Low-energy constants and condensates from the V-A spectrum

S. Peris (U.A. Barcelona)

D. Boito (T.U. Munich) & M. Golterman (San Francisco State U.)

M. Jamin (ICREA/IFAE, Barcelona) & K. Maltman (York U. & U. Adelaide)

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Introduction

Knecht, de Rafael '98
Davier, Girlanda, Hoecker, Stern '98

- Let $\rho_{V,A}$ be the non-strange $I=1$, $J=0+1$, vector and axial spectral functions, *without* the π pole.

  Define

$$\widehat{\Pi}^{(\mathbf{w})}_{V-A}(Q^2) = \int_{4m_\pi^2}^{\infty} dt\, \mathbf{w}(t/s_0)\, \frac{\rho_V(t)-\rho_A(t)}{t+Q^2} \quad , \quad \mathbf{w}(\mathbf{x}) \text{ polynomial}$$

  i.e. $\widehat{\Pi}^{(\mathbf{1})}_{V-A}$ is the $\langle VV-AA\rangle$ correlator $\Pi_{V-A}(Q^2)$ minus the pion pole.

  GOAL:

- Calculate L_{10}^{eff} and C_{87}^{eff} related to the similar $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$ LECs in ChPT:

$$\widehat{\Pi}^{(\mathbf{1})}_{V-A}(Q^2) = -\,8\,L_{10}^{eff} - \,16\,C_{87}^{eff}\,Q^2 + \cdots\ , \quad Q^2 \to 0$$

- the dim-6,8 condensates $C_{\{6,8\};V-A}$; $\quad C_{\{2,4\};V-A} \sim \{\alpha_s m_q^2, \alpha_s m_\pi^2\}$, known.

$$\Pi_{V-A}(Q^2) = \frac{C_{2,V-A}}{Q^2} + \frac{C_{4,V-A}}{Q^4} + \frac{C_{6,V-A}}{Q^6} + \frac{C_{8,V-A}}{Q^8} + \cdots\ , \quad Q^2 \to \infty$$

- in massless Pert. Theory all these terms vanish $\Rightarrow$ they contain non perturbative information about QCD.

Long/Short-distance Constraints

If s_0 large enough , $\mathbf{w}(t)$ polynomial

Floratos, Narison, de Rafael '79
Cata, Golterman, SP '05

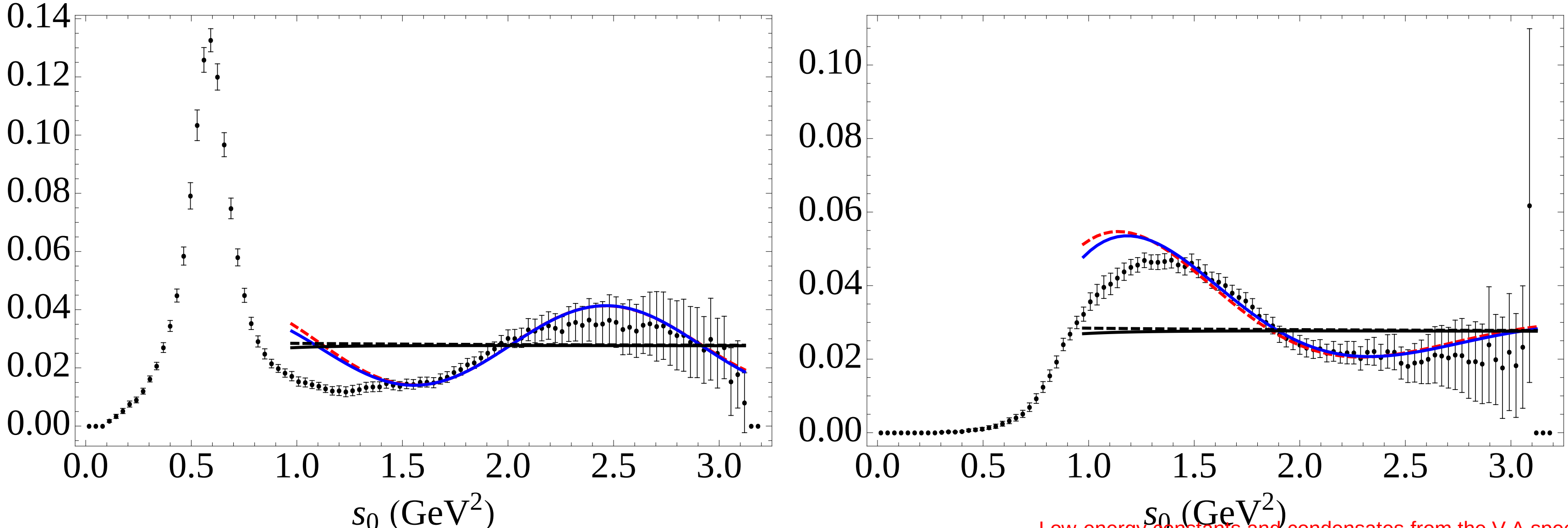
$$\int_{4m_\pi^2}^{s_0} dt \ \mathbf{w}(t) \ \rho^{Exp.}_{V-A}(t) - 2\, f_\pi^2 \ \mathbf{w}(m_\pi^2) + \underbrace{\int_{s_0}^{\infty} dt \ \mathbf{w}(t) \ \rho^{DV}_{V-A}(t)}_{\text{Duality Violations}}$$

