Dark Energy, Dark Matter and Gravity

General Relativity, Dark Energy & Dark Matter

(R) theories of gravity with non-minimal curvature-matter coupling

Implications, Energy Conditions, Stability & traversable wormholes

Dark Energy - Dark Matter Interaction & Equivalence Principle

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General Relativity
($\gamma = \beta = 1$)

- **GR has survived all tests so far…**

  [C. Will, gr-qc/0510072; S. Turyshev, M. Shao, K. Nordtvedt, gr-qc/0601035]
  [O.B., J. Páramos, S. Turyshev, gr-qc/0602016]

- **Parametrized Post-Newtonian Formalism (U-gravitational potential, v_i velocity)**

$$g_{00} = -1 + 2U - 2\beta U^2 + \ldots, \quad g_{ij} = (1 + 2\gamma U)\delta_{ij} + \ldots, \quad g_{0i} = -\frac{1}{2}(4\gamma + 3)v_i + \ldots$$

- **Local (solar system) tests**

Mercury´s perihelion shift: $|2\gamma - \beta - 1| < 3 \times 10^{-3}$ **[Shapiro 1990]**

Lunar Laser Ranging: $4\beta - \gamma - 3 = (4.4 \pm 4.5) \times 10^{-4}$ **[Williams, Turyshev, Boggs 2004]**

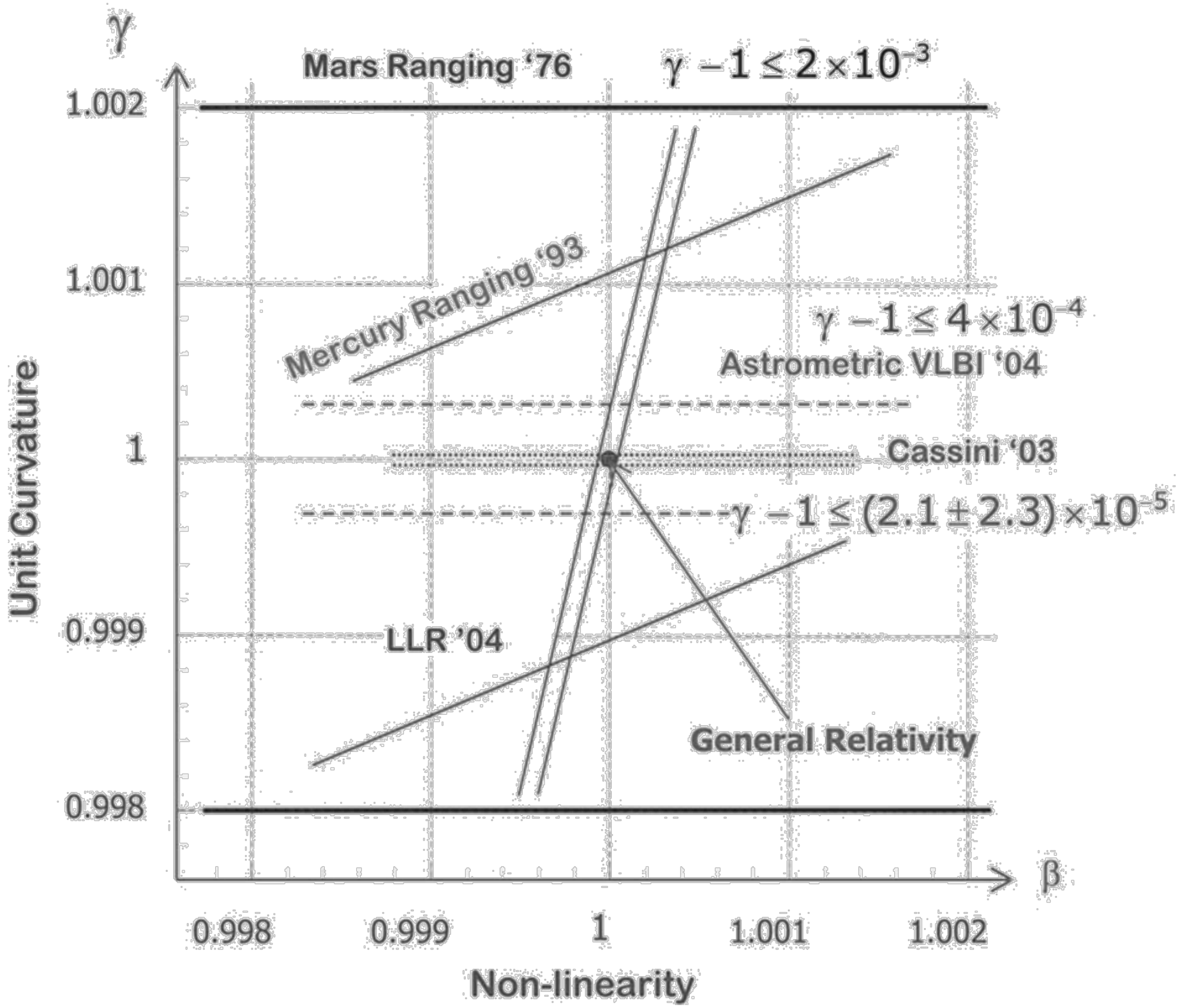
LBLI light deflection: $|\gamma - 1| < 4 \times 10^{-4}$ **[Eubanks et al. 1997]**

Cassini Experiment: $\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$ **[Bertotti, Iess, Tortora 2003]**

Cassini-Huygens Radiometric Experiment

B. Bertotti, L. Iess and P. Tortora, Nature **425** (2003) 374

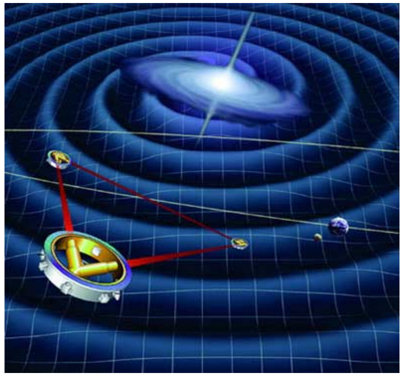
Summary of the General Relativity Tests



Partially confirmed predictions:

Gravitational waves – PSR B1913+16

(LIGO, …, LISA)



Lense-Thirring Effect

(Gravity Probe-B)

19% Accuracy - May 2011

BepiColombo Mission to Mercury (ESA/ISAS)

$$\frac{\Delta J_2}{J_2} < 10^{-9}, \frac{\Delta \gamma}{\gamma} < 2.5 \times 10^{-6}, \frac{\Delta \beta}{\beta} < 5 \times 10^{-6}$$

$$\frac{\Delta \eta_1}{\eta_1} < 2 \times 10^{-5}, \eta_1 = -1 - \beta + 2\gamma$$

Cosmological Tests of General Relativity

- Outstanding challenges (GR + Quantum Field Theory)
  - Singularity Problem
  - Cosmological Constant Problem
  - Underlying particle physics theory for Inflation

- Theory provides in the context of the Big Bang model an impressive picture of the history of the Universe
  - Nucleosynthesis ($N_\nu < 4$, $\Omega_B h^2 = 0.023 \pm 0.001$)
  - Cosmic Microwave Background Radiation
  - Large Scale Structure
  - Gravitational lensing
  - …

- Required entities (missing links):
  - Dark Matter
  - Dark Energy

Dark Matter

- **Evidence:**

  Flatness of the rotation curve of galaxies
  Large scale structure
  Gravitational lensing
  N-body simulations and comparison with observations
  Merging galaxy cluster 1E 0657-56
  Massive Clusters Collision Cl 0024+17
  Dark core of the cluster A520

- **Cold Dark Matter (CDM) Model**

  Weakly interacting non-relativistic massive particle at decoupling

- **Candidates:**

  Neutralinos (SUSY WIMPS), axions, scalar fields, self-interacting scalar particles (adamastor particle), *etc.*

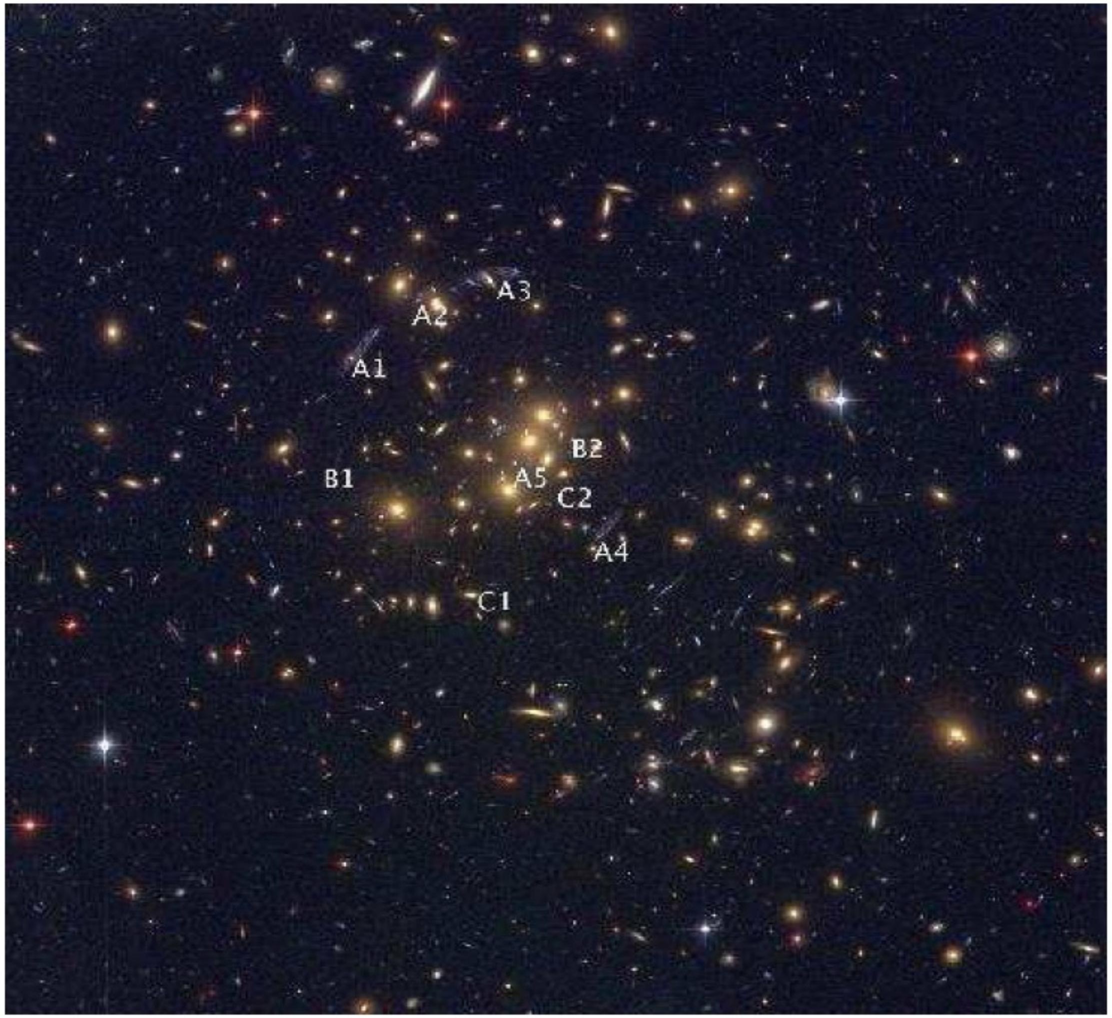
Merging Galaxy Cluster 1E 0657-56

[Clowe et al., astro-ph/0608407]

