Exotic Quarkonium Spectroscopy

X(3872), Z(10610), and Z(10650) in Non-Relativistic Effective Theory

Thomas Mehen
Duke University
Durham, North Carolina, USA

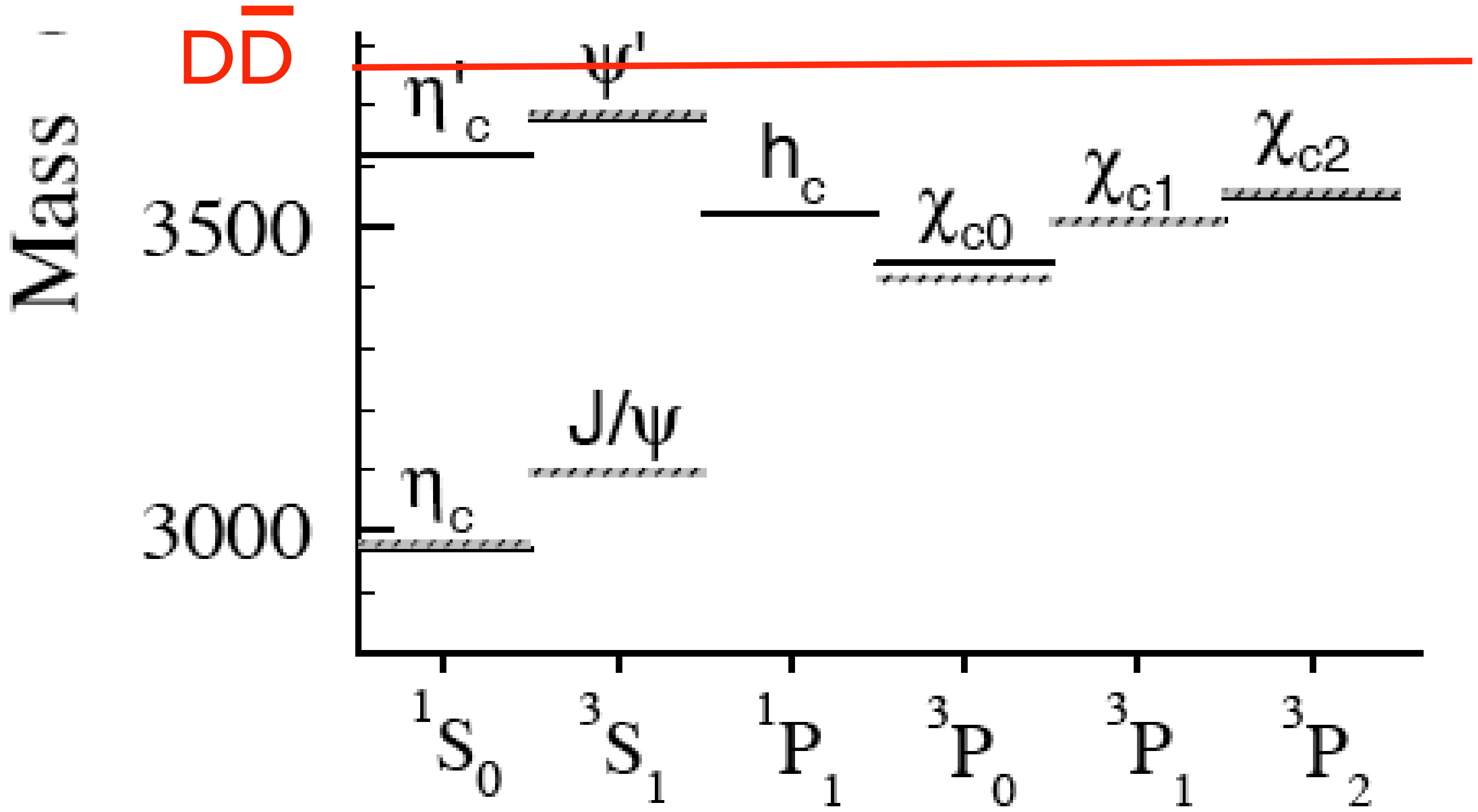
PhiPsi13, U. Of Rome, Italy, Sep. 11, 2013

- Below $D\bar{D}$ threshold:

complete HQS multiplets

$$(\eta_c, J/\psi) \quad (\eta_c', \psi')$$

$$(h_c, \chi_{cJ})$$



$c\bar{c}$ meson masses & (most) transitions described by potential model

- Above $D\bar{D}$ threshold:

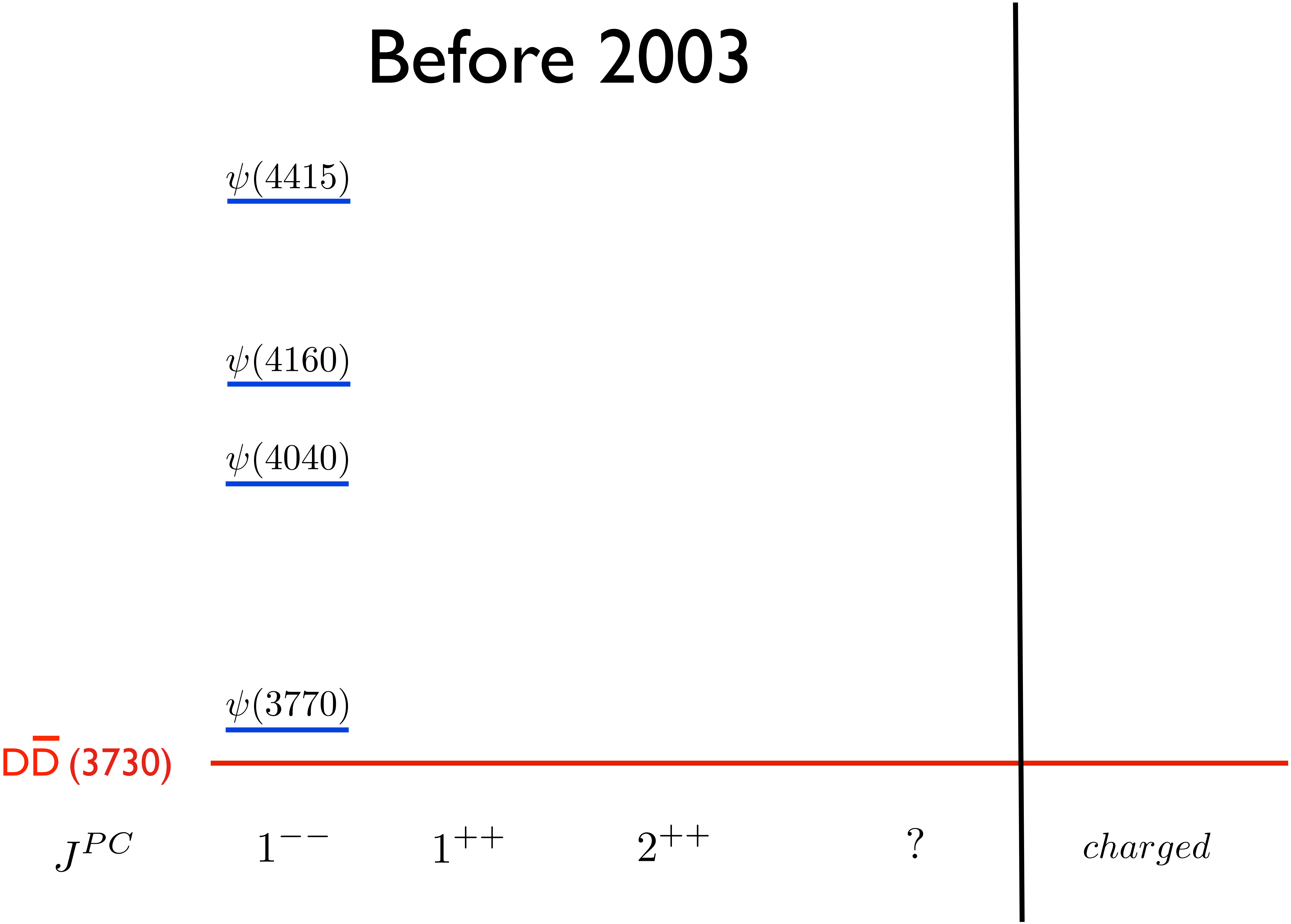
X(3872): bound state of $D^0\bar{D}^{*0} + c.c.$

new 1^{--} states: Y(4008), Y(4260),Y(4360),Y(4660)

charged states! $Z_c^{\pm}(3900) \to J/\psi\pi^{\pm}$ (2013) (BESIII, Belle,CLEO-c)

others whose J^{PC}, nature unclear

Before 2003



$\psi(4415)$

$\psi(4160)$

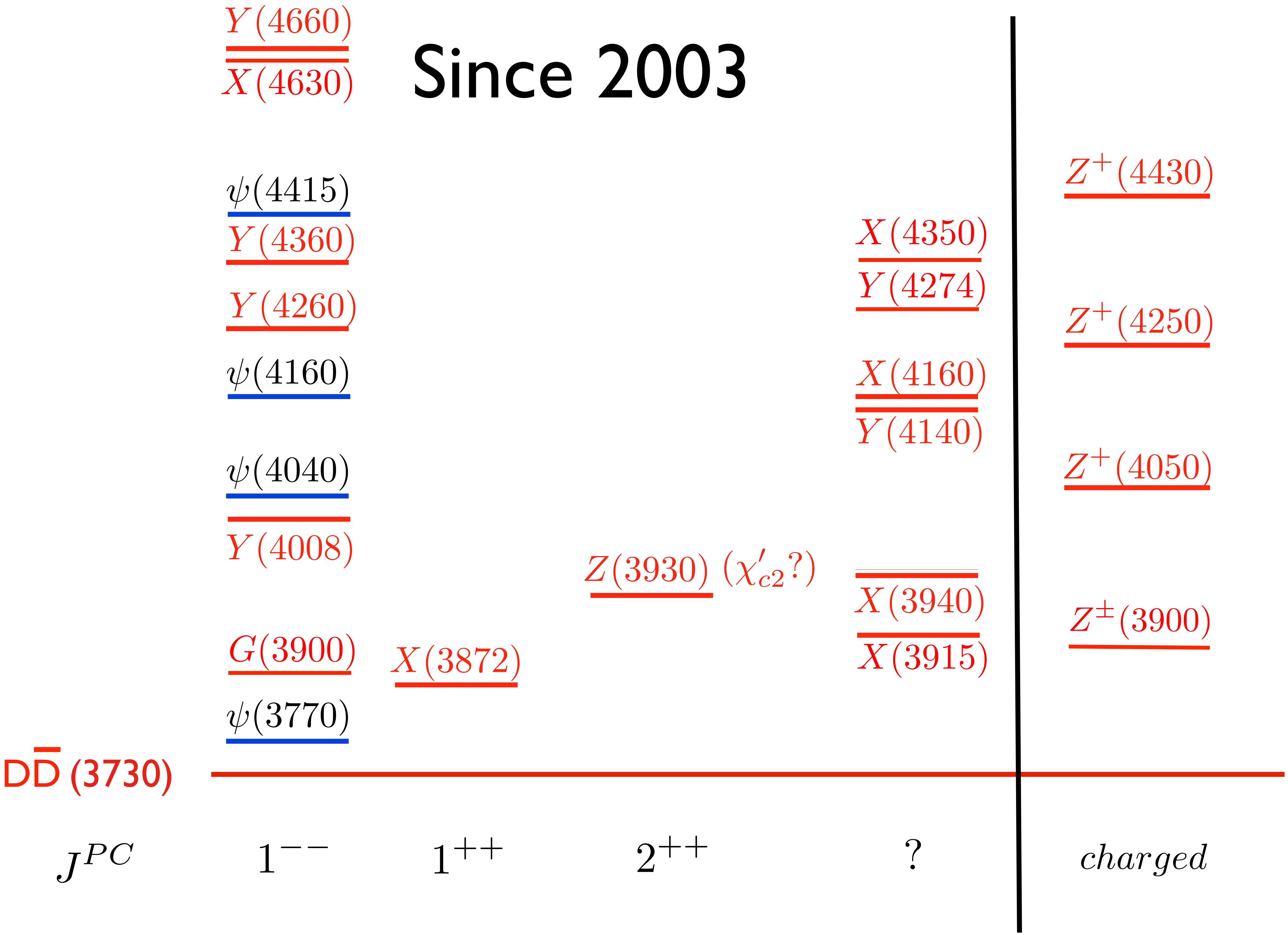
$\psi(4040)$

$\psi(3770)$

$D\bar{D}$ (3730)

J^{PC} 1^{--} 1^{++} 2^{++} $?$ $charged$

Since 2003



$Y(4660)$

$X(4630)$

$\psi(4415)$

$Y(4360)$

$Y(4260)$

$\psi(4160)$

$\psi(4040)$

$Y(4008)$

$G(3900)$

$\psi(3770)$

$X(3872)$

$Z(3930)$ $(\chi'_{c2}?)$

$X(4350)$

$Y(4274)$

$X(4160)$

$Y(4140)$

$X(3940)$

$X(3915)$

$Z^{+}(4430)$

$Z^{+}(4250)$

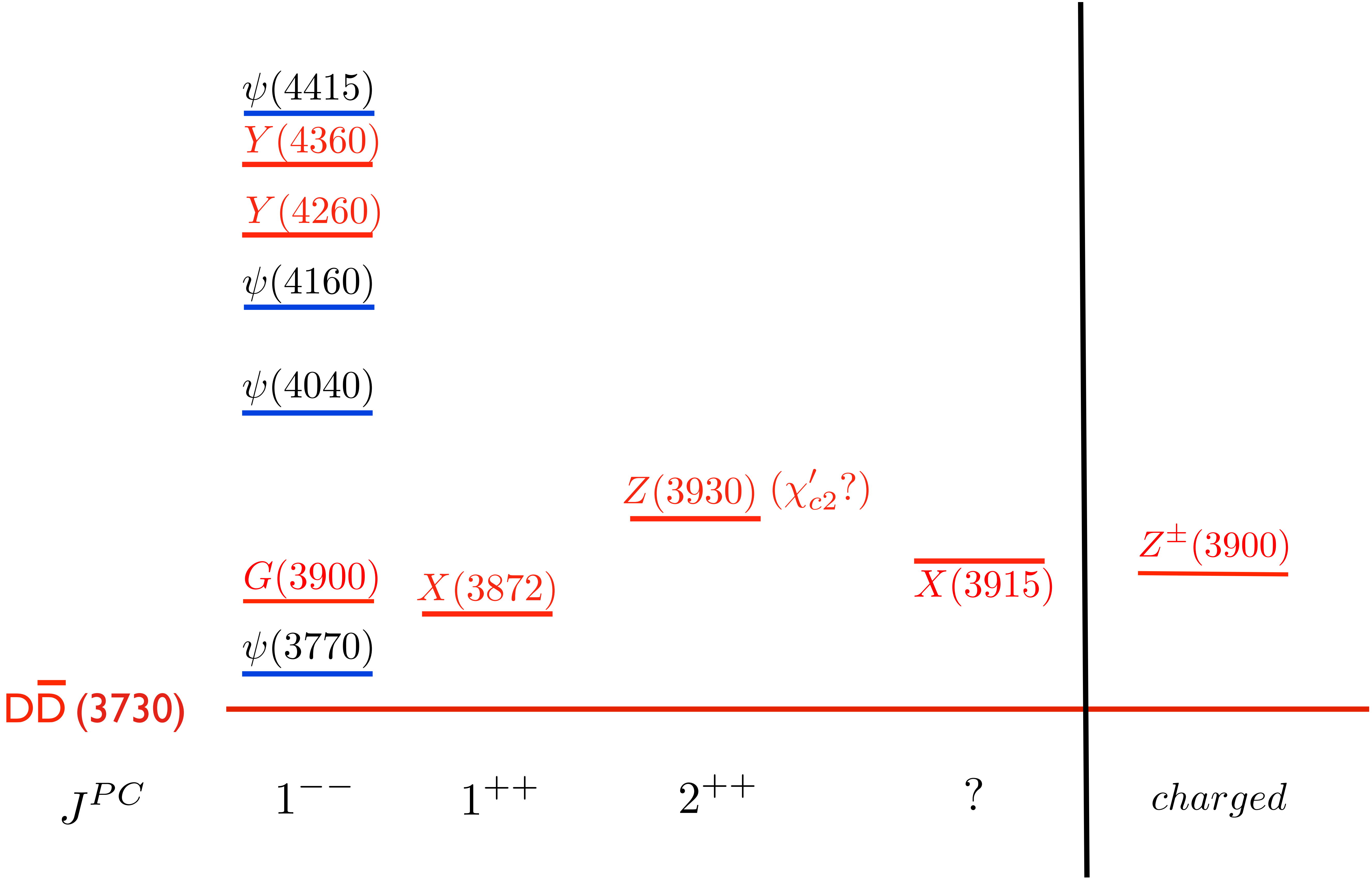
$Z^{+}(4050)$

$Z^{\pm}(3900)$

D$\overline{\text{D}}$ (3730)

J^{PC} 1^{--} 1^{++} 2^{++} $?$ *charged*

Confirmed States (>1 Expt.)



$\psi(4415)$

$Y(4360)$

$Y(4260)$

$\psi(4160)$

$\psi(4040)$

$Z(3930)$ $(\chi'_{c2}?)$

$Z^{\pm}(3900)$

$G(3900)$

$X(3872)$

$X(3915)$

$\psi(3770)$

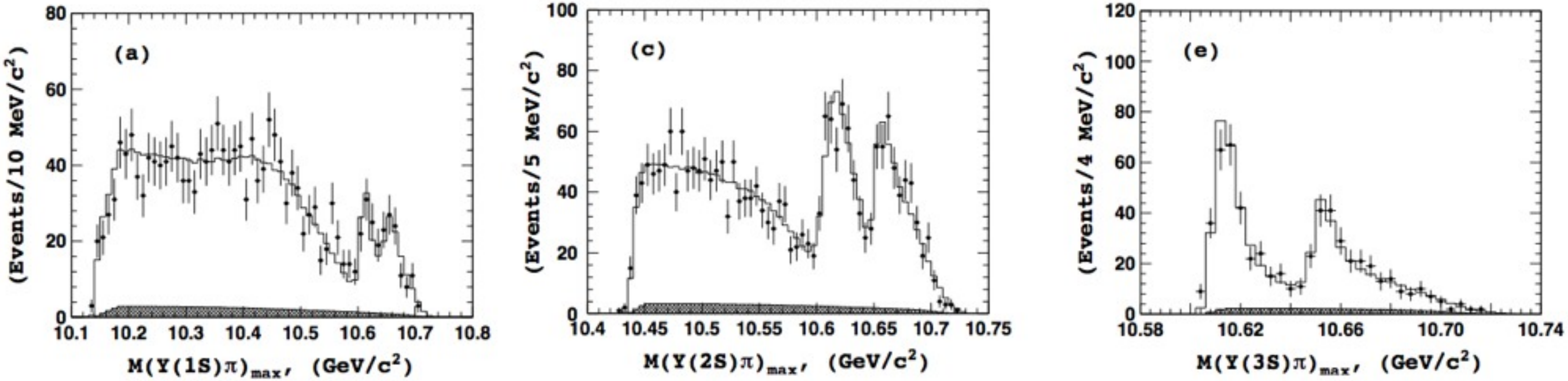
D$\overline{\text{D}}$ (3730)

