Measurement of τ mass at BESIII

Xiaohu Mo

On behalf of BESIII Collaboration

The 9th International Workshop on $e^+ e^-$ collisions from phi to psi 2013
Sapienza University, Rome, Italy, 9th-12th, September, 2013

Motivation of high accurate τ mass measurement

Elementary parameter in SM (PDG2012)

- **M_e=0.510998910±0.000000013 (2.6×10^{-8})**
- **M_μ=105.658367±0.000004 (3.8×10^{-8})**
- **M_τ=1776.82 ±0.16 (9.0×10^{-5})**

Lepton universality testing

$$\left(\frac{g_\tau}{g_\mu}\right)^2 = \frac{\tau_\mu}{\tau_\tau}\left(\frac{m_\mu}{m_\tau}\right)^5 \frac{B(\tau \to e\nu_e\nu_\tau)}{B(\mu \to e\nu_e\nu_\mu)}(1+\Delta_e)$$

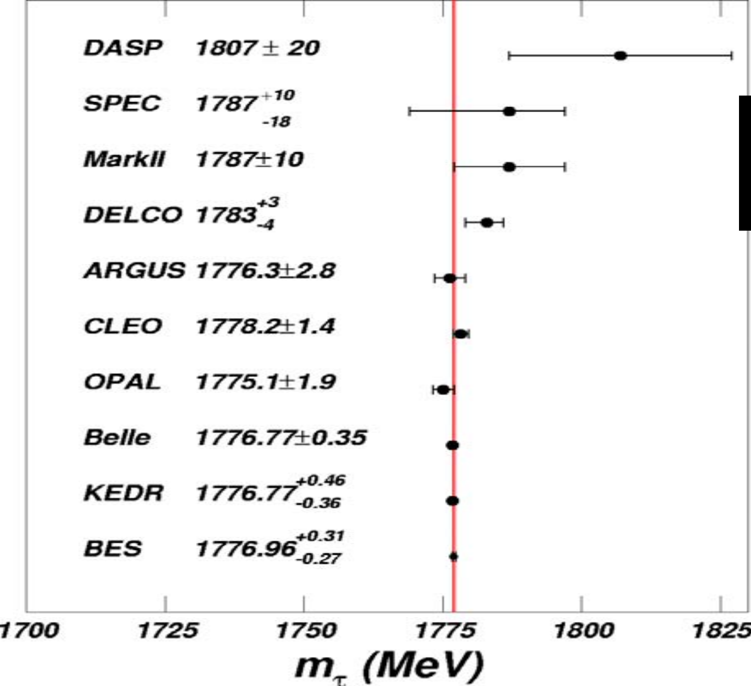
g_τ and g_μ : coupling constants; τ_τ and τ_μ :life time of τ and μ ; *B* $(\tau \to e\nu_e\nu_\tau)$ and *B* $(\mu \to e\nu_e\nu_\mu)$: decay branching ratio; Δ_e:correct factor (phase factor, radiative correction factor of QED， correct factor of propagator of W-meson etc.)

Yoshio Koideo (1981) equality testing

$$m_e + m_\mu + m_\tau = \frac{2}{3}\left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2$$

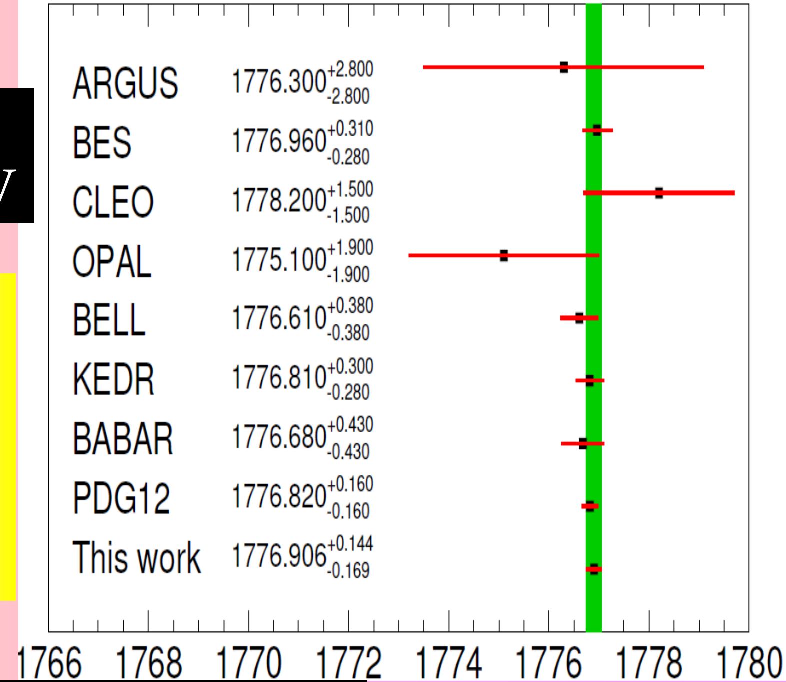
$$\Delta f_m = \sqrt{\sum_{i=e,\mu,\tau}\left(m_i - \frac{2}{3}\sum_{k=e,\mu,\tau}\sqrt{m_i m_k}\right)^2 \cdot \left(\frac{\delta m_i}{m_i}\right)^2}$$

→ $\Delta f_m \cong 1/3\ \delta m_\tau$



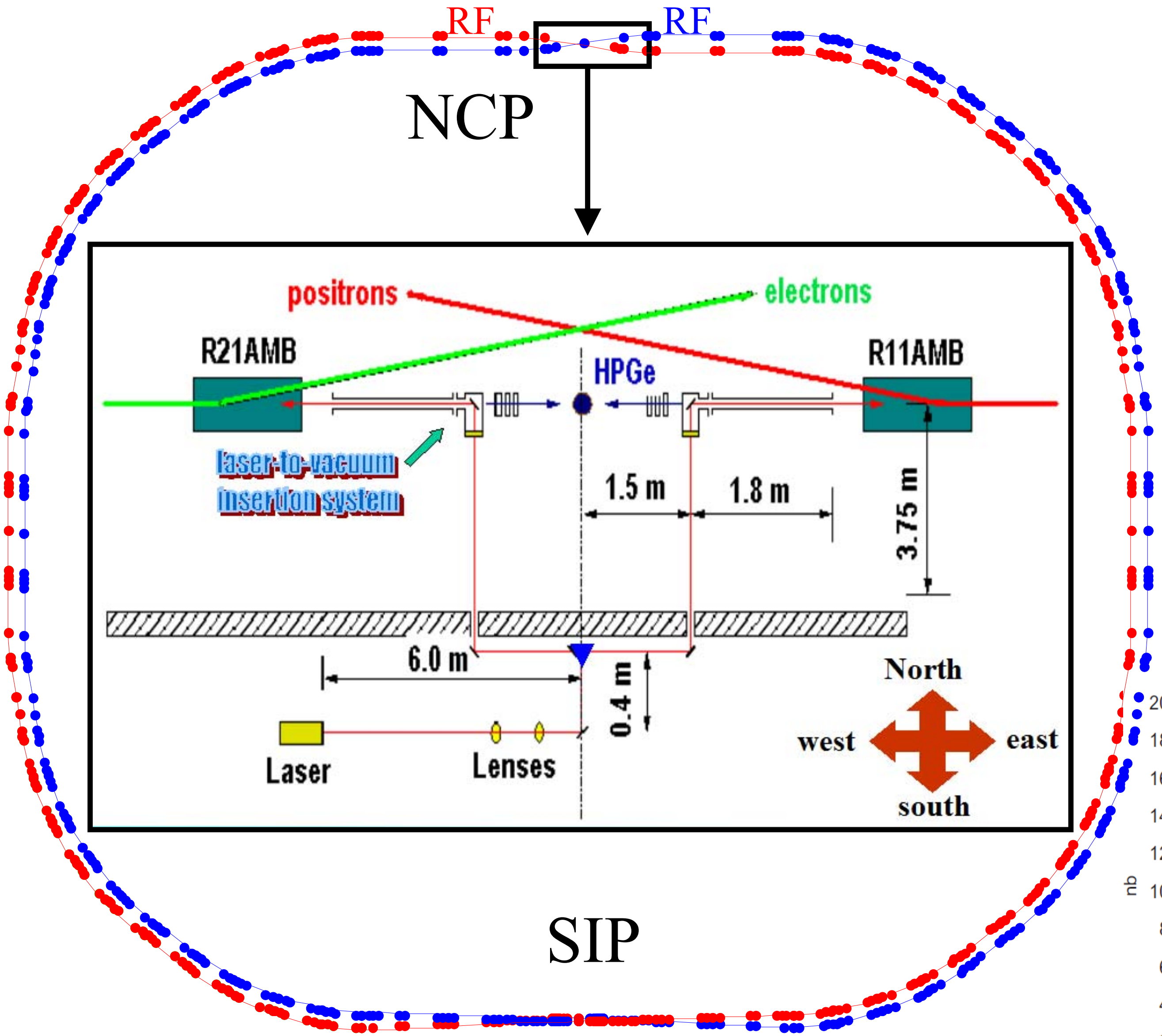
PDG2012:
1776.82 ±0.16 MeV

m_τ



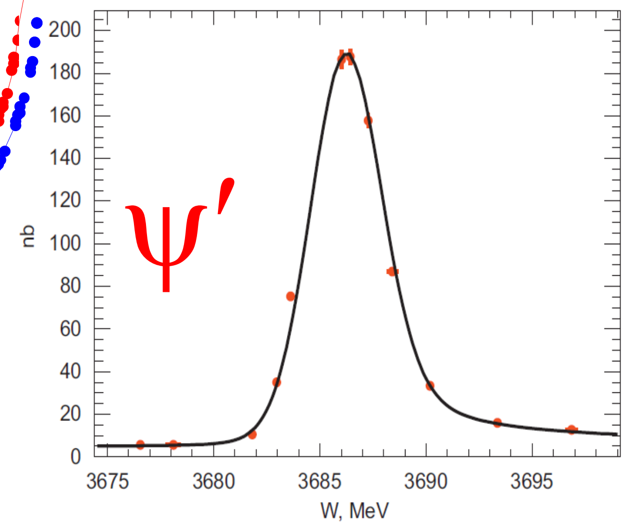
Method: Pseudo-mass and threshold scan

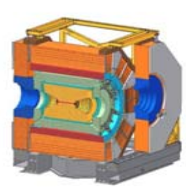
τ lepton mass measurement [value+statistic +systematic error]	Year	Ex. Group	Data sample	Method
1776.68 ± 0.12 ± 0.41	**2009**	**Babar**	423 fb^{-1}	Pseudo-mass
1776.81+(+0.25−0.23) ± 0.15	**2007**	**KEDR**	6.7 pb^{-1}	Scan
1776.61 ± 0.13 ± 0.35	**2007**	**Belle**	414 fb^{-1}	Pseudo-mass
1776.96+(+0.18−0.21)+(+0.25−0.17)	**1996**	**BES**	5.1 pb^{-1}	Scan



For BEPCII
Energy accuracy
Improvement :
$10^{-3} \rightarrow 5\times 10^{-5}$

$\delta m_{\psi'}/m_{\psi'} = 2\times 10^{-5}$;
$\delta\Delta / \Delta =$ 6%.





Content

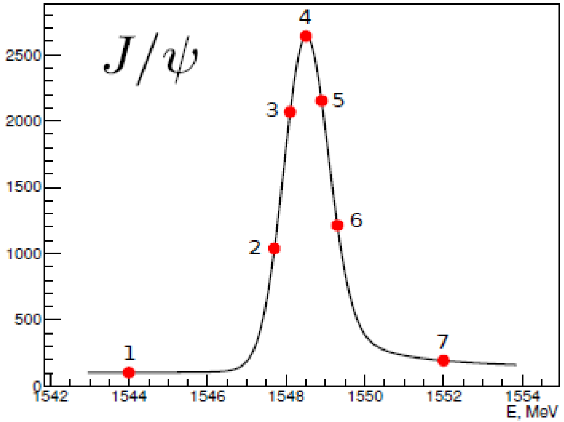
1. Scan optimization
2. Data taking
3. Analysis
   1) Energy point determination
   2) Resonance scan $(E, \Delta E)$
   3) Events selection
   4) τ mass fit
4. Summary

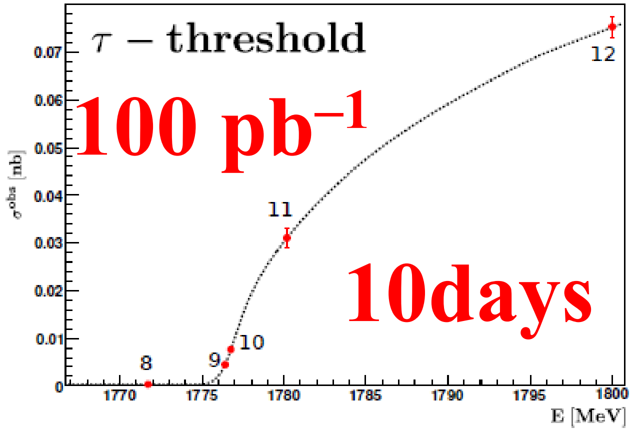
Most accurate measurement at BESIII!