“Bullet” Cluster

Massive Clusters Collision Cl 0024+17

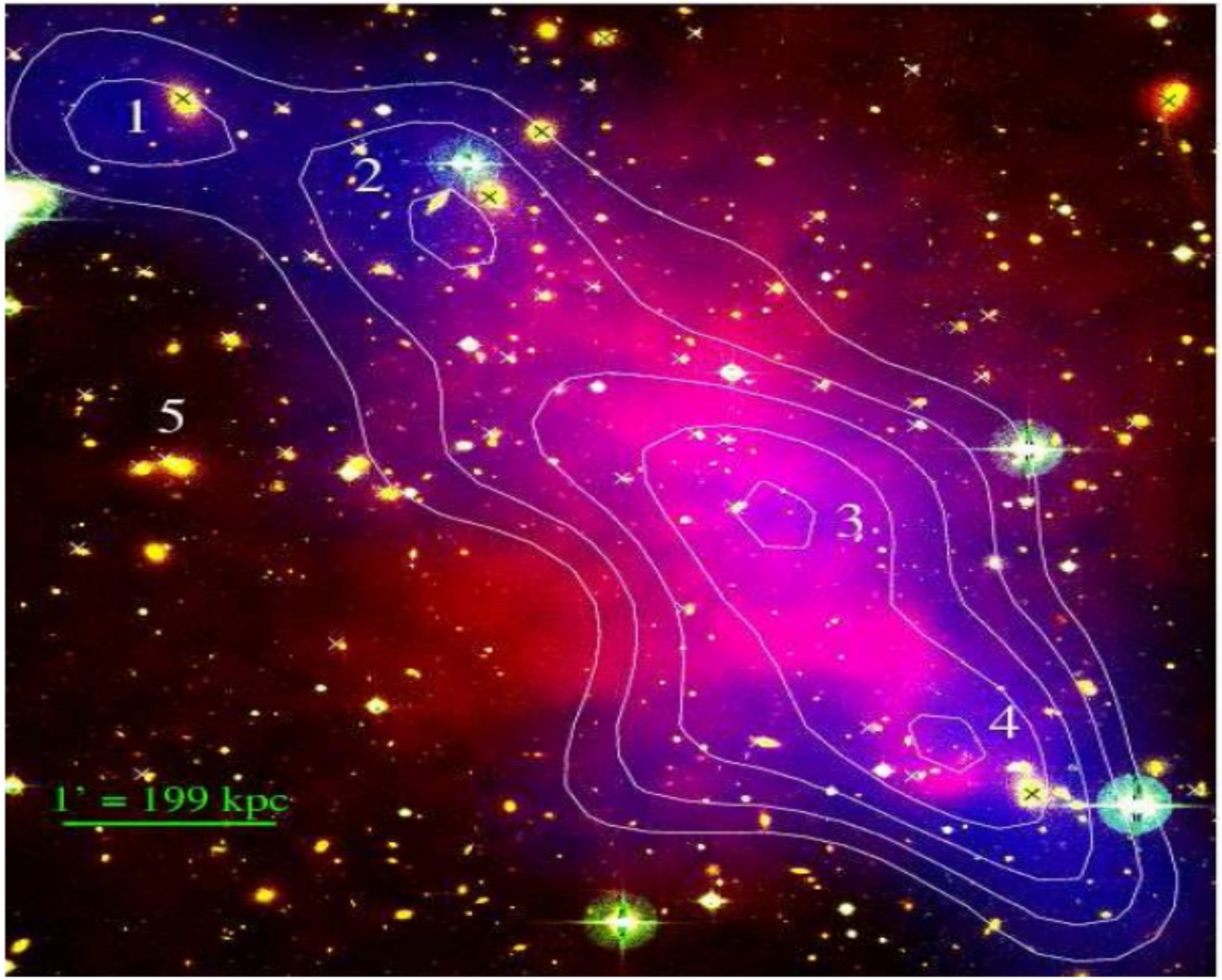
[Jee et al., astro-ph/0705.2171]



Ring-like dark matter structure

Dark core of the Abell 520

[Mahdavi et al., 0706.3048(astro-ph)]



Collisional dark matter ?

Self-Interacting Dark Matter

[Spergel, Steinhardt 2000]

Motivation: "cuspy core" problem

Model: $$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m_\phi^2 \phi^2 - \frac{g}{4!}\phi^4 + g' v \phi^2 h$$

Higgs decay width

$$\Gamma(h \to \phi\phi) = 5.23 \left(\frac{m_h}{115 \text{ GeV}}\right)^{-1} g'^2 \text{ GeV}$$

[Bento, O.B., Rosenfeld, Teodoro 2000]
[Silveira, Zee 1988]
[Bento, O.B., Rosenfeld 2001]

Unified model for dark energy – dark matter: $g' \Phi^2 H^2$

[O.B., Rosenfeld 2008]

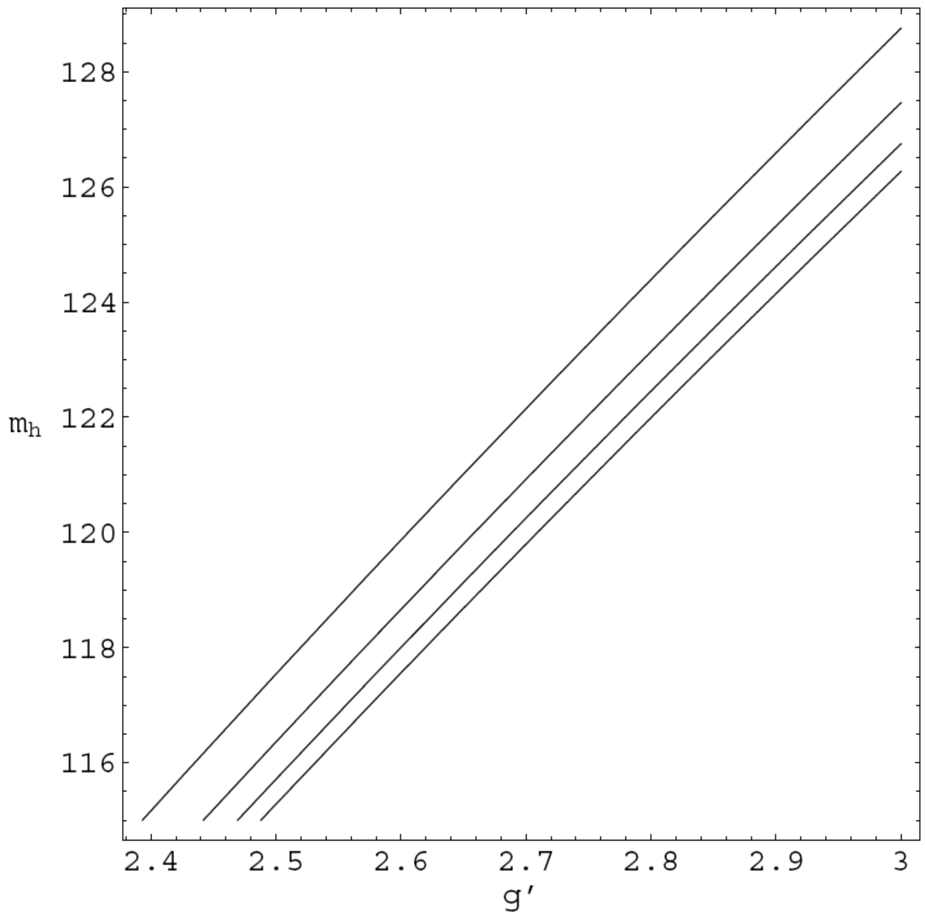


FIG. 2. Contour of $\Omega_\phi h^2 = 0.3$ as a function of m_h (in GeV) and g', for $m_\phi = 0.5$ GeV (top), 1.0, 1.5 and 2 GeV (bottom).

[Bento, O.B., Rosenfeld 2001]

Dark Energy

- **Evidence:**

  Dimming of type Ia Supernovae with *z > 0.35*

  Accelerated expansion (negative deceleration parameter): $q_0 \equiv -\frac{\ddot{a}a}{\dot{a}^2} \leq -0.47$

  [Perlmutter et al. 1998; Riess et al. 1998, …]

- **Homogeneous and isotropic expanding geometry**

  Driven by the vacuum energy density Ω_Λ and matter density Ω_M

  Equation of state: $p = \omega\rho \qquad \omega \leq 1$

- **Friedmann and Raychaudhuri equations imply:** $q_0 = \frac{1}{2}(3\omega + 1)\Omega_m - \Omega_\Lambda$

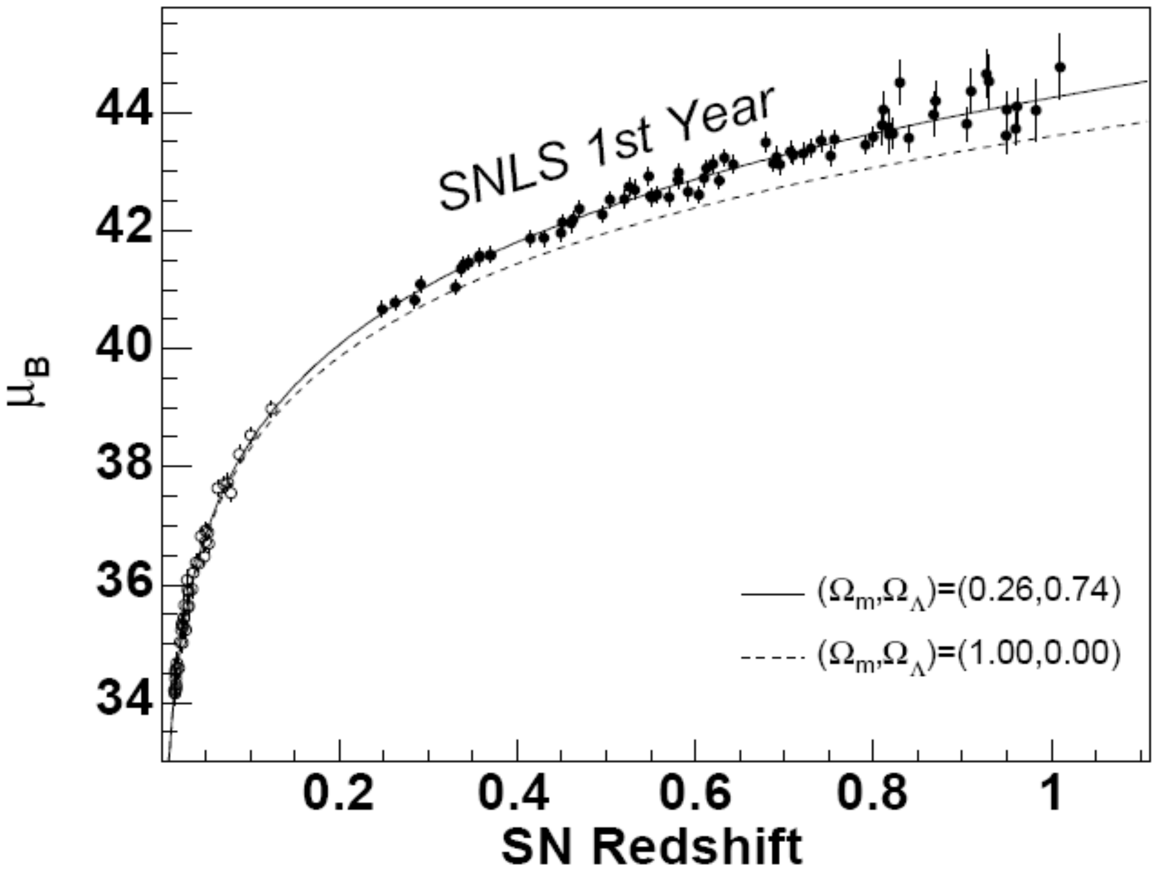
  $q_0 < 0$ suggests an invisible smooth energy distribution

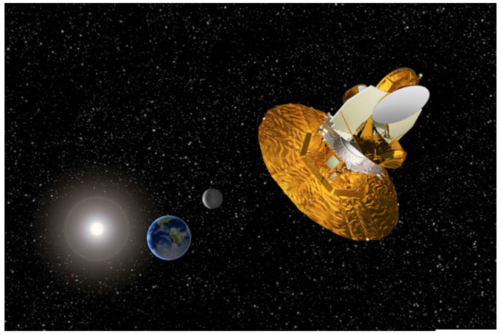
- **Candidates:**

  Cosmological constant, quintessence, more complex equations of state, etc.

Supernova Legacy Survey (SNLS)

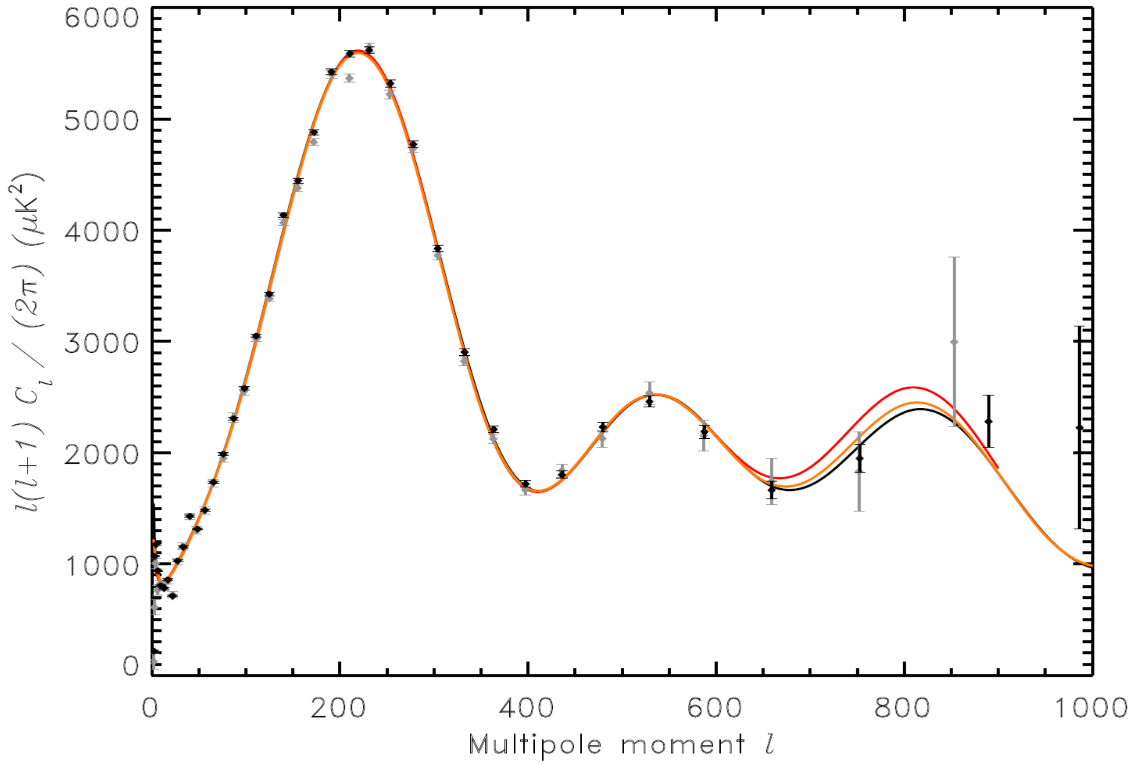
[Astier et al., astro-ph/0510447]





