
T and CPT tests in the entangled neutral meson systems at e^+e^- colliders



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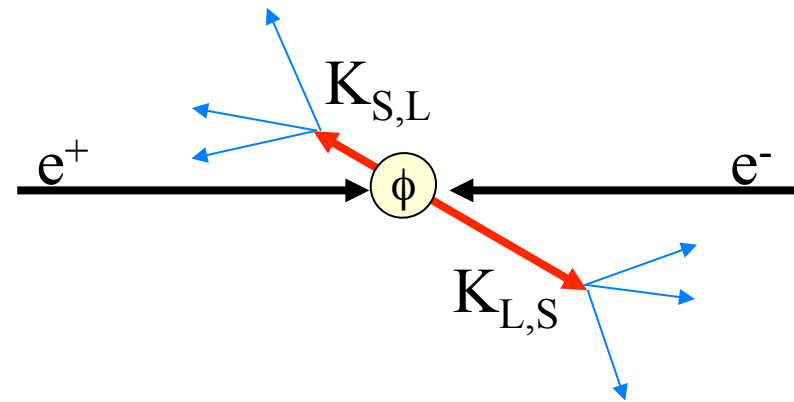
Entangled neutral kaons at a ϕ -factory

Production of the vector meson ϕ
in e^+e^- annihilations:

- $e^+e^- \rightarrow \phi$ $\sigma_\phi \sim 3 \mu\text{b}$
 $W = m_\phi = 1019.4 \text{ MeV}$
- $\text{BR}(\phi \rightarrow K^0\bar{K}^0) \sim 34\%$
- $\sim 10^6$ neutral kaon pairs per
 pb^{-1} produced in an
antisymmetric quantum state
with $J^{PC} = 1^{--}$:

$$\mathbf{p}_K = 110 \text{ MeV}/c$$

$$\lambda_S = 6 \text{ mm} \quad \lambda_L = 3.5 \text{ m}$$



$$\begin{aligned}
 |i\rangle &= \frac{1}{\sqrt{2}} \left[|K^0(\vec{p})\rangle |\bar{K}^0(-\vec{p})\rangle - |\bar{K}^0(\vec{p})\rangle |K^0(-\vec{p})\rangle \right] \\
 &= \frac{N}{\sqrt{2}} \left[|K_S(\vec{p})\rangle |K_L(-\vec{p})\rangle - |K_L(\vec{p})\rangle |K_S(-\vec{p})\rangle \right]
 \end{aligned}$$

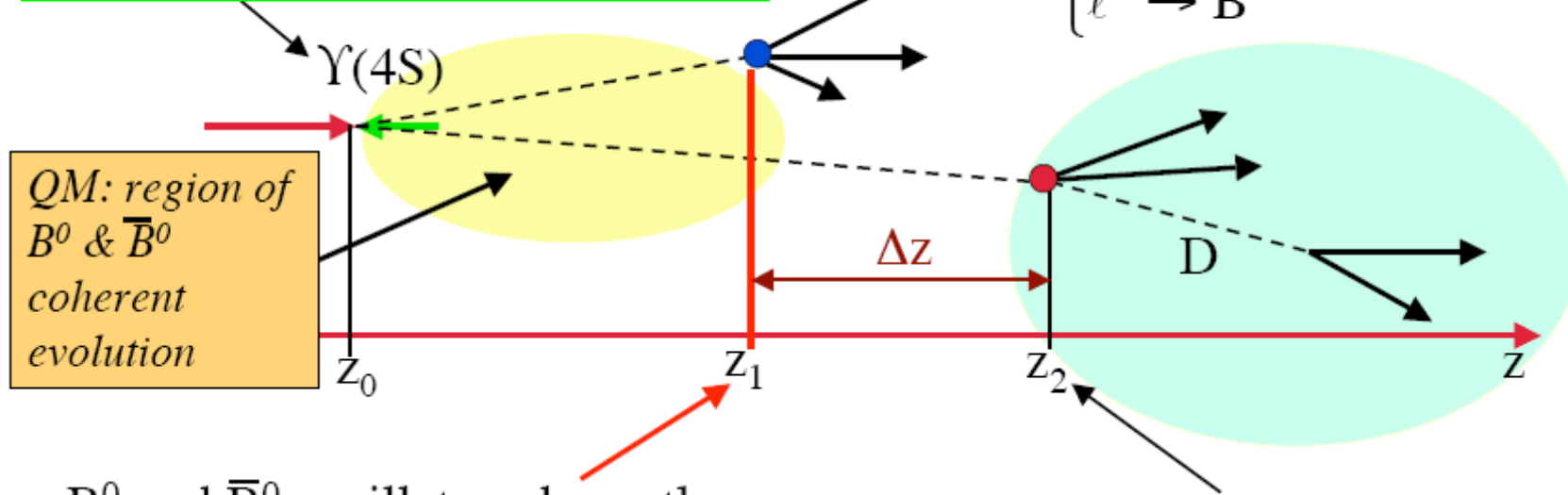
$$N = \sqrt{(1 + |\varepsilon_S|^2)(1 + |\varepsilon_L|^2)} / (1 - \varepsilon_S \varepsilon_L) \cong 1$$

Entangled B meson pairs

B

$$|i\rangle = \frac{1}{\sqrt{2}} [|B^0(\vec{p})\rangle |\bar{B}^0(-\vec{p})\rangle - |\bar{B}^0(\vec{p})\rangle |B^0(-\vec{p})\rangle]$$

$\Upsilon(4S)$ produced with $\beta\gamma = 0.425$ by the asymmetric collider



B^0 and \bar{B}^0 oscillate coherently. When the **first** decays, the other is known to be of the opposite flavour, at the same proper time

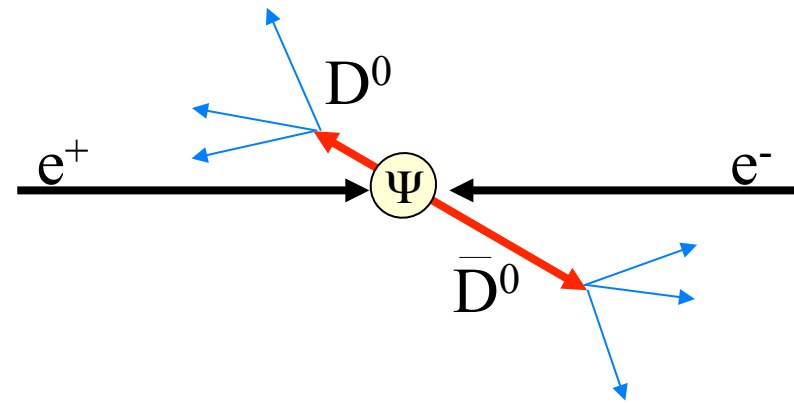
Then the other B^0 oscillates freely before decaying after a time given by $\Delta t \approx \Delta z / c \beta \gamma$

N.B. : production vertex position z_0 not very well known : only Δz is available !

Entangled neutral D mesons at a τ -charm factory

Production of the vector meson $\Psi(3770)$ in e^+e^- annihilations:

$$e^+e^- \rightarrow \Psi(3770) \rightarrow D^0\bar{D}^0$$



$$|i\rangle = \frac{1}{\sqrt{2}} \left[|D^0(\vec{p})\rangle |\bar{D}^0(-\vec{p})\rangle - |\bar{D}^0(\vec{p})\rangle |D^0(-\vec{p})\rangle \right]$$

**Entanglement imposed by the
Einstein-Podolsky-Rosen correlation
as a TOOL for discrete symmetries tests !**

Time reversal: introduction

The three discrete symmetries of QM, C (charge conjugation), P (parity), and T (time reversal) are known to be violated in nature both singly and in pairs. Only CPT appears to be an exact symmetry of nature.

CPT theorem (Luders, Jost, Pauli, Bell 1955 -1957):

Exact CPT invariance holds for any quantum field theory which assumes:

(1) Lorentz invariance (2) Locality (3) Unitarity (i.e. conservation of probability).

→ Automatic connection between CP-violation and T-violation in the Standard Model or any field theoretic extension

Even though CPT invariance has been confirmed by all present experimental tests, particularly in the neutral kaon system with stringent limits to possible CPT violation effects, **the theoretical connection between CP and T symmetries does not imply an experimental identity between them.**

T and CPT described by ANTIUNITARY rather than unitary operators, introducing many intriguing subtleties.

Time Reversal: introduction

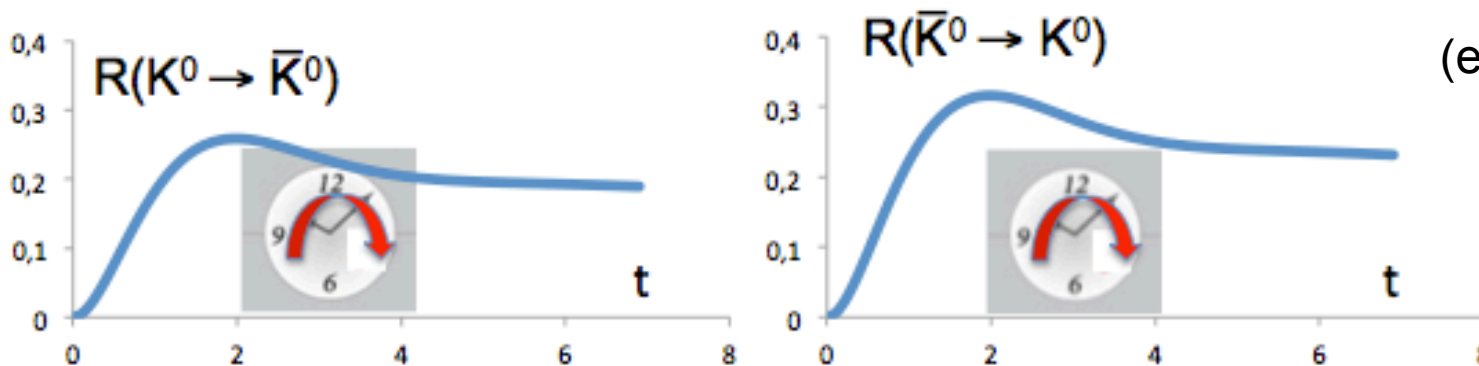
The observation of **motion reversal**, i.e. exchange of *in* \leftrightarrow *out* and reversal of all momenta and spins **without reversing** $t \rightarrow -t$, tests **time reversal T**, i.e. the symmetry of the dynamics responsible for the observed process **under the reversal** $t \rightarrow -t$

T symmetry \rightarrow motion reversal symmetry

Observation of motion reversal asymmetry \rightarrow T symmetry is violated

Time reversal symmetry can be tested e.g. in the case of

- (i) T-odd observable for a non degenerate stationary state: e.g. electric dipole moment of neutron;
- (ii) transition between stable particles: e.g. neutrino oscillations
- (iii) transition between unstable particles: e.g. K^0 oscillations

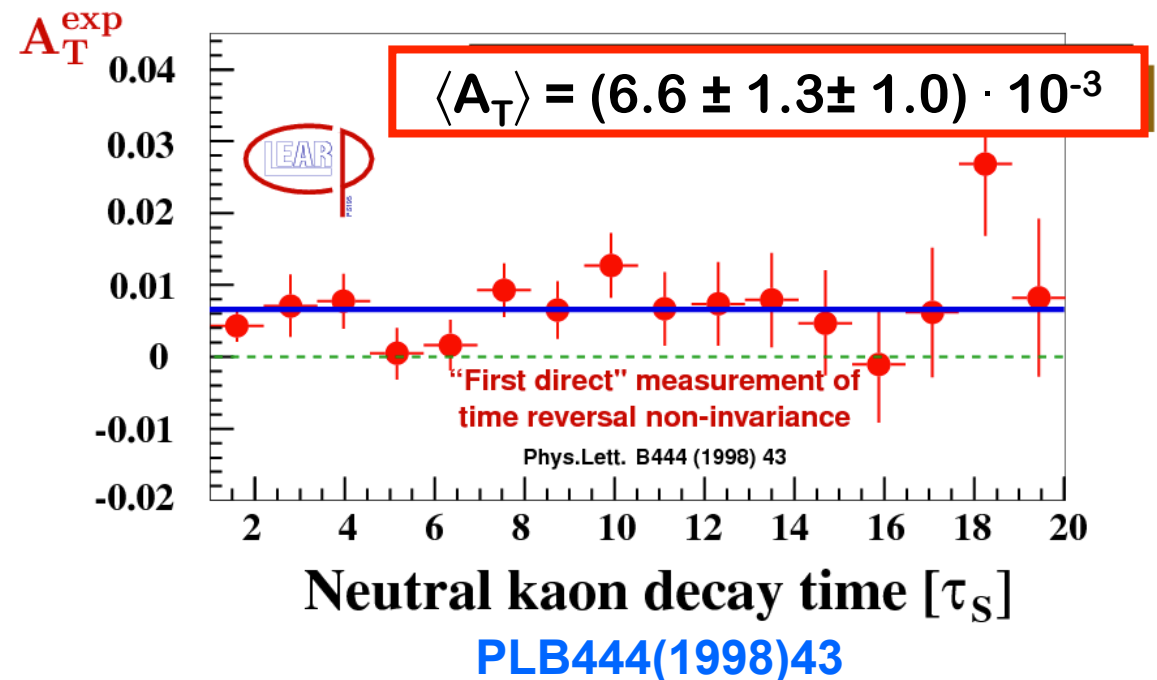
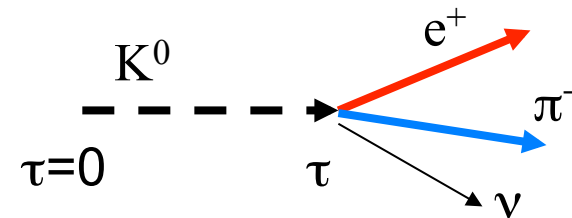


(e.g. Kabir asymmetry, PRD 1970)

Test of Time Reversal symmetry using Kabir's asymmetry

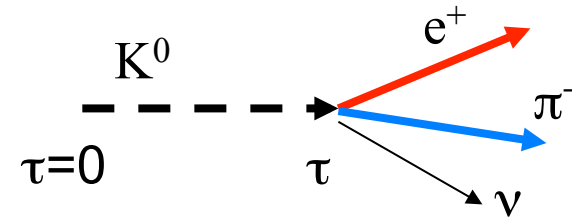
- Only one evidence of T violation: Kabir asymmetry, comparing a process with its T-conjugated one, i.e. $K^0 \rightarrow \bar{K}^0$ vs $\bar{K}^0 \rightarrow K^0$ performed by the CPLEAR experiment

$$A_T = \frac{P(\bar{K}^0 \rightarrow K^0) - P(K^0 \rightarrow \bar{K}^0)}{P(\bar{K}^0 \rightarrow K^0) + P(K^0 \rightarrow \bar{K}^0)}$$



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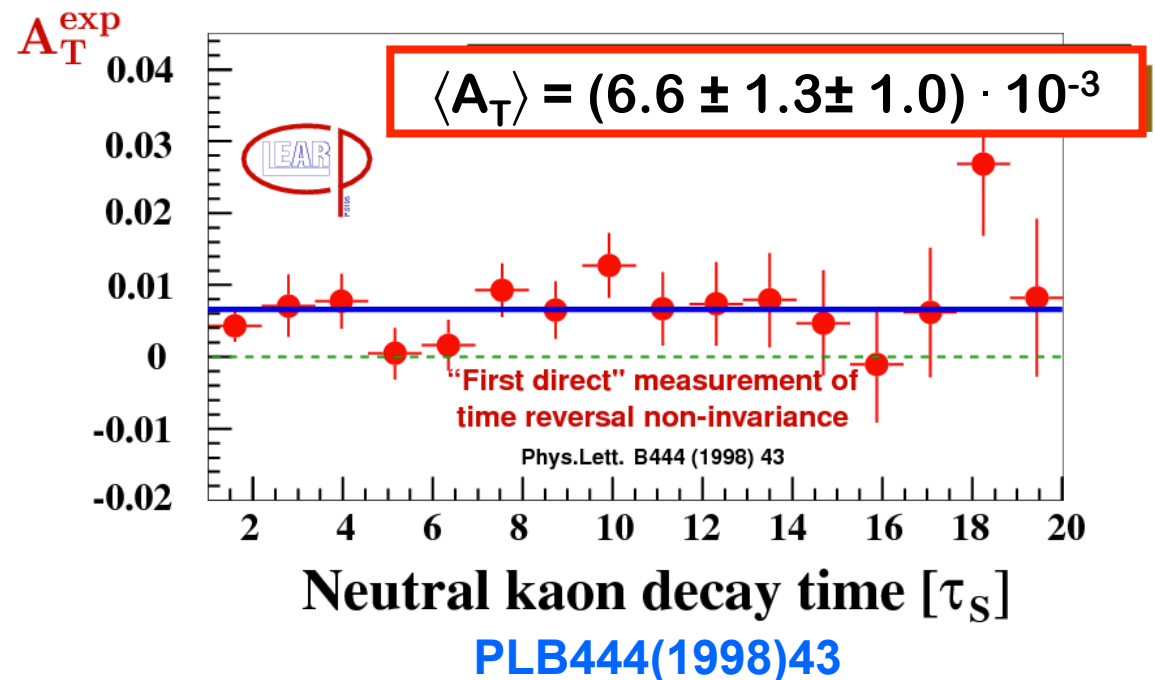
$$= 4\Re\varepsilon$$

assumption: no CPT violation
in semileptonic decay:

$$\Re(y - x_-) = 0$$

$$\varepsilon = \frac{H_{12} - H_{21}}{2(\lambda_S - \lambda_L)}$$

$$\delta = \frac{H_{11} - H_{22}}{2(\lambda_S - \lambda_L)}$$



Test of Time Reversal symmetry using Kabir's asymmetry

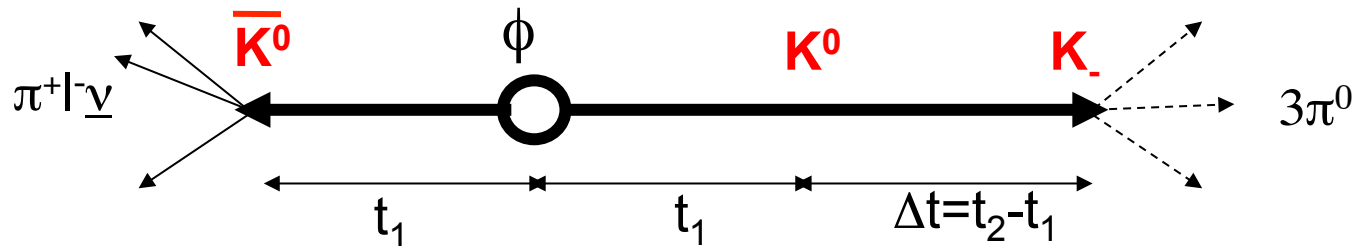
- A direct evidence for T violation would mean an experiment that, considered by itself, clearly shows T violation INDEPENDENT and unconnected to the results for CP violation and CPT invariance
- Remarks on the CPLEAR result as “direct” test:
 - 1) $K^0 \rightarrow \bar{K}^0$ is a CPT-even transition, so $CP \equiv T$ in this case !
CP and T cannot be distinguished (not independent)
T test: $K^0 \rightarrow \bar{K}^0$ vs $\bar{K}^0 \rightarrow K^0$
CP test: $K^0 \rightarrow \bar{K}^0$ vs $\bar{K}^0 \rightarrow K^0$
 - 2) $A_T \propto \Re \varepsilon \propto \Delta\Gamma = \Gamma_S - \Gamma_L$; if $\Delta\Gamma \sim 0$ the TRV effect vanishes (in B meson system $\Delta\Gamma \sim 0$: no TRV through $B^0 \rightarrow \bar{B}^0$ transition); decay plays an essential role.
- L. Wolfenstein IJMP(1999),PRL (1999): “it is not as direct a test of TRV as one might like”
Bernabeu PLB (1999), NPB (2000), H. Quinn (JPPS (2008); Bernabeu, Martinez Vidal, Villanueva JHEP (2012)

Entanglement in neutral meson pairs

- Entangled states in QM: the INDIVIDUAL STATE of each neutral meson is NOT DEFINED BEFORE the observation of the decay of its orthogonal partner.
- transitions involving also “CP states” K_+ ($\pi\pi$ decay) and K_- ($3\pi^0$ decay)

$$\begin{aligned}
 |i\rangle &= \frac{1}{\sqrt{2}} \left[|K^0(\vec{p})\rangle |\bar{K}^0(-\vec{p})\rangle - |\bar{K}^0(\vec{p})\rangle |K^0(-\vec{p})\rangle \right] \\
 &= \frac{1}{\sqrt{2}} \left[|K_+(\vec{p})\rangle |K_-(-\vec{p})\rangle - |K_-(-\vec{p})\rangle |K_+(\vec{p})\rangle \right]
 \end{aligned}$$