J^{PC} 1^{--} 1^{++} 2^{++} $?$ $charged$

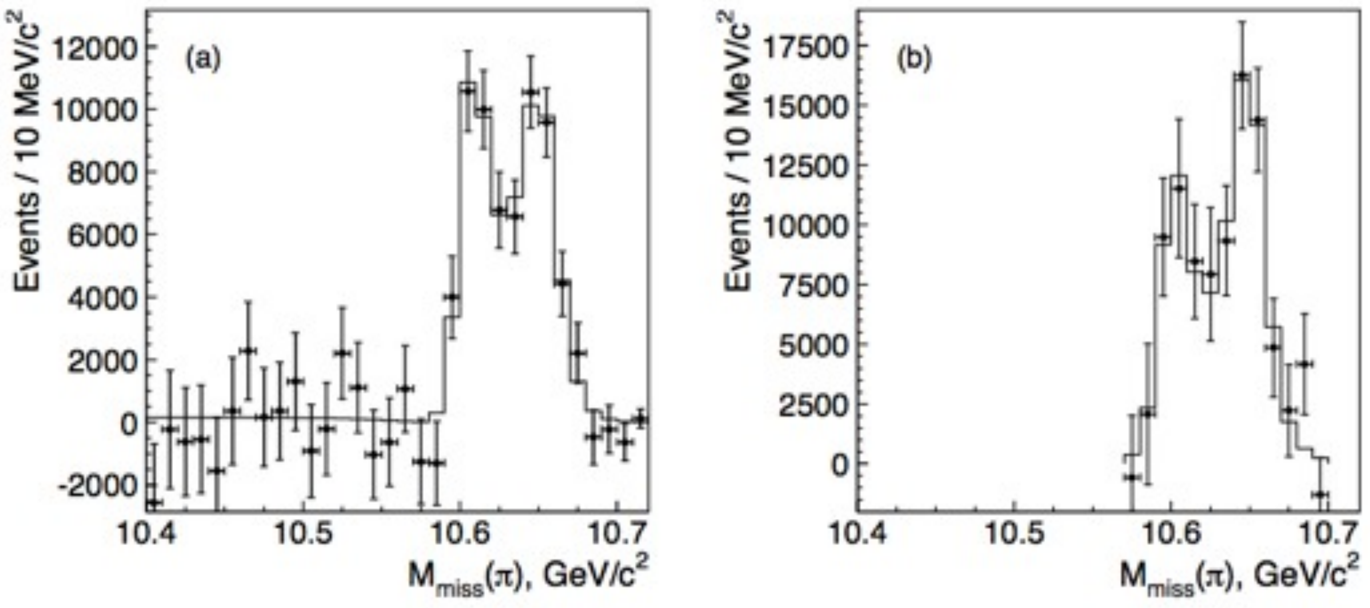
New Bottomonium Resonances

- Z(10610) and Z(10650) : resonant structures in

$$\Upsilon(5S) \to \Upsilon(nS)\pi^+\pi^- \ (n = 1, 2, \text{ or } 3)$$



$$\Upsilon(5S) \to h_b(mP)\pi^+\pi^- \ (m = 1 \text{ or } 2)$$



Belle, PRL 108 (2012) 122001

$\Upsilon(5S) \to Z_b\pi \to \Upsilon(nS)\pi^+\pi^-$ charged, quark content: $b\bar{b}u\bar{d}$, $b\bar{b}\bar{u}d$

- quantum numbers: $I^G(J^P) = 1^+(1^+)$
- $B\bar{B}^*$ threshold: 10604 MeV $B^*\bar{B}^*$ threshold: 10658 MeV
- large widths ~ 15 MeV (unlike X(3872))

Molecular hypothesis

A.E. Bondar, et.al., PRD 84: 054010 (2011)

$$|Z_b'\rangle \sim |B^*\bar{B}^*\rangle$$

$$|Z_b\rangle \sim |B^*\bar{B} - B\bar{B}^*\rangle$$

$$\begin{aligned} & i\,\epsilon_{ijk}\,(\bar{\chi}_{\bar{b}}\sigma^j\psi_q)(\bar{\psi}_{\bar{Q}}\sigma^k\chi_b) \\ & = (\bar{\chi}_{\bar{b}}\chi_b)(\bar{\psi}_{\bar{Q}}\sigma^i\psi_q) - (\bar{\chi}_{\bar{b}}\sigma^i\chi_b)(\bar{\psi}_{\bar{Q}}\psi_q) \\ & \sim 0^-_{\bar{b}b} \otimes 1^-_{\bar{Q}q} - 1^-_{\bar{b}b} \otimes 0^-_{\bar{Q}q}\,, \end{aligned}$$

$$\begin{aligned} & (\bar{\chi}_{\bar{b}}\sigma^i\psi_q)(\bar{\psi}_{\bar{Q}}\chi_b) + (\bar{\chi}_{\bar{b}}\psi_q)(\bar{\psi}_{\bar{Q}}\sigma^i\chi_b) \\ & = -(\bar{\chi}_{\bar{b}}\chi_b)(\bar{\psi}_{\bar{Q}}\sigma^i\psi_q) - (\bar{\chi}_{\bar{b}}\sigma^i\chi_b)(\bar{\psi}_{\bar{Q}}\psi_q) \\ & \sim 0^-_{\bar{b}b} \otimes 1^-_{\bar{Q}q} + 1^-_{\bar{b}b} \otimes 0^-_{\bar{Q}q}\,, \end{aligned}$$

$$|Z_b'\rangle = \frac{1}{\sqrt{2}}\left(0^-_{\bar{b}b} \otimes 1^-_{\bar{Q}q} - 1^-_{\bar{b}b} \otimes 0^-_{\bar{Q}q}\right)$$

$$|Z_b\rangle = \frac{1}{\sqrt{2}}\left(0^-_{\bar{b}b} \otimes 1^-_{\bar{Q}q} + 1^-_{\bar{b}b} \otimes 0^-_{\bar{Q}q}\right)$$

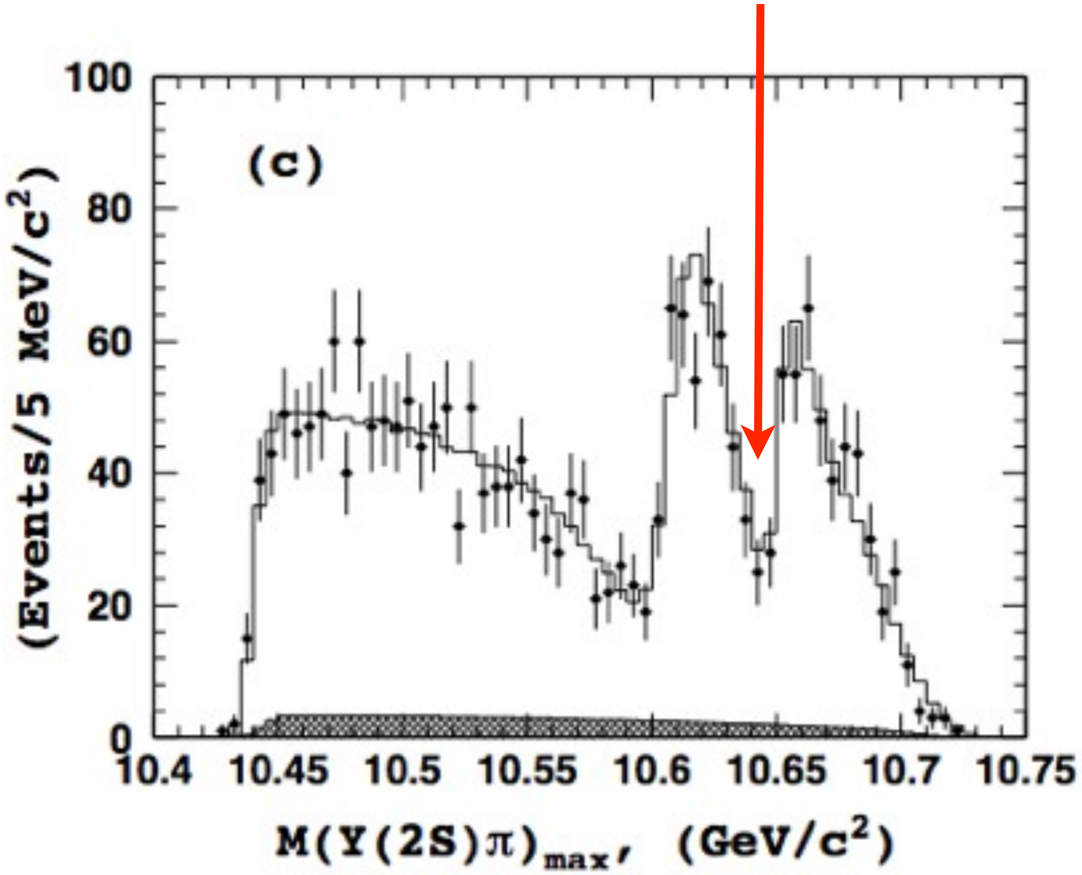
If Z_b, Z_b' are equal (orthogonal) mixtures of $S_{b\bar{b}} = 0, 1$ then

- $\Gamma[Z_b^{(\prime)} \to \Upsilon\pi]$ and $\Gamma[Z_b^{(\prime)} \to h_b\pi]$ can have similar rates

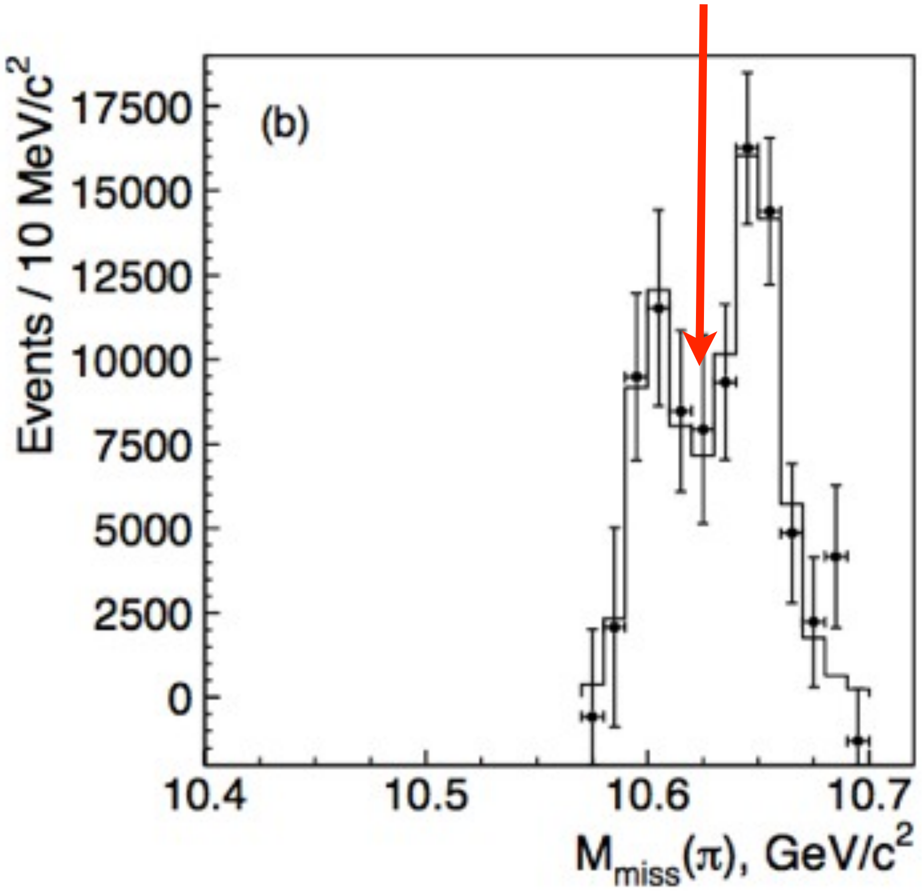
  otherwise one must be suppressed by $O(\Lambda_{\rm QCD}/m_b)^2 \sim 10^{-2}$

- Interference effects in $\Upsilon(5S) \to \Upsilon(nS)\pi\pi, h_b(mP)\pi\pi$

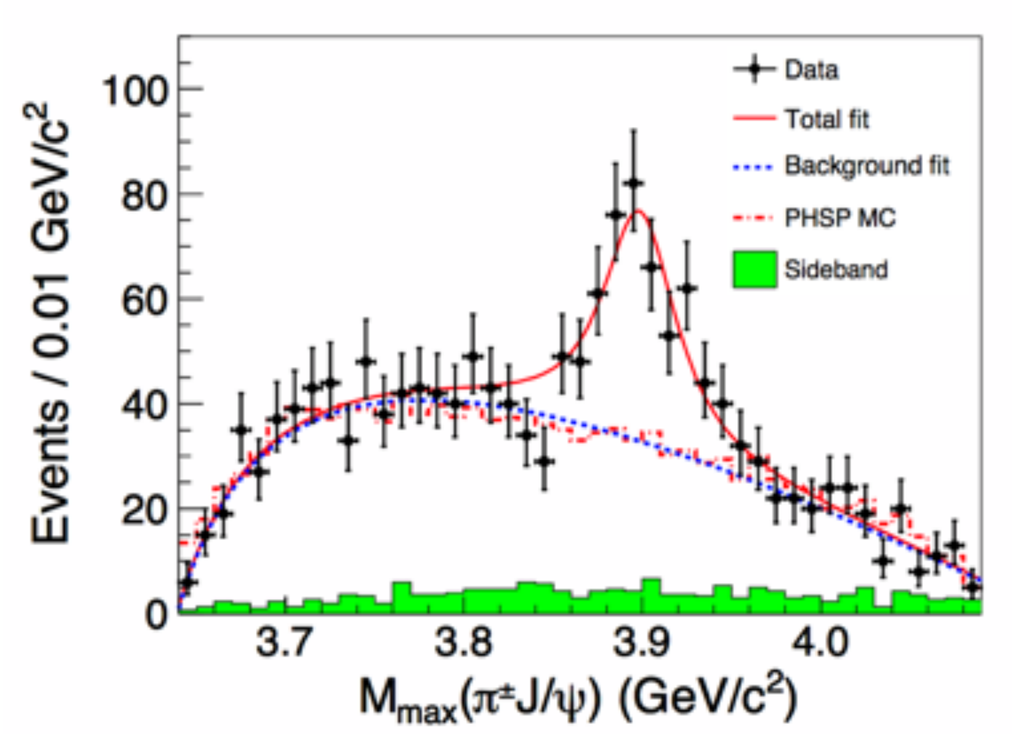
destructive interference



constructive interference

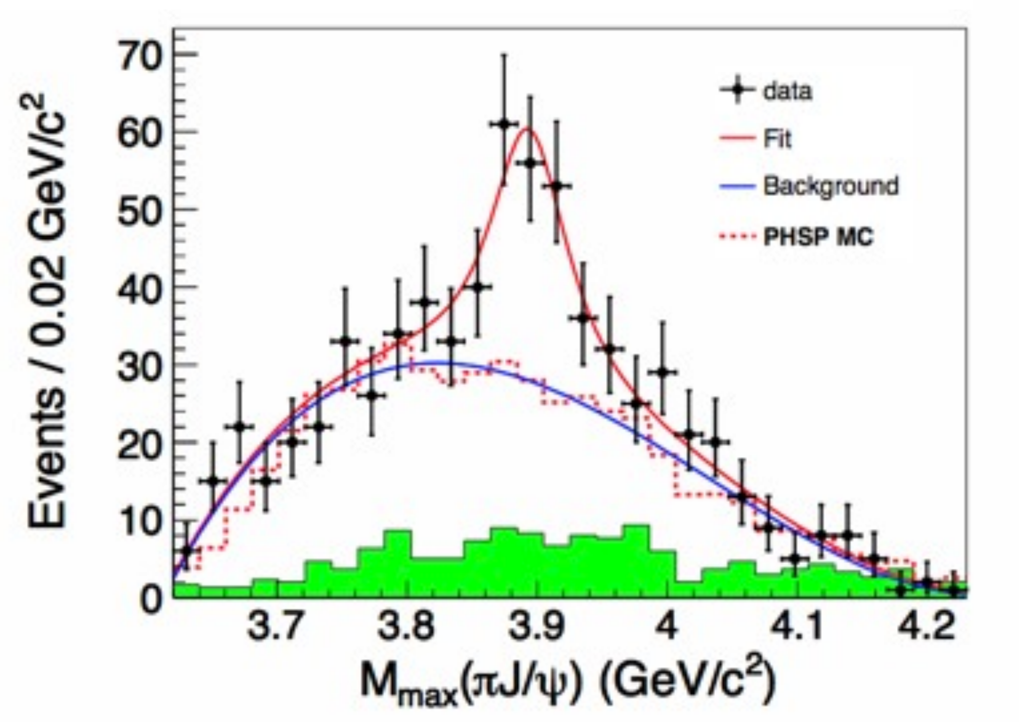


- Discovery of Zc(3900) - $e^+e^- \rightarrow Y(4260) \rightarrow J/\psi\pi^+\pi^-$



$$M_{Z_C^+} = 3899 \pm 3.6 \pm 4.9\,\mathrm{MeV}$$
$$\Gamma_{Z_c^+} = 46 \pm 10 \pm 20\,\mathrm{MeV}$$

M. Ablikim et. al. (BESIII), PRL 110,252001 (2013)



$$M_{Z_C^+} = 3894.5 \pm 6.6 \pm 4.5\,\mathrm{MeV}$$
$$\Gamma_{Z_c^+} = 63 \pm 24 \pm 26\,\mathrm{MeV}$$

Z.Q. Liu et. al. (BELLE), PRL 110,252002 (2013)

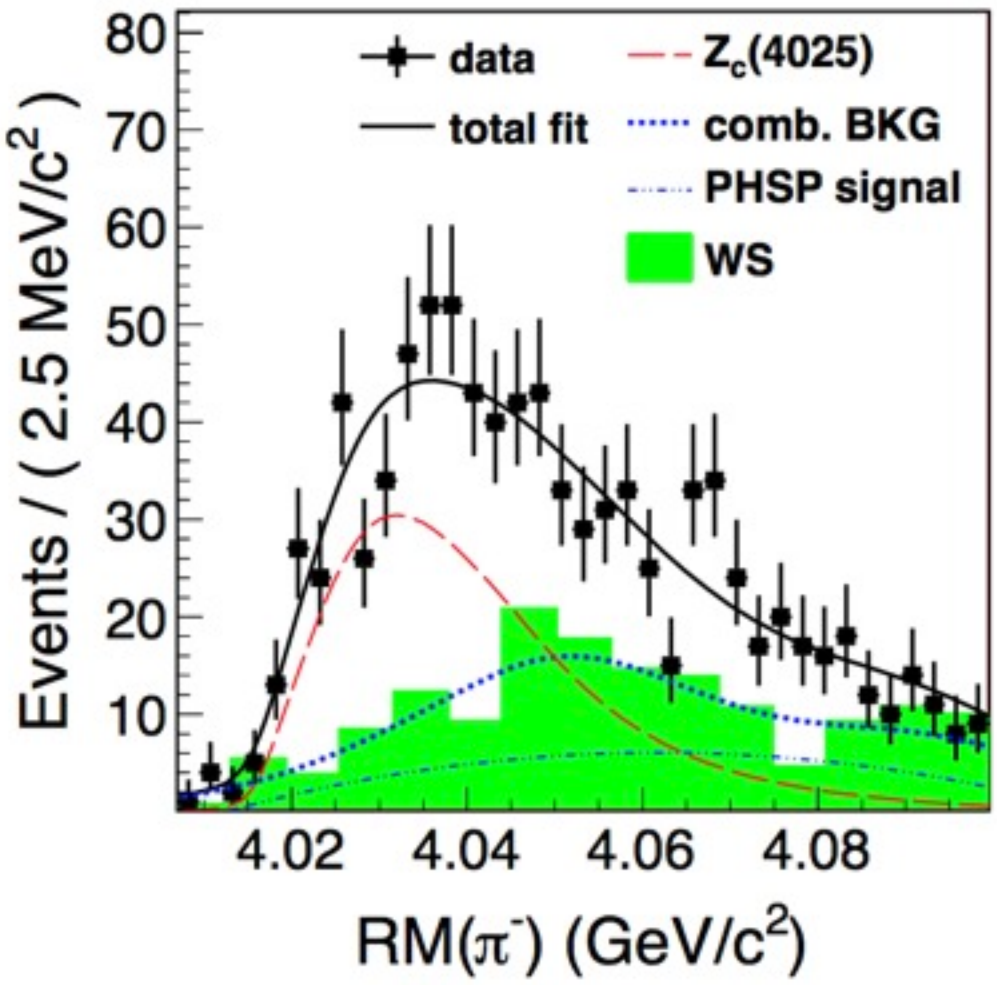
- also confirmed by CLEO-c

T. Xiao, et. al. (BELLE), arXiv:1303.6608

- unconfirmed resonances $Z_c^+(4430) \rightarrow \psi(2s)\pi^+, \quad Z_c^+(4050), Z_c^+(4250) \rightarrow \chi_{c1}\pi^+,$

(Belle 2007-2008)

- Z$_c$(4025) in $e^+e^- \rightarrow (D^*\bar{D}^*)^{\pm}\pi^{\mp}$ $\sqrt{s} = 4.26\,\mathrm{GeV}$



$$m(Z_c^+(4025)) = (4026.3 \pm 2.6)\,\mathrm{MeV}/c^2$$
$$\Gamma(Z_c^+(4025)) = (24.8 \pm 5.6)\,\mathrm{MeV}.$$

