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TESTING THE DYNAMICS OF GENERAL RELATIVITY WITH GRAVITATIONAL WAVES

Outline

- Motivation
- Coalescence of compact binary systems
- Model independent tests: TIGER
- A few examples
- Outlook

Motivation

- General Relativity (GR) passed all experimental tests to date with flying colours
- Static regime:
  - Mercury perihelion precession;
  - Gravitational lensing;
- Weak field regime:
  - Hulse-Taylor binary;
- Tests in the “strong field” regime and of its full dynamics have yet to be performed

The strong field regime

- GR is a scale free theory, so what is it meant by “strong field”?

$$\boxed{\frac{2GM}{Rc^2} \approx 1}$$

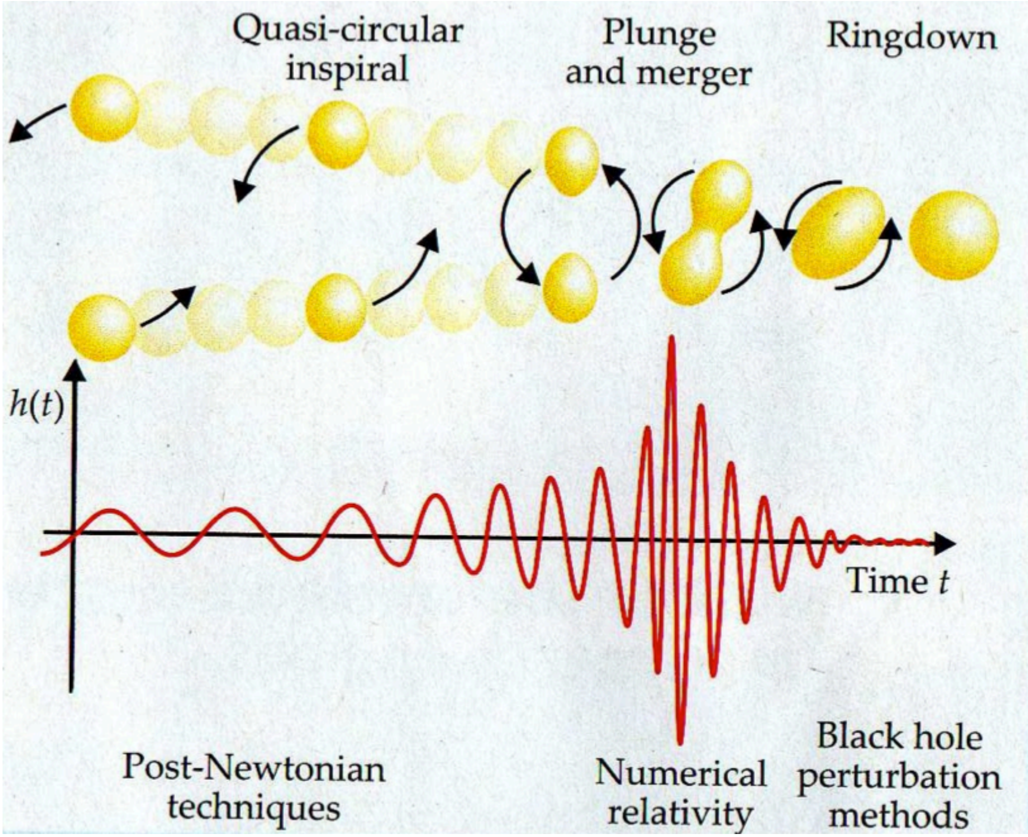
compactness

	Binary pulsars	Coalescing binaries at last stable orbit
compactness	$O(10^{-6})$	$O(10^{-1})$
typical velocity	10^{-3} c	0.1 c

- Gravitational Waves (GW) from coalescing binary systems probe the full dynamical regime of GR

GW from compact binaries coalescence

- The GW emission from CBC is divided in three stages:
  - inspiral
  - merger
  - ringdown
- Each stage offers different insights into the dynamics of GR



Testing GR during the inspiral

- Consider the inspiral phase
- in the PN approximation, the phase is given by:

$$\Phi(v(t)) = \left(\frac{v}{c}\right)^{-5} \sum_{n=0}^{7} \left[\psi_n + \psi_n^{(l)} \ln\left(\frac{v}{c}\right)\right] \left(\frac{v}{c}\right)^n$$

- in GR, the ψ_n are **uniquely** defined functions of component masses and spins

- Alternative theories either change one or more of these functions or add terms not present in GR

Phase modifications in selected alternative gravities

- Massive Graviton Theories
  - modify the 1PN term
- Brans-Dicke Theories
  - introduce a -1PN term
- See Yunes & Pretorius 2010, Yunes & Siemens 2013 for a comprehensive list

Two approaches to testing GR

Model independent tests:

- do not assume any specific alternative model, but verify the consistency of GR with the observed data

- If GR is violated, what is the underlying "true" theory?

Model dependent tests:

- pick an alternative theory (MG)
- decide which theory describe the observed data better

- What is a viable MG? Should we test them all?

How do we decide which theory fits the data better?

Which theory fits better?

- Within the Bayesian framework, the "goodness of fit" is quantified by the odds ratio
- Given some data d and two hypotheses H_i and H_j the odds ratio is defined as:

$$O^i_j = \underbrace{\frac{P(\mathcal{H}_i|\mathrm{I})}{P(\mathcal{H}_j|\mathrm{I})}}_{\text{prior odds}} \frac{P(d|\mathcal{H}_i,\mathrm{I})}{P(d|\mathcal{H}_j,\mathrm{I})} = \frac{P(\mathcal{H}_i|\mathrm{I})}{P(\mathcal{H}_j|\mathrm{I})} \underset{\text{Bayes factor}}{B^i_j}$$

Which theory fits better?