- decay as filtering measurement
- entanglement -> preparation of state

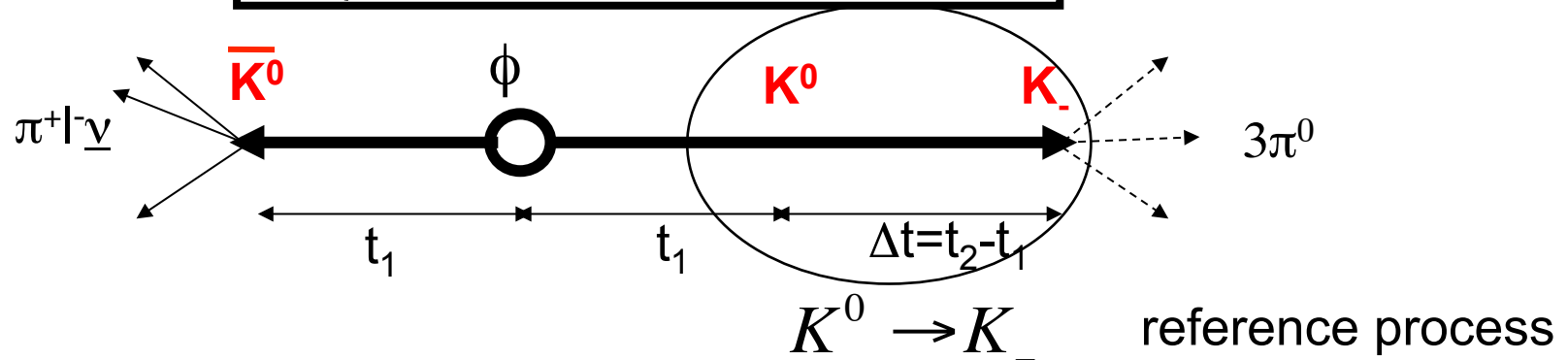


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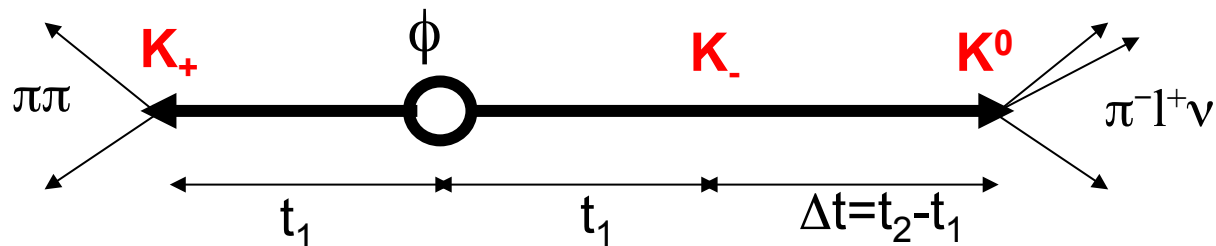
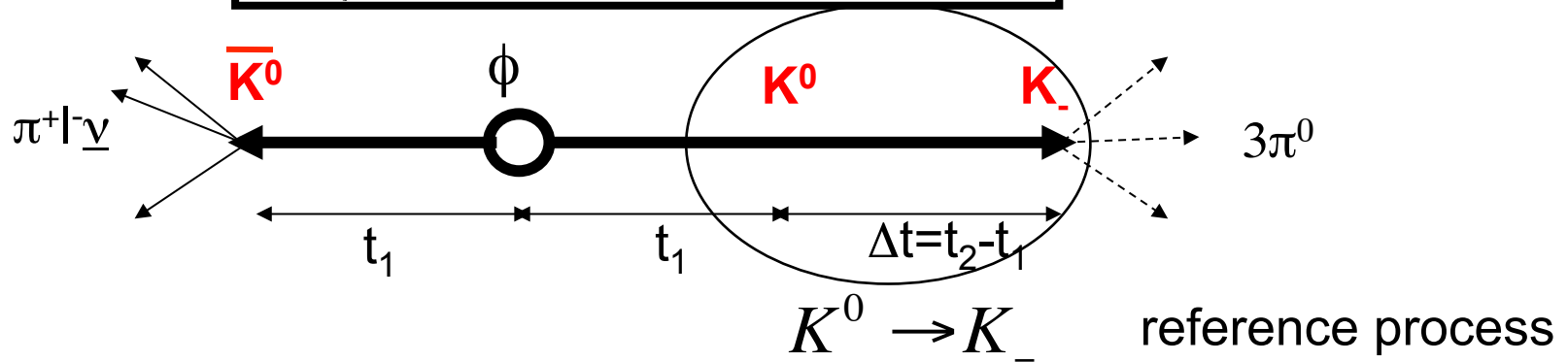


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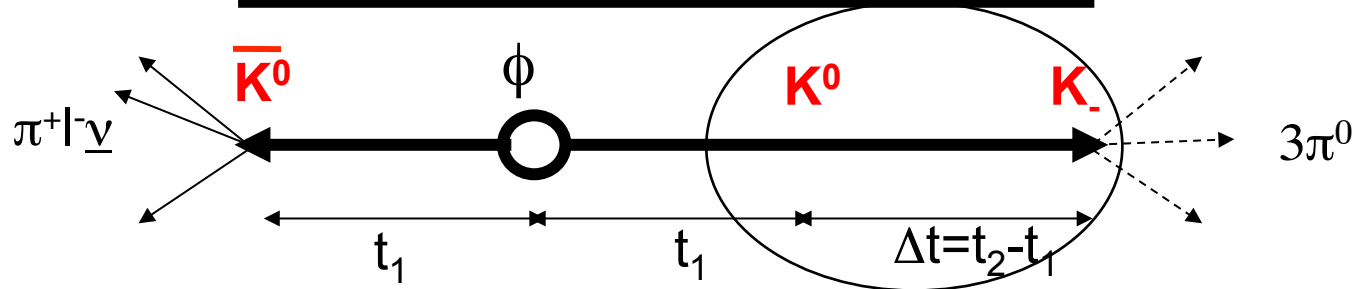


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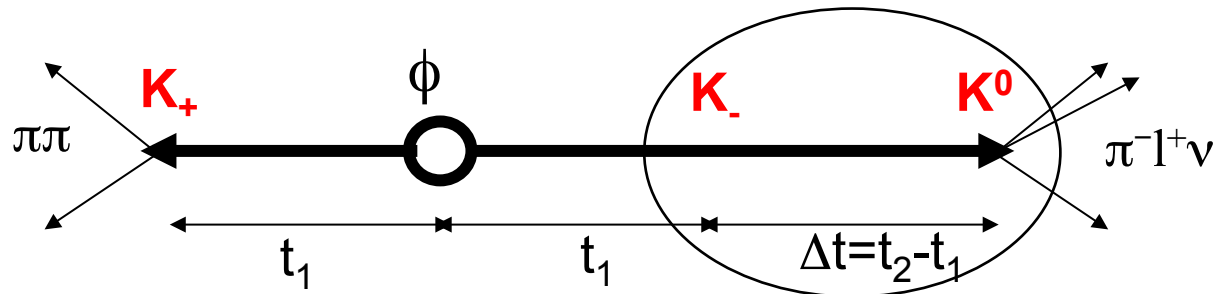
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- decay as filtering measurement
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$K^0 \rightarrow K_-$ reference process

$K_- \rightarrow K^0$ T-conjugated process



Direct test of Time Reversal symmetry with neutral kaons

T symmetry test

Reference		T -conjugate	
Transition	Final state	Transition	Final state
$\bar{K}^0 \rightarrow K_-$	$(\ell^+, \pi^0 \pi^0 \pi^0)$	$K_- \rightarrow \bar{K}^0$	$(\pi^0 \pi^0 \pi^0, \ell^-)$
$K_+ \rightarrow K^0$	$(\pi^0 \pi^0 \pi^0, \ell^+)$	$K^0 \rightarrow K_+$	$(\ell^-, \pi \pi)$
$\bar{K}^0 \rightarrow K_+$	$(\ell^+, \pi \pi)$	$K_+ \rightarrow \bar{K}^0$	$(\pi^0 \pi^0 \pi^0, \ell^-)$
$K_- \rightarrow K^0$	$(\pi \pi, \ell^+)$	$K^0 \rightarrow K_-$	$(\ell^-, \pi \pi)$

One can define the following ratios of probabilities:

$$\begin{aligned}
 R_1(\Delta t) &= P [K^0(0) \rightarrow K_+(\Delta t)] / P [K_+(0) \rightarrow K^0(\Delta t)] \\
 R_2(\Delta t) &= P [K^0(0) \rightarrow K_-(\Delta t)] / P [K_-(0) \rightarrow K^0(\Delta t)] \\
 R_3(\Delta t) &= P [\bar{K}^0(0) \rightarrow K_+(\Delta t)] / P [K_+(0) \rightarrow \bar{K}^0(\Delta t)] \\
 R_4(\Delta t) &= P [\bar{K}^0(0) \rightarrow K_-(\Delta t)] / P [K_-(0) \rightarrow \bar{K}^0(\Delta t)] .
 \end{aligned}$$

Any deviation from $R_i=1$ constitutes a violation of T-symmetry

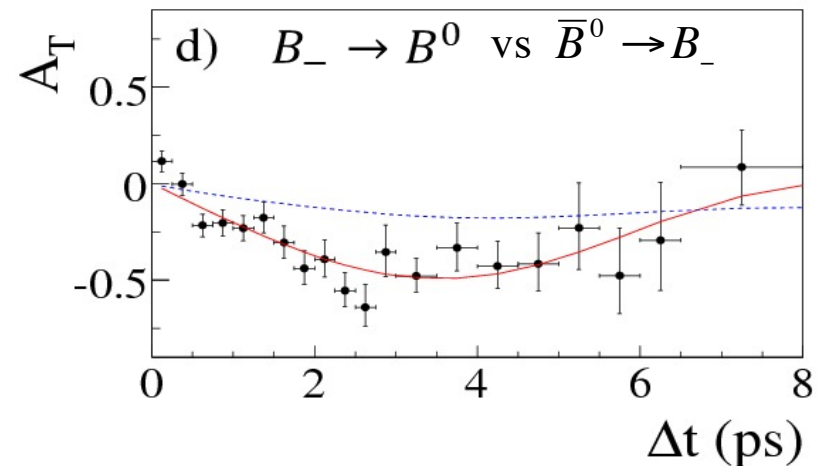
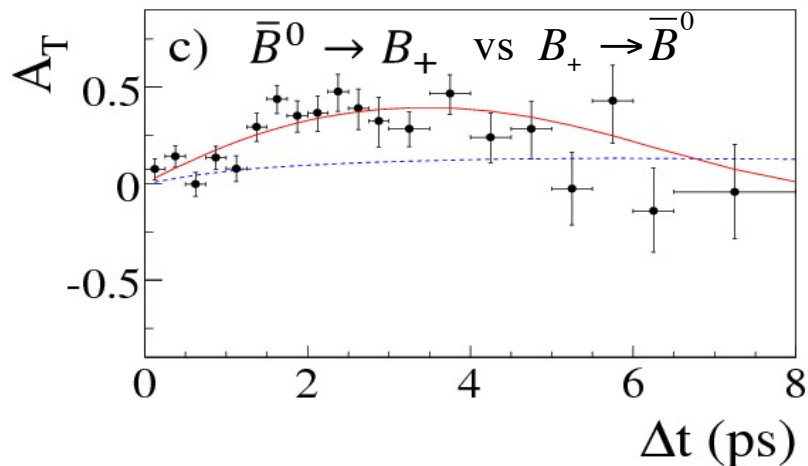
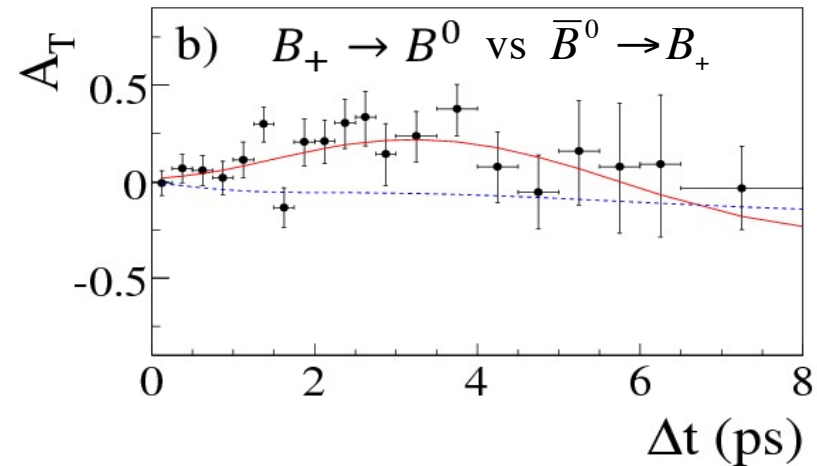
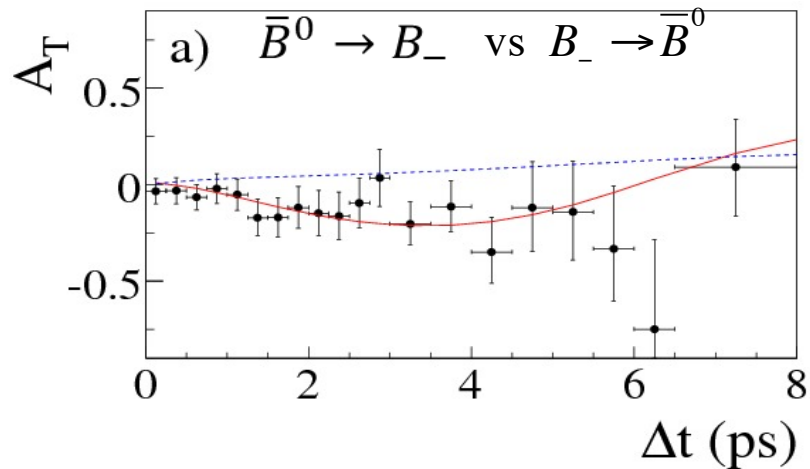
[J. Bernabeu, A.D.D., P. Villanueva: NPB 868 \(2013\) 102](#)

Test feasible at KLOE-2 with $L=O(10 \text{ fb}^{-1})$ (but quite challenging !!)

Direct test of Time Reversal symmetry in neutral B mesons

Direct T violation observed at BABAR
 in the B's with significance of 14σ
[Babar coll. PRL 109 \(2012\) 211801](#)

$$I_i(\Delta\tau) \sim e^{-\Gamma\Delta\tau} \left\{ C_i \cos(\Delta m \Delta\tau) + S_i \sin(\Delta m \Delta\tau) + C'_i \cosh(\Delta\Gamma\Delta\tau) + S'_i \sinh(\Delta\Gamma\Delta\tau) \right\}$$

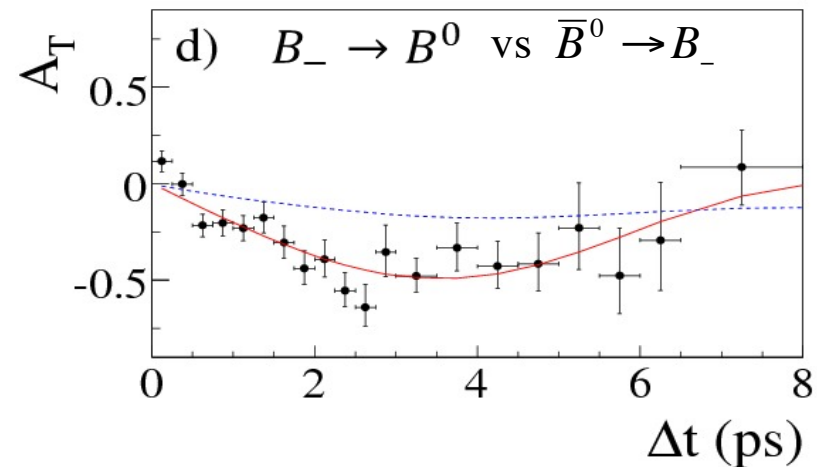
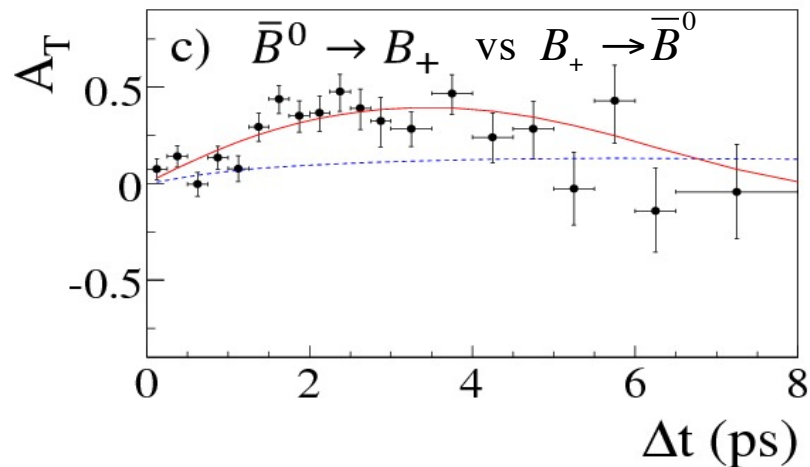
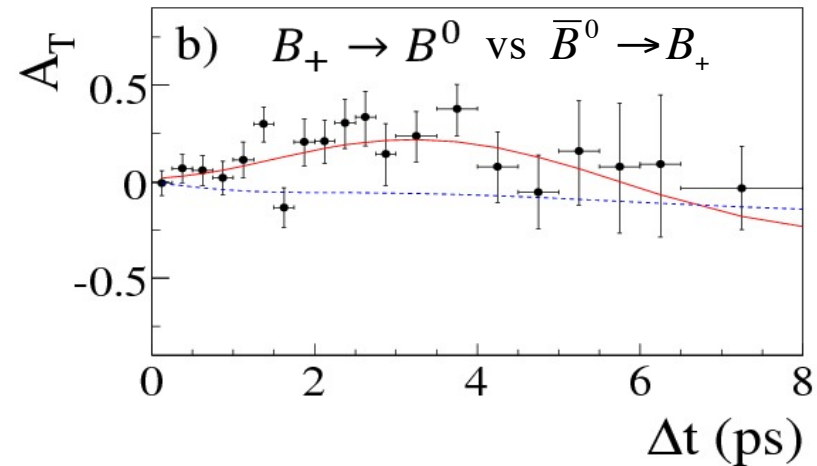
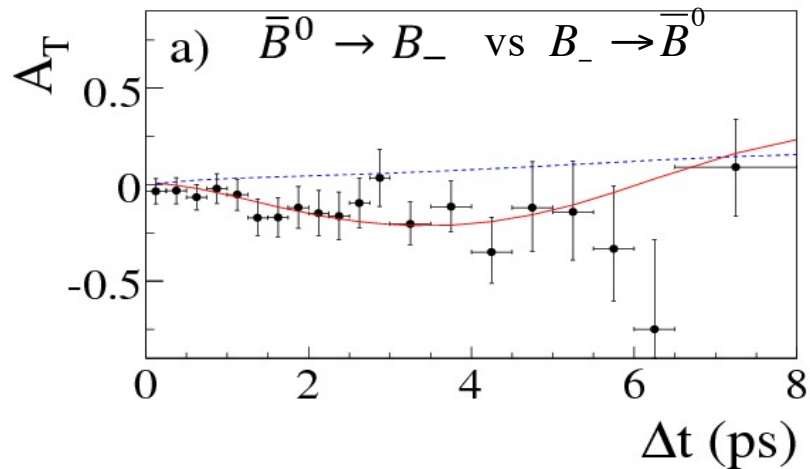


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Babar coll. PRL 109 (2012) 211801

ΔS_T^+	=	$-1.37 \pm 0.14 \pm 0.06$
ΔS_T^-	=	$1.17 \pm 0.18 \pm 0.11$
ΔC_T^+	=	$0.10 \pm 0.16 \pm 0.08$
ΔC_T^-	=	$0.04 \pm 0.16 \pm 0.08$



CPT: introduction

CPT theorem (Luders, Jost, Pauli, Bell 1955 -1957):

Exact CPT invariance holds for any quantum field theory which assumes:

(1) Lorentz invariance (2) Locality (3) Unitarity (i.e. conservation of probability).

Testing the validity of the CPT symmetry probes the most fundamental assumptions of our present understanding of particles and their interactions.

Extension of CPT theorem to a theory of quantum gravity far from obvious (e.g. CPT violation appears in some models with space-time foam backgrounds).

No predictive theory incorporating CPT violation => only phenomenological models to be constrained by experiments.

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neutral K system $\delta = \frac{1}{2} \frac{(m_{\bar{K}^0} - m_{K^0}) - (i/2)(\Gamma_{\bar{K}^0} - \Gamma_{K^0})}{\Delta m + i\Delta\Gamma/2}$

$$\left| m_{K^0} - m_{\bar{K}^0} \right| / m_K < 10^{-18}$$

neutral B system $z = -2\delta$

$$\left| m_{B^0} - m_{\bar{B}^0} \right| / m_B < 10^{-14}$$

proton- anti-proton

$$\left| m_p - m_{\bar{p}} \right| / m_p < 10^{-8}$$

CPT and Lorentz invariance violation (SME)

“Anti-CPT theorem” (Greenberger 2002):

Any unitary, local, point-particle quantum field theory that violates CPT invariance necessarily violates Lorentz invariance.

Kostelecky et al. developed a phenomenological effective model providing a framework for CPT and Lorentz violations, based on spontaneous breaking of CPT and Lorentz symmetry, which might happen in quantum gravity (e.g. in some models of string theory)

Standard Model Extension (SME) [Kostelecky PRD61, 016002, PRD64, 076001]

CPT violation in neutral kaons according to SME:

- CPTV only in mixing, not in decay, at first order (i.e. $B_I = y = x_- = 0$)
- δ cannot be a constant (momentum dependence)

$$\varepsilon_{S,L} = \varepsilon \pm \delta$$

$$\delta = i \sin \phi_{SW} e^{i\phi_{SW}} \gamma_K \left(\Delta a_0 - \vec{\beta}_K \cdot \Delta \vec{a} \right) / \Delta m$$

where Δa_μ are four parameters associated to SME lagrangian terms and related to CPT and Lorentz violation.