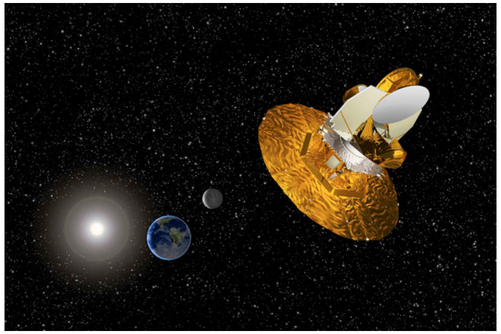
WMAP 3 Year Results

D.N. Spergel et al., astro-ph/0603449



$$(\Omega_m h^2, \Omega_b h^2, h, n_s, \tau, \sigma_8) \quad =$$

$$(0.127^{+0.007}_{-0.013}, 0.0223^{+0.0007}_{-0.0009}, 0.73^{+0.03}_{-0.03}, 0.951^{+0.015}_{-0.019}, 0.09^{+0.03}_{-0.03}, 0.74^{+0.05}_{-0.06})$$



WMAP 3 Year Results

D.N. Spergel et al., astro-ph/0603449

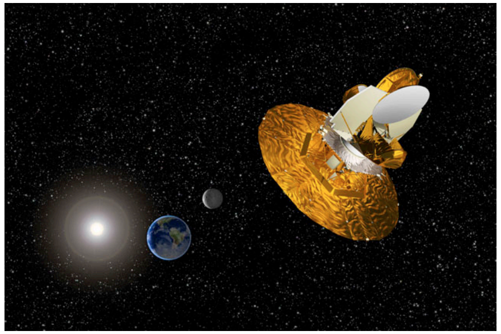
ΛCDM Model

Parameter	WMAP+ SDSS	WMAP+ LRG	WMAP+ SNLS	WMAP + SN Gold	WMAP+ CFHTLS
$100\Omega_b h^2$	$2.233^{+0.062}_{-0.086}$	$2.242^{+0.062}_{-0.084}$	$2.233^{+0.069}_{-0.088}$	$2.227^{+0.065}_{-0.082}$	$2.255^{+0.062}_{-0.083}$
$\Omega_m h^2$	$0.1329^{+0.0056}_{-0.0075}$	$0.1337^{+0.0044}_{-0.0061}$	$0.1295^{+0.0056}_{-0.0072}$	$0.1349^{+0.0056}_{-0.0071}$	$0.1408^{+0.0034}_{-0.0050}$
h	$0.709^{+0.024}_{-0.032}$	$0.709^{+0.016}_{-0.023}$	$0.723^{+0.021}_{-0.030}$	$0.701^{+0.020}_{-0.026}$	$0.687^{+0.016}_{-0.024}$
A	$0.813^{+0.042}_{-0.052}$	$0.816^{+0.042}_{-0.049}$	$0.808^{+0.044}_{-0.051}$	$0.827^{+0.045}_{-0.053}$	$0.846^{+0.037}_{-0.047}$
τ	$0.079^{+0.029}_{-0.032}$	$0.082^{+0.028}_{-0.033}$	$0.085^{+0.028}_{-0.032}$	$0.079^{+0.028}_{-0.034}$	$0.088^{+0.026}_{-0.032}$
n_s	$0.948^{+0.015}_{-0.018}$	$0.951^{+0.014}_{-0.018}$	$0.950^{+0.015}_{-0.019}$	$0.946^{+0.015}_{-0.019}$	$0.953^{+0.015}_{-0.019}$
σ_8	$0.772^{+0.036}_{-0.048}$	$0.781^{+0.032}_{-0.045}$	$0.758^{+0.038}_{-0.052}$	$0.784^{+0.035}_{-0.049}$	$0.826^{+0.022}_{-0.035}$
Ω_m	$0.266^{+0.026}_{-0.036}$	$0.267^{+0.018}_{-0.025}$	$0.249^{+0.024}_{-0.031}$	$0.276^{+0.023}_{-0.031}$	$0.299^{+0.019}_{-0.025}$

WMAP 3 + SNLS: $w = -0.97^{+0.07}_{-0.09}$

$$\omega = \frac{p}{\rho}$$

$$\Omega_k = -0.015^{+0.020}_{-0.016} \qquad \Omega_\Lambda = 0.72 \pm 0.04$$



WMAP 5 Year Results

E. Komatsu et al., 0803.0547 [astro-ph]

Summary of the cosmological parameters of ΛCDM model and the corresponding 68% intervals

Class	Parameter	*WMAP* 5-year ML[a]	*WMAP*+BAO+SN ML	*WMAP* 5-year Mean[b]	*WMAP*+BAO+SN Mean
Primary	$100\Omega_b h^2$	2.268	2.263	2.273 ± 0.062	2.265 ± 0.059
	$\Omega_c h^2$	0.1081	0.1136	0.1099 ± 0.0062	0.1143 ± 0.0034
	Ω_Λ	0.751	0.724	0.742 ± 0.030	0.721 ± 0.015
	n_s	0.961	0.961	$0.963^{+0.014}_{-0.015}$	$0.960^{+0.014}_{-0.013}$
	τ	0.089	0.080	0.087 ± 0.017	0.084 ± 0.016
	$\Delta^2_{\mathcal{R}}(k_0{}^e)$	2.41×10^{-9}	2.42×10^{-9}	$(2.41 \pm 0.11) \times 10^{-9}$	$(2.457^{+0.092}_{-0.093}) \times 10^{-9}$
Derived	σ_8	0.787	0.811	0.796 ± 0.036	0.817 ± 0.026
	H_0	72.4 km/s/Mpc	70.3 km/s/Mpc	$71.9^{+2.6}_{-2.7}$ km/s/Mpc	70.1 ± 1.3 km/s/Mpc
	Ω_b	0.0432	0.0458	0.0441 ± 0.0030	0.0462 ± 0.0015
	Ω_c	0.206	0.230	0.214 ± 0.027	0.233 ± 0.013
	$\Omega_m h^2$	0.1308	0.1363	0.1326 ± 0.0063	0.1369 ± 0.0037
	$z_{\rm reion}{}^f$	11.2	10.5	11.0 ± 1.4	10.8 ± 1.4
	$t_0{}^g$	13.69 Gyr	13.72 Gyr	13.69 ± 0.13 Gyr	13.73 ± 0.12 Gyr

Summary of the 95% confidence limits on deviations from the simple (flat, Gaussian, adiabatic, power-law) ΛCDM model

Section	Name	Type	*WMAP* 5-year	*WMAP*+BAO+SN
§ 3.2	Gravitational Wave[a]	No Running Ind.	$r < 0.43$[b]	$r < 0.20$
§ 3.1.3	Running Index	No Grav. Wave	$-0.090 < dn_s/d\ln k < 0.019$[c]	$-0.0728 < dn_s/d\ln k < 0.0087$
§ 3.4	Curvature[d]		$-0.063 < \Omega_k < 0.017$[e]	$-0.0175 < \Omega_k < 0.0085$[f]
	Curvature Radius[g]	Positive Curv.	$R_{\rm curv} > 12\ h^{-1}$Gpc	$R_{\rm curv} > 23\ h^{-1}$Gpc
		Negative Curv.	$R_{\rm curv} > 23\ h^{-1}$Gpc	$R_{\rm curv} > 33\ h^{-1}$Gpc
§ 3.5	Gaussianity	Local	$-9 < f_{NL}^{\rm local} < 111$[h]	N/A
		Equilateral	$-151 < f_{NL}^{\rm equil} < 253$[i]	N/A
§ 3.6	Adiabaticity	Axion	$\alpha_0 < 0.16$[j]	$\alpha_0 < 0.067$[k]
		Curvaton	$\alpha_{-1} < 0.011$[l]	$\alpha_{-1} < 0.0037$[m]
§ 4	Parity Violation	Chern-Simons[n]	$-5.9^\circ < \Delta\alpha < 2.4^\circ$	N/A
§ 5	Dark Energy	Constant w[o]	$-1.37 < 1 + w < 0.32$[p]	$-0.11 < 1 + w < 0.14$
		Evolving $w(z)$[q]	N/A	$-0.38 < 1 + w_0 < 0.14$[r]
§ 6.1	Neutrino Mass[s]		$\sum m_\nu < 1.3$ eV[t]	$\sum m_\nu < 0.61$ eV[u]
§ 6.2	Neutrino Species		$N_{\rm eff} > 2.3$[v]	$N_{\rm eff} = 4.4 \pm 1.5$[w] (68%)

[a]In the form of the tensor-to-scalar ratio, r, at $k = 0.002$ Mpc^{-1}

[b]Dunkley et al. (2008)

[c]Dunkley et al. (2008)

[d](Constant) dark energy equation of state allowed to vary ($w \neq -1$)

[e]With the HST prior, $H_0 = 72 \pm 8$ km/s/Mpc. For $w = -1$, $-0.052 < \Omega_k < 0.013$ (95% CL)

[f]For $w = -1$, $-0.0181 < \Omega_k < 0.0071$ (95% CL)

[g]$R_{\rm curv} = (c/H_0)/\sqrt{|\Omega_k|} = 3/\sqrt{|\Omega_k|}\ h^{-1}$Gpc

[h]Cleaned V+W map with $l_{\rm max} = 500$ and the *KQ75* mask, after the point source correction

[i]Cleaned V+W map with $l_{\rm max} = 700$ and the *KQ75* mask, after the point source correction

[j]Dunkley et al. (2008)

[k]In terms of the adiabaticity deviation parameter, $\delta_{adi}^{(c,\gamma)} = \sqrt{\alpha}/3$ (Eq. [39]), the axion-like dark matter and photons are found to obey the adiabatic relation (Eq. [36]) to 8.6%.

[l]Dunkley et al. (2008)

[m]In terms of the adiabaticity deviation parameter, $\delta_{adi}^{(c,\gamma)} = \sqrt{\alpha}/3$ (Eq. [39]), the curvaton-like dark matter and photons are found to obey the adiabatic relation (Eq. [36]) to 2.0%.

[n]For an interaction of the form given by $(\phi/M)F_{\alpha\beta}\tilde{F}^{\alpha\beta}$, the polarization rotation angle is $\Delta\alpha = M^{-1}\int \frac{dt}{a}\dot{\phi}$

[o]For spatially curved universes ($\Omega_k \neq 0$)

[p]With the HST prior, $H_0 = 72 \pm 8$ km/s/Mpc

[q]For a flat universe ($\Omega_k = 0$)

[r]$w_0 \equiv w(z = 0)$

[s]$\sum m_\nu = 94(\Omega_\nu h^2)$ eV

[t]Dunkley et al. (2008)

[u]For $w = -1$. For $w \neq -1$, $\sum m_\nu < 0.66$ eV (95% CL)

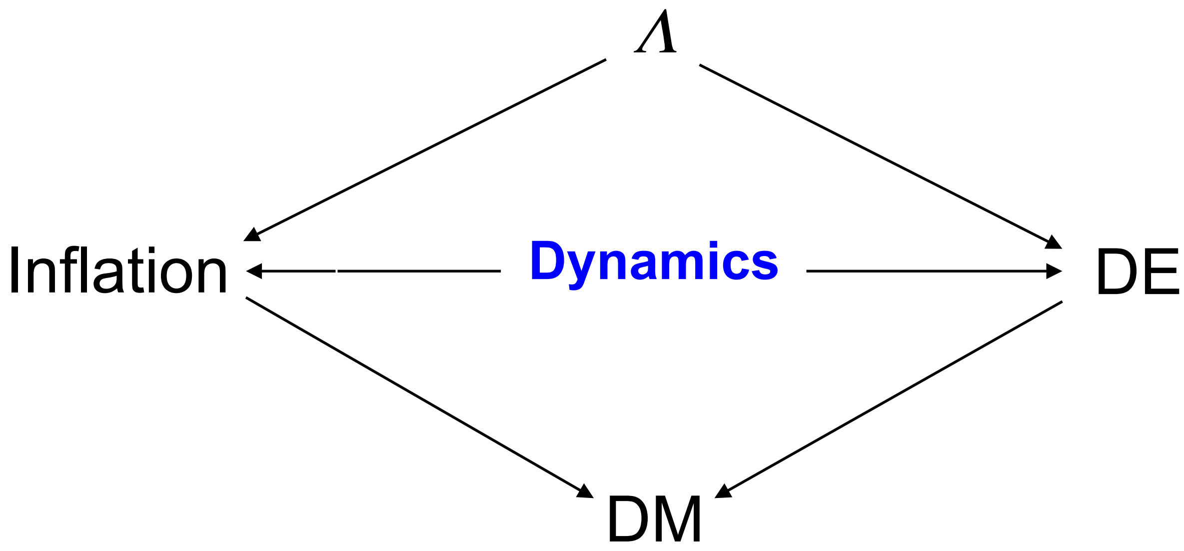
[v]Dunkley et al. (2008)

[w]With the HST prior, $H_0 = 72 \pm 8$ km/s/Mpc. The 95% limit is $1.9 < N_{\rm eff} < 7.8$

Dark Energy -- Dark Matter

"Quintessential Inflation"

[Peebles, Vilenkin 99; Dimopoulos, Valle 02; Rosenfeld, Frieman 05, O.B., Duvvuri 06, …]



Dark Energy – Dark Matter interaction

[Amendola 2000, ..., O.B., Gil Pedro, Le Delliou 2007]

