Introduction to Kalman controls for MultiSAS

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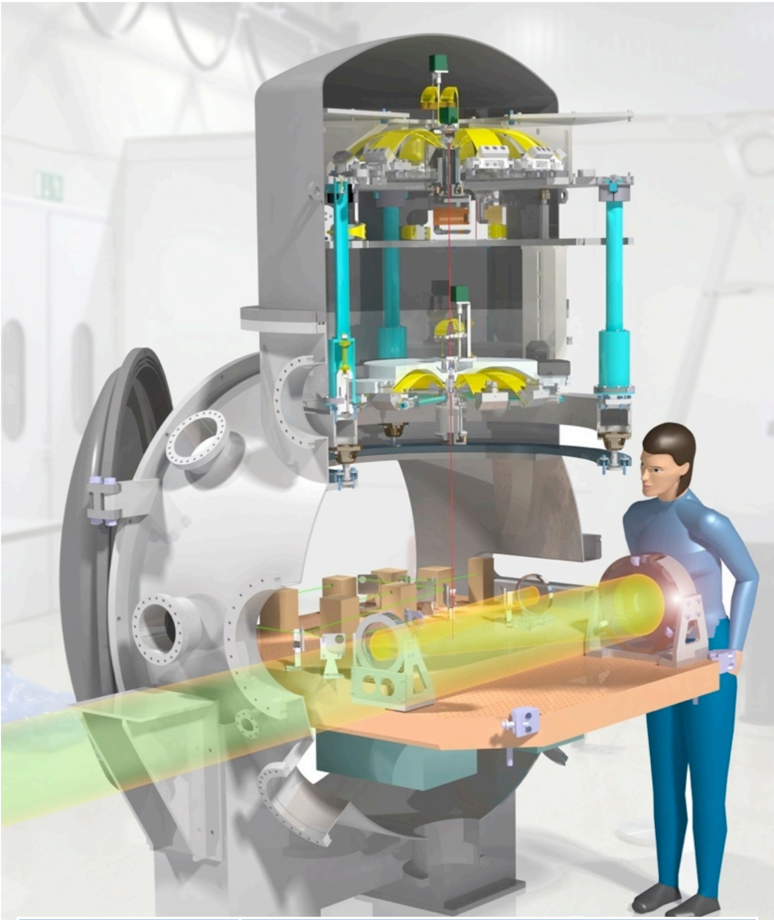
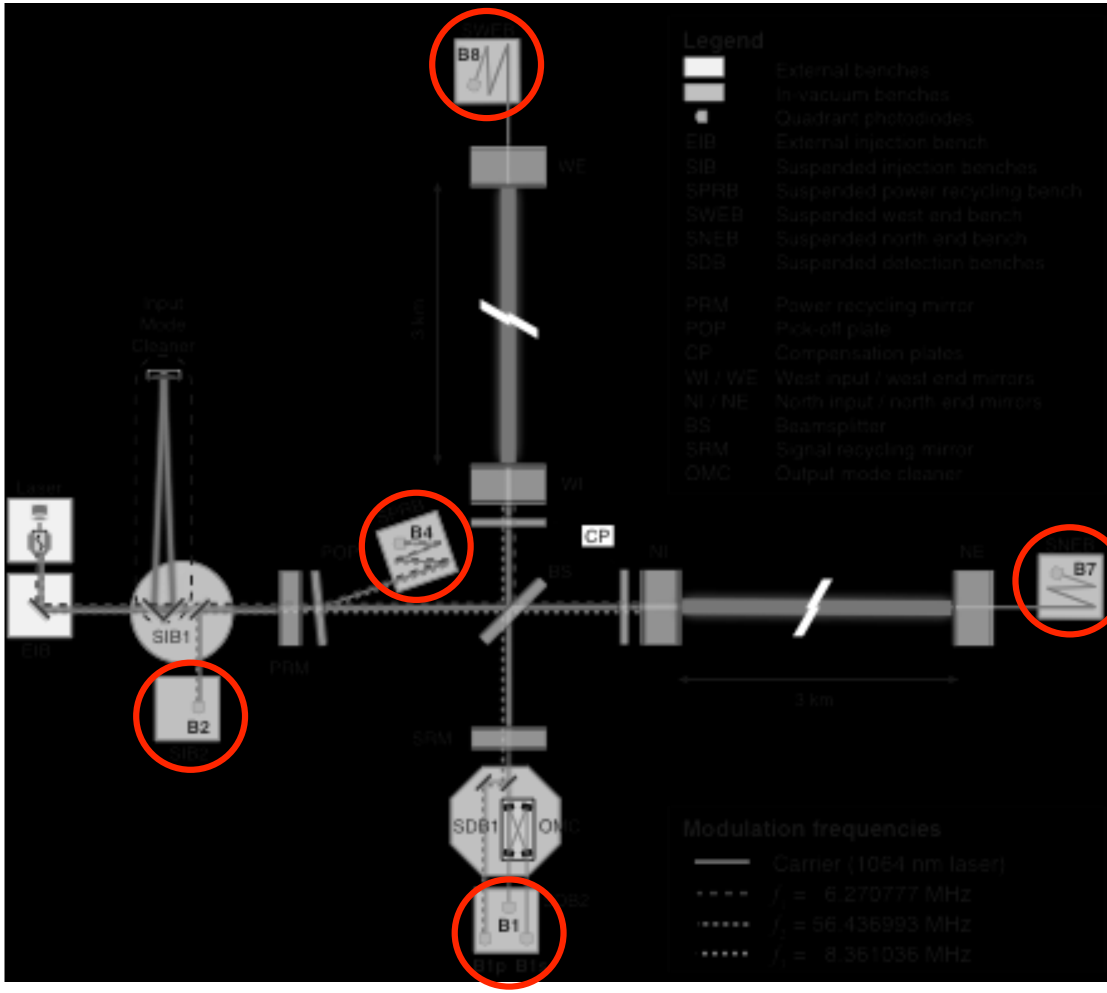
Reminder: MultiSAS is a multi-stage seismic isolation system for aVirgo in-vacuum optical benches

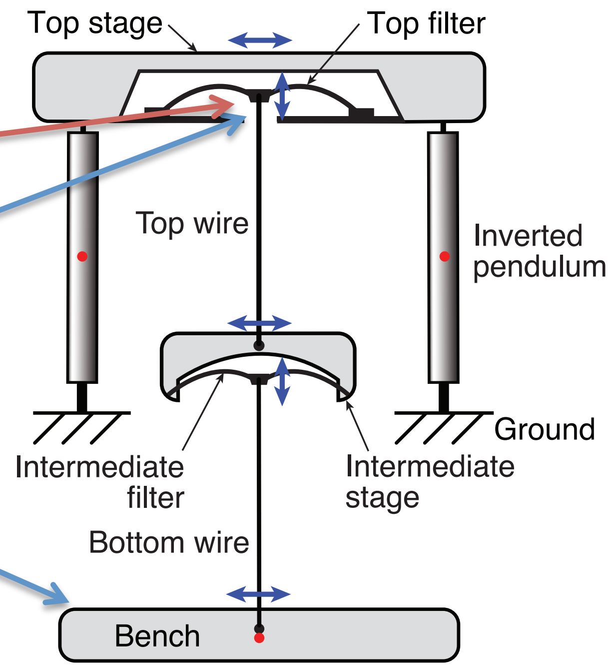




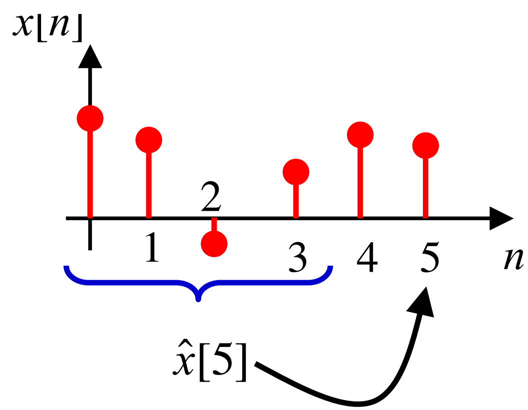
	MultiSAS Requirements M. Mantovani, VIR-0101A-12
δx	$2 \cdot 10^{-12}$ m/√Hz @ 10 Hz
$\delta\theta$	$3.3 \cdot 10^{-15}$ rad/√Hz @ 10 Hz
x_{rms}	$24 \cdot 10^{-6}$ m
θ_{rms}	$0.033 \cdot 10^{-6}$ rad

The “optimal” control problem

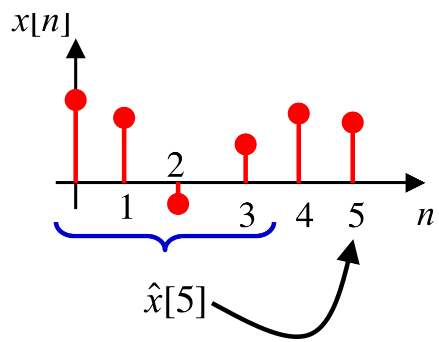
- Minimize motion of the bench
  - by applying a force at the top stage only
  - given (displacement) sensor on top stage
  - and (inertial) sensor on bench
- MultiSAS dynamics governs relationship between the inputs
- Some states not measured directly
- Measurement noise



- Lets first consider a simpler (1D) example
  - How can we make an “optimal” prediction given the past measurements

