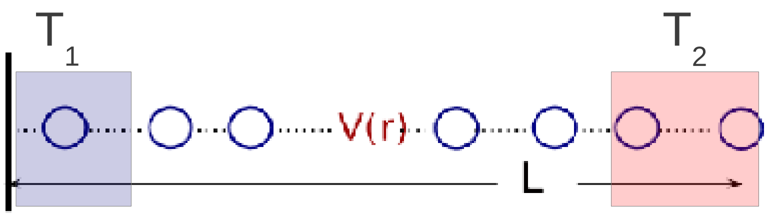
<u>*Poster session: “The effect of heat fluxes on the vibrational modes of an oscillator” - Paolo De Gregorio*</u>



*The 1st longitudinal mode can be model as a mass-spring system (rod mass negligible).*

*One dimensional model with identical particle interacting with their first and second neighbors via Lennard-Jones potential.*



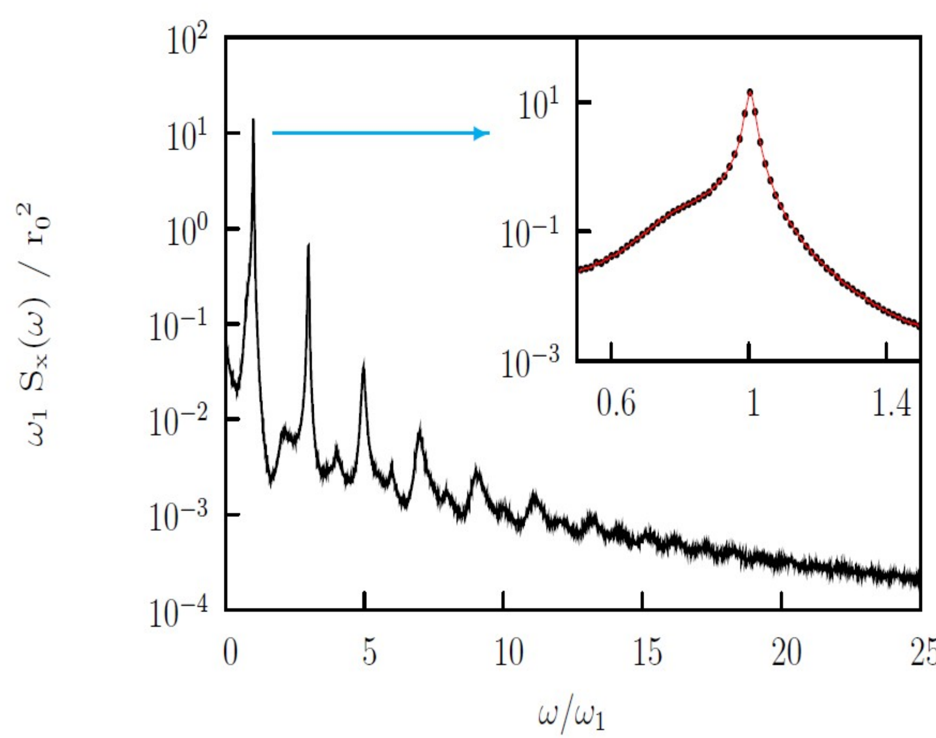

*Left end has been clamped with neighbors thermostated to $T_1$, right end free, with last two particle thermostated to $T_2$*

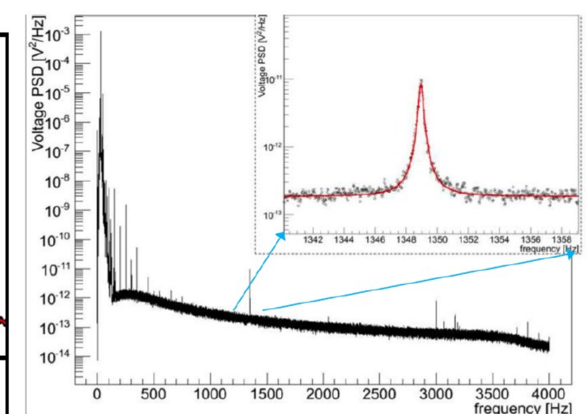
*The time evolution of the length chain is estimated with molecular dynamic simulation*