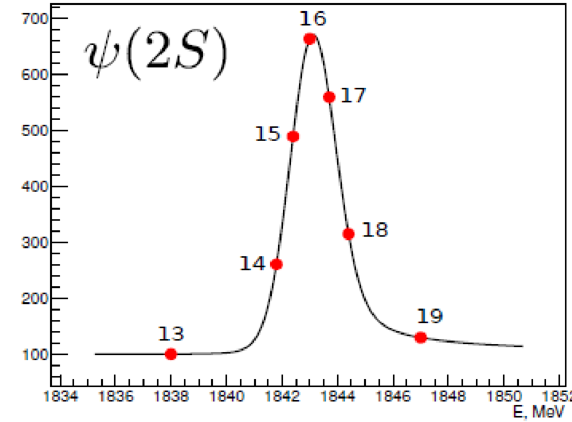
BINP, Hawaii University,
IHEP, Tsinghua University.

Statistic optimization for scan data taking

1. N free parameters fit, N energy points is enough
2. The optimal position can be obtained by single parameter scan
3. Luminosity allocation can be determined analytically or by simulation method
4. The uncertainty of τ mass is proportional to the inverse of square root of luminosity







First circle:

J/ψ scan (7 pts) → τ scan (5 pts) → ψ' scan (7 pts)

Second circle:

J/ψscan (7pts) →τscan (pt.9&10) → ψ'scan (7 pts)

Final uncertainty

(sta. ⊕ sys.)< 0.1MeV

τ scan plan /anticipated

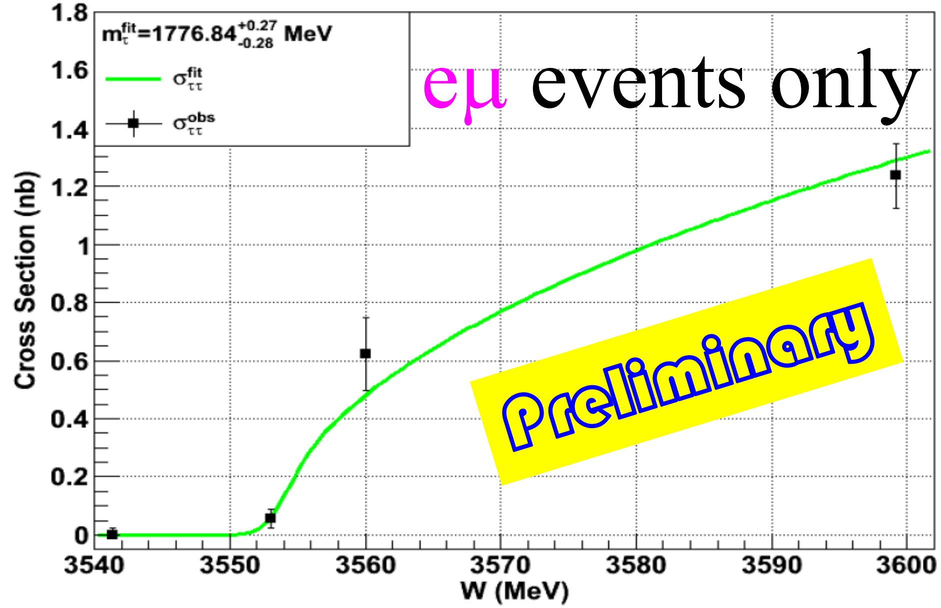
Energy region	order	Beam energy (MeV)	C.M.S energy (MeV)	Integrated lum. (pb^{-1})
Energy points for J/psi scan	1	1544.0	3088.00	
	2	1547.7	3095.40	
	3	1548.1	3096.20	
	4	1548.5	3097.00	
	5	1548.9	3097.80	
	6	1549.3	3098.60	
	7	1552.0	3104.00	
Energy points for tau scan	8	1771.0	3542.00	14
	9	1776.4	3552.80	14+25
	10	1776.65	3553.30	14+12
	11	1780.2	3560.40	7
	12	1792.0	3584.00	14
Energy points for psi' scan	13	1838.0	3676.00	
	14	1841.8	3683.60	
	15	1842.4	3684.80	
	16	1843.0	3686.00	
	17	1843.7	3687.40	
	18	1844.4	3688.80	
	19	1847.0	3694.00	

actual τ mass scan

Job	Beam energy (MeV)	Begin Run number	Begin time	End Run number	End time	Int. lum. (pb^{-1})
J/psi scan	1544.0 1547.7 1548.1 1548.5 1548.9 1549.3 1552.0	24937	2011/12/22; 10:40:12	24978	2011/12/23; 10:01:35	1.428
Energy points for tau scan	1771.0	24984	2011/12/23; 13:12:26	25015	2011/12/24; 14:47:46	4.035
	1776.9	25019	2011/12/23; 18:24:10	25094	2011/12/26; 08:32:34	4.914
	1780.4	25098	2011/12/26; 09:46:24	25141	2011/12/27; 09:38:34	3.671
	1800.0	25142	2011/12/27; 10:26:46	25243	2011/12/29; 08:48:20	9.056
psi' scan	1838.0 1841.9 1842.5 1843.1 1843.8 1844.5 1847.0	25244	2011/12/29; 09:04:32	25337	2011/12/31; 02:31:32	7.245

4/14 5/(39+26) 3.7/7 9/14

28% 7.7% 53% 64%



21.75 pb^{-1} (τ) + 8.677pb^{-1}(J/ψ &ψ′)=30.348pb^{-1} /9.7days

Preliminary results:

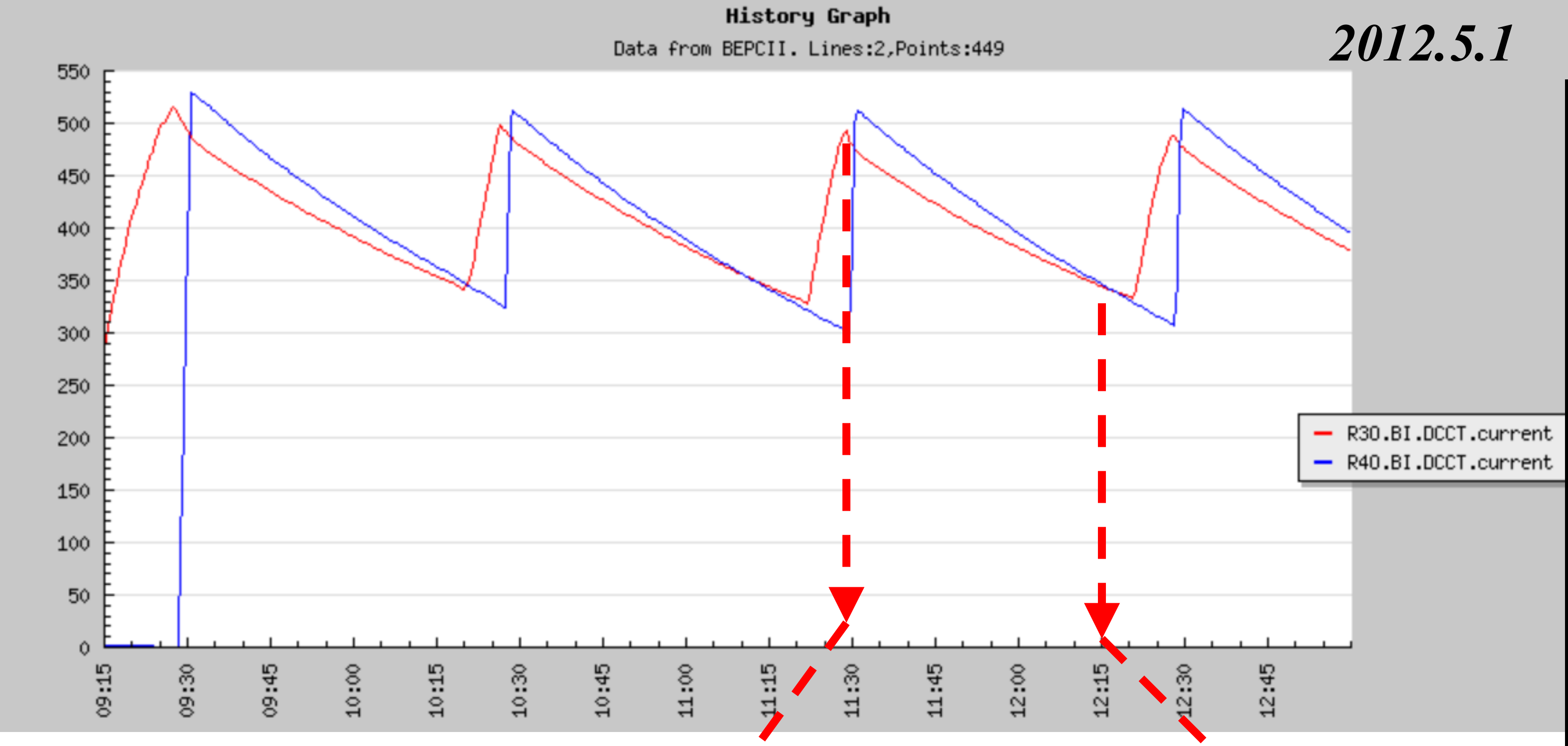
(1776.84 ±0.30) MeV

0.31MeV (5 pb^{-1}) → < 0.1MeV (66 pb^{-1})

Testing scan !!

Details about data analysis

1) *Energy point determination*
2) *Resonance scan* $(E, \Delta E)$
3) *Events selection*
4) τ *mass fit*



Under the condition of stable running, the data taking time for **BESIII** and **BEMS** almost coincide with each other; but for unstable situation, whatever due to accelerator or detector, the overlap of two kinds of time are complicated, and should be considered carefully, especially for scan data taking.

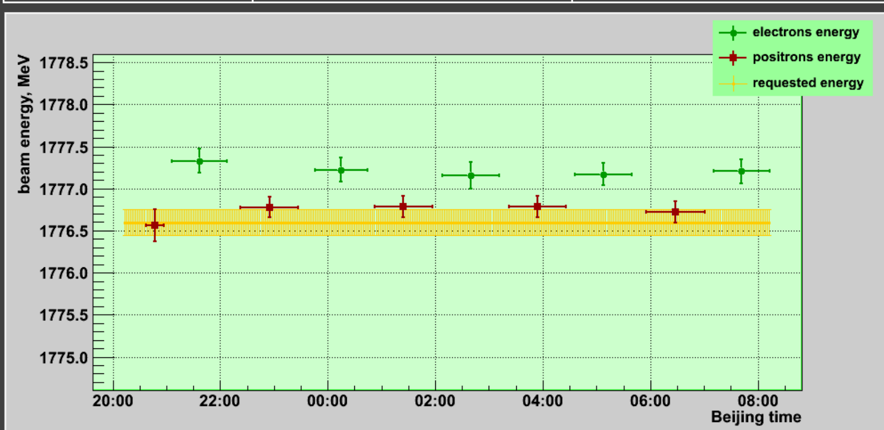
Two conditions for BEMS data taking control:

1.Accelerator status, beam injection;

2. Time duration of counting, 40-60 minutes.

Beam Energy Measurement System Status (Sun Dec 25 08:13:20 2011)

	Electrons	Positrons
Energy, MeV:	1777.207 ± 0.141	1776.722 ± 0.132
Energy spread, keV:	976 ± 199	1227 ± 168
Measured from:	Sun Dec 25 07:10:25 2011	Sun Dec 25 05:55:33 2011
Measured until:	Sun Dec 25 08:13:01 2011	Sun Dec 25 07:00:35 2011