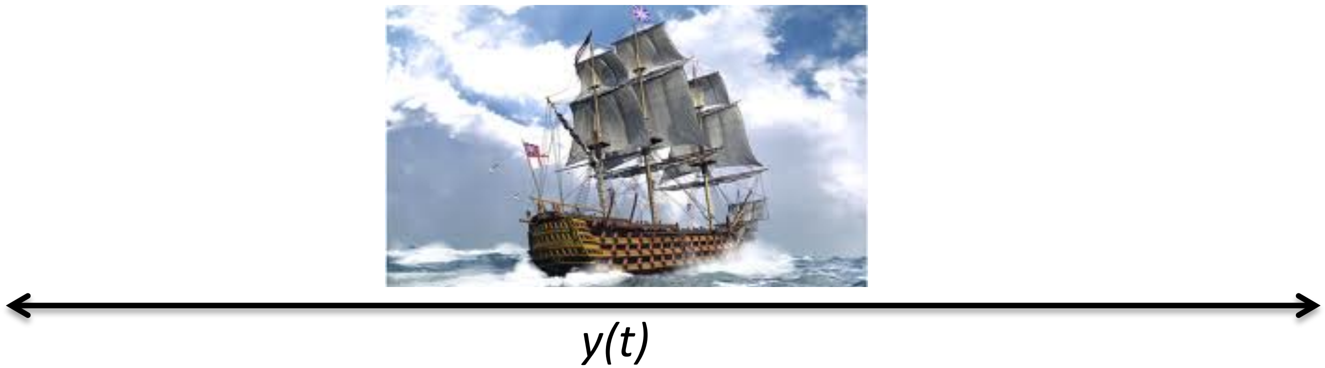


Wiener vs Kalman filtering



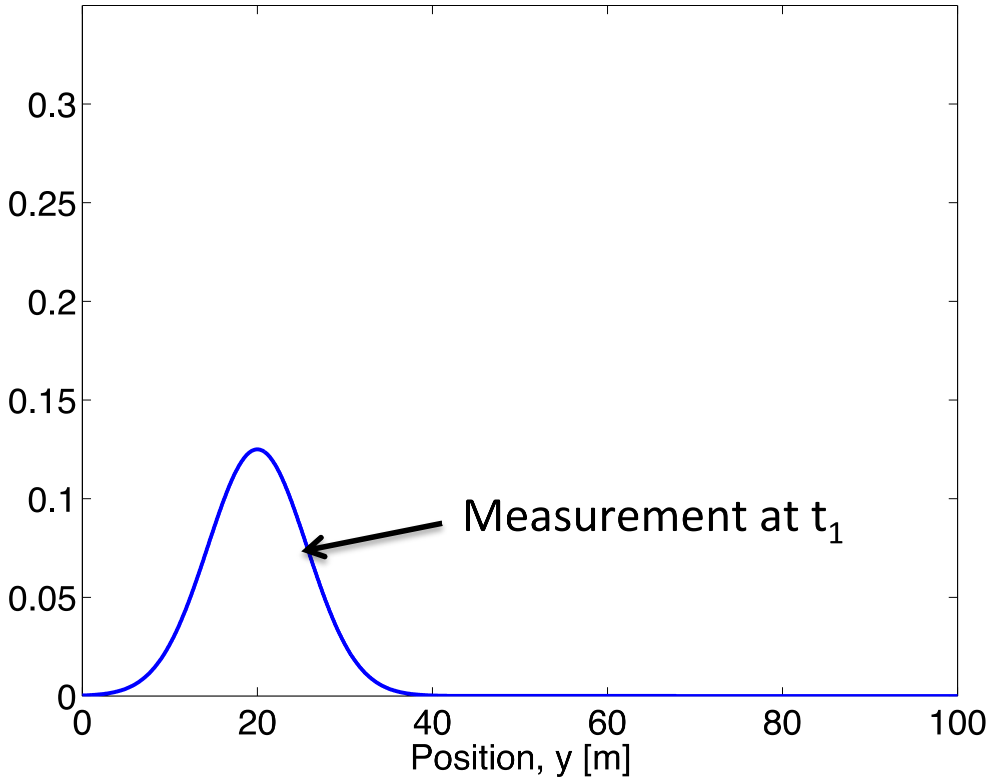
- Least squares
  - Minimizes the sum of squares of the errors
  - Has no “knowledge” of the system
- Wiener filter
  - x[n] = s[n] + w[n] -> “estimate s[n] so as to minimize the error”
  - Stationary processes – The statistical properties of the inputs don’t change in t
  - Causal, length grows, (generally) non-recursive
  - For discrete samples reduces to least squares solution
- Kalman filter
  - Generalization for Wiener filter to non-stationary processes – The signal is characterized by a dynamical model
  - Recursive – don’t need to re-evaluate all data at each step
  - Uses prior knowledge of the system
    - Requires a dynamic (state space) model

Kalman filter – conceptual description
Boat in 1-D space



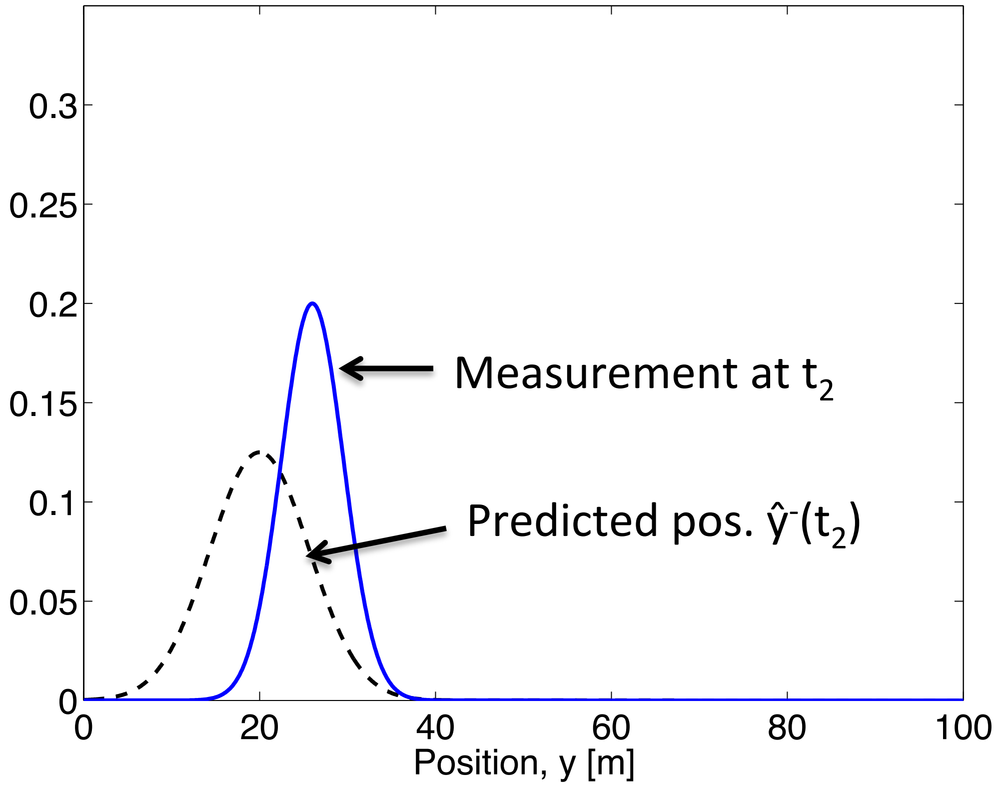
- Lost in 1-D space
- Position *y(t)*
- Assume Gaussian distributed measurements

Kalman filter – conceptual description
Boat in 1-D space



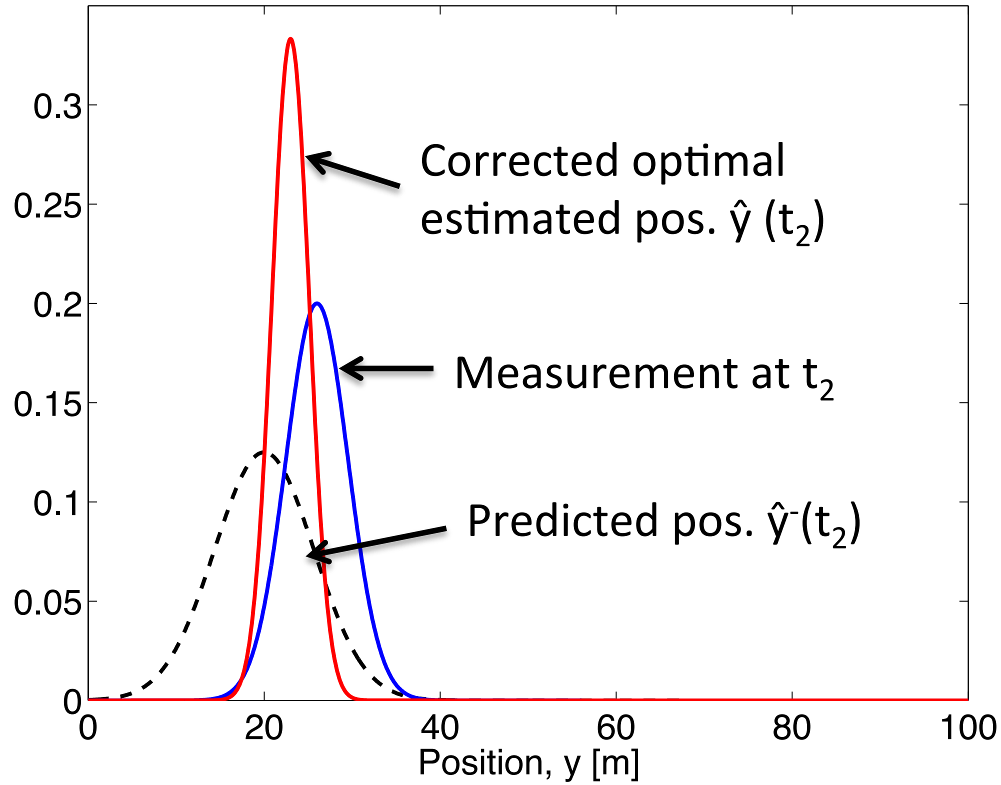
- Sextant measurement at t_1: Mean = z_1 and Variance = σ_{z1}
- Optimal estimate of position is: $\hat{y}(t_1) = z_1$
- Variance of error in estimate: $\sigma^2_x(t_1) = \sigma^2_{z1}$
- Boat in same position at time t_2 - <u>Predicted</u> position is z_1