Dark Energy – Dark Matter Unification

[Kamenschik, Moschella, Pasquier 2001]

[Bilic, Tupper, Viollier 2002; Bento, O.B., Sen 2002]

[O.B., Rosenfeld 2008]

Generalized Chaplygin gas model

- **Unified model for Dark Energy and Dark Matter**

Generalized d-brane

$$\mathcal{L} = -A^{\frac{1}{1+\alpha}} \left(1 - (g^{\mu\nu}\Theta_{,\mu}\Theta_{,\nu})^{\frac{\alpha+1}{2\alpha}}\right)^{\frac{\alpha}{\alpha+1}}$$

$\alpha = 1$: d-brane

Generalized Chaplygin gas

$$p = -\frac{A}{\rho^{\alpha}}$$

$\alpha = 1$: Chaplygin gas

$p \ll \rho$

Dust

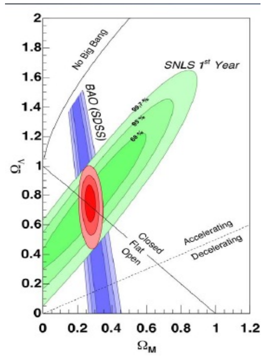
$p = \alpha\rho$

$\alpha = 1$: stiff matter

$p = -\rho$

De Sitter

[Bento, O.B., Sen 2002]



Dark Energy - Dark Matter Unification: Generalized Chaplygin Gas Model

- CMBR Constraints [Bento, O. B., Sen 2003, 2004; Amendola et al. 2004, Barreiro, O.B., Torres 2008]
- SNe Ia [O. B., Sen, Sen, Silva 2004; Bento, O.B., Santos, Sen 2005]
- Gravitational Lensing [Silva, O. B. 2003]
- Structure Formation *

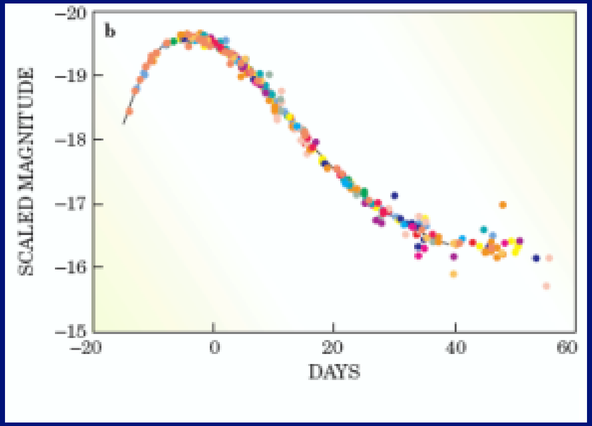
  [Sandvik, Tegmark, Zaldarriaga, Waga 2004; Bento, O. B., Sen 2004; Avelino et al. 2004; Bilic, Tupper, Viollier 2005; …]
- Gamma-ray bursts [O. B., Silva 2006, Barreiro, O.B., Torres 2010]
- Cosmic topology [Bento, O. B., Rebouças, Silva 2006]
- Inflation [O.B., Duvvuri 2006]
- Coupling with electromagnetic coupling [Bento, O.B., Torres 2007]
- Coupling with neutrinos [Bernardini, O.B. 2007, 2008, 2010]

Background tests: $\alpha \leq 0.35, \quad 0.8 \leq A_s \leq 0.9 \qquad A_s \equiv \frac{A}{\rho_{Ch0}^{1+\alpha}}$

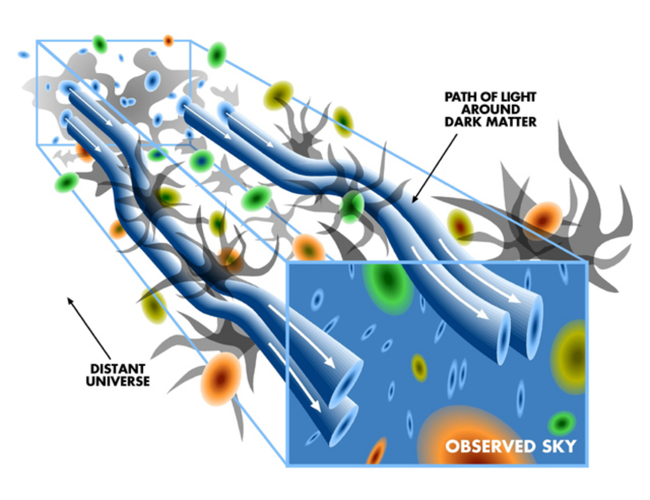
Structure formation and BAO: $\alpha \leq 0.2$

Large Dark Energy-Matter Surveys

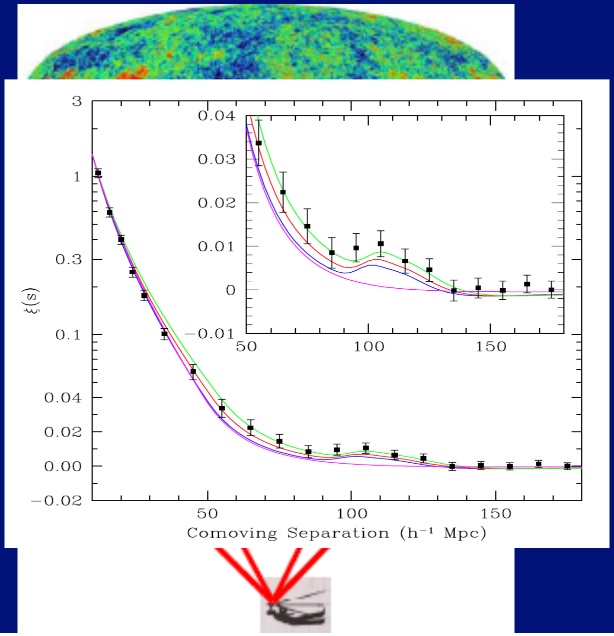
Euclid



Supernovae **Standard Candles**
Luminosity Distance



Cosmic Shear **Evolution of DM perts.**



Baryon Acoustic Oscillations
Standard ruler
Angular diameter distance

Modified Newtonian Dynamics (MOND)

[Milgrom 1983, Bekenstein, Milgrom 1984, ..., Bekenstein 2004]

[O.B., Böhmer, Harko, Lobo, Phys. Rev. D 75 (2007) 104016]

- **Model:** $$S=\int\left\{\frac{1}{2}f_1(R)+\left[1+\lambda f_2(R)\right]\mathcal{L}_m\right\}\sqrt{-g}\,d^4x$$

$f_i(R)$ – arbitrary functions of R ($F_i = \dfrac{df_i(R)}{dR}$)

- **Energy-momentum tensor of matter is not necessarily conserved:**

$$\nabla^\mu T^{(m)}_{\mu\nu}=\frac{\lambda F_2}{1+\lambda f_2}\left[g_{\mu\nu}\mathcal{L}_m-T^{(m)}_{\mu\nu}\right]\nabla^\mu R$$

- **Motion is non-geodesic:** $$\frac{Du^\alpha}{ds}\equiv\frac{du^\alpha}{ds}+\Gamma^\alpha_{\mu\nu}u^\mu u^\nu=f^\alpha$$

- **For a perfect fluid:** $$T^{(m)}_{\mu\nu}=(\epsilon+p)\,u_\mu u_\nu-pg_{\mu\nu}$$

$$f^\alpha=\frac{1}{\epsilon+p}\left[\frac{\lambda F_2}{1+\lambda f_2}(\mathcal{L}_m+p)\nabla_\nu R+\nabla_\nu p\right]h^{\alpha\nu}\quad h_{\mu\lambda}=g_{\mu\lambda}-u_\mu u_\lambda$$

Modified Newtonian Dynamics (MOND)

Dark Matter

[Milgrom 1983, Bekenstein, Milgrom 1984, ..., Bekenstein 2004]

Motivation: Flatness Rotation Curve of Galaxies

$$\vec{a} = \mu\left(\frac{|\vec{g}|}{a_0}\right)\vec{g} = -\mu\left(\frac{|\vec{g}|}{a_0}\right)\nabla\phi$$

$$\mu(x) = \begin{cases} 1 \;\; if \;\; x \gg 1 \\ x \;\; if \;\; x \ll 1 \end{cases}$$

$a_0 \approx 1.2 \times 10^{-10}\ m/s^2$ - universal acceleration

Tully-Fisher Law: $L_H \propto v_c^4$ **as** $L_H \propto M = (Ga_0)^{-1} v_c^4$

TeVeS[2] version: F-function problem

$$S_s = -\frac{1}{2}\int\left[\sigma^2 h^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta} + \frac{1}{2}G\ell^{-2}\sigma^4 F(kG\sigma^2)\right](-g)^{1/2}d^4x$$

MOND

Tensor-Vector-Scalar field theory, $S = S_g + S_s + S_v + S_m$:

$$S_g = (16\pi G)^{-1} \int g^{\alpha\beta} R_{\alpha\beta} (-g)^{1/2} d^4x$$

$$S_s = -\frac{1}{2} \int \left[\sigma^2 h^{\alpha\beta} \phi_{,\alpha} \phi_{,\beta} + \frac{1}{2} G \ell^{-2} \sigma^4 F(kG\sigma^2) \right] (-g)^{1/2} d^4x$$

$$S_v = -\frac{K}{32\pi G} \int \left[g^{\alpha\beta} g^{\mu\nu} \mathfrak{U}_{[\alpha,\mu]} \mathfrak{U}_{[\beta,\nu]} - 2(\lambda/K)(g^{\mu\nu} \mathfrak{U}_\mu \mathfrak{U}_\nu + 1) \right] (-g)^{1/2} d^4x$$

$$S_m = \int \mathcal{L}(\tilde{g}_{\mu\nu}, f^\alpha, f^\alpha{}_{|\mu}, \cdots)(-\tilde{g})^{1/2} d^4x$$

Conformal transformation to the physical metric: $(-\tilde{g})^{1/2} = e^{-2\phi}(-g)^{1/2}$

Consistency

- **PPN:** $\beta = 1, \gamma = 1$ **(see however O.B., Páramos 2006)**

  i) (Potentially) compatible

  [Skordis, Mota, Ferreira, Boehm 2005]

- **CMBR**

  ii) Problem with the third peak $$\frac{P_{\Lambda CDM}}{P_{MOND}} \cong 2 \times 10^2$$

  [Slosar, Melchiorri, Silk 2005]

- **Gravitational lensing – great potential for testing**

  [Zhao, Bacon, Taylor, Horne 2005]

Can MOND take a bullet ?

[Angus, Shan, Zhao, Famaey 2006]

- **Dark halo made of neutrinos:** $m_\nu = (2-3)eV$
- **Not quite !** [Takahashi, Chiba, 2007]

Neutrino oscillations: $\Delta m_\nu^{\ 2} \leq 10^{-3} eV^2$

Tremaine-Gunn bound: $\rho_{\nu Max.} = 4.8\times10^{-27}\left(\frac{m_\nu}{2eV}\right)^4\left(\frac{T_X}{keV}\right)^{3/2} g/cm^3$

Core density (Hernquist profile): $M(<r) = \frac{M_0 r^2}{(r+r_0)^2}$ $\rho_{core} = \frac{3M(<r_0)}{4\pi r^3}$

$$\rho_{core} < \rho_{\nu Max.}: \quad m_\nu > 6.1\left(\frac{M_0}{10^{14}M_{sun}}\right)^{1/4}\left(\frac{r_0}{100kpc}\right)^{-3/4}\left(\frac{T_X}{keV}\right)^{-3/8} eV$$

A1689: $M_0 = (6.2\pm1.2)\times10^{14}M_{sun}$ $r_0 = (125\pm52)kpc$ $T_X = (9.00\pm0.13)keV$

$$m_\nu > (3.6\pm1.1)eV$$

Beyond General Relativity:
f(R) gravity with nonminimal curvature-matter coupling

[O.B., Böhmer, Harko, Lobo 2007]

Action:

$$S = \int \left[\frac{1}{2}f_1(R) + f_2(R)\mathcal{L}_m\right]\sqrt{-g}d^4x\,,$$

Field equations:

$$(f_1' + 2\mathcal{L}_m f_2')R_{\mu\nu} - \frac{1}{2}f_1 g_{\mu\nu} - \Delta_{\mu\nu}(f_1' + 2\mathcal{L}_m f_2') = f_2 T_{\mu\nu}$$

$$\Delta_{\mu\nu} = \nabla_\mu\nabla_\nu - g_{\mu\nu}g^{\alpha\beta}\nabla_\alpha\nabla_\beta$$

Effective energy-momentum tensor non-conservation:

$$\nabla^\mu T_{\mu\nu} = \frac{f_2'}{f_2}\left[g_{\mu\nu}\mathcal{L}_m - T_{\mu\nu}\right]\nabla^\mu R\,.$$

Eq. motion test particle:
(Perfect fluid)

$$u^\nu\nabla_\nu u^\lambda = \frac{1}{\epsilon + p}\left(\frac{f_2'}{f_2}(\mathcal{L}_m + p)\nabla_\nu R + \nabla_\nu p\right)h^{\nu\lambda}$$

$$\equiv f^\lambda\,.$$

f(R) theory of gravity with non-minimal curvature-matter coupling (II)