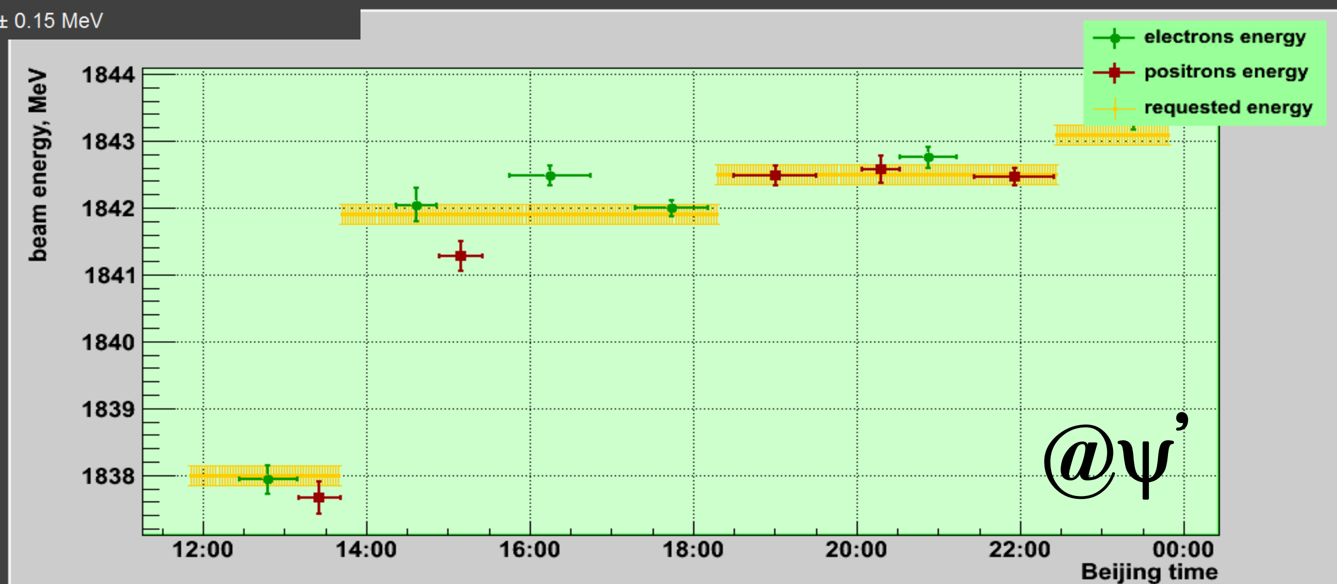


Requested energy is: 1776.60 ± 0.15 MeV

For resonance scans, the situation is rather … …

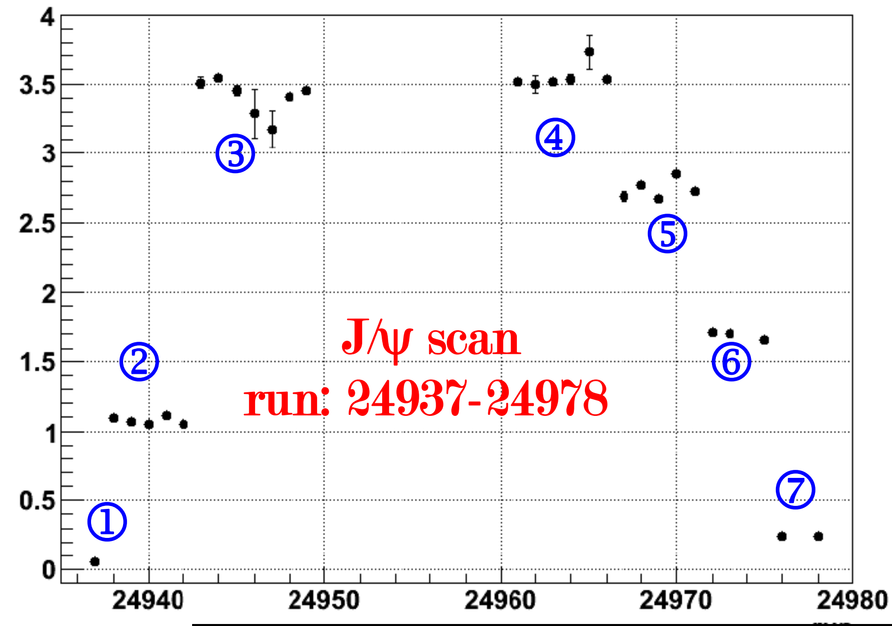
m Energy Measurement System Status (Thu Dec 29 23:50:15 2011)

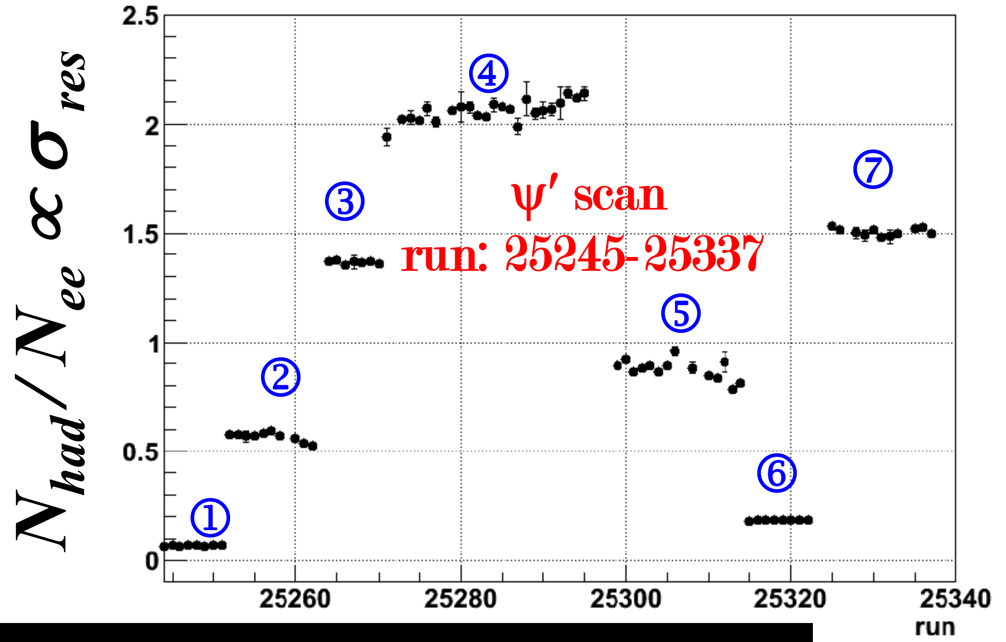
	Electrons	Positrons
	1843.323 ± 0.147	1842.458 ± 0.131
	1139 ± 226	605 ± 190
	Thu Dec 29 22:57:11 2011	Thu Dec 29 21:26:32 2011
	Thu Dec 29 23:49:57 2011	Thu Dec 29 22:25:04 2011



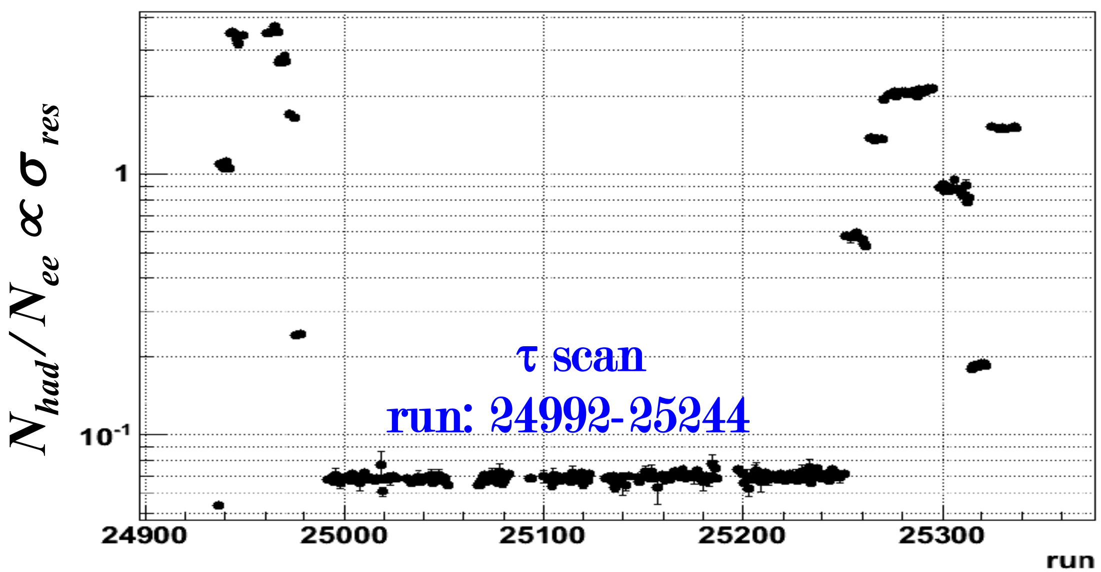
Requested energy is: 1843.10 ± 0.15 MeV

For τ scan, the data time is comparative long, and the running status is fairly stable





Classification of run by the ratio of N_{had}/N_{ee}

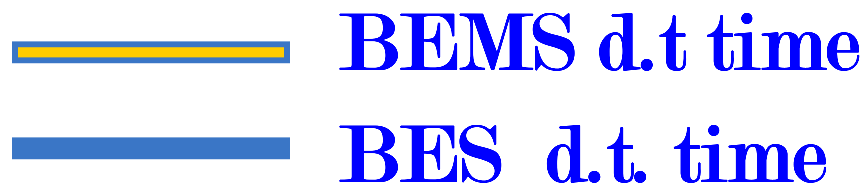


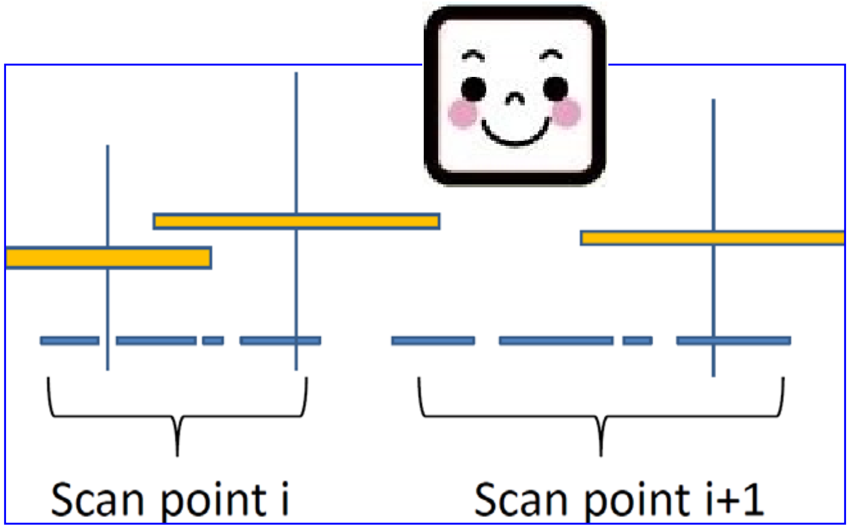
Time matching

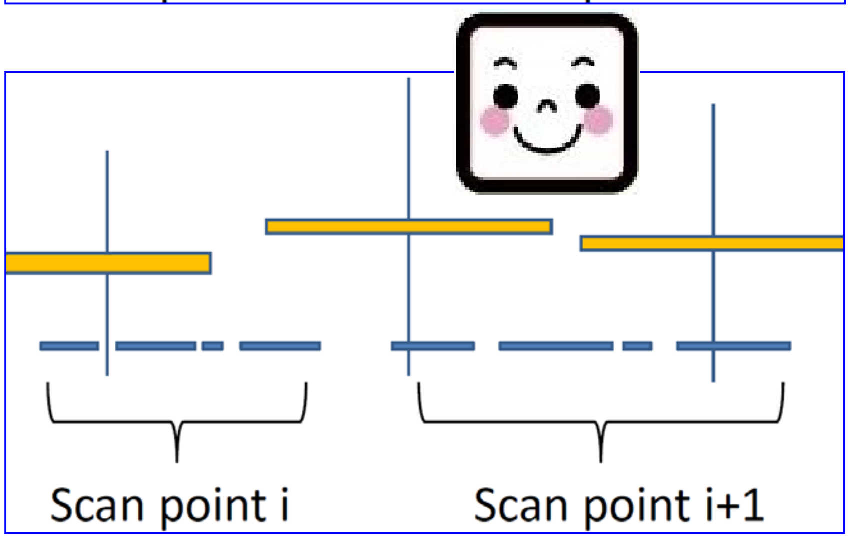
1) For each scan point, find run0, run1 are the starting and ending run # respectively; and T0 is starting time of run0; T1 is ending time of run1;

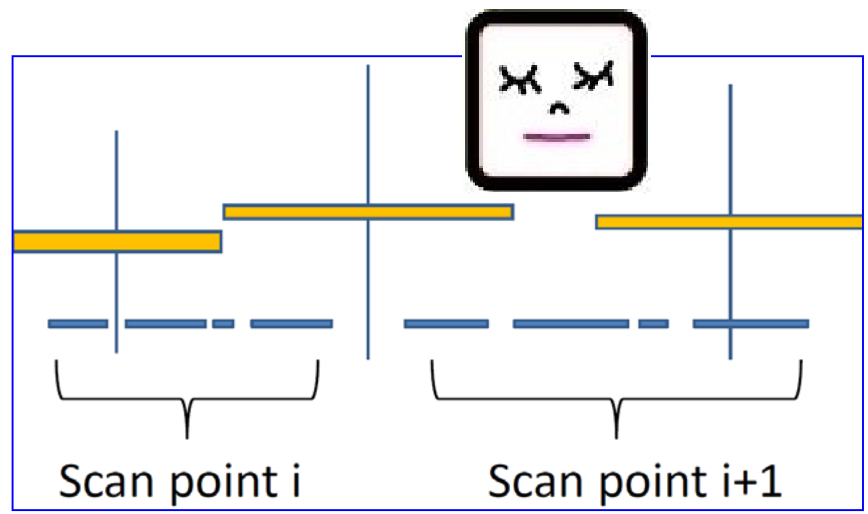
2) Select E(e+/e-) whose starting time or ending time falls >=T0 and <=T1;

3) Abandon E(e+/e-) of the scan point i whose central time <T1(i-1) or >T0(i+1).