$$= -\frac{1}{2\pi i} \oint_{|z|=s_0} dz \ \mathbf{w}(z) \ \Pi^{OPE}_{V-A}(z)$$

$$\rho^{DV}_{V/A}(\mathbf{t}) = \ \mathrm{e}^{-\ \delta_{V/A} -\ \gamma_{V/A}\mathbf{t}} \ \sin\left(\alpha_{V/A} + \beta_{V/A}\mathbf{t}\right) \quad , \quad \mathbf{t} \text{ large}, \quad (8 \text{ parameters})$$

OPAL V and A spectra:

Boito et al. '11, '12



L_{10}^{eff} and C_{87}^{eff}

Gonzalez-Alonso, Pich, Prades '08, '10a, '10b

Using

$$\widehat{\Pi}^{(\mathbf{w})}_{V-A}(Q^2) = \int_{4m_\pi^2}^{\infty} dt\ \mathbf{w}(t/s_0)\ \frac{\rho_{V-A}(t)}{t+Q^2} = \int_{4m_\pi^2}^{s_0} dt\, \mathbf{w}(t/s_0)\ \frac{\rho^{OPAL}_{V-A}(t)}{t+Q^2} + \int_{s_0}^{\infty} dt\, \mathbf{w}(t/s_0)\ \frac{\rho^{DV}_{V-A}(t)}{t+Q^2}$$

with $\mathbf{w}_k(x) = (1-x)^k, \quad k = 0,1,2$ and $\rho_{V-A} \approx \rho^{DV}_{V-A}$ for $t \geq s_0 \simeq 1.5\ \mathrm{GeV}^2$

one finds

$$L_{10}^{eff} = \left\{ \begin{array}{c} -\frac{1}{8}\ \widehat{\Pi}^{(\mathbf{w_0})}_{V-A}(0) \\ -\frac{1}{8}\ \left(\widehat{\Pi}^{(\mathbf{w_1})}_{V-A}(0) + \frac{2f_\pi^2}{s_0} \right) \\ -\frac{1}{8}\ \left(\widehat{\Pi}^{(\mathbf{w_2})}_{V-A}(0) + \frac{4f_\pi^2}{s_0} \left[1 - \frac{m_\pi^2}{2s_0} + \cdots \right] \right) \end{array} \right\} = \left\{ \begin{array}{l} -(6.52 \pm 0.14) \times 10^{-3} \\ -(6.52 \pm 0.11) \times 10^{-3} \\ -(6.45 \pm 0.09) \times 10^{-3} \end{array} \right.$$

$$C_{87}^{eff} = -\frac{1}{16}\ \frac{d}{dQ^2} \widehat{\Pi}^{(\mathbf{w_0})}_{V-A}(0) = (8.47 \pm 0.29) \times 10^{-3}\ \mathrm{GeV}^{-2}$$

Comparison with previous results

Central values for L_{10}^{eff} and C_{87}^{eff} compatible with previous results found in recent literature (e.g. Gonzalez-Alonso et al. '10) but our errors $\sim$ twice as large.

★We believe errors in previous determinations underestimated★

Main reasons:

- We use OPAL data, although ALEPH data is formally more precise, because ALEPH correlations are presently *incomplete*. (Boito et al. '11)
- Previous work either did not consider DV at all, or used an overly simplified form for ρ_{V-A}^{DV} (e.g. with only 4 parameters), not compatible with the spectrum observed in V, A or both. In contrast, we needed 4 (Vector)+ 4 (Axial)= 8 parameters.

