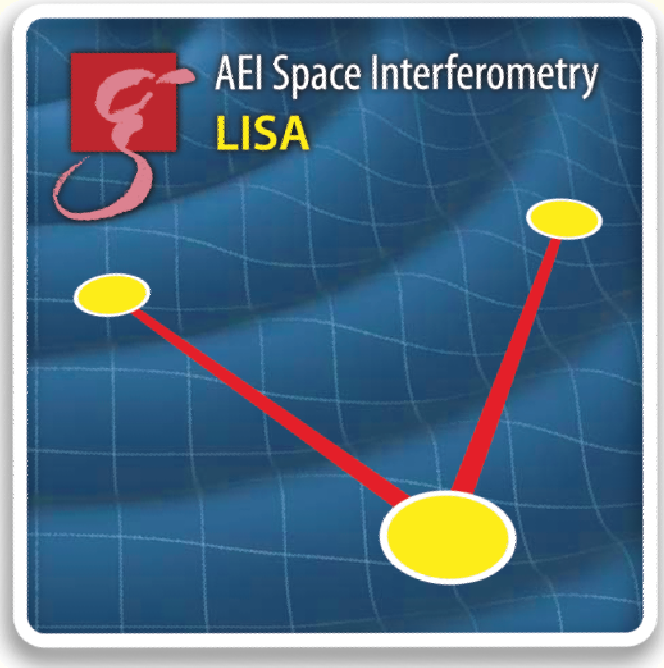
Some ideas on…



Alternative Configurations for Space-Based Gravitational Wave Detectors

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Leibniz Universität Hannover

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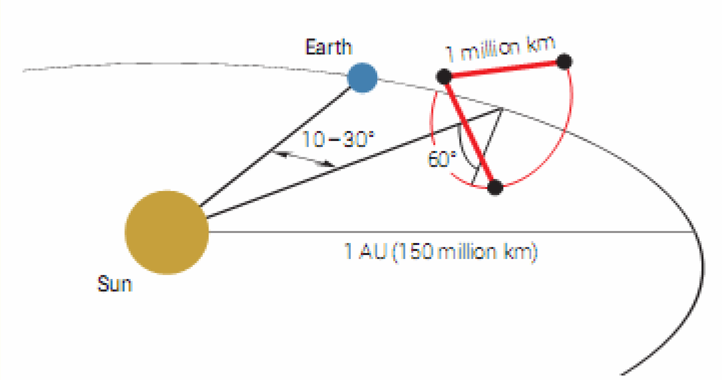
… in the vicinity of Lagrangian points

Space Interferometry at AEI Hannover

Submission of eLISA whitepaper for L2 slot

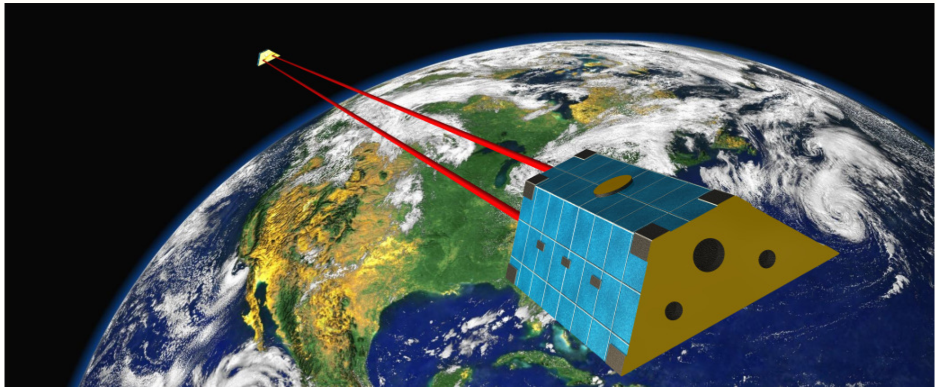
- "Gravitational Universe"
- deadline TODAY 12:00 noon
- talk/update by Guido Müller
- www.elisascience.org



[eLISA whitepaper 2013]

GRACE Follow-On to be launched in 2017

- measuring Earth's gravity field with unprecedented precision
- Laser Ranging Interferometer (LRI) as technical demonstrator
- heritage from LISA, learn for LISA
- last month: 5 PDRs for german units of LRI
- next week milestone LRI PDR



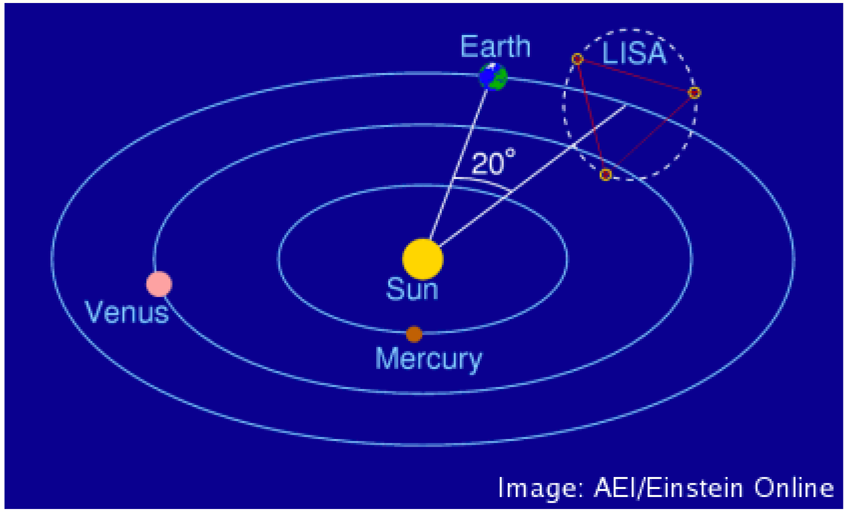
… just a short update.

Outline

Introduction

- May 2011: downscaled LISA mission proposal as an ESA-only mission (NGO)
  - Constellation with 2 arms (4 links) likely
  - Various orbits discussed:
    - Heliocentric, classical LISA
    - Heliocentric 90°
    - SAGITTARIUS/OMEGA like
    - Fly-away options
    - Halo orbits at Lagrangian Points L1 or L2

- GW mission proposals:
  - LAGOS, LISA, DECIGO, BBO,…
  - SAGITTARIUS/OMEGA, LISA2020,…
  - LAGRANGE



Halo orbits not considered so far for GW missions

Lagrangian Point Framework

- Under the assumption that $m_\odot \gg m_{\text{sat}} \quad \wedge \quad m_{\oplus☾} \gg m_{\text{sat}}$
and of a circular motion of Earth+Moon around the Sun,
the potential for a satellite in a co-rotating frame can be written as

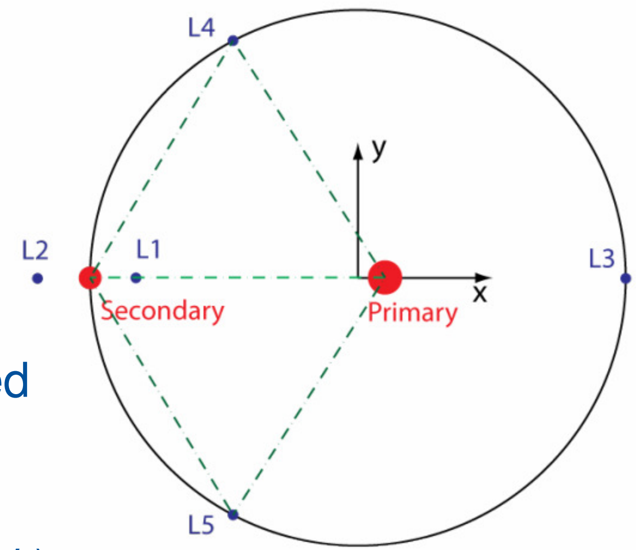
$$U(\vec{p}_{\text{sat}}) = \frac{1}{2}(x^2+y^2) + \frac{1}{2}\mu(1-\mu) + \frac{1-\mu}{|\vec{p}_{\text{sat}} - \vec{p}_\odot|} + \frac{\mu}{|\vec{p}_{\text{sat}} - \vec{p}_{\oplus☾}|} \tag{2.7}$$

$$= \frac{1}{2}(x^2+y^2) + \frac{1}{2}\mu(1-\mu) + \sum_{n=2}^{\infty} c_n \cdot \rho^n \cdot P_n\left(\frac{x}{\rho}\right), \tag{2.8}$$

$$\mu = \frac{m_{\oplus☾}}{m_\odot + m_{\oplus☾}} \approx 3.0404 \cdot 10^{-6},$$

- This is called the *Circular Restricted Three Body problem* (CR3BP). Good approximation for the real situation.

