Status and prospects of the muonium experiment at J-PARC

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Energy Levels of the Hydrogen Atom

- **Bound States**
  - n=1: 1s, -13.6 eV
  - n=2: 2s, 2p
  - n=3: 3s, 3p, 3d
  - n=4: 4s, 4p, 4d
  - n=5: 5s, 5p, 5d
  - n=6: 6s, 6p, 6d

- **Continuum**

- **Dirac**
- **Lamb**
- **HFS**

- Fine Structure (Dirac eqn)
- Lamb Shift (QED effect)

- **Hyperfine Structure**
  - 24 MHz
  - 178 MHz
  - 59 MHz

- **10.97 GHz**
- **1.06 GHz**
- **1.42 GHz**
Energy diagram of Muonium (Mu) Atoms

\[ 2^2S_{1/2} \]

- \( F=0 \)
- \( F=1 \)

\[ 2^2P_{1/2} \]

- \( F=0 \)
- \( F=1 \)

\[ 2^2P_{3/2} \]

- \( F=1 \)
- \( F=2 \)

Hyperfine Structure

Fine Structure (Dirac eqn)

Lamb Shift (QED effect)

- \( \Delta \nu (\text{Fermi}) = \frac{16}{3} \alpha^2 c R_\infty \frac{m_e}{m_\mu} \left[ 1 + \frac{m_e}{m_\mu} \right]^{-3} \)

(QED weak hadronic correction not included)
HyperFine Structure (HFS) of Muonium Atoms (Mu)

\[ H = a \mathbf{I} \cdot \mathbf{J} + \mu^e_B g_J \mathbf{J} \cdot \mathbf{H} - \mu^\mu_B g^\mu \mathbf{I} \cdot \mathbf{H} \]

HFS

Zeeman Splitting

\[ \Delta \nu = \nu_{12} + \nu_{34} \]

\[ \mu_\mu / \mu_p \propto \nu_{12} - \nu_{34} \propto m_p / m_\mu \]

- Precise test of bound-state QED
- Magnetic moment mass of muon
Comparison: Muonium (Mu) and Hydrogen (H) Atoms

\[ \Delta HFS_{\text{M}}^{\text{th}} = 4.46330288(55) \text{ GHz (120ppb)} \]

\[ \Delta HFS_{\text{M}}^{\text{ex}} = 4.463302765(53) \text{ GHz (12ppb)} \]

\[ \Delta HFS_{\text{H}}^{\text{th}} = 1.4204031(8) \text{ GHz (560ppb)} \]

\[ \Delta HFS_{\text{H}}^{\text{ex}} = 1.4204057517667(9) \text{ GHz (0.6ppt)} \]


<table>
<thead>
<tr>
<th>Term</th>
<th>Fractional contribution</th>
<th>( \Delta E ) (kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_F )</td>
<td>1.0000000000</td>
<td>4459031.88(50)(3)</td>
</tr>
<tr>
<td>( \alpha_e )</td>
<td>0.001159652</td>
<td>5170.925(1)</td>
</tr>
<tr>
<td>QED2</td>
<td>−0.000195815</td>
<td>−873.145</td>
</tr>
<tr>
<td>QED3</td>
<td>−0.000005923</td>
<td>−26.411</td>
</tr>
<tr>
<td>QED4</td>
<td>−0.0000000123(49)</td>
<td>−0.548(218)</td>
</tr>
<tr>
<td>Hadronic</td>
<td>0.0000000054(1)</td>
<td>0.241(4)</td>
</tr>
<tr>
<td>Weak</td>
<td>−0.0000000015</td>
<td>−0.067</td>
</tr>
<tr>
<td>Total</td>
<td>1.000957830(49)</td>
<td>4463302.88(55)</td>
</tr>
</tbody>
</table>
Why Muonium HFS measurement is so important?