[O.B., Böhmer, Harko, Lobo 2007]

$$S=\int\left[\frac{1}{2}f_1(R)+f_2(R)\mathcal{L}_m\right]\sqrt{-g}d^4x\,;$$

- **Implications:** $\vec{a}=\vec{a}_N+\vec{f}$

- **MOND-like behaviour: extra force and Tully-Fisher law** ($L\sim v_\infty^4$)

If $a_N \ll a$: $\vec{a}_N\approx\frac{a}{a_E}\vec{a}$ $\quad\frac{1}{a_E}\equiv\frac{1}{2f}\left(1-\frac{f^2}{a^2}\right)$

Hence $a\approx\sqrt{a_E a_N}$ and as $a_N = GM/r^2$ follows that

$$a\ \approx\ \sqrt{a_E GM}/r\ =\ v_{tg}^2/r \qquad v_{tg}^2\rightarrow v_\infty^2=\sqrt{a_E GM}$$

and the Tully-Fisher law as **L~M**

f(R) theory of gravity with non-minimal curvature-matter coupling (II)

[O.B., Böhmer, Harko, Lobo 2007]

$$S = \int \left[\frac{1}{2} f_1(R) + f_2(R)\mathcal{L}_m\right] \sqrt{-g} d^4x \; ;$$

- **Implications:** $\vec{a} = \vec{a}_N + \vec{f}$
- **MOND-like behaviour: extra force and Tully-Fisher law** ($L \sim v_\infty^4$)

If a_N<<a: $\vec{a}_N \approx \frac{a}{a_E}\vec{a}$ $\qquad \frac{1}{a_E} \equiv \frac{1}{2f}\left(1 - \frac{f^2}{a^2}\right)$

Hence $a \approx \sqrt{a_E a_N}$ and as $a_N = GM/r^2$ follows that

$$a \approx \sqrt{a_E GM}/r = v_{tg}^2/r \qquad v_{tg}^2 \rightarrow v_\infty^2 = \sqrt{a_E GM}$$

and the Tully-Fisher law as **L~M**

f(R) theory of gravity with non-minimal curvature-matter coupling (III)

[O.B., Böhmer, Harko, Lobo, Phys. Rev. D 75 (2007) 104016]

- Implications: $\vec{a} = \vec{a}_N + \vec{f}$

- Pioneer-like acceleration:

$$a_E = \frac{f^2 r^2}{GM} + 2f \qquad f \sim GM\alpha/r$$

$$f \rightarrow 0, \ a_E \approx \alpha^2 \qquad \alpha - const.$$

[Anderson, Laing, Lau, Liu, Nieto, Turyshev 2002]

[O.B., Páramos 2004]

- However, most likely the Pioneer anomalous acceleration is due to on-board thermal effects

[O.B., Francisco, Gil, Páramos 2008, 2011]

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f(R) theory of gravity with non-minimal curvature-matter coupling (IV)

[O.B., Böhmer, Harko, Lobo, Phys. Rev. D 75 (2007) 104016]

- Stellar stability
  [O.B., Páramos, Phys. Rev. D 77 (2008)]

- On the non-trivial gravitational coupling to matter
  [O.B., Páramos, Class. Quant. Grav. 25 (2008)]

- Non-minimal coupling of perfect fluids to curvature
  [O.B., Lobo, Páramos, Phys. Rev. D 78 (2008)]

- Non-minimal curvature-matter couplings in modified gravity (Review)
  [O.B., Páramos, Harko, Lobo, arXiv:0811.2876 [gr-qc]]

- A New source for a braneworld cosmological constant from a modified gravity model in the bulk [O.B., Carvalho, Laia, Nucl. Phys. B 807 (2009)]

- Energy Conditions and Stability in f(R) theories of gravity with non-minimal coupling to matter [O.B., Sequeira, Phys. Rev. B 79 (2009)]

- Mimicking dark matter through a non-minimal gravitational coupling with matter
  [O.B., Páramos, JCAP 1003, 009 (2010)]

- Accelerated expansion from a non-minimal gravitational coupling to matter
  [O.B., Frazão, Páramos, Phys. Rev. D 81 (2010)]

- Reheating via a generalized non-minimal coupling of curvature to matter
  [O.B., Frazão, Páramos, Phys. Rev. D 83 (2011)]

f(R) theory of gravity with non-minimal curvature-matter coupling (V)

[O.B., Böhmer, Harko, Lobo, Phys. Rev. D 75 (2007) 104016]

- Mimicking the cosmological constant: Constant curvature spherical solutions in non-minimally coupled model
[O.B., Páramos, Phys. Rev. D 84 (2011)]

- On the dynamics of perfect fluids in non-minimally coupled gravity
[O.B., Martins, Phys. Rev. D 85 (2012)]

- Mimicking dark matter in clusters through a non-minimal gravitational coupling with matter: the case of the Abell cluster A586
[O.B., Frazão, Páramos, Phys. Rev. D 86 (2012)]

- Traversable Wormholes and Time Machines in non-minimally coupled curvature-matter f(R) theories
[O.B., Ferreira, Phys. Rev. D 85 (2012)]

- More general clusters, the bullet cluster, ...

Energy Conditions and Stability

[O.B., Sequeira, Phys. Rev. B79 (2009)]

• Physical Viability

- Match GR Parametrized Post-Newtonian behaviour at solar system
  [O.B., Páramos, Class. Quant. Grav. 25 (2008)]

- Can lead to a phenomenologically consistent cosmology if the Energy Conditions are satisfied:

  Strong Energy Condition (SEC) (Gravity is attractive)

  Null Energy Condition (NEC) (Gravity is attractive)

  Dominant Energy Condition (DEC) ($v_{sound} \leq c$)

  Weak Energy Condition (WEK) (Positive energy density)

- Instability Free

  Dolgov-Kawasaki instability

-Ghost free, well posed Cauchy problem, correct cosmological perturbations, ...

Action and Field equations

Field equations:

$$(f_1' + 2\mathcal{L}_m f_2')R_{\mu\nu} - \frac{1}{2} f_1 g_{\mu\nu} - \Delta_{\mu\nu}(f_1' + 2\mathcal{L}_m f_2') = f_2 T_{\mu\nu}$$

$$G_{\mu\nu} = \hat{k}\left(\hat{T}_{\mu\nu} + T_{\mu\nu}\right)$$

Effective energy-momentum tensor:

$$\hat{T}_{\mu\nu} = \frac{1}{2}\left(\frac{f_1}{f_2} - \frac{f_1' + 2\mathcal{L}_m f_2'}{f_2} R\right) g_{\mu\nu} + \frac{1}{f_2}\Delta_{\mu\nu}\left(f_1' + 2\mathcal{L}_m f_2'\right)$$

Effective gravitational coupling: $\hat{k} = \dfrac{f_2}{f_1' + 2\mathcal{L}_m f_2'}$

Effective quantities

Perfect fluid:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p g_{\mu\nu}$$

Robertson-Walker metric:

$$ds^2 = dt^2 - a^2(t)ds_3^2$$

$$\Delta_{\mu\nu} h(R, \mathcal{L}_m) = (\nabla_\mu \nabla_\nu - g_{\mu\nu}\Box) h(R, \mathcal{L}_m)$$

$$= (\partial_\mu \partial_\nu - g_{\mu\nu}\partial_0\partial_0)h - (\Gamma^0_{\mu\nu} + g_{\mu\nu}3H)\partial_0 h$$

Effective energy-density:

$$\hat{\rho} = \frac{1}{2}\left(\frac{f_1}{f_2} - \frac{f_1' + 2\mathcal{L}_m f_2'}{f_2}R\right) - 3H\frac{f_1'' + 2\mathcal{L}_m f_2''}{f_2}\dot{R}$$

Effective pressure:

$$\hat{p} = -\frac{1}{2}\left(\frac{f_1}{f_2} - \frac{f_1' + 2\mathcal{L}_m f_2'}{f_2}R\right) + (\ddot{R} + 2H\dot{R})\frac{f_1'' + 2\mathcal{L}_m f_2''}{f_2} + \frac{f_1''' + 2\mathcal{L}_m f_2'''}{f_2}\dot{R}^2$$

Effective gravitational coupling:

$$\hat{k} = \frac{f_2}{f_1' + 2\mathcal{L}_m f_2'} > 0$$

Kinematical Quantities

- **Flat Robertson-Walker metric**

$$R = -6\left(H^2 + \frac{\ddot{a}}{a}\right)$$

- **Deceleration (q), jerk (j), snap (s) parameters**

$$q = -\frac{1}{H^2}\frac{\ddot{a}}{a}, \quad j = \frac{1}{H^3}\frac{\dddot{a}}{a}, \quad s = \frac{1}{H^4}\frac{\ddddot{a}}{a}$$

$$\dot{R} = -6H^3(j - q - 2)\,, \quad \ddot{R} = -6H^4(s + q^2 + 8q + 6)\,,$$

Energy Conditions

- Raychaudhuri eq. for the expansion parameter for a congruence of timelike geodesics

$$\frac{d\theta}{d\tau} = -\frac{1}{3}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}u^\mu u^\nu$$

- Raychaudhuri eq. for a congruence of null geodesics

$$\frac{d\theta}{d\tau} = -\frac{1}{2}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}k^\mu k^\nu$$

- Condition for attractive gravity

$$\frac{d\theta}{d\tau} < 0$$

Energy Conditions

- **SEC** $$R_{\mu\nu}u^{\mu}u^{\nu} \geq 0$$
- **NEC** $$R_{\mu\nu}k^{\mu}k^{\nu} \geq 0$$

Warrant that gravity is geometrically attractive

$$G_{\mu\nu} = \hat{k}\left(\hat{T}_{\mu\nu} + T_{\mu\nu}\right) \qquad R_{\mu\nu} = \hat{k}\left(\hat{T}_{\mu\nu} + T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}(\hat{T} + T)\right)$$

- **SEC**

$$\rho + 3p - \left(\frac{f_1}{f_2} - \frac{f_1' + 2\mathcal{L}_m f_2'}{f_2}R\right) + 3(\ddot{R} + H\dot{R})\frac{f_1'' + 2\mathcal{L}_m f_2''}{f_2} + 3\frac{f_1''' + 2\mathcal{L}_m f_2'''}{f_2}\dot{R}^2 \geq 0$$

- **NEC**

$$\rho + p + (\ddot{R} - H\dot{R})\frac{f_1'' + 2\mathcal{L}_m f_2''}{f_2} + \frac{f_1''' + 2\mathcal{L}_m f_2'''}{f_2}\dot{R}^2 \geq 0$$

Energy Conditions

- **DEC**

$$\rho - p + \left(\frac{f_1}{f_2} - \frac{f_1' + 2\mathcal{L}_m f_2'}{f_2} R\right) - (\ddot{R} + 5H\dot{R})\frac{f_1'' + 2\mathcal{L}_m f_2''}{f_2} - \frac{f_1''' + 2\mathcal{L}_m f_2'''}{f_2}\dot{R}^2 \geq 0$$

- **WEC**

$$\rho + \frac{1}{2}\left(\frac{f_1}{f_2} - \frac{f_1' + 2\mathcal{L}_m f_2'}{f_2} R\right) - 3H\frac{f_1'' + 2\mathcal{L}_m f_2''}{f_2}\dot{R} \geq 0$$

Transformations

(GR → f(R) theory with non-minimal coupling)

$$\rho \longrightarrow \rho + \hat{\rho} \qquad\qquad p \longrightarrow p + \hat{p}$$

Energy Conditions

- **Models**

$$f_1(R) = R + \epsilon R^n \qquad f_2(R) = 1 + \lambda R^m$$

- **Energy conditions:**

$$\frac{\hat{\epsilon}|R|^n}{1+\hat{\lambda}|R|^m}\left(a - \alpha_n - \frac{2\hat{\lambda}\mathcal{L}_m}{\hat{\epsilon}}\alpha_m|R|^{m-n}\right) \geq b$$

$$\hat{\epsilon} = (-1)^n\epsilon \qquad \hat{\lambda} = (-1)^m\lambda$$

		$\hat{\epsilon} > 0$	$\hat{\epsilon} < 0$
$\hat{\lambda} > 0$		$\frac{2\hat{\lambda}\mathcal{L}_m}{\hat{\epsilon}} \leq \frac{a-\alpha_n}{\alpha_m}\|R\|^{n-m}$	$\frac{2\hat{\lambda}\mathcal{L}_m}{\|\hat{\epsilon}\|} \leq \frac{\alpha_n-a}{\alpha_m}\|R\|^{n-m}$
$\hat{\lambda} < 0$	$1-\|\hat{\lambda}\|\|R\|^m > 0$	$\frac{2\|\hat{\lambda}\|\mathcal{L}_m}{\hat{\epsilon}} \geq \frac{\alpha_n-a}{\alpha_m}\|R\|^{n-m}$	$\frac{2\|\hat{\lambda}\|\mathcal{L}_m}{\|\hat{\epsilon}\|} \geq \frac{a-\alpha_n}{\alpha_m}\|R\|^{n-m}$
	$1-\|\hat{\lambda}\|\|R\|^m < 0$	$\frac{2\|\hat{\lambda}\|\mathcal{L}_m}{\hat{\epsilon}} \leq \frac{\alpha_n-a}{\alpha_m}\|R\|^{n-m}$	$\frac{2\|\hat{\lambda}\|\mathcal{L}_m}{\|\hat{\epsilon}\|} \leq \frac{a-\alpha_n}{\alpha_m}\|R\|^{n-m}$