Scan energy	Runs	E_{e^-}	ΔE_{e^-}	E_{e^+}	ΔE_{e^+}	E_{CM}	ΔE_{CM}
1554.0	24937	1544.542	0.135	1544.312	0.217	3088.667	0.256
1547.7	24938-24942	1547.917	0.099	1547.548	0.106	3095.278	0.145
1548.1	24943-24949	1548.692	0.103	1548.171	0.086	3096.676	0.135
1548.5	24959-24966	1549.079	0.109	1548.714	0.075	3097.606	0.133
1548.9	24967-24971	1549.451	0.081	1549.014	0.114	3098.278	0.140
1549.3	24972-24975	1549.566	0.101	1549.438	0.083	3098.817	0.131
1552.0	24976-24978	1552.186	0.088	1551.936	0.107	3103.934	0.139
1771.0	24983-25015	1771.558	0.067	1771.069	0.053	3542.413	0.085
1777.0	25016-25094	1777.307	0.060	1776.730	0.046	3553.822	0.075
1780.4	25100-25141	1780.926	0.055	1780.431	0.065	3561.142	0.085
1800.0	25143-25243	1800.526	0.044	1799.878	0.044	3600.186	0.062
1838.0	25244-25251	1838.183	0.256	1837.940	0.157	3675.901	0.300
1841.9	25252-25262	1842.234	0.112	1841.642	0.281	3683.653	0.303
1842.5	25264-25270	1842.825	0.201	1842.511	0.112	3685.113	0.230
1843.1	25271-25295	1843.560	0.113	1843.000	0.152	3686.337	0.189
1843.8	25325-25337	1844.148	0.126	1843.648	0.095	3687.573	0.158
1844.5	25299-25314	1844.700	0.177	1844.342	0.140	3688.819	0.226
1847.0	25315-25322	1847.141	0.189	1846.597	0.156	3693.515	0.245

$$E_{cm}^{\sqrt{s}} \approx 2E_{beam}\left(1-\frac{\alpha^2}{8}\right)$$

Most

$\delta L/L < 2$ %

Bhabha

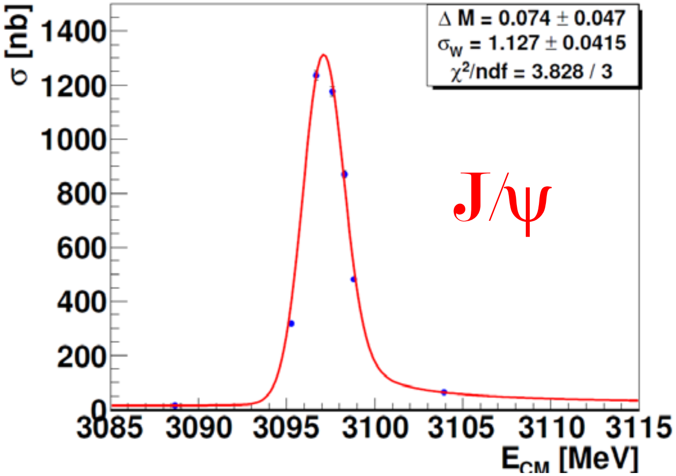
$e^+e^- \to e^+e^-$

Di-gamma

$e^+e^- \to \gamma\gamma$

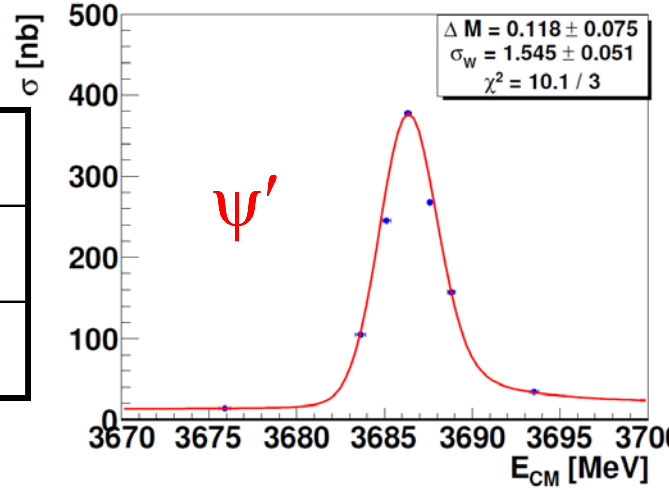
Scan	E_{CMS} (GeV)	$\mathcal{L}_{Bhabha}$ (nb^{-1})	$\mathcal{L}_{\gamma\gamma}$ (nb^{-1})	$\mathcal{L}_{Bhabha}/\mathcal{L}_{\gamma\gamma}$
J/ψ	3.0887	77.95 ± 0.81	78.5 ± 1.9	0.993 ± 0.026
	3.0953	223.6 ± 2.6	219.3 ± 3.1	1.020 ± 0.019
	3.0967	247.4 ± 2.1	243.1 ± 3.3	1.018 ± 0.016
	3.0976	202.6 ± 1.8	206.5 ± 3.1	0.981 ± 0.017
	3.0983	223.2 ± 2.2	223.5 ± 3.2	0.999 ± 0.017
	3.0988	213.9 ± 2.2	216.9 ± 3.1	0.986 ± 0.018
	3.1039	312.9 ± 2.4	317.3 ± 3.8	0.986 ± 0.014
τ	3.5424	4283.4 ± 26.5	4252.1 ± 18.9	1.007 ± 0.008
	3.5538	5595.9 ± 34.4	5566.7 ± 22.8	1.005 ± 0.007
	3.5611	3873.0 ± 24.0	3889.2 ± 17.9	0.996 ± 0.008
	3.6002	9581.3 ± 58.5	9553.0 ± 33.8	1.003 ± 0.007
ψ'	3.6759	788.2 ± 5.5	787.0 ± 7.2	1.001 ± 0.012
	3.6837	835.4 ± 6.2	823.1 ± 7.4	1.015 ± 0.012
	3.6851	836.7 ± 6.0	832.4 ± 7.5	1.005 ± 0.012
	3.6863	1209.4 ± 8.0	1184.3 ± 9.1	1.021 ± 0.010
	3.6876	1672.8 ± 11.1	1660.7 ± 11.0	1.007 ± 0.009
	3.6888	788.7 ± 5.6	767.7 ± 7.2	1.027 ± 0.012
	3.6935	1497.3 ± 9.8	1470.8 ± 10.3	1.018 ± 0.010

Fit results for J/ψ & ψ'

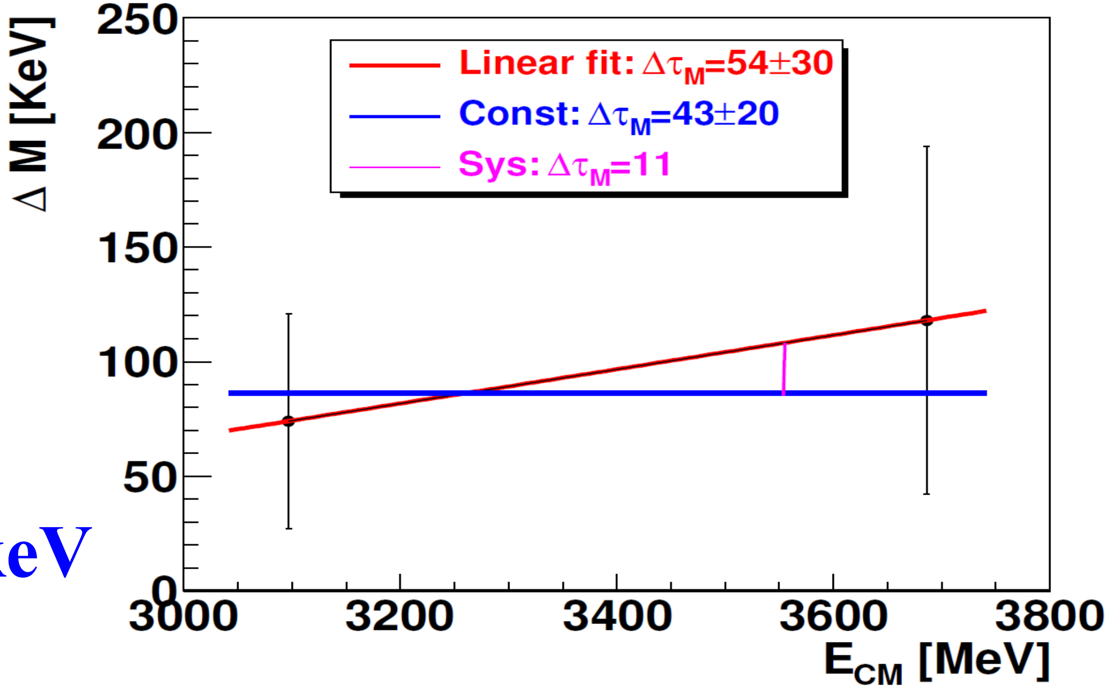


$\Delta M = M_{fit} - M_{PDG}$

Scan	ΔM / keV
J/ψ	74±47±33
ψ′	118±76±9



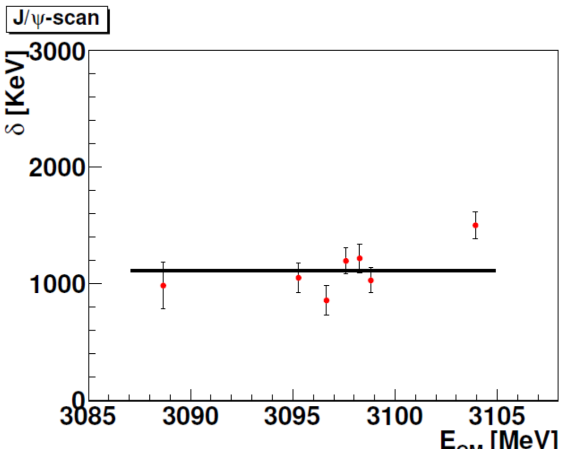
$\delta M_{\tau} = 54 \pm 30$ (stat.) ± 33 (sys.) keV

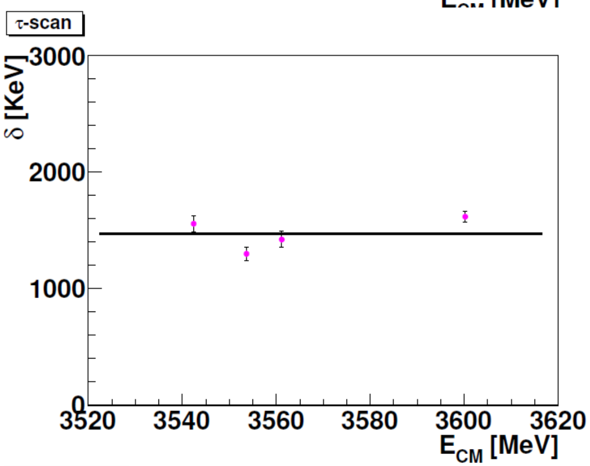


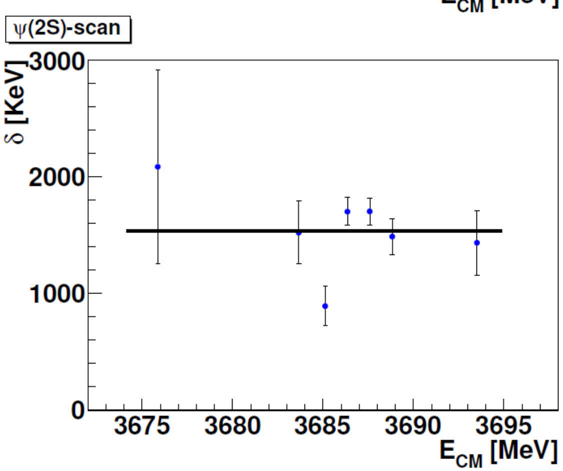
$\Delta E_{Scale} \approx 90$ keV

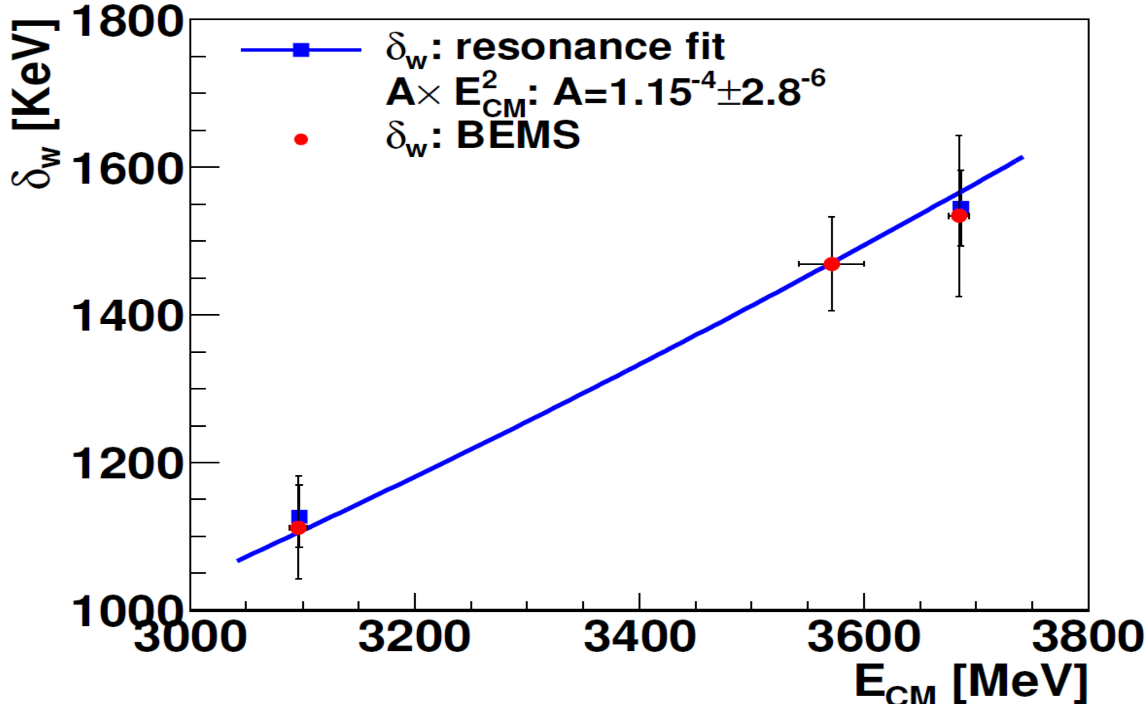
effect of ΔM on δM τ due to energy scale

Fit results for J/ψ, τ and ψ' from BEMS data









Scan	ΔM / keV
Measured from BEMS	
J/ψ	**1111.9±69.7**
τ	**1469.2±63.7**
ψ′	**1534.3±108.8**
Fit from Scan Curve	
J/ψ	**1127±42±42**
ψ′	**1545±51±22**

effect of Δ_{Spread} on δM $_\tau$ due to energy spread

$\Delta E_{Spread} \approx 12$ keV

Event Selection

Partial information, not the full list !