- **g-2 E821(BNL) 0.5ppm 3σ deviation**

-measurement of the deviation of muon spin direction ($\omega_s$) and muon momentum direction ($\omega_c$) 

\[ \omega_a \propto \frac{(g-2)}{2} = a_\mu \]

\[ \Rightarrow \bar{\omega}_a = \frac{e}{mc} \left[ a_\mu \bar{B} - \left( a_\mu - \frac{1}{\sqrt{2} - 1} \right) \beta \times \bar{E} \right] \]

- The ratio to proton NMR frequency is important!

\[ a_\mu \text{ an independent precise muon mass measurement is required} \]

\[ \mu_\mu / \mu_p \text{ accuracy from direct measurement 0.12ppm} \]

\[ \Delta \nu = 4.463\,302\,765(53) \text{ GHz} \quad (12 \text{ ppb}) \]

\[ \mu_\mu / \mu_p = 3.183\,345\,24(37) \quad (120 \text{ ppb}) \]

**Uncertainty for \( \mu_\mu / \mu_p \) (frequency sweep)**

- Microwave Power
- Gas Density Extrapolation
- Muon Stopping Distribution
- Field Measurement
- Statistical Error

LAMPF
(Los Alamos exp. in the 1990s)

Statistical & Systematic Uncertainties

\[ \Delta \nu = 4.463 \, 302 \, 765(53) \, \text{GHz} \, (12 \, \text{ppb}) \]

\[ \mu_\mu / \mu_p = 3.183 \, 345 \, 24(37) \, (120 \, \text{ppb}) \]

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Uncertainty for \( \mu_\mu / \mu_p \) (frequency sweep)

- Microwave Power
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- Statistical Error

- coaxial pipe
- lower gas density : longer cavity
- beam profile monitor
- higher homogeneity of the magnetic field
- higher muon intensity
J-PARC = Japan Proton Accelerator Research Complex

Need to have high-power proton beams
→ MW-class proton accelerator
(current frontier is about 0.1 MW)

Materials & Life Science from RCS
Nuclear & Particle Physics from MR
R&D toward Transmutation from LINAC
Muon beam from intense proton beam at J-PARC
Beam Intensity @ J-PARC MUSE H line $1 \times 10^8 / \text{s}$ (expected)

Pulsed beam

Total beam time ca. 100 days

Total Muon $1 \times 10^{15}$

100 times more statics than the Los Alamos experiment.
Schematic of the Experiment

μ⁺ beam → RF → Mu atoms → e⁺
$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$

$\pi^+ \rightarrow \mu^+ + \nu_\mu$

$\mu^+ + \text{Kr} \rightarrow \text{Mu} (\mu^+e^-) + \text{Kr}^+$

**Schematic of the Experiment**

- magnet
- detector
- gas chamber
- RF cavity
- beam profile monitor
- beam
- $\mu^+$ beam
- $\mu^+$
- $\pi^+$
- $\nu_\mu$
- $e^+$
- $\nu_e$
- $\bar{\nu}_\mu$
- $\text{Kr}$
- $\text{Mu}$ (Mu atoms)
Gas Chamber
1.7 T high-precision superconducting magnet

Bore diameter = $\phi 925$ mm

Requirement: 1 ppm homogeneity + absolute calibration in 300 mm x $\phi 200$ mm spheroidal region

- Field strength to be measured by water NMR probes.
  - 24 probes x 24 positions
- Field correction by shim coils and insertion of iron shims.
  - Iterative adjustment
RF Cavity & Gas Chamber

- RF resonator
- Kr gas vessel

- RF coaxial pipe
- RF antenna
- Tuning bar
- Cavity
- Gas Chamber
- Thermometer, etc.
- Gas Handling

Dimensions:
- 187 mm
- 440 mm
- 304 mm
Pressure shift of the resonance line

Large shift
- Due to atomic collision
- Linear dependence

\[ \nu(P) = \nu(0) (1 + aP + bP^2) \]

\[ a_{12} = -1.14 \times 10^{-8} \text{ Torr}^{-1} @ 0 \text{ deg. C} \]
\[ a_{34} = -1.01 \times 10^{-8} \text{ Torr}^{-1} @ 0 \text{ deg. C} \]

pressure: precise measurement
temperature: sub-K control
Pressure shift of the resonance line