- If $O_j^i > 1$, H_i is favored over H_j
- If $O_j^i < 1$, H_j is favored over H_i
- Estimates of the values of any parameters $\vec{\theta}$ that describe each model are obtained by computing the posterior probability distribution

$$p(\vec{\theta}|d, \mathcal{H}_i, \mathrm{I}) = \frac{p(\vec{\theta}|\mathcal{H}_i, \mathrm{I})p(d|\vec{\theta}, \mathcal{H}_i, \mathrm{I})}{p(d|\mathcal{H}_i, \mathrm{I})}$$

Testing the validity of GR: desiderata

- If we aim at testing GR, without assuming any specific alternative, an analysis method should:
  - be as generic as possible
  - introduce parameterized deformations in GR waveforms
  - have as many free “test parameters” as possible
  - be flexible with regards to waveform approximants
  - work in noisy data and low SNR regime
  - be robust against:
    - unknown GR effects (e.g. matter effects in BNS)
    - unknown features of the detector (e.g. calibration errors)

Test Infrastructure for GEneral Relativity: TIGER

TIGER, overview

- Recall the PN expansion:

$$\Phi(v(t)) = \left(\frac{v}{c}\right)^{-5} \sum_{n=0}^{7} \left[\psi_n + \psi_n^{(l)} \ln\left(\frac{v}{c}\right)\right] \left(\frac{v}{c}\right)^n$$

- Define the following hypotheses:
  - $\mathcal{H}_{\rm GR}$: the ψ_n are the unique functions of the component masses and spins as predicted by GR
  - $\mathcal{H}_{\rm modGR}$: some or all of the ψ_n do not have the GR predicted functional dependence

However, no waveform can be used to parametrise $\mathcal{H}_{\rm modGR}$

TIGER, overview

- Introduce some auxiliary hypotheses:
  - $H_{i_1 i_2 \ldots i_k}$ is the hypothesis that the phase coefficients $\psi_{i_1}, \ldots, \psi_{i_k}$ do not have the dependence on masses and spins as in GR, but all other coefficients, *do have the dependence predicted by GR*
- Then $\mathcal{H}_{\mathrm{modGR}} = \bigvee_{i_1 < i_2 < \ldots < i_k} H_{i_1 i_2 \ldots i_k}$
- We can compute $O_{\mathrm{GR}}^{\mathrm{modGR}}$ using the rules of probability theory

A two parameters example

- For clarity consider the limited case in which we wish to test only two terms of the PN serie: ψ_1, ψ_2
- In this case $\mathcal{H}_{\text{modGR}} = H_1 \vee H_2 \vee H_{12}$
- Since they are mutually exclusive:

$$
(2)\, O_{\mathrm{GR}}^{\mathrm{modGR}} \equiv \frac{P(\mathcal{H}_{\mathrm{modGR}}|d,\mathrm{I})}{P(\mathcal{H}_{\mathrm{GR}}|d,\mathrm{I})} \equiv \frac{P(H_1 \vee H_2 \vee H_{12}|d,\mathrm{I})}{P(\mathcal{H}_{\mathrm{GR}}|d,\mathrm{I})}
$$

$$
= \frac{P(H_1|\mathrm{I})}{P(\mathcal{H}_{\mathrm{GR}}|\mathrm{I})} B_{\mathrm{GR}}^{1} + \frac{P(H_2|\mathrm{I})}{P(\mathcal{H}_{\mathrm{GR}}|\mathrm{I})} B_{\mathrm{GR}}^{2} + \frac{P(H_{12}|\mathrm{I})}{P(\mathcal{H}_{\mathrm{GR}}|\mathrm{I})} B_{\mathrm{GR}}^{12}
$$

A two parameters example

- If a priori none of the auxiliary hypotheses is preferred:

$$\frac{P(H_1|\mathrm{I})}{P(\mathcal{H}_{\mathrm{GR}}|\mathrm{I})} = \frac{P(H_2|\mathrm{I})}{P(\mathcal{H}_{\mathrm{GR}}|\mathrm{I})} = \frac{P(H_{12}|\mathrm{I})}{P(\mathcal{H}_{\mathrm{GR}}|\mathrm{I})}$$

- We still need to fix the overall prior odds:

$$\frac{P(\mathcal{H}_{\mathrm{modGR}}|\mathrm{I})}{P(\mathcal{H}_{\mathrm{GR}}|\mathrm{I})} = \frac{P(H_1 \vee H_2 \vee H_{12}|\mathrm{I})}{P(\mathcal{H}_{\mathrm{GR}}|\mathrm{I})} = \alpha$$

- Finally:

$$^{(2)}O_{\mathrm{GR}}^{\mathrm{modGR}} = \frac{\alpha}{3}\left[B_{\mathrm{GR}}^{1} + B_{\mathrm{GR}}^{2} + B_{\mathrm{GR}}^{12}\right]$$

The general case

- For N_T coefficients and $\mathcal{N}$ sources:

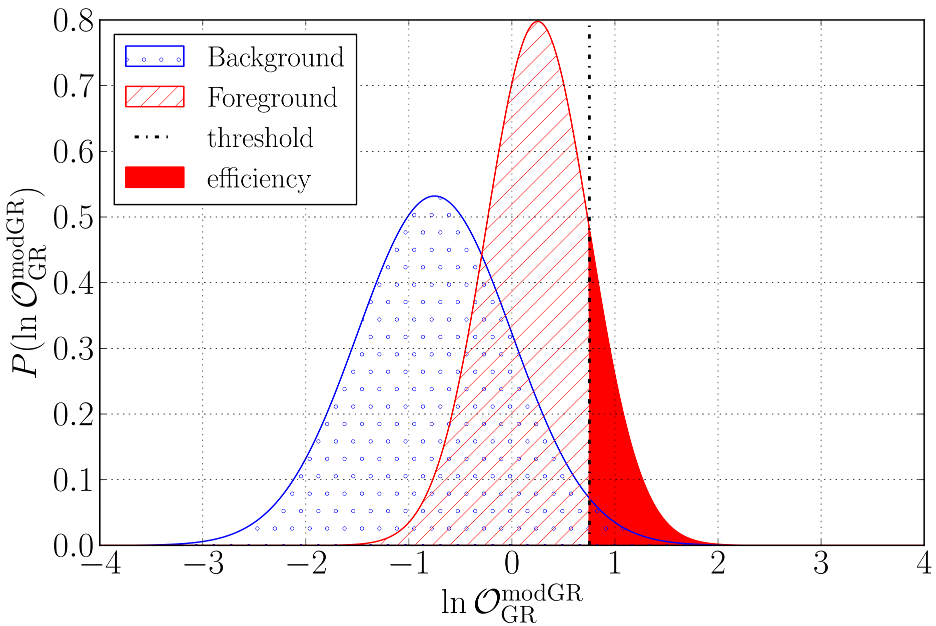
$$^{(N_T)}\mathcal{O}_{\mathrm{GR}}^{\mathrm{modGR}} = \frac{\alpha}{2^{N_T}-1} \sum_{k=1}^{N_T} \sum_{i_1<i_2<\ldots<i_k} \prod_{A=1}^{\mathcal{N}} {}^{(A)}B_{\mathrm{GR}}^{i_1 i_2 \ldots i_k}$$

- Note that, while $\mathcal{H}_{\mathrm{modGR}}$ cannot be parametrised, each of the $H_{i_1 i_2 \ldots i_k}$ can
- A convenient parametrisation is

$$\psi_i = \psi_i^{\mathrm{GR}}(m_1, m_2, \vec{S}_1, \vec{S}_2)\ [1 + \delta\chi_i]$$

Dealing with noise

- Noise can mimic a GR deviation
  - compute a background distribution for GR signals
  - pick a *false alarm probability* to define a threshold in log Odds
  - for given GR violation, the *efficiency* is the fraction of catalogs above the threshold



$$\zeta = \int_{\ln \mathcal{O}_\beta}^{\infty} P(\ln \mathcal{O} | \kappa', \mathcal{H}_{\mathrm{alt}}, \mathrm{I})\, d\ln \mathcal{O}$$

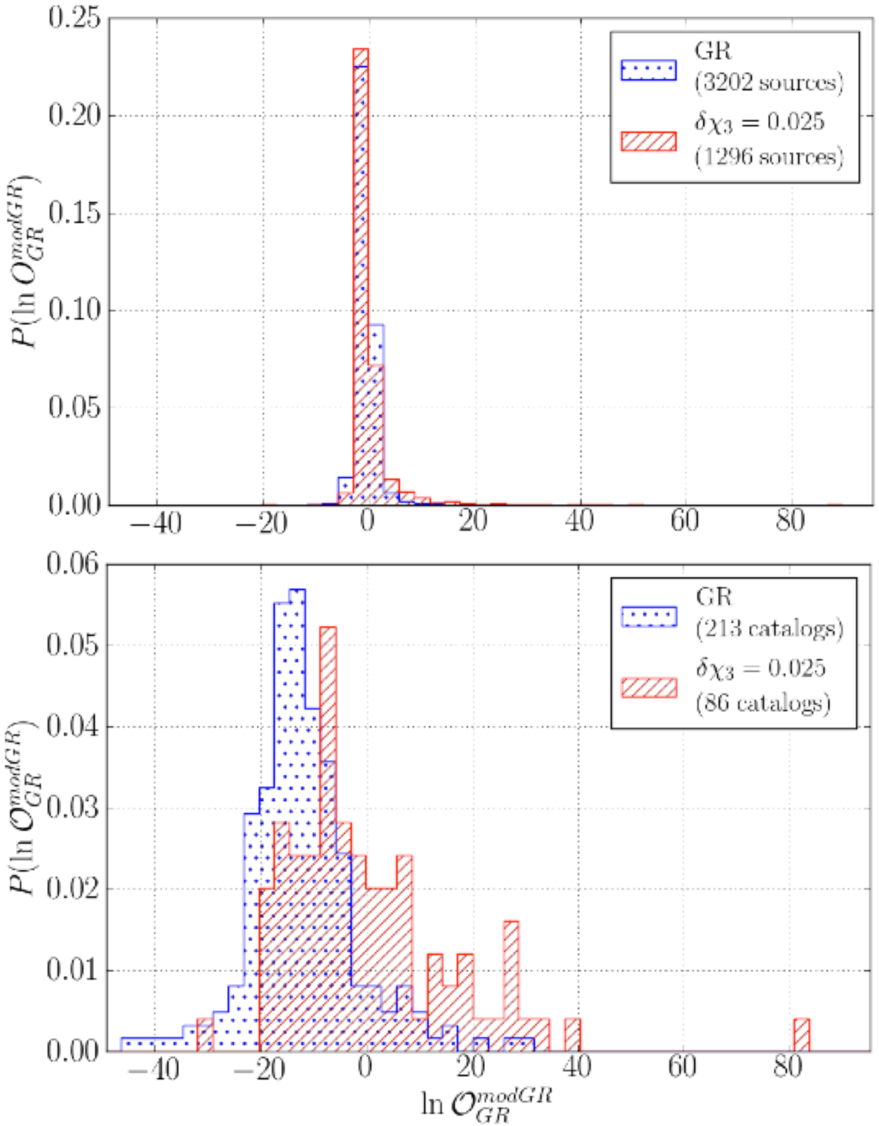
Testing the 1.5PN coefficient

- The 1.5PN phase coefficient ψ_3 is of particular interest since:
  - not accessible in binary pulsars experiments
  - encodes the first non-linear interactions of GR (tail effects)
- What sensitivity do we expect from Advanced instruments?