Kalman filter – conceptual description
Boat in 1-D space



- So we have the prediction $\hat{y}^{-}(t_2)$
- GPS measurement at t_2: Mean = z_2 and Variance = σ_{z2}
- Need to <u>correct</u> the prediction due to measurement to get $\hat{y}(t_2)$
- Closer to more trusted measurement – linear interpolation?

Kalman filter – conceptual description
Boat in 1-D space



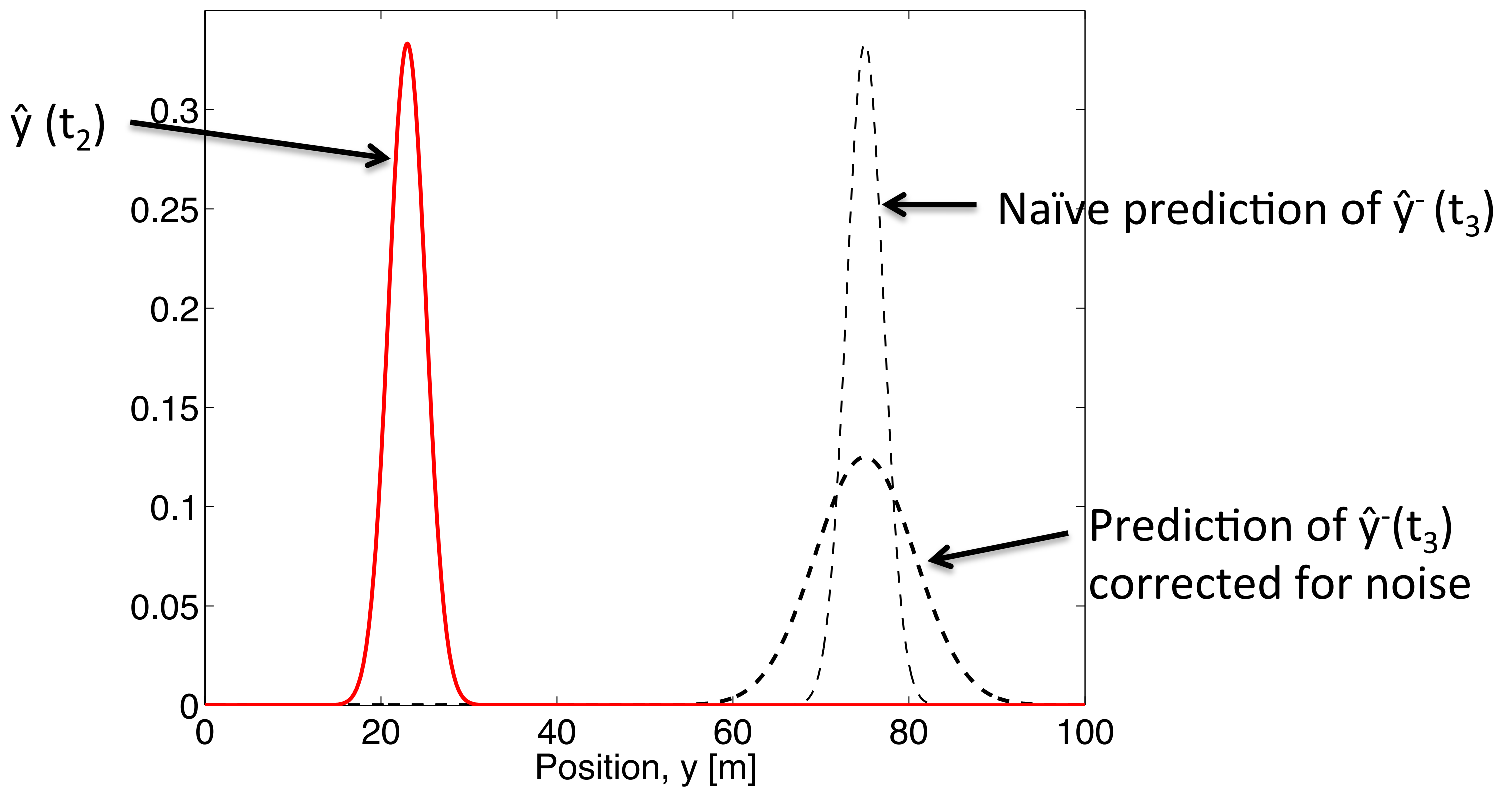
- Corrected mean is the new optimal estimate of position
- New variance is smaller than either of the previous two variances

Kalman filter – conceptual description
Boat in 1-D space

- Now we add a physical model
- The boat moves with velocity $v = dy/dt$

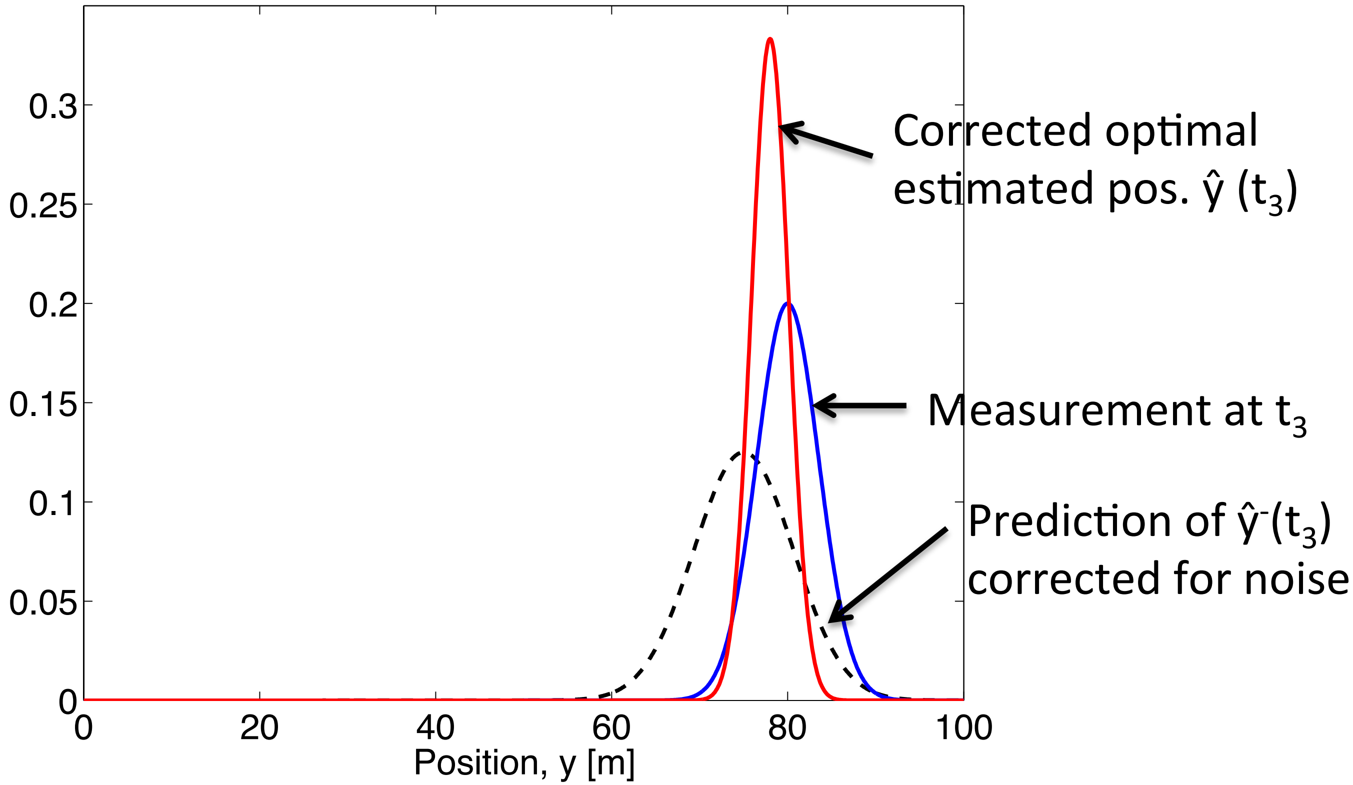
v

Kalman filter – conceptual description
Boat in 1-D space



- At time t_3, boat moves with velocity *v = dy/dt*
- Naïve approach: Shift probability to the right to predict new position
- This would work if we knew the velocity exactly (perfect model)
- Better to assume imperfect model by adding Gaussian noise