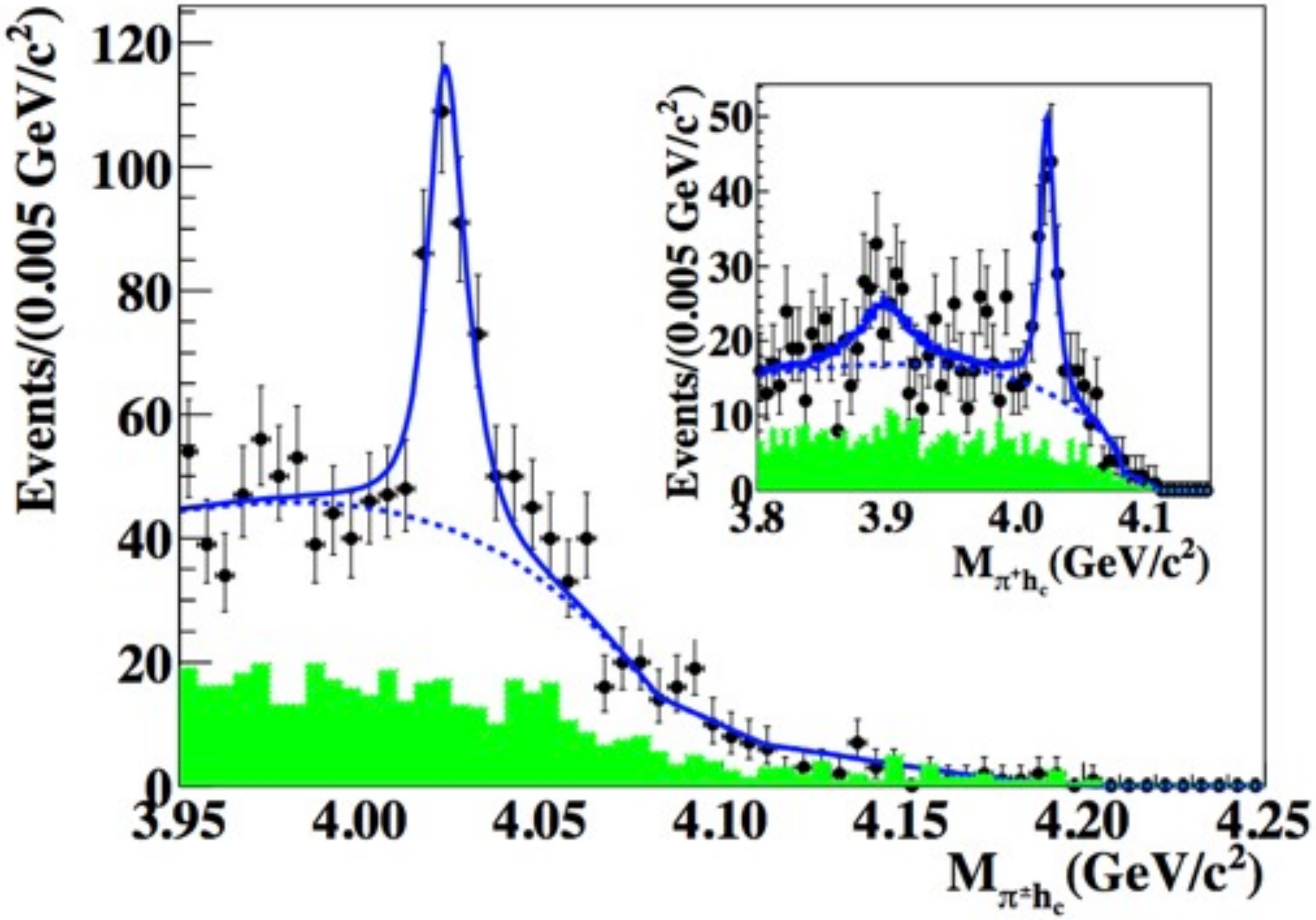
M. Ablikim et. al. (BESIII), arXiv:1308.2760

- manifestly exotic states: $Z_c^+(3900), Z_c^+(4025) \sim (c\bar{c}u\bar{d})$
- only $Z_c^+(4025)$ within a few MeV of open charm thresholds

$$m_{(DD^*)^+} = 3876\,\mathrm{MeV} \qquad m_{(D^*D^*)^+} = 4021\,\mathrm{MeV}$$

- search for Z_c states in $e^+e^- \rightarrow h_c\pi^+\pi^-$ see $Z_c^+(4020) \rightarrow h_c\pi^+$

no statistically significant evidence for $Z_c^+(3900)$



$$m_Z = 4022.9 \pm 0.8 \pm 0.27\,\mathrm{MeV}$$

$$\Gamma_Z = 7.9 \pm 2.7 \pm 2.6\,\mathrm{MeV}$$

$$J^P = 1^-$$

M. Ablikim et. al. (BESIII), arXiv:1309.1896

$Z_c(4020) = Z_c(4025)$? similar mass, very different width...

Theoretical Interpretations of $Z_c(3900)$

- molecular S-wave $D\bar{D}^*$ state charm(-ing) cousin of $Z_b(10610)$

  24 MeV above threshold? Predicts $J^P=1^+$

  Q. Wang, C. Hanhart, Q. Zhao, arXiv:1303.6355

- tetraquarks

diquark-diquark	$(Qq)_{\bar{3}}(\overline{Q}\overline{q})_3$	$J^P=1^+$	L. Maiani, et. al. PRD87 (2013) 111102
hadro-charmonium	$(\overline{Q}Q)_1(\overline{q}q)_1$	$J^P=1^+$	M.B.Voloshin, PRD87 (2013) 091501
Born-Oppenheimer	$(\overline{Q}Q)_8(\overline{q}q)_8$	$J^P=1^-$	E. Braaten, arXiv:1305.6905

- Exptal. observations that could elucidate structure

  - measuring J^P
  - relative rates to para-(η_c, h_c) and ortho-charmonium (ψ, χ_c)
  - rates to $D\bar{D}^*$
  - partner states

X(3872), Z(10610) and Z(10650) in Non-Relativistic Effective Theory

S.Fleming, M. Kusunoki, T.M., U. van Kolck, PRD 76:034006 (2007)
S.Fleming, T.M., PRD 78:094019 (2008)
D. Canham, H.-W. Hammer, R.P. Springer, PRD80:014009 (2009)
H.-W. Hammer, T.M., E. Braaten, PRD 82:034018 (2010)
T.M., R. Springer, PRD 83:094001 (2011)
T.M., J. Powell, PRD 84:114013 (2011)
T.M., S. Fleming, PRD 85:014002 (2012)
T.M., J. Powell, PRD88:034017 (2013)
A. Margaryan, R.P. Springer, PRD88:014017 (2013)

- X(3872)

  Case for Molecular State $D^0\bar{D}^{0*} + D^{*0}\bar{D}^0$

- XEFT: Effective theory for X(3872) Production/Decay

  KSW-like theory of DD^* bound states

  Universal Predictions (LO)

  $X(3872) \to D^0\bar{D}^0\pi^0$

  $D^{+0}\bar{D}^{*0} \to X(3872)\pi^+$

  $D^{(*)}X(3872) \to D^{(*)}X(3872)$

  Range, Pion Corrections (NLO)

  $X(3872) \to D^0\bar{D}^0\pi^0$

  Factorization Thms. for Decay to $Q\bar{Q}$

  $X(3872) \to \chi_{cJ}\pi(\pi)$

  $X(3872) \to \psi(2S)\gamma$

  $\psi(4040) \to X(3872)\gamma$

  $\psi(4160) \to X(3872)\gamma$

- New Bottomonium Resonances, Z_b(10160) and Z_b(10650)

  Heavy Quark Symmetry predictions for binding energies, widths,
  lineshapes in $\Upsilon(5S) \to B^{(*)}\bar{B}^{(*)}\pi$

X(3872)

- shallow bound state of $D^0\bar{D}^{0*}+\bar{D}^0 D^{0*}$
- Decays: $X(3872)\rightarrow J/\psi\pi^+\pi^-$ $\quad X(3872)\rightarrow J/\psi\pi^+\pi^-\pi^0$

$\rightarrow D^0\bar{D}^0\pi^0$ $\quad \rightarrow J/\psi\gamma$ (C=1)

$\Gamma_X < 1.2\,\mathrm{MeV}$ $\quad \rightarrow D^0\bar{D}^0\gamma$ $\quad \rightarrow \psi(2S)\gamma$

- angular distributions in $J/\psi\pi^+\pi^-$ require $\boxed{J^{PC}=1^{++}}$

LHCb, PRL 110 (2013) 222001

S-wave coupling to $D\bar{D}^*+\bar{D}D^*$

- $$\frac{Br[X(3872)\rightarrow J/\psi\pi^+\pi^-\pi^0]}{Br[X(3872)\rightarrow J/\psi\pi^+\pi^-]}=0.8\pm 0.3$$

X(3872) is mixed state
w/ I=0 and I=1

- extremely close to threshold:

$$\boxed{M_X - (M_{D^{*0}} + M_{D^0}) = -0.16 \pm 0.26\,\mathrm{MeV}}$$

$$m_X = 3871.68 \pm 0.17\,\mathrm{MeV}$$ (from PDG)

$$m_{D^0} = 1864.86 \pm 0.13\,\mathrm{MeV}$$

$$m_{D^{*0}} = 2006.98 \pm 0.15\,\mathrm{MeV}$$

unique among proposed molecules:

$$Z^+(4430) : (D_1^0 D^{*+}) \quad E_B = -0.4 \pm 5.4\,\mathrm{MeV}$$

$$Y(4660) : (\psi' f_0) \quad E_B = 2 \pm 25\,\mathrm{MeV}$$

- Universality: $\psi_{DD^*}(r) \propto \dfrac{e^{-r/a}}{r}$ $\quad a = 11.2^{+\infty}_{-4.8}\,\mathrm{fm} \quad B.E. = \dfrac{1}{2\mu_{DD^*}a^2}$

Long distance physics of X(3872) calculable in terms of scattering length, known properties of D mesons - Effective Range Theory (ERT)

(M. B.Voloshin, E. Braaten, et. al.)

- Attempts to extract resonance parameters from line shapes in $X(3872) \to J/\psi\,\pi^+\pi^-$ and $D^0\bar{D}^0\pi^0$ yield similar scattering lengths

E. Braaten & J. Stapleton, PRD 81:014019 (2010)

C. Hanhart, et. al., PRD 76:034007 (2007)

Y.S. Kalashnikova & A.V. Nefediev, PRD 80:074004 (2009)

XEFT

S.Fleming, M.Kusunoki, T.M., U.van Kolck, PRD76:034006 (2007)

- Non-Relativistic Propagators

D D^* π $\sim \frac{1}{Q^2}$

- Contact interactions, Pion Exchange

$C_0 \sim Q^{-1}$ $C_2\, p^2 \sim Q^0$ $B_1\, \epsilon \cdot p_\pi \sim Q^{-1}$ $\sim Q^0$

- Power Counting

$p_D \sim p_\pi \sim \mu \sim \gamma \sim Q \qquad \gamma \equiv \sqrt{-2\mu_{DD^*}\mathrm{B.E.}} \leq 34\,\mathrm{MeV}$

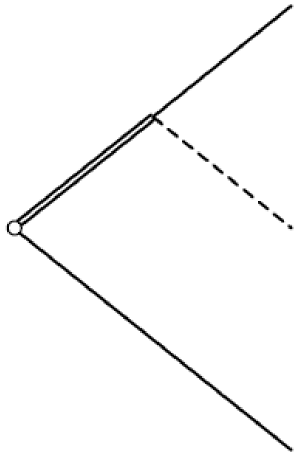
- $m_\pi \approx \Delta_H \approx 140\,\mathrm{MeV}$ are large scales in X-EFT

similar to KSW theory of NN force

D. Kaplan, M. Savage, M. Wise, PLB 424:390 (1998), NPB 534:329 (1998)

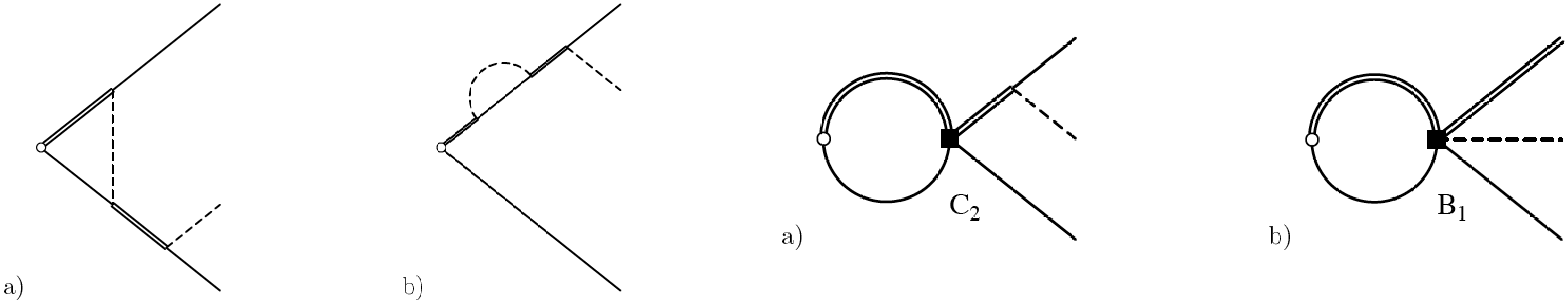
LO - reproduce ERT prediction for $X(3872) \to D^0 \bar{D}^0 \pi^0$

M.B.Voloshin, PLB 579: 316 (2004)

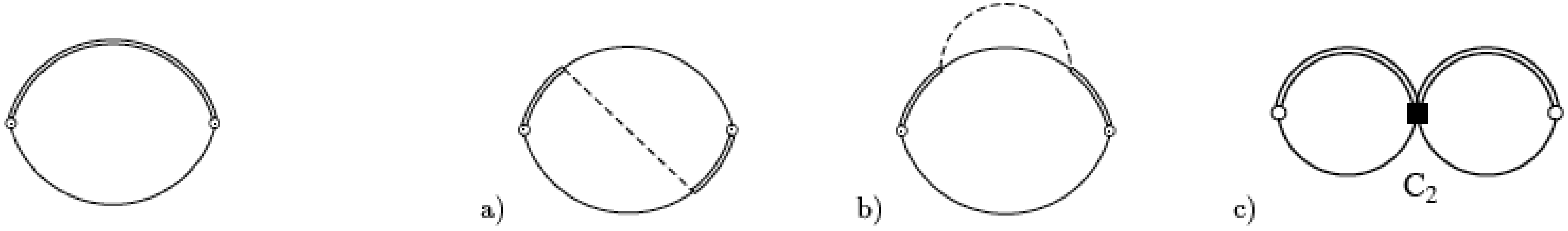


$$\frac{d\Gamma_{\rm LO}}{dp_D^2\, dp_{\bar D}^2} = \frac{g^2}{32\pi^3 f_\pi^2} 2\pi\gamma(\vec{p}_\pi\cdot\vec{\epsilon}_X)^2\left[\frac{1}{p_D^2+\gamma^2}+\frac{1}{p_{\bar D}^2+\gamma^2}\right]^2$$

NLO - range corrections, non-analytic corr. from π^0 exchange



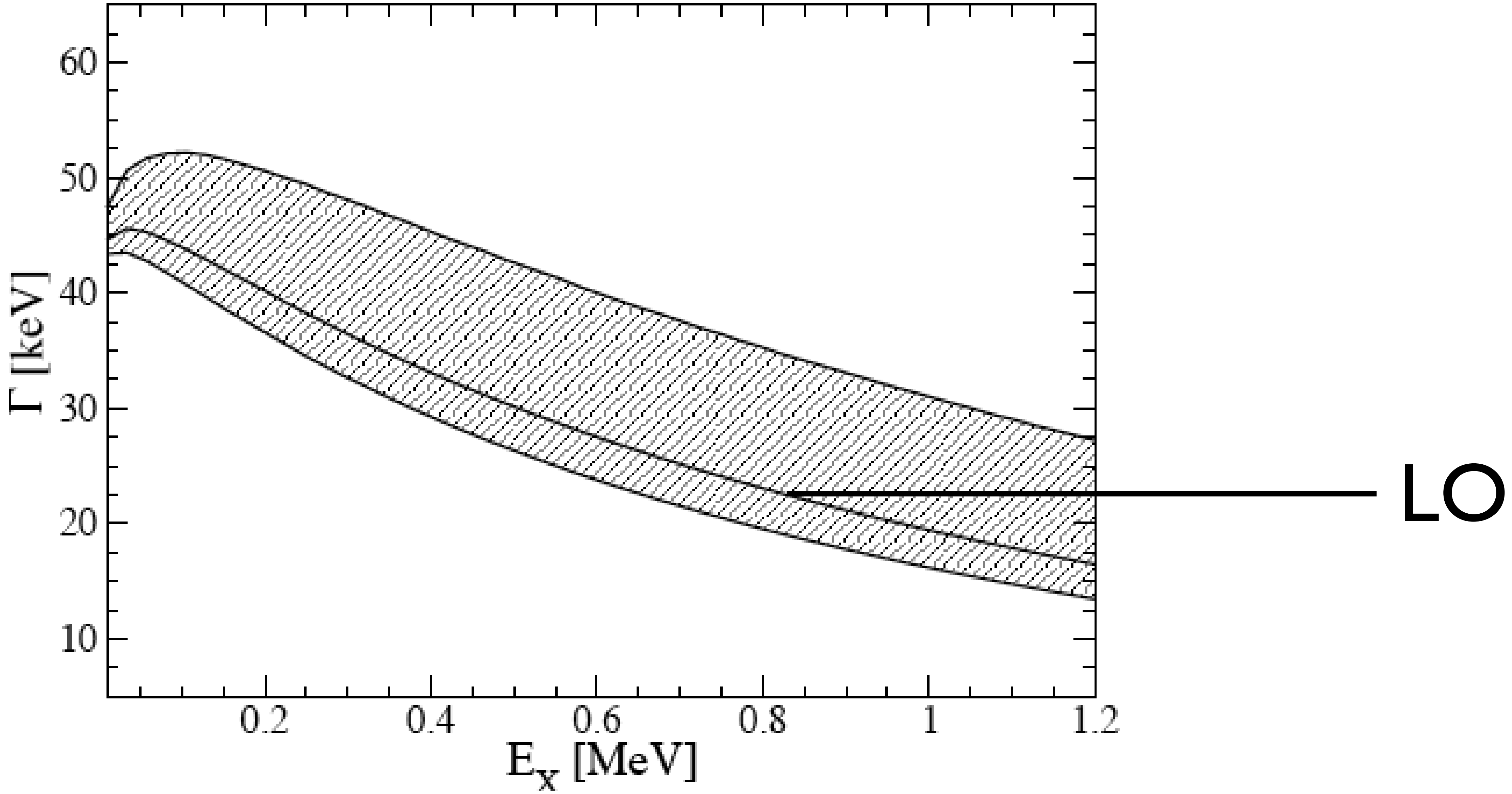
Wavefunction Renormalization



LO

NLO

$X(3872) \to D^0 \bar{D}^0 \pi^0$ at NNLO



$g = 0.6 \qquad 0 \leq r_0 \leq (100\,\mathrm{MeV})^{-1} \qquad -1 \leq \eta \leq 1$

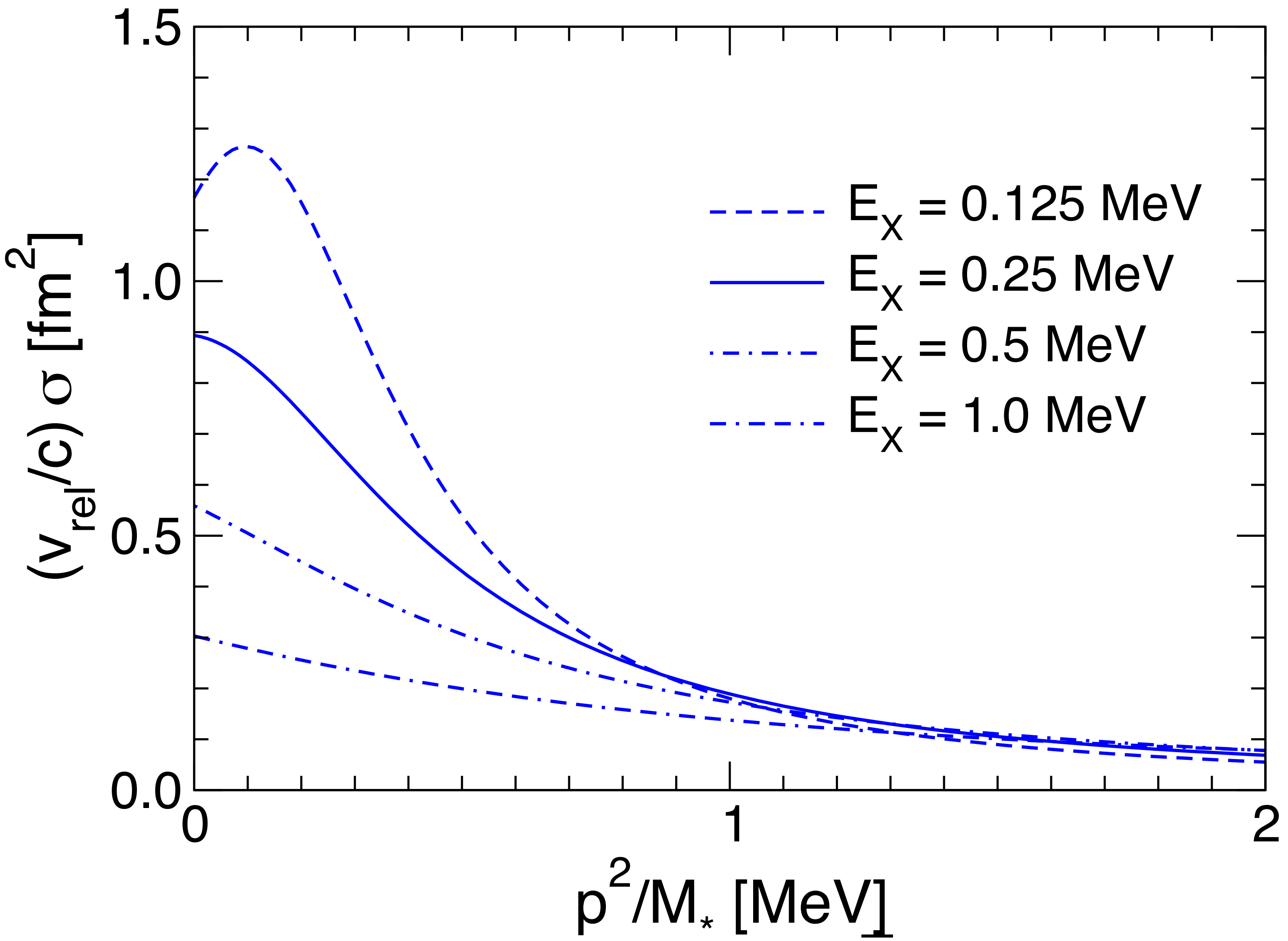
$$\left(\frac{g M_{DD^*}}{f_\pi} C_2(\Lambda_{\mathrm{PDS}}) + B_1(\Lambda_{\mathrm{PDS}})\right)(\Lambda_{\mathrm{PDS}} - \gamma) = \frac{\eta}{(100\,\mathrm{MeV})^3}$$