*Average PSD of the last particle*

*$\omega_1$: resonant frequency of first mode*

*$r_0$ inter-particle average distance*



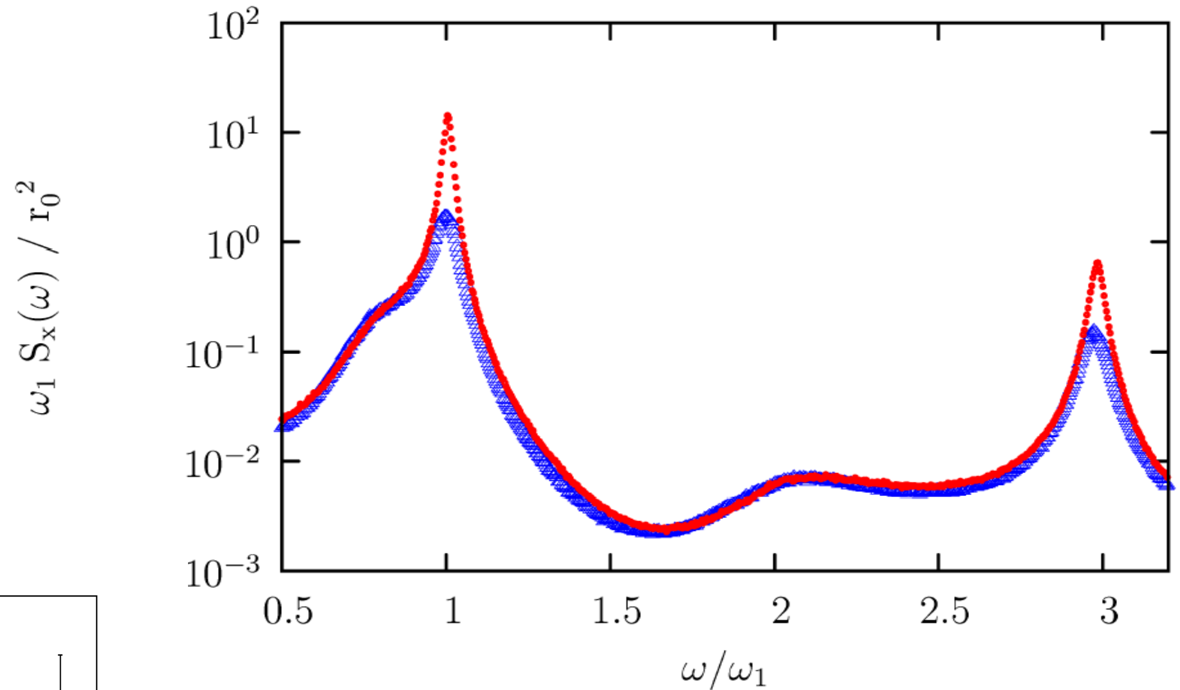
# Numerical simulation

- *1-dimensional model reproduces thermo-elastic properties at equilibrium (elastic modulus and $\omega_{res}$), and also out of equilibrium.*