Kalman filter – conceptual description
Boat in 1-D space



- Now we take a measurement at t_3
- Need to once again correct the prediction
- Corrected mean is the new optimal estimate of position
- New variance is smaller than either of the previous two variances

Kalman filter – conceptual description
Lessons learnt

Prediction

Make a prediction based on previous data and model

Measurement

Take a measurement

Correction

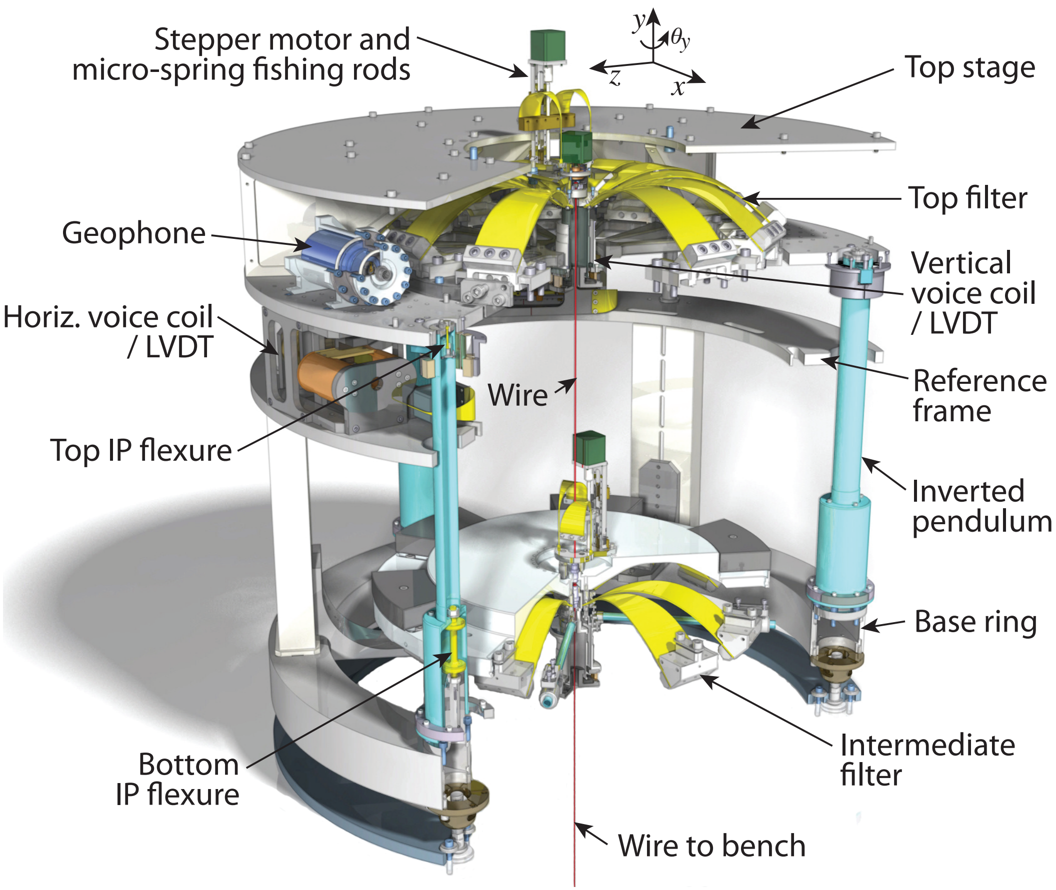
Use measurement to correct prediction by 'blending' prediction and residual

Optimal estimate = Prediction + (Kalman gain) * (Measurement - Prediction)

Residual = (Measurement - Prediction)

Low measurement variance		Low process variance
(good measurements)	↔	(good model)
Kalman gain increases		Kalman gain decreases
Rely more heavily on measurements		Rely more heavily on prediction

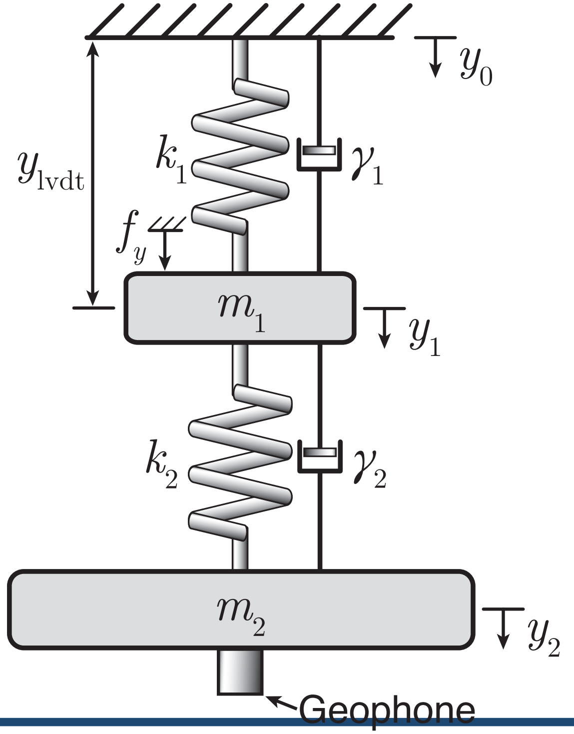
MultiSAS Control



MultiSAS vertical control
Step 1: Define (state space) model

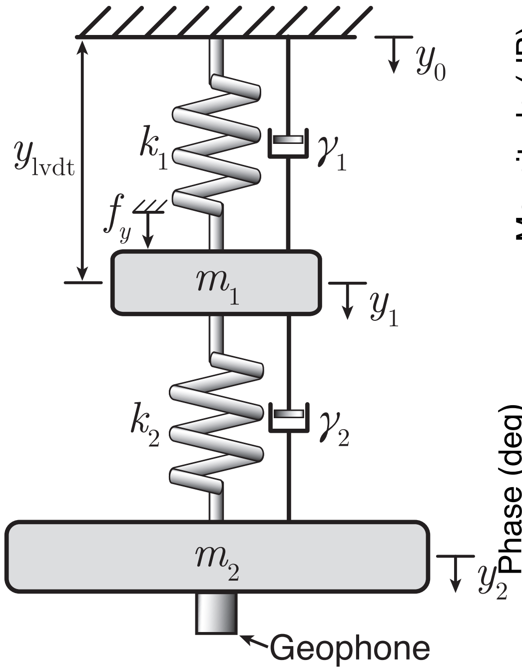
$$\underbrace{\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{v}_1 \\ \dot{v}_2 \end{bmatrix}}_{\dot{\mathbf{x}}_p} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-(k_1+k_2)}{m_1} & \frac{k_2}{m_1} & \frac{-(\gamma_1+\gamma_2)}{m_1} & \frac{\gamma_2}{m_1} \\ \frac{k_2}{m_2} & \frac{-k_2}{m_2} & \frac{\gamma_2}{m_2} & \frac{-\gamma_2}{m_2} \end{bmatrix}}_{A_p} \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ v_1 \\ v_2 \end{bmatrix}}_{\mathbf{x}_p} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \frac{k_1}{m_1} \\ 0 \end{bmatrix}}_{B_p} u_y + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \frac{k_1 y_0 + \gamma_1 v_0}{m_1} \\ 0 \end{bmatrix}}_{B_u u_d}, \tag{5.14}$$

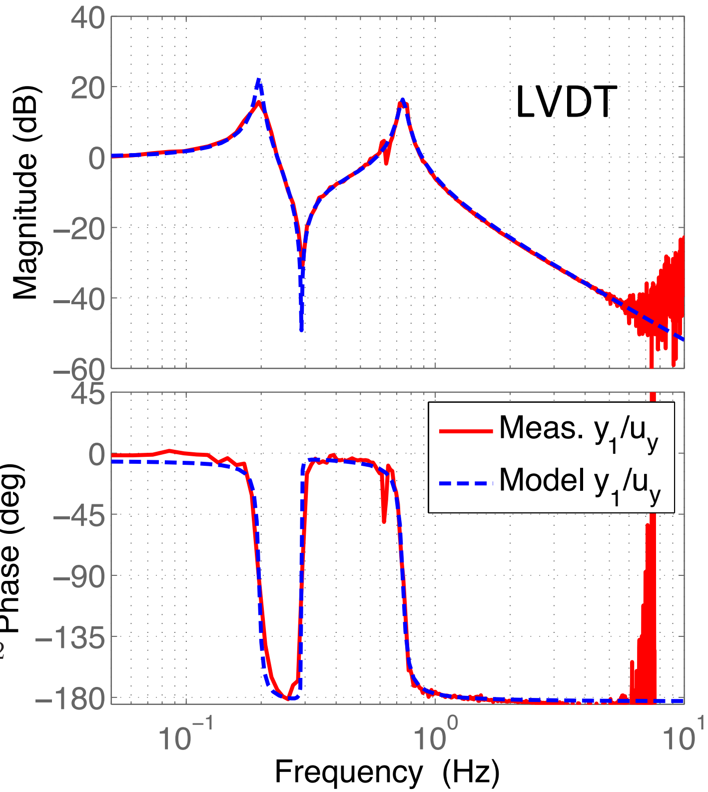
$$\underbrace{\begin{bmatrix} y_{\text{lvdt}} \\ y_{\text{geo}} \end{bmatrix}}_{\mathbf{y}_p} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{C_p} \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ v_1 \\ v_2 \end{bmatrix}}_{\mathbf{x}_p}. \tag{5.15}$$