The LEC $L^r_{10}(M_\rho)$ in ChPT

Even though $Q^2 = 0$, Nature's result for L^{eff}_{10} contains terms to all orders in m_π, m_K.

$\Rightarrow$ The precise relation between $L^r_{10}(M_\rho)$ and L^{eff}_{10} contains $\mathcal{O}(p^6)$ (and higher) LECs.

(Amoros et al. '00)

$\Rightarrow$ Even restricted at $\mathcal{O}(p^6)$, to get $L^r_{10}(M_\rho)$ from L^{eff}_{10} one needs to

either

- a) make model assumptions (systematic error?) about those $\mathcal{O}(p^6)$ LECs:

$$L^r_{10}(M_\rho) = -(4.06 \pm 0.39) \times 10^{-3}$$ (Glez-Alonso et al. '08)

or

- b) determine them with help from the lattice freedom to change quark masses:

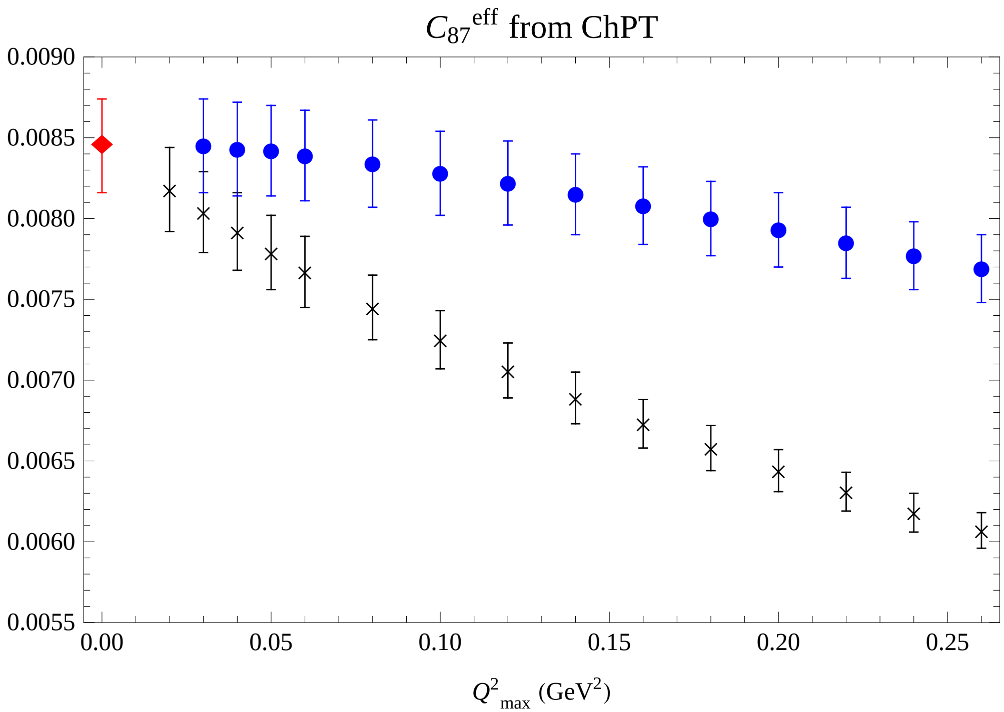
$$L^r_{10}(M_\rho) = -(3.1 \pm 0.8) \times 10^{-3}$$ (This work and Boyle et al. RBC/UKQCD)

(some model assumptions in a) violated by lattice results $\Rightarrow$ larger error.)

Recent refinement: look at ud-us flavor combination of $\langle VV - AA \rangle$

$$L^r_{10}(M_\rho) = -(3.5 \pm 0.3) \times 10^{-3}$$ (K. Maltman's talk at lattice 2013)

$C_{87}^r(M_\rho)$ *and the convergence of ChPT*



Red point: Value at $Q^2 = 0$.

Black crosses: $\mathcal{O}(p^6)$ ChPT fit up to Q^2_{max}.

Blue points: $\mathcal{O}(p^6)$ ChPT fit up to Q^2_{max} with an $\mathcal{O}(p^8)$ term ppal to Q^4.

$\Rightarrow$ Clear evidence for significant $\mathcal{O}(p^8)$ terms.
Much less impact in L_{10}^{eff}.