- **SEC**

$$a^{SEC} = -1\,, \quad b^{SEC} = -(\rho + 3p)\,,$$

$$\alpha_n^{SEC} = -n\left[1 + 3(n-1)(\ddot{R} + H\dot{R})R^{-2} + 3(n-1)(n-2)R^{-3}\dot{R}^2\right]$$

$$\alpha_n^{SEC} = n\left[-1 + \frac{q^2 + 7q + j + s + 4}{2(q-1)^2}(n-1) - \frac{(j-q-2)^2}{2(q-1)^3}(n-1)(n-2)\right]$$

- **NEC**

$$a^{NEC} = 0\,, \quad b^{NEC} = -(\rho + p)\,,$$

$$\alpha_n^{NEC} = -n(n-1)\left[(\ddot{R} - H\dot{R})R^{-2} + (n-2)R^{-3}\dot{R}^2\right]$$

$$\alpha_n^{NEC} = n(n-1)\left[\frac{q^2 + 9q - j + s + 8}{6(q-1)^2} - \frac{(j-q-2)^2}{6(q-1)^3}(n-2)\right]$$

- **DEC**

$$a^{DEC} = 1\,, \quad b^{DEC} = -(\rho - p)\,,$$

$$\alpha_n^{DEC} = n\left[1 + (n-1)(\ddot{R} + 5H\dot{R})R^{-2} + (n-1)(n-2)R^{-3}\dot{R}^2\right]$$

$$\alpha_n^{DEC} = n\left[1 - \frac{q^2 + 3q + 5j + s - 4}{6(q-1)^2}(n-1) + \frac{(j-q-2)^2}{6(q-1)^3}(n-1)(n-2)\right]$$

- **WEC**

$$a^{WEC} = \frac{1}{2}, \quad b^{WEC} = -\rho,$$

$$\alpha_n^{WEC} = n\left[\frac{1}{2} + 3(n-1)HR^{-2}\dot{R}\right]$$

$$\alpha_n^{WEC} = n\left[\frac{1}{2} - \frac{j-q-2}{2(1-q)^2}(n-1)\right]$$

- **Positive gravitational coupling**

$$a^{AG} = 1, \quad b^{AG} = 0,$$

$$\alpha_n^{AG} = -\frac{n}{6H^2(1-q)}.$$

- **Dolgov-Kawasaki criterion** (a^{DK}=b^{DK}=0)

$$\frac{\alpha_n^{DK}}{\alpha_m^{DK}} = \frac{n(n-1)}{m(m-1)}$$

Dolgov-Kawasaki Instability

- **Dynamical eq. for the scalar curvature**

$$3(f_1'' + 2\mathcal{L}_m f_2'')\Box R + 3(f_1''' + 2\mathcal{L}_m f_2''')\nabla^\mu R \nabla_\mu R + 12 f_2'' \nabla^\mu \mathcal{L}_m \nabla_\mu R + 6 f_2' \Box \mathcal{L}_m + (f_1' + 2\mathcal{L}_m f_2')R - 2f_1 = f_2 T$$

- **Perturbative equation**

$$R = R_0 + R_1 \qquad T = T_0 + T_1 \qquad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\frac{\partial^2 R_1}{\partial t^2} - \nabla^2 R_1 + m_{eff}^2 R_1 = -\frac{f_2}{3(f_1'' + 2\mathcal{L}_m f_2'')} T_1$$

$$m_{eff}^2 = \frac{1}{3(f_1'' + 2\mathcal{L}_m f_2'')}\left[f_1' + f_2'(T_0 - 2\mathcal{L}_m) - (f_1'' + 2\mathcal{L}_m f_2'')R_0\right]$$

- **Stability Criterion** $\quad f_1(R) = R + \epsilon\varphi_1(R) \qquad f_2(R) = 1 + \lambda\varphi_2(R)$

$$f_1''(R) + 2\mathcal{L}_m f_2''(R) \geq 0$$

Dolgov-Kawasaki Criterion

$$f_1''(R) + 2\mathcal{L}_m f_2''(R) \geq 0$$

- **Models**

$$f_1(R) = R + \epsilon R^n \qquad\qquad f_2(R) = 1 + \lambda R^m$$

- **Stability conditions**

$$\hat{\epsilon} n(n-1)|R|^{n-2} + 2\hat{\lambda}\mathcal{L}_m m(m-1)|R|^{m-2} \geq 0$$

$$\hat{\epsilon} = \begin{cases} (-1)^n \epsilon, & \text{if } R < 0 \\ \epsilon, & \text{if } R > 0 \end{cases}, \quad \hat{\lambda} = \begin{cases} (-1)^m \lambda, & \text{if } R < 0 \\ \lambda, & \text{if } R > 0 \end{cases}$$

Dolgov-Kawasaki Criterion

- **Results**

$$f_1(R) = R + \epsilon R^n \qquad\qquad f_2(R) = 1 + \lambda R^m$$

$$\frac{\hat{\epsilon}|R|^n}{1+\hat{\lambda}|R|^m}\left(a - \alpha_n - \frac{2\hat{\lambda}\mathcal{L}_m}{\hat{\epsilon}}\alpha_m|R|^{m-n}\right) \geq b$$

a=b=0	$\hat{\epsilon} > 0$	$\hat{\epsilon} < 0$
$\hat{\lambda} > 0$	$\frac{2\hat{\lambda}\mathcal{L}_m}{\hat{\epsilon}} \leq \frac{a-\alpha_n}{\alpha_m}\|R\|^{n-m}$	$\frac{2\hat{\lambda}\mathcal{L}_m}{\|\hat{\epsilon}\|} \leq \frac{\alpha_n-a}{\alpha_m}\|R\|^{n-m}$
$\hat{\lambda} < 0$	$\frac{2\|\hat{\lambda}\|\mathcal{L}_m}{\hat{\epsilon}} \geq \frac{\alpha_n-a}{\alpha_m}\|R\|^{n-m}$	$\frac{2\|\hat{\lambda}\|\mathcal{L}_m}{\|\hat{\epsilon}\|} \geq \frac{a-\alpha_n}{\alpha_m}\|R\|^{n-m}$

Dolgov-Kawasaki Criterion

$$f_1''(R) + 2\mathcal{L}_m f_2''(R) \geq 0$$

- **Models**

$$f_1(R) = \sum_{n=1}^{k} a_n R^n \qquad\qquad f_2(R) = 1 + \lambda \sum_{m=1}^{k'} b_m R^m$$

- **Stability condition**

$$\sum_{n=2}^{k} a_n n(n-1) R^{n-2} + 2\lambda\mathcal{L}_m \sum_{m=2}^{k'} b_m m(m-1) R^{m-2} \geq 0$$

<table>
<tr><td>R>0</td><td colspan="2">$k' = 2$</td><td colspan="2">$k' = 3$</td></tr>
<tr><td rowspan="2">$k = 2$</td><td colspan="2" rowspan="2">$a_2 + 2\lambda\mathcal{L}_m b_2 \geq 0$</td><td>$a_3 > 0$</td><td>$R \geq -\frac{a_2+2\lambda\mathcal{L}_m b_2}{3a_3}$</td></tr>
<tr><td>$a_3 < 0$</td><td>$R \leq \frac{a_2+2\lambda\mathcal{L}_m b_2}{3\vert a_3\vert}$</td></tr>
<tr><td rowspan="2">$k = 3$</td><td>$\lambda\mathcal{L}_m b_3 > 0$</td><td>$R \geq -\frac{a_2+2\lambda\mathcal{L}_m b_2}{6\lambda\mathcal{L}_m b_3}$</td><td>$a_3 + 2\lambda\mathcal{L}_m b_3 > 0$</td><td>$R \geq -\frac{a_2+2\lambda\mathcal{L}_m b_2}{3(a_3+2\lambda\mathcal{L}_m b_3)}$</td></tr>
<tr><td>$\lambda\mathcal{L}_m b_3 < 0$</td><td>$R \leq \frac{a_2+2\lambda\mathcal{L}_m b_2}{6\vert\lambda\mathcal{L}_m b_3\vert}$</td><td>$a_3 + 2\lambda\mathcal{L}_m b_3 < 0$</td><td>$R \leq \frac{a_2+2\lambda\mathcal{L}_m b_2}{3\vert a_3+2\lambda\mathcal{L}_m b_3\vert}$</td></tr>
</table>

The Action of a Perfect Fluid: the GR story (I)

Action(s):

$$S_m = \int d^4x \sqrt{-g}\, p$$

$$S_m = -\int d^4x \sqrt{-g}\, \rho$$

$$\longrightarrow \qquad T_{\mu\nu} = (\rho + p)\, U_\mu U_\nu + p g_{\mu\nu}$$

[Schutz 1970; Hawking & Ellis 1973; Alba & Lusanna 2002; Brown 2003]

Actually, the most general action involves energy density, ρ, matter current, J^μ, entropy per particle (constant), s, and Lagrangian coords., φ, θ, α^A:

$$S_m = \int d^4x \left[-\sqrt{-g}\, \rho(n, s) + J^\mu \left(\varphi_{,\mu} + s\theta_{,\mu} + \beta_A \alpha^A_{,\mu}\right)\right]$$

$$\longrightarrow \qquad T^{\mu\nu} = \rho U^\mu U^\nu + \left(n\frac{\partial \rho}{\partial n} - \rho\right)(g^{\mu\nu} + U^\mu U^\nu)$$

Perfect fluid:

$$p = n\frac{\partial \rho}{\partial n} - \rho$$

The Action of a Perfect Fluid: the GR story (II)

Eqs. Motion:

$$\frac{\delta S}{\delta J^\mu} = \mu U_\mu + \varphi_{,\mu} + s\theta_{,\mu} + \beta_A \alpha^A_{,\mu} = 0\,,$$

$$\frac{\delta S}{\delta \varphi} = -J^\mu_{,\mu} = 0\,,$$

$$\frac{\delta S}{\delta \theta} = -(sJ^\mu)_{,\mu} = 0\,,$$

$$\frac{\delta S}{\delta s} = -\sqrt{-g}\frac{\partial \rho}{\partial s} + \theta_{,\mu} J^\mu = 0\,,$$

$$\frac{\delta S}{\delta \alpha^A} = -(\beta_A J^\mu)_{,\mu} = 0\,,$$

$$\frac{\delta S}{\delta \beta_A} = \alpha^A_{,\mu} J^\mu = 0\,.$$

where n is the particle number density, $n = |J|/\sqrt{-g}$ **,** $J^\mu = \sqrt{-g}\, n U^\mu$

such that, $T^{\mu\nu}_{;\nu} = 0$ **implies the covariant conservation,** $(nU^\mu)_{;\mu} = 0$ **.**

First Law of Thermodynamics:

$$d\rho = \mu\, dn + nT ds$$

$$\mu = \partial\rho/\partial n = (\rho + p)/n$$

The Action of a Perfect Fluid: the modified gravity model with non-minimal coupling to curvature story (III)

[O.B., Lobo, Páramos Phys. Rev. 78 (2008)]

In the context of the **f(R)** model with non-minimal couplig to curvature:

$$S = \int \left[\frac{1}{2} f_1(R) + [1 + \lambda f_2(R)]\, \mathcal{L}_m\right] \sqrt{-g}\; d^4x$$

If $\mathcal{L}_m = p \longrightarrow f^\mu = \dfrac{h^{\mu\nu}\nabla_\nu p}{\rho + p}$ which for a const. **p**, the motion is geodesic.

[Sotiriou & Faraoni 2008]

If however, $\mathcal{L}_m = -\rho \longrightarrow f^\mu = \left(-\dfrac{\lambda F_2}{1 + \lambda f_2}\nabla_\nu R + \dfrac{1}{\rho + p}\nabla_\nu p\right) h^{\mu\nu}$

that is, the motion is non-geodesic!

The Action of a Perfect Fluid: the modified gravity model with non-minimal coupling to curvature story (IV)

[O.B., Lobo, Páramos Phys. Rev. 78 (2008)]

Indeed, the degeneracy has been lifted, however, in order to warrant that the non-minimal coupling proposal is consistently applied, one should consider the action

$$S'_m = \int d^4x \left[-\sqrt{-g}\,[1+\lambda f_2(R)]\,\rho(n,s) + J^\mu \left(\varphi_{,\mu} + s\theta_{,\mu} + \beta_A \alpha^A_{,\mu}\right)\right]$$

instead the most obvious one

$$S = \int d^4x\,[1+\lambda f_2(R)]\left[-\sqrt{-g}\,\rho(n,s) + J^\mu \left(\varphi_{,\mu} + s\theta_{,\mu} + \beta_A \alpha^A_{,\mu}\right)\right]$$

as in this way the affected eqs. of motion are (others remain unchanged):

$$\frac{\delta S}{\delta J^\mu} = \mu\,[1+f_2(R)]\,U_\mu + \varphi_{,\mu} + s\theta_{,\mu} + \beta_A \alpha^A_{,\mu} = 0$$

$$\frac{\delta S}{\delta s} = -\sqrt{-g}\,[1+f_2(R)]\,\frac{\partial \rho}{\partial s} + \theta_{,\mu} J^\mu = 0$$

General Relativity admits Closed Timelike Curves (CTCs): A typology

Sols. with a “bite” on the light cone – rotation, cosmic string

[Gödel 1949; Gott 1991; Deser, Jackiw, ‘t Hooft 1992; Deser 1993]

Traversable wormhole sols.

[Morris, Thorne, Yurtsever 1988; Lobo 2006; Garattini, Lobo 2007; ...]

Warp drive sols.