CPT and Lorentz invariance violation (SME)

$$\delta = i \sin \phi_{SW} e^{i\phi_{SW}} \gamma_K \left(\Delta a_0 - \vec{\beta}_K \cdot \Delta \vec{a} \right) / \Delta m$$

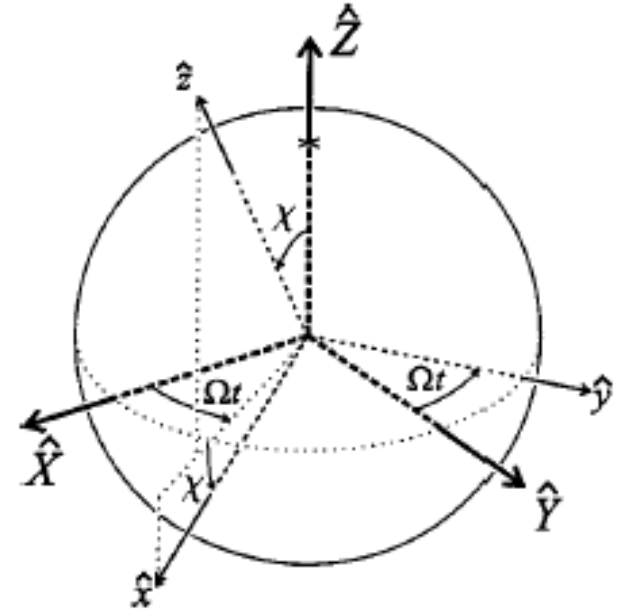
δ depends on sidereal time t since laboratory frame rotates with Earth.

For a ϕ -factory there is an additional dependence on the polar and azimuthal angle θ, ϕ of the kaon momentum in the laboratory frame:

$$\begin{aligned} \delta(\vec{p}, t) = & \frac{i \sin \phi_{SW} e^{i\phi_{SW}}}{\Delta m} \gamma_K \left\{ \Delta a_0 \right. \\ & + \beta_K \Delta a_z (\cos \theta \cos \chi - \sin \theta \sin \phi \sin \chi) \\ & + \beta_K \left[-\Delta a_x \sin \theta \sin \phi + \Delta a_y (\cos \theta \sin \chi + \sin \theta \cos \phi \cos \chi) \right] \sin \Omega t \\ & \left. + \beta_K \left[+\Delta a_y \sin \theta \sin \phi + \Delta a_x (\cos \theta \sin \chi + \sin \theta \cos \phi \cos \chi) \right] \cos \Omega t \right\} \end{aligned}$$

Ω : Earth's sidereal frequency

χ : angle between the z lab. axis and the Earth's rotation axis



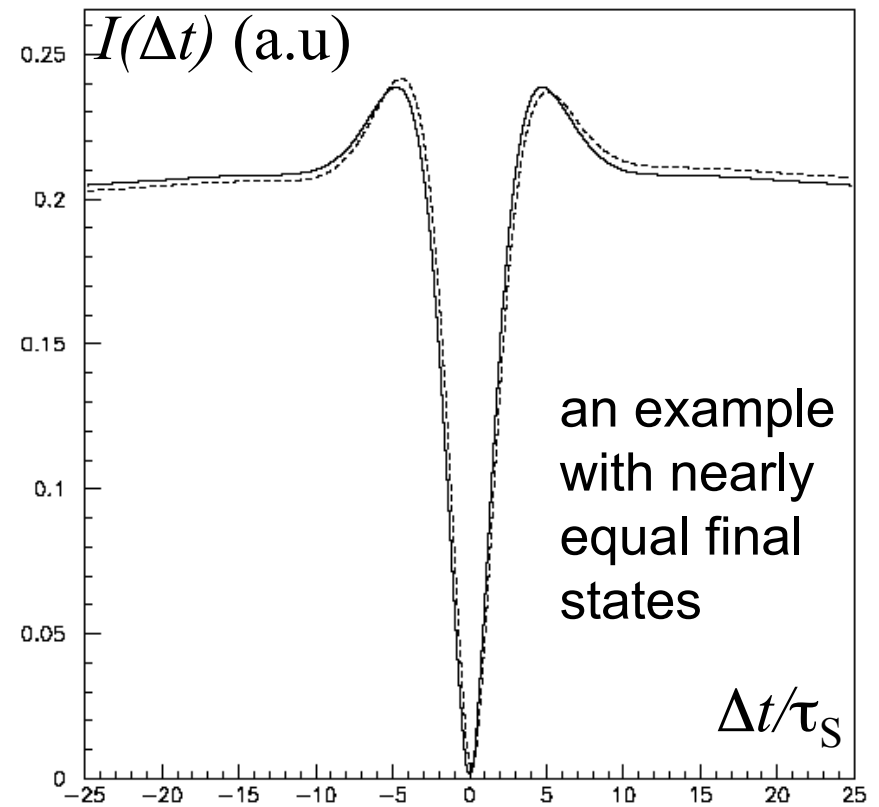
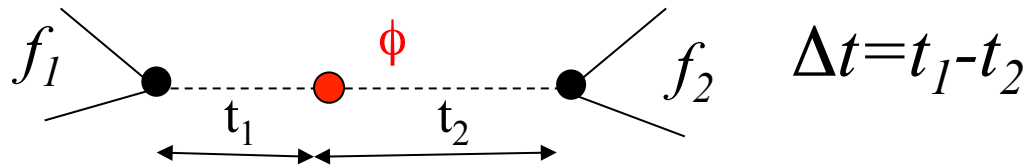
(in general z lab. axis is non-normal to Earth's surface)

Exploiting EPR correlations

$$|i\rangle = \frac{1}{\sqrt{2}} \left[|K^0\rangle |\bar{K}^0\rangle - |\bar{K}^0\rangle |K^0\rangle \right]$$

$$\eta_i = |\eta_i| e^{i\phi_i} = \langle f_i | T | K_L \rangle / \langle f_i | T | K_S \rangle$$

$$I(f_1, f_2; \Delta t) \propto \left\{ |\eta_1|^2 e^{-\Gamma_L \Delta t} + |\eta_2|^2 e^{-\Gamma_S \Delta t} - 2 |\eta_1| |\eta_2| e^{-(\Gamma_S + \Gamma_L) \Delta t / 2} \cos(\Delta m \Delta t + \phi_2 - \phi_1) \right\}$$

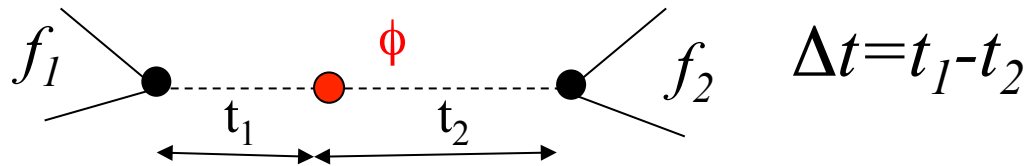


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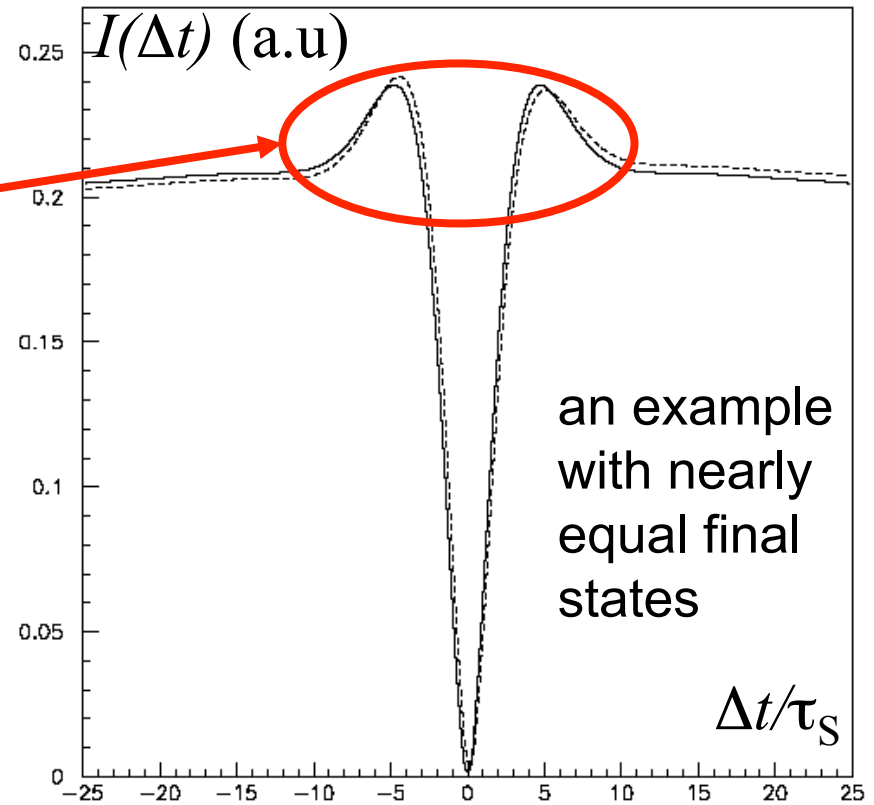
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η_2/η_1
from the asymmetry at **small** Δt

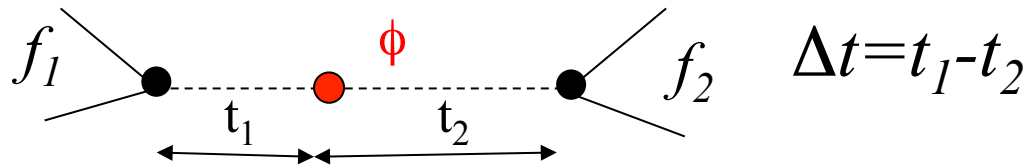


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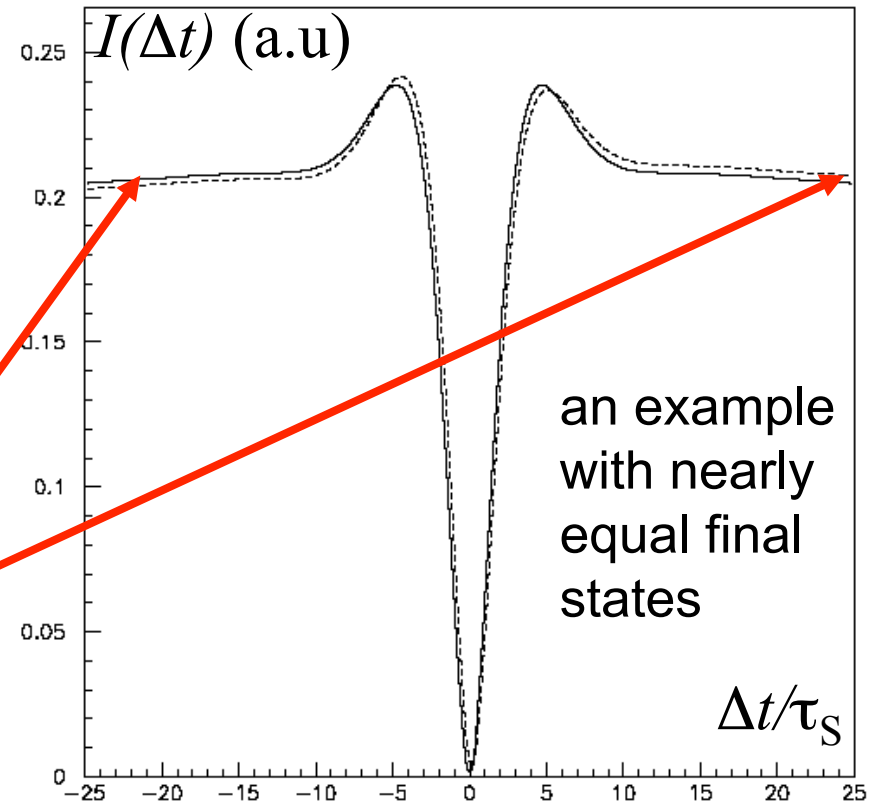
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η_2/η_1
from the asymmetry at **small** Δt

$|\eta_2/\eta_1|^2$
from the asymmetry at **large** Δt

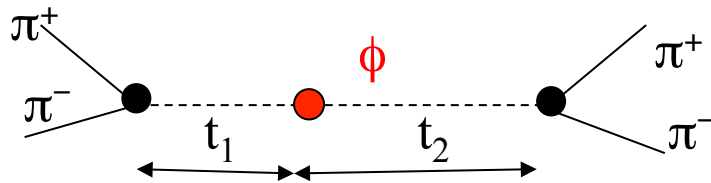


Exploiting EPR correlations

$$|i\rangle = \frac{1}{\sqrt{2}} \left[|K^0\rangle |\bar{K}^0\rangle - |\bar{K}^0\rangle |K^0\rangle \right]$$

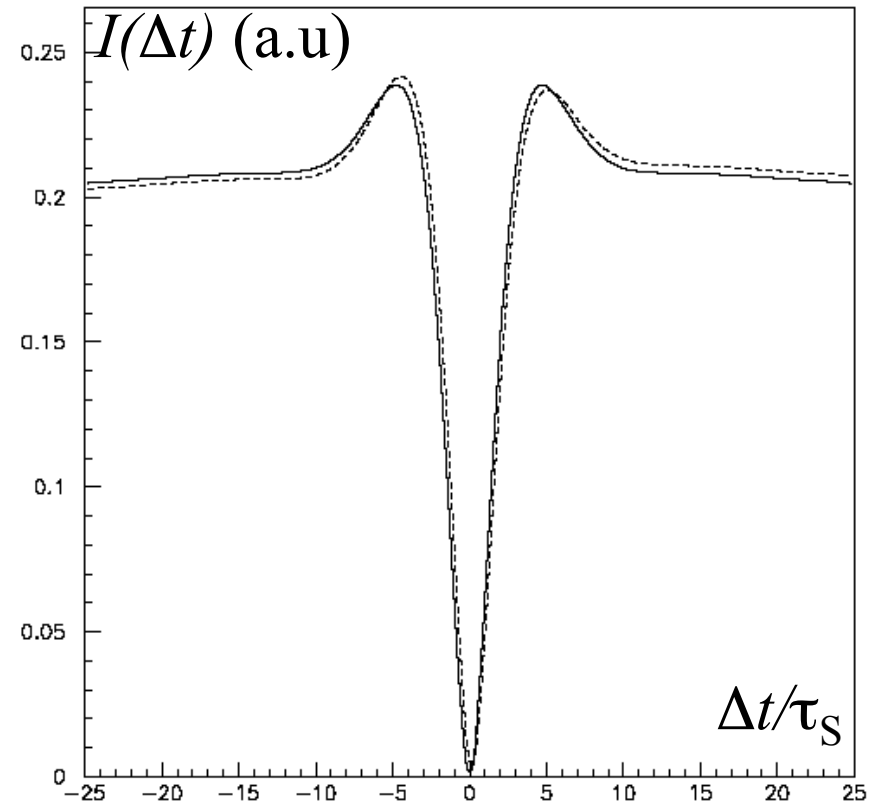
$$\eta_i = |\eta_i| e^{i\phi_i} = \langle f_i | T | K_L \rangle / \langle f_i | T | K_S \rangle$$

$$I(f_1, f_2; \Delta t) \propto \left\{ |\eta_1|^2 e^{-\Gamma_L \Delta t} + |\eta_2|^2 e^{-\Gamma_S \Delta t} - 2|\eta_1||\eta_2| e^{-(\Gamma_S + \Gamma_L)\Delta t/2} \cos(\Delta m \Delta t + \phi_2 - \phi_1) \right\}$$



$$\eta_{+-}^{(1)} = \varepsilon \left(1 - \delta(+\vec{p}, t) / \varepsilon \right)$$

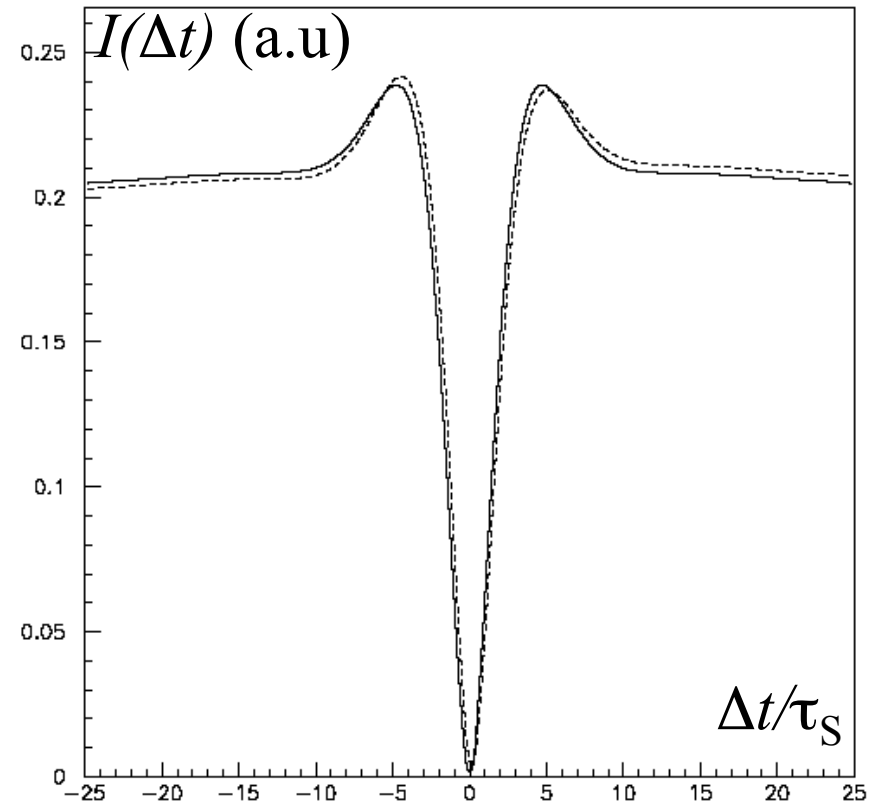
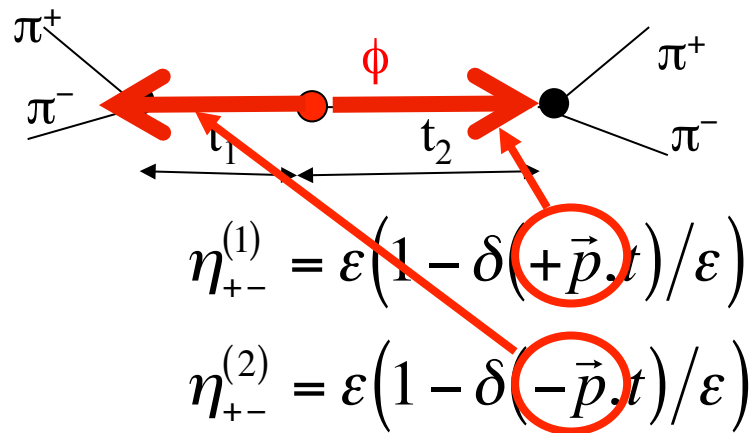
$$\eta_{+-}^{(2)} = \varepsilon \left(1 - \delta(-\vec{p}, t) / \varepsilon \right)$$



$$|i\rangle = \frac{1}{\sqrt{2}} \left[|K^0\rangle |\bar{K}^0\rangle - |\bar{K}^0\rangle |K^0\rangle \right]$$

$$\eta_i = |\eta_i| e^{i\phi_i} = \langle f_i | T | K_L \rangle / \langle f_i | T | K_S \rangle$$

$$I(f_1, f_2; \Delta t) \propto \left\{ |\eta_1|^2 e^{-\Gamma_L \Delta t} + |\eta_2|^2 e^{-\Gamma_S \Delta t} - 2|\eta_1||\eta_2| e^{-(\Gamma_S + \Gamma_L)\Delta t/2} \cos(\Delta m \Delta t + \phi_2 - \phi_1) \right\}$$

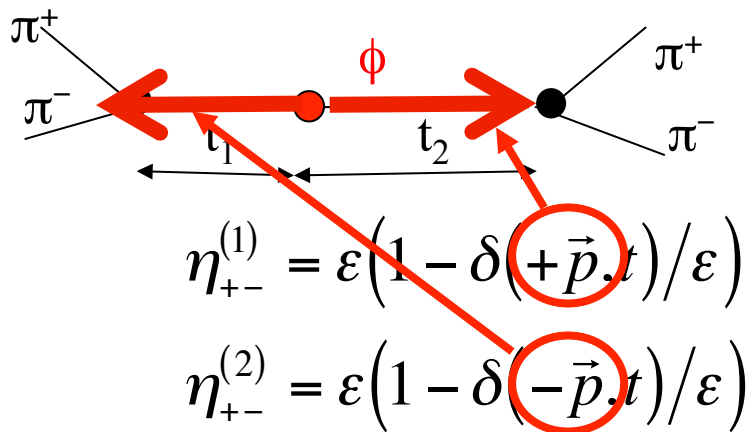


Exploiting EPR correlations

$$|i\rangle = \frac{1}{\sqrt{2}} \left[|K^0\rangle |\bar{K}^0\rangle - |\bar{K}^0\rangle |K^0\rangle \right]$$

$$\eta_i = |\eta_i| e^{i\phi_i} = \langle f_i | T | K_L \rangle / \langle f_i | T | K_S \rangle$$

$$I(f_1, f_2; \Delta t) \propto \left\{ |\eta_1|^2 e^{-\Gamma_L \Delta t} + |\eta_2|^2 e^{-\Gamma_S \Delta t} - 2|\eta_1||\eta_2| e^{-(\Gamma_S + \Gamma_L)\Delta t/2} \cos(\Delta m \Delta t + \phi_2 - \phi_1) \right\}$$