- The equilibrium points of the CR3BP are the so-called Lagrangian Points L1-L5.
  - *Satellites at Lagrangian Points will stay there.*
  - Earth/Moon – Sun system L1 & L2: 1.5 Mkm *(comm-link)*

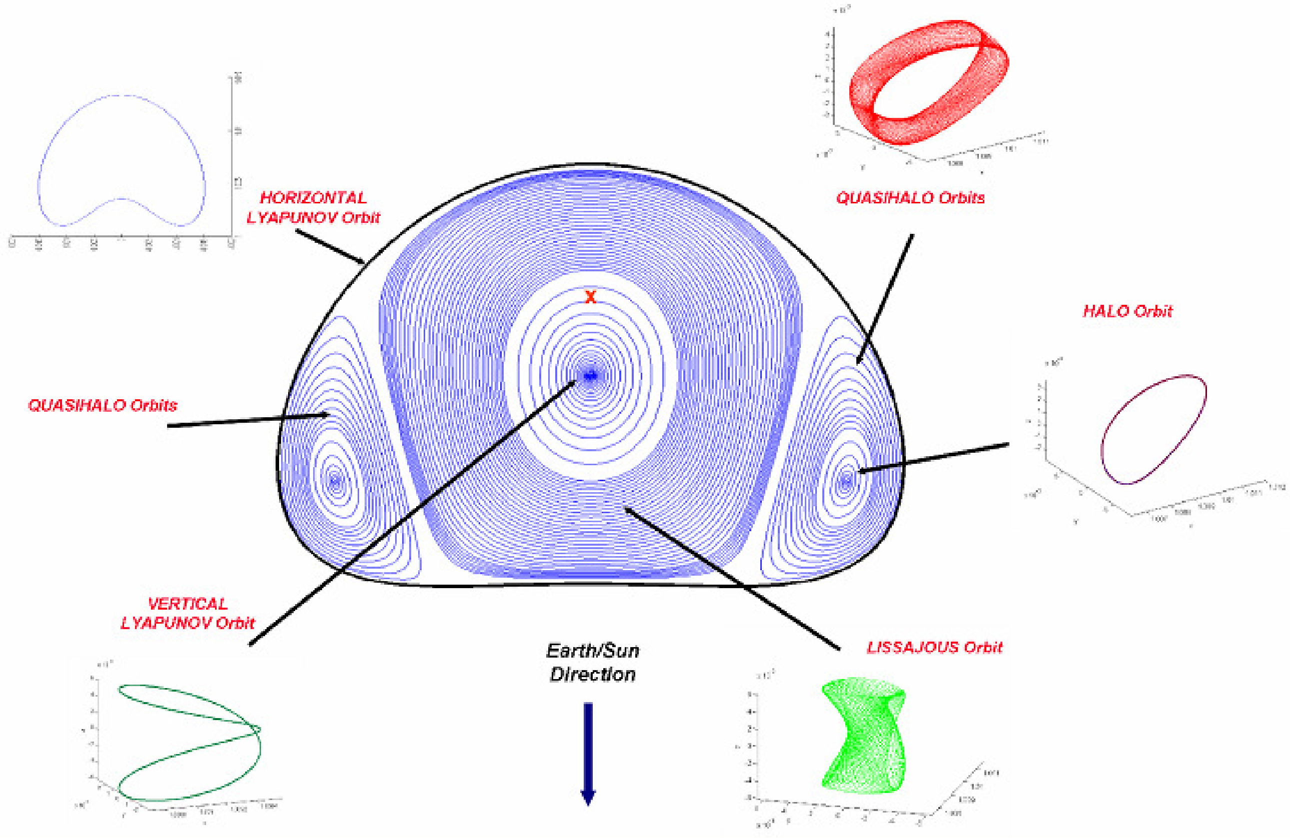


1950: A.C. Clark “broadcast TV signals to colonies on the Moon”

Lagrangian Point Dynamics

- Analytical approaches to find bounded solutions for the CR2BP equations of motion, usually exploiting a perturbation series.
- Poincare map: intersection of orbit trajectories with the ecliptic plane and with particular energy value

- **Lissajous orbits**
  - Unequal in-plane and out-of-plane frequencies
- **Special cases: zero frequency**
  - Horizontal Lyapunov (in-plane)
  - Vertical Lyapunov
- **Halo-Orbits (equal frequencies)**
  - Potato-chip form
  - Periodic
- **Quasi-Halo orbits**
  - Orbits around Halo-Orbits



[Kolemen et al., 2007]

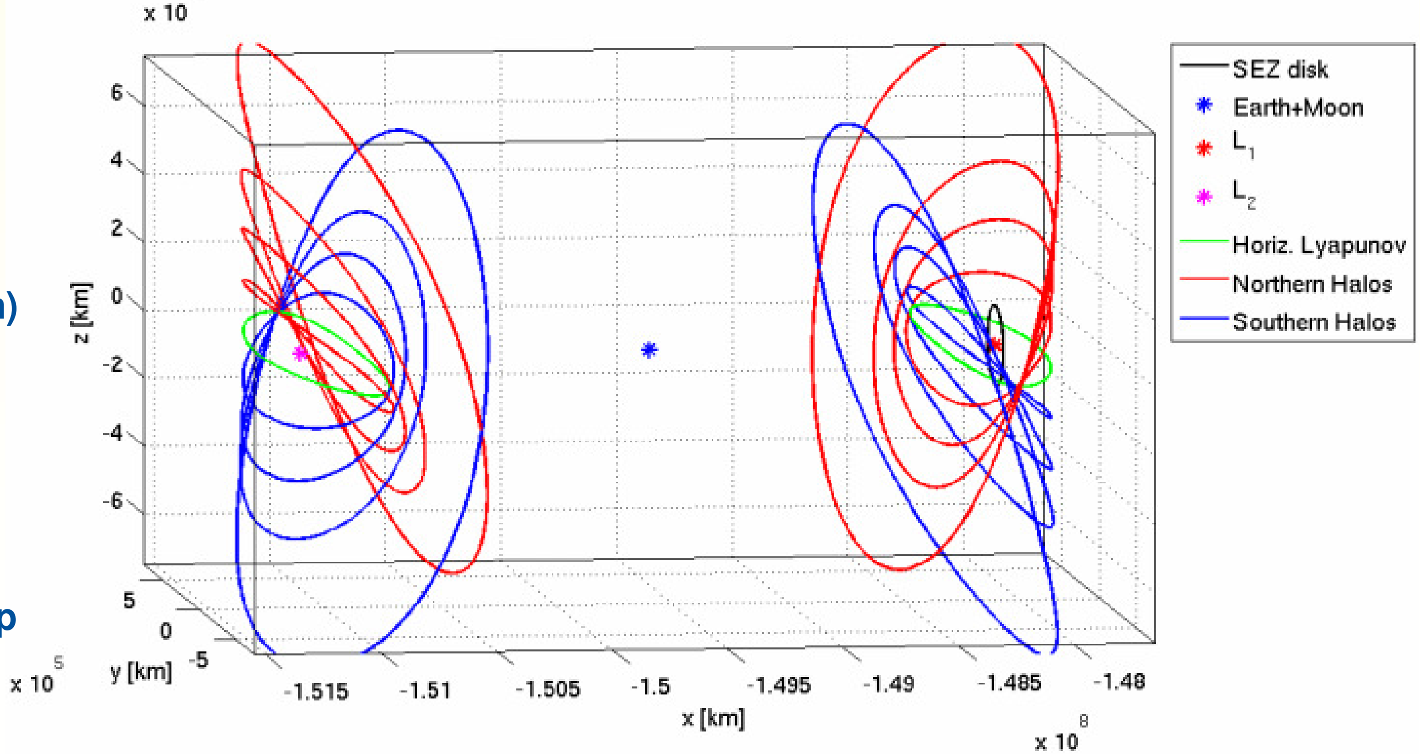
1960s: relay satellite for Apollo 17 on L2 Lissajous orbit (planned)