No ChPT calculation of terms $\mathcal{O}(p^8)$: use $\mathcal{O}(p^6)$ and *estimate* the error:

$$C_{87}^r(M_\rho) \simeq (4 \pm 1) \times 10^{-3}\ \text{GeV}^2$$

cfr. $\quad C_{87}^r(M_\rho) \simeq (4.89 \pm 0.19) \times 10^{-3}\ \text{GeV}^2 \qquad$ (Glez-Alonso et al. '08)

Dim-6,8 condensates

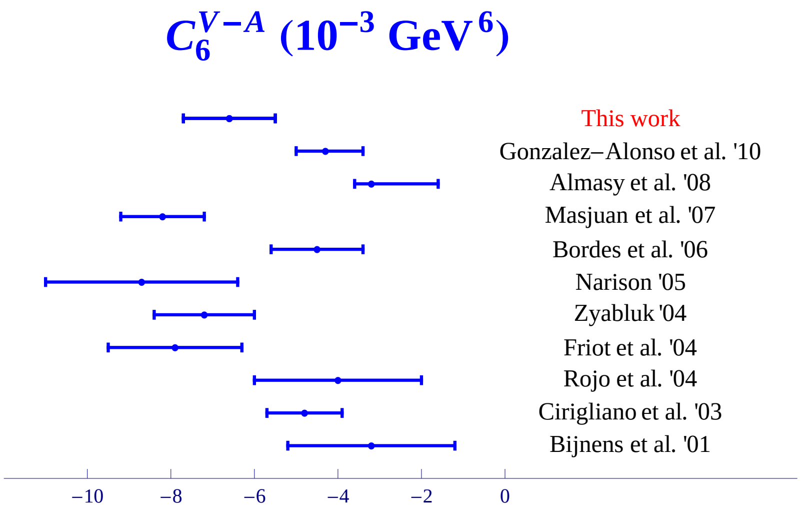
Choose $\mathbf{w}(t)$ in FESR (recall $s_0 \simeq 1.5\ \mathrm{GeV}^2$):

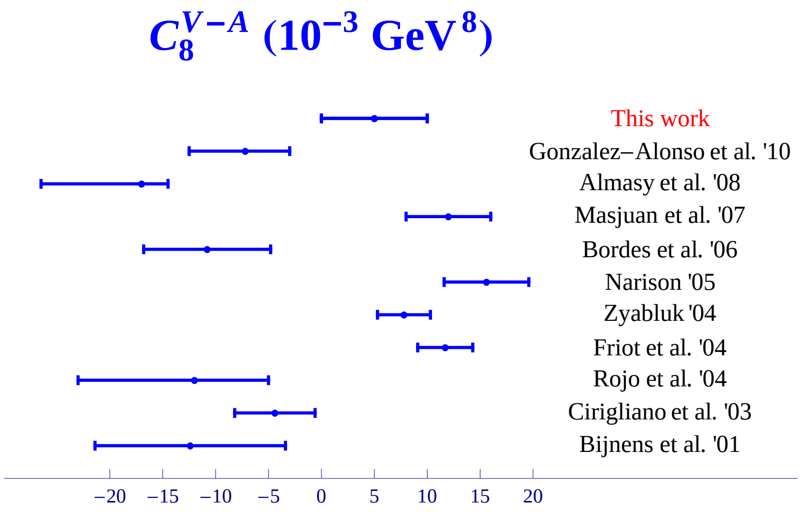
$$\mathbf{w}(t) = (t - s_0)^2 \quad \longrightarrow \quad C_{6,V-A} = (-6.6 \pm 1.1) \times 10^{-3}\ \mathrm{GeV}^6$$

$$\mathbf{w}(t) = (t - s_0)^2 (t + 2 s_0) \quad \longrightarrow \quad C_{8,V-A} = (5 \pm 5) \times 10^{-3}\ \mathrm{GeV}^8$$

Observations:

- Sensitive to details in (upper end) of spectrum (large errors).
- Sensitive to details in fits, in particular to the inclusion of DVs.
- No evidence for breakdown of Vacuum Saturation Approx. with $\langle \overline{q} q(m_\tau) \rangle = -(272\ \mathrm{MeV})^3$ (Beneke, Jamin '08).





Summary and Outlook

- First-ever fully self-consistent determination of “LECs" $L_{10}^{eff}, C_{87}^{eff}$ and condensates $C_{6,8;V-A}$ including quark-hadron DVs in both V and A channels.
  (DVs: physically motivated model; good description of data.)