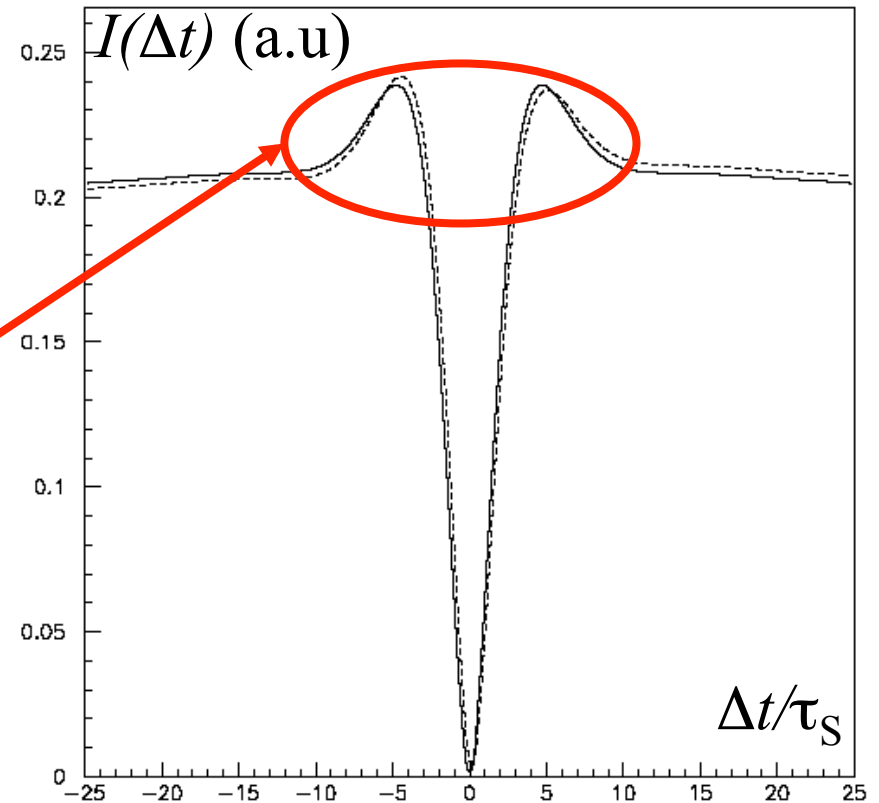


$$\Im(\Delta\delta/\varepsilon)$$

from the asymmetry at **small** Δt

$$\Re(\Delta\delta/\varepsilon) \approx 0 \text{ because } \Delta\delta \perp \varepsilon$$

from the asymmetry at **large** Δt

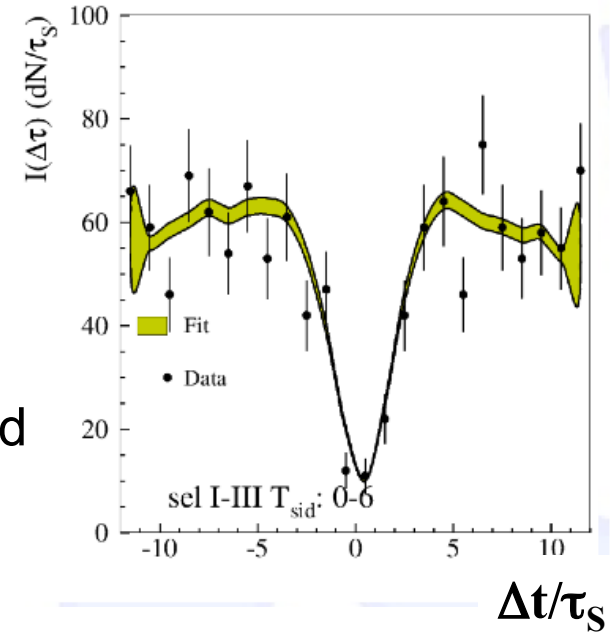
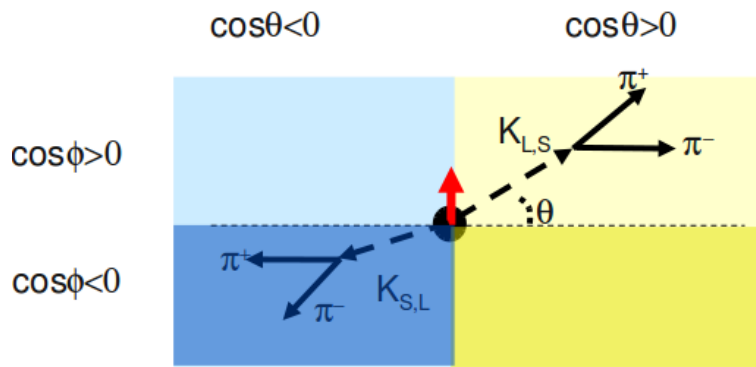


Measurement of Δa_μ at KLOE

K

The analysis is performed in
 4 bins of sidereal time
 x 2 bins for the ϕ quadrant of
 the forward kaon
 x 30 bins of Δt = 240 bins

Example:
 1 bin sidereal time
 (0-4 hours)
 for quadrant
 ($\cos\theta > 0$ $\cos\phi > 0$).
 Data: black points
 Fit result: green band
 (stat. err. only)



with $L=1.7 \text{ fb}^{-1}$ [KLOE final result \(2013\)](#)

$$\Delta a_0 = \left(-6.0 \pm 7.7_{STAT} \pm 3.1_{SYST} \right) \times 10^{-18} \text{ GeV}$$

$$\Delta a_X = \left(0.9 \pm 1.5_{STAT} \pm 0.6_{SYST} \right) \times 10^{-18} \text{ GeV}$$

$$\Delta a_Y = \left(-2.0 \pm 1.5_{STAT} \pm 0.5_{SYST} \right) \times 10^{-18} \text{ GeV}$$

$$\Delta a_Z = \left(-3.1 \pm 1.7_{STAT} \pm 0.6_{SYST} \right) \times 10^{-18} \text{ GeV}$$

see E. Czerwinski's talk

CPT and Lorentz invariance violation (SME)

B

$$z = \frac{\gamma_B \left(\Delta a_0^B - \vec{\beta}_B \cdot \Delta \vec{a}^B \right)}{\Delta m - i \Delta \Gamma / 2}$$

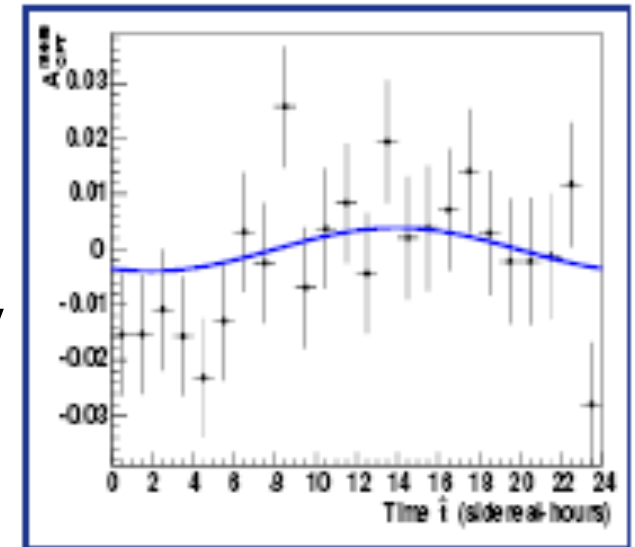
boosted B's at B-factory (almost fixed direction)
=> cannot distinguish between Δa_0^B and Δa_Z^B

searching for a dependence of the form

$$z = z_0 + z_1 \cos(\Omega t + \phi)$$

A_{CPT}
dilepton
asymmetry

$L \sim 232 \text{ fb}^{-1}$



Babar

[PRL 100 (2008) 131802]

$$\Delta a_0^B - 0.30 \Delta a_Z^B \cong (-3.0 \pm 2.4) (\Delta m / \Delta \Gamma) \times 10^{-15} \text{ GeV}$$

$$\Delta a_X^B \cong (-22 \pm 7) (\Delta m / \Delta \Gamma) \times 10^{-15} \text{ GeV}$$

$$\Delta a_Y^B \cong (-14^{+10}_{-13}) (\Delta m / \Delta \Gamma) \times 10^{-15} \text{ GeV}$$

i.e. $\sim O(10^{-13} \text{ GeV})$

CPT and Lorentz invariance violation (SME)

D

$$\xi = \frac{\gamma_D \left(\Delta a_0^D - \vec{\beta}_D \cdot \Delta \vec{a}^D \right)}{\Delta \lambda}$$

boosted D's from photoproduction at fixed target experiment

=> cannot distinguish between Δa_0^D and Δa_Z^D

D* -> Dπ
D in right-sign
hadronic decays

$$A_{CPT}(t) = \frac{I(D^0 \rightarrow K^- \pi^+(t)) - I(\bar{D}^0 \rightarrow K^+ \pi^-(t))}{I(D^0 \rightarrow K^- \pi^+(t)) + I(\bar{D}^0 \rightarrow K^+ \pi^-(t))}$$

FOCUS at FNAL [PLB 556 (2003) 7]

$$f(x, y, \delta) \left[\Delta a_0^D + 0.6 \Delta a_Z^D \right] \cong (1.0 \pm 1.1) \times 10^{-16} \text{ GeV}$$

$$f(x, y, \delta) \Delta a_X^D \cong (-1.6 \pm 2.0) \times 10^{-16} \text{ GeV}$$

$$f(x, y, \delta) \Delta a_Y^D \cong (-1.6 \pm 2.0) \times 10^{-16} \text{ GeV}$$

i.e. $\sim O(10^{-12} \text{ GeV})$

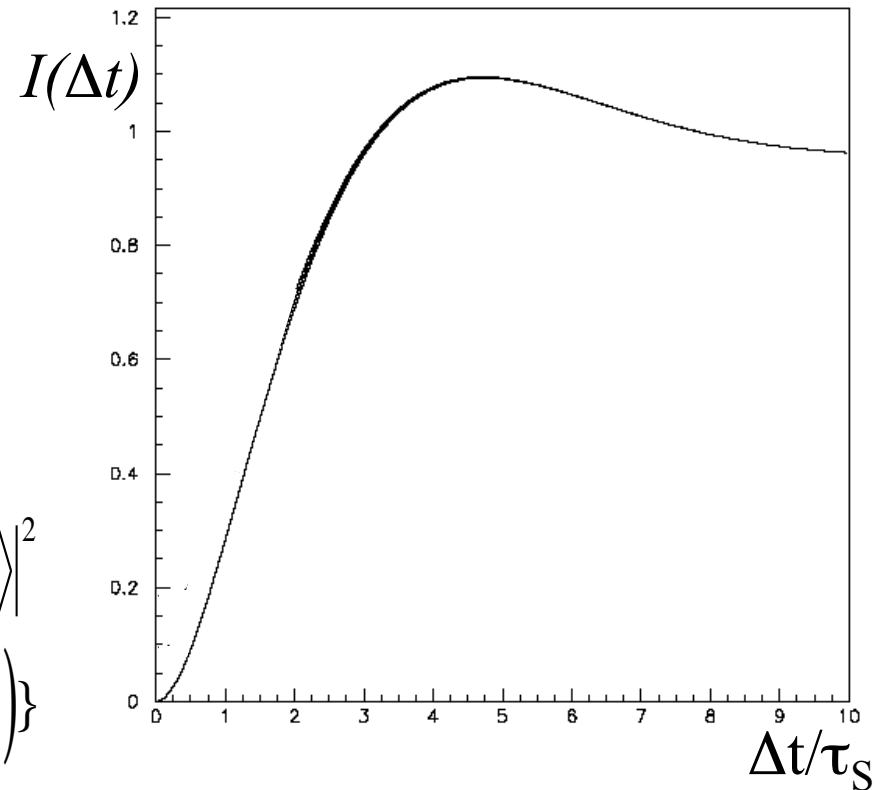
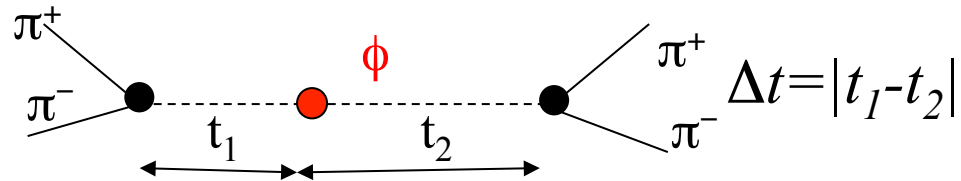
$$f(x, y, \delta) = xy/3 + 0.06(x \cos \delta + y \sin \delta)$$

Testing the EPR entanglement !!!

Test of quantum coherence: $\phi \rightarrow K_S K_L \rightarrow \pi^+ \pi^- \pi^+ \pi^-$

K

$$|i\rangle = \frac{1}{\sqrt{2}} \left[|K^0\rangle |\bar{K}^0\rangle - |\bar{K}^0\rangle |K^0\rangle \right]$$



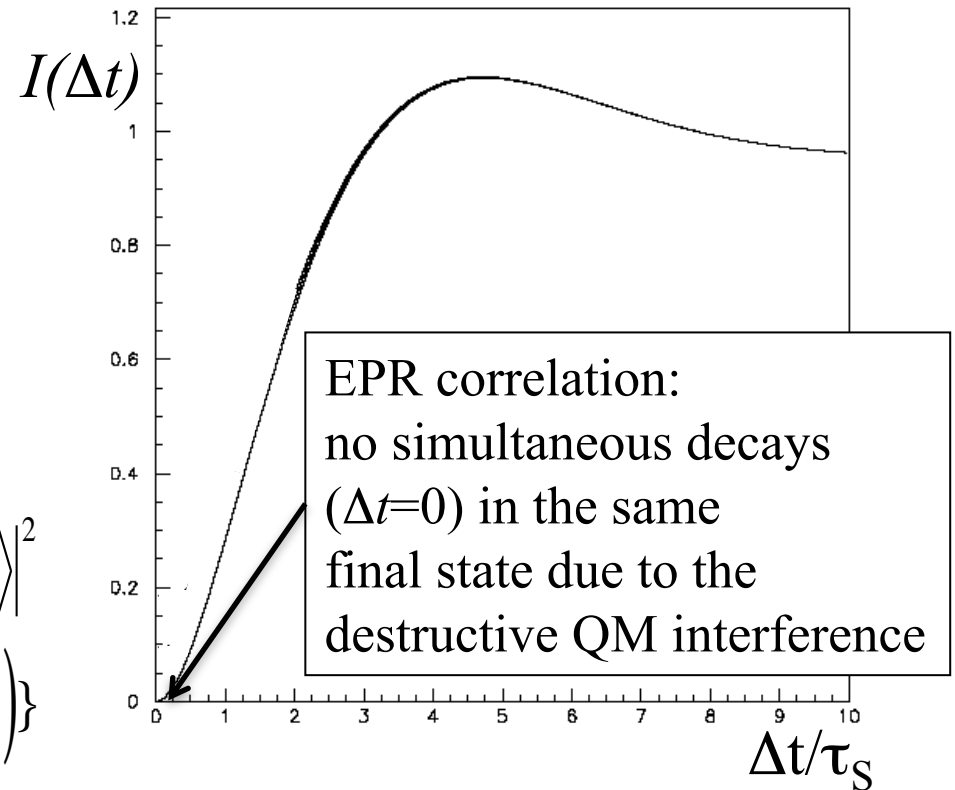
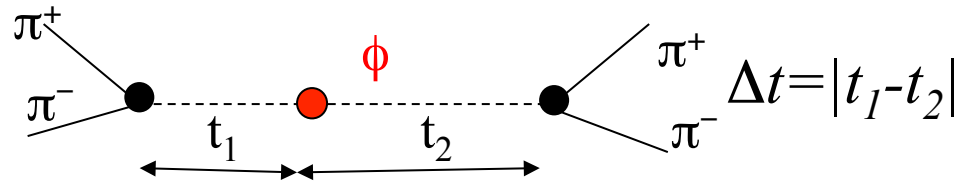
$$I(\pi^+ \pi^-, \pi^+ \pi^-; \Delta t) =$$

$$\frac{N}{2} \left\{ \left| \langle \pi^+ \pi^-, \pi^+ \pi^- | K^0 \bar{K}^0(\Delta t) \rangle \right|^2 + \left| \langle \pi^+ \pi^-, \pi^+ \pi^- | \bar{K}^0 K^0(\Delta t) \rangle \right|^2 \right. \\ \left. - 2 \Re \left(\langle \pi^+ \pi^-, \pi^+ \pi^- | K^0 \bar{K}^0(\Delta t) \rangle \langle \pi^+ \pi^-, \pi^+ \pi^- | \bar{K}^0 K^0(\Delta t) \rangle^* \right) \right\}$$

Test of quantum coherence: $\phi \rightarrow K_S K_L \rightarrow \pi^+ \pi^- \pi^+ \pi^-$

K

$$|i\rangle = \frac{1}{\sqrt{2}} \left[|K^0\rangle |\bar{K}^0\rangle - |\bar{K}^0\rangle |K^0\rangle \right]$$



$$I(\pi^+ \pi^-, \pi^+ \pi^-; \Delta t) =$$

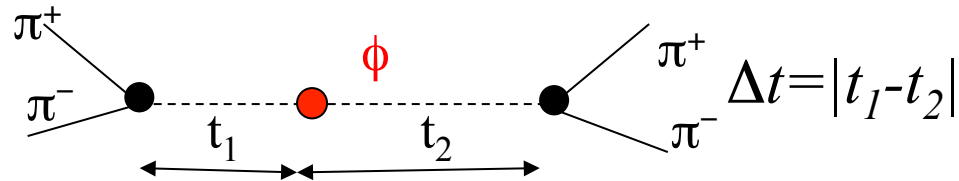
$$\frac{N}{2} \left\{ \left| \langle \pi^+ \pi^-, \pi^+ \pi^- | K^0 \bar{K}^0(\Delta t) \rangle \right|^2 + \left| \langle \pi^+ \pi^-, \pi^+ \pi^- | \bar{K}^0 K^0(\Delta t) \rangle \right|^2 \right.$$

$$\left. - 2 \Re \left(\langle \pi^+ \pi^-, \pi^+ \pi^- | K^0 \bar{K}^0(\Delta t) \rangle \langle \pi^+ \pi^-, \pi^+ \pi^- | \bar{K}^0 K^0(\Delta t) \rangle^* \right) \right\}$$

Test of quantum coherence: $\phi \rightarrow K_S K_L \rightarrow \pi^+ \pi^- \pi^+ \pi^-$

K

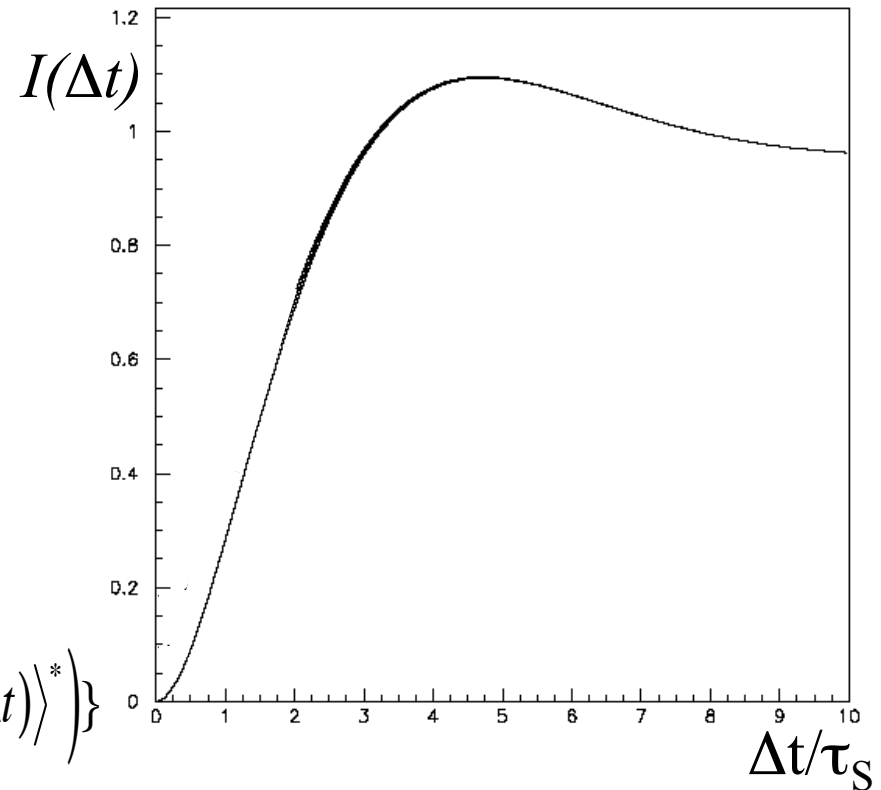
$$|i\rangle = \frac{1}{\sqrt{2}} \left[|K^0\rangle |\bar{K}^0\rangle - |\bar{K}^0\rangle |K^0\rangle \right]$$



$$I(\pi^+ \pi^-, \pi^+ \pi^-; \Delta t) =$$

$$\frac{N}{2} \left\{ \left| \langle \pi^+ \pi^-, \pi^+ \pi^- | K^0 \bar{K}^0(\Delta t) \rangle \right|^2 + \left| \langle \pi^+ \pi^-, \pi^+ \pi^- | \bar{K}^0 K^0(\Delta t) \rangle \right|^2 \right.$$

$$\left. - (1 - \xi_{00}) 2 \Re \left(\langle \pi^+ \pi^-, \pi^+ \pi^- | K^0 \bar{K}^0(\Delta t) \rangle \langle \pi^+ \pi^-, \pi^+ \pi^- | \bar{K}^0 K^0(\Delta t) \rangle^* \right) \right\}$$