Simulation

- Simulated binary neutron star sources observed by Advanced LIGO/Virgo network:
  - m_1, m_2 in [1, 2] $M_\odot$
  - Zero spins
  - Distributed uniformly in sky position
  - Random orientations of inspiral plane
  - Uniformly in co-moving volume
  - distances in [100,400] Mpc
- Waveform: TaylorF2 [analytic, frequency domain]
- Simulated catalogs of sources: 15 per catalog
- Use 3 testing parameters: $\psi_1,\ \psi_2,\ \psi_3$

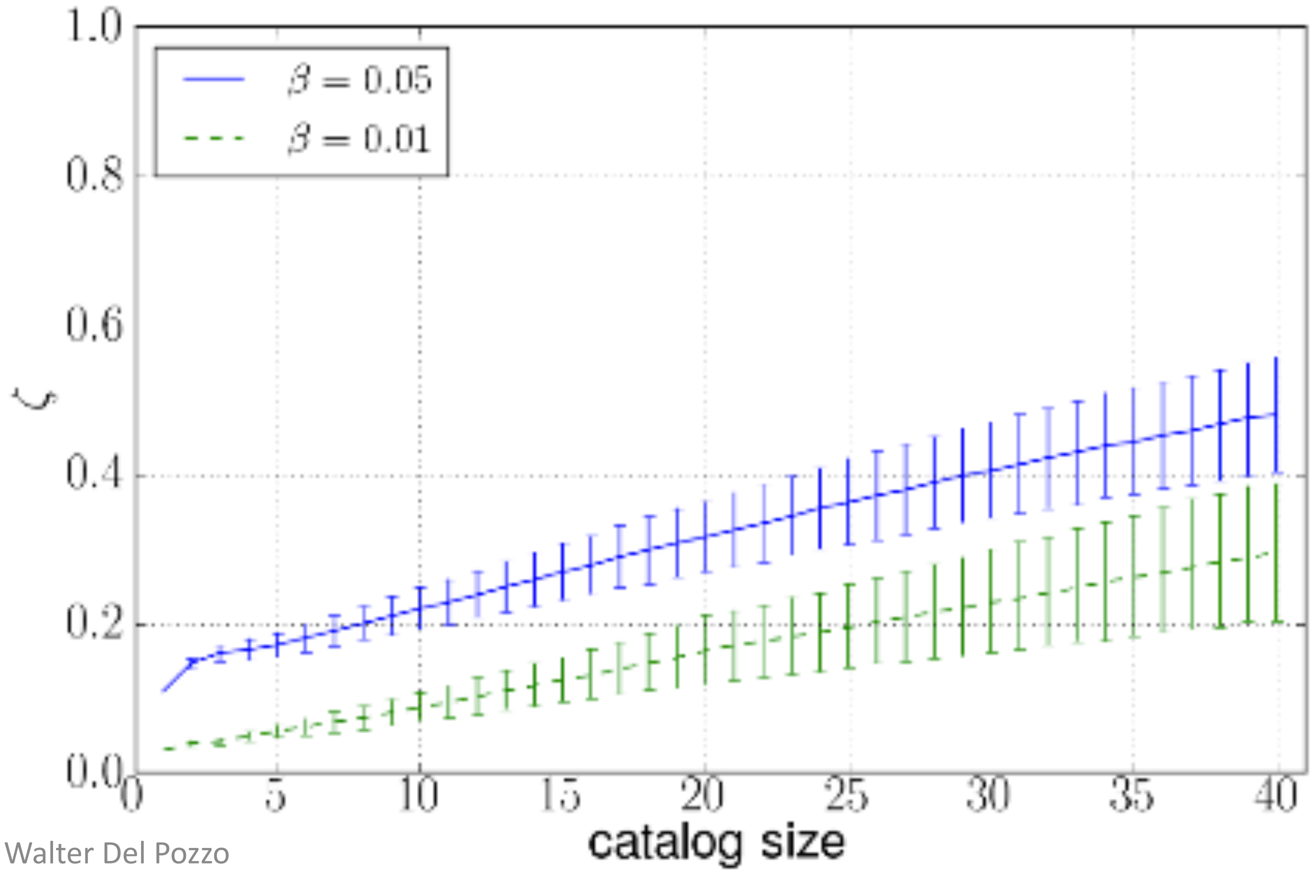
A 2.5% violation at $(v/c)^3$



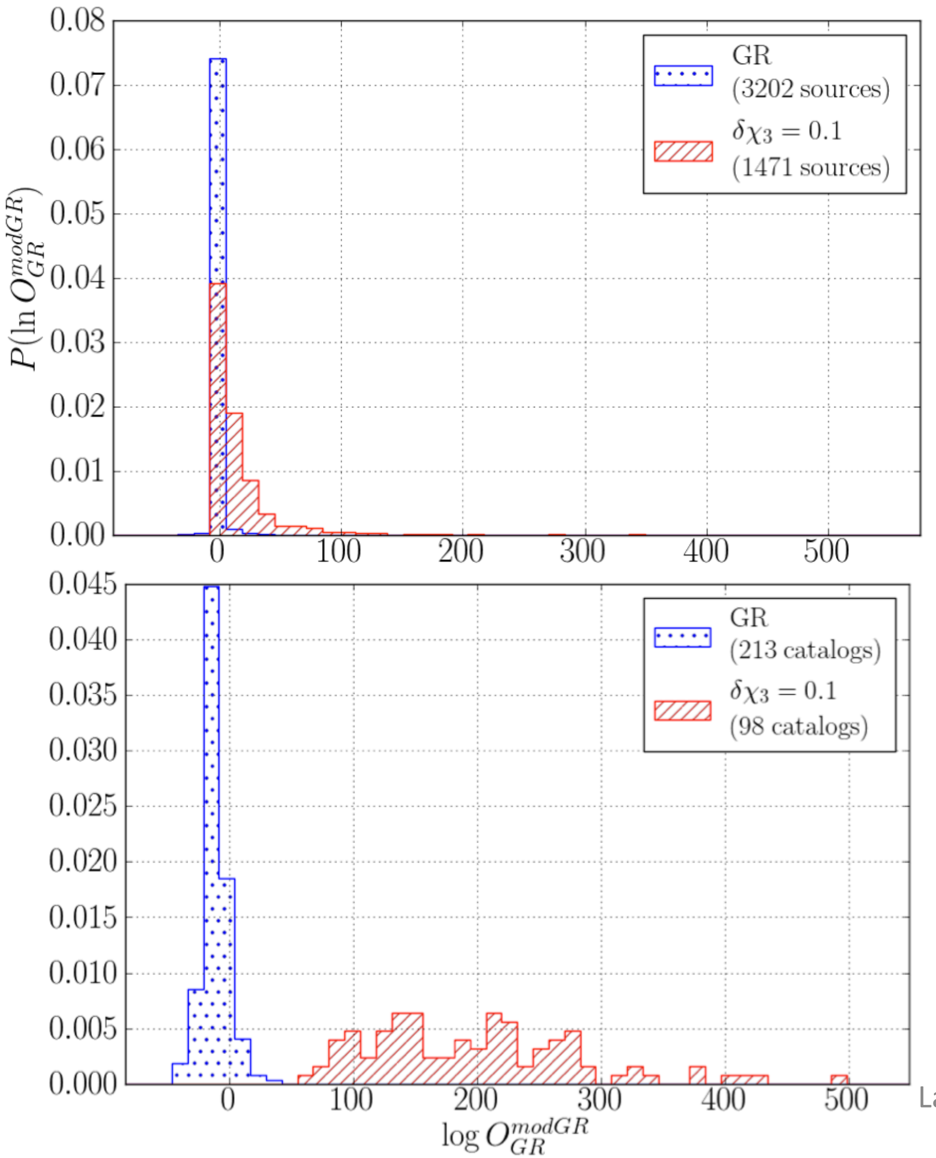
- In blue TaylorF2 (GR) signals
- In red TaylorF2+2.5% deviation at $(v/c)^3$
- Top: single sources, the distributions overlap
- Bottom: 15 sources per catalog, the distributions begin to be distinguishable

A 2.5% violation at $(v/c)^3$

- Efficiency as a function of the number of sources per catalog



A 10% violation at $(v/c)^3$



- In blue TaylorF2 (GR) signals
- In red TaylorF2+10% deviation at $(v/c)^3$
- Top: single sources, moderate separation
- Bottom: 15 sources per catalog, complete separation

Outlook

- GW observations are the only means of testing the full dynamical regime of GR
- TIGER is suited for detecting deviations from the predictions of GR
- the background computation allows for robustness against unknown effects (e.g. waveform systematics, detector calibration uncertainties, etc.)
- The pipeline is fully integrated with the LIGO Algorithms Library (LAL) and ready to use
- For further details/examples:
  - Li, Del Pozzo, et al 2012a
  - Li, Del Pozzo, et al 2012b
  - Agathos, Del Pozzo, et al 2013