- Corrections dominated by counterterms, pion loops are negligible
- Agrees well with recent calculation with nonperturbative pions

Baru, et. al., PRD84:074029 (2011)

Other Universal Cross Sections

- **D Meson Coalesence** H.-W. Hammer, T.M., E. Braaten, PRD 82:034018 (2010)



$$D^{+0}\bar{D}^{*0} \to X(3872)\pi^+$$

- also $X(3872)\pi \to D\bar{D}^*, \pi X(3872) \to \pi X(3872)$
- only input is X(3872) binding energy
- D-X(3872) scattering (three-body calculations)

D. Canham, H.-W. Hammer, R.P. Springer, PRD80:014009 (2009)

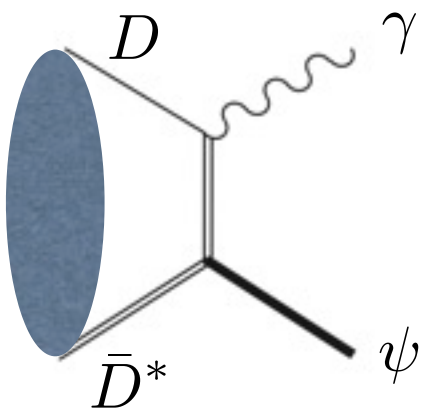
X(3872) Decays involving Quarkonia

S.Fleming, T.M., PRD 78:094019 (2008), PRD 85:014002 (2012), T.M., R. Springer, PRD 83:094001 (2011),
A. Margaryan, R.P. Springer, PRD88:014017 (2013)

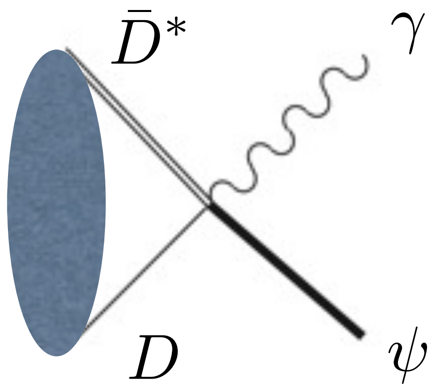
- Factorization Approach

unknown parameter ↓ calculate in HHChiPT ↓

$$\Gamma[X(3872) \to \psi(2S)\gamma] = |\psi_{DD}(0)|^2 \times \sigma[D^0 \bar{D}^{*0} + c.c. \to \psi(2S)\gamma]$$



meson exchange



short-distance

- Predict relative rates for $\Gamma[X(4872) \to \chi_{cJ}\pi^0] \equiv \Gamma_J$

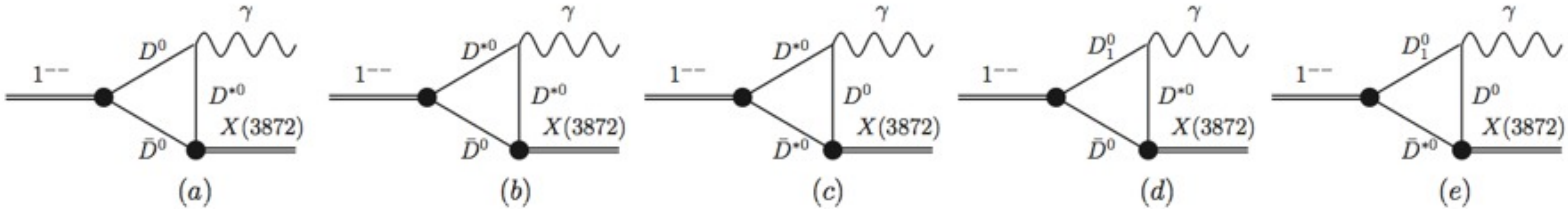
$$\Gamma_0 : \Gamma_1 : \Gamma_2 :: 3.5^{+0.6}_{-0.5} : 1.2^{+0.18}_{-0.14} : 1$$

- Angular distributions in $X(3872) \to \psi(2S)\gamma \quad \psi(4040) \to X(3872)\gamma \quad \psi(4160) \to X(3872)\gamma$

can be used to disentangle meson exchange, short distance contributions

- Hadronic Loops

F. K. Guo, et. al., PLB 725 (2013) 127



NREFT loop is finite

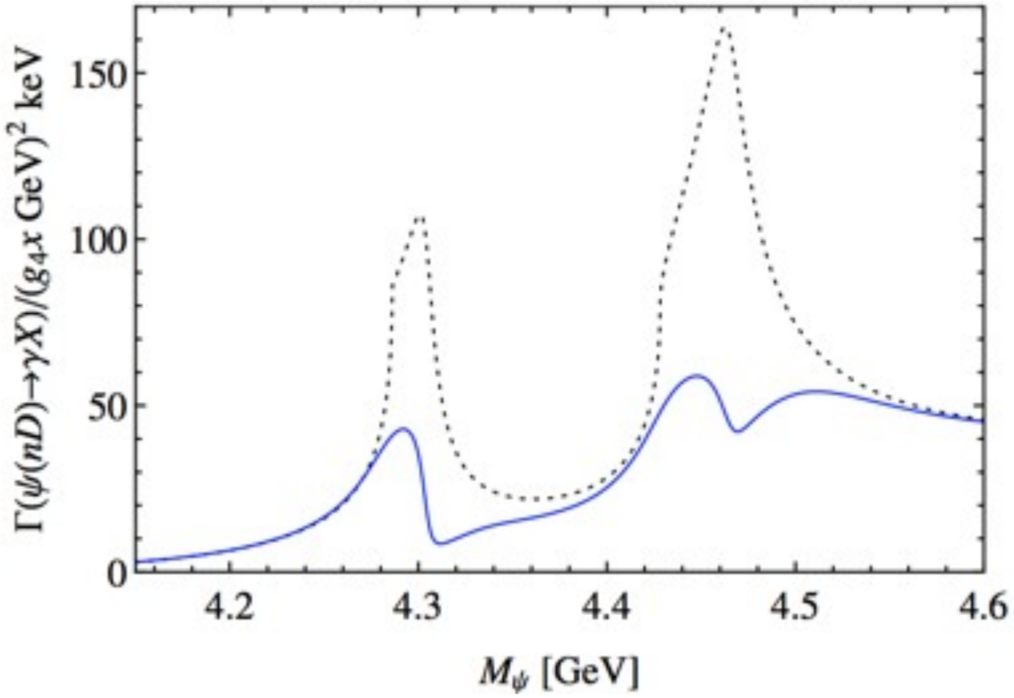
predictive if hadronic couplings can be estimated

Calculate rates $\psi(nS), \psi(nD), Y(4260) \rightarrow X(3872)\gamma$

Estimates for $\psi(4160) \rightarrow X(3872)\gamma$

compatible with factorization approach

A. Margaryan, R.P. Springer, PRD88 (2013) 014017



- assuming Y(4260) is D_1D molecule predict large rate for $Y(4260) \rightarrow X(3872)\gamma$

  $Y(4260) \rightarrow X(3872)\gamma$ recently observed by BESIII !

- current EFT calculations do not include charged mesons; argued to be important in

F. Aceti, E. Oset, PRD86 (2013) 113017

work in progress J.Z. Lin, T.M., R.P. Springer

Heavy Quark Spin Symmetry Predictions for $Z_b(10610)$ & $Z_b(10650)$

A.E. Bondar, et.al., PRD 84: 054010 (2011)

M.B Voloshin, PRD 84: 031502 (2011)

Hamiltonian

$$H_s = \mu\,(\vec{s}_b \cdot \vec{s}_{\bar{q}}) + \mu\,(\vec{s}_{\bar{b}} \cdot \vec{s}_q) = \frac{\mu}{2}\,(\vec{S}_H \cdot \vec{S}_{SLB}) - \frac{\mu}{2}\,(\vec{\Delta}_H \cdot \vec{\Delta}_{SLB}) \ ,$$

Quark Model Wavefunctions $S_{Q\bar{Q}} \otimes S_{q\bar{q}}$

$$W_2 : \quad 1_{Q\bar{Q}} \otimes 1_{q\bar{q}}\Big|_{J=2}$$

$$W_1 : \quad 1_{Q\bar{Q}} \otimes 1_{q\bar{q}}\Big|_{J=1}$$

$$W'_{b0} : \quad \frac{\sqrt{3}}{2}\, 0_{Q\bar{Q}} \otimes 0_{q\bar{q}} + \frac{1}{2}\, 1_{Q\bar{Q}} \otimes 1_{q\bar{q}}\Big|_{J=0}$$

$$W_0 : \quad \frac{\sqrt{3}}{2}\, 1_{Q\bar{Q}} \otimes 1_{q\bar{q}}\Big|_{J=0} - \frac{1}{2}\, 0_{Q\bar{Q}} \otimes 0_{q\bar{q}}$$

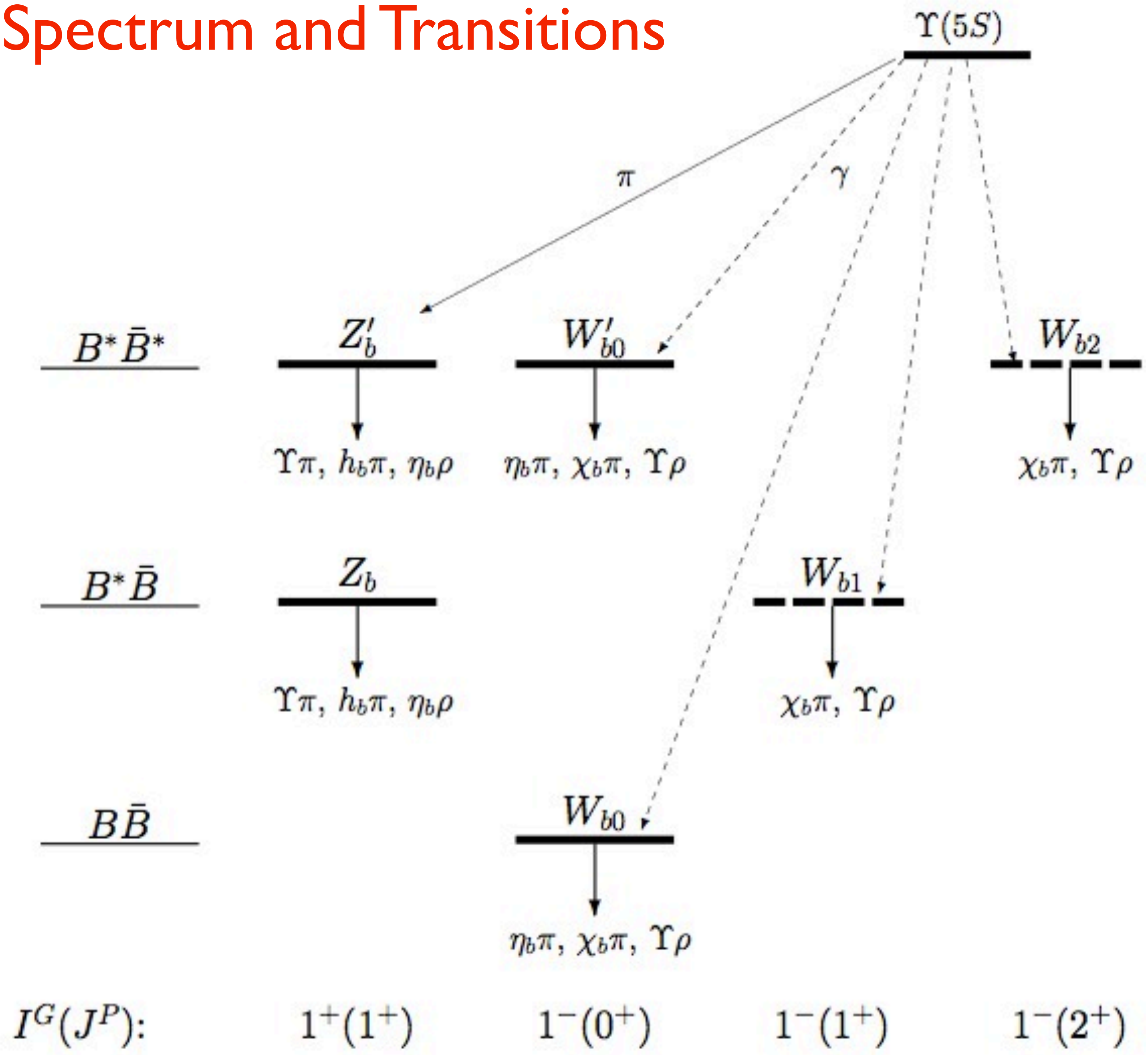
$$Z' : \quad \frac{1}{\sqrt{2}}\, 0_{Q\bar{Q}} \otimes 1_{q\bar{q}} - \frac{1}{\sqrt{2}}\, 1_{Q\bar{Q}} \otimes 0_{q\bar{q}}$$

$$Z : \quad \frac{1}{\sqrt{2}}\, 0_{Q\bar{Q}} \otimes 1_{q\bar{q}} + \frac{1}{\sqrt{2}}\, 1_{Q\bar{Q}} \otimes 0_{q\bar{q}} \, .$$

binding should only depend on $S_{q\bar{q}}$

expect similar states in other channels

Spectrum and Transitions



Strong Decay Widths

$$\Gamma(W_{b2}) = \Gamma(W_{b1}) =$$

$$= \frac{3}{2}\Gamma(W_{b0}) - \frac{1}{2}\Gamma(W_{b0}')$$

Radiative Decays

$$f(W_{b0}\gamma) : f(W_{b0}'\gamma) : f(W_{b1}\gamma) : f(W_{b2}\gamma) = \frac{3}{4}\omega_0^3 : \frac{1}{4}\omega_2^3 : 3\,\omega_1^3 : 5\omega_2^3$$

Effective Field Theory

T.M., J. Powell, PRD 84:114013 (2011)

$$\begin{aligned}
\mathcal{L} \;=\; & \mathrm{Tr}[H_a^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{2M}\right)_{ba} H_b] + \frac{\Delta}{4}\mathrm{Tr}[H_a^\dagger\, \sigma^i\, H_a\, \sigma^i] \\
+ \; & \mathrm{Tr}[\bar{H}_a^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{2M}\right)_{ab} \bar{H}_b] + \frac{\Delta}{4}\mathrm{Tr}[\bar{H}_a^\dagger\, \sigma^i\, \bar{H}_a\, \sigma^i] \\
- \; & \frac{C_{00}}{4}\mathrm{Tr}[\bar{H}_a^\dagger H_a^\dagger H_b \bar{H}_b] - \frac{C_{01}}{4}\mathrm{Tr}[\bar{H}_a^\dagger \sigma^i H_a^\dagger H_b \sigma^i \bar{H}_b] \\
- \; & \frac{C_{10}}{4}\mathrm{Tr}[\bar{H}_a^\dagger \tau^A_{aa'} H_{a'}^\dagger H_b \tau^A_{bb'} \bar{H}_{b'}] - \frac{C_{11}}{4}\mathrm{Tr}[\bar{H}_a^\dagger \tau^A_{aa'} \sigma^i H_{a'}^\dagger H_b \tau^A_{bb'} \sigma^i \bar{H}_{b'}] \,. \\
= \; & -2\, C_{11} \left(W_{0+}^{A\,\dagger} W_{0+}^{A} + Z_{+}^{A\,i\,\dagger} Z_{+}^{A\,i} + W_{1}^{A\,i\,\dagger} W_{1}^{A\,i} + \sum_{\lambda} W_{2\,\lambda}^{A\,\dagger} W_{2\,\lambda}^{A} \right) \\
& -2\, C_{10} \left(W_{0-}^{A\,\dagger} W_{0-}^{A} + Z_{-}^{A\,i\,\dagger} Z_{-}^{A\,i} \right) \,,
\end{aligned}$$

interpolating fields

$$W_{0+}^A = \tfrac{1}{2} W_0'^{\,A} + \tfrac{\sqrt{3}}{2} W_0^A \qquad W_{0-}^A = \tfrac{\sqrt{3}}{2} W_0'^{\,A} - \tfrac{1}{2} W_0^A$$

$$\begin{aligned}
Z^{A\,i} &= \frac{1}{\sqrt{2}}(V_a^i \tau^A_{ab} \bar{P}_b - P_a \tau^A_{ab} \bar{V}_b^i) & W_0^A &= P_a \tau^A_{ab} \bar{P}_b & W_1^{A\,i} &= \frac{1}{\sqrt{2}}(V_a^i \tau^A_{ab} \bar{P}_b + P_a \tau^A_{ab} \bar{V}_b^i) \\
Z'^{\,A\,i} &= \frac{i}{\sqrt{2}}\, \epsilon^{ijk} V_a^j \tau^A_{ab} \bar{V}_b^k & W_0'^{\,A} &= \frac{1}{\sqrt{3}} V_a^i \tau^A_{ab} \bar{V}_b^i & W_2^{A\,\lambda} &= \epsilon^{\lambda}_{ij} V_a^i \tau^A_{ab} \bar{V}_b^j \,,
\end{aligned}$$

Solve coupled channel problem in EFT

$$
\begin{aligned}
T_{Z'Z'} &= \frac{4\pi}{M}\frac{-\gamma_+ + \sqrt{M(\Delta-E)-i\epsilon}}{(\gamma_+ - \sqrt{M(\Delta-E)-i\epsilon})(\gamma_+ - \sqrt{M(2\Delta-E)-i\epsilon}) - \gamma_-^2} \\
T_{Z'Z} &= T_{ZZ'} = \frac{4\pi}{M}\frac{\gamma_-}{(\gamma_+ - \sqrt{M(\Delta-E)-i\epsilon})(\gamma_+ - \sqrt{M(2\Delta-E)-i\epsilon}) - \gamma_-^2} \\
T_{ZZ} &= \frac{4\pi}{M}\frac{-\gamma_+ + \sqrt{M(2\Delta-E)-i\epsilon}}{(\gamma_+ - \sqrt{M(\Delta-E)-i\epsilon})(\gamma_+ - \sqrt{M(2\Delta-E)-i\epsilon}) - \gamma_-^2},
\end{aligned}
$$

HQSS predictions   Decay Rates, Binding energies

$$\Gamma[W_1] = \Gamma[W_2] = \frac{3}{2}\Gamma[W_0] - \frac{1}{2}\Gamma[W_0']$$

M.B Voloshin, PRD 84: 031502 (2011)