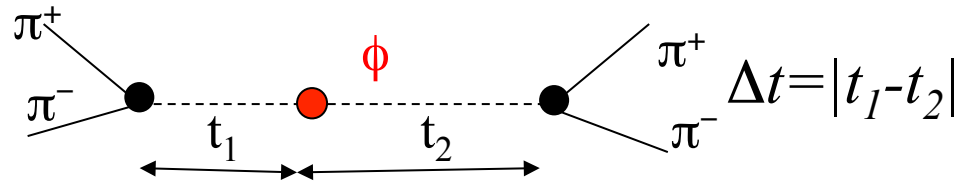


ξ decoherence parameter (QM predicts $\xi=0$)

Test of quantum coherence: $\phi \rightarrow K_S K_L \rightarrow \pi^+ \pi^- \pi^+ \pi^-$

K

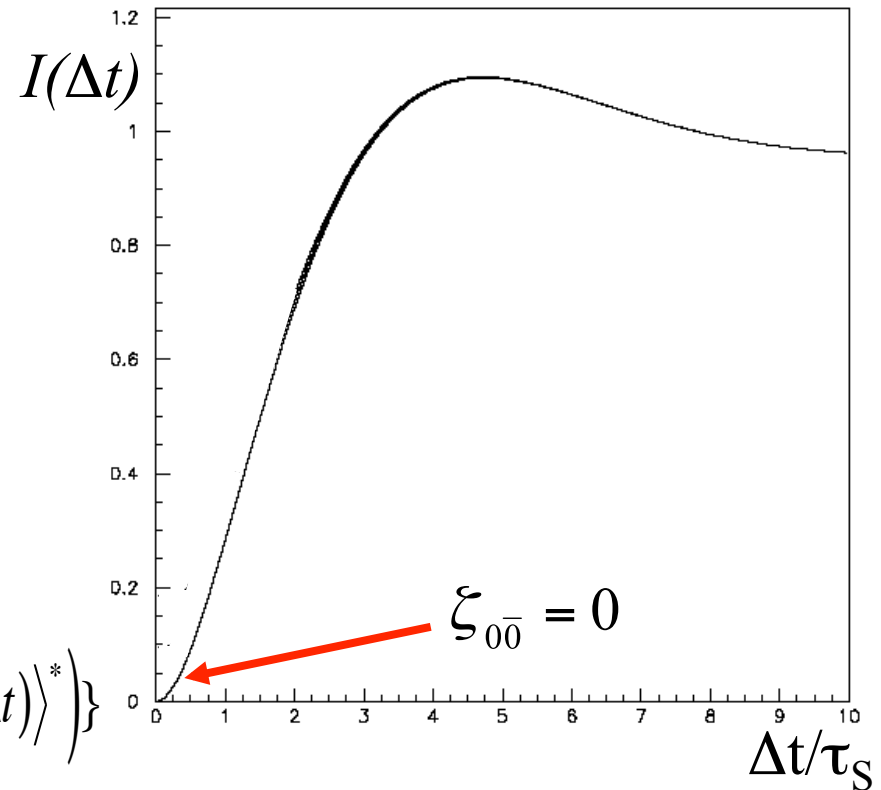
$$|i\rangle = \frac{1}{\sqrt{2}} \left[|K^0\rangle |\bar{K}^0\rangle - |\bar{K}^0\rangle |K^0\rangle \right]$$



$$I(\pi^+ \pi^-, \pi^+ \pi^-; \Delta t) =$$

$$\frac{N}{2} \left\{ \left| \langle \pi^+ \pi^-, \pi^+ \pi^- | K^0 \bar{K}^0(\Delta t) \rangle \right|^2 + \left| \langle \pi^+ \pi^-, \pi^+ \pi^- | \bar{K}^0 K^0(\Delta t) \rangle \right|^2 \right.$$

$$\left. - (1 - \xi_{0\bar{0}}) 2 \Re \left(\langle \pi^+ \pi^-, \pi^+ \pi^- | K^0 \bar{K}^0(\Delta t) \rangle \langle \pi^+ \pi^-, \pi^+ \pi^- | \bar{K}^0 K^0(\Delta t) \rangle^* \right) \right\}$$

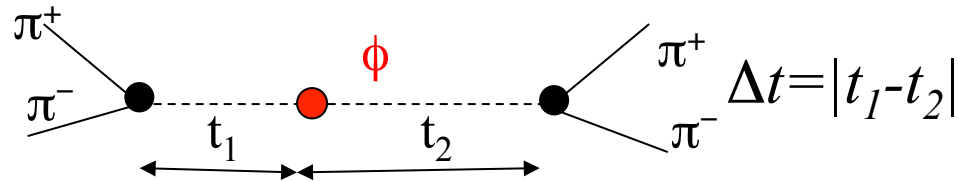


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Test of quantum coherence: $\phi \rightarrow K_S K_L \rightarrow \pi^+ \pi^- \pi^+ \pi^-$

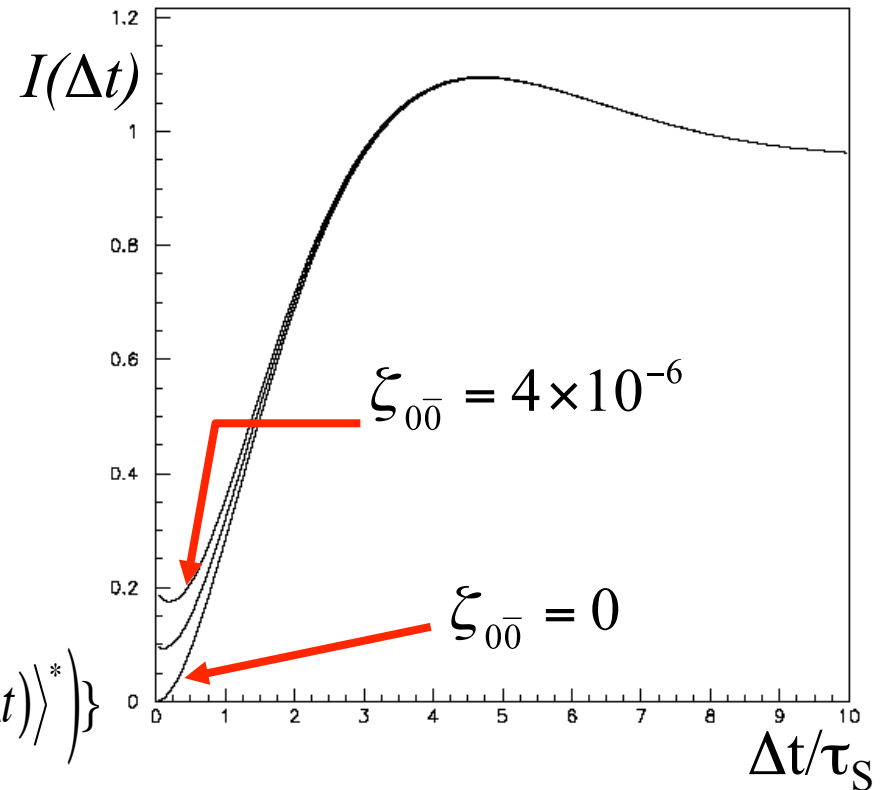
K

$$|i\rangle = \frac{1}{\sqrt{2}} \left[|K^0\rangle |\bar{K}^0\rangle - |\bar{K}^0\rangle |K^0\rangle \right]$$



$$I(\pi^+ \pi^-, \pi^+ \pi^-; \Delta t) =$$

$$\frac{N}{2} \left\{ \left| \langle \pi^+ \pi^-, \pi^+ \pi^- | K^0 \bar{K}^0(\Delta t) \rangle \right|^2 + \left| \langle \pi^+ \pi^-, \pi^+ \pi^- | \bar{K}^0 K^0(\Delta t) \rangle \right|^2 \right. \\ \left. - (1 - \xi_{0\bar{0}}) 2 \Re \left(\langle \pi^+ \pi^-, \pi^+ \pi^- | K^0 \bar{K}^0(\Delta t) \rangle \langle \pi^+ \pi^-, \pi^+ \pi^- | \bar{K}^0 K^0(\Delta t) \rangle^* \right) \right\}$$

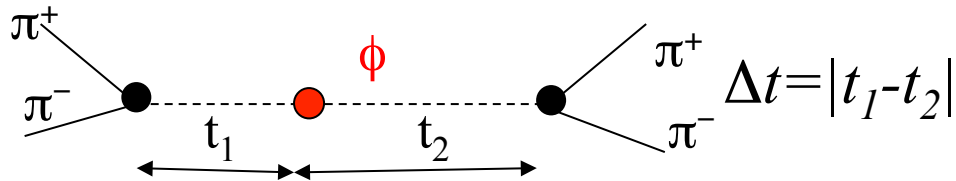


ξ decoherence parameter (QM predicts $\xi=0$)

Test of quantum coherence: $\phi \rightarrow K_S K_L \rightarrow \pi^+ \pi^- \pi^+ \pi^-$

K

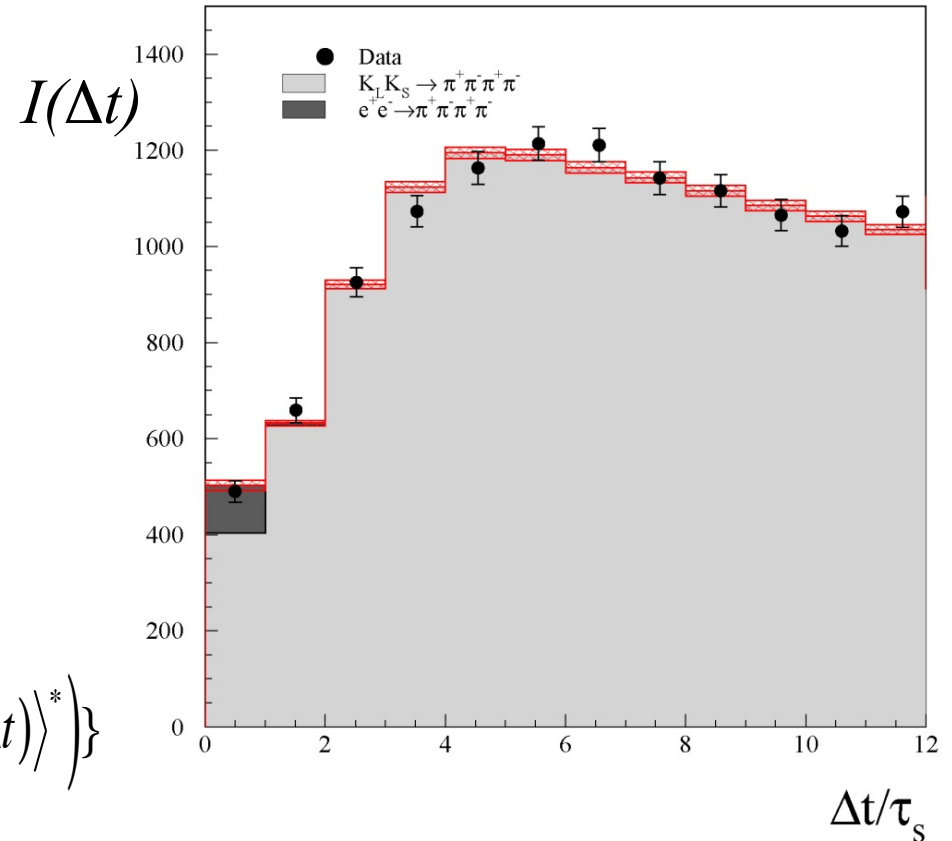
$$|i\rangle = \frac{1}{\sqrt{2}} \left[|K^0\rangle |\bar{K}^0\rangle - |\bar{K}^0\rangle |K^0\rangle \right]$$



$$I(\pi^+ \pi^-, \pi^+ \pi^-; \Delta t) =$$

$$\frac{N}{2} \left\{ \left| \langle \pi^+ \pi^-, \pi^+ \pi^- | K^0 \bar{K}^0(\Delta t) \rangle \right|^2 + \left| \langle \pi^+ \pi^-, \pi^+ \pi^- | \bar{K}^0 K^0(\Delta t) \rangle \right|^2 \right.$$

$$\left. - (1 - \zeta_{00}) \Re \left(\langle \pi^+ \pi^-, \pi^+ \pi^- | K^0 \bar{K}^0(\Delta t) \rangle \langle \pi^+ \pi^-, \pi^+ \pi^- | \bar{K}^0 K^0(\Delta t) \rangle^* \right) \right\}$$



ζ decoherence parameter (QM predicts $\zeta=0$)

Most precise test of quantum coherence in an entangled system:

$$\zeta_{00} = (1.4 \pm 9.5_{\text{STAT}} \pm 3.8_{\text{SYST}}) \times 10^{-7}$$

terms $\zeta_{00}/|\eta_{+-}|^2 \Rightarrow$ enhanced sensitivity due to CP violation

KLOE result: PLB 642(2006) 315 L=1.5 fb⁻¹ : J.Phys.Conf.Ser.171:012008,2009.

Test of quantum coherence in neutral B mesons

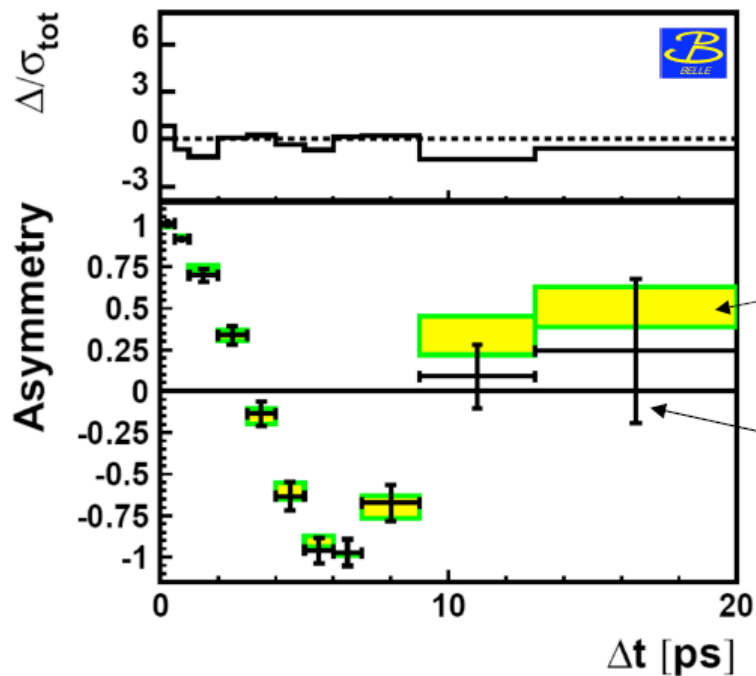
B

Δt dependent rates: opposite sign di-lepton vs same sign di-lepton

$$A(\Delta t) = \frac{I(\ell^\pm, \ell^\mp; \Delta t) - I(\ell^\pm, \ell^\pm; \Delta t)}{I(\ell^\pm, \ell^\mp; \Delta t) + I(\ell^\pm, \ell^\pm; \Delta t)} = \cos(\Delta m \Delta t)$$

↑
QM prediction

After correcting for Δt resolution and selection efficiency by a deconvolution procedure:



fitted value:
 $\Delta m_d = (0.501 \pm 0.009) \text{ ps}^{-1}$
 $\chi^2 = 5.2 \text{ (11 dof)}$
 QM (error from Δm_d)

Data

$L \sim 150 \text{ fb}^{-1}$

BELLE PRL 99 131802 (2007)

$$\xi_{00} = 0.029 \pm 0.057$$

no enhanced sensitivity due to CP violation here $\left| \frac{\langle f_{CP} | T | B_H \rangle}{\langle f_{CP} | T | B_L \rangle} \right| \sim O(1)$
 (η not very small)

$\phi \rightarrow K_S K_L \rightarrow \pi^+ \pi^- \pi^+ \pi^-$: CPT violation in entangled K states **K**

In presence of decoherence and CPT violation induced by quantum gravity (CPT operator “ill-defined”) the definition of the particle-antiparticle states could be modified. This in turn could induce a breakdown of the correlations imposed by Bose statistics (EPR correlations) to the kaon state:

[Bernabeu, et al. PRL 92 (2004) 131601, NPB744 (2006) 180].

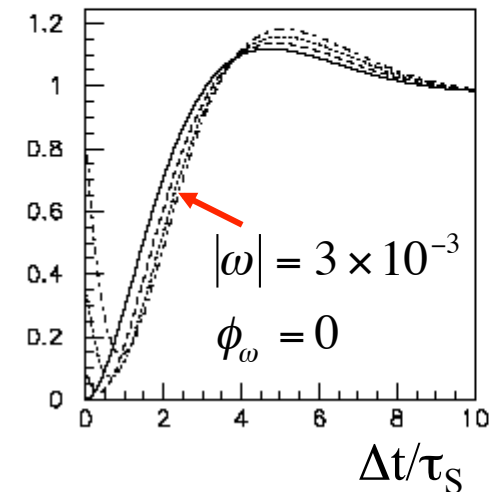
$$|i\rangle \propto (|K^0\rangle|\bar{K}^0\rangle - |K^0\rangle|\bar{K}^0\rangle) + \omega(|K^0\rangle|\bar{K}^0\rangle + |K^0\rangle|\bar{K}^0\rangle)$$

$$\propto (|K_S\rangle|K_L\rangle - |K_L\rangle|K_S\rangle) + \omega(|K_S\rangle|K_S\rangle - |K_L\rangle|K_L\rangle)$$

at most one expects:

$$|\omega|^2 = O\left(\frac{E^2/M_{PLANCK}}{\Delta\Gamma}\right) \approx 10^{-5} \Rightarrow |\omega| \sim 10^{-3}$$

$I(\pi^+\pi^-, \pi^+\pi^-; \Delta t)$ (a.u.)