Collision with reactive contaminant gases

Mu is an isotope of H.
Mu behaves chemically as an hydrogen radical (H·).

\[ \frac{\partial \Delta \nu (H)}{\partial P} \approx 16 \text{ Hz} / \text{ mbar} \]

\[ \partial \Delta \nu (0.8 \text{ atm, 300 ppm}) \approx 4 \text{ Hz} \]

\[ \text{H}_2 \text{ gas} \]

O\(_2\) gas highly reactive (unpaired electrons)

Shift of resonance lines
Quenching of Mu polarization signal vanishes

purity: < O(ppm)
gas chromatograph or Qmass detector
to-do list of designing
• gas-handling system
• measurement of gas purity
• temperature control
Our cavity is long enough to stop most of muons in it at a few atms. Extrapolation to zero density gives the intrinsic resonance frequency.

Pressure shift of the resonance line

Extrapolation to zero density gives the intrinsic resonance frequency.

160mm (LAMPF)

68%

304mm (J-PARC)

94%

Longer cavity allows reliable measurements at lower pressure.
RF Cavity & Gas Chamber
RF Cavity

two transitions

two resonance modes

\[ \nu_{12} = 1.906 \text{ GHz} \]

\[ \Delta \nu = \nu_{12} + \nu_{34} \]

\[ \nu_{34} = 2.556 \text{ GHz} \]

\[ B (T) \]

Energy (GHz)
two transitions

\[ \nu_{12} = 1.906 \text{ GHz} \]

\[ \Delta \nu = \nu_{12} + \nu_{34} \]

\[ \nu_{34} = 2.556 \text{ GHz} \]

Energy (GHz)

0 1.0 2.0
B (T)

‘magic’ magnetic field = 1.7 T

RF Cavity

two resonance modes

TM110

TM210

Input

Monitor

Monitor

Input

E

H

Input

Monitor

Monitor
frequency tuning by physically moving tuning bars.

alumina tuning bar
20 x 100 x 5 mm³

RF Cavity

RF Input
Monitor
RF simulations

RF field

frequency characteristics

Simulation by CST studio (Microwave Studio)
The RF Cavity page contains diagrams and graphs illustrating the frequency characteristics of the TM110 mode. The diagrams show the resonance frequency tuning by tuning bars and the S11 parameter with respect to frequency and displacement. The cavity manufactured is also shown in the image.
Beam Profile Monitor

Prototype Front BPM

20 scintillators each for x & y
6 mm x 184 mm

0.1 – 0.2 mm thickness !!
Beam Profile Monitor

- 0.15 mmt, w/o Al coated
- 0.15 mmt, w/ Al coated
- 0.2 mmt, w/o Al coated
- 0.2 mmt, w/ Al coated

- Beam Profile Monitor
- 2013/3/27
- Japanese Physical Society 68th Annual Congress

- Average number of photons

- J-PARC/MLF (Material and Life Science Experimental Facility)
- Pulse beam. Muon momentum is 27 MeV/c, 25 Hz, 105 μ⁺/pulse

- Test with 90Sr source
- Test with pulsed μ⁺ beam
  27 MeV/c, 25 Hz, 105 μ⁺/pulse @ J-PARC/MLF
Schematic of the Experiment

\[ \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \]

\[ \pi^+ \rightarrow \mu^+ + \nu_\mu \]

\[ \mu^+ \text{ beam} \]

beam profile monitor

gas chamber

RF cavity

Mu atoms

\[ \mu^+ + \text{Kr} \rightarrow \text{Mu} (\mu^+e^-) + \text{Kr}^+ \]

detector
Schematic of the Experiment

\[ \pi^+ \rightarrow \mu^+ + \nu_\mu \]

\[ \mu^+ \text{ beam} \]

\[ \mu^+ + \text{Kr} \rightarrow \text{Mu} (\mu^+e^-) + \text{Kr}^+ \]

\[ \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \]

max. 50 MeV
Detection of muon decay

Requirement: Suitable for high intensity pulsed beam
- \((1 \times 10^8 /s) / (25\ \text{pulses/s}) \sim 4 \times 10^6\ \text{muons/pulse}\)
- Highly segmented positron counter
  - \(2\sim 4\ \text{layers of scintillating fiber hodoscope}\)
- Expected event rate \(\sim 3\ \text{MHz/cm}^2\) (‘old muonium’ method)
Scintillation fiber + MPPC + ASIC-based ASD + FPGA MHTDC
- Prototype has been developed and beam test performed.