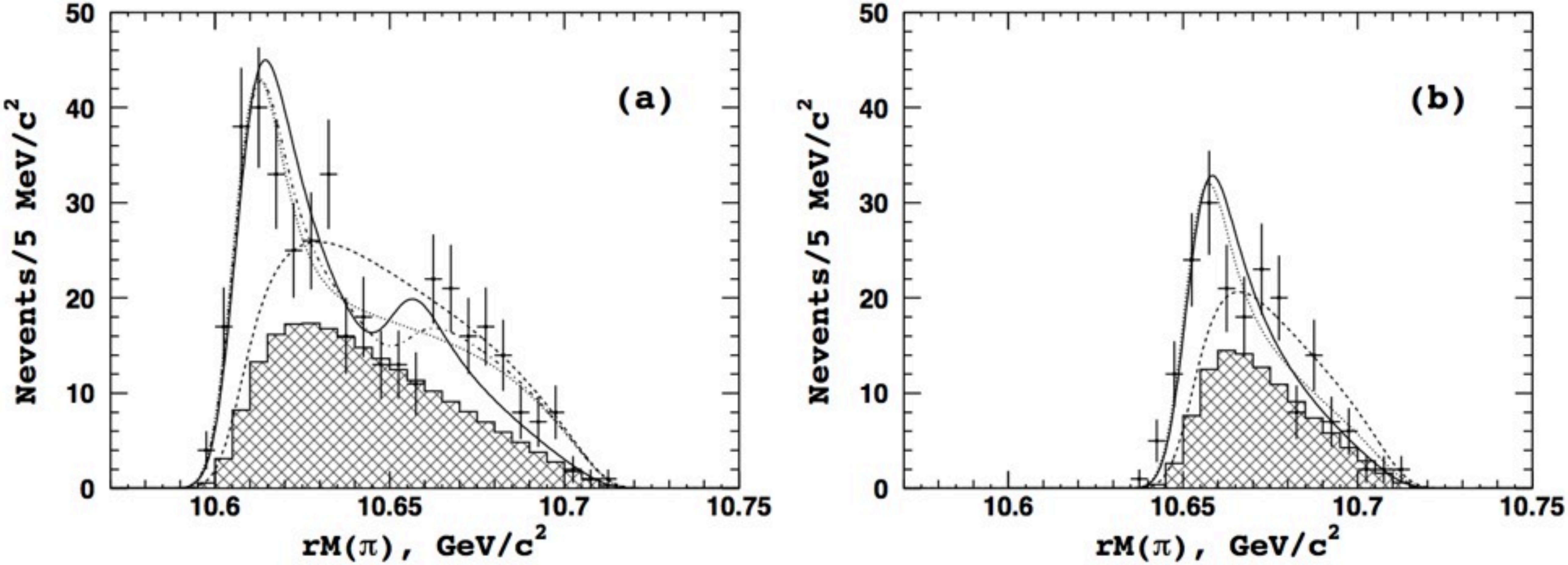
$$\Gamma[Z] = \Gamma[Z'] = \frac{1}{2}(\Gamma[W_0] + \Gamma[W_0'])$$

(new)

Factorization approach yields predictions partial widths

Z(10610) and Z(10650) in $\Upsilon(5S) \to B^*\bar{B}^{(*)}\pi$

Belle, arXiv:1209.6450, R. Mizyuk's QWG2013 talk



BF[Υ(5S) → $B^{(*)}\bar{B}^{(*)}\pi$]	Belle 121.4 fb^{-1}
$B\bar{B}$	<0.60 % at 90% C.L.
$B\bar{B}^*$ + $B\bar{B}^*$	(4.25 ± 0.44 ± 0.69) %
$B^*\bar{B}^*$	(2.12 ± 0.29 ± 0.36) %

Explicit calculations of 2-body Decays

$$\Gamma[W_0 \to \pi\eta_b] = \frac{m_\eta k_\pi E_\pi^2}{8\pi m_{W_0} f_\pi^2}\left[g_{\pi\Upsilon} - 2gg_\Upsilon \frac{k_\pi^2}{E_\pi(E_\pi+\Delta)}\right]^2 \times \mathcal{O}_1 \qquad (36)$$

$$\Gamma[W_0' \to \pi\eta_b] = \frac{3m_\eta k_\pi E_\pi^2}{8\pi m_{W_0'} f_\pi^2}\left[g_{\pi\Upsilon} - 2gg_\Upsilon \frac{k_\pi^2}{E_\pi^2}\left(1+\frac{1}{3}\frac{\Delta}{E_\pi-\Delta}\right)\right]^2 \times \mathcal{O}_2$$

$$\Gamma[Z \to \pi\Upsilon] = \frac{m_\Upsilon k_\pi E_\pi^2}{4\pi m_Z f_\pi^2}\left[\left[g_{\pi\Upsilon} - 2gg_\Upsilon \frac{k_\pi^2}{E_\pi^2}\left(1-\frac{\Delta}{3}\frac{E_\pi-2\Delta}{E_\pi^2-\Delta^2}\right)\right]^2 + \frac{2}{9}\left[gg_\Upsilon \frac{k_\pi^2}{E_\pi^2}\frac{\Delta}{E_\pi-\Delta}\right]^2\right] \times \mathcal{O}_3$$

$$\Gamma[Z' \to \pi\Upsilon] = \frac{m_\Upsilon k_\pi E_\pi^2}{4\pi m_{Z'} f_\pi^2}\left[\left[g_{\pi\Upsilon} - 2gg_\Upsilon \frac{k_\pi^2}{E_\pi^2}\left(1+\frac{1}{3}\frac{\Delta}{E_\pi-\Delta}\right)\right]^2 + \frac{2}{9}\left[gg_\Upsilon \frac{k_\pi^2}{E_\pi^2}\frac{\Delta}{E_\pi-\Delta}\right]^2\right]$$

corrections to HQSS from phase space, kinematics

$$\Gamma[W_0 \to \pi\eta_b(3S)] \;:\; \Gamma[W_0' \to \pi\eta_b(3S)] : \Gamma[Z \to \pi\Upsilon(3S)] : \Gamma[Z' \to \pi\Upsilon(3S)]$$
$$= 0.26 \;:\; 2.0 \;:\; 0.62 \;:\; 1 \qquad (\lambda_\Upsilon = 0)\,,$$

$$\Gamma[W_0 \to \pi\eta_b(3S)] \;:\; \Gamma[W_0' \to \pi\eta_b(3S)] : \Gamma[Z \to \pi\Upsilon(3S)] : \Gamma[Z' \to \pi\Upsilon(3S)]$$
$$= 0.12 \;:\; 2.1 \;:\; 0.41 \;:\; 1 \qquad (|\lambda_\Upsilon| = \infty)\,.$$

EFT analysis of $\Upsilon(5S) \to B^{(*)}\bar{B}^{(*)}\pi$

T.M., J. Powell, PRD 88 (2013) 034017

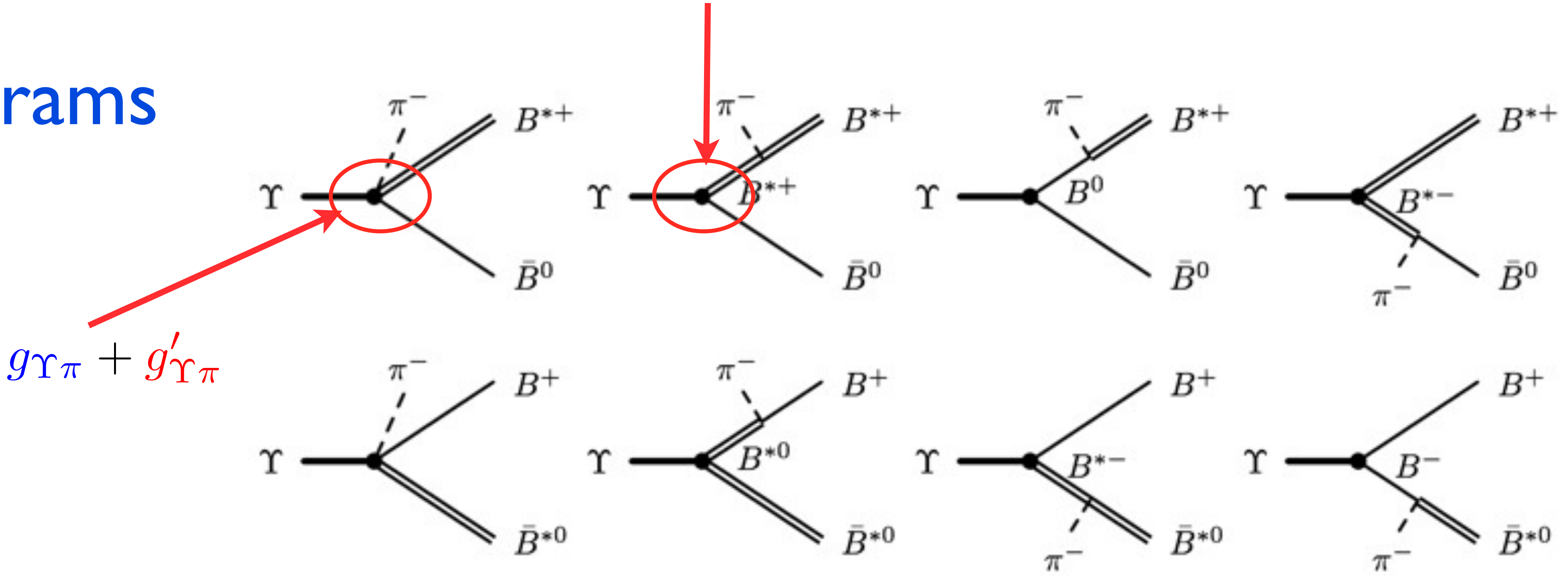
- $140\,\mathrm{MeV} < E_\pi < 270\,\mathrm{MeV}$ $p_B < 1\,\mathrm{GeV}$ non-relativistic, chiral theory can be applied

Use data on $\Upsilon(5S) \to B^{(*)}\bar{B}^{(*)}$ to determine $\Upsilon(5S)B^{(*)}\bar{B}^{(*)}$ couplings

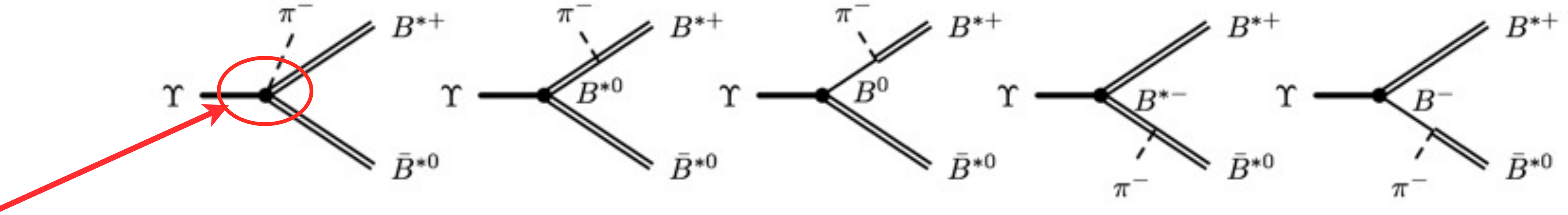
requires HQSS violating contact interactions

- ## Tree Diagrams

$\Upsilon(5S) \to B^*\bar{B}\pi$



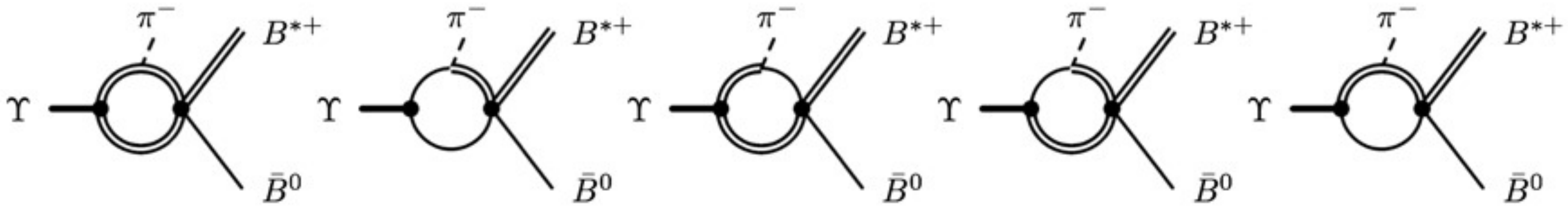
$\Upsilon(5S) \to B^*\bar{B}^*\pi$



$g_{\Upsilon\pi} - g'_{\Upsilon\pi}$

Determine $g_{\Upsilon\pi}, g'_{\Upsilon\pi},$ by reproducing measured branching fractions

Loop corrections to $\Upsilon(5S) \to B^* \bar{B} \pi$

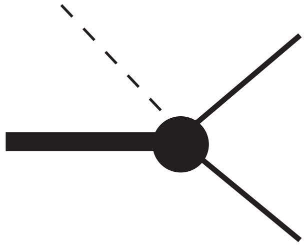


Power counting Loop Q^5 Propagators Q^{-6}

Derivatives Q^2 C$_\pm$ Contact Q^{-1}

$\longrightarrow Q^0$ same order as tree graphs

Dressing amplitudes

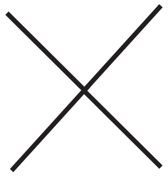


$$i\mathcal{M} = \begin{pmatrix} i\mathcal{M}_{B^*B^*} \\ i\mathcal{M}_{BB^*} \end{pmatrix}$$

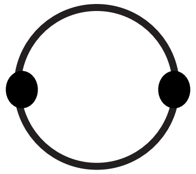
leading contact interactions or loops

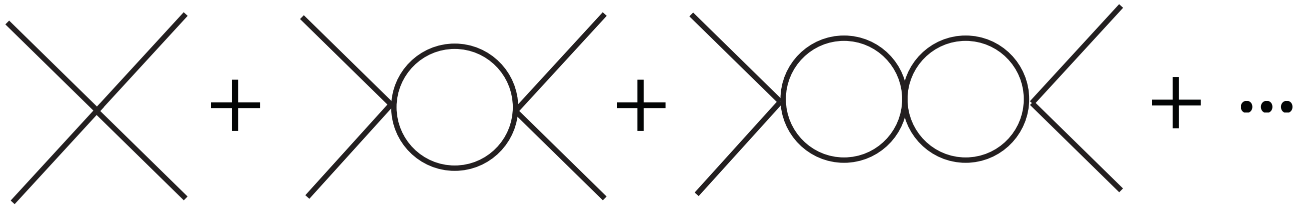
T-matrix

$$C = \begin{pmatrix} C_+ & C_- \\ C_- & C_+ \end{pmatrix} =$$

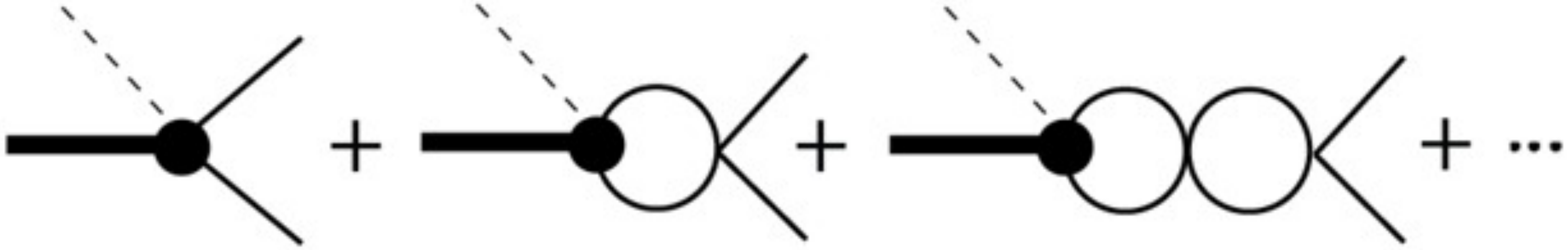


$$\Sigma_Z = \begin{pmatrix} \Sigma_{B^*B^*}(E) & 0 \\ 0 & \Sigma_{BB^*}(E) \end{pmatrix} =$$





$$i\,T = \ (1 - C\,\Sigma_Z + C\,\Sigma_Z\,C\,\Sigma_Z + \ldots)\times -iC$$



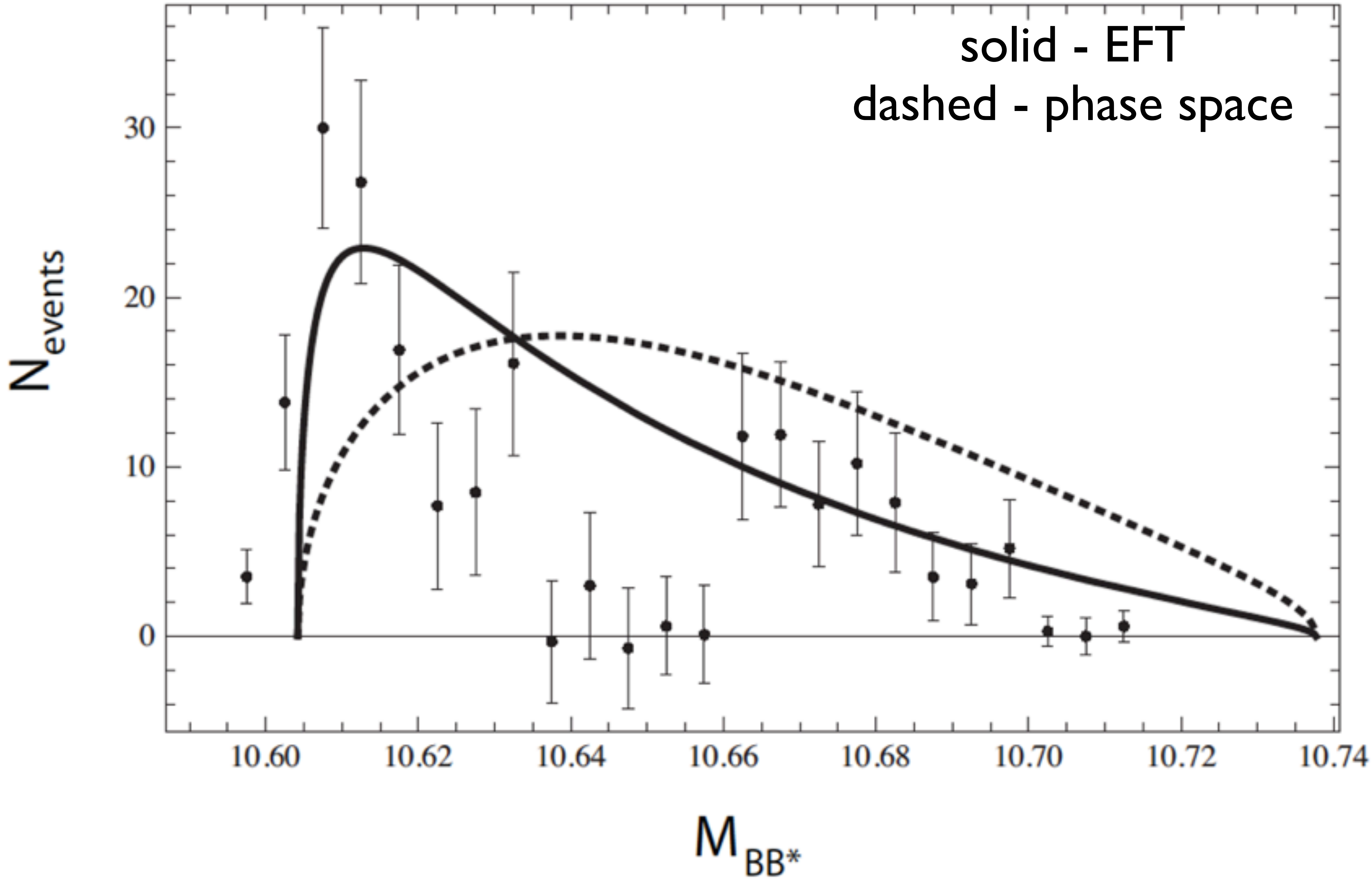
$$i\mathcal{M}^{\text{dressed}} = (1 - C\,\Sigma_Z + C\,\Sigma_Z\,C\,\Sigma_Z + \ldots)\,i\mathcal{M}$$

$$= (1 + T_Z\Sigma_Z)\,i\mathcal{M} \quad = -T_Z\,C^{-1}\,i\mathcal{M}$$

contact interactions

loop graphs

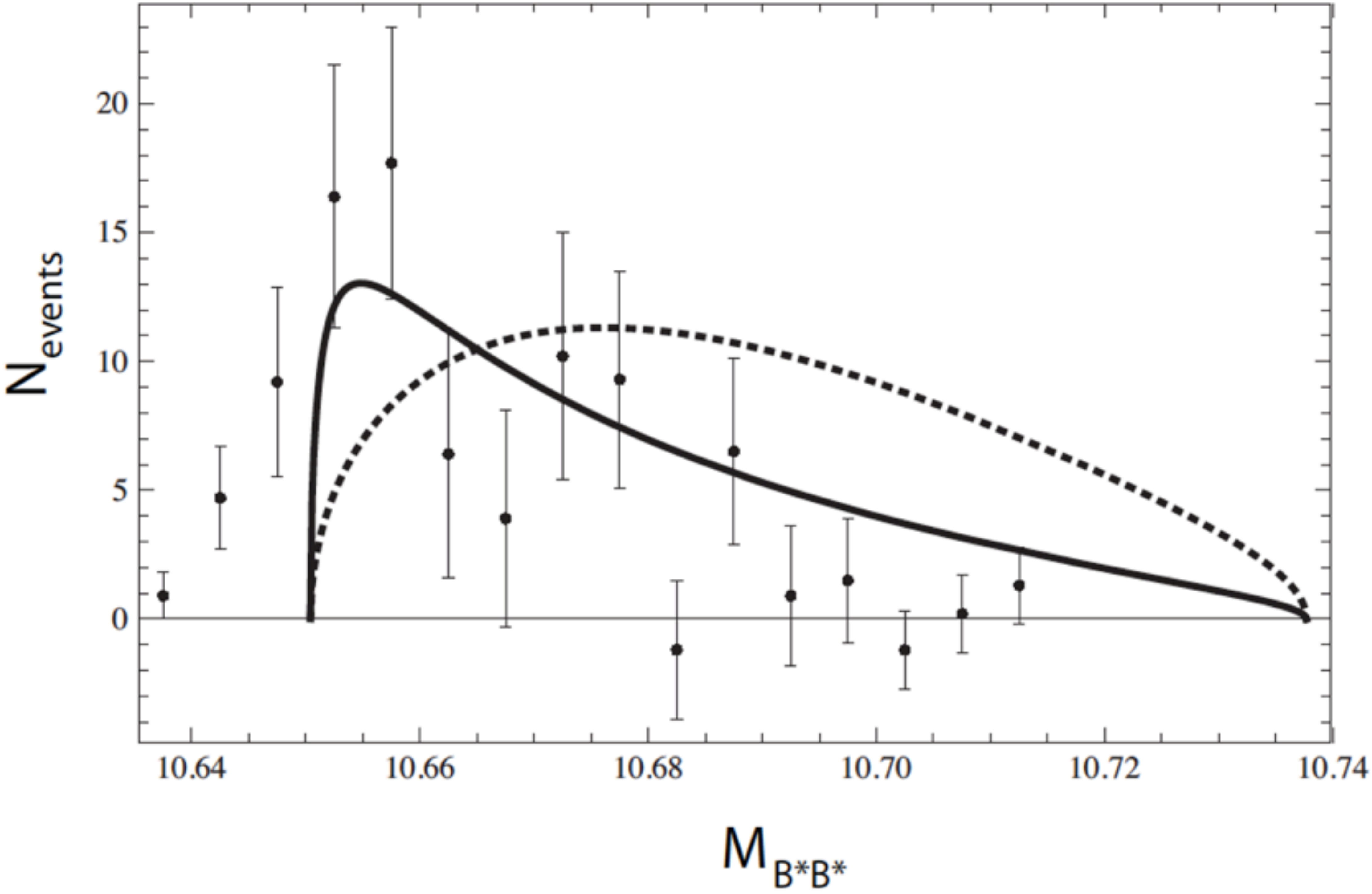
Lineshape for $\Upsilon(5S) \to B^* \bar{B} \pi$



fit T-matrix to poles determined in $\Upsilon(5S) \to \Upsilon(nS)\pi^+\pi^-, h_b(nP)\pi^+\pi^-$

undetermined coupling constants to total rate

Lineshape for $\Upsilon(5S) \to B^* \bar{B}^* \pi$



Conclusions

- Plethora of XYZ states in charmonium, bottomonium

  states close to open meson thresholds: X(3872), $Z_b(10610)$, and $Z_b(10650)$

  EFT's for shallow bound states similar to those developed for nuclear physics (deuteron) can be used to study the properties of these states

- Numerous processes calculated, many untested predictions

- EFT calculation predicted large rate for $Y(4260) \to X(3872)\gamma$

  recently observed by BESIII !