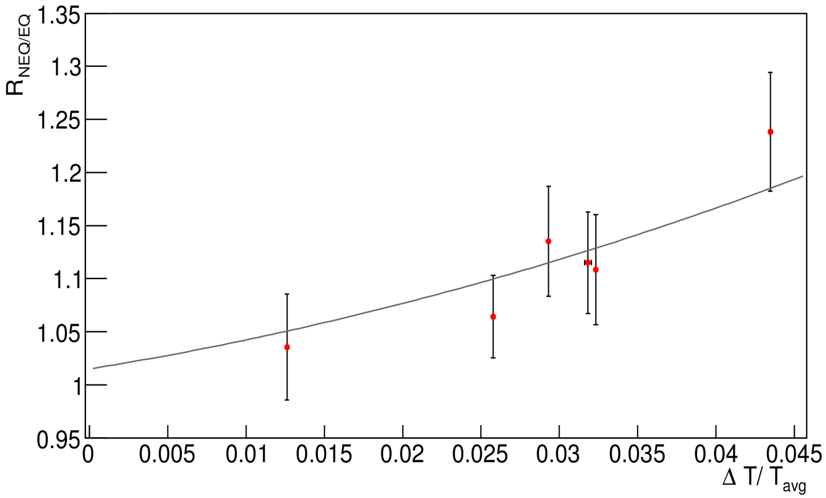
Phys. Rev. E 85, 066605 (2012)
Phys. Rev. B 84, 224103 (2011)

*NESS: $T_2 > T_1$*

*first and second mode*
*blue: equilibrium at T*
*red: NESS T2-T1 =0.2T*




<u>*$T_{eff}$ in NESS is not a valid measure of thermostated temperature*</u>



# *Possible theoretical explanation: mode-mode correlation*

*The equilibrium probability distribution can be described by the canonical distribution of a chain of N harmonic oscillators.*

*Dynamics: sum of independent damped oscillators forced by thermal noise*

*The different modes are orthogonal*

$$\langle x_l(t)^2 \rangle = \frac{k_b T}{m_l \omega_l^2}$$

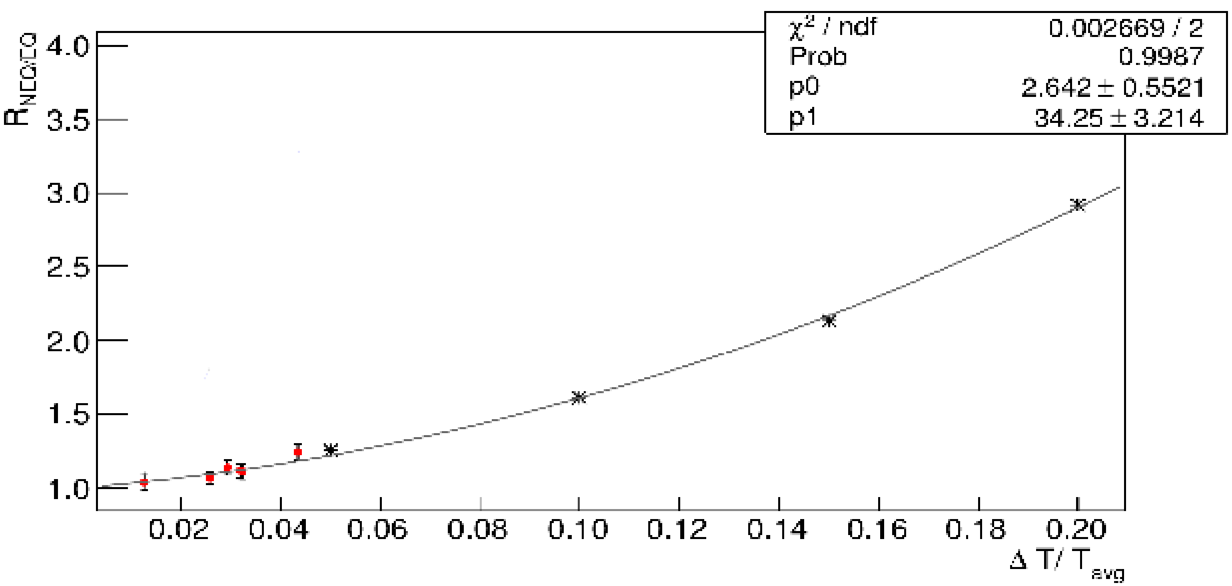
*It is usually to define the heat rate via cross-terms:*

$$J = \frac{-1}{N} \sum_{i \neq j}^{N} j_{ik} x_i v_k$$

*So if a heat flux is present:* $\langle x_i v_j \rangle \neq 0 \quad \Rightarrow$ *<u>correlations between modes</u>*