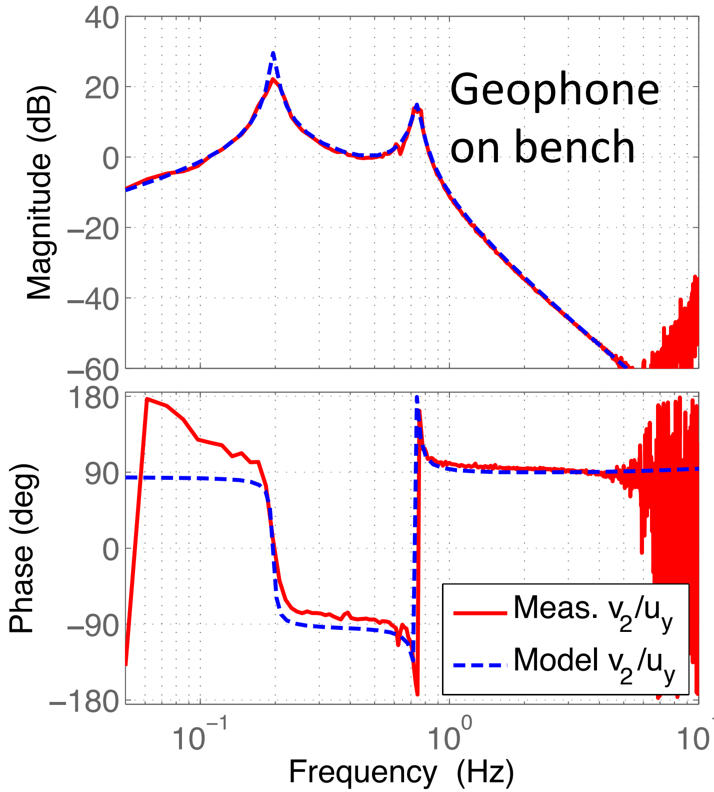


1. Define model, derive equations of motion, create state space model
2. Generate Kalman observer K_{est} and LQR gain matrix K
3. Combine K_{est} and K to make LQR (MISO regulator)

MultiSAS vertical control
Step 1: Define (state space) model

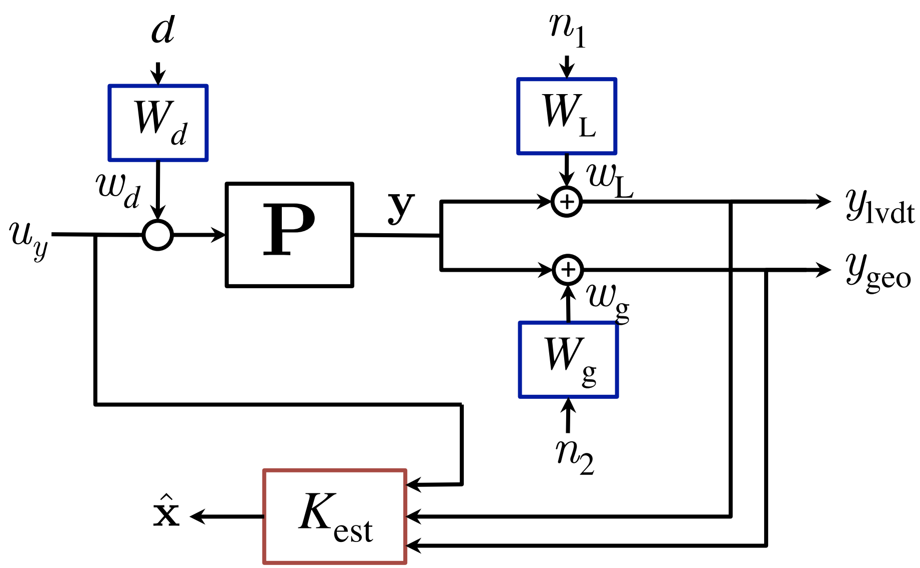


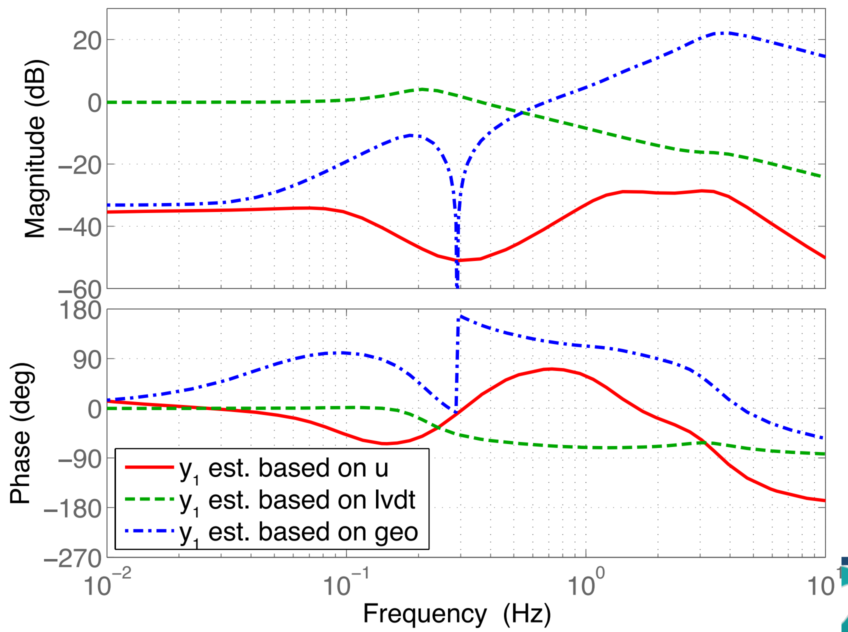




MultiSAS vertical control
Step 2: Find Kalman state estimator (observer)

- Provide:
  - Measurement noise covariance Q
  - Process noise covariance R
- Shaping filters are used to account for non-Gaussian noise
- K_{est} is the Kalman gain matrix
  - Can be adaptive to varying noise covariance's
- Observes all the states of the system
  - Also those that are not measureable
- Blends the LVDT and geophone signals according to sensitivity and dynamics



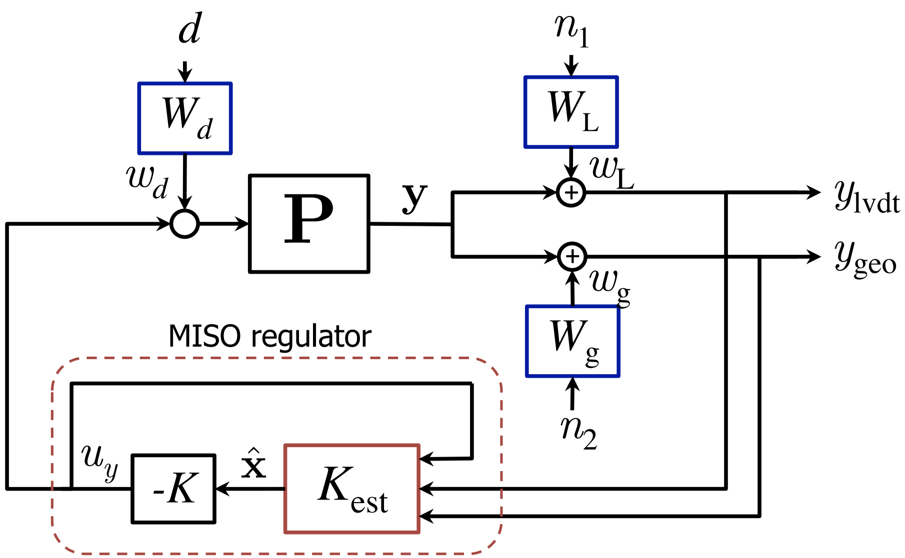


MultiSAS vertical control
Step 3: Find LQR gain matrix

- Linear Quadratic Regulator
- Minimizes a quadratic cost function in order to weigh the (observed) states and output in an optimal way
- In the case of MultiSAS, provides a single output to the force actuator
- LQR + Kalman observer