Halo Orbits

- **Halo orbits in 3rd order approximation:**

$$
\begin{aligned}
x &= -A_y/k \cdot \cos(\tau_1) + A_y^2 \cdot x_{2000} + A_z^2 \cdot x_{0200} + (A_y^2 \cdot x_{2020} + A_z^2 \cdot \cdot x_{0202}) \cdot \cos(2\tau_1) \\
&\quad + (A_y^3 \cdot x_{3030} + A_y \cdot A_z^2 x_{1212}) \cdot \cos(3\tau_1) \\
y &= (A_y + A_y \cdot A_z^2 \cdot y_{1210} + A_y^3 \cdot y_{3010}) \cdot \sin(\tau_1) + (A_y^2 \cdot y_{2020} + A_z^2 \cdot y_{0202}) \cdot \sin(2\tau_1) \\
&\quad + (A_y^3 \cdot y_{3030} + A_y \cdot A_z^2 \cdot y_{1212}) \cdot \sin(3\tau_1) \\
z &= A_z \cdot \cos(\tau_1) + A_y \cdot A_z \cdot z_{111(-1)} + A_y \cdot A_z \cdot z_{1111} \cdot \cos(2\tau_1) \\
&\quad + (A_z^3 \cdot z_{3030} + A_y^2 \cdot A_z \cdot z_{2121}) \cdot \cos(3\tau_1) \qquad (2.82)
\end{aligned}
$$

- **Two families (Northern and Southern)**
- **Orbit period ~180 days**
- **Usually stable thermal environment (observation of the Sun)**
- **Various missions e.g. LISA Pathfinder**
- **Orbit injection from Earth is relatively cheap in terms of fuel (~3.5 km/s).**

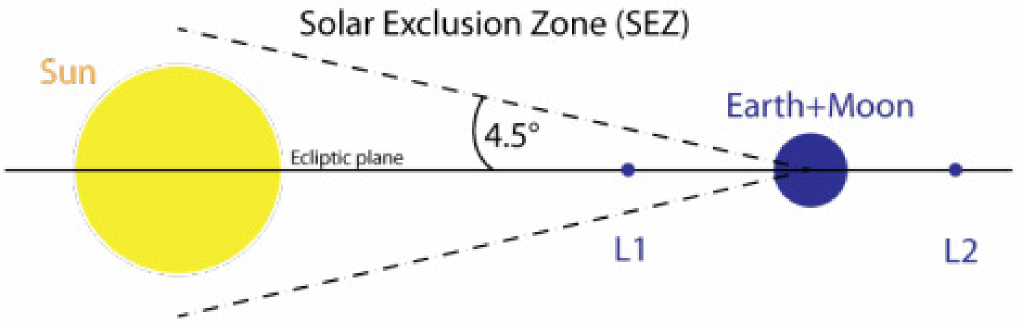


NGO: approx. 11 km/s/SC from LEO

Configuration Objectives

Q: Are there orbits in the vicinity of the LP, which form a triangular S/C constellation with

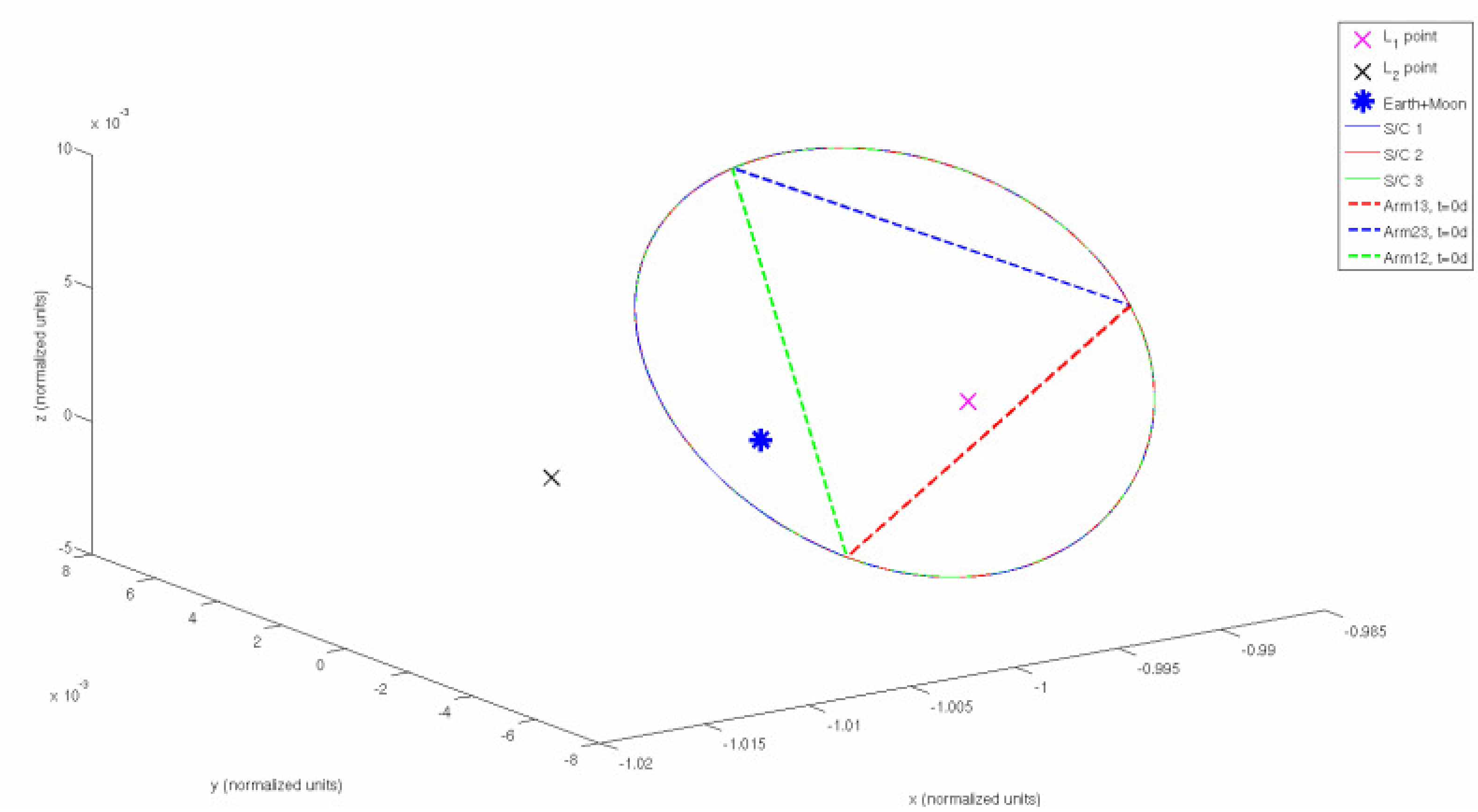
- ~million km armlength
  - due to meaurement band
- mission duration > 2yrs
- stable inner angle (between the arms)
  - picometer stable beam steering is challenging
- two or three interferometer arms
  - telescope (=mass) reduction
- avoidance of the Solar Exclusion Zone
  - communication down-link