- HQSS predictions for the $Z_b(10610)$, $Z_b(10650)$ using quark model

  can also be obtained using EFT, include corrections

- Line shape for $\Upsilon(5S) \to B^* \bar{B}^{(*)} \pi$ computed using EFT

  far from clear that EFT is working, does illustrate importance of FSI

  incorporate exptal. resolution, width of the $\Upsilon(5S)$

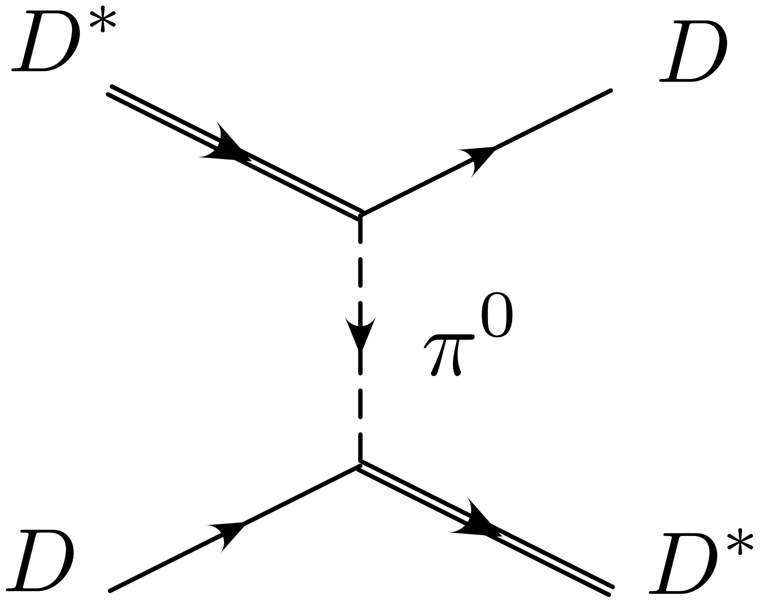
  range corrections

Additional Slides

Energy Scales from Pion Exchange in the X(3872)

- π^0 exchange

$\Delta \equiv m_{D^*} - m_D \approx 142\,\mathrm{MeV}$ $\qquad$ $m_{\pi^0} \approx 135\,\mathrm{MeV}$



$$\frac{g^2}{2f^2}\frac{\vec{q}\cdot\epsilon\,\vec{q}\cdot\epsilon^*}{\vec{q}^{\,2}-\Delta^2+m_\pi^2} = \frac{g^2}{2f^2}\frac{\vec{q}\cdot\epsilon\,\vec{q}\cdot\epsilon^*}{\vec{q}^{\,2}-\mu^2}$$

oscillatory rather than Yukawa-like potential

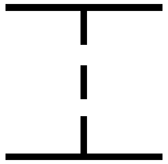
M. Suzuki, PRD 72:114013 (2005)

- $\mu^2 \equiv \Delta^2 - m_\pi^2 \approx (44\,\mathrm{MeV})^2$ - new long-distance scale
- binding momentum: $\gamma \equiv \sqrt{-2\mu_{DD^*}\mathrm{B.E.}} \leq 34\,\mathrm{MeV}$
- $X(3872) \to D^0\bar{D}^0\pi^0$: $\qquad T_\pi \leq 6\,\mathrm{MeV} \quad T_D \leq 3.2\,\mathrm{MeV}$

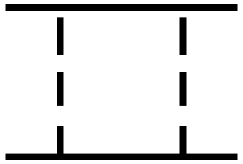
⟶ Non-relativistic D^0, D^{*0}, π^0

- Perturbative Pions and the X(3872)

Nuclear Physics: NN scattering



$$= \frac{g_A^2}{2f^2}\, A\left(\frac{p}{m_\pi}\right),$$



$$= \left(\frac{g_A^2}{2f^2}\right)^2 \frac{M m_\pi}{4\pi}\, B\left(\frac{p}{m_\pi}\right)$$

Expansion parameter: $\frac{g_A^2 M_N m_\pi}{8\pi f^2} \sim \frac{1}{2}$

NLO ~30% accuracy, fails at NNLO

S. Fleming, T.M., I. Stewart, NPA 677, 313 (2000)

X(3872): $g_A = 1.25 \to g \sim 0.5 - 0.7$ $m_\pi \to \mu$

$$\boxed{\frac{g^2 M_D \mu}{8\pi f^2} \sim \frac{1}{20} - \frac{1}{10}}$$

XEFT computation of $X(3872) \rightarrow \text{Quarkonia} + X$

1) include quarkonia explicitly in HHChiPT Lagrangian

HQSS, other symmetries are used to constrain form of Lagrangian

2) compute $D^0 \bar{D}^{*0} + c.c. \rightarrow \text{Quarkonia} + X$

3) match onto XEFT and compute decay of X(2872)

Reproduces X(3872) factorization theorems

E. Braaten, M. Kusunoki, PRD 72:014012 (2005)

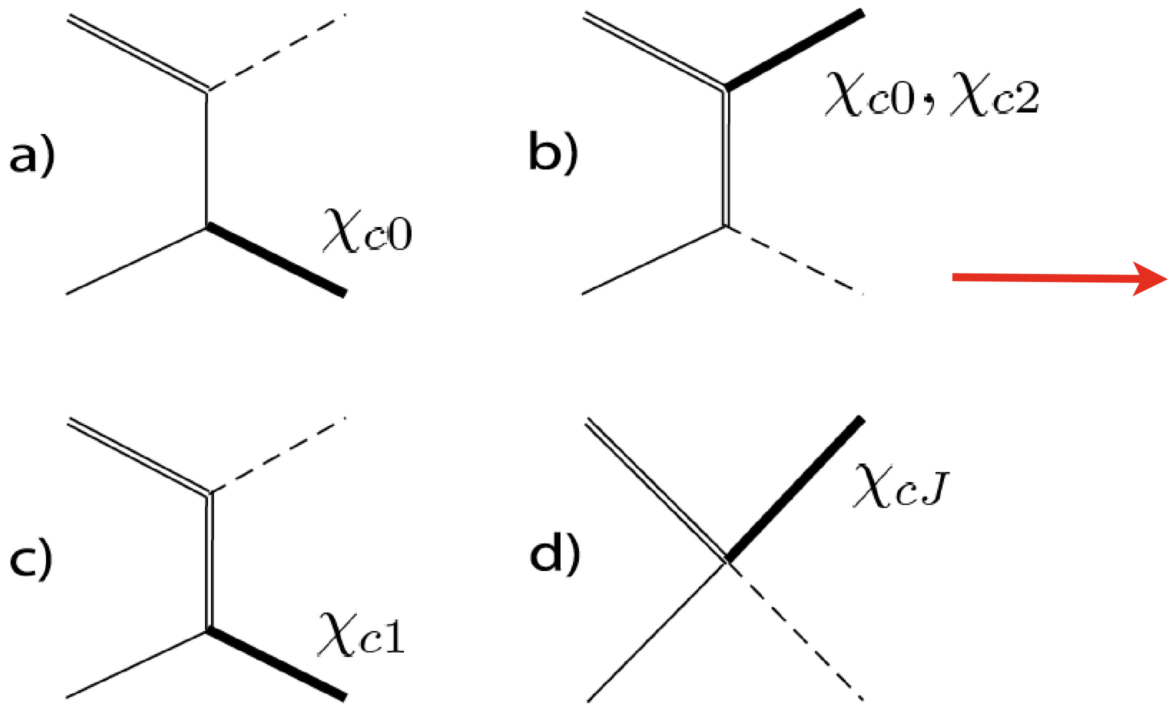
E. Braaten, M. Lu, PRD 74:054020 (2006)

ignorance of short-distance structure of X(3872) reflected in XEFT matrix elements, also unknown HHChiPT couplings limit predictive power

- Example: $X(3872) \to \chi_{c,J}\pi^0$ in X-EFT

$$\chi^i = \sigma^j \chi^{ij} = \sigma^j \left(\chi_2^{ij} + \frac{1}{\sqrt{2}} \epsilon^{ijk} \chi_1^k + \frac{\delta^{ij}}{\sqrt{3}} \chi_0 \right)$$

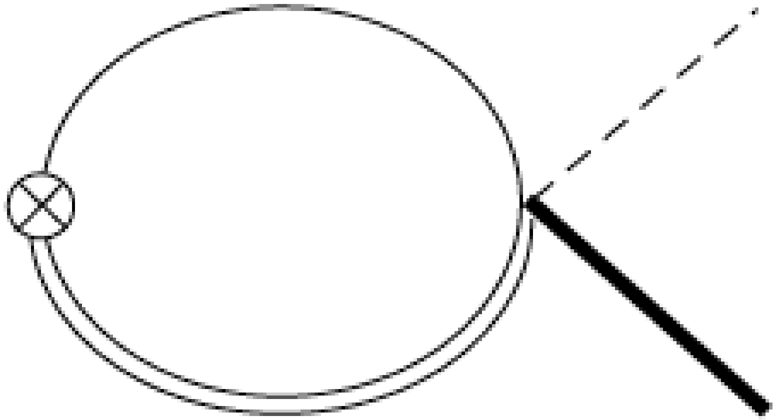
$$\mathcal{L}_\chi = i\frac{g_1}{2}\mathrm{Tr}[\chi^{\dagger\, i} H_a \sigma^i \, \bar{H}_a] + \frac{c_1}{2}\mathrm{Tr}[\chi^{\dagger\, i} H_a \sigma^j \, \bar{H}_b]\epsilon_{ijk} A^k_{ab} + \mathrm{h.c.}$$



$$\mathcal{L} = i\frac{C_{\chi,0}(E_{\pi,0})}{4\sqrt{m_\pi}}(\vec{V}\bar{P} + \bar{\vec{V}}P) \cdot \frac{\vec{\nabla}\pi^0}{f_\pi}\chi^\dagger_{c0}$$

- calculation of $X(3872) \to \chi_{c,J}\pi^0$ in X-EFT

Interpolating field for X(3872)



$$\Gamma[X(3872) \to \chi_{c,J}\pi^0] =$$

$$\frac{1}{3}\sum_{\lambda}|\langle 0|\frac{1}{\sqrt{2}}\vec{\epsilon}_{\lambda}\cdot(\vec{V}\bar{P}+\vec{\bar{V}}P)|X,\lambda\rangle|^2 \; \frac{m_{\chi_{cJ}}}{m_X}\,\frac{p^3_{\pi,J}}{72\pi f_\pi^2}\alpha_J|C_{\chi,J}(E_{\pi,J})|^2$$

XEFT matrix element

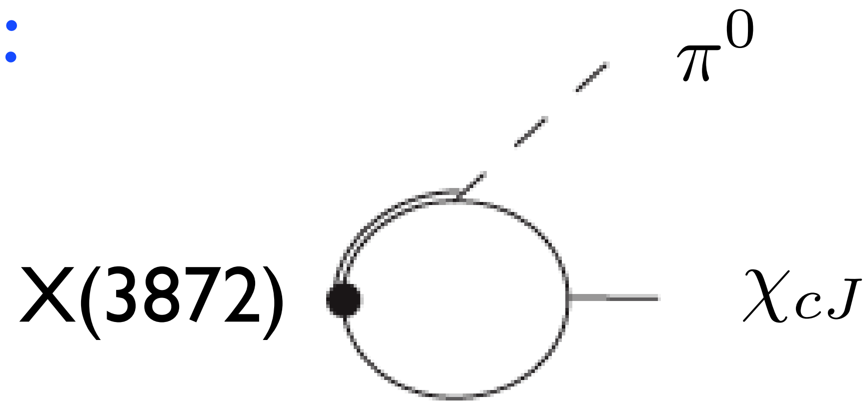
$$\propto |\mathcal{M}(D^0\bar{D}^{0^*}+c.c.\to\chi_{cJ}\pi^0)|^2$$

- Predict relative rates to χ_{cJ} for $J = 0, 1, 2$

S. Dubynskiy, M.B. Voloshin, PRD 77:014013 (2008)

S. Fleming, T.M., PRD 78:094019 (2008) PRD 85:014002 (2012)

- Comparison w/ direct evaluation:



$$\int d^4l \, \frac{1}{E_X - \Delta + l_0 - \frac{l^2}{2m_{D^*}}} \frac{1}{-l_0 - \frac{l^2}{2m_{D^*}}} \frac{1}{E_X + l_0 - E_\pi - \frac{(l-p_\pi)^2}{2m_D}}$$

$$= \int d^3l \, \frac{2\mu_{DD^*}}{l^2+\gamma^2} \frac{1}{E_\pi - \Delta \underbrace{- \frac{l^2}{2m_{D^*}} - \frac{(l-p_\pi)^2}{2m_D}}_{O(Q^2/m\_D)}} \approx \frac{1}{E_\pi - \Delta} \int d^3l \, \frac{2\mu_{DD^*}}{l^2+\gamma^2}$$

- direct evaluation + multipole expansion is equivalent to matching procedure described above

- drops contributions coming from integrand from

$$l \sim \sqrt{2\mu_{DD^*}(E_\pi - \Delta)} \sim 750\,\mathrm{MeV}$$

outside range of X-EFT !

Analysis of $X(3872) \to \psi(2S)\gamma$

T.M., R. Springer, PRD 83:094001 (2011)

$$\begin{aligned}\mathcal{L} &= \frac{e\beta}{2}\mathrm{Tr}[H_1^\dagger H_1\,\vec{\sigma}\cdot\vec{B}\,Q_{11}] + \frac{eQ'}{2m_c}\mathrm{Tr}[H_1^\dagger\,\vec{\sigma}\cdot\vec{B}\,H_1] + h.c. \\ &+ i\frac{g_2}{2}\mathrm{Tr}[J^\dagger H_1\vec{\sigma}\cdot\overleftrightarrow{\partial}\,\bar{H}_1] + i\frac{ec_1}{2}\mathrm{Tr}[J^\dagger H_1\vec{\sigma}\cdot\vec{E}\bar{H}_1] + h.c.\end{aligned}$$

charmonium superfield $J = \eta_c + \vec{\psi}\cdot\vec{\sigma}$

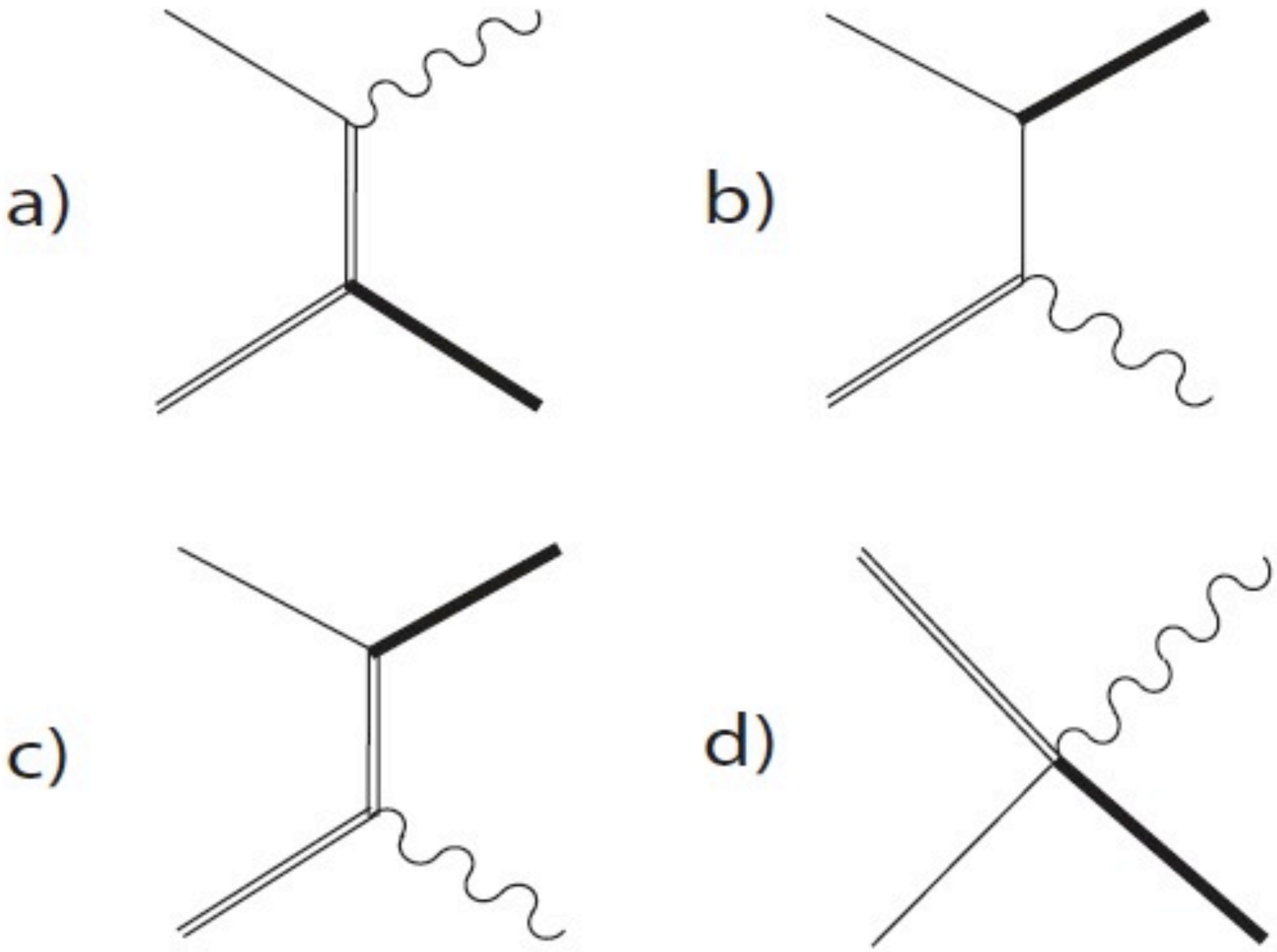
$\beta_+ (\beta_-)$ coupling for $D^{*0} \to D^0\gamma\,(D^{*0} \to D^{*0}\gamma)$

$$\beta_\pm = \beta \pm \frac{1}{m_c} \quad r_\beta = \beta_+/\beta_-$$

g_2 P-wave coupling of charmonia to D mesons

c_1 contact interaction coupling charmonia, D mesons, E-field

$D^0\bar{D}^{*0} + c.c. \to \psi(2S)\gamma$



$$a) = -\frac{g_2\,e\,\beta_+}{3}\frac{1}{E_\gamma+\Delta}(\vec{k}\cdot\vec{\epsilon}_\psi^{\,*}\,\vec{\epsilon}_{D^*}\cdot\vec{k}\times\vec{\epsilon}_\gamma^{\,*} - \vec{k}\cdot\vec{\epsilon}_{D^*}\,\vec{\epsilon}_\psi^{\,*}\cdot\vec{k}\times\vec{\epsilon}_\gamma^{\,*})$$

$$b) = \frac{g_2\,e\,\beta_+}{3}\frac{1}{\Delta-E_\gamma}\vec{k}\cdot\vec{\epsilon}_\psi^{\,*}\,\vec{\epsilon}_{D^*}\cdot\vec{k}\times\vec{\epsilon}_\gamma^{\,*}$$

$$c) = \frac{g_2\,e\,\beta_-}{3}\frac{1}{E_\gamma}\vec{k}\cdot\vec{\epsilon}_{D^*}\,\vec{\epsilon}_\psi^{\,*}\cdot\vec{k}\times\vec{\epsilon}_\gamma^{\,*}$$

$$d) = -e\,c_1\,E_\gamma\,\vec{\epsilon}_{D^*}\cdot\vec{\epsilon}_\psi^{\,*}\times\vec{\epsilon}_\gamma^{\,*}$$