KLOE result:

PLB 642(2006) 315
J.Phys.Conf.Ser.171:012008,2009.

$$\Re\omega = \left(-1.6_{-2.1}^{+3.0}{}_{STAT} \pm 0.4_{SYST}\right) \times 10^{-4}$$

$$\Im\omega = \left(-1.7_{-3.0}^{+3.3}{}_{STAT} \pm 1.2_{SYST}\right) \times 10^{-4}$$

$$|\omega| < 1.0 \times 10^{-3} \quad \text{at } 95\% \text{ C.L.}$$

CPT violation in entangled B states

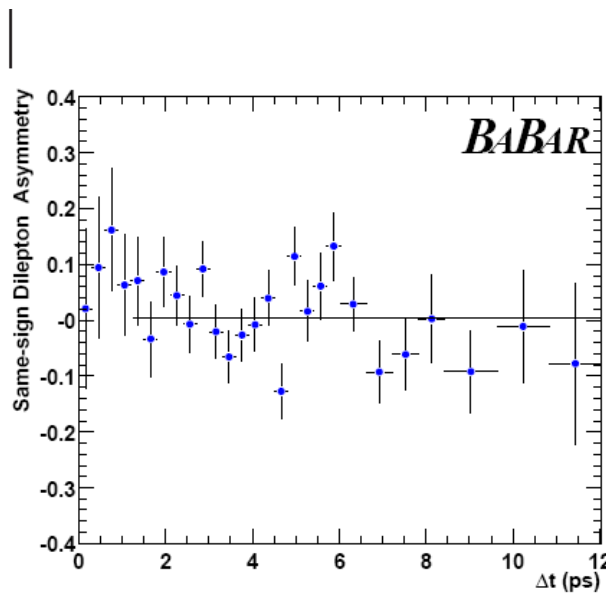
B

Observable asymmetry of Δt dependent rates: same sign di-lepton

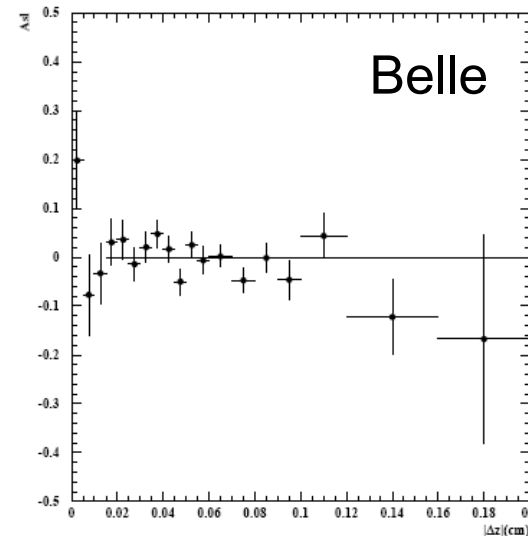
$$A_{sl}(\Delta t) = \frac{I(\ell^+, \ell^+; \Delta t) - I(\ell^-, \ell^-; \Delta t)}{I(\ell^+, \ell^+; \Delta t) + I(\ell^-, \ell^-; \Delta t)}$$

- For $\omega=0$ equal sign di-lepton time asymmetry A_{sl} is exactly time independent
- For $\omega \neq 0$ A_{sl} acquires a time dependence $A_{sl}(0) \propto |\omega|^2$

$L \sim 20 \text{ fb}^{-1}$



(a) Babar, $\Delta t = \frac{|\Delta z|}{1.53 \text{ ps}} \Gamma^{-1}$



$L \sim 90 \text{ fb}^{-1}$

(b) Belle, $\Delta t = \frac{|\Delta z|}{0.0186 \text{ cm}} \Gamma^{-1}$

Alvarez, Bernabeu, Nebot JHEP 0611, 087:

$$-0.0084 \leq \Re \omega \leq 0.0100 \quad \text{at } 95\% \text{ C.L.}$$

Conclusions

- Neutral meson systems are unique and excellent laboratories for the study of discrete symmetries
- A direct test of the T symmetry, independently from CP violation and CPT invariance constraints, has been recently performed by **Babar** for **B** mesons.
- Several parameters related to possible CPT violation (together with Lorentz symmetry breaking or decoherence) have been recently measured at **KLOE** for **K** mesons and at **Belle** and **Babar** for **B** mesons, with very high precision, especially for kaons. In some cases the precision reaches the interesting Planck's scale region.
- At e⁺e⁻ colliders entanglement imposed by EPR correlations plays a crucial role, both as a tool and as a QM property to be tested
- All results are consistent with no CPT violation.
- Improvements in the precision of the tests are expected at the next generation of experiments at flavor factories, KLOE-2 at DAFNE, Belle-II at Super KEKB, Super Tau-Charm factory, BES-III

Spare slides

CPT test

$$\delta = \frac{1}{2} \frac{(m_{\bar{K}^0} - m_{K^0}) - (i/2)(\Gamma_{\bar{K}^0} - \Gamma_{K^0})}{\Delta m + i\Delta\Gamma/2}$$

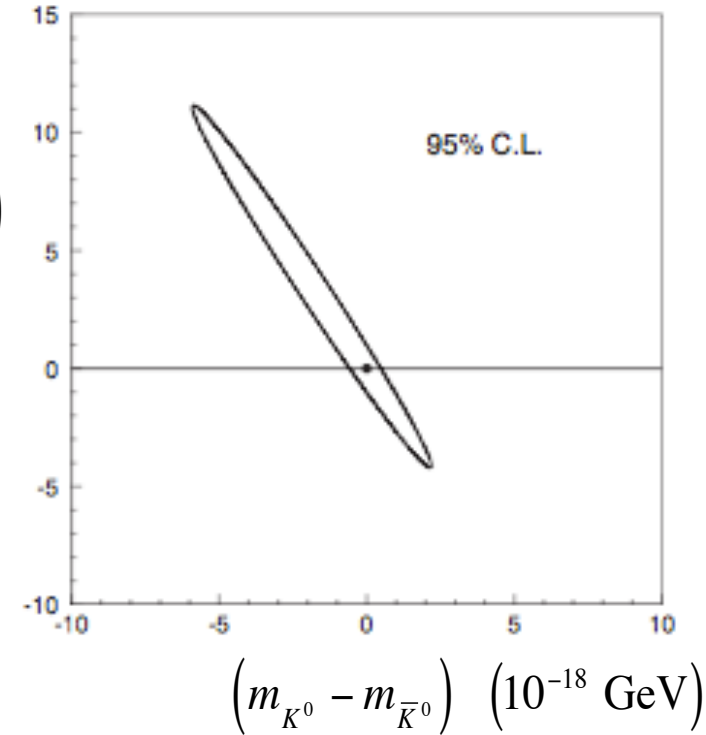
$$\begin{aligned} &(\Gamma_{K^0} - \Gamma_{\bar{K}^0}) \\ &(10^{-18} \text{ GeV}) \end{aligned}$$

Combining $\text{Re}\delta$ and $\text{Im}\delta$ results



$$\text{Re } \delta = (0.30 \pm 0.33) \times 10^{-3}$$

$$\text{Im } \delta = (-1.5 \pm 1.6) \times 10^{-5}$$



Assuming $(\Gamma_{\bar{K}^0} - \Gamma_{K^0}) = 0$, i.e. no CPT viol. in decay:

$$\left| m_{\bar{K}^0} - m_{K^0} \right| < 4.8 \times 10^{-19} \text{ GeV} \quad \text{at 95\% c.l.}$$

CPT test: the “standard” picture

From the study of the time evolution of neutral B mesons with opposite flavor (and also other) decays

$$A_{CPT} = \frac{P(B^0 \rightarrow B^0) - P(\bar{B}^0 \rightarrow \bar{B}^0)}{P(B^0 \rightarrow B^0) + P(\bar{B}^0 \rightarrow \bar{B}^0)}$$

PDG av.

BABAR PRL 96, 251802 (2006)

BELLE PRD85, 071105(R) (2012)

$$\text{Re } z = (1.9 \pm 3.7 \pm 3.3) \times 10^{-2}$$

$$\text{Im } z = (-0.8 \pm 0.4) \times 10^{-2}$$

Assuming $(\Gamma_{B^0} - \Gamma_{\bar{B}^0}) = 0$, i.e. no CPT viol. in decay:

$$\left| m_{B^0} - m_{\bar{B}^0} \right| < \sim 5 \times 10^{-14} \text{ GeV} \quad \text{at 95\% c.l.}$$

CPT and Lorentz invariance violation (SME)

K

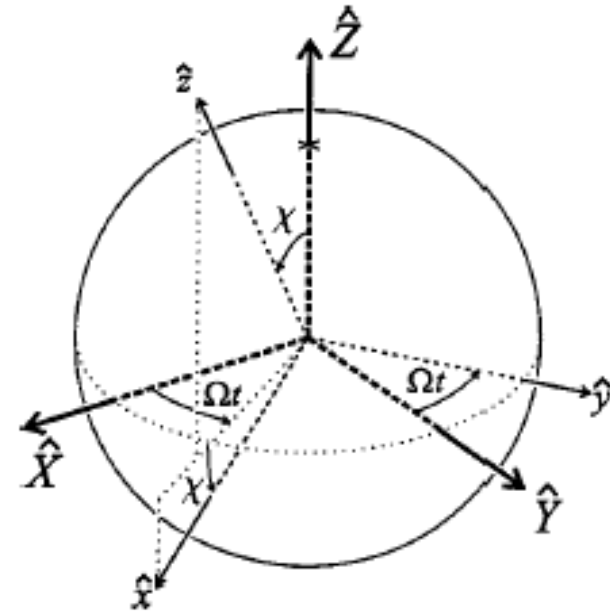
$$\delta = i \sin \phi_{SW} e^{i\phi_{SW}} \gamma_K \left(\Delta a_0 - \vec{\beta}_K \cdot \Delta \vec{a} \right) / \Delta m$$

δ depends on sidereal time t since laboratory frame rotates with Earth.

For a ϕ -factory there is an additional dependence on the polar and azimuthal angle θ, ϕ of the kaon momentum in the laboratory frame:

$$\begin{aligned} \delta(\vec{p}, t) = & \frac{i \sin \phi_{SW} e^{i\phi_{SW}}}{\Delta m} \gamma_K \left\{ \Delta a_0 \right. \\ & + \beta_K \Delta a_z (\cos \theta \cos \chi - \sin \theta \sin \phi \sin \chi) \\ & + \beta_K \left[-\Delta a_x \sin \theta \sin \phi + \Delta a_y (\cos \theta \sin \chi + \sin \theta \cos \phi \cos \chi) \right] \sin \Omega t \\ & \left. + \beta_K \left[+\Delta a_y \sin \theta \sin \phi + \Delta a_x (\cos \theta \sin \chi + \sin \theta \cos \phi \cos \chi) \right] \cos \Omega t \right\} \end{aligned}$$

Ω : Earth's sidereal frequency χ : angle between the z lab. axis and the Earth's rotation axis



(in general z lab. axis is non-normal to Earth's surface)

CPT and Lorentz invariance violation (SME)

K

$$\delta = i \sin \phi_{SW} e^{i\phi_{SW}} \gamma_K \left(\Delta a_0 - \vec{\beta}_K \cdot \Delta \vec{a} \right) / \Delta m$$

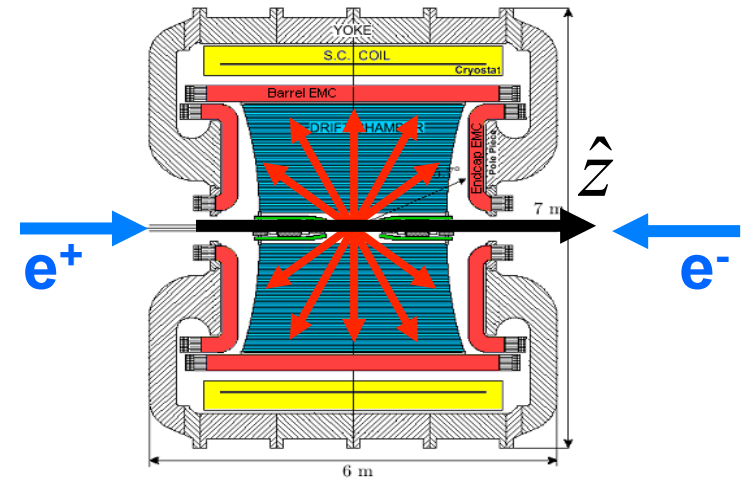
δ depends on sidereal time t since laboratory frame rotates with Earth.

For a ϕ -factory there is an additional dependence on the polar and azimuthal angle θ, ϕ of the kaon momentum in the laboratory frame:

$$\begin{aligned} \delta(\vec{p}, t) = & \frac{i \sin \phi_{SW} e^{i\phi_{SW}}}{\Delta m} \gamma_K \left\{ \Delta a_0 \right. \\ & + \beta_K \Delta a_Z (\cos \theta \cos \chi - \sin \theta \sin \phi \sin \chi) \\ & + \beta_K \left[-\Delta a_X \sin \theta \sin \phi + \Delta a_Y (\cos \theta \sin \chi + \sin \theta \cos \phi \cos \chi) \right] \sin \Omega t \\ & \left. + \beta_K \left[+\Delta a_Y \sin \theta \sin \phi + \Delta a_X (\cos \theta \sin \chi + \sin \theta \cos \phi \cos \chi) \right] \cos \Omega t \right\} \end{aligned}$$

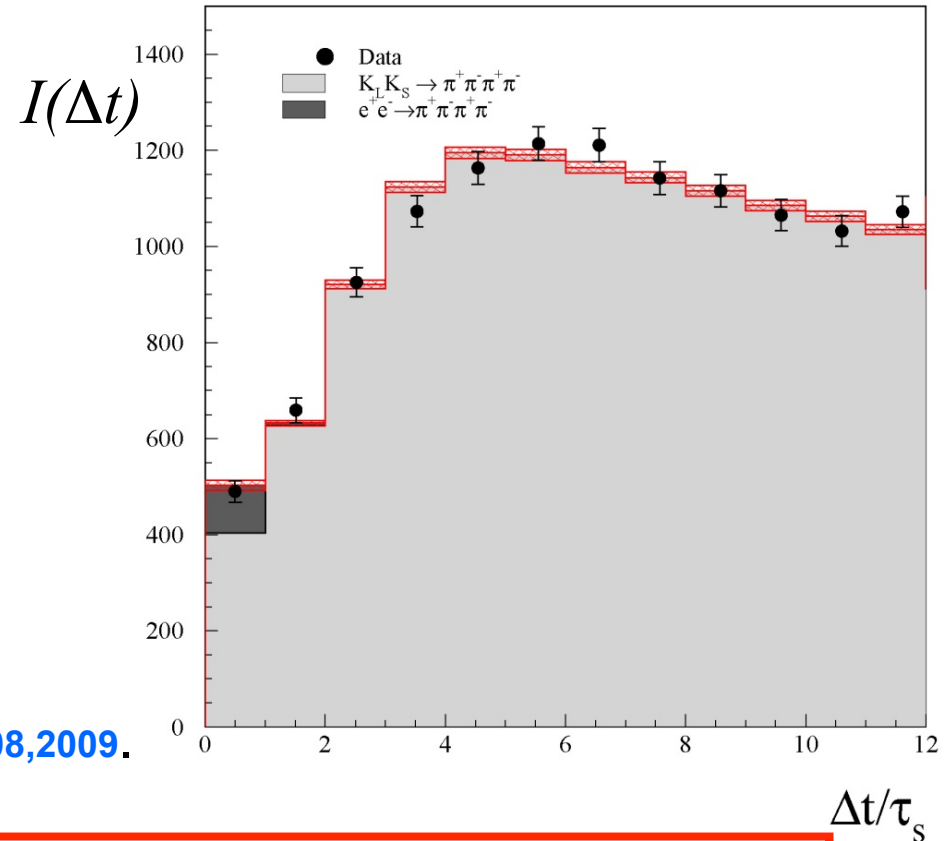
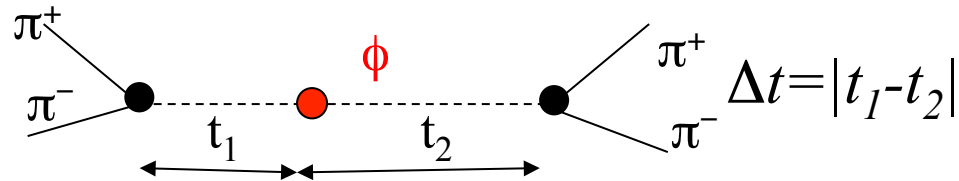
Ω : Earth's sidereal frequency χ : angle between the z lab. axis and the Earth's rotation axis

At DAΦNE K mesons are produced with angular distribution $dN/d\Omega \propto \sin^2\theta$



Neutral kaon interferometry: $\phi \rightarrow K_S K_L \rightarrow \pi^+ \pi^- \pi^+ \pi^-$

$$|i\rangle = \frac{1}{\sqrt{2}} \left[|K^0\rangle |\bar{K}^0\rangle - |\bar{K}^0\rangle |K^0\rangle \right]$$



Most precise test of quantum coherence in an entangled system:

$$\xi_{0\bar{0}} = (1.4 \pm 9.5_{\text{STAT}} \pm 3.8_{\text{SYST}}) \times 10^{-7}$$

ξ decoherence parameter (QM predicts $\xi=0$)

PLB 642(2006) 315 L=1.5 fb⁻¹ : J.Phys.Conf.Ser.171:012008,2009.

Quantum gravity effects might induce:

1) decoherence and CPT violation
(at most $\gamma = O(m_K^2/M_{\text{Planck}}) \sim 2 \times 10^{-20}$ GeV)

$$\gamma = (0.7 \pm 1.2_{\text{STAT}} \pm 0.3_{\text{SYST}}) \times 10^{-21} \text{ GeV}$$

2) decoherence and CPT violation induce modification of the initial correlation of the kaon pair (at most $\omega = O(m_K^2/M_{\text{Planck}}/\Delta\Gamma) \sim 1 \times 10^{-3}$)

$$|i\rangle \propto (K^0 \bar{K}^0 - \bar{K}^0 K^0) + \omega (K^0 \bar{K}^0 + \bar{K}^0 K^0)$$

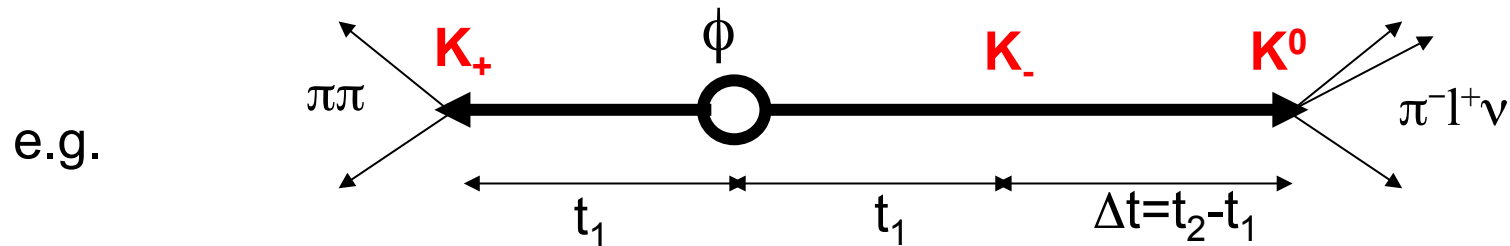
$$\Re \omega = (-1.6^{+3.0}_{-2.1 \text{ STAT}} \pm 0.4_{\text{SYST}}) \times 10^{-4}$$

$$\Im \omega = (-1.7^{+3.3}_{-3.0 \text{ STAT}} \pm 1.2_{\text{SYST}}) \times 10^{-4}$$

Entanglement in neutral meson pairs

- EPR correlations at a ϕ -factory (or B-factory) can be exploited to study other transitions involving also “CP states” K_+ and K_-

$$\begin{aligned}
 |i\rangle &= \frac{1}{\sqrt{2}} \left[|K^0(\vec{p})\rangle |\bar{K}^0(-\vec{p})\rangle - |\bar{K}^0(\vec{p})\rangle |K^0(-\vec{p})\rangle \right] \\
 &= \frac{1}{\sqrt{2}} \left[|K_+(\vec{p})\rangle |K_-(-\vec{p})\rangle - |K_-(-\vec{p})\rangle |K_+(\vec{p})\rangle \right]
 \end{aligned}$$



$$I(\pi\pi, l^+; \Delta t) = C(\pi\pi, l^+) \times P[K_-(0) \rightarrow K^0(\Delta t)]$$

In general with $f_{\bar{X}}$ decaying before f_Y , i.e. $\Delta t > 0$:

$$I(f_{\bar{X}}, f_Y; \Delta t) = C(f_{\bar{X}}, f_Y) \times P[K_X(0) \rightarrow K_Y(\Delta t)]$$

with

$$C(f_{\bar{X}}, f_Y) = \frac{1}{2(\Gamma_S + \Gamma_L)} |\langle f_{\bar{X}} | T | \bar{K}_X \rangle \langle f_Y | T | K_Y \rangle|^2$$

Direct test of symmetries with neutral kaons

Reference	T -conjugate	CP -conjugate	CPT -conjugate
$K^0 \rightarrow K^0$	$K^0 \rightarrow K^0$	$\bar{K}^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow \bar{K}^0$
$K^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K^0$	$\bar{K}^0 \rightarrow K^0$	$K^0 \rightarrow \bar{K}^0$
$K^0 \rightarrow K_+$	$K_+ \rightarrow K^0$	$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$
$K^0 \rightarrow K_-$	$K_- \rightarrow K^0$	$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$
$\bar{K}^0 \rightarrow K^0$	$K^0 \rightarrow \bar{K}^0$	$K^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K^0$
$\bar{K}^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow \bar{K}^0$	$K^0 \rightarrow K^0$	$K^0 \rightarrow K^0$
$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$	$K^0 \rightarrow K_+$	$K_+ \rightarrow K^0$
$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$	$K^0 \rightarrow K_-$	$K_- \rightarrow K^0$
$K_+ \rightarrow K^0$	$K^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_+$
$K_+ \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow K^0$	$K^0 \rightarrow K_+$
$K_+ \rightarrow K_+$	$K_+ \rightarrow K_+$	$K_+ \rightarrow K_+$	$K_+ \rightarrow K_+$
$K_+ \rightarrow K_-$	$K_- \rightarrow K_+$	$K_+ \rightarrow K_-$	$K_- \rightarrow K_+$
$K_- \rightarrow K^0$	$K^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_-$
$K_- \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow K^0$	$K^0 \rightarrow K_-$
$K_- \rightarrow K_+$	$K_+ \rightarrow K_-$	$K_- \rightarrow K_+$	$K_+ \rightarrow K_-$
$K_- \rightarrow K_-$	$K_- \rightarrow K_-$	$K_- \rightarrow K_-$	$K_- \rightarrow K_-$

Direct test of symmetries with neutral kaons

Conjugate=
reference

Reference	<i>T</i> -conjugate	<i>CP</i> -conjugate	<i>CPT</i> -conjugate
$K^0 \rightarrow K^0$	$K^0 \rightarrow K^0$	$\bar{K}^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow \bar{K}^0$
$K^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K^0$	$\bar{K}^0 \rightarrow K^0$	$K^0 \rightarrow \bar{K}^0$
$K^0 \rightarrow K_+$	$K_+ \rightarrow K^0$	$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$
$K^0 \rightarrow K_-$	$K_- \rightarrow K^0$	$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$
$\bar{K}^0 \rightarrow K^0$	$K^0 \rightarrow \bar{K}^0$	$K^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K^0$
$\bar{K}^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow \bar{K}^0$	$K^0 \rightarrow K^0$	$K^0 \rightarrow K^0$
$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$	$K^0 \rightarrow K_+$	$K_+ \rightarrow K^0$
$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$	$K^0 \rightarrow K_-$	$K_- \rightarrow K^0$
$K_+ \rightarrow K^0$	$K^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_+$
$K_+ \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow K^0$	$K^0 \rightarrow K_+$
$K_+ \rightarrow K_+$	$K_+ \rightarrow K_+$	$K_+ \rightarrow K_+$	$K_+ \rightarrow K_+$
$K_+ \rightarrow K_-$	$K_- \rightarrow K_+$	$K_- \rightarrow K_-$	$K_- \rightarrow K_+$
$K_- \rightarrow K^0$	$K^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_-$
$K_- \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow K^0$	$K^0 \rightarrow K_-$
$K_- \rightarrow K_+$	$K_+ \rightarrow K_-$	$K_+ \rightarrow K_+$	$K_+ \rightarrow K_-$
$K_- \rightarrow K_-$	$K_- \rightarrow K_-$	$K_- \rightarrow K_-$	$K_- \rightarrow K_-$