[Alcubierre 1994; Lobo, Visser 2003; ...]

Krasnikov tube

[Krasnikov 1998; ...]

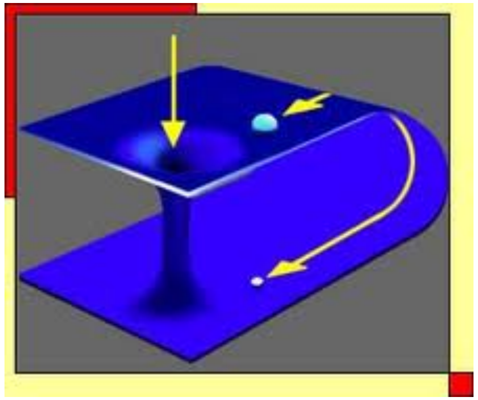
→ **Weak Energy Condition Violation**

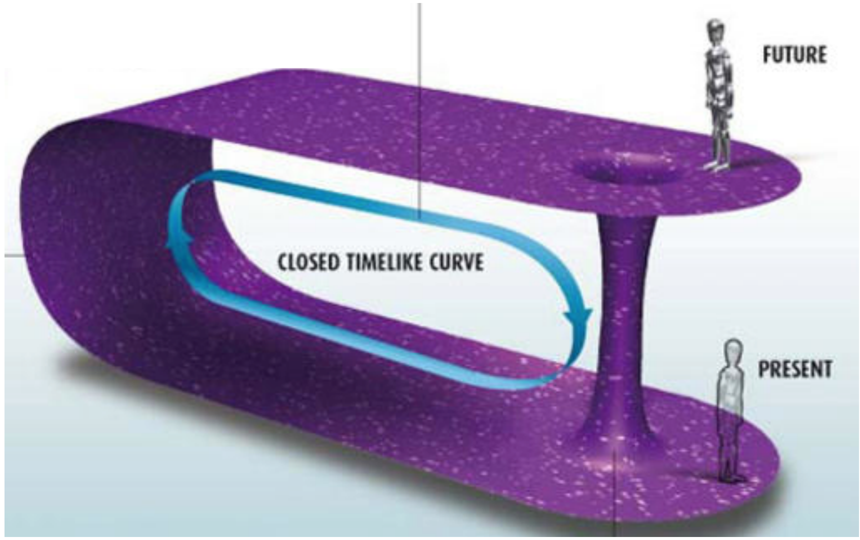
Wormhole geometry

[Morris, Thorne 1998]

"Wormholes in Spacetime and their use for Interstellar Travel: a tool for teaching General Relativity"

- Avoid singularities and horizons
- Ensure well behaved physical properties and metric components
- Ensure stability





[Morris, Thorne, Yurtsever 1988]

"Wormholes, Time Machines and the Weak Energy Condition"

Time travel paradoxes

- The killing of an ancestror paradox
- The Oedipus time traveller complex
- ...

→ Serious threats to Causality

- Creation of information paradox

Time traveller from the future conveys the secret of time travelling to a researcher, who in turn publishes it. Later, the researcher travels back in time and convey the secret to his/her younger person. So the information has appeared from "nowhere".

Putative Solutions

Novikov's self-consistent principle [Novikov 1990]

CTCs might exist, but that they cannot entail any type of causality violation or time paradox

It assumes either that there is only one timeline or that alternative timelines (such as in the Many-Worlds Interpretation of Quantum Mechanics) are not accessible

Hawking's chronology protection conjecture [Hawking 1992]

"It seems that there is a Chronology Protection Agency which prevents the appearance of CTCS so to make the universe safe for historians"

Quantum Mechanics (Many Worlds Interpretation)?

[Deustch 1991, Deustch, Lockwood 1994]

Systems can travel from one time in one world to another time in another world, but no system travels to an earlier time in the same world

Emergent Gravity Solution [O.B. 2012]

CTCs do not actually exist as they require conditions for which the affective and emergent GR and other classical models of gravity are no longer valid

Traversable Wormholes and Time Machines in non-minimally coupled curvature-matter f(R) theories

[O.B., Ferreira, Phys. Rev. D 85 (2012)]

Wormhole sols. → **WEC violation** $\xrightarrow{\text{GR}}$ $T_{\mu\nu}^{matter} k^\mu k^\nu \leq 0 \rightarrow \rho \leq 0$

NEC violation → $\hat{T}_{\mu\nu} k^\mu k^\nu \leq 0 \quad (T_{\mu\nu}^{matter} k^\mu k^\nu \geq 0)$

Non-minimally coupled theory

Effective energy-momentum tensor:

$$\hat{T}_{\mu\nu} = \frac{1}{2}\left(\frac{f_1}{f_2} - \frac{f_1' + 2\mathcal{L}_m f_2'}{f_2} R\right) g_{\mu\nu} + \frac{1}{f_2}\Delta_{\mu\nu}\left(f_1' + 2\mathcal{L}_m f_2'\right)$$

Field equations: $G_{\mu\nu} = \hat{k}\left(\hat{T}_{\mu\nu} + T_{\mu\nu}\right)$

Effective gravitational coupling: $\hat{k} = \dfrac{f_2}{f_1' + 2\mathcal{L}_m f_2'}$

How to build a Time Machine

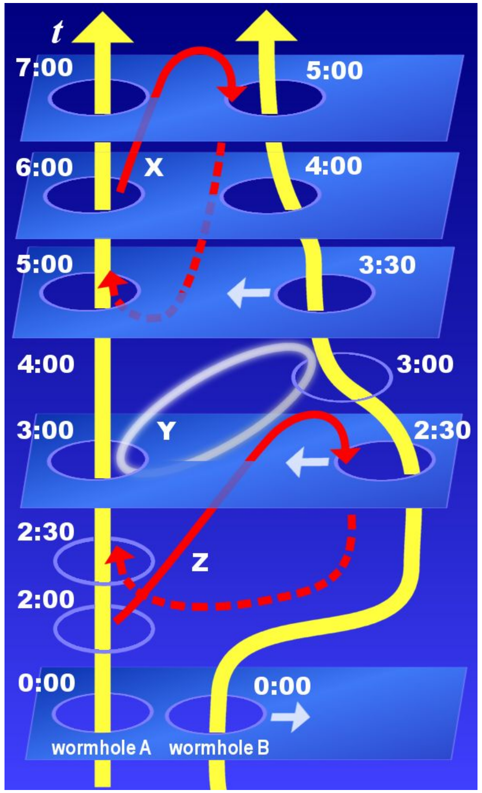
Recipe:

[Morris, Thorne, Yurtsever 1988]

Acquire a traversable wormhole;

Create a red-shift between the two wormhole mouths by, for example, acelerating one of the wormhole mouths;

Bring the mouths close together adiabatically.

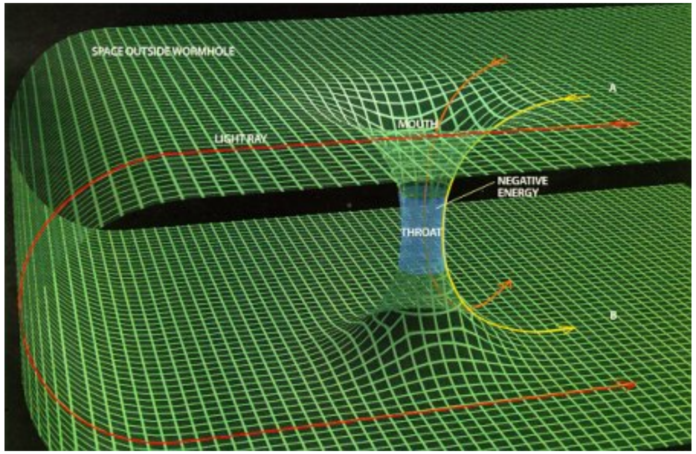


Set up

Wormhole Geometry

Static spherically symmetric space-time:

$$ds^2 = -e^{2\Phi(r)}dt^2 + \frac{dr^2}{1 - \frac{b(r)}{r}} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$



Further constraints:

$$\frac{b(r) - b'(r)r}{b^2(r)} > 0 \qquad 1 - \frac{b(r)}{r} \geqslant 0 \qquad (r - b(r))\,\Phi'(r) \to 0 \;\text{ as }\; r \to r_0$$

Equations solved for $f_1(R) = f_2(R) = R$ **in the presence of a fluid with stress-energy tensor:**

$$T_{\mu\nu} = (\rho + p_t)\,U_\mu U_\mu + p_t g_{\mu\nu} + (p_r - p_t)\chi_\mu\chi_\nu$$

Results

- System of 3 non-linear second order diff eqs. with 5 unknown functions

Cases considered:

Isotropic pressure: $p_r = p_t$

Two Energy densities

- Function b(r) obtained everywhere [Garcia, Lobo 2010]

- Pressure and Redshift obtained:

At infinity, where the geometry should be assymptotically flat

Near the wormhole throat where NEC violation must occur

Solutions

Energy Density:

Case 1: Constant and localized close the wormhole throat

$$\rho_1(r) = \begin{cases} \rho_0, & r < r_2 \\ 0, & r > r_2 \end{cases}$$

Case 2: Exponentially decaying

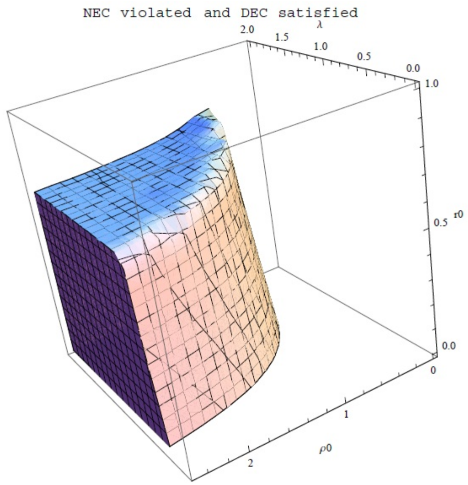
$$\rho_2(r) = \frac{\rho_0 r_0}{r} e^{-\frac{r - r_0}{\sqrt{2\lambda}}}$$

Results:

Assymptotically flat wormhole solutions with ordinary matter if:

$$\lambda > \frac{1}{2\rho_0}$$

$$\rho_0 > \frac{1}{2\lambda}\left(1 + \frac{r_0}{\sqrt{2\lambda + r_0^2}}\right)$$



Solutions

[O.B., Ferreira, Phys. Rev. D 85 (2012)]

Case 1

There is a discontinuity at an arbitrary scale which gives rise to problems associated with singularities

Case 2

One obtains differentiable and well behaved wormhole solutions which respect all the constraints to be traversable: Time Machines

Violation of the Equivalence Principle

- Expected at cosmological level as the fall of matter will depend on the local curvature as different bodies would feel a different coupling to gravity:

$$f_2(R) = 1 + \lambda \varphi_2(R)$$

- Similar to what is expected if dark energy couples to dark matter (c.f. below)

[O.B., Gil Pedro, Le Delliou 2007, 2009, 2012]

- Means an IR breaking of GR and presumably a bearing on the cosmological constant problem at low energies

[O.B. 2009]

Dark Energy – Dark Matter Interaction

[O.B., Gil Pedro, Le Delliou Phys. Lett. B654 (2007); ; GR&R (2009, 2012)]

- **Evolution equations:** ($p_{DE} = \omega_{DE}\rho_{DE}$)

$$\dot\rho_{DM} + 3H\rho_{DM} = \zeta H \rho_{DM}$$

$$\dot\rho_{DE} + 3H\rho_{DE}(1+\omega_{DE}) = -\zeta H \rho_{DM}$$

- **For** $\frac{\rho_{DE}}{\rho_{DM}} = \frac{\Omega_{DE_0}}{\Omega_{DM_0}} a^\eta$

$$\zeta = -\frac{(\eta + 3\omega_{DE})\Omega_{DE_0}}{\Omega_{DE_0} + \Omega_{DM_0} a^{-\eta}}$$

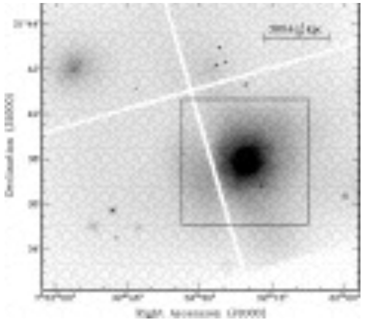
- **From which follows:**

$$\begin{aligned} \rho_{DM} &= a^{-3}\rho_{DM_0} e^{\int_1^a \zeta \frac{da}{a}} \\ &= a^{-3}\rho_{DM_0}\left[\Omega_{DE_0} a^\eta + \Omega_{DM_0}\right]^{-\frac{(\eta+3\omega_{DE})}{\eta}} \\ \rho_{DE} &= a^{\eta-3}\rho_{DE_0} e^{\int_1^a \zeta \frac{da}{a}} \\ &= a^{\eta-3}\rho_{DE_0}\left[\Omega_{DE_0} a^\eta + \Omega_{DM_0}\right]^{-\frac{(\eta+3\omega_{DE})}{\eta}} \end{aligned}$$

- **Bias parameter:**

$$b = \frac{\rho_B}{\rho_{DM}} = b_0 \left[\frac{\Omega_{DE_0} a^\eta + \Omega_{DM_0}}{\Omega_{DE_0} + \Omega_{DM_0}}\right]^{\frac{(\eta+3\omega_{DE})}{\eta}}$$