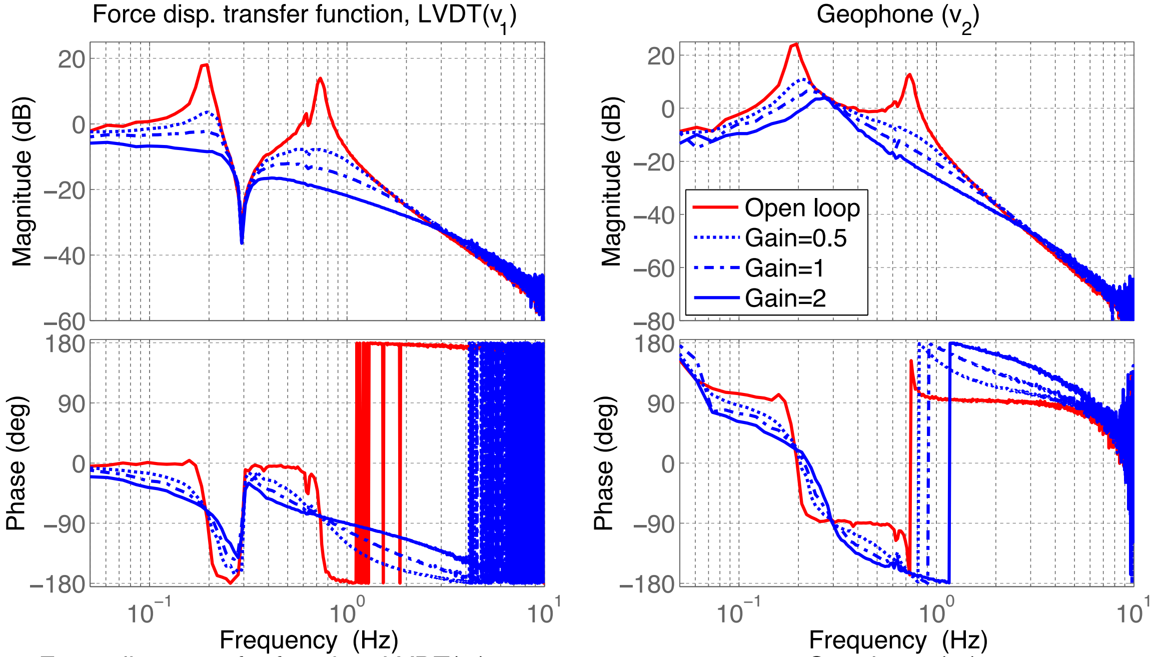
  =

  Linear Quadratic Gaussian (LQG) Control

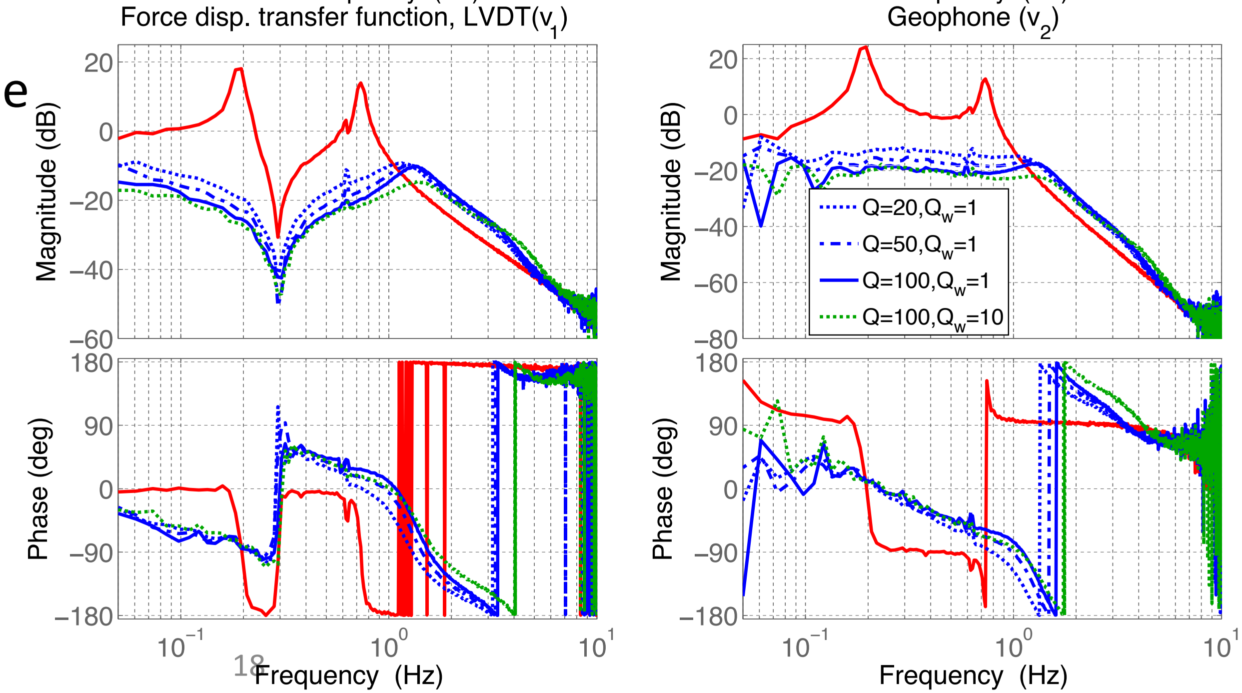


Testing control performance with noise injection

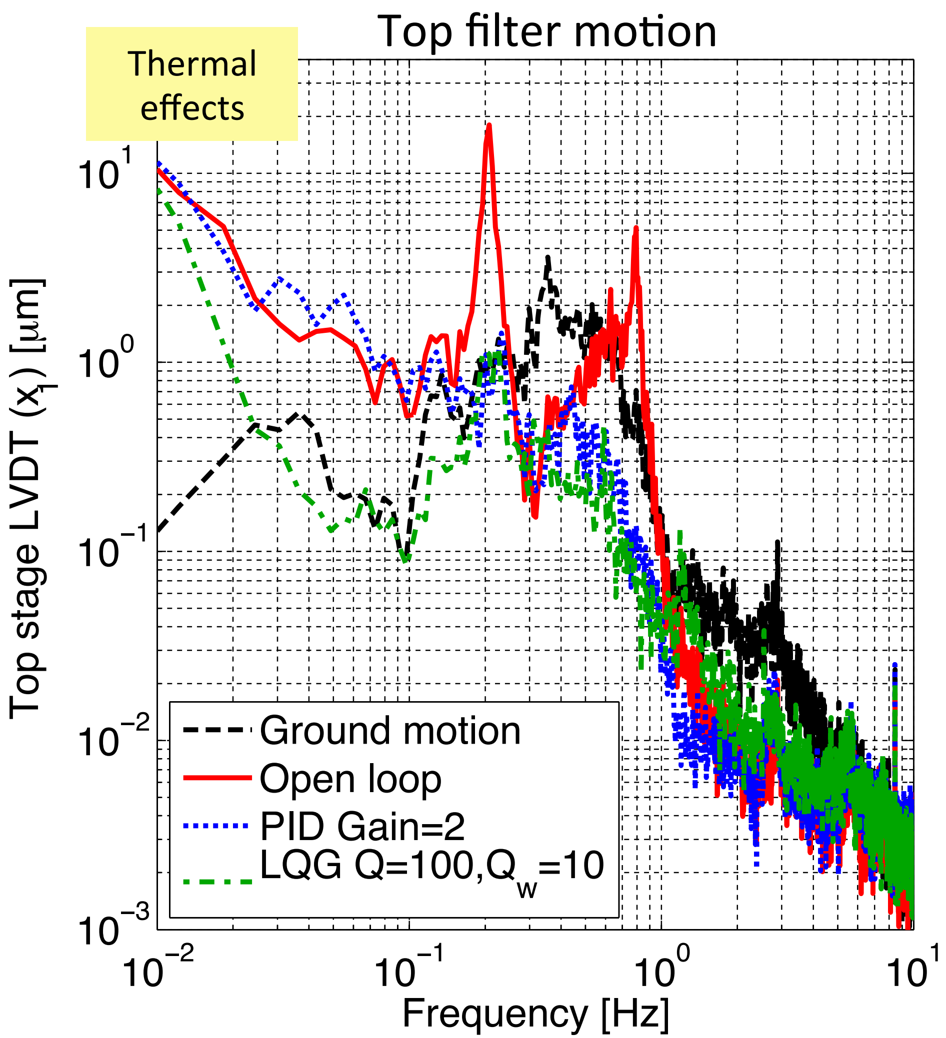
- “Traditional” PID
  - With sensor corrected LVDT (Trillium ground noise subtracted)