PID	p (GeV/c)	EMC	TOF	MUC	other
e	$p_{min} < p < p_{max}$	$0.8 < E/p < 1.05$	$\lvert\Delta tof(e)\rvert < 0.2$ $0 < tof < 4.5$		
μ	$p_{min} < p < p_{max}$	$E/p < 0.7$ $0.1 < E < 0.3$	$\lvert\Delta tof(\mu)\rvert < 0.2$	($depth > 80 \times p$-50 or $depth > 40$) and $numhits > 1$	
π	$p_{min} < p < p_{max}$	$E/p < 0.6$	$\lvert\Delta tof(\pi)\rvert < 0.2$ $0 < tof < 4.5$		not μ
K	$p_{min} < p < p_{max}$	$E/p < 0.6$	$\lvert\Delta tof(K)\rvert < 0.2$ $0 < tof < 4.5$		not μ

No good photon: $N_\gamma=0$

Good photon:

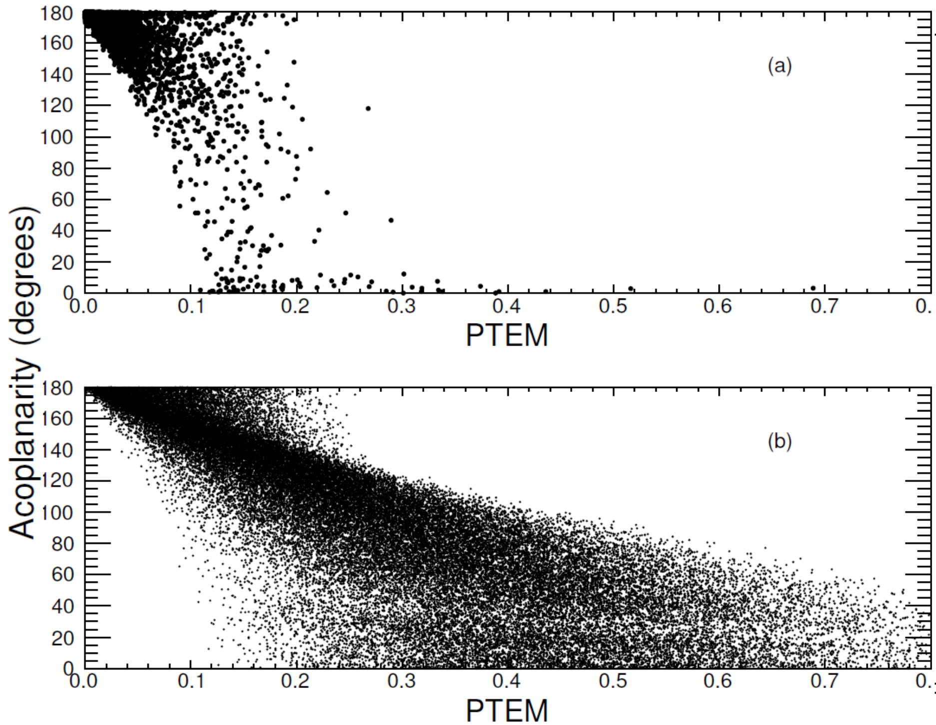
1) 0<TDC<14, (unit: 50ns)
2) |cosθ|<0.8, E>25MeV
3) 0.84<| cosθ |<0.92, E>50MeV
4) θγc >20

6

The detection efficiency for different final states at different scan points

scan point	Efficiency (%)								
	ee	$e\mu$	eh	$\mu\mu$	μh	hh	$e\rho$	$\mu\rho$	$\pi\rho$
2	17.1	21.8	32.4	14.2	15.3	25.6	9.9	5.5	9.1
3	17.6	23.2	34.9	14.0	16.9	29.3	10.4	6.1	8.9
4	17.8	23.1	36.2	13.9	17.7	34.5	10.8	5.3	12.8

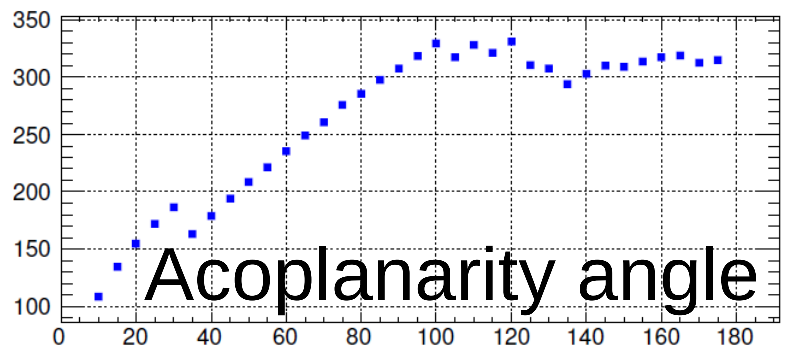
Event Selection and optimization

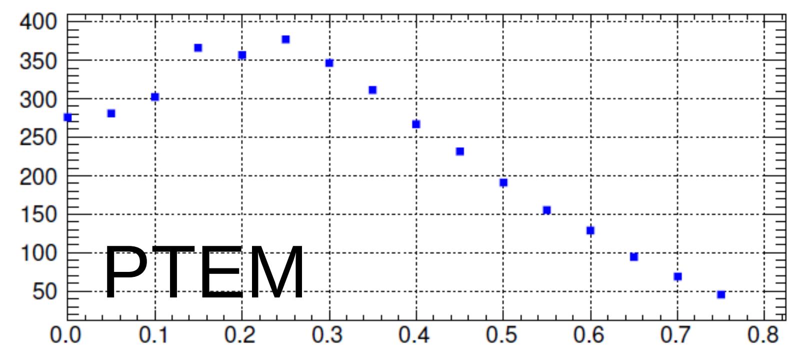


final state	acoplanarity angle	PTEM
ee	$\theta_{acop} > 10$	PTEM>0.3
$e\mu$	$\theta_{acop} < 160$	PTEM>0.1
$e\pi$	$\theta_{acop} < 170$	PTEM>0.1
eK	$\theta_{acop} < 170$	
$\mu\mu$	$\theta_{acop} < 140$	
μh	$\theta_{acop} < 140$	
hh	$\theta_{acop} < 160$	
$e\rho$	$\theta_{acop} < 170$	
$\mu\rho$	$\theta_{acop} < 150$	
$\pi\rho$		

$$PTEM = \frac{P_T}{E_{miss}^{max}} = \frac{(\vec{P}_1 + \vec{P}_2)_T}{W - |\vec{P}_1| - |\vec{P}_2|}$$

$$\frac{S}{\sqrt{S+B}}$$





MC sample

The number of observed events and that of normalized MC samples are consistent within errors.

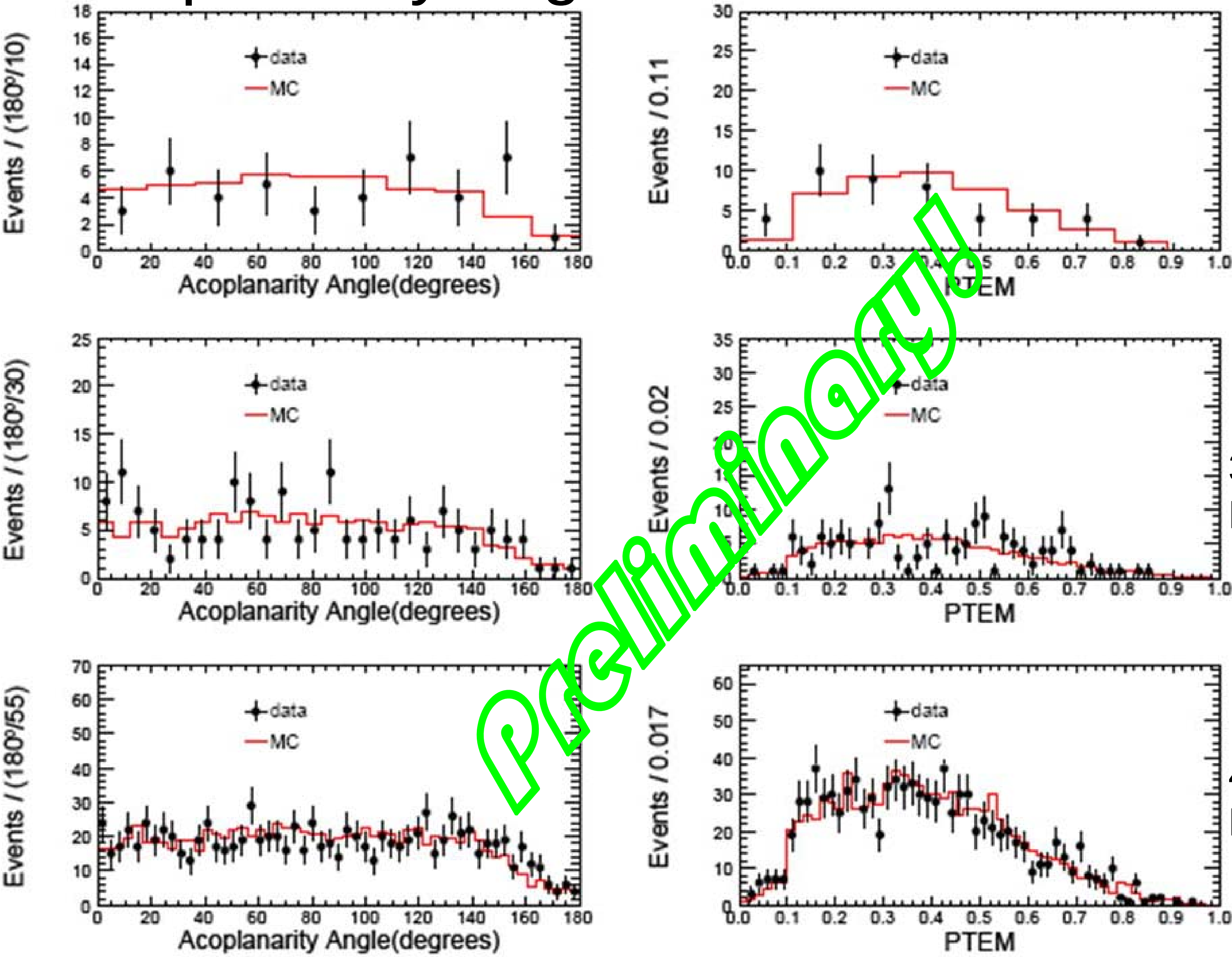
final state	1		2		3		4		total	
	Data	MC	Data	MC	Data	MC	Data	MC	Data	MC
ee	0	0	4	3.7	13	12.2	84	76.1	101	91.9
$e\mu$	0	0	8	9.2	35	31.3	168	192.7	211	233.1
$e\pi$	0	0	8	8.6	33	29.6	202	184.5	243	222.7
ek	0	0	0	0.5	2	1.8	16	16.9	18	19.3
$\mu\mu$	0	0	2	2.9	8	9.2	49	56.3	59	68.4
$\mu\pi$	0	0	4	3.9	11	14.0	89	86.7	104	104.7
μk	0	0	0	0.2	3	0.8	7	9.0	10	10.1
$\pi\pi$	0	0	1	2.0	5	7.7	57	54.0	63	63.8
πk	0	0	1	0.3	0	0.8	10	8.2	11	9.3
kk	0	0	0	0.0	1	0.1	1	0.3	2	0.4
$e\rho$	0	0	3	6.1	19	20.6	142	132.0	164	158.7
$\mu\rho$	0	0	8	3.3	18	11.8	52	62.3	68	78.5
$\pi\rho$	0	0	5	3.4	15	10.8	97	96.0	117	110.2
Total	0	0	44	44.2	153	150.8	974	976.1	1171	1171.1

Preliminary!

Total consistency is fairly well!