The natural idea…

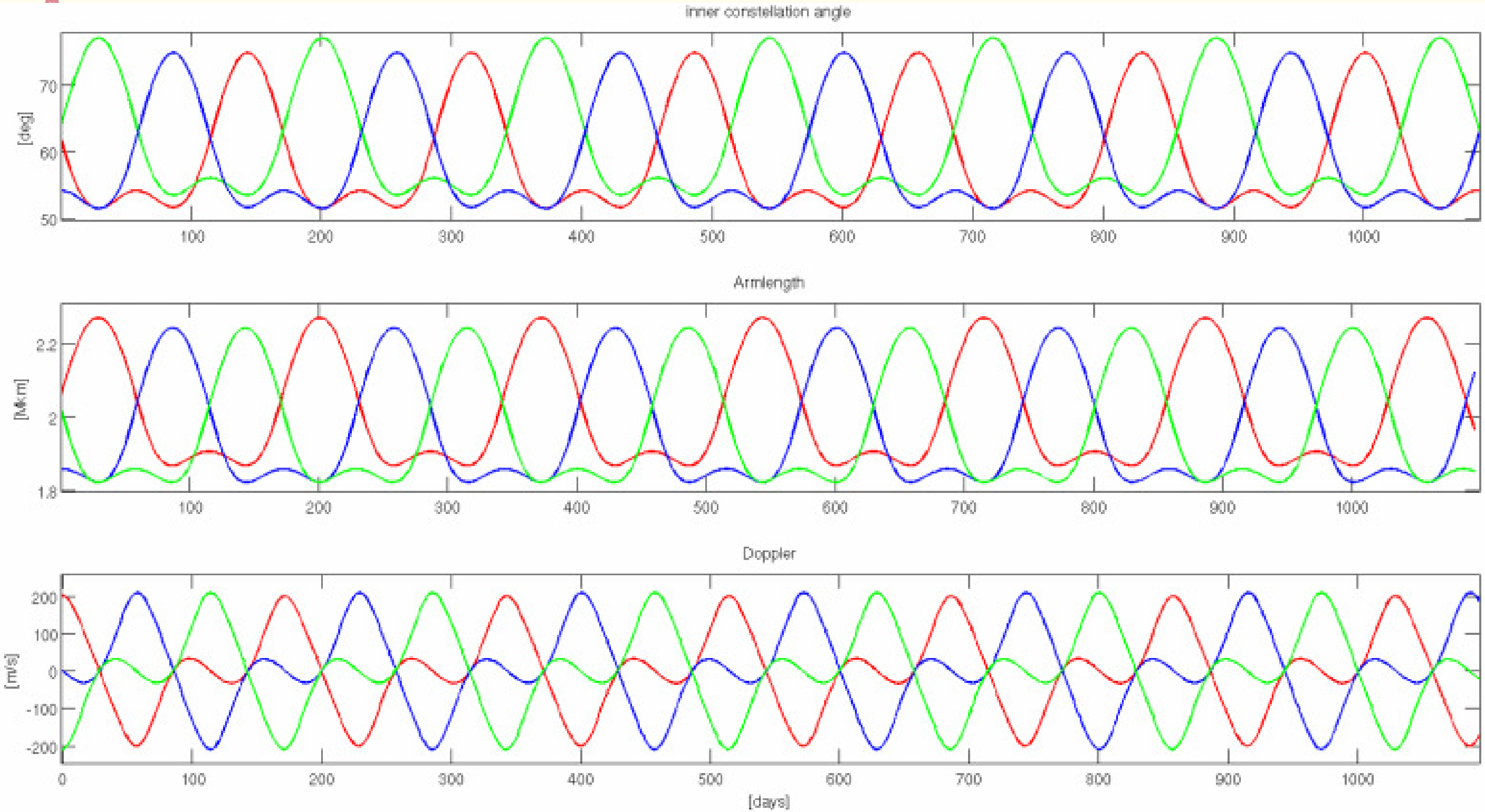
Configuration HHH60 – nominal Halo

- Classical 3-arm LISA on a nominal Halo orbit
- Parameter estimation of GW source benefits from higher revolution frequency (2/yr)



plot is deceptive….

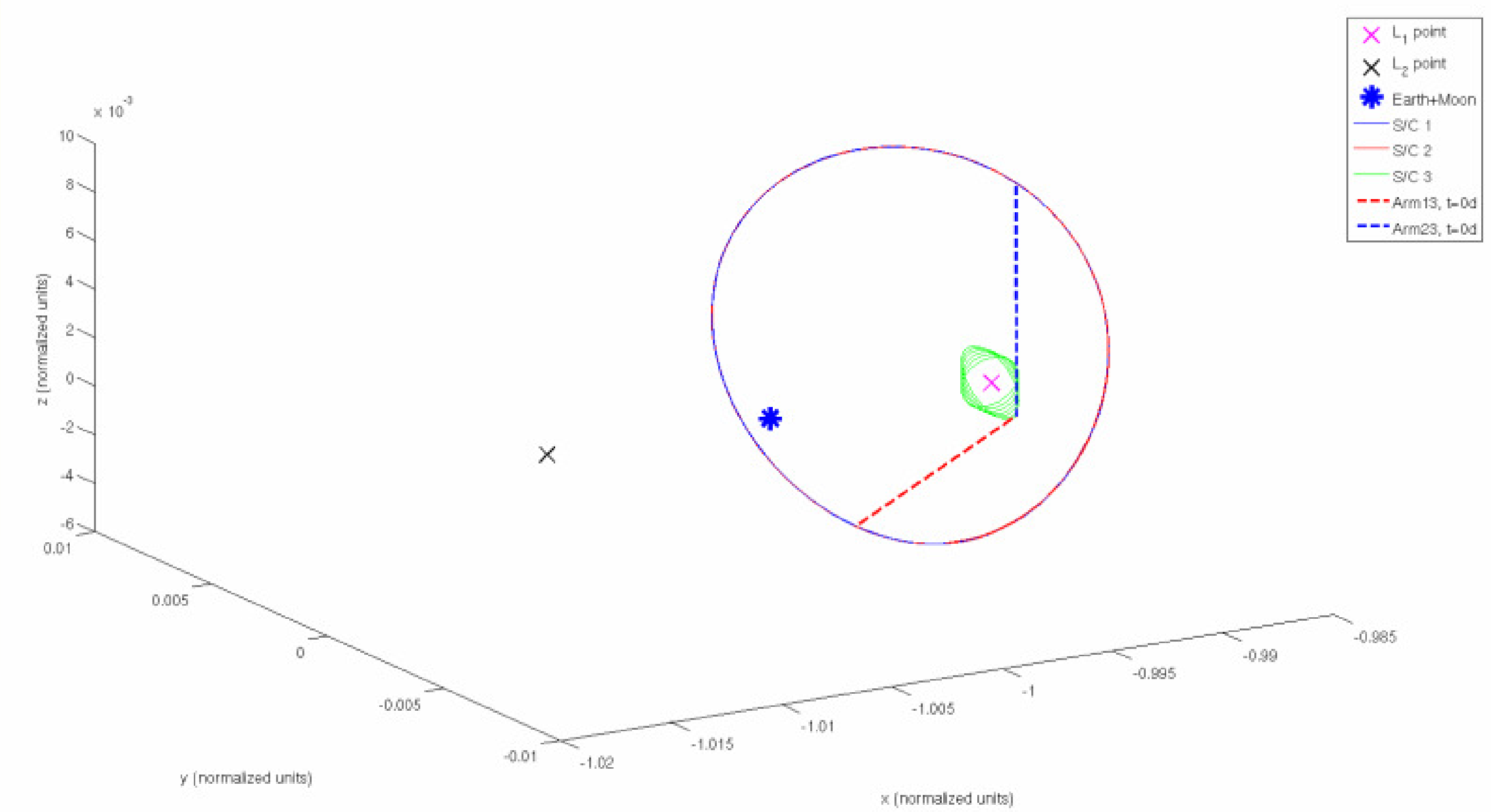
Configuration HHH60 – nominal Halo



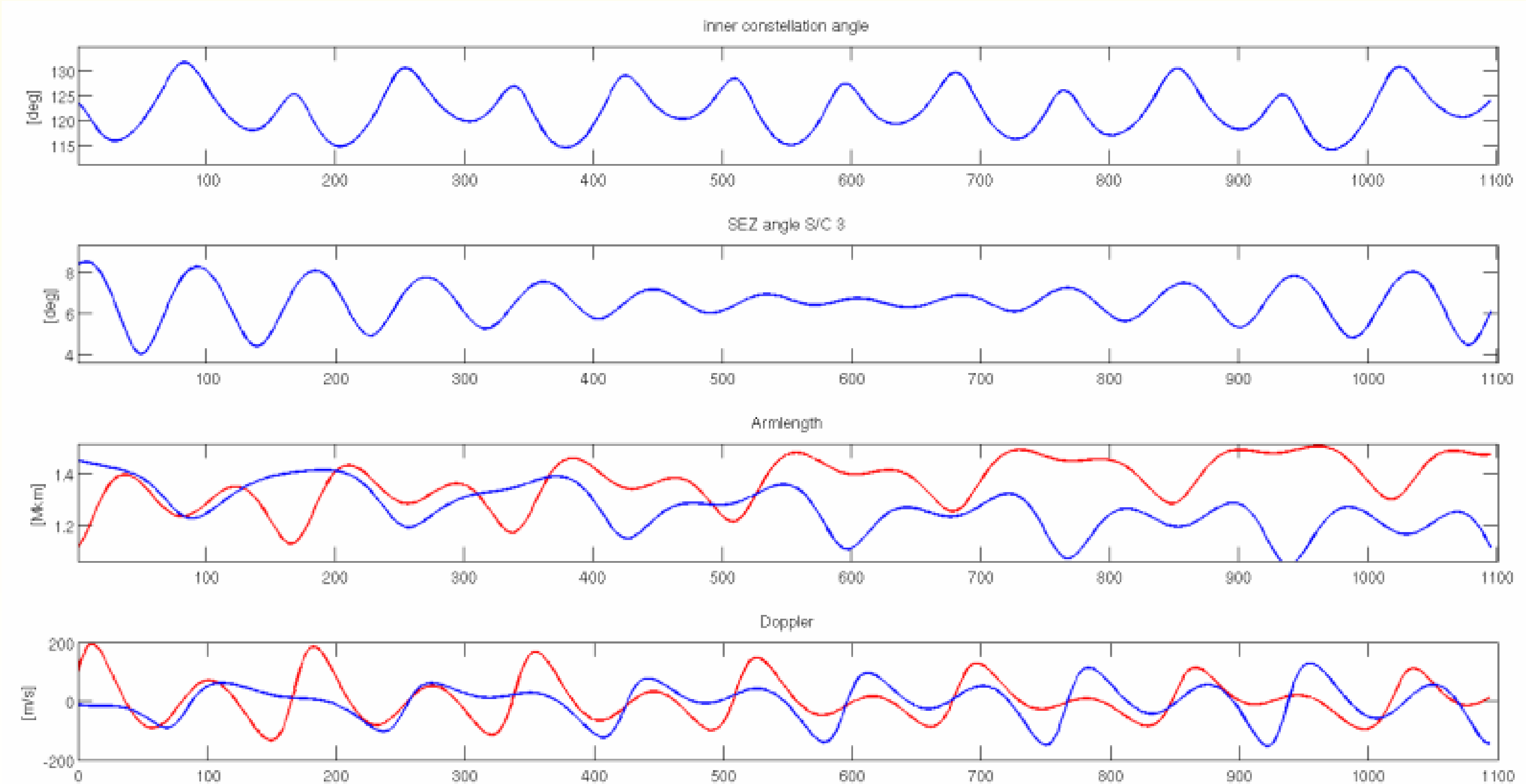
- Orbital velocity changes along the orbit
- Halo orbits are slightly elliptical and non-planar