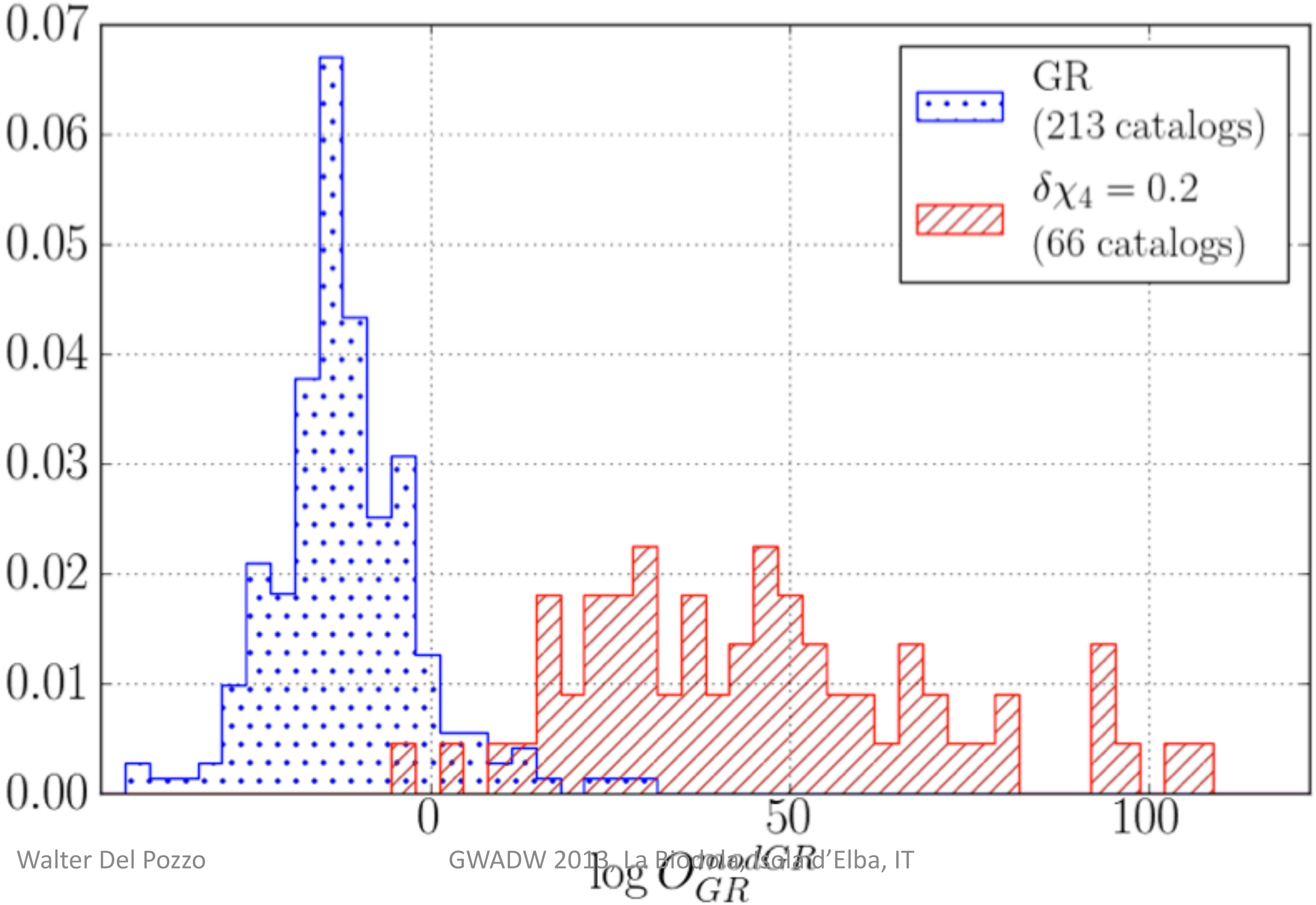
Outlook

- However:
  - the very nature of the test does not allow for an identification of the nature of the deviation
    - use PPE-like approach (Yunes & Pretorius 2010)
    - test all known theories and rank them according to the odd ratio
  - The pipeline is tailor made for binary neutron star systems
    - no reliable waveforms yet for binary black holes or neutron star black holes