Under this hypothesis we obtain:

$$\frac{\langle x_1^2 \rangle_{NESS}}{\langle x_1^2 \rangle_{EQ}} - 1 \propto \Delta T^2$$



# Conclusion

- ✗ *We studied the effective temperature of a macroscopic oscillator at room temperature in and out of equilibrium (w/o heat flux)*
- ✗ *Experimentally in  NESS the effective temperature increases with the heat flux.*
- ✗ *Molecular dynamics simulations of 1-dimensional oscillator chain show similar results.*
- ✗ *the results are interpreted in terms of new flux-mediated correlations between modes in non equilibrium state, absent at equilibrium.*

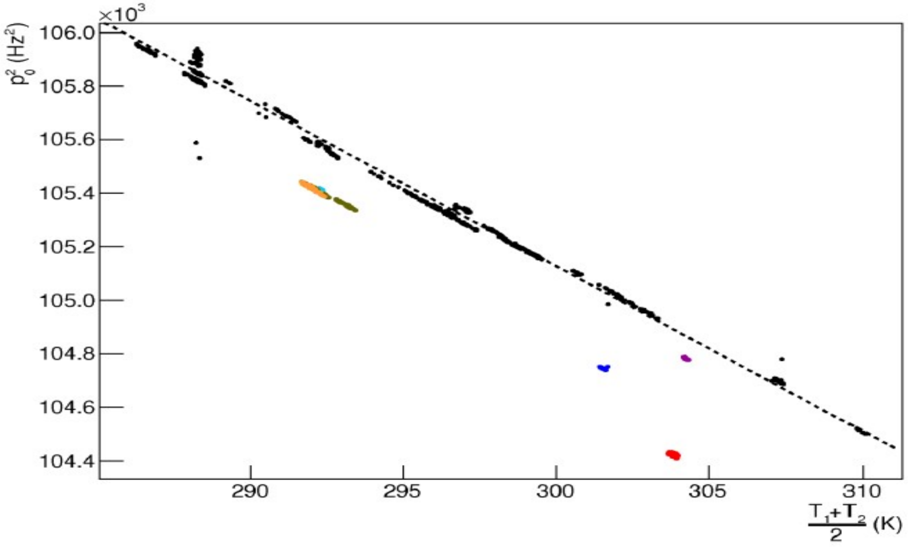
*...work in progress*

- ✗ Studies on the statistics of mean energy distribution and correlations between modes
- ✗ Cryogenics experimental setup is ongoing

# *Backup*

# *Taverage in transversal mode*

*The corresponding NESS $T_{avg}$ can not be defined as (T1+T2)/2*




*Square resonant frequency as function of function (T1+T2)/2. The non equilibrium transverse mode average temperature defined as the correspond resonant frequency value in the equilibrium case. $T_{avg}$ error ~ 1K*

| $\Delta T$ | $R_{NEQ}/R_{EQ}$ |
|---|---|
| 3.83±0.02(stat)±0.2(syst) | 1.32±0.30(stat)±0.03(syst) |
| 7.54±0.02(stat)±0.2(syst) | 1.76±0.036(stat)±0.04(syst) |
| 8.84±0.03(stat)±0.2(syst) | 2.11±0.41(stat)±0.05(syst) |
| 9.31±0.06(stat)±0.2(syst) | 2.14±0.43(stat)±0.05(syst) |
| 9.44±0.02(stat)±0.2(syst) | 2.08±0.04(stat)±0.18(syst) |
| 13.21±0.04(stat)±0.2(syst) | 3.31±0.64(stat)±0.08(syst) |