Data 1171
MC 1171.1

Acoplanarity angle　　PTEM

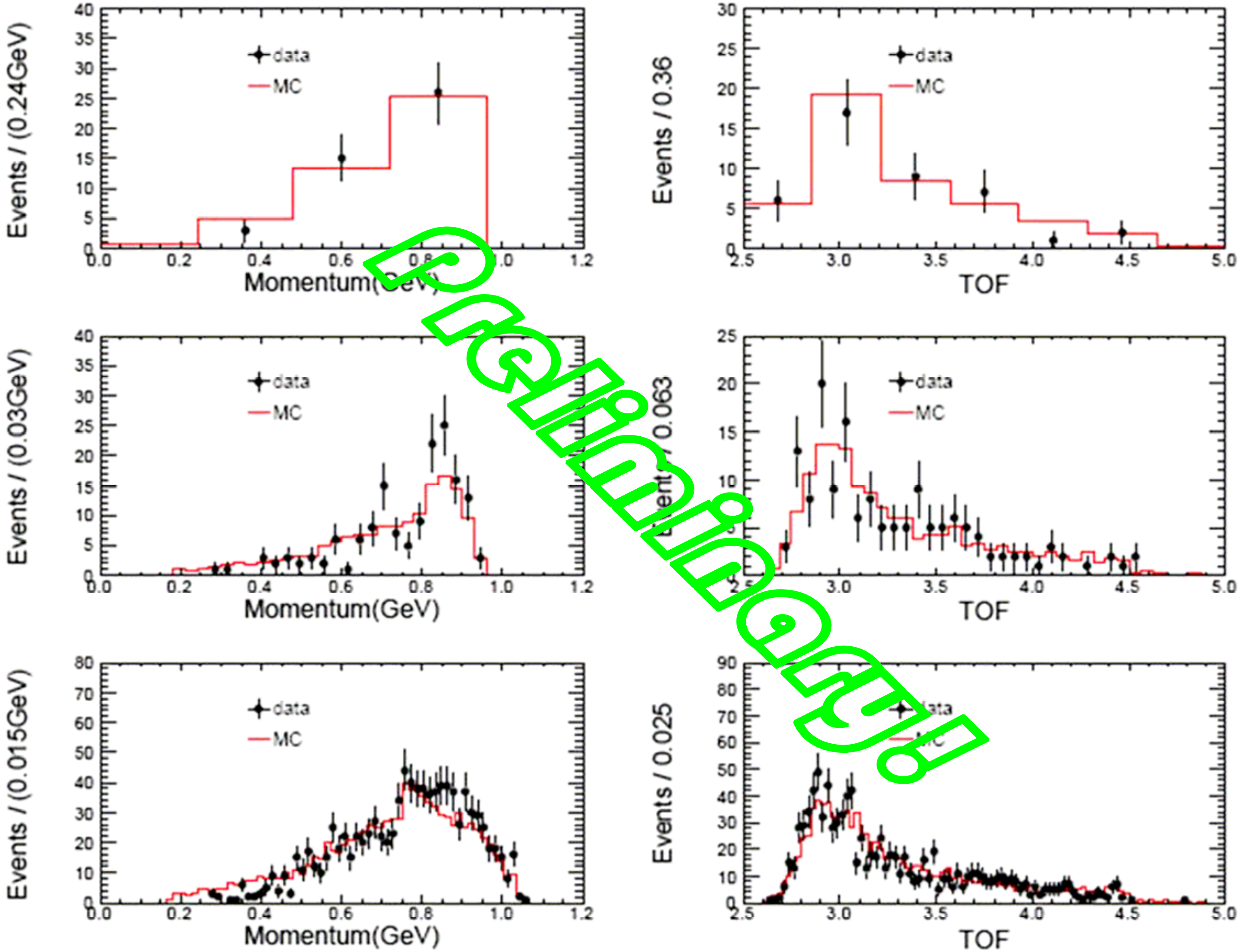


2-point
Ecm=3553.8MeV

3-point
Ecm=3561.1MeV

4-point
Ecm=3600.2MeV

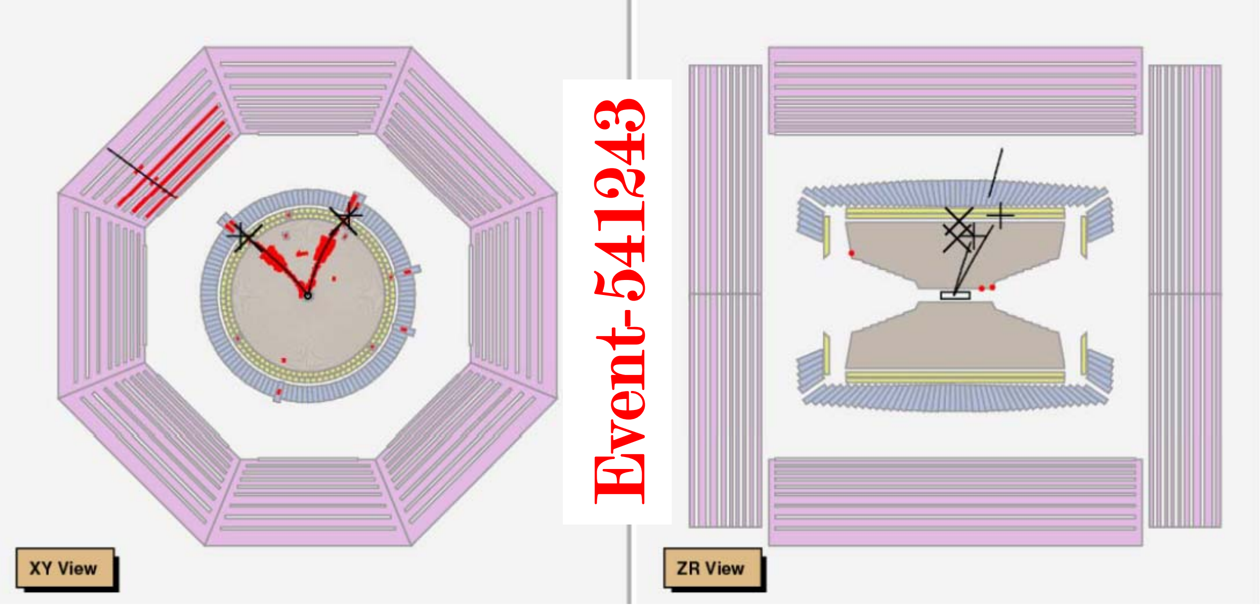
Momentum of charged tracks   TOF of charged tracks



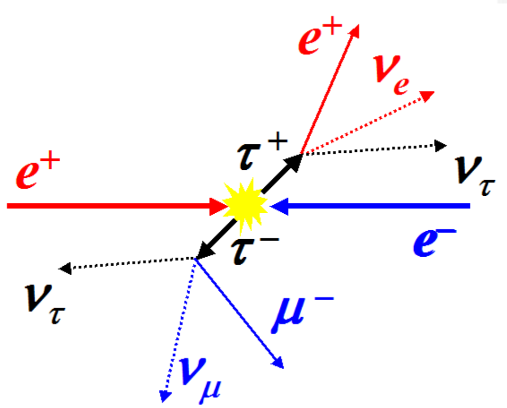
2-point
Ecm=3553.8MeV

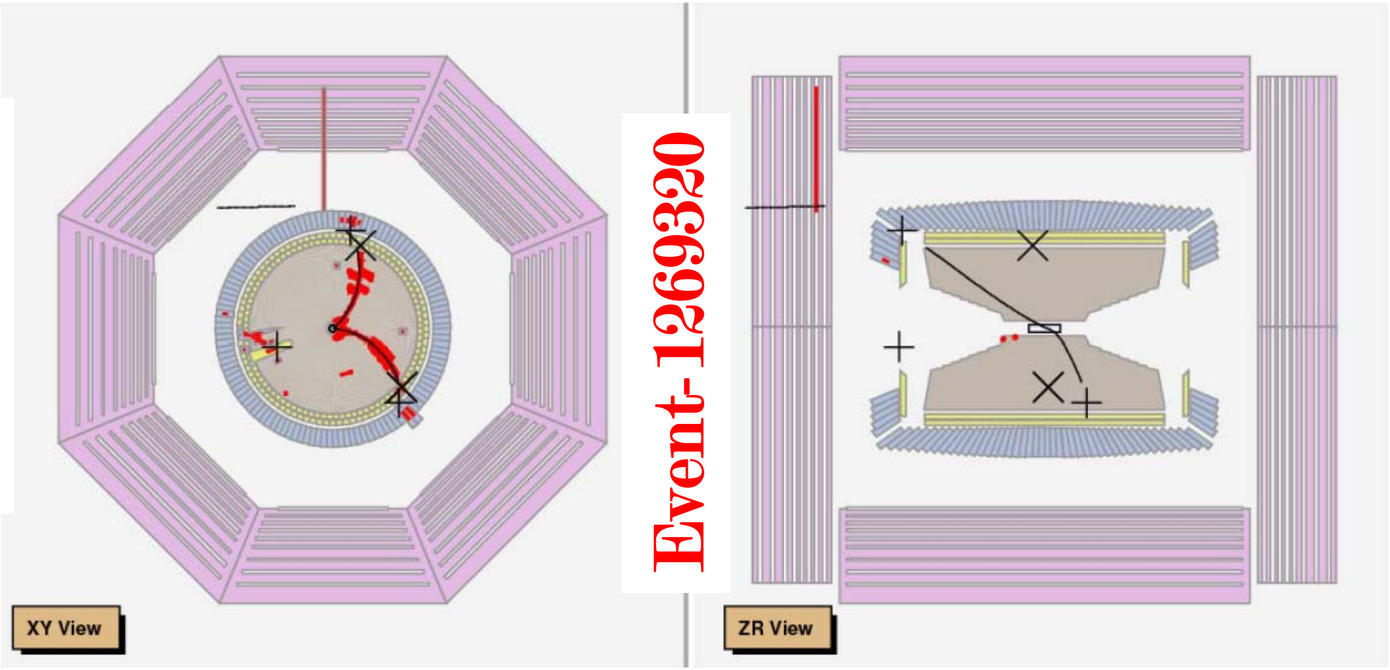
3-point
Ecm=3561.1MeV

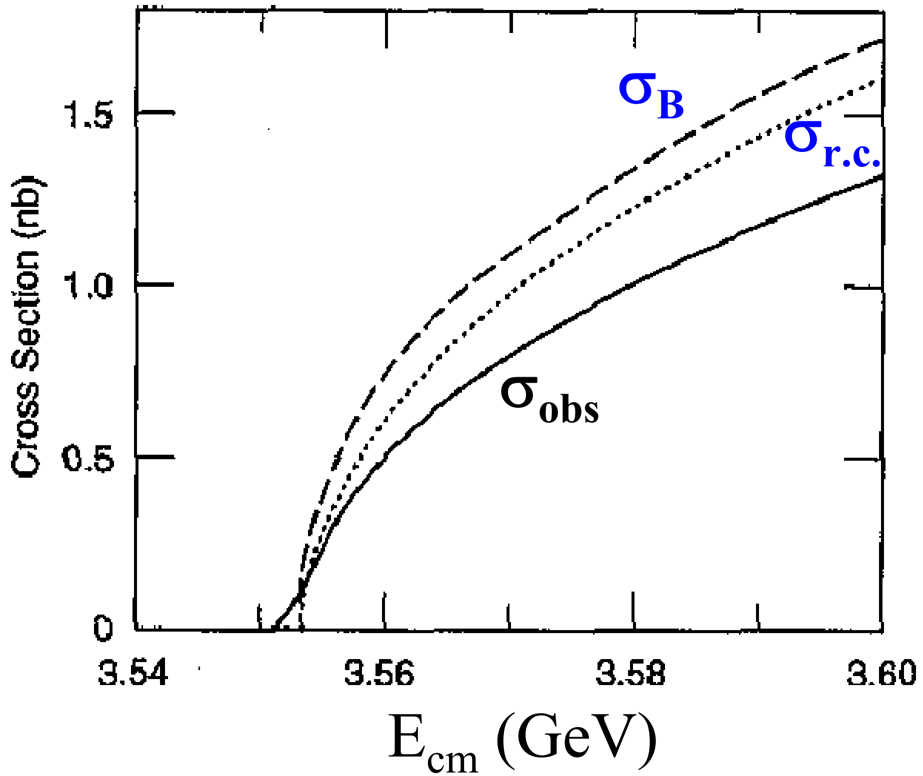
4-point
Ecm=3600.2MeV



Event display for τ-pair decay to e μ final state @ τ-pair threshold







BES:PRD53(1995)20

$$LF = \prod_{i=1}^{n} P_i \,, \quad P_i = \frac{\mu_i^{N_i} e^{-\mu_i}}{N_i!}$$

$$\mu_i(m_\tau, s_i) = \mathcal{L}_i \cdot \left[\varepsilon \cdot \mathcal{B}_f \cdot \sigma_{obs}(m_\tau, s_i) + \sigma_{BG}\right]$$

$$G(\sqrt{s}, \sqrt{s'}) = \frac{1}{\sqrt{2\pi}\Delta} \cdot \exp\left[-\frac{(\sqrt{s'} - \sqrt{s})^2}{2\Delta^2}\right]$$

$$\sigma_{obs}(m_\tau, s_i) = \int_0^\infty \sigma_{r.c.}(m_\tau, s') \cdot G(\sqrt{s}, \sqrt{s'})\, d\sqrt{s'}$$

$$\sigma_{r.c.}(m_\tau, s) = \int_0^{1-\frac{4m_\tau^2}{s}} dx\, F(x) \frac{\sigma_B[m_\tau, s(1-x)]}{|1 - \Pi[s(1-x)]|^2}$$

F(x): E.A.Kuraev,V.S.Fadin , Sov.J.Nucl.Phys. 41(1985)466;
Π(s): F.A. Berends et al. , Nucl. Phys. B57 (1973)381.