- all diagrams O(Q) in HHChiPT counting
- contact interaction gives naive coupling, a)-c) give rise to new spin structures
- b) enhanced by $\frac{E_\gamma}{E_\gamma-\Delta} \sim 4.7 \text{ and } \propto \vec{k}\cdot\vec{\epsilon}_\psi^{\,*}$

- Decay Rate

$$\Gamma[X(3872)\to\psi(2S)(\vec{\epsilon}_\psi)\gamma] = \sum_\lambda |\langle 0|\frac{1}{\sqrt{2}}\epsilon^i(\lambda)\,(V^i\,\bar{P}+\bar{V}^i\,P)|X(3872,\lambda)\rangle|^2$$

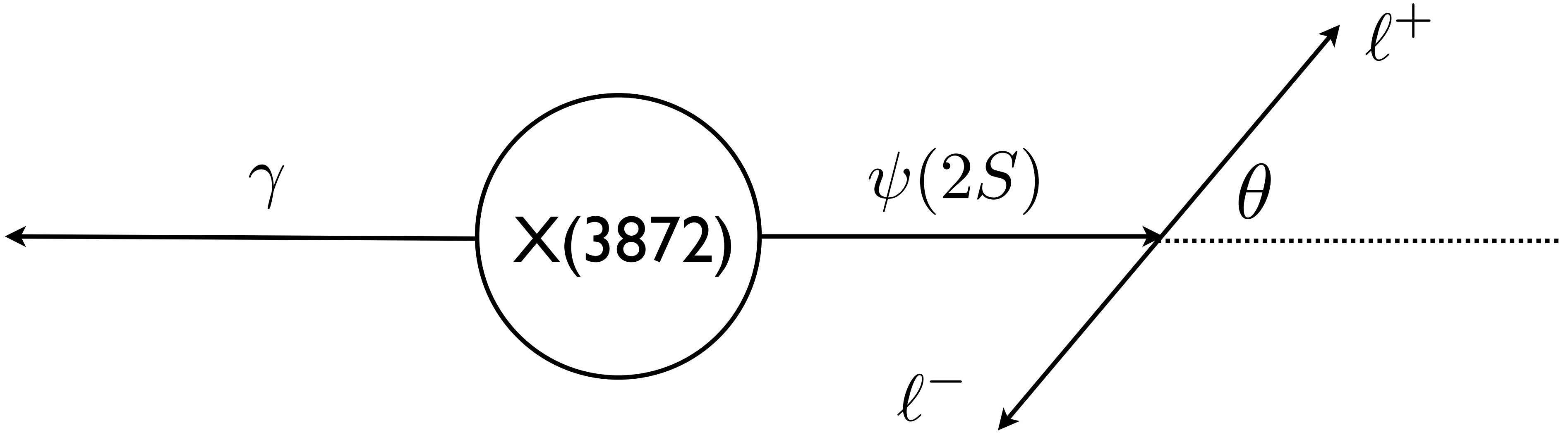
$$\times\frac{E_\gamma}{36\pi}\frac{m_\psi}{m_X}\,[(A+C)^2+(B-C)^2]$$

$$A=\frac{g_2 e\beta_+}{3}\frac{2E_\gamma^3}{\Delta^2-E_\gamma^2}\quad B=\frac{g_2 e}{3}\frac{\beta_+E_\gamma^2+\beta_-E_\gamma(E_\gamma+\Delta)}{E_\gamma+\Delta}\quad C=-ec_1E_\gamma$$

- $\Gamma[X\to\psi\gamma]$ no longer $\propto E_\gamma^3$ because of diagrams a)-c)

- Absolute rate unknown

- Polarization $\psi(2S) \to \ell^+\ell^-$ $\boxed{\frac{d\Gamma}{d\cos\theta} \propto 1 + \alpha\cos^2\theta}$ $\alpha = \frac{1 - 3f_L}{1 + f_L}$



contact interaction

i) $g_2\beta \ll c_1$ d) only

$$f_L = \frac{1}{2}, \boxed{\alpha = -\frac{1}{3}}$$

$$\mathcal{M} \propto \vec{\epsilon}_X \cdot \vec{\epsilon}_\psi^{\,*} \times \vec{\epsilon}_\gamma^{\,*}$$

constituent decay

ii) $g_2\beta \gg c_1$ a-c) only b) dominates

$$f_L = \frac{4E_\gamma^4}{4E_\gamma^4 + (2E_\gamma + \Delta)^2(E_\gamma - \Delta)^2} = 0.92$$

$$\boxed{\alpha = -0.91}$$

- Polarization measurement would shed light on relative importance of decay mechanisms

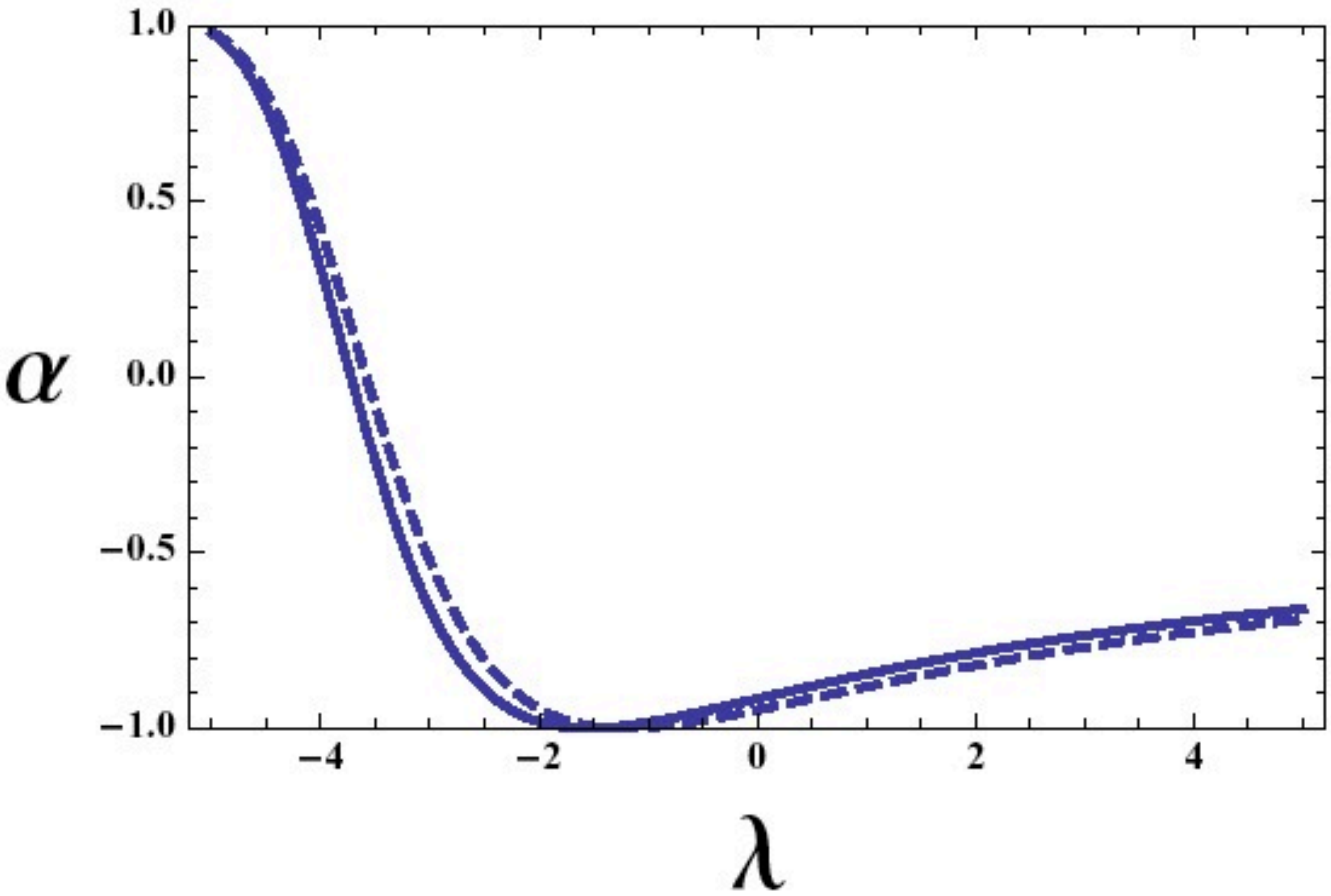
- Polarization as function of $\lambda \equiv \frac{3c_1}{g_2\beta_+} \approx 1.3\frac{c_1}{\mathrm{GeV}^{-5/2}} \sim O(1)$

$g_2 \approx 0.81\,\mathrm{GeV}^{-3/2}$ from $\psi' \to J/\psi\,\pi^0\,(\eta)$

(Guo, et. al. arXiv: 0907.0521[hep-ph])

$\beta = (356\,\mathrm{MeV})^{-1}$ from $D^* \to D\gamma$

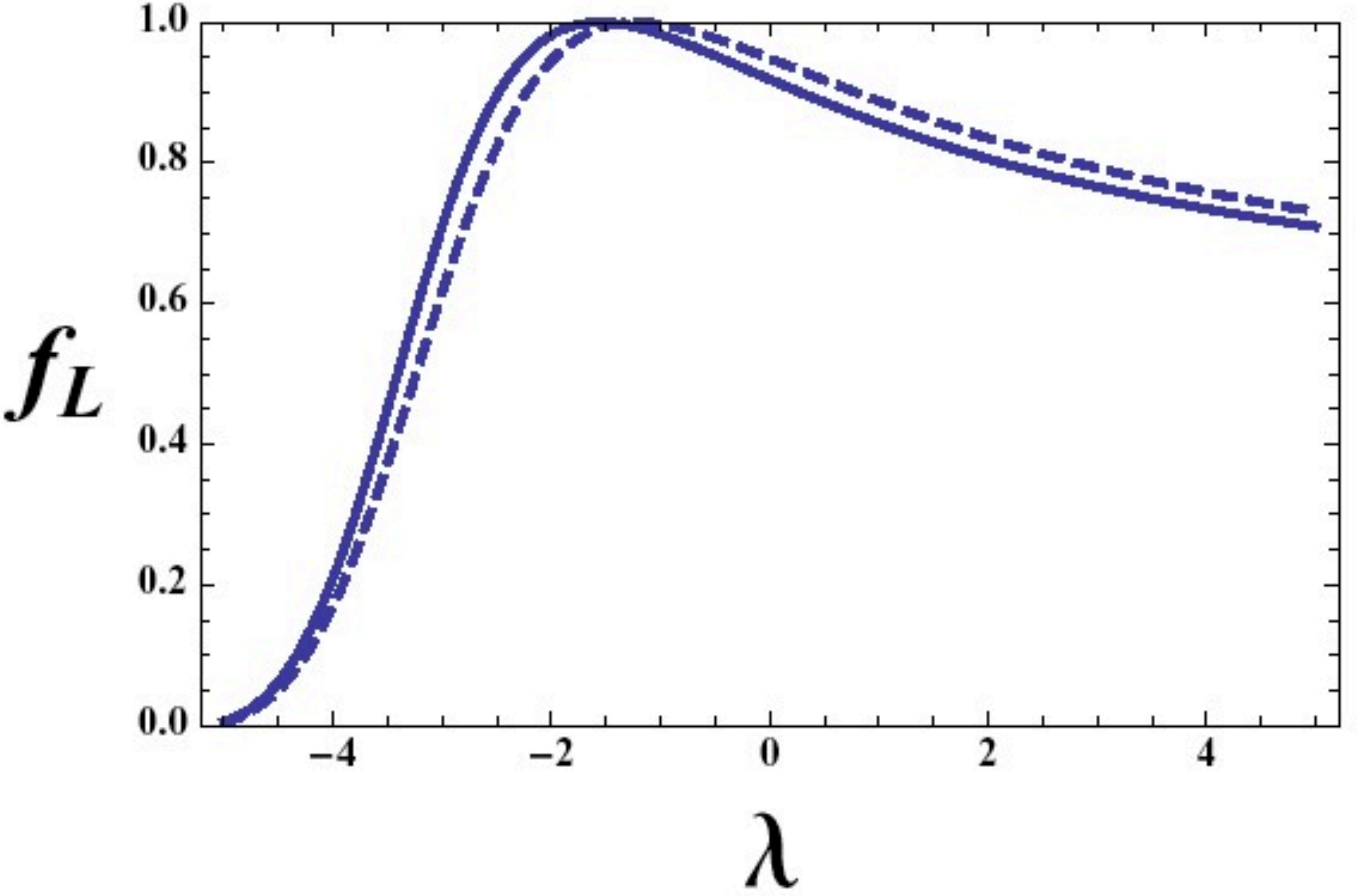
(Hu & T.M., et. al. PRD73:054003 (2006))



- Longitudinal Polarization $(\alpha < -0.5)$ for $-3.5 \leq \lambda \leq 5$

(solid line - $r_\beta = 1.0$, dotted line - $r_\beta = 0.66$, includes Λ/m_c corrections)

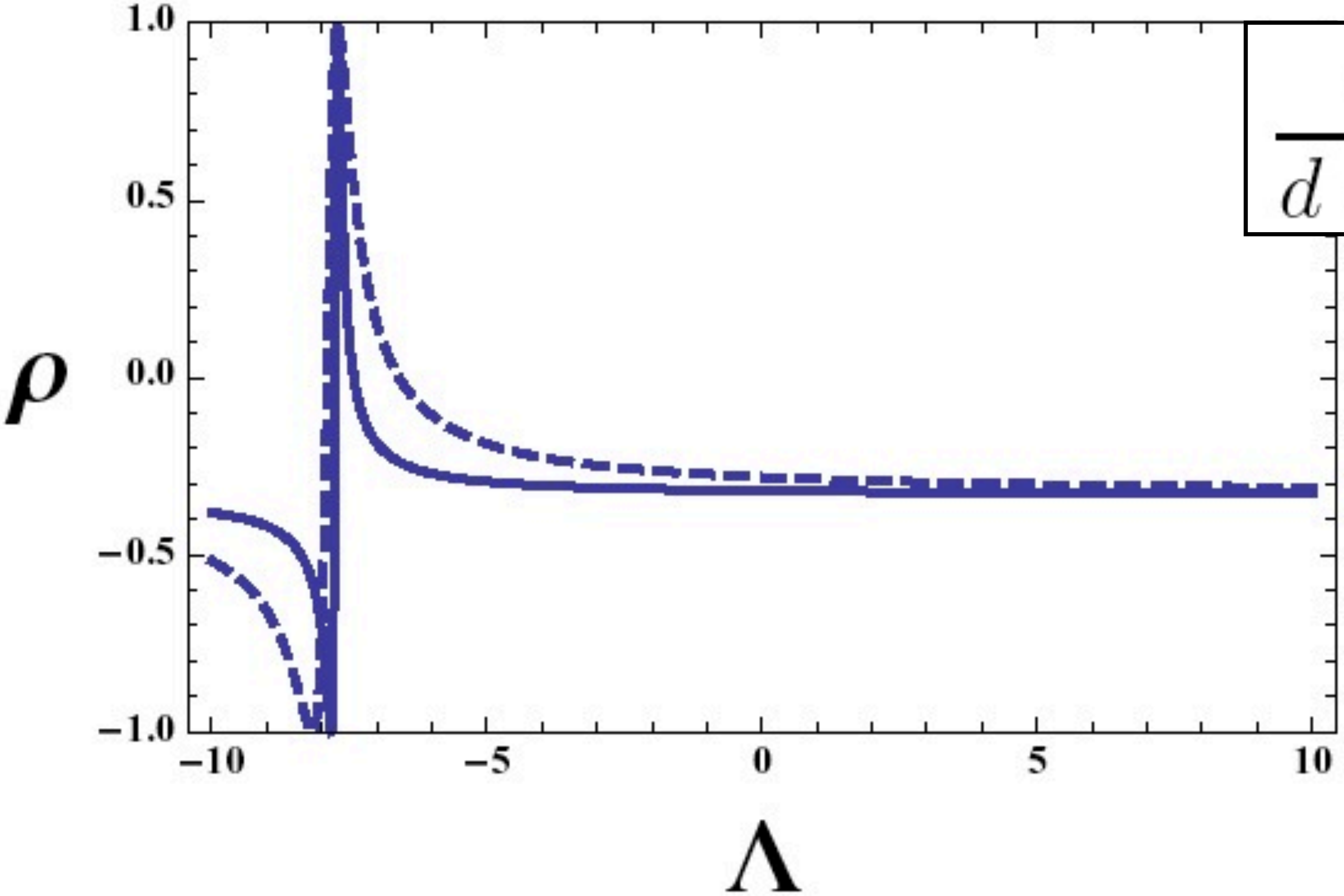
- Longitudinal Polarization vs. λ



- $e^+e^- \to \psi(4040) \to X(3872)\gamma$ (BES?)