Configuration HHL120 - nominal

- Two arm configuration utilizing 2 S/C on a large Halo, center S/C on a “circular” Lissajous orbit
- 120° inner constellation angle
- 1.35 Mkm armglength



Configuration HHL120 - nominal



- Secular drift due to unequal orbital periods for Halo and Lissajous orbits.

approx. 18° inner angle variation

Orbit Corrections

Result: No suitable orbits found when utilizing the analytical (or nominal) orbits, which require only ~1 m/s/yr orbit corrections to suppress the natural orbit instability.

New Approach: Introduce orbit corrections every X days, which are assumed to be instantaneous ΔV(elocity) thrusts on all S/C (at the same time) ⇨(short) measurement gaps

How to compute the orbit corrections?

Optimization method

- Define figure of merit (FoM) or cost function: function describing quality of constellation
- Target points / waypoints defined on nominal orbit trajectory
- Constrained Least-Squares fit in the CR3BP with gradient descent for FoM

decrease FoM

minimize total ΔV

$$\min_{\delta\vec{x}} ||\widehat{C} \cdot \delta\vec{x} - \vec{d}_c||^2 \quad \text{such that} \quad \begin{cases} \widehat{A} \cdot \delta\vec{x} \leqslant \vec{b} \\ \widehat{A}_e \cdot \delta\vec{x} = \vec{b}_e \\ \vec{l} \leqslant \delta\vec{x} \leqslant \vec{u} \end{cases}$$

thrust times equal on the 3 S/Cs

target points and thrust times

ensure small steps to stay in linearized regime

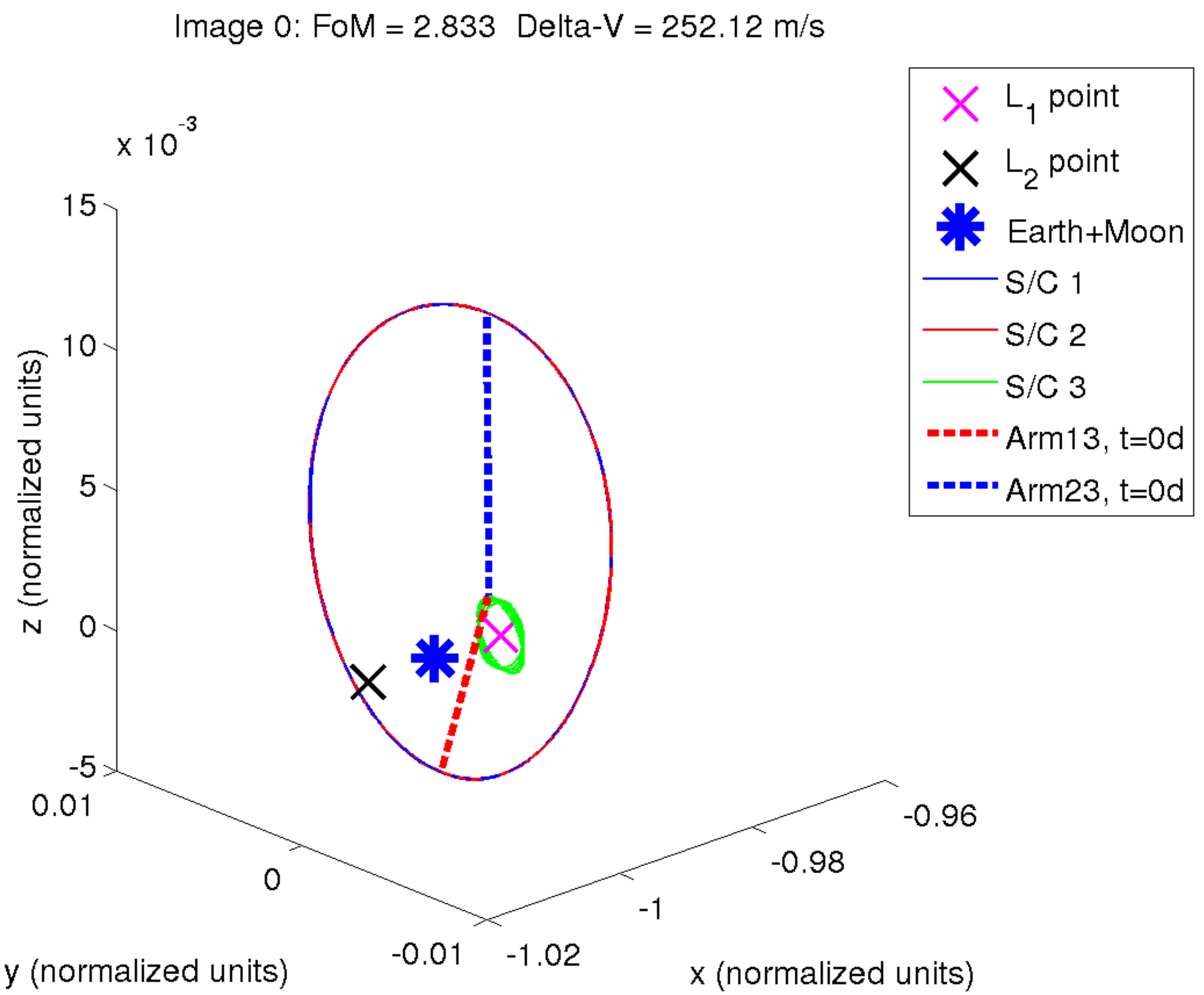
- Iterative optimization process:

$$\vec{x}^{(s+1)} = \vec{x}^{(s)} + \delta\vec{x}^{(s)}$$

- Target points yield ΔV corrections

a very simple approach, but capable of handling large dim problems

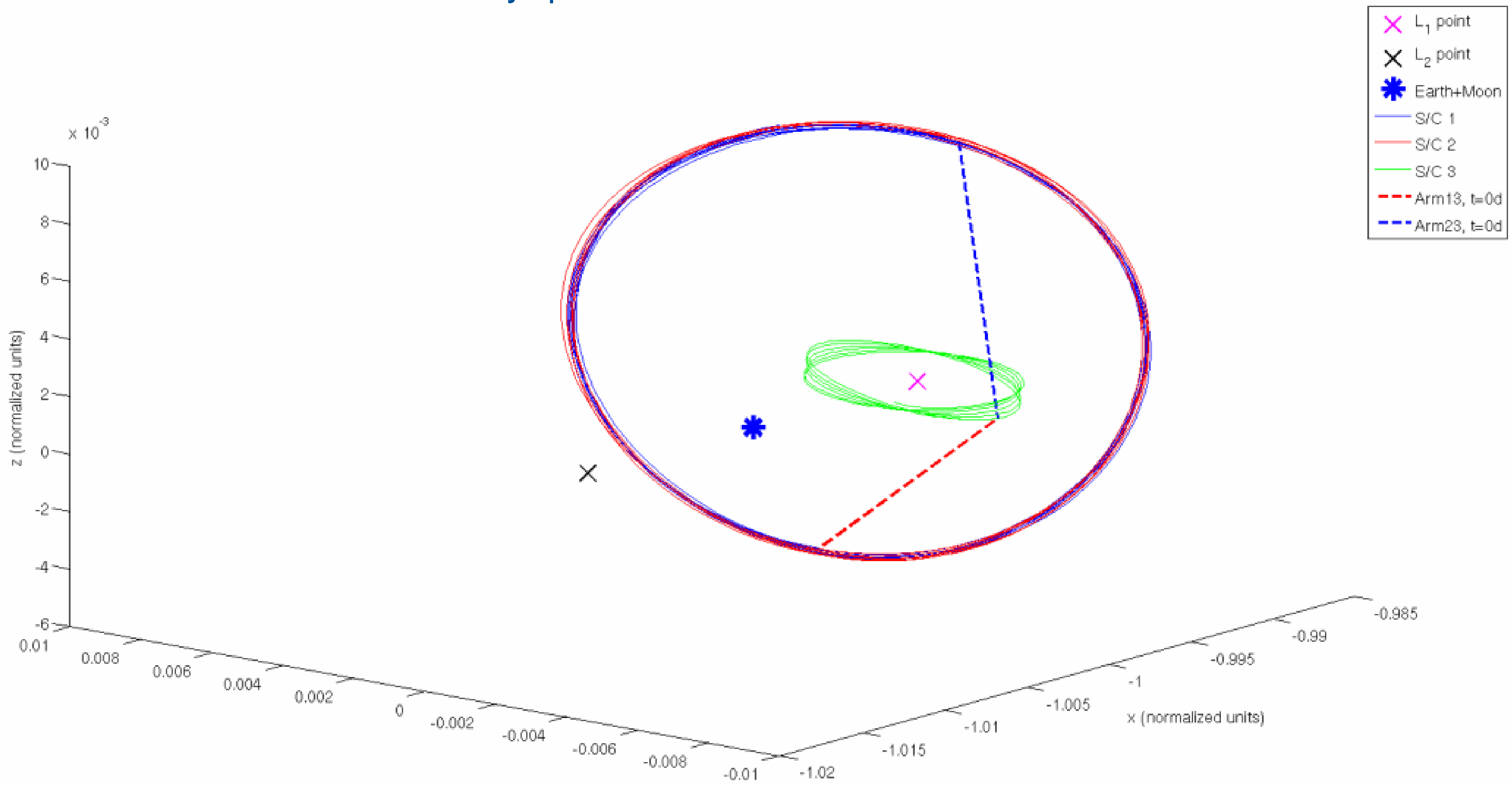
Visualized optimization process



FoM decreases

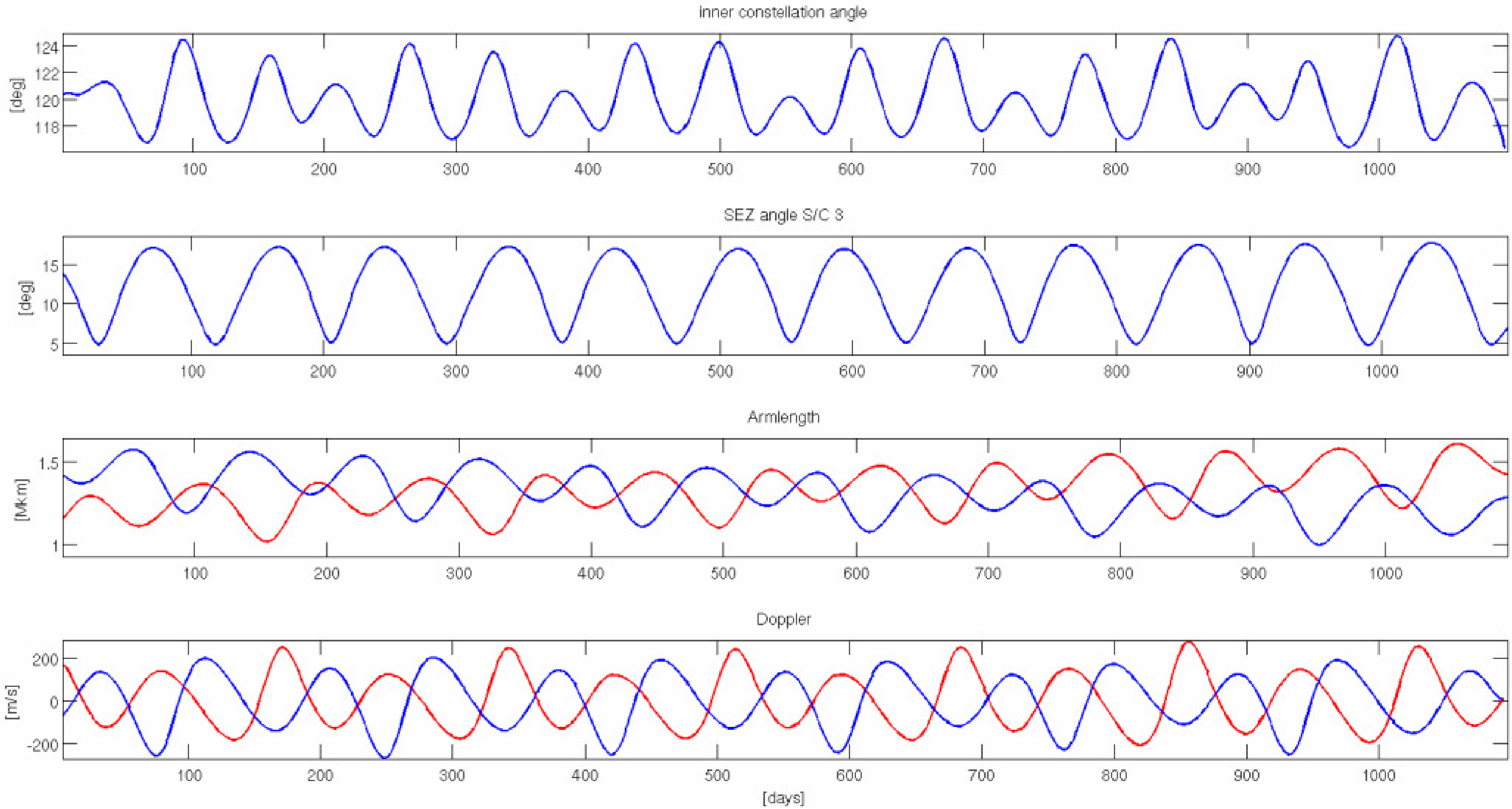
Optimized HHL120

- shape of Lissajous orbit changed to Halo-like orbit
- orbit corrections every ~28 days for 3 year period
- thrust below 30 m/s/yr per S/C



Optimized HHL120

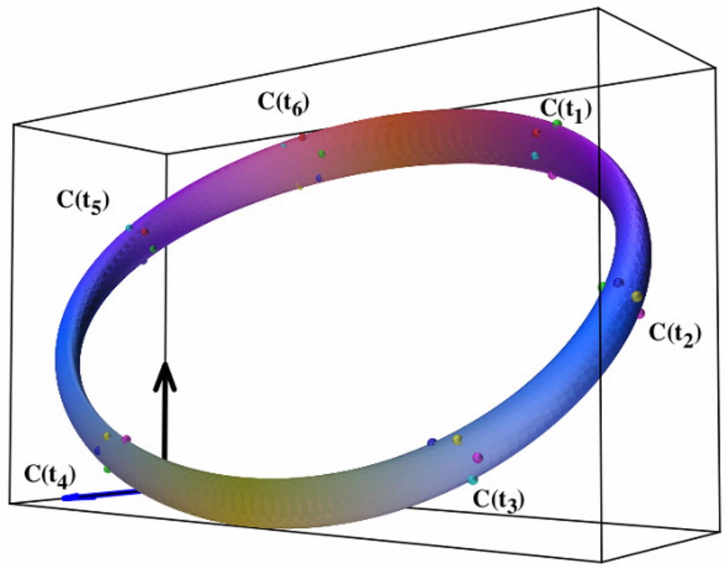
- Variation in inner constellation angle reduced from 18° to 8.3° (peak-peak)
- Lissajous orbit avoids SEZ



best result so far

Example: Quasi-Halo orbits

- Paper by Howell and Barden (1999) on formation flying using Quasi-Halo orbits
  - Deployed circular constellation of 6 S/C on a rotating ring around an inner Halo orbit
  - Quasi-Halo orbits form a torus around a nominal Halo orbit
  - Control effort (ΔV) scales with radius of the circle, 1000 km radius seams feasible
  - Deformation of the constellation appears



[Howell and Barden, 1999]

- Computation of analytical trajectories relatively non-trivial
  - here: no simulations performed
- Not very usefule for LISA-like missions due to small armlength