Theoretical accuracy of cross section at the level of 0.1%

$$\sigma(W)=\frac{1}{\sqrt{2\pi}\Delta_E}\int_0^{+\infty} dW' e^{-(W'-W)^2/2\Delta_E^2}\int_0^{\beta^2} dx F_i(x,W')\,\sigma^0\left(W'\sqrt{1-x}\right)$$

Energy Spread

ISR correction

Coulomb Correction

FSR Correction

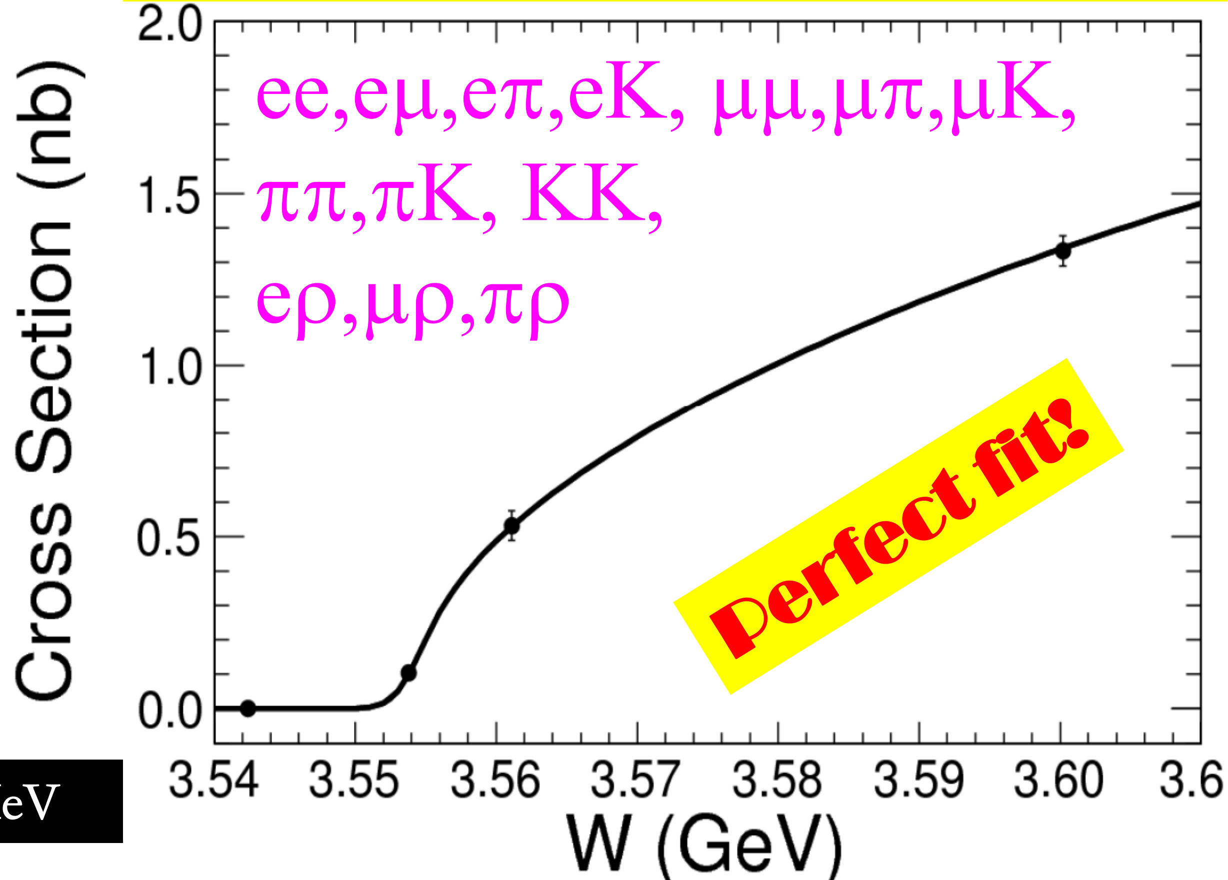
Vacuum Polarization Correction

$$\sigma^0(W)=\frac{4\pi\alpha^2}{3W^2}\,\frac{\beta\left(3-\beta^2\right)}{2}\,\frac{F_c(\beta)F_r(\beta)}{\left[1-\Pi(W)\right]^2}$$

τ mass measurement

Hawaii university of USA; Tsihua university of China; IHEP at Beijing; BINP at Novosibirsk

Systematic errors	
Source	keV
Theo.	10
E_{Spread}	12
E_{Scale}	90
$E_{Selection}$	50
Eff.	48
Bg. shape	40
Lum.	6
Sum	122



PDG2012: 1776.82 ±0.16 MeV

$$M_\tau = 1776.91 \pm 0.12\,(\text{stat.}) \pm 0.12\,(\text{sys.})\ \text{MeV}$$

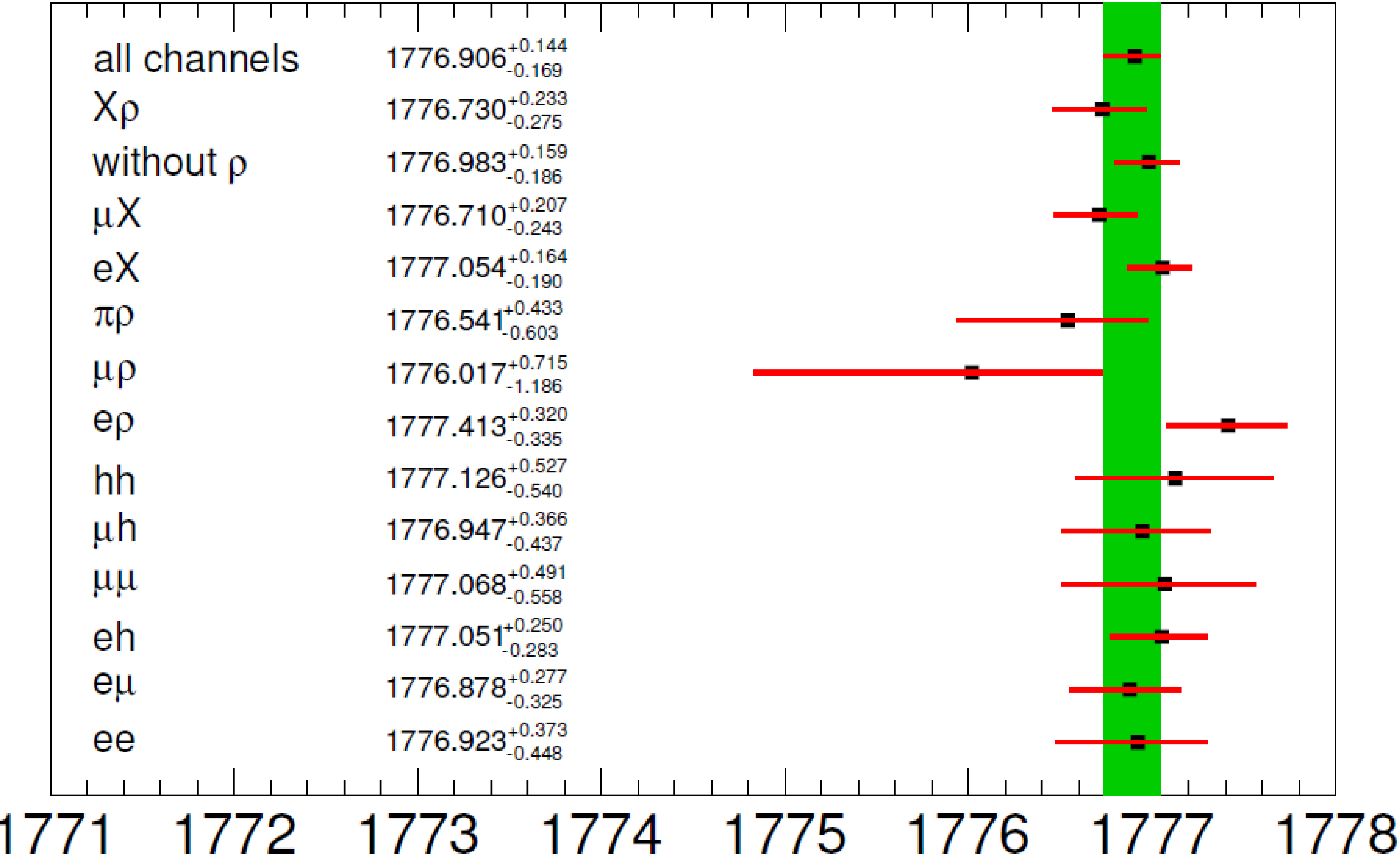
$$\delta M_\tau = 0.171\ \text{MeV},\ \delta M_\tau / M_\tau = 9.6 \times 10^{-5}.$$

Summary

1. **BESIII: M_τ = 1776.91 ± 0.17 MeV,**
   PDG12: M_τ = 1776.82 ± 0.16 MeV;
2. **Universality will be tested at the level of 0.5‰; and Koideo identity is established within the error of 57 keV;**
3. **Experience is acquired from the present data analysis, more detailed plan are made for the finer τ mass scan in the future.**


谢谢
THANKS

Backup


all channels
1776.906 +0.144 -0.169
Xρ
1776.730 +0.233 -0.275
without ρ
1776.983 +0.159 -0.186
μX
1776.710 +0.207 -0.243
eX
1777.054 +0.164 -0.190
πρ
1776.541 +0.433 -0.603
μρ
1776.017 +0.715 -1.186
eρ
1777.413 +0.320 -0.335
hh
1777.126 +0.527 -0.540
μh
1776.947 +0.366 -0.437
μμ
1777.068 +0.491 -0.558
eh
1777.051 +0.250 -0.283
eμ
1776.878 +0.277 -0.325
ee
1776.923 +0.373 -0.448
1771
1772
1773
1774
1775
1776
1777
1778

Tau mass scan is actually an accelerator experiment

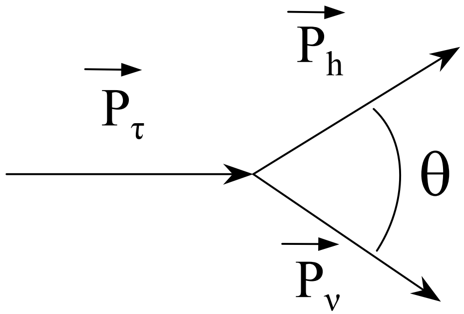
Tau mass scan

Expected luminosity (speed of data accumulation) is 0.3 pb-1/hour. Expected time of hitting to energy point is 1 day (3 steps x 4+4 measurements of electron/positron beam energy). Then it will take about 29 days to do experiment.

Beam energy, Mev	1771.0	1776.6	1777.0	1780.4	1800.0	Psi prime scan	1776.6	1777.0	J/psi scan
Integrated luminosity, pb-1	14	14	14	7	14		25	12	
Time of data aquisition, days	3	3	3	2	3	4	4,5	2,5	4

Old plan 10 days→New plan: 29 days

Pseudomass method



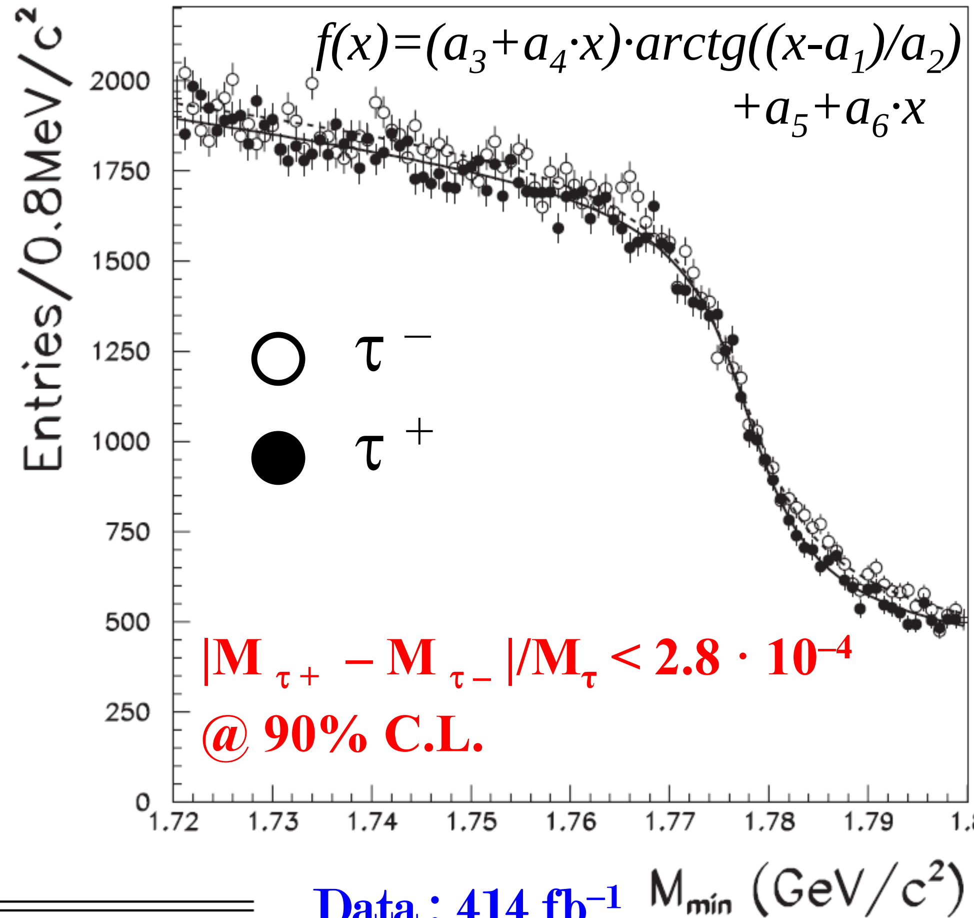
All in CMS

$\mathbf{M_{min} \le M_\tau}$

$\mathbf{M_{min}^2 = M_h^2 + 2(E_\tau - E_h)(E_h - P_h)}$

$E_\tau = E_{beam}$: beam energy, run dependence is corrected
E_h : hadron system energy
P_h : hadron system momentum
M_h : mass of the hadron system

Source of systematics	σ, MeV/c^2
Beam energy and tracking system	0.26
Edge parameterization	0.18
Limited MC statistics	0.14
Fit range	0.04
Momentum resolution	0.02
Model of $\tau \to 3\pi\nu_\tau$	0.02
Background	0.01
Total	0.35



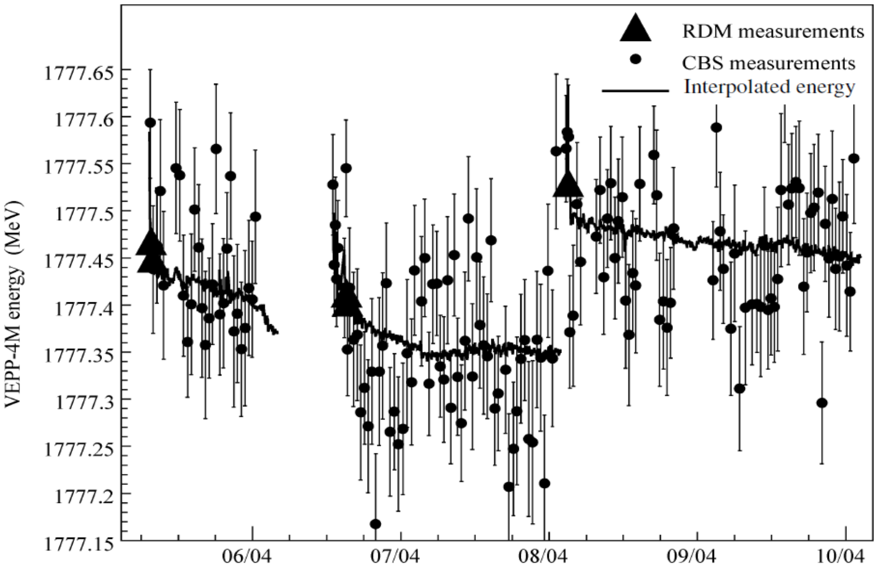
$|M_{\tau+} - M_{\tau-}|/M_\tau < 2.8 \cdot 10^{-4}$
@ 90% C.L.