$\psi(4040)$ produced with polarization transverse to beam axis (LO)

same (crossed) graphs as $X(3872) \to \psi(2S)\gamma$



$$\boxed{\frac{d\sigma}{d\cos\theta} \propto 1 + \rho\cos^2\theta}$$

θ - angle from beam axis

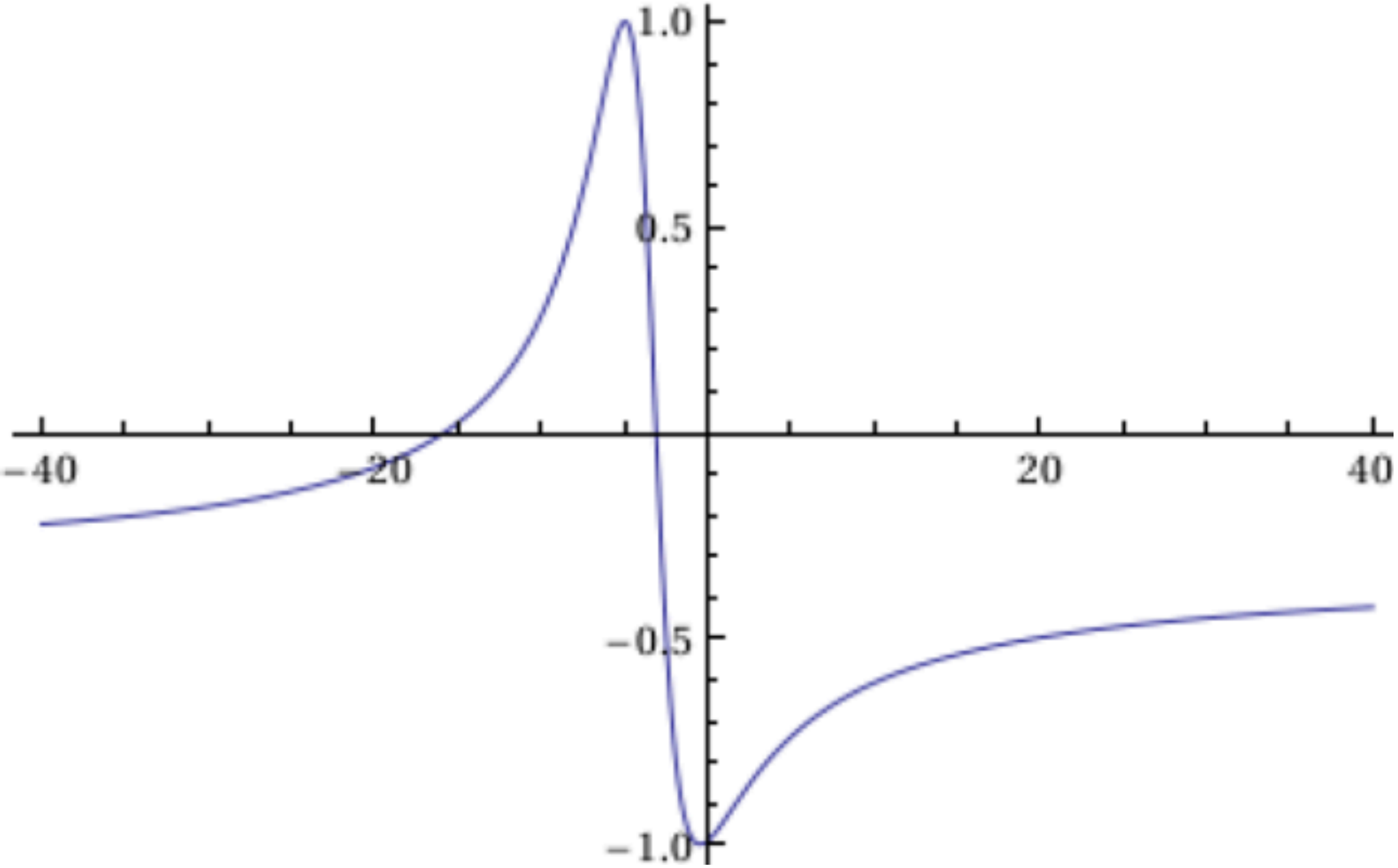
$$\Lambda \equiv \frac{3\tilde{c}_1}{\tilde{g}_2\beta_+}$$

- $J^{PC} = 2^{-+}$ predicts $\rho = 0.08$

  molecule predicts $\rho \approx -1/3$ for most of parameter space

- $\psi(4160) \to X(3871)\gamma$ Margaryan, Springer, to appear

$\psi(4160)$ D-wave state



$$\mathcal{L} = -i\frac{g}{2}Tr(J^{\mu\nu}J\sigma_\mu\partial_\nu\bar{J} - J^{\mu\nu}\bar{J}\sigma_\mu\partial_\nu J) + i\frac{ec}{2}Tr(J^{\mu\nu}J\sigma_\mu E_\nu\bar{J}) \qquad \lambda = \frac{5}{2}\frac{c_1}{g\beta_+}$$

$\Upsilon(5S) \to B\bar{B}\pi$

$$\frac{d^2\Gamma[\Upsilon(5S) \to B^+\bar{B}^0\pi^-]}{dE_B dE_{\bar{B}}} = \frac{g^2(g_\Upsilon - 2g_2)^2 m_B m_{\bar{B}}}{12\pi^3 f^2} \frac{p_B^2 p_{\bar{B}}^2 - (\vec{p}_B \cdot \vec{p}_{\bar{B}})^2}{(E_\pi - \Delta)^2}$$

$$\Gamma[\Upsilon(5S) \to B\bar{B}\pi] \sim 1\,\mathrm{keV} \quad < 0.2 - 0.5\,\mathrm{MeV}$$

$\Upsilon(5S) \to B^*\bar{B}\pi$

$$\begin{aligned}\frac{d^2\Gamma[\Upsilon(5S) \to B^{+*}\bar{B}^0\pi^-]}{dE_B dE_{\bar{B}}} = {} & \frac{m_B m_{B^*}}{192\pi^3 f^2}\left(3|A_1|^2 + |A_2|^2(p_B^2)^2 + |A_3|^2(p_{\bar{B}}^2)^2 + (|A_4|^2 + |A_5|^2)p_B^2\, p_{\bar{B}}^2\right. \\ & -\mathrm{Re}[A_1^*(A_2\, p_B^2 + A_3\, p_{\bar{B}}^2 + (A_4 + A_5)\vec{p}_B \cdot \vec{p}_{\bar{B}}] \qquad (7) \\ & +\mathrm{Re}[A_2^* A_3 + A_4^* A_5](\vec{p}_B \cdot \vec{p}_{\bar{B}})^2 + \mathrm{Re}[A_2^*(A_4 + A_5)]\, p_B^2\, \vec{p}_B \cdot \vec{p}_{\bar{B}} \\ & \left. +\mathrm{Re}[A_3^*(A_4 + A_5)] p_{\bar{B}}^2\, \vec{p}_B \cdot \vec{p}_{\bar{B}}\right)\ ,\end{aligned}$$

$$A_1^{\mathrm{tree}} = (g_{\Upsilon\pi} + g'_{\Upsilon\pi})\frac{E_\pi}{f} - \frac{2g(g_\Upsilon - 2g_2)}{f\, E_\pi}\vec{p}_\pi \cdot \vec{p}_B - \frac{2gg_\Upsilon}{f\,(E_\pi - \Delta)}\vec{p}_\pi \cdot \vec{p}_{\bar{B}}$$

$$A_2^{\mathrm{tree}} = -\frac{2g(g_\Upsilon - 2g_2)}{f\, E_\pi} + \frac{2g(g_\Upsilon + g_1 + 3g_2)}{f(E_\pi + \Delta)}$$

$$A_3^{\mathrm{tree}} = -\frac{2g(g_2 - g_1)}{f\,(E_\pi - \Delta)}$$

$$A_4^{\mathrm{tree}} = -\frac{2g(g_\Upsilon - 2g_2)}{f\, E_\pi} - \frac{2g(g_\Upsilon + g_2 - g_1)}{f\,(E_\pi - \Delta)}$$

$$A_5^{\mathrm{tree}} = \frac{2g(g_\Upsilon + g_1 + 3g_2)}{f\,(E_\pi + \Delta)} + \frac{2gg_\Upsilon}{f\,(E_\pi - \Delta)}\,.$$

$\Upsilon(5S) \to B^*\bar{B}^*\pi$

Similar lengthy expression

$$i\mathcal{M}^{1-\mathrm{loop}} = \begin{pmatrix} i\mathcal{M}^{1-\mathrm{loop}}_{B^*B^*} \\ i\mathcal{M}^{1-\mathrm{loop}}_{BB^*} \end{pmatrix} = \boxed{\begin{pmatrix} C_+ & C_- \\ C_- & C_+ \end{pmatrix}} \begin{pmatrix} L_{Z'}1\, p_\pi \cdot \epsilon_\Upsilon\, p_\pi \cdot \epsilon_{Z'} + L^2_{Z'}\, p_\pi^2\, \epsilon_\Upsilon \cdot \epsilon_{Z'} \\ L^1_Z\, p_\pi \cdot \epsilon_\Upsilon\, p_\pi \cdot \epsilon_Z + L^2_Z\, p_\pi^2\, \epsilon_\Upsilon \cdot \epsilon_Z \end{pmatrix}$$

$$L^1_Z = \frac{g m_B^{3/2}}{4\sqrt{2}\pi f}\left[-(g_\Upsilon + g_1 + 3g_2)\overline{F}(b_{BB}, E_\pi + \Delta) + (g_\Upsilon - 2g_2)\overline{F}(b_{BB^*}, E_\pi)\right.$$
$$\left.+(g_2 - g_1)\overline{F}(b_{B^*B^*}, E_\pi - \Delta)\right]$$

$$\overline{F}(b, E) = F(E/b)/\sqrt{b}.$$

$$\begin{aligned} F(x) &= \int_0^1 dy \frac{y}{\sqrt{-1 + xy - i\epsilon}} \\ &= i\left(\frac{4 - (4 + 2x)\sqrt{1-x}}{3x^2}\right) \qquad (x < 1) \\ &= \frac{(4 + 2x)\sqrt{x-1} + i4}{3x^2} \qquad (x > 1)\,. \end{aligned}$$

Scales in loop integrals

$$m_B b_{B^{(*)}B^{(*)}} \sim m_B E_\pi \sim 1\,\mathrm{GeV}^2$$

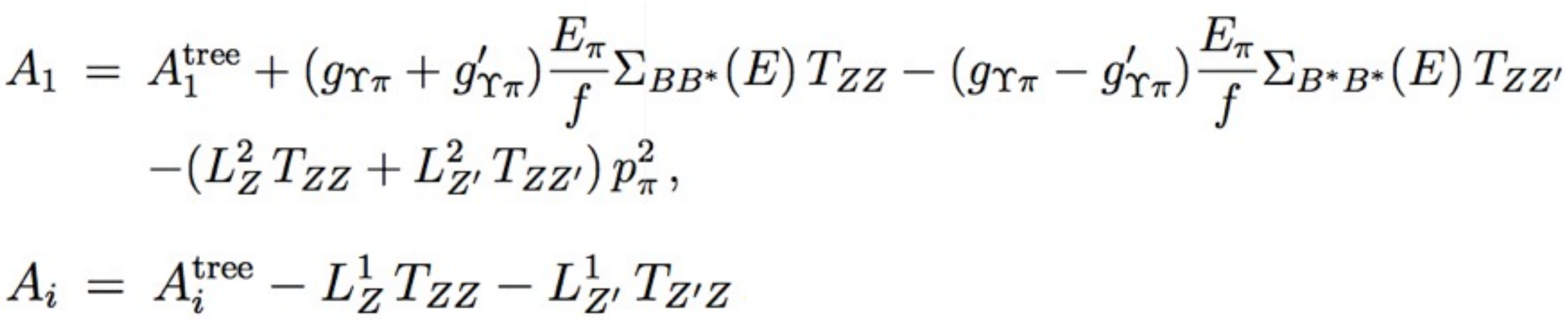
$$p_\pi^2 \lesssim 0.05\,\mathrm{GeV}^2$$

drop $O\left(\frac{p_\pi^2}{m_M b_{BB}}\right)$ in integrals

single channel - Watson’s Theorem

$$i\mathcal{M}^{\rm dressed} = i\mathcal{M}(1 + \Sigma\, T) = i\mathcal{M}\left(1 + i\frac{pM}{4\pi}\,T\right) = i\mathcal{M}\,e^{i\delta}\cos\delta$$

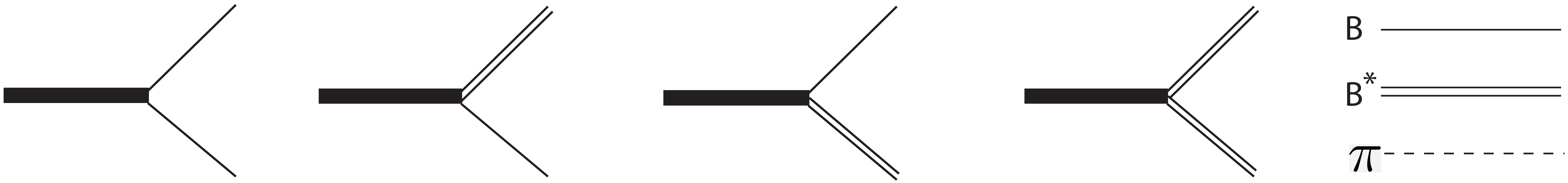
$\Upsilon(5S) \to B^*\bar{B}\pi$



$$\begin{aligned} A_1 &= A_1^{\rm tree} + (g_{\Upsilon\pi} + g'_{\Upsilon\pi})\frac{E_\pi}{f}\Sigma_{BB^*}(E)\,T_{ZZ} - (g_{\Upsilon\pi} - g'_{\Upsilon\pi})\frac{E_\pi}{f}\Sigma_{B^*B^*}(E)\,T_{ZZ'} \\ &\quad -(L^2_Z\,T_{ZZ} + L^2_{Z'}\,T_{ZZ'})\,p_\pi^2\,, \\ A_i &= A_i^{\rm tree} - L^1_Z\,T_{ZZ} - L^1_{Z'}\,T_{Z'Z} \end{aligned}$$

Similar expression for $\Upsilon(5S) \to B^*\bar{B}^*\pi$

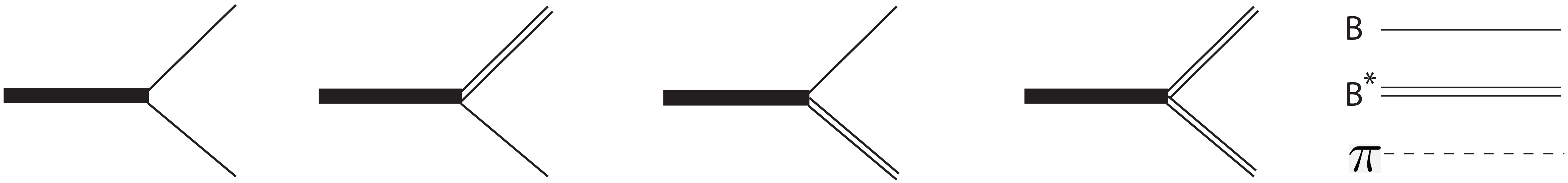
Tree-level diagrams for $\Upsilon(5S) \to B^{(*)}\bar{B}^{(*)}$



B mesons on top half of diagram and B anti-mesons on the bottom

$$\begin{aligned}\mathcal{L}_{\mathrm{HH}\chi\mathrm{PT}} = {} & \mathrm{tr}(H_a^\dagger i\partial_0 H_a) + \tfrac{1}{4}\Delta\,\mathrm{tr}(H_a^\dagger \sigma_i H_a \sigma^i) + \mathrm{tr}(\bar{H}_a^\dagger i\partial_0 \bar{H}_a) + \tfrac{1}{4}\Delta\,\mathrm{tr}(\bar{H}_a^\dagger \sigma_i \bar{H}_a \sigma^i) \\ & + g\ \mathrm{tr}(\bar{H}_a \bar{H}_b^\dagger \boldsymbol{\sigma}) \cdot \mathbf{A}_{ab} - g\ \mathrm{tr}(H_a^\dagger H_b \boldsymbol{\sigma}) \cdot \mathbf{A}_{ab} \\ & + \tfrac{1}{2}[g_\Upsilon\ \mathrm{tr}(\Upsilon \bar{H}_a^\dagger \boldsymbol{\sigma} \cdot i\overleftrightarrow{\partial} H_a^\dagger) + g_{\Upsilon\pi}\,\mathrm{tr}(\Upsilon \bar{H}_a^\dagger H_b^\dagger) A_{ab}^0] \\ & + \frac{g_1}{4}\,\mathrm{tr}[(\Upsilon\sigma^i + \sigma^i\Upsilon)\bar{H}_a^\dagger i\overleftrightarrow{\partial}^i H_a^\dagger] + \frac{g_2}{4}\,\mathrm{tr}[(\sigma^i\Upsilon\sigma^j + \sigma^j\Upsilon\sigma^i)\bar{H}_a^\dagger \sigma^i i\overleftrightarrow{\partial}^j H_a^\dagger] \\ & + \frac{g'_{\Upsilon\pi}}{4}\,\mathrm{tr}[(\Upsilon\sigma^i + \sigma^i\Upsilon)\bar{H}_a^\dagger \sigma^i H_a^\dagger] A^0 + \mathrm{h.\,c.}\,.\end{aligned}$$

Tree-level diagrams for $\Upsilon(5S) \to B^{(*)}\bar{B}^{(*)}$



B mesons on top half of diagram and B anti-mesons on the bottom

HQSS conserving

$$\begin{aligned}
\mathcal{L}_{\mathrm{HH}\chi\mathrm{PT}} = {} & \mathrm{tr}(H_a^\dagger i\partial_0 H_a) + \tfrac{1}{4}\Delta\,\mathrm{tr}(H_a^\dagger \sigma_i H_a \sigma^i) + \mathrm{tr}(\bar{H}_a^\dagger i\partial_0 \bar{H}_a) + \tfrac{1}{4}\Delta\,\mathrm{tr}(\bar{H}_a^\dagger \sigma_i \bar{H}_a \sigma^i) \\
& + g\,\mathrm{tr}(\bar{H}_a \bar{H}_b^\dagger \boldsymbol{\sigma}) \cdot \mathbf{A}_{ab} - g\,\mathrm{tr}(H_a^\dagger H_b \boldsymbol{\sigma}) \cdot \mathbf{A}_{ab} \\
& + \tfrac{1}{2}[g_\Upsilon\,\mathrm{tr}(\Upsilon \bar{H}_a^\dagger \boldsymbol{\sigma} \cdot i\overleftrightarrow{\partial} H_a^\dagger) + g_{\pi\Upsilon}\,\mathrm{tr}(\Upsilon \bar{H}_a^\dagger H_b^\dagger) A_{ab}^0] \\
& + \frac{g_1}{4}\mathrm{tr}[(\Upsilon\sigma^i + \sigma^i\Upsilon)\bar{H}_a^\dagger i\overleftrightarrow{\partial}^i H_a^\dagger] + \frac{g_2}{4}\mathrm{tr}[(\sigma^i\Upsilon\sigma^j + \sigma^j\Upsilon\sigma^i)\bar{H}_a^\dagger \sigma^i i\overleftrightarrow{\partial}^j H_a^\dagger] \\
& + \frac{g'_{\Upsilon\pi}}{4}\mathrm{tr}[(\Upsilon\sigma^i + \sigma^i\Upsilon)\bar{H}_a^\dagger \sigma^i H_a^\dagger] A^0 + \mathrm{h.c.}\,.
\end{aligned}$$

HQSS violating

Decay Rates

$$\Gamma[\Upsilon(5S)\to B\bar{B}] = \frac{p_B^3}{6\pi}\frac{m_B^2}{m_{\Upsilon(5S)}}(g_\Upsilon + g_1 + 3g_2)^2$$

$$\Gamma[\Upsilon(5S)\to B\bar{B}^*] = \Gamma[\Upsilon(5S)\to B^*\bar{B}] = \frac{p_B^3}{3\pi}\frac{m_B m_{B^*}}{m_{\Upsilon(5S)}}(g_\Upsilon - 2g_2)^2$$

$$\Gamma[\Upsilon(5S)\to B^*\bar{B}^*] = \frac{p_B^3}{6\pi}\frac{m_{B^*}^2}{m_{\Upsilon(5S)}}\left(\tfrac{20}{3}g_\Upsilon^2 + 3(\tfrac{1}{3}g_\Upsilon - g_1 + g_2)^2\right) .$$

HQSS

$$\Gamma[\Upsilon(5S)\to B\bar{B}] : \Gamma[\Upsilon(5S)\to B\bar{B}^* + \bar{B}B^*] : \Gamma[\Upsilon(5S)\to B^*\bar{B}^*] :: 1 : 4 : 7,$$

weight by phase space $\propto p_B^3$ $1 : 3.2 : 4.3.$

experiment $1 : 2.5 : 6.9$

Couplings

$$g_\Upsilon = 0.112\,\mathrm{GeV}^{-3/2} \quad g_1 = -0.048\,\mathrm{GeV}^{-3/2} \quad g_2 = 0.012\,\mathrm{GeV}^{-3/2}$$

HQSS expectation

$$g_1, g_2 \sim 0.1\,g_\Upsilon - 0.2\,g_\Upsilon$$

$g_1 \approx 2-3$ times too big g_2 OK

More Predictions for Partial Widths

$$\Gamma[W_0 \to \pi \eta_b(3S)] \; : \; \Gamma[W_0' \to \pi \eta_b(3S)] : \Gamma[Z \to \pi \Upsilon(3S)] : \Gamma[Z' \to \pi \Upsilon(3S)]$$

$$= 0.26 : 2.0 : 0.62 : 1 \qquad (\lambda_\Upsilon = 0)\,,$$

$$\Gamma[W_0 \to \pi \eta_b(3S)] \; : \; \Gamma[W_0' \to \pi \eta_b(3S)] : \Gamma[Z \to \pi \Upsilon(3S)] : \Gamma[Z' \to \pi \Upsilon(3S)]$$

$$= 0.12 : 2.1 : 0.41 : 1 \qquad (|\lambda_\Upsilon| = \infty)\,.$$

$$\Gamma[W_0 \to \pi \chi_{b1}(2P)] \; : \; \Gamma[W_0' \to \pi \chi_{b1}(2P)] : \Gamma[Z \to \pi h_b(2P)] : \Gamma[Z' \to \pi h_b(2P)]$$

$$= 0.72 : 0.57 : 0.66 : 1 \qquad (g_{\pi\chi}/g_\chi = 0\,\text{GeV}^{-1})\,,$$

$$\Gamma[W_1 \to \pi \chi_{bJ}(2P)] \; : \; \Gamma[W_2 \to \pi \chi_{bJ}(2P)] : \frac{3}{2}\Gamma[W_0 \to \pi \chi_{b1}(2P)] - \frac{1}{2}\Gamma[W_0' \to \pi \chi_{b1}(2P)]$$

$$= 0.81 : 1 : 0.43 \qquad (g_{\pi\chi}/g_\chi = 0\,\text{GeV}^{-1})\,. \qquad (42)$$