Bonus Slides

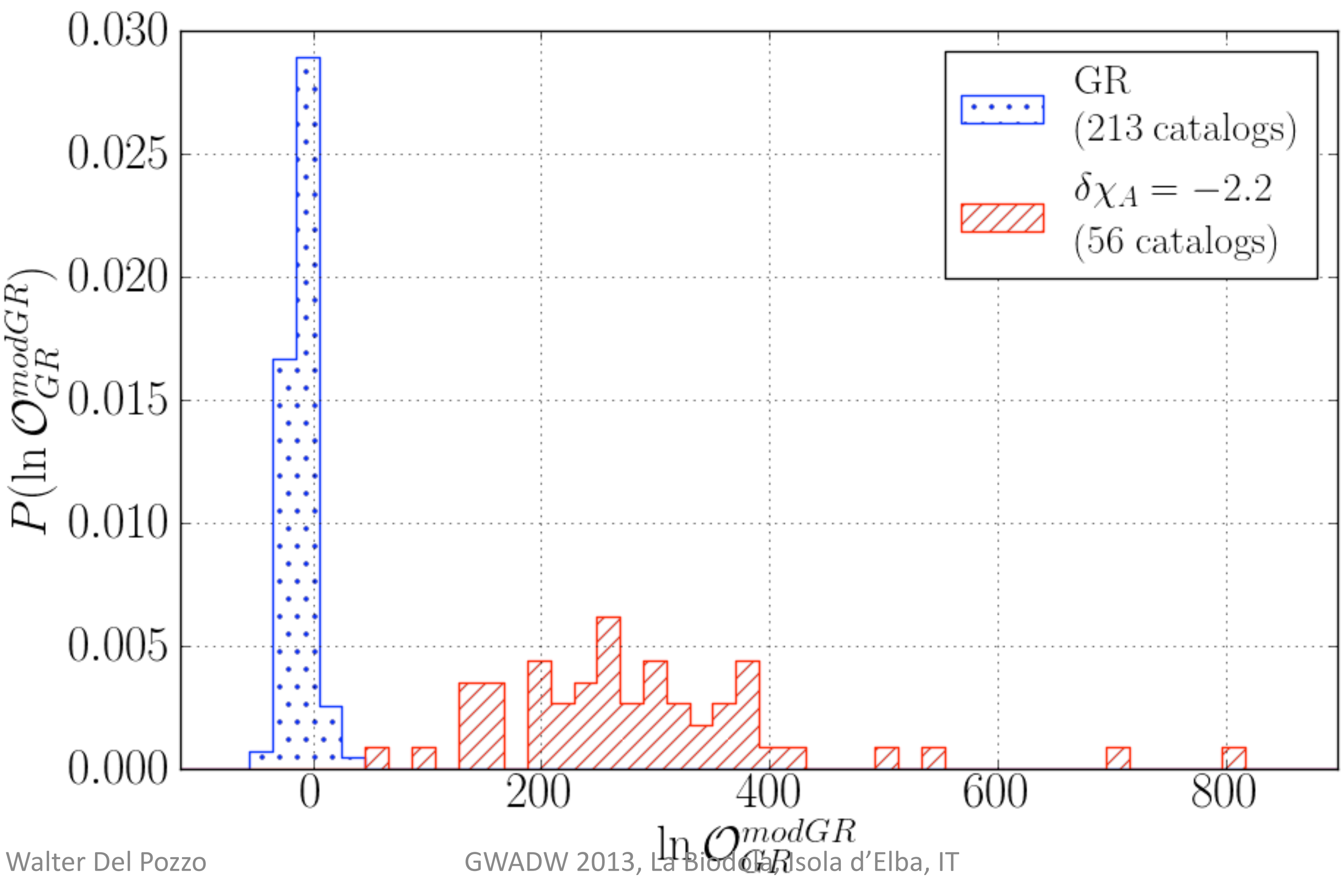
A deviation in a non-test parameter

- 2PN



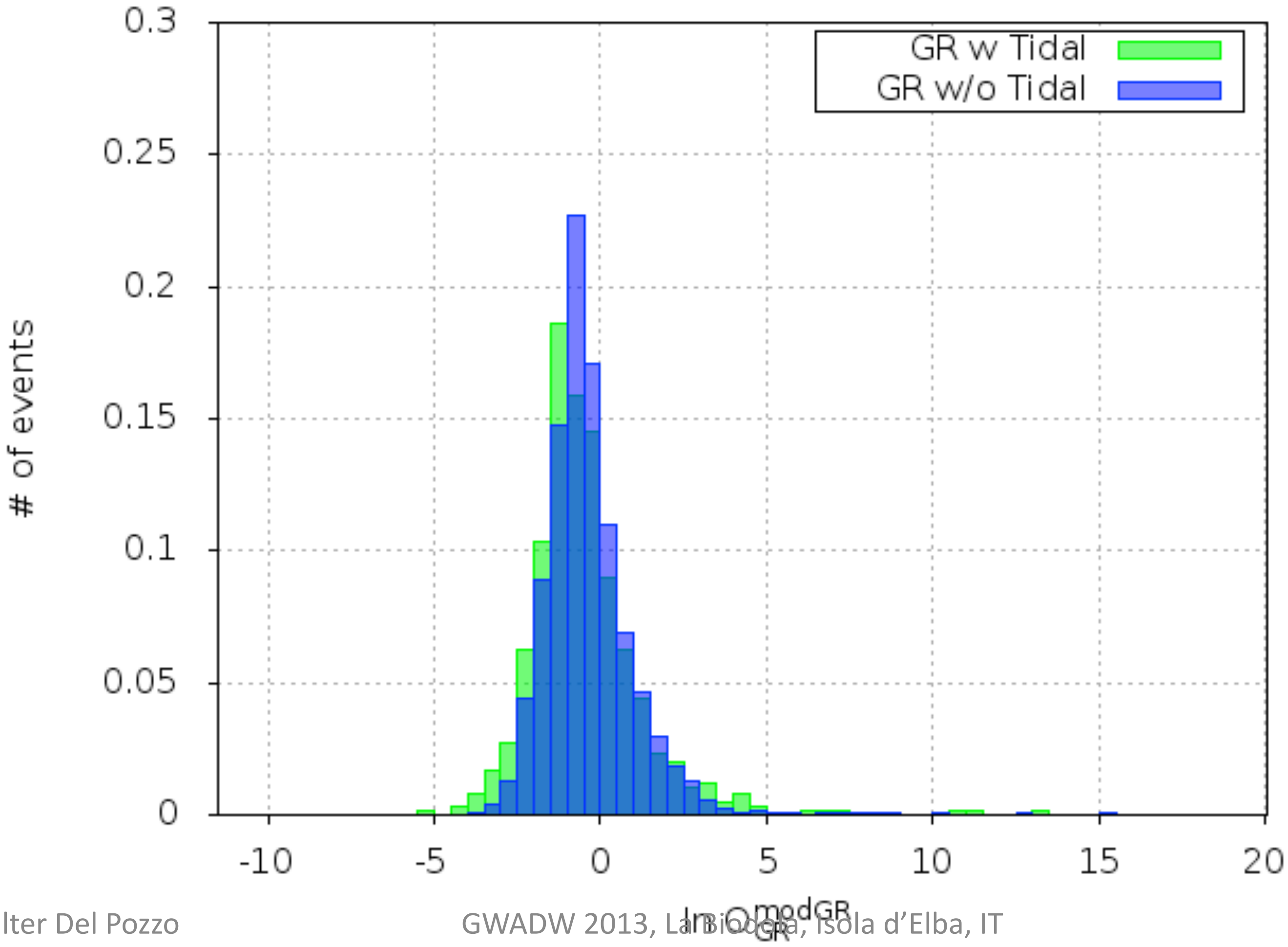