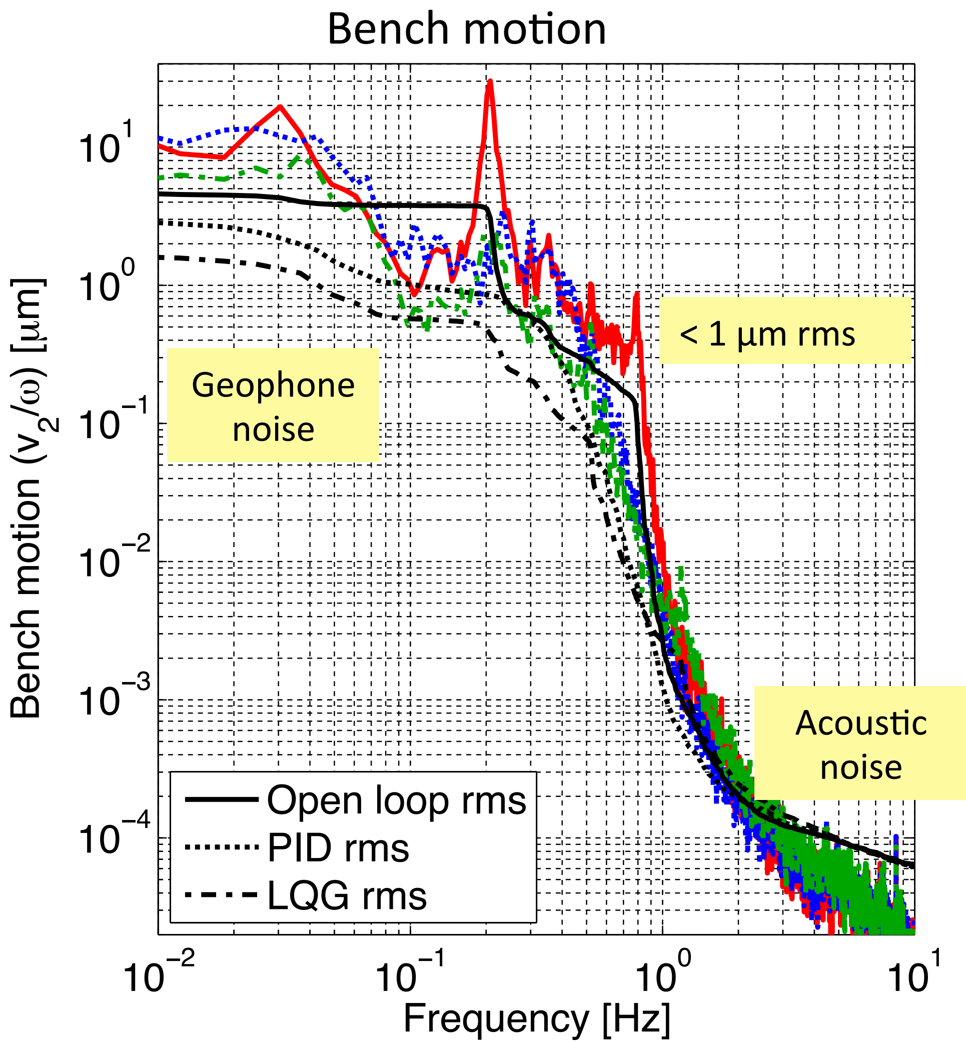


- LQG with LVDT and geophone



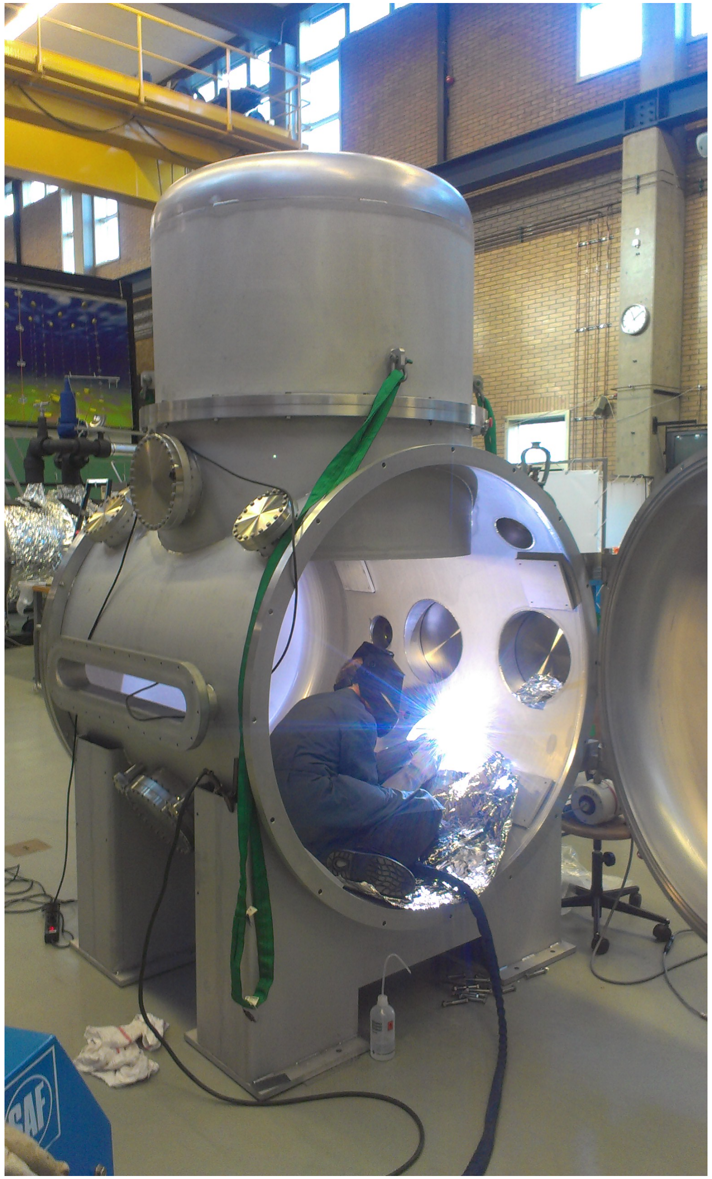
Control performance with environmental noise only





Current status and planning

- Prototype
  - MultiSAS (in air) performance tests complete
  - Installation and testing of MiniTower (vacuum chamber) complete
  - Installation of MultiSAS into MiniTower underway
  - Long term tests continuing
  - Optimal control design continuing
- Advanced Virgo
  - Production of five units started
  - Installation of first system (SIB2) on April 2014
  - Ready for IFO commissioning by end of October 2014



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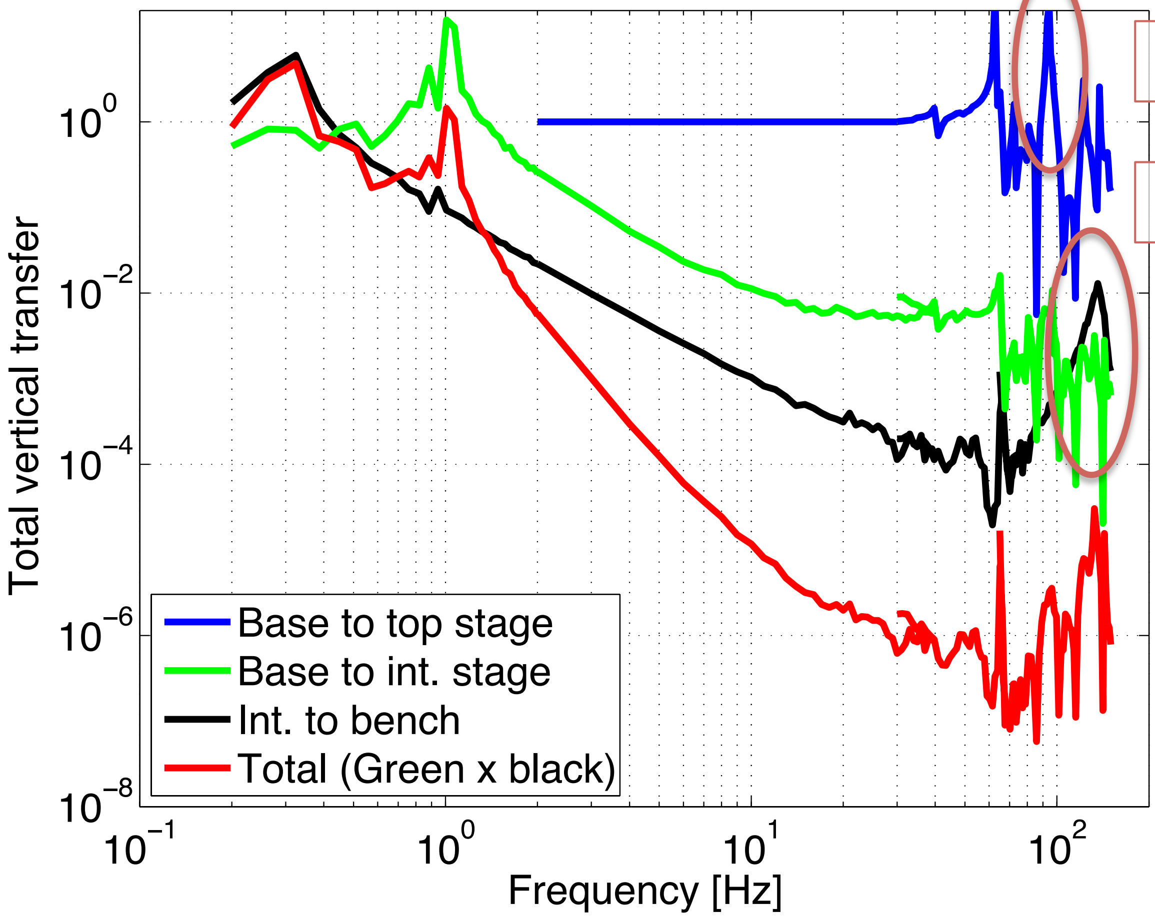
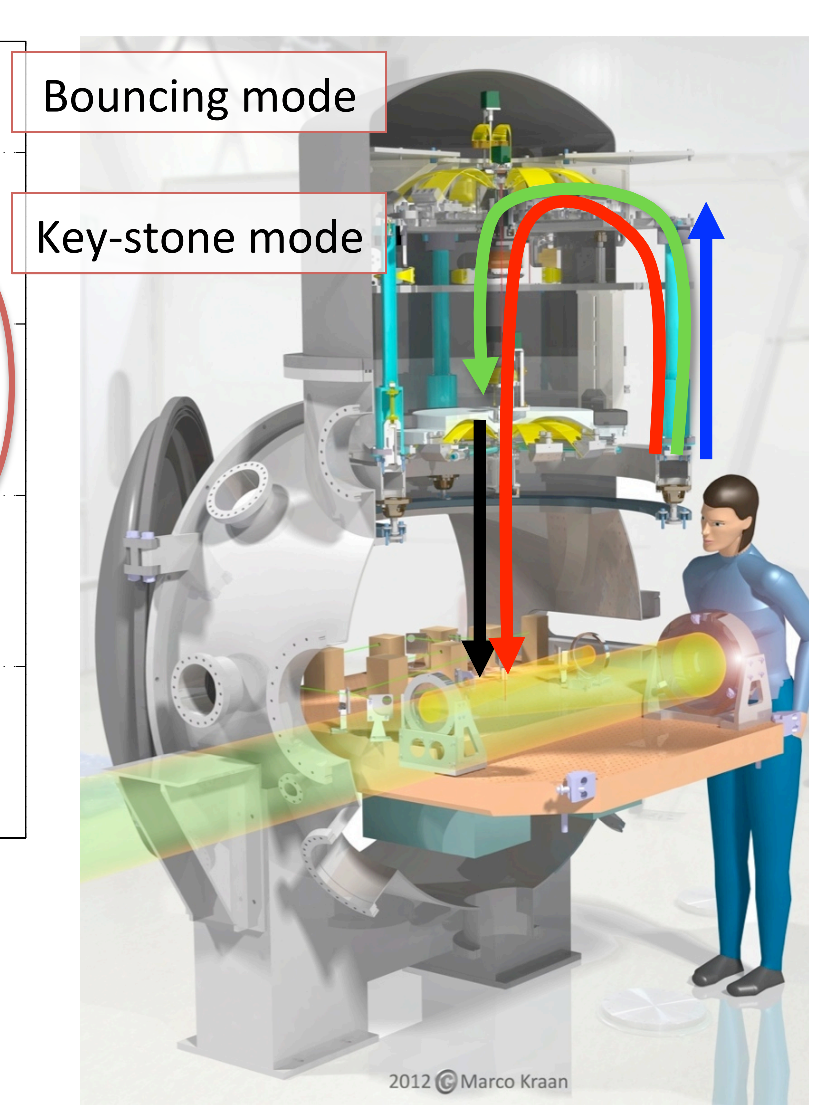
Summary