Simulated positron hits per stopped muon

Prototype of the detector
Evaluation of uncertainty by resonance-line simulations

Calculate transition probability.

RF field map
fluctuation of RF power effect of tuning bar

B field map
fluctuation of B homogeneity

detection efficiency map

muonium distribution map
fluctuation of beam intensity statistical uncertainty

Repeat calculation for every muonium.

Center of the resonance line determined by fitting.

plotting

fitting
Selection of long-lived muonium narrows the resonance spectra, but reduces the statistics.

Less statistics is not always bad: it suppresses the high count rate at the detector.
Effect of drift or fluctuation of the RF power

A set of simulation examples for the case where the RF power is changed by 0.1% over the frequency scanning range.

Narrower spectrum is more robust against drifts & fluctuations of e.g. RF power.

RF power 100.0% → → → → → → 100.1%
Effect of misalignment of the muon distribution

A set of simulation examples for the case where the muon stopping position is displaced.

Constant displacement does not shift the center frequency of the resonance line.

Displacement during the scan (dependent on the frequency) shifts the center frequency.
\[ \Delta \nu = 4.463\, 302\, 765(53)\, \text{GHz} \quad (12 \text{ ppb}) \]
\[ \mu_\mu/\mu_p = 3.183\, 345\, 24(37) \quad (120 \text{ ppb}) \]

LAMPF
(Exp. at Los Alamos in the 1990s)

Statistical & Systematic Uncertainties

- Microwave Power
- Gas Density Extrapolation
- Muon Stopping Distribution
- Field Measurement
- Statistical Error

- Coaxial pipe
- Lower gas density: longer cavity
- Beam profile monitor
- Higher homogeneity of the magnetic field
- Higher muon intensity

Uncertainty for \( \mu_\mu/\mu_p \) (frequency sweep)
Collaborators

Univ. of Tokyo
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KEK

RIKEN
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Osaka University
M. Aoki

University of Yamanashi
E. Torikai
Summary

• Microwave spectroscopy of Mu GS-HFS (the ground-state hyperfine structure of the muonium atom) determines $\mu_\mu$ and $m_\mu$ (magnetic moment and mass of the muon), important properties also for $(g-2)_\mu$.

• Experiment planned at J-PARC (KEK-JAEA joint facility) aims at one-digit more precision = O(ppb): O(Hz) uncertainty out of O(GHz) resonances.

• Development is on-going in various parts of the exp.: magnet, pure-gas chamber, RF cavity, muon profile monitor, positron detector etc.

• Evaluation of systematic uncertainties are under way.
Fine.

Grazie per vostro attenzione.
Gratias ago pro audientia vestra.
Спасибо за внимание.
Merci de votre attention.
Thank you for your attention.
경청해 주셔서 감사합니다.
ご清聴ありがとうございました。

鳥居 寛之
Hiroyuki A. TORII
Microwave Spectroscopy Experiment of Mu-HFS

\[ \nu_{12} = 1.897\,539\,800(35) \text{ GHz (18 ppb)} \]
\[ \nu_{34} = 2.565\,762\,965(43) \text{ GHz (17 ppb)} \]
\[ \Delta \nu = \nu_{12} + \nu_{34} \]
\[ \Delta \nu = 4.463\,302\,765(53) \text{ GHz (12 ppb)} \]
\[ \mu_{\mu}/\mu_p = 3.18\,334\,524(37) \text{ (120 ppb)} \]