Direct test of symmetries with neutral kaons

Conjugate=
reference

already in the
table with
conjugate as
reference

Reference	<i>T</i> -conjugate	<i>CP</i> -conjugate	<i>CPT</i> -conjugate
$K^0 \rightarrow K^0$	$K^0 \rightarrow K^0$	$\bar{K}^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow \bar{K}^0$
$K^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K^0$	$\bar{K}^0 \rightarrow K^0$	$K^0 \rightarrow \bar{K}^0$
$K^0 \rightarrow K_+$	$K_+ \rightarrow K^0$	$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$
$K^0 \rightarrow K_-$	$K_- \rightarrow K^0$	$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$
$\bar{K}^0 \rightarrow K^0$	$K^0 \rightarrow \bar{K}^0$	$K^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K^0$
$\bar{K}^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow \bar{K}^0$	$K^0 \rightarrow K^0$	$K^0 \rightarrow K^0$
$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$	$K^0 \rightarrow K_+$	$K_+ \rightarrow K^0$
$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$	$K^0 \rightarrow K_-$	$K_- \rightarrow K^0$
$K_+ \rightarrow K^0$	$K^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_+$
$K_+ \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow K^0$	$K^0 \rightarrow K_+$
$K_+ \rightarrow K_+$	$K_+ \rightarrow K_+$	$K_- \rightarrow K_+$	$K_- \rightarrow K_+$
$K_+ \rightarrow K_-$	$K_- \rightarrow K_+$	$K_- \rightarrow K_-$	$K_- \rightarrow K_+$
$K_- \rightarrow K^0$	$K^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_-$
$K_- \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow K^0$	$K^0 \rightarrow K_-$
$K_- \rightarrow K_+$	$K_+ \rightarrow K_-$	$K_- \rightarrow K_+$	$K_+ \rightarrow K_-$
$K_- \rightarrow K_-$	$K_- \rightarrow K_-$	$K_- \rightarrow K_-$	$K_- \rightarrow K_-$

Direct test of symmetries with neutral kaons

Conjugate=
reference



already in the
table with
conjugate as
reference



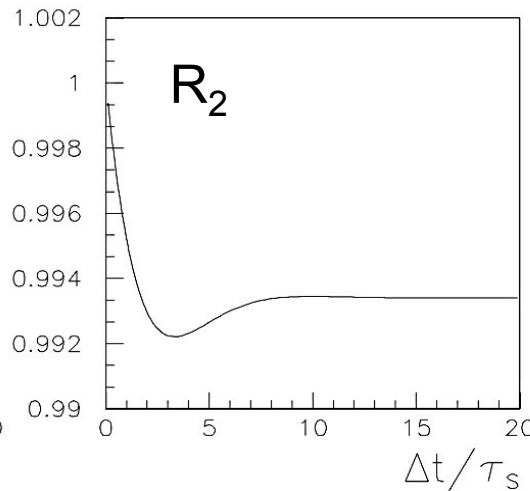
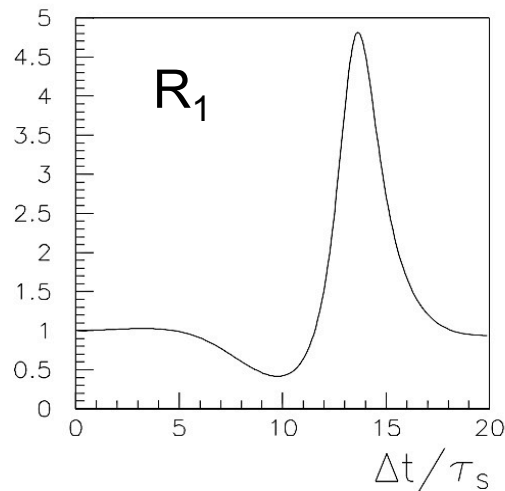
Two identical
conjugates
for one reference



Reference	<i>T</i> -conjugate	<i>CP</i> -conjugate	<i>CPT</i> -conjugate
$K^0 \rightarrow K^0$	$K^0 \rightarrow K^0$	$\bar{K}^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow \bar{K}^0$
$K^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K^0$	$\bar{K}^0 \rightarrow K^0$	$K^0 \rightarrow \bar{K}^0$
$K^0 \rightarrow K_+$	$K_+ \rightarrow K^0$	$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$
$K^0 \rightarrow K_-$	$K_- \rightarrow K^0$	$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$
$\bar{K}^0 \rightarrow K^0$	$K^0 \rightarrow \bar{K}^0$	$K^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K^0$
$\bar{K}^0 \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow \bar{K}^0$	$K^0 \rightarrow K^0$	$K^0 \rightarrow K^0$
$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$	$K^0 \rightarrow K_+$	$K_+ \rightarrow K^0$
$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$	$K^0 \rightarrow K_-$	$K_- \rightarrow K^0$
$K_+ \rightarrow K^0$	$K^0 \rightarrow K_+$	$K_+ \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_+$
$K_+ \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_+$	$K_+ \rightarrow K^0$	$K^0 \rightarrow K_+$
$K_+ \rightarrow K_+$	$K_+ \rightarrow K_+$	$K_+ \rightarrow K_+$	$K_+ \rightarrow K_+$
$K_+ \rightarrow K_-$	$K_- \rightarrow K_+$	$K_- \rightarrow K_+$	$K_- \rightarrow K_+$
$K_- \rightarrow K^0$	$K^0 \rightarrow K_-$	$K_- \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_-$
$K_- \rightarrow \bar{K}^0$	$\bar{K}^0 \rightarrow K_-$	$K_- \rightarrow K^0$	$K^0 \rightarrow K_-$
$K_- \rightarrow K_+$	$K_+ \rightarrow K_-$	$K_+ \rightarrow K_+$	$K_+ \rightarrow K_-$
$K_- \rightarrow K_-$	$K_- \rightarrow K_-$	$K_- \rightarrow K_-$	$K_- \rightarrow K_-$

Direct test of Time Reversal symmetry with neutral kaons

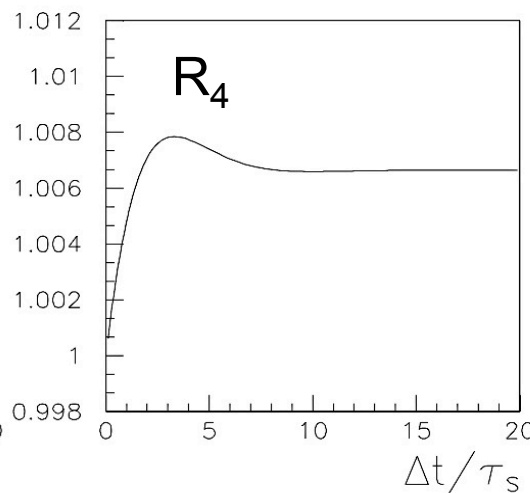
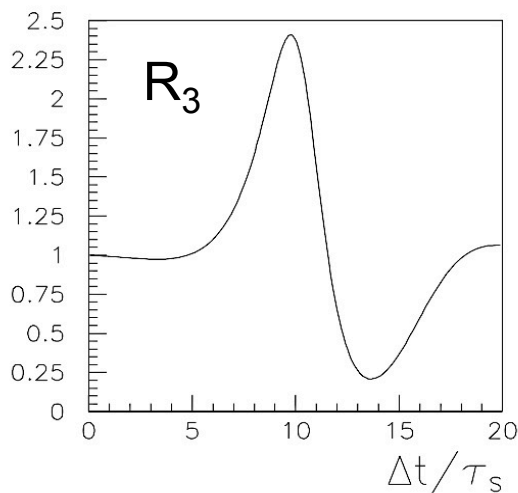
Any deviation from $R_i=1$ constitutes a direct evidence of T-symmetry violation



$$R_i(\Delta t=0)=1$$

$$R_2(\Delta t \gg \tau_S)=1-4\text{Re}(\epsilon)$$

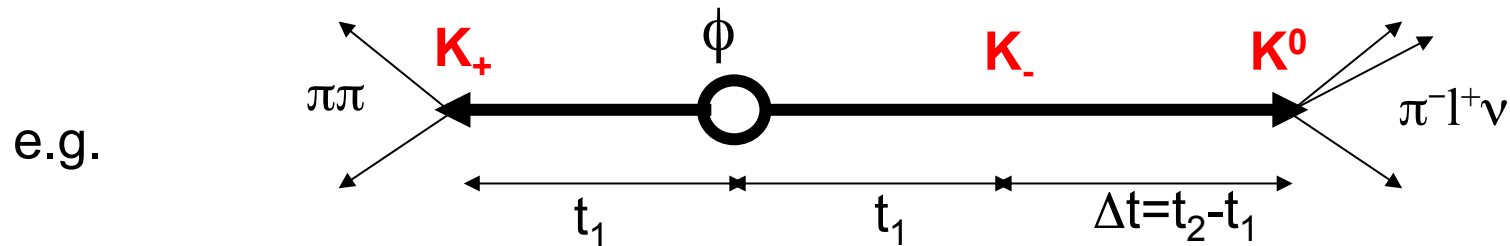
$$R_4(\Delta t \gg \tau_S)=1+4\text{Re}(\epsilon)$$



Direct test of Time Reversal symmetry with neutral kaons

- EPR correlations at a ϕ -factory can be exploited to study other transitions involving also “CP states” K_+ and K_-

$$\begin{aligned}
 |i\rangle &= \frac{1}{\sqrt{2}} [|K^0\rangle |\bar{K}^0\rangle - |\bar{K}^0\rangle |K^0\rangle] \\
 &= \frac{1}{\sqrt{2}} [|K_+\rangle |K_-\rangle - |K_-\rangle |K_+\rangle]
 \end{aligned}$$



$$I(\pi\pi, l^+; \Delta t) = C(\pi\pi, l^+) \times P[K_-(0) \rightarrow K^0(\Delta t)]$$

In general with $f_{\bar{X}}$ decaying before f_Y , i.e. $\Delta t > 0$:

$$I(f_{\bar{X}}, f_Y; \Delta t) = C(f_{\bar{X}}, f_Y) \times P[K_X(0) \rightarrow K_Y(\Delta t)]$$

with

$$C(f_{\bar{X}}, f_Y) = \frac{1}{2(\Gamma_S + \Gamma_L)} |\langle f_{\bar{X}} | T | \bar{K}_X \rangle \langle f_Y | T | K_Y \rangle|^2$$

Direct test of Time Reversal symmetry with neutral kaons

$$R_1^{\text{exp}}(\Delta t) \equiv \frac{I(\ell^-, \pi\pi; \Delta t)}{I(3\pi^0, \ell^+; \Delta t)} = R_1(\Delta t) \times \frac{C(\ell^-, \pi\pi)}{C(3\pi^0, \ell^+)}$$

$$R_2^{\text{exp}}(\Delta t) \equiv \frac{I(\ell^-, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^+; \Delta t)} = R_2(\Delta t) \times \frac{C(\ell^-, 3\pi^0)}{C(\pi\pi, \ell^+)}$$

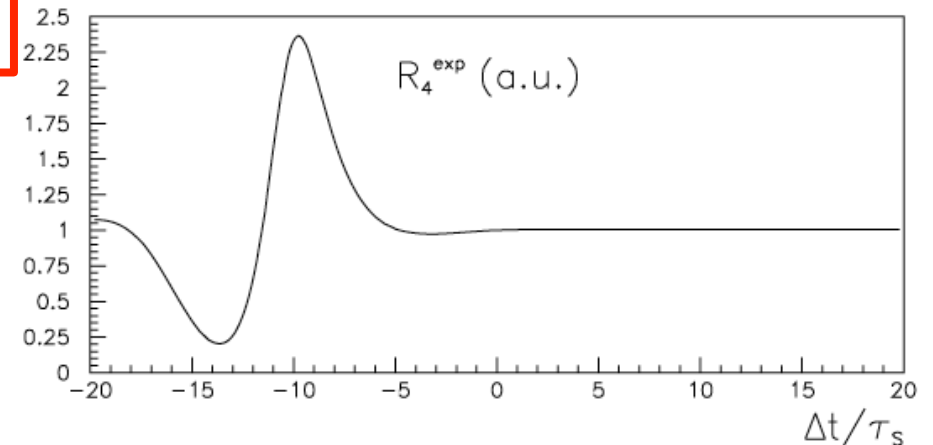
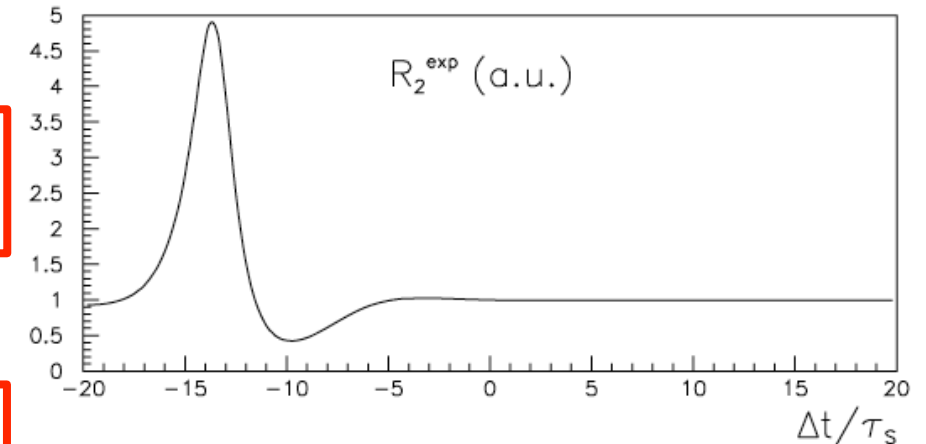
$$R_3^{\text{exp}}(\Delta t) \equiv \frac{I(\ell^+, \pi\pi; \Delta t)}{I(3\pi^0, \ell^-; \Delta t)} = R_3(\Delta t) \times \frac{C(\ell^+, \pi\pi)}{C(3\pi^0, \ell^-)}$$

$$R_4^{\text{exp}}(\Delta t) \equiv \frac{I(\ell^+, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^-; \Delta t)} = R_4(\Delta t) \times \frac{C(\ell^+, 3\pi^0)}{C(\pi\pi, \ell^-)}$$

In practice two measurable ratios with $\Delta t < 0$ or > 0

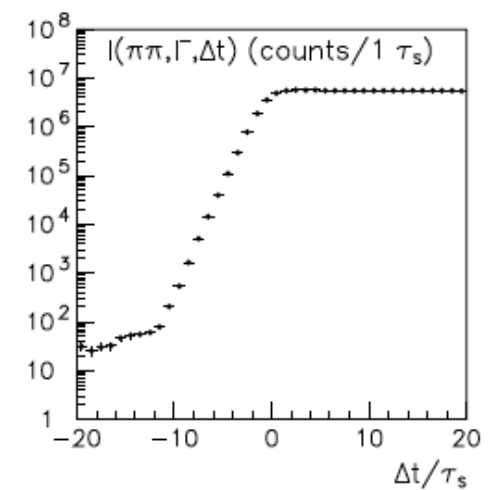
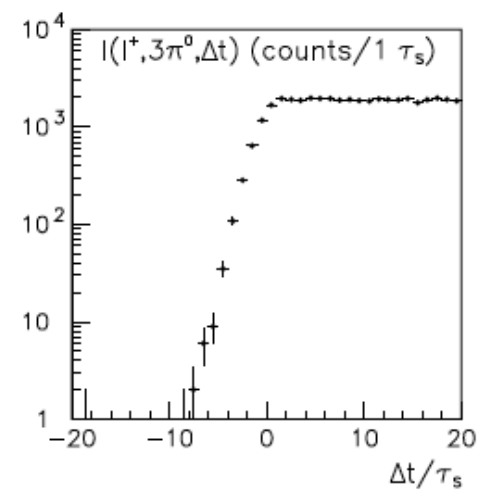
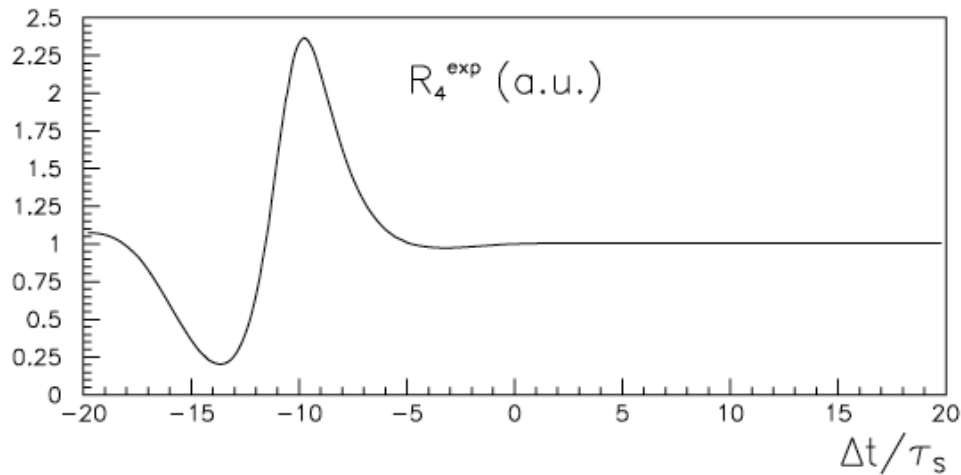
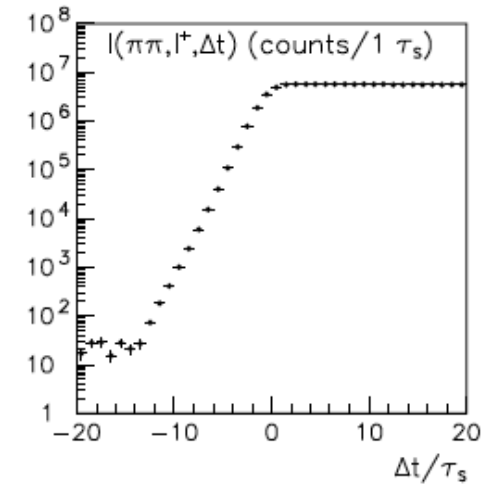
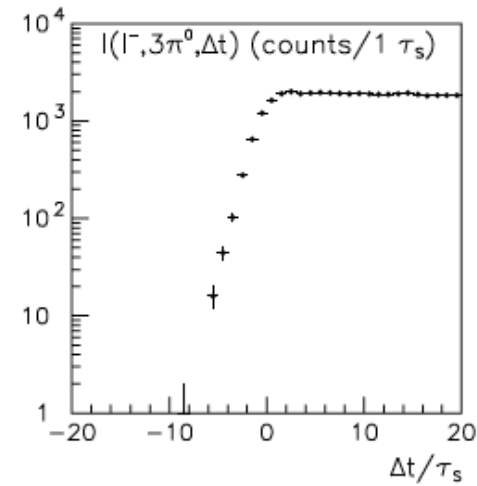
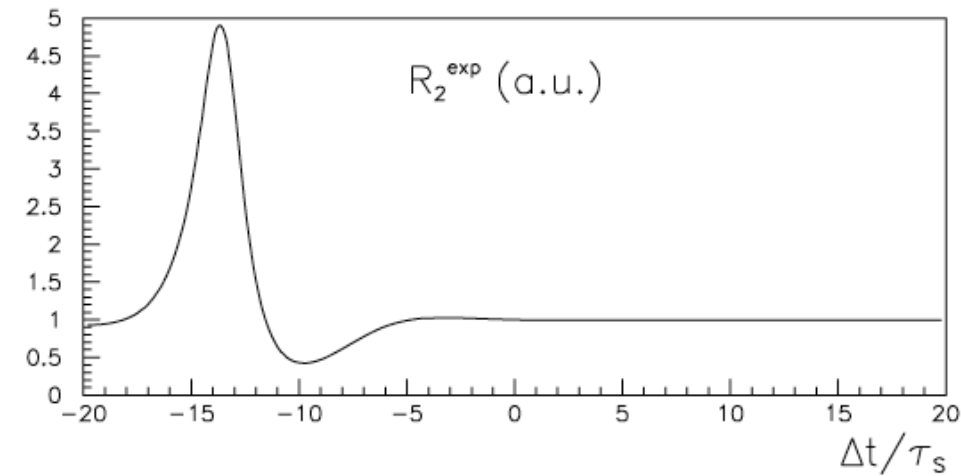
$$R_2^{\text{exp}}(-\Delta t) = \frac{1}{R_3^{\text{exp}}(\Delta t)} = \frac{1}{R_3(\Delta t)} \times \frac{C(3\pi^0, \ell^-)}{C(\ell^+, \pi\pi)},$$

$$R_4^{\text{exp}}(-\Delta t) = \frac{1}{R_1^{\text{exp}}(\Delta t)} = \frac{1}{R_1(\Delta t)} \times \frac{C(3\pi^0, \ell^+)}{C(\ell^-, \pi\pi)}.$$