A deviation in a non-PN order

- 1.25PN

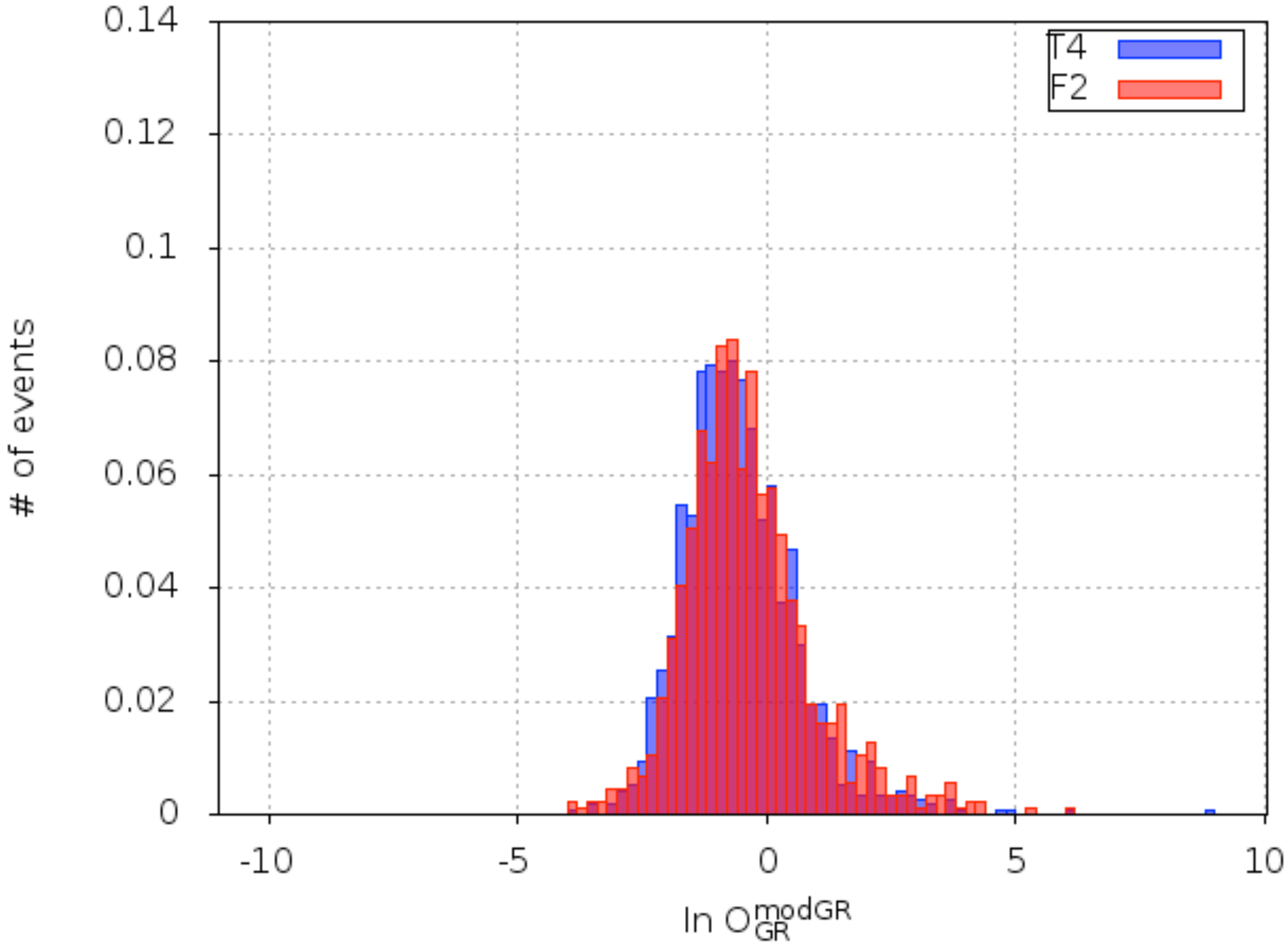


Finite size effects