Backup

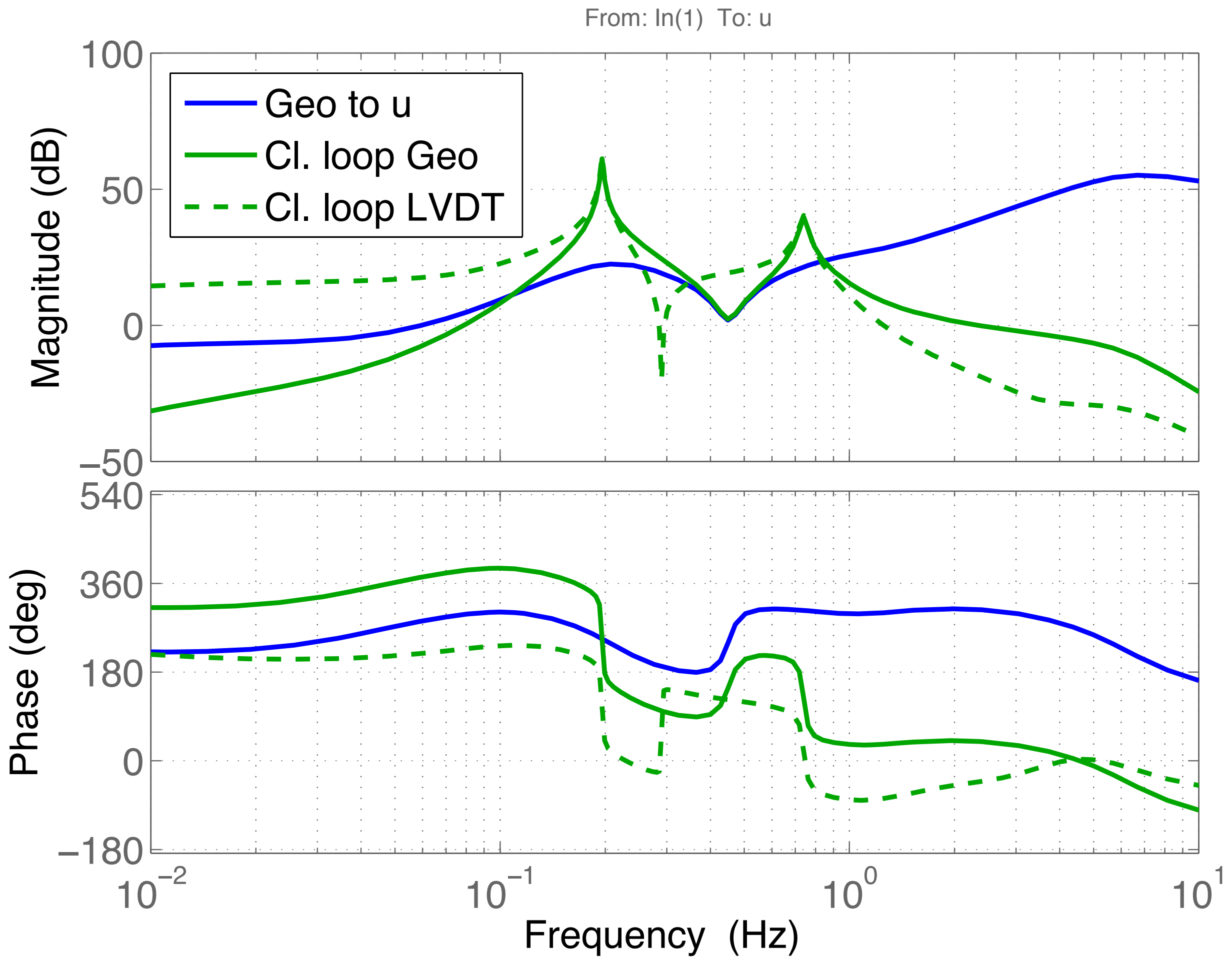
Vertical transfer function measured in stages



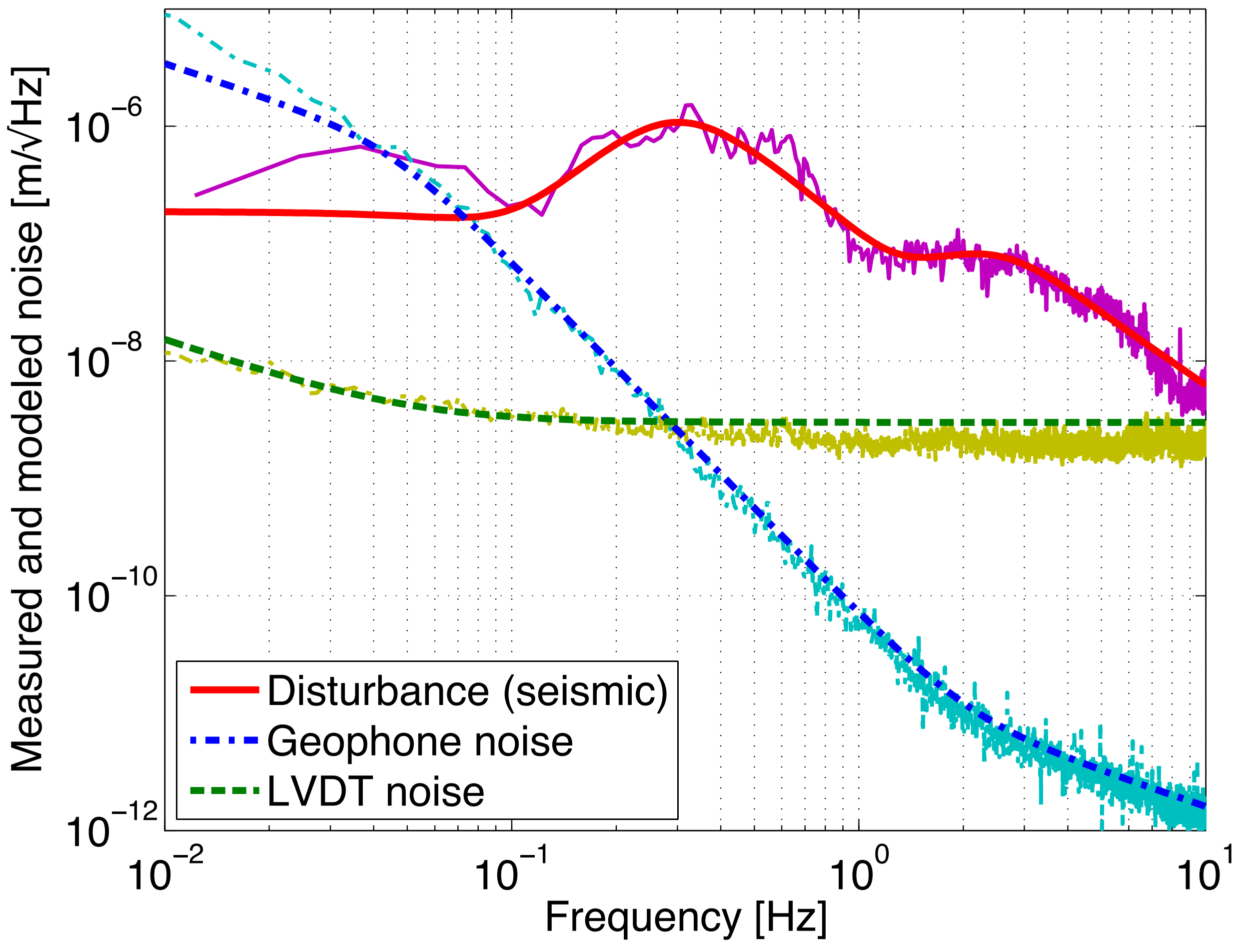


- Attenuation of 10^{-6} @ 30 Hz
- Top-stage bouncing and key-stone modes visible
- Other structures associated with support frame

Open-loop transfer functions



Kalman observer shaping filters



Horizontal control