- While central values for $L_{10}^{eff}, C_{87}^{eff}$ are data dominated, errors do depend on DVs.

- For $C_{6,8;V-A}$, central values already sensitive to DVs and upper end of spectrum.

- $L^r_{10}(M_\rho), C^r_{87}(M_\rho)$ require additional knowledge about specific chiral corrections. Lattice (by tuning quark masses) may help!

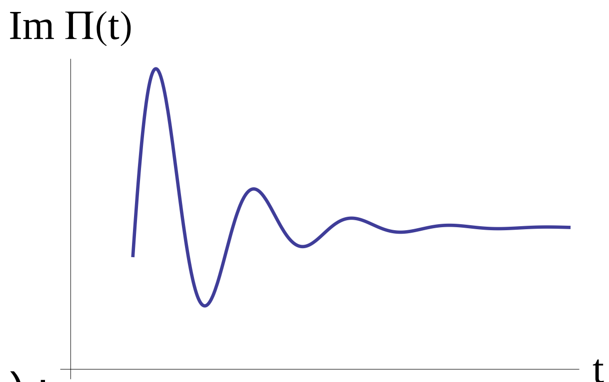
- Analysis made with OPAL. Once Aleph data with correct correlations is available, better errors are in principle possible.

Inclusive data from BaBar and Belle very important !

BACK-UP SLIDES

Duality Violations(I)

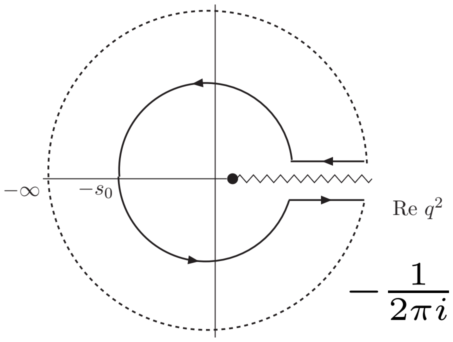
- OPE valid in euclidean, but not in minkowski. We know that spectrum $\neq$ OPE



- We expect (@ large t) :

$$\mathrm{Im}\Pi_{DV} \sim \mathrm{Im}(\Pi - \Pi_{OPE}) \sim \kappa \underbrace{e^{-\gamma t}}_{\text{OPE asympt.}} \underbrace{\sin(\alpha + \beta t)}_{\text{Regge}}$$

- $\Pi_{DV}(s) \to 0$ as $|s| \to \infty$. Then:



(Cata-Golterman-S.P. '05)

$$-\frac{1}{2\pi i}\oint_{|z|=s_0} dz\, w(z)\, \Pi_{DV}(z) = - \underbrace{\int_{s_0}^{\infty} ds}_{\text{extrapolation!}} \; w(s)\, \frac{1}{\pi}\mathrm{Im}\Pi_{DV}(s)$$

D. Violations(II)

Blok-Shifman-Zhang '98; Cata-Golterman-SP '05'08; Jamin '11

Explicit realization only models, no theory. Take $\Lambda_{QCD} = 1$; $F \sim 0.1$, decay constant.

- 1 resonance ($M \to M + i\Gamma/2$):

$$\frac{F^2}{q^2 - n} \longrightarrow \frac{F^2}{q^2 - n - i\sqrt{n}\,\Gamma}$$

- Regge-like tower: $n = 1, 2, 3, \ldots$

$$\Pi(q^2) \sim \sum_n^{\infty} \frac{F^2}{z+n} \quad , \quad z = \underbrace{(-q^2)^{\zeta}}_{\text{cut},\ q^2>0} \quad , \quad \zeta \simeq 1 - \mathcal{O}(\frac{1}{N_c})$$

$$\sim \quad \psi(z) = \frac{d \log \Gamma(z)}{dz}$$

- For $q^2 < 0 \longrightarrow \Pi(q^2) \sim \log z + \sum \frac{c_n}{z^n}$

- For $q^2 > 0 \longrightarrow \psi(z) = \psi(-z) - \frac{1}{z} - \pi \cot(\pi z)$,

$$\mathrm{Im}\Pi(q^2) \sim \mathrm{Im}(\log z) + \mathrm{Im} \sum \frac{c_n}{z^n} + \underline{\underline{F^2 \ \mathrm{e}^{\frac{-q^2}{N_c}} \ \sin(\alpha + \beta\, q^2)}} \qquad F \sim 0.1 \quad ; \quad \alpha, \beta \sim 1$$