Maybe useful for a DECIGO like detector

OGO: Octahedral Gravitational Observatory

Octahedron configuration for a displacement noise-cancelling gravitational wave detector in space

Yan Wang,[1] Stanislav Babak,[2] Antoine Petiteau,[2,3] David Keitel,[1] Markus Otto,[1] Simon Barke,[1] Fumiko Kawazoe,[1] Alexander Khalaidovski,[1] Vitali Müller,[1] Daniel Schütze,[1] Holger Wittel,[1] Karsten Danzmann,[1] and Bernard F. Schutz[2]

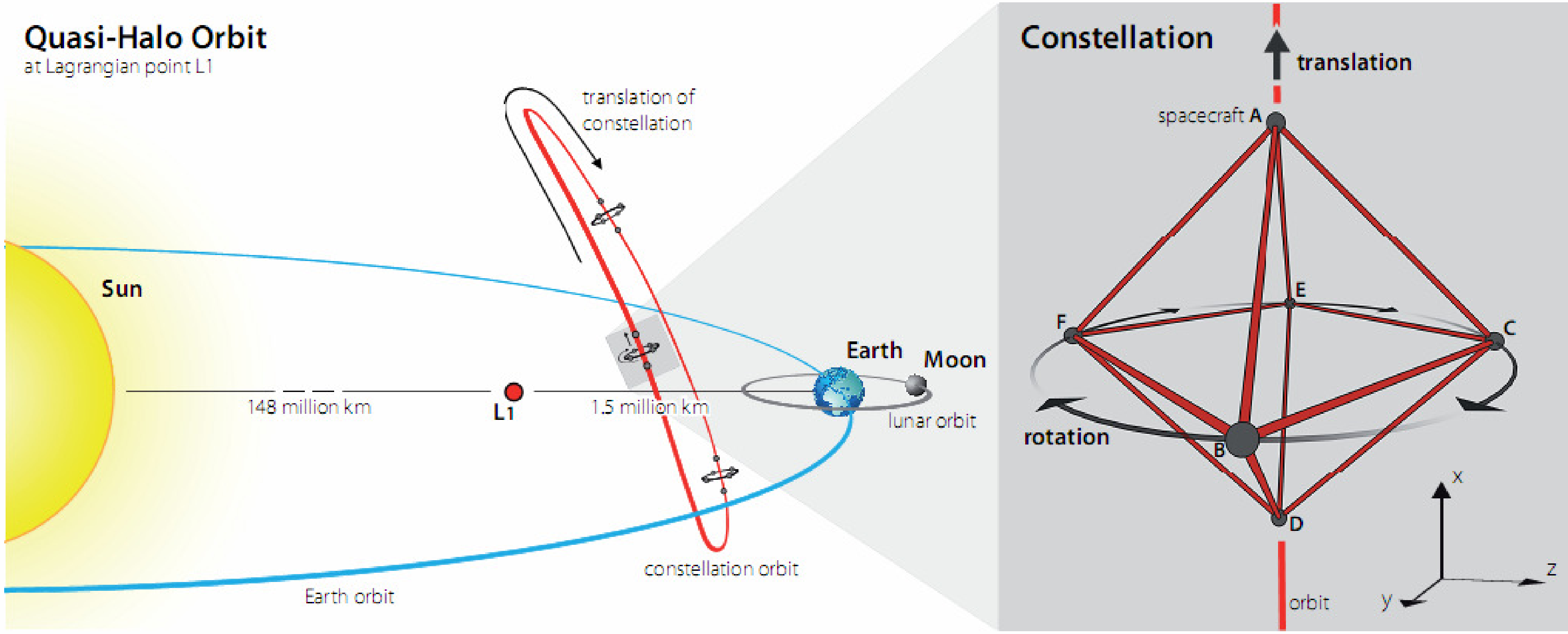
- OGO started as PhD students' project on an IMPRS Lecture week on Mallorca, Nov 2011
- Paper ready to be submitted (sent around on LSC P&P mailing list for final comments)
- Inspired by Displacement-Noise Free Interferometry (DFI) scheme

[1] S. Kawamura and Y. Chen. Displacement-noise-free gravitational-wave detection. *Phys. Rev. Lett.* **93**, 211103 (2004).
[2] Y. Chen., A. Pai and K. Somiya. Interferometers for displacement-noise-free gravitational-wave detection. *Phys. Rev. Lett.* **97**, 151103 (2006).
[3] Y. Chen and S. Kawamura. Displacement- and timing-noise-free gravitational-wave detection. *Phys. Rev. Lett.* **96**, 231102 (2006).

- transferred DFI to a space-borne observatory, used LISA-like link concept with Time-Delay Interferometry (TDI) techniques
- Basic Idea: To have more independent measurements than noise channels
  ⇨ cancellation of acceleration and laser frequency noise.

orbits for octahedral S/C constellation…

OGO concept

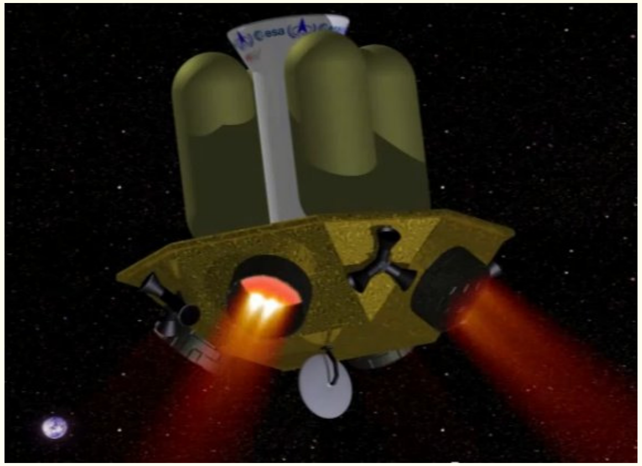


- spacecraft A and D on inner Halo orbit; B,C,E,F on Quasi-Halo orbits.
- 12 arms, 24 links or science measurement channels vs. 23 noise sources

- 6 testmasses with maximum 6x3=18 components for acceleration noise
- side length / armlength: 1414 km
- problems might be constellation deformations, blinding of arms, thermal stability

proper orbit simulations and formation control required

OGO concept II



- Computation of laser frequency and acceleration noise free combinations of the 24 channels

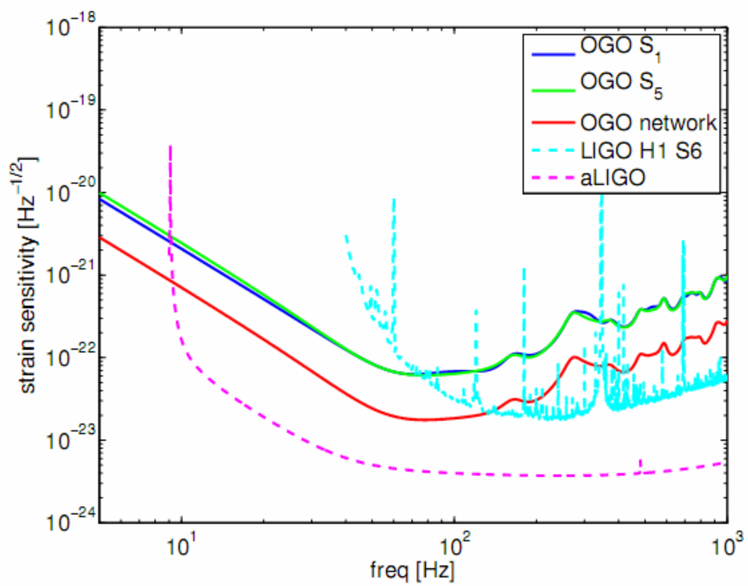
$$s^{\text{tot}}_{IJ} = h_{IJ} + b_{IJ} + \mathcal{D}p_I - p_J + \mathcal{D}\left(\vec{a}_I \cdot \hat{n}_{IJ}\right) - \left(\vec{a}_J \cdot \hat{n}_{IJ}\right)$$

  - Assumption of perfect regular octahedron, deformations absorbed in low-frequency term

- Naive idea:
  - Only shot noise would lead to a flat sensitivity at low frequencies: $\epsilon \equiv \omega L/2$

$$\sqrt{\widetilde{S}_{h,X}} \sim \sqrt{\widetilde{S}_{\mathrm{n},X}/\mathcal{T}_X} \sim \epsilon^0 \ \text{for TDI-X}$$