\[
\frac{m_{\mu}}{m_e} = \left( \frac{g_\mu}{2} \right) \left( \frac{\mu_p}{\mu_\mu} \right) \left( \frac{\mu_B^e}{\mu_p} \right)
\]

\[ g_\mu = 2 \left( 1 + a_\mu \right), \quad a_\mu = 0.011\,659\,23(8.5) \text{ (4 ppb for } g_\mu) \]
\[ \mu_B^e/\mu_p = 1.521\,032\,202(15) \text{ (1 ppb)} \]
\[ m_{\mu}/m_e = 206.768\,277(24) \text{ (120ppb)} \]

\[ \Delta \nu_{\text{Fermi}} = \frac{16}{3} \alpha^2 c R_\infty \frac{m_e}{m_{\mu}} \left[ 1 + \frac{m_e}{m_{\mu}} \right]^{-3} \]
\[ \Delta \nu_{\mu}(\text{th}) = \Delta \nu_{\text{Fermi}} F (\alpha, m_e/m_{\mu}) \]

LAMPF
( Los Alamos exp. in the 1990s)
Phys. Rev. Lett. 82, 711

Phenomenological analysis indicates that

\begin{align*}
\Delta \nu_{\mu}\text{(theoretical)} & = \Delta \nu_{\mu}\text{(Fermi)} \times F (\alpha, m_e/m_{\mu}) \\
\Delta \nu_{\mu}\text{(LAMPF)} & = \Delta \nu_{\mu}\text{(Fermi)} \times F (\alpha, m_e/m_{\mu}) \\
\end{align*}

Muon mass to be determined

Magnetic moment to be determined

Contribution to \((g - 2)\) exp.

Contribution to the value of fine structure constant \(\alpha\)
Fine structure constant $\alpha$  
CODATA 2010  
(QED consistency)

The 2010 recommended value represents a  
halving of the confidence interval of the recommended value by about twice its uncertainty; 
the relative uncertainty increases to  
17.5, 11, 25.

The 2010 recommended value yields a  
recommendation that the smallest and largest values would differ  
by an amount that is larger than the spread of the measured values of 
8.

Taking into account the historic difficulty in measuring 
the fine structure constant, the uncertainties quoted in the table  
are between  
4.0, 3.0, 3.2.

The largest normalized residual, that of JILA-10, is now  
reduced to  
4.0, 4.4, 3.0.

The 2010 recommended value is not 
the recommended value by about twice its uncertainty; 
the relative uncertainty increases to  
17.5, 11, 25.

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reduced to  
4.0, 4.4, 3.0.
Microwave Spectroscopy Experiment of Mu-HFS

Fine structure constant $\alpha$

given by CODATA ($a_e = (g - 2)/2$ of $e^-$)

$$\Delta \nu(\text{exp}) = 4.463\ 302\ 765(53)\ \text{GHz (12 ppb)}$$

Comparison gives a more precise value of the mass ratio $m_\mu/m_e$.

One of the most precise tests of Bound-state QED by comparing theory & exp.

$$\Delta \nu_{\text{Mu}}(\text{th}) = \Delta \nu(\text{Fermi}) F(\alpha, m_e/m_\mu)$$

$$\Delta \nu(\text{Fermi}) = \frac{16}{3} \alpha^2 c R_\infty \frac{m_e}{m_\mu} \left[ 1 + \frac{m_e}{m_\mu} \right]^{-3}.$$ 

$m_\mu/m_e = 206.768\ 2670(55)\ (27\ \text{ppb})$

$\mu_\mu/\mu_p = 3.18\ 334\ 5396(94)\ (30\ \text{ppb})$

cf. Ps HFS gives a precision of a few ppm.
Pressure shift of the resonance line of Mu HFS

\[ \nu(P) = \nu(0) \left( 1 + aP + bP^2 \right) \]

- \( a_{12} = -1.14 \times 10^{-8} \text{ Torr}^{-1} \) @ 0 deg. C
- \( a_{34} = -1.01 \times 10^{-8} \text{ Torr}^{-1} \) @ 0 deg. C

- \( b = (9.7 \pm 2.0) \times 10^{-15} \text{ Torr}^{-2} \)

cited in Liu’s thesis

- \( \frac{da}{dT} = 1 \times 10^{-11} \text{ deg.C}^{-1} \text{ Torr}^{-1} \)


\( T^{3/10} \) dependence?