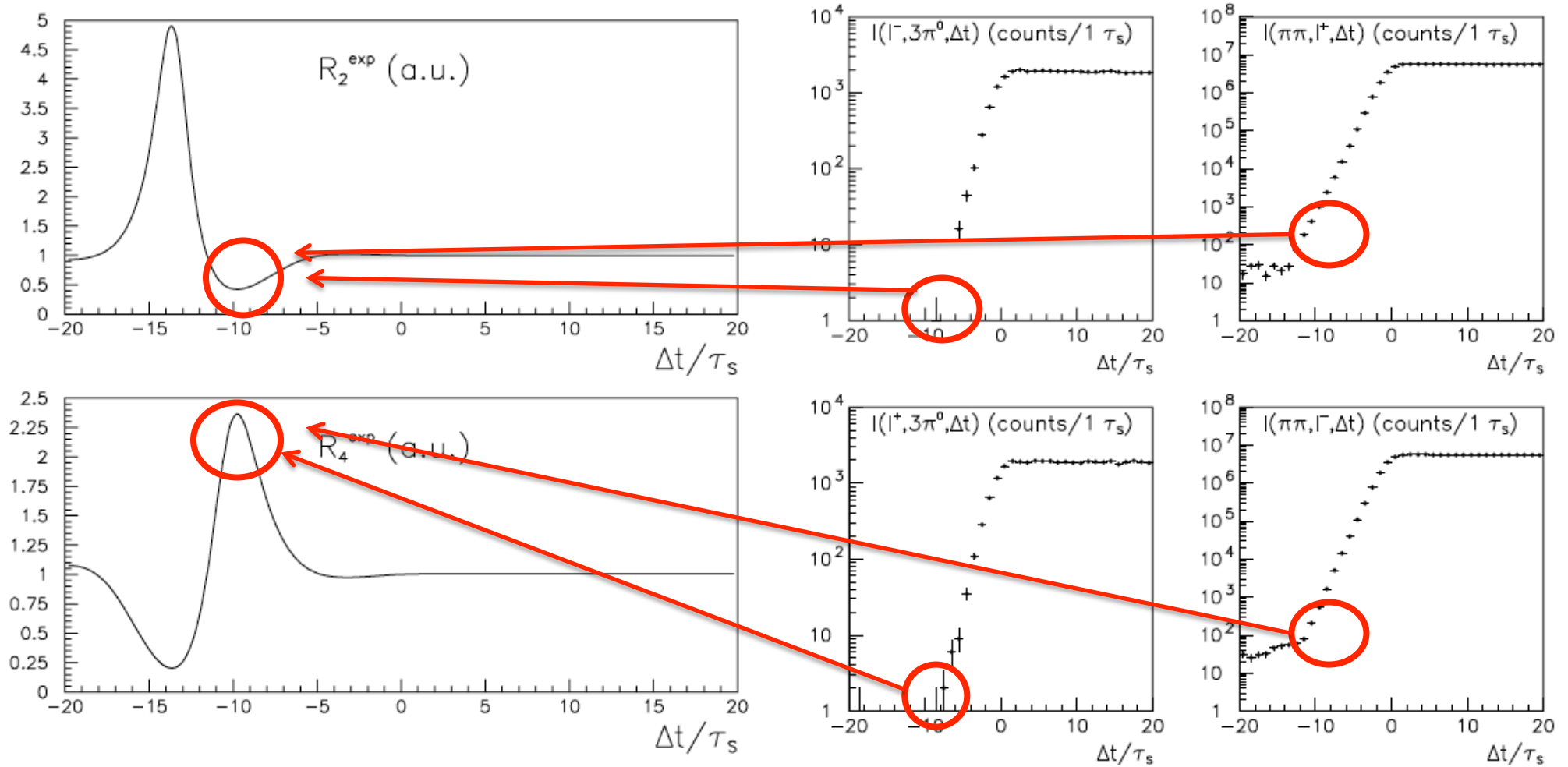
Direct test of Time Reversal symmetry with neutral kaons

toy MC with $L=10 \text{ fb}^{-1}$



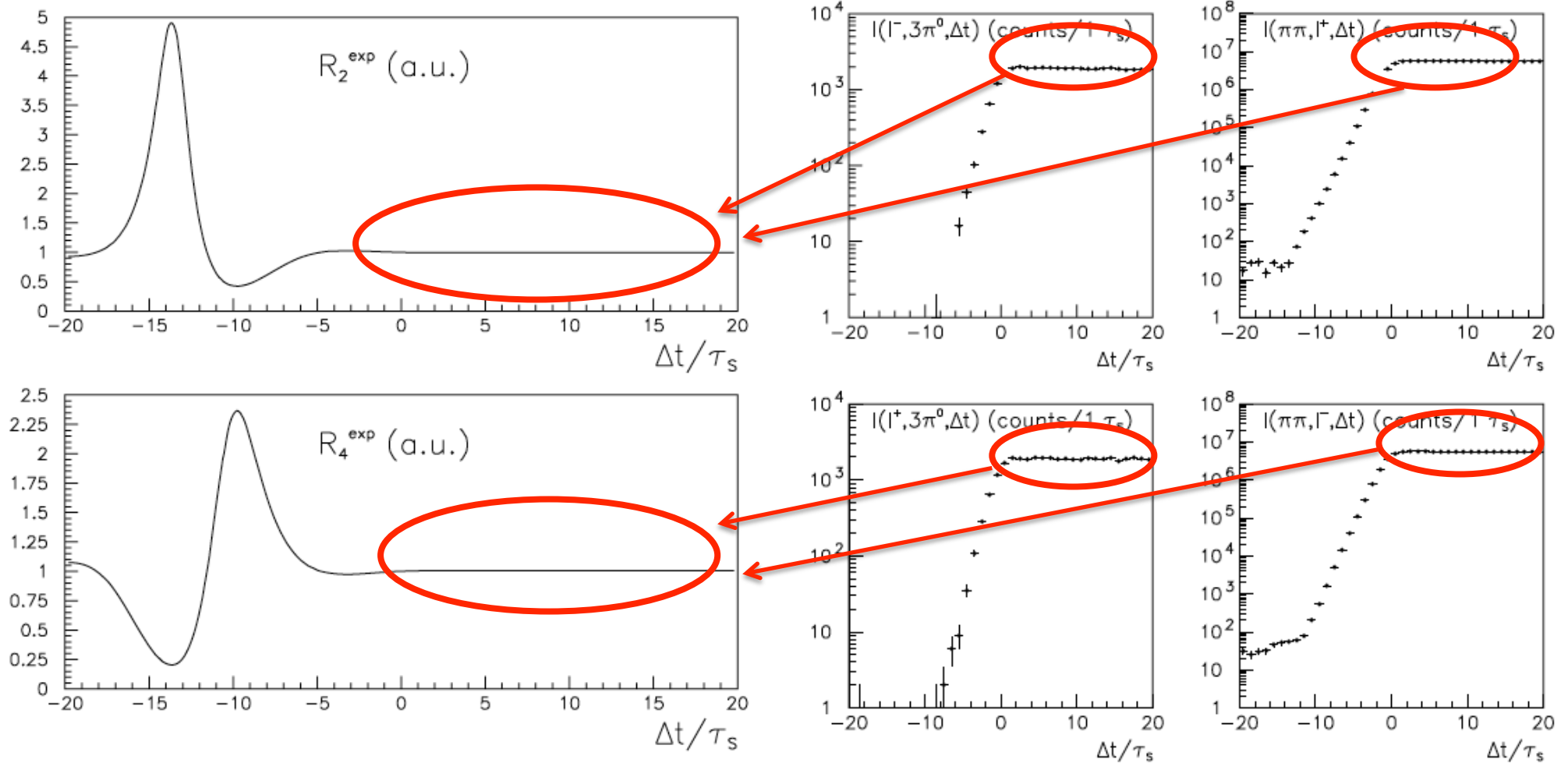
Direct test of Time Reversal symmetry with neutral kaons

toy MC with $L=10 \text{ fb}^{-1}$



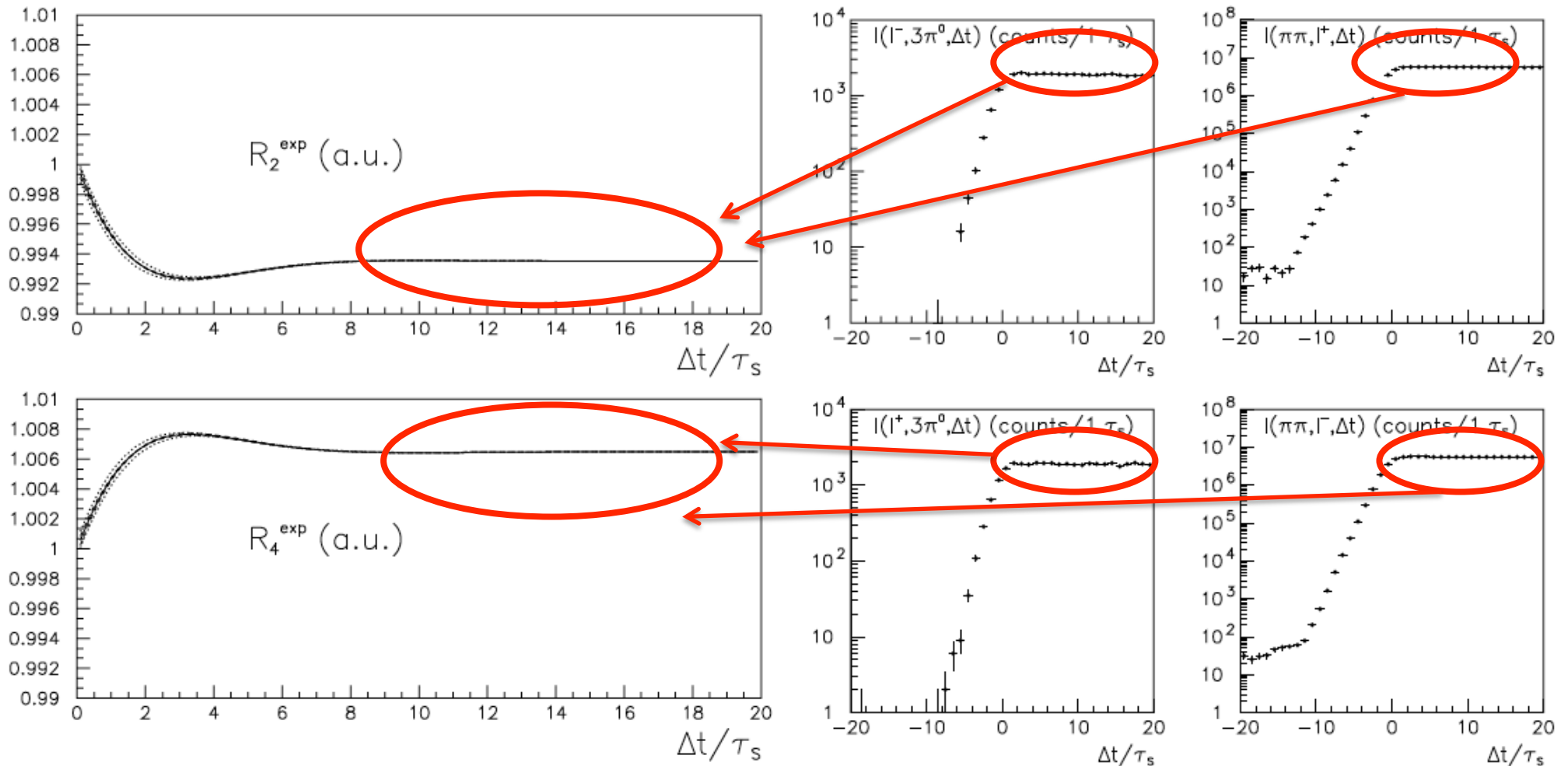
Direct test of Time Reversal symmetry with neutral kaons

toy MC with $L=10 \text{ fb}^{-1}$



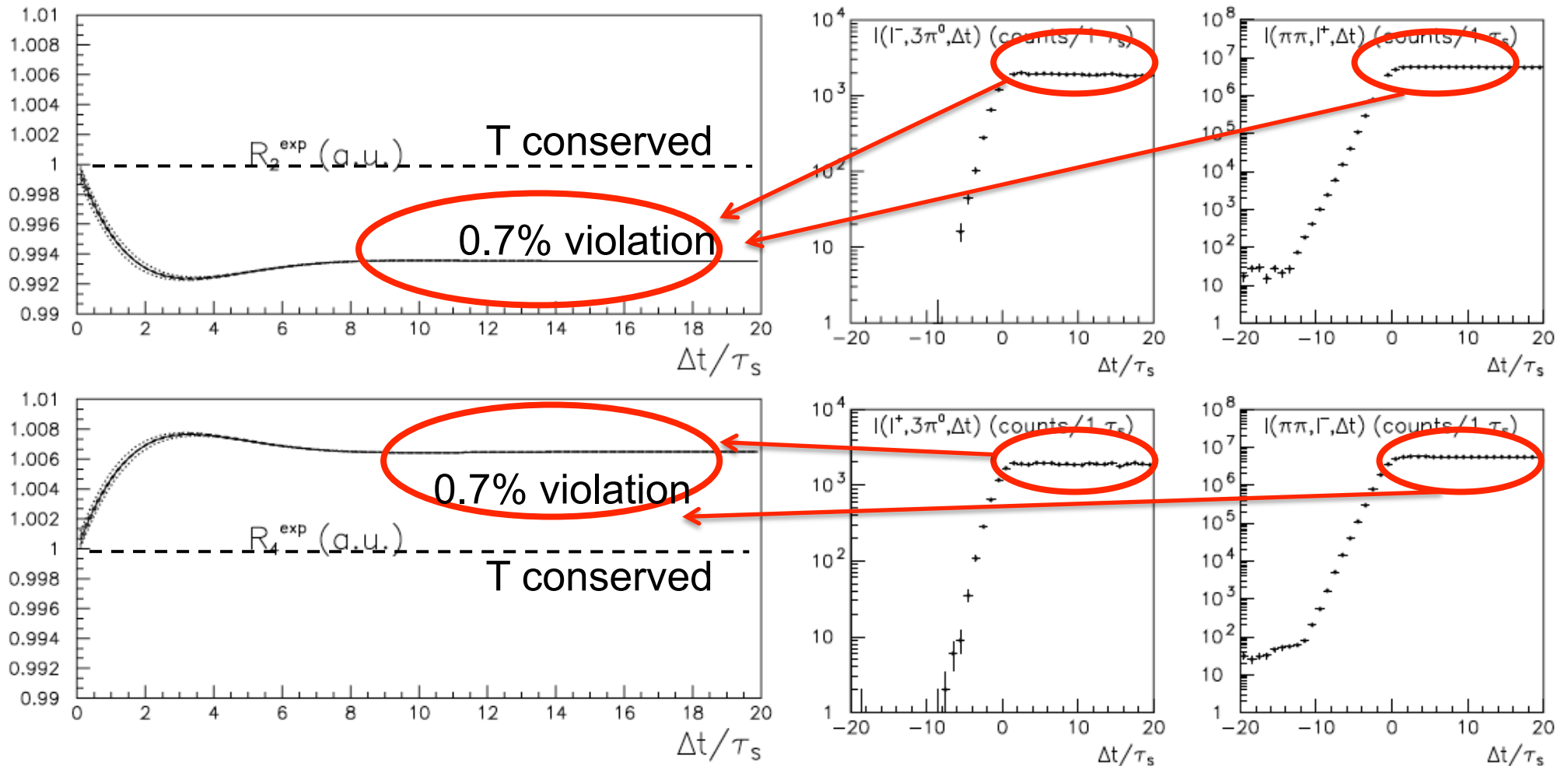
Direct test of Time Reversal symmetry with neutral kaons

toy MC with $L=10 \text{ fb}^{-1}$



Direct test of Time Reversal symmetry with neutral kaons

toy MC with $L=10 \text{ fb}^{-1}$



$$R_2(\Delta t \gg \tau_S) = 1 - 4\text{Re}(\epsilon) \sim 0.993$$

$$R_4(\Delta t \gg \tau_S) = 1 + 4\text{Re}(\epsilon) \sim 1.007$$

Direct test of Time Reversal symmetry with neutral kaons

Integrating in a Δt region between 0 and $300 \tau_S \Rightarrow$
stat. significance of 4.4, 6.2, 8.8 σ with $L=5, 10, 20 \text{ fb}^{-1}$ (full efficiency)

But, in the “plateau” region one needs to measure the absolute value of R_i

$$\frac{C(\ell^-, 3\pi^0)}{C(\pi\pi, \ell^+)} \simeq \frac{C(\ell^+, 3\pi^0)}{C(\pi\pi, \ell^-)} \simeq \frac{\text{BR}(K_L \rightarrow 3\pi^0) \Gamma_L}{\text{BR}(K_S \rightarrow \pi\pi) \Gamma_S} \equiv D.$$

$$R_2(\Delta t) = \frac{R_2^{\text{exp}}(\Delta t)}{D},$$

$$R_4(\Delta t) = \frac{R_4^{\text{exp}}(\Delta t)}{D}.$$

It is needed to measure the constant D with at least 0.1% precision,
i.e. BRs and K_S, K_L lifetimes

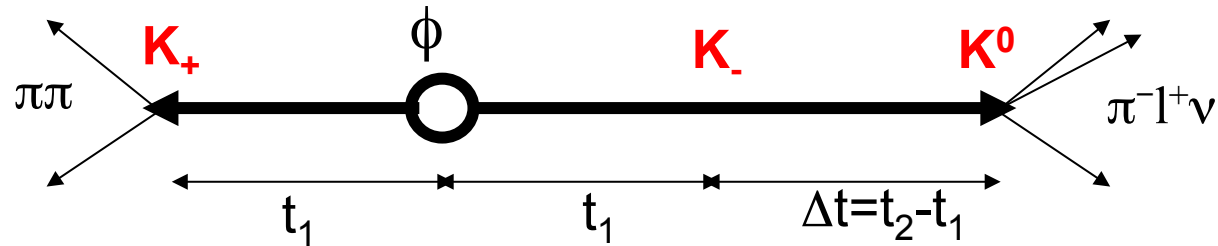
T test could be feasible at KLOE-2 with $L=O(10 \text{ fb}^{-1})$

(but quite difficult !!)

The kaon states

$|K_{+(-)}\rangle \equiv$ state filtered by the decay in $\pi\pi(3\pi^0)$ (pure CP = +1(-1) state)

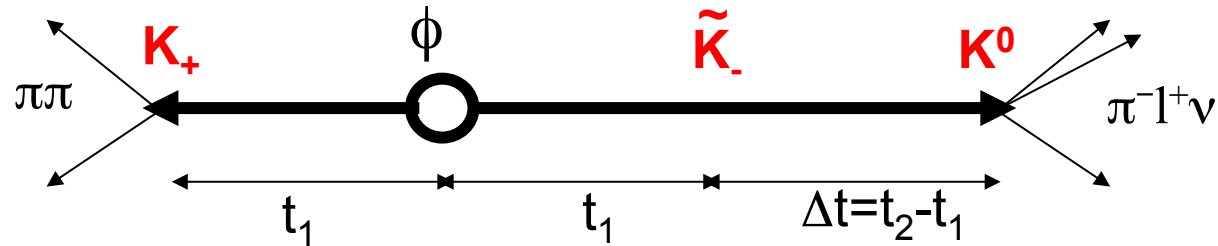
$|K_{- (+)}\rangle \equiv$ state orthogonal to $|K_{+(-)}\rangle$ which cannot decay in $\pi\pi(3\pi^0)$



The kaon states

$|K_{+(-)}\rangle \equiv$ state filtered by the decay in $\pi\pi(3\pi^0)$ (pure CP = +1(-1) state)

$|\tilde{K}_{-(+)}\rangle \equiv$ state orthogonal to $|K_{+(-)}\rangle$ which cannot decay in $\pi\pi(3\pi^0)$



state orthogonal to K_+ cannot decay in $\pi\pi$

state orthogonal to K_- cannot decay in $3\pi^0$

$$|\tilde{K}_-\rangle \equiv \tilde{N}_- [|K_L\rangle - \eta_{\pi\pi} |K_S\rangle]$$

$$|\tilde{K}_+\rangle \equiv \tilde{N}_+ [|K_S\rangle - (\eta_{3\pi^0}^{-1}) |K_L\rangle]$$

$$|K_+\rangle = N_+ [|K_S\rangle + \alpha |K_L\rangle]$$

$$|K_-\rangle = N_- [|K_L\rangle + \beta |K_S\rangle]$$

where

$$\alpha = \frac{\eta_{\pi\pi}^* - \langle K_L | K_S \rangle}{1 - \eta_{\pi\pi}^* \langle K_S | K_L \rangle},$$

where

$$\beta = \frac{(\eta_{3\pi^0}^{-1})^* - \langle K_S | K_L \rangle}{1 - (\eta_{3\pi^0}^{-1})^* \langle K_L | K_S \rangle},$$

need to assume $|K_+\rangle \equiv |\tilde{K}_+\rangle$
 $|K_-\rangle \equiv |\tilde{K}_-\rangle$

\Rightarrow

$$\eta_{\pi\pi} + (\eta_{3\pi^0}^{-1})^* \simeq \langle K_S | K_L \rangle \simeq \epsilon_L + \epsilon_S^*.$$

not valid if direct CP violation is present
assumption: direct CPV negligible