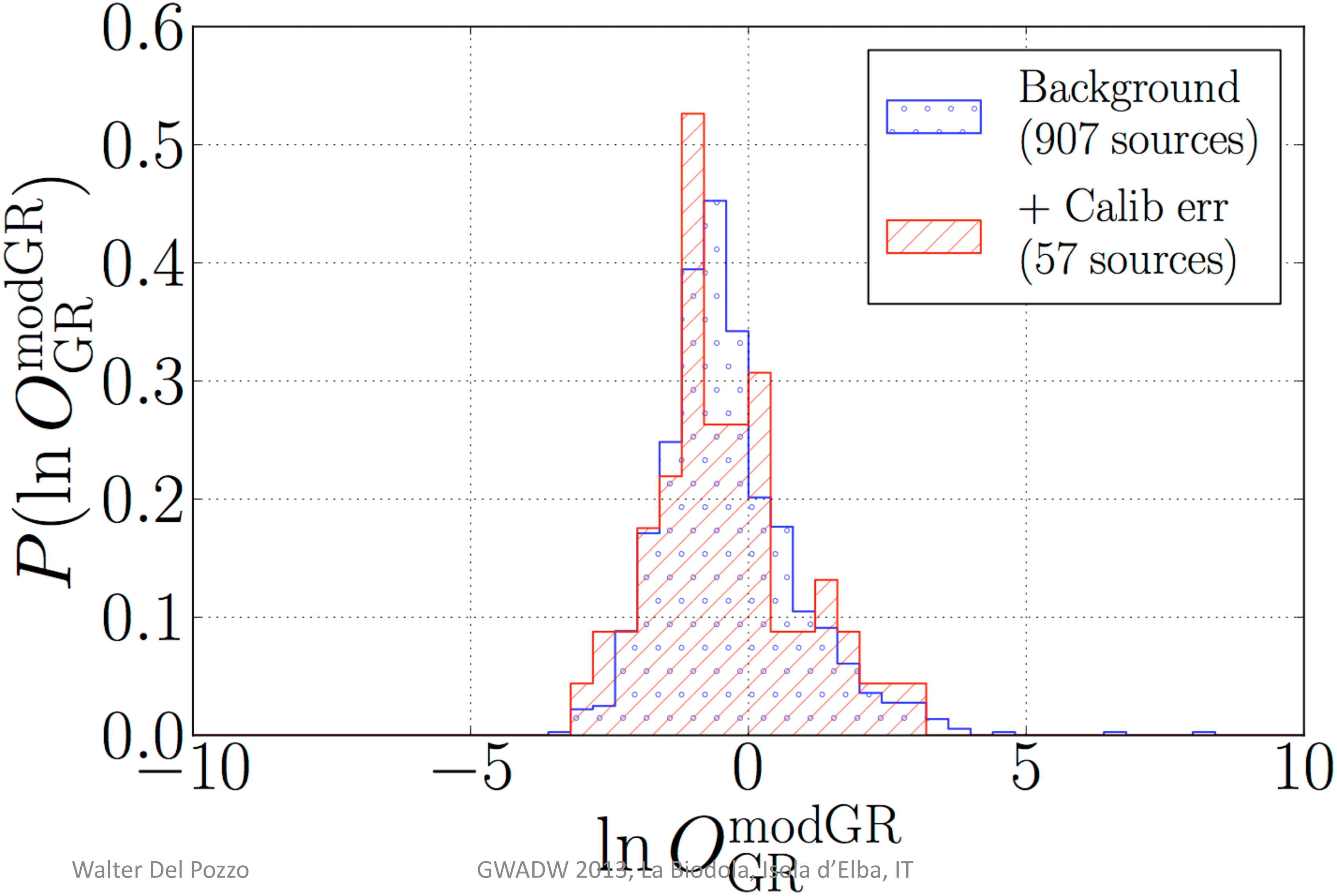
- Comparison of the backgrounds:



Waveform mismatch



Calibration errors



A note on parameter estimation