Figure 4.6: Pressure extrapolation of the old muonium data. Plot a shows the extrapolation for \( \nu_{12} \), and Plot b shows that for \( \nu_{34} \). The transition frequencies in vacuum and the pressure shift coefficients are also shown in the figures.

from Liu’s thesis
Collisional phase shift

Phase shift due to collision $\Rightarrow$ shift and width of resonance

$$\eta_j(b) = \int_{-\infty}^{\infty} V_j(R(t)) \, dt$$

$$\Delta - i \gamma = -i N v \int_{b_1}^{\infty} (1 - e^{i \eta(b)}) 2 \pi b db \equiv -i N v (\sigma_\Delta - i \sigma_\gamma)$$

Shift is linear with the buffer-gas density.

\[ V_j(R) = -\frac{C_6}{R^6} = -\frac{\alpha \langle j | \mu^2 | j \rangle}{R^6} \]

van der Waals attraction

impact approx. (sudden collision)

binary collision

random collision

ab initio calculation by Bakalov for antiprotonic helium atoms
Collisional shift of Sr atoms with rare gases

\[ ^1S_0 \rightarrow ^3P_1 \]

\[ f_0 = 434 \, 829 \, 121 \, 312 \, 334 \, \text{Hz}, \]

N. Shiga, et al., PRA 80, 030501 (2009)

optical lattice clock

Slide from T. Ido, NICT, Tokyo
Positronium Hyperfine Splitting (Ps-HFS)

Energy difference between two spin eigenstates of the ground state Ps

$\rightarrow$ Ps-HFS (203 GHz)

\[ 2^1S_0 \rightarrow 1^3S_1 \text{ (o-Ps)} \]
\[ 2^1P_1 \rightarrow 2^3P \]
\[ 1^3S_1 \rightarrow 1^3P \]

$0.84 \text{ meV}$

Slide from A. Ishida, ICEPP, Univ. Tokyo
Muon beam and Magnet

- **H-Line**: The highest intensity pulsed muon beam at J-PARC

  ![Simulated muon beam by G4Beamline](image)

  Simulation Result:
  - Profile at final focus
    - $\sigma_x = 13$ mm, $\sigma_y = 13$ mm
    - $\theta_x = 161.5$ mrad, $\theta_y = 137.4$ mrad
    - 93.6% transmission efficiency
    - Leakage field 0.5 Gauss
  - (Requirement < 1.7 Gauss)


- **Magnet**: 1.7 T high precision superconducting magnet

  ![Magnet at J-PARC](image)

  Requirement to the magnet:
  - 1ppm homogeneity in z300 mm, r100 mm region

  Specification of the magnet:
  - Field strength 1.7 T
  - Bore diameter 925 mm

  Field correction is performed by main coil, iron shim, and shim coil

  Field strength is monitored by NMR probes

Beam intensity: $1 \times 10^8$ muons / s
\[ \rightarrow (1 \times 10^8 /s) / (25 \text{ pulses/s}) \sim 4 \times 10^6 \text{ muons / pulse} \]

Acceptance of the detector ($z = 700$ mm)
$5 \times 10^{-5}$ $e^+ / \text{cm}^2 / \text{muon}$ (@ 700 mm)
\[ \rightarrow 200 e^+ / \text{cm}^2 / \text{pulse} (90 \text{ MHz} / \text{cm}^2) \]
\[ \rightarrow 3 e^+ / \mu\text{s} / \text{cm}^2 (3 \text{ MHz} / \text{cm}^2) \text{ for “Old Muoniums”} \]

900 segments / layer (1 segment = 1 cm$^2$)