- However:
  - Transfer function for OGO $\mathcal{T}_{1,2,\ldots,7} \sim \epsilon^3$
  - Sensitivity: $\sqrt{\widetilde{S}_{h,1,2,\ldots,7}/\mathcal{T}_{1,2,\ldots,7}} \sim \epsilon^{-2}$

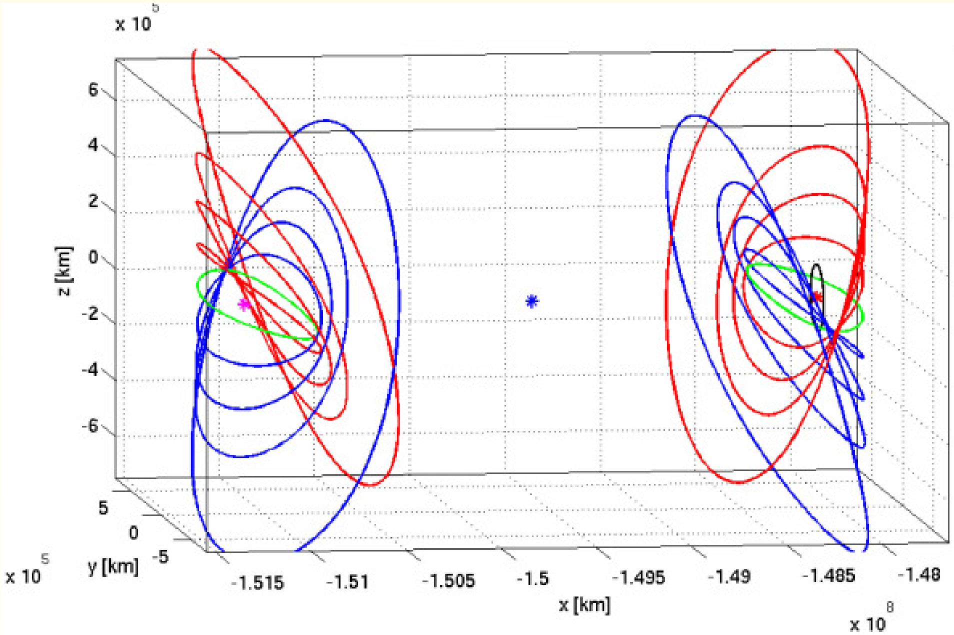


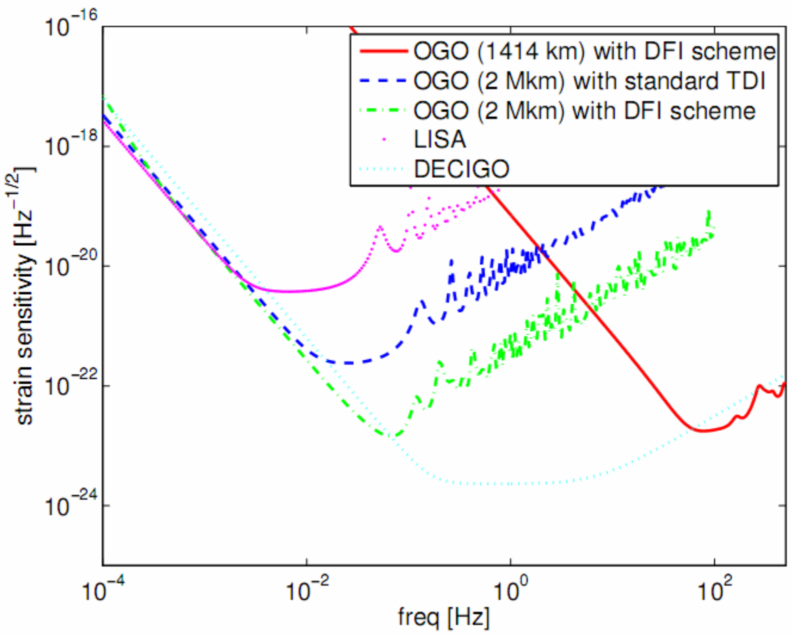
- No drag-free system required
- Each S/C has a testmass (likely with octahedral/pyramid shape) and 4 telescopes

consistent with original DFI results.

Large OGO

- 2 Mkm OGO
  - 4 S/C on very large Halo L1/L2
  - 1 S/C on small L1 Halo
  - 1 S/C on small L2 Halo





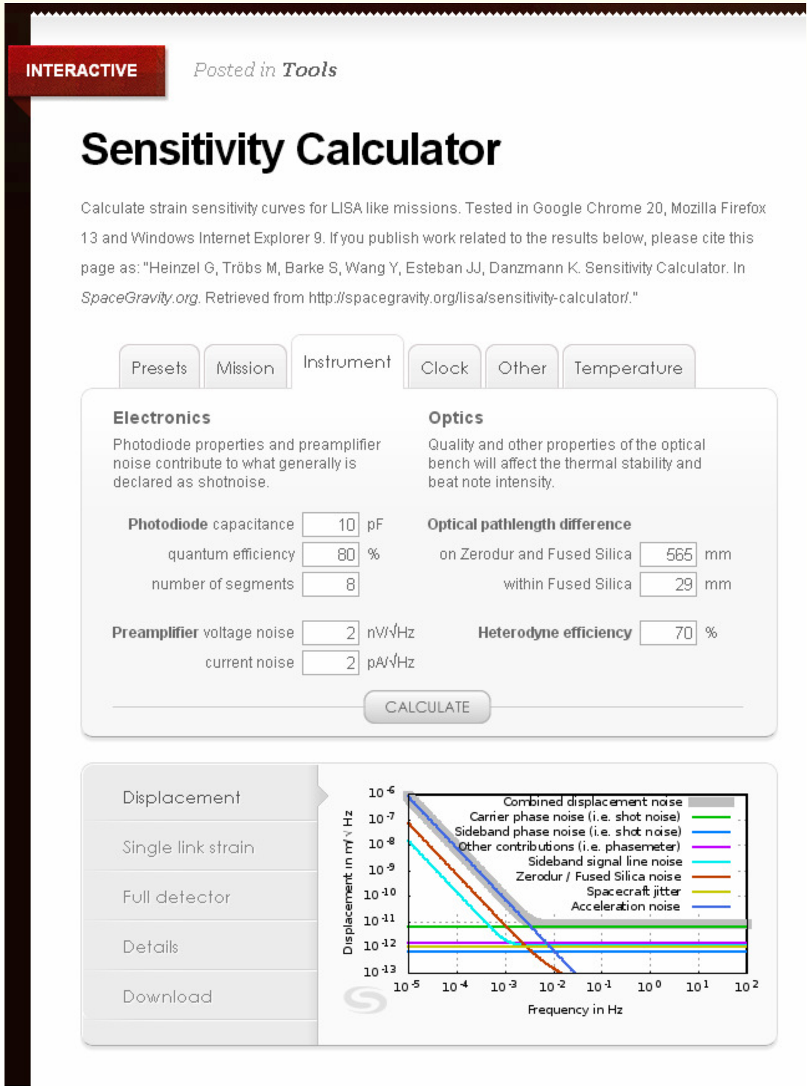
- Slightly asymmetric 2 Mkm constellation and large distortions using the analytical (nominal) trajectories.

Analysis of DFI scheme with deformations outstanding.

Sensitivity calculator

A sensitivity curve computed from laser power and interferometer armlength is not the whole truth.

- Started web-sensitivity calculator considering various technical aspects
  - http://spacegravity.org/lisa/sensitivity-calculator/
  - User: “Space”, Pass: “Gravity”
- Considers / Input:
  - Mission parameters (armlength, power …)
  - Electric noise
  - Clock noise (temperature dependent)
  - Pathlength OB noise (temperature dependent)
  - Pre-defined setups (LISA, eLISA, OGO)
- Useful for educational purpose or playing around with mission parameters.



OGO: PM 0.1 nrad/√Hz; Temp. stability: 1 nK/√Hz

Results & Conclusion

Ideas on alternative configurations

- Lagrangian points:
  - rich dynamics (Halo, Lissajous, Quasi-Halo orbits)
  - close to Earth, easy to reach in terms of fuel, higher GW modulation
  - natural orbits not suited, therefore orbit corrections required
  - problem: holding of TM during correction?
  - 2-arm constellation with 1.4 Mkm and 120° inner constellation angle seems promising
    - ± 4.2° variation (no fundamental limit, yet)
    - 200 m/s Doppler shift
    - thermal stability issue
  - other constellations (HHH60, HHL90) analyzed, but result not as good as for HHL120.

- OGO mission concept
  - a gedankenexperiment for a DFI space-based detector (without drag free system)

- Sensitivity calculator also considering technical details

Thank you for you attention.