LATTICE INPUT ON THE INCLUSIVE τ DECAY V_{us} PUZZLE

KM, P. Boyle, L. Del Debbio, N. Garron, R. Hudspith, E. Kerrane, J. Zanotti

PHIPSI 2013, Rome, September 12/13

OUTLINE

- *FB τ and τ-EM sum rules for V_{us} (including OPE issues and the inclusive τ decay determination puzzle)*
- *Lattice results c.f. the OPE for the relevant correlators*
- *Lattice-motivated preliminary analyses: impact on V_{us}*

THE V, A CURRENT-CURRENT CORRELATORS

- $J = 0, 1$ scalar correlators (spin decomposition of V, A current-current 2-point functions)
  - $\Pi^{(J)}_{ij;V/A}(Q^2)$ for flavor $ij = ud, us$ V, A currents
  - $\Pi_{EM}(Q^2)$ for EM current (pure $J = 1$)

- Data on corresponding spectral functions $\rho(s)$
  - $\rho_{EM}(s)$ from $\sigma[e^+e^- \rightarrow hadrons]$
  - $\rho^{(J)}_{ij;V/A}(s)$ from hadronic τ decay ratio $R_{ij;V/A}$

$$R_{ij;V/A} \equiv \frac{\Gamma[\tau \rightarrow \nu_\tau \, \text{hadrons}_{ij;V/A}\,(\gamma)]}{\Gamma[\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e(\gamma)]}$$

FLAVOR-BREAKING τ, τ-EM FESRs FOR V_{us}

- The "conventional" flavor-breaking τ FESR
  - FB combination $\delta R_\tau \equiv \frac{R_{ud;V+A}}{|V_{ud}|^2} - \frac{R_{us;V+A}}{|V_{us}|^2}$
  - FESR relation (Cauchy's Theorem), valid for any $s_0 > 0$, analytic $w(s)$, $\Pi = \Pi^{(0+1)}_{ij;V/A}$ or $s\,\Pi^{(0)}$

$$\int_0^{s_0} ds\, w(s)\, \rho(s) = \frac{-1}{2\pi i} \oint_{|s|=s_0} ds\, w(s)\, \Pi(s)$$

  - External V_{ud}, OPE for $[\delta R_\tau] \Rightarrow$

$$|V_{us}| = \sqrt{\frac{R_{us;V+A}}{\frac{R_{ud;V+A}}{|V_{ud}|^2} - [\delta R_\tau]^{OPE}}}\ .$$

- Details/complications

  * Bad $D = 2$, $J = 0$ OPE behavior forces $J = 0$ dR/ds subtraction

  * Known π, K subtractions dominant; phenomenology for small chirally suppressed us residual

  * $J = 0$ subtraction $\rightarrow |Vij|^2 \rho^{(0+1)}_{ij;V/A}(s)$, $R_{ij} \rightarrow R^w_{ij}(s_0)$ $\Rightarrow |V_{us}|$ from FESRs **with arbitrary** $w(s)$**,** s_0, correlator

$$\Delta\Pi_\tau \equiv \Pi^{(0+1)}_{ud;V+A} - \Pi^{(0+1)}_{us;V+A}$$

  * Self-consistency tests: $|V_{us}|$ independent of $w(s)$, s_0? (Impact of slow $D = 2$ OPE convergence?)

- The "conventional" $\Delta\Pi_\tau$ $|V_{us}|$ determination
  - * Kinematic weight choice $w_\tau(s)$, 4- (or 5)-loop CIPT $D=2$, $s_0=m_\tau^2$ only
  - * w_τ, $s_0=m_\tau^2 \Rightarrow |V_{us}|$ from BFs only
  - * Downside: no stability tests **[systematics??]**
  - * Results typically 3σ low c.f. 3-family unitarity expectation $|V_{us}|=0.2255(10)$
  - * E.g.: 4-loop CIPT $D=2$, preliminary BaBar $B[\tau^-\to K^- n\pi^0\nu_\tau]$ updates [Adametz thesis]

$$|V_{us}|=0.2176(25)_{exp}(10??)_{th}$$

- Issues for the "conventional" analysis
  - * Varying $w(s)$, s_0 exposes significant stability problems [more later]
  - * us data problem? Theory problem? Both?
  - * Dependence of $|V_{us}|$ on choice of scheme for truncated $D=2$ OPE series (CIPT vs FOPT) much larger than previously estimated theory error
  - * CIPT vs FOPT suggests slow convergence of truncated $J=0+1$ $D=2$ series a potential issue
  - * To be investigated using lattice data below

- Mixed τ-EM FB V_{us} sum rules [PLB 672 (2009) 257]
  - FESRs for the FB correlator combination

$$\Delta\Pi_{\tau-EM} \equiv 9\Pi_{EM} - 5\Pi^{(0+1)}_{ud;V} + \Pi^{(0+1)}_{ud;A} - \Pi^{(0+1)}_{us;V+A}$$

  - Constructed to kill LO $D = 2$ OPE coefficient
  - Strong HO $D = 2$, $D = 4$ OPE suppression for free
  - $D = 2, 4$ suppression NOT due to hidden symmetry as VSA version of $D = 6$ not suppressed

- Generic $D=2,4$ OPE, $\Delta\Pi = \Delta\Pi_\tau$, $\Delta\Pi_{\tau-EM}$, $\Pi