GCG: $\eta = 3(1+\alpha)$



Estimates

[O.B., Gil Pedro, Le Delliou (Phys. Lett. B654,165 (2007))]

- **X-ray, velocity dispersion and weak gravitational lensing (WGL):**

$$M_{Cluster} = (4.3 \pm 0.7) \times 10^{14}\, M_{\odot} \qquad \sigma_v = (1243 \pm 58)\, kms^{-1}$$

[Cypriano, Neto, Sodré, Kneib 2005]

- **WGL concerns a spherical region with 422 kpc radius and N_{Gal}=25 galaxies (within a $570h_{70}^{-1}$ kpc region with 31 galaxies); hence with the known coords.:**

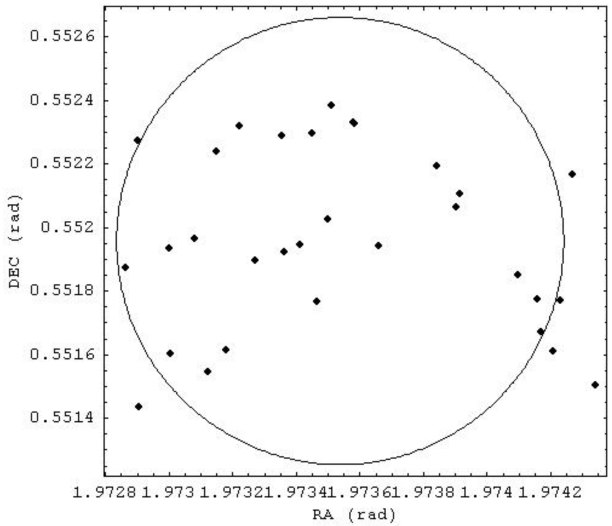
$$< R >= 309 \text{ kpc} \quad \left(< R >= \frac{2}{N_{gal}(N_{gal}-1)} \sum_{i=1}^{N_{gal}} \sum_{j=1}^{i} r_{ij} \right)$$

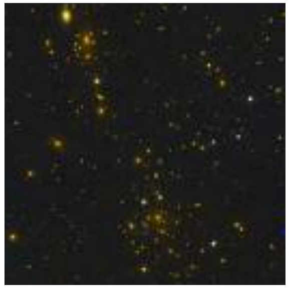
and therefore:

$$\rho_K = (2.14 \pm 0.55) \times 10^{-10} Jm^{-3}$$

$$\rho_W = (-2.83 \pm 0.92) \times 10^{-10} Jm^{-3}$$

- **From which we get:** $$\frac{\rho_K}{\rho_W} \simeq -0.76 \pm 0.14$$





Cosmic Virial Theorem and the Abell cluster A586

[O.B., Gil Pedro, Le Delliou Phys. Lett. B654 (2007)]

- **Generalized Cosmic Virial Theorem (Layzer-Irvine eq.):**

$$\dot{\rho}_{DM} + H(2\rho_K + \rho_W) = -\frac{(\eta + 3\omega_{DE})H}{1 + \Omega_{DM_0}/\Omega_{DE_0} a^{-\eta}} \rho_W$$

where

$$\rho_K \equiv M dK/dV = d(MK)/dV \propto a^{-2}$$

$$\rho_W \equiv M dW/dV = d(MW)/dV \propto a^{\zeta - 1}$$

$$W = -2\pi G a^2 \rho_{DM} \int dr \xi(r) r$$

($\xi(r)$ - auto-correlation function)

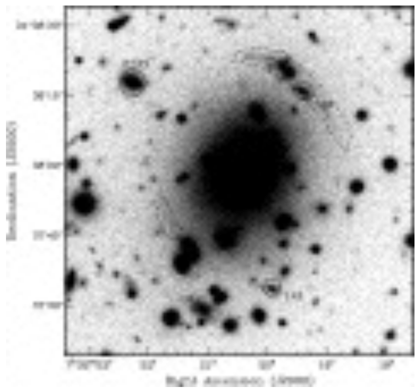
- **Abell cluster A586 – spherical and close to stationary equilibrium:**

$$2\rho_K + \rho_W = \zeta \rho_W$$

- **Moreover:**

$$\rho_K = M \frac{d}{dV} K \simeq M \frac{K}{V} \simeq \frac{9}{8\pi} \frac{M_{Cluster}}{R^3_{Cluster}} \sigma_v^2$$

$$\rho_W = M \frac{d}{dV} W \simeq M \frac{W}{V} \simeq -\frac{3}{8\pi} \frac{G}{< R >} \frac{M^2_{Cluster}}{R^3_{Cluster}}$$



Dark Energy – Dark Matter Interaction

[O.B., Gil Pedro, Le Delliou Phys. Lett. B654 (2007)]

- Interaction requires: $\eta \neq -3\omega_{DE}$
- For $\omega_{DE} = -1, \; \Omega_{DE_0} = 0.72, \; \Omega_{DM_0} = 0.24 \; , \; z = 0.1708$

$$\eta = 3.82^{+0.5}_{-0.47}$$

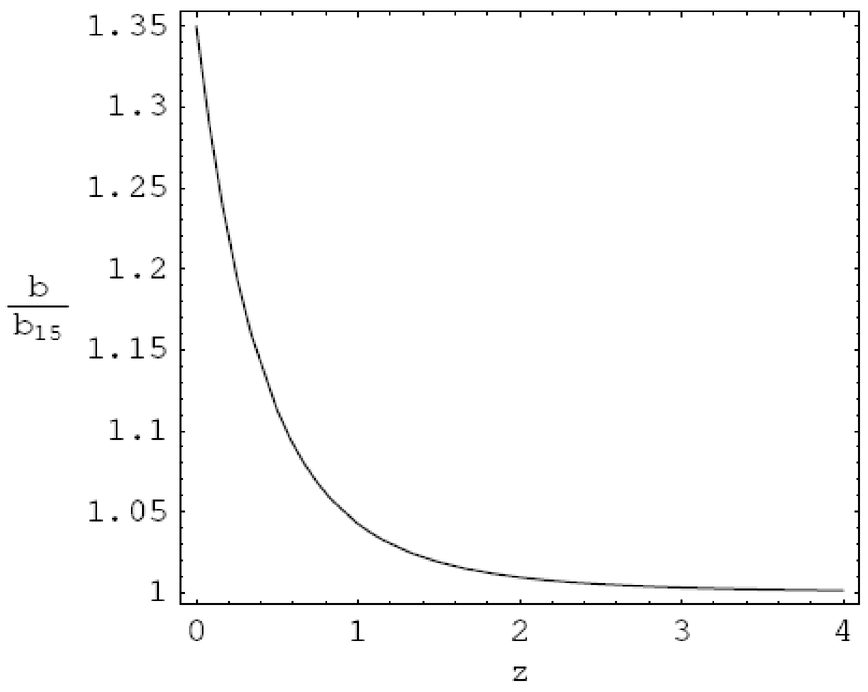
- GCG: $\alpha = 0.27^{+0.17}_{-0.16}$
- Data is consistent with interaction !
- Same methodology used for 33 relaxed galaxy clusters (optical, X-ray, gravitational lensing) suggests evidence for the interaction of DE and DM

  [Abdalla, Abramo, Sodré, Wang 2008]
- Gamma ray bursts (not so clear) [Barreiro, O.B., Torres 2010]
- Realistic density profiles (A586, A1689) (idem) [O.B., Gil Pedro, Le Delliou 2012]

Dark Energy – Dark Matter Interaction and the Equivalence Principle (EP)

[O.B., Gil Pedro, Le Delliou Phys. Lett. B654 (2007)]

- Bias parameter evolution indicates a possible violation of the Equivalence Principle

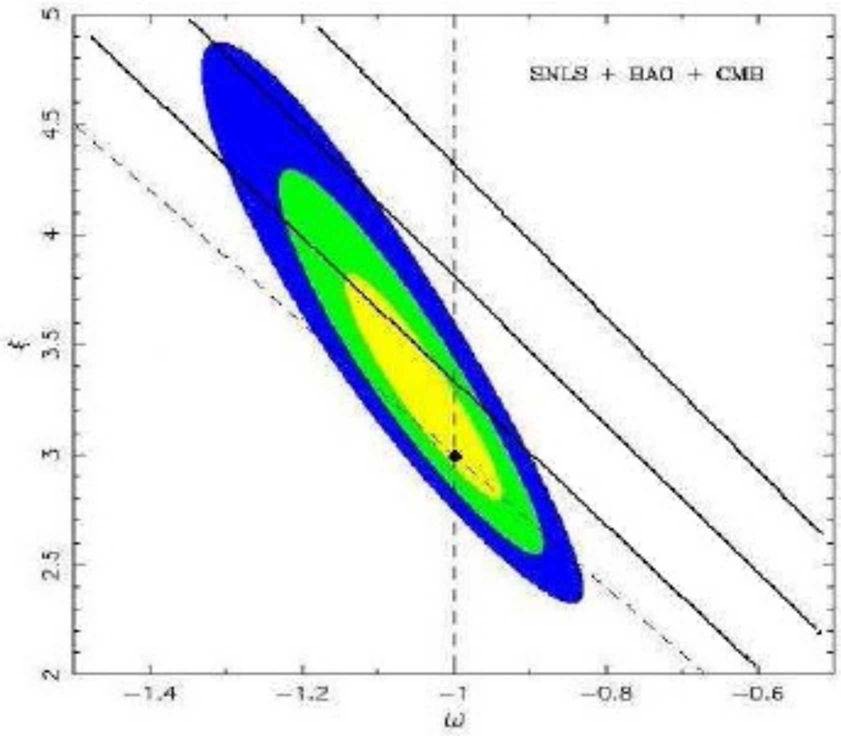


Consistency of the Abell cluster A586 results - I

[O.B., Gil Pedro, Le Delliou GR&G 2009]

- Consistency with interaction results from SN Ie, CMB and BAO:

  [Guo,Ohta, Tsujikawa 2007]



Consistency of the Abell cluster **A586** results - II

[O.B., Gil Pedro, Le Delliou, GR&G 2009]

- **DM** gravitational fall into **DM** from a simulation of the tidal stream of the Sagittarius dwarf galaxy which allows for:

$$\frac{G_{(DM-DM)}}{G_{(DM-B)}} \leq 1.1$$

[Kesden, Kamionkowski 2006]

- Assuming **DE-DM** interaction implies a change of the **DM** number density, then:

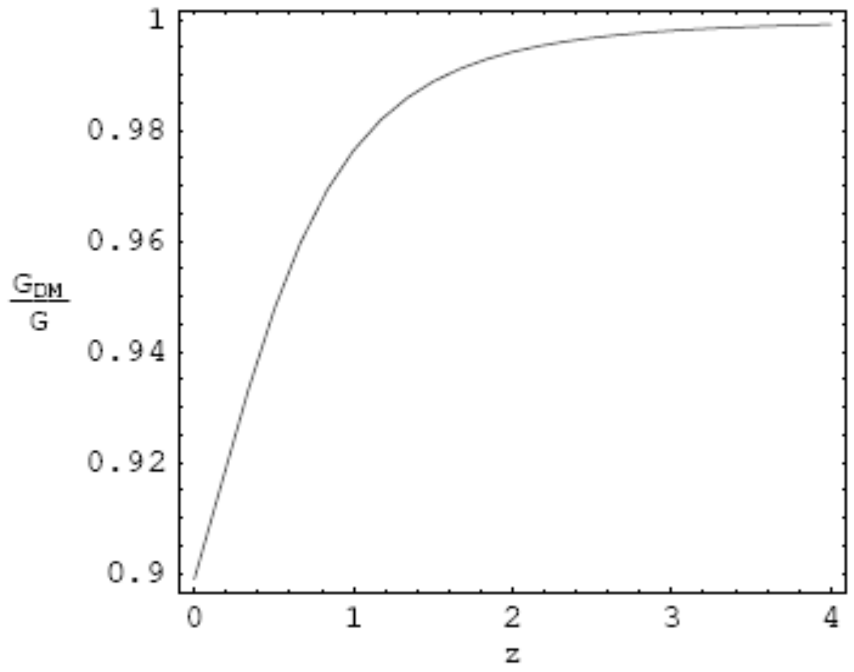
$$G_{DM} = G(1 + \delta\zeta)$$

$$\zeta = -\frac{(\eta + 3\omega_{DE})\Omega_{DE_0}}{\Omega_{DE_0} + \Omega_{DM_0} a^{-\eta}}$$

Consistency of the Abell cluster A586 results

[O.B., Gil Pedro, Le Delliou GR&G 2009]

- **For** $\omega_{DE_0} = -1$ **and** $\eta = 3.82^{+0.5}_{-0.47}$ **:**



which for z=0.17 ($\delta = 0.163$ **) yields** $G_{DM}/G = 0.92$

Conclusions

- f(R) modified graviy theories of with non-minimal curvature-matter coupling have interesting phenomenological features

- They are shown to be physically consistent and all energy conditions depend on the geometry, matter Langragian and parameters (ε, λ)

$$f_1(R) = R + \epsilon\varphi_1(R) \qquad f_2(R) = 1 + \lambda\varphi_2(R)$$

- All energy conditions, positive effective gravitational coupling and stabiltiy conditions can be expressed through a single type inequality

- For a perfect fluid, the degeneracy found in GR at the Lagrangian density level is lifted

- It admits closed timelike curves and traversable wormholes

- It leads to violation of the Equivalence Principle at cosmological scales

- It successfuly mimicks dark matter and dark energy