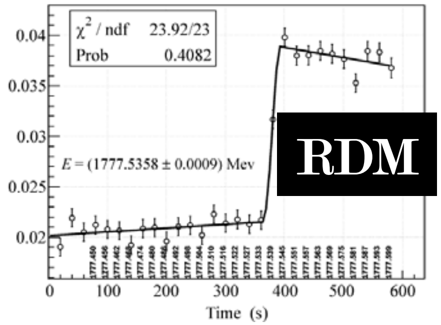
Data : 414 fb^{-1}

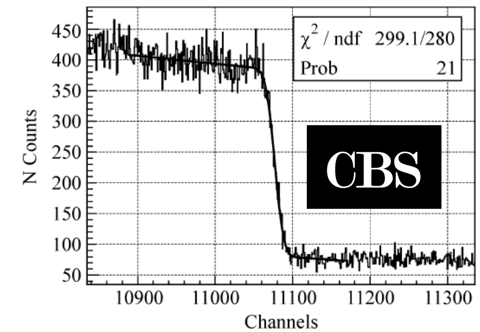
M_τ = 1776.61
± 0.13(stat.)
± 0.35(sys.) MeV

(Belle:PRL99,011801)

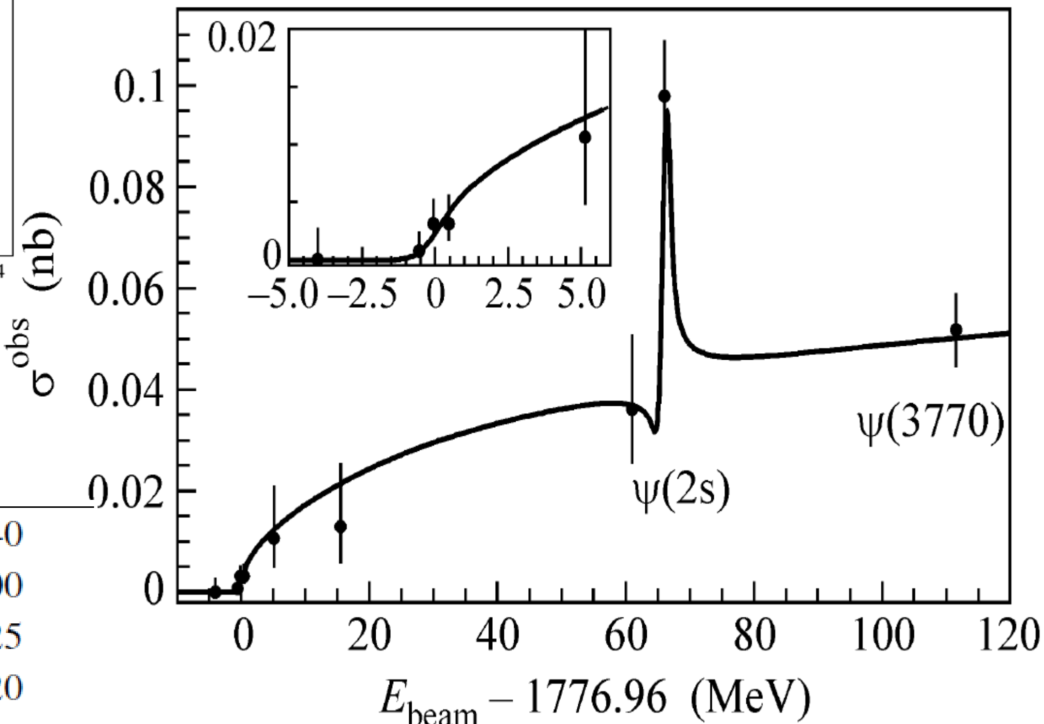
Threshold scan method







8 points, 6.7 pb^{-1}, for τ
1 points, 0.8 pb^{-1}, at ψ′



Beam energy determination	40
Detection efficiency variations	100
Energy spread determination accuracy	25
Energy dependence of the background	20
Luminosity measurement instability	90
Beam energy spread variation	15
Cross section calculation (r.c., interference)	30
Sum in quadrature	150

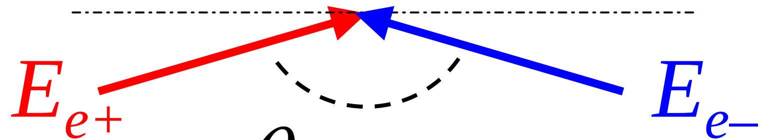
$$M_\tau = 1776.81^{+0.25}_{-0.23} \pm 0.15 \text{ MeV}$$

$$\sigma M_\tau / M_\tau = 1.64 \times 10^{-4}$$

KDER : JETPL85_347

CM energy setting

$$S = (E_{e^+} + E_{e^-})^2 - (p_{e^+} + p_{e^-})^2$$



$$\theta = \pi - \alpha$$

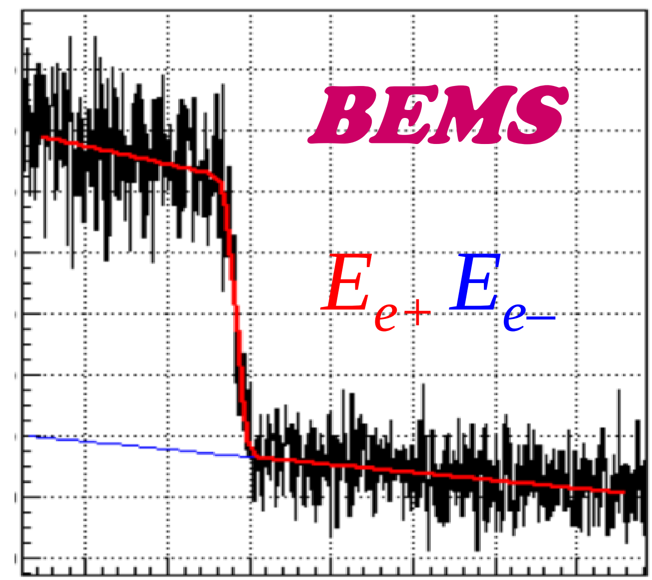
$$\alpha \approx 22 mrad$$

$$E_{cm}^{AA} = (E_{e^+} + E_{e^-}) \cdot \cos\frac{\alpha}{2}$$

$$E_{cm}^{GA} = 2\sqrt{E_{e^+} E_{e^-}} \cdot \cos\frac{\alpha}{2}$$

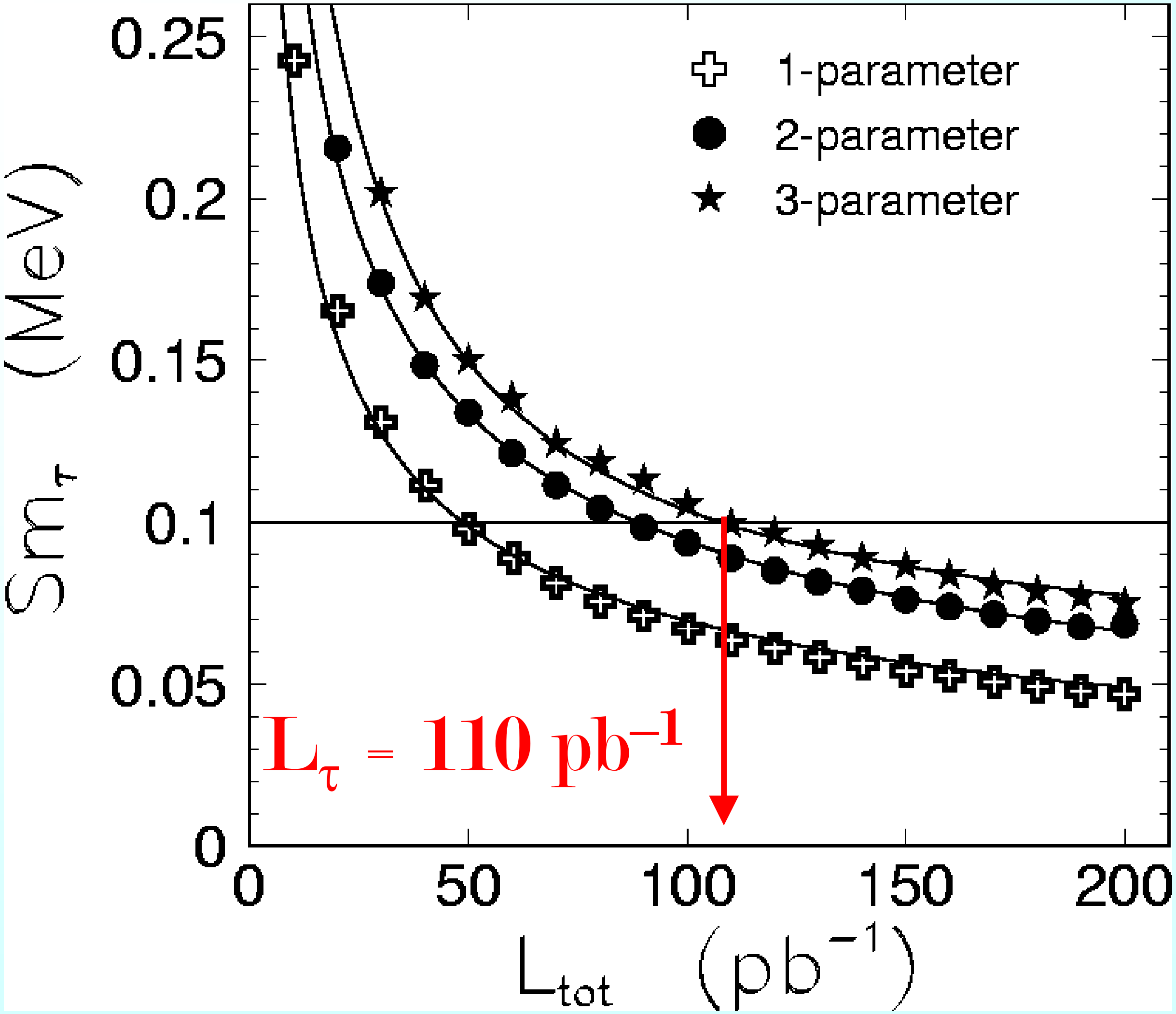
$$E_{cm}^{GA} \approx E_{cm}^{GA} \approx 2E_{beam}\left(1 - \frac{\alpha^2}{8}\right)$$

$E_{cm}{}^{GA} \approx E_{cm}{}^{GA} \approx E_{cm}{}^{\sqrt{s}}$
α-effect $\approx 6\times10^{-5}$,
0.11MeV @τ threshold



$$E_{cm}^{\sqrt{S}} \approx 2E_{beam}\left(1 - \frac{\alpha^2}{8}\right)$$

$$E_{cm}^{\sqrt{S}} = \sqrt{2m_e^2 + 2E_{e^+}E_{e^-} - 2\sqrt{E_{e^+}^2 - m_e^2}\sqrt{E_{e^-}^2 - m_e^2} \cdot \cos(\pi - \alpha)}$$



Optimization study:
Chin. Phys. C 2009, 33:501-507 ;
Y.K.Wang,
J.Y.Zhang,
X.H.Mo,
C.Z.Yuan.

Only based on e μ event !!
Only Statistics uncertainty !!

$1 = M_\tau$, $2 = \varepsilon$, $3 = \sigma_{BG}$;

$L_1 : L_2 = 3:1$, $L_1 : L_{tot} = 10\%$, $\delta M_\tau \propto (\sqrt{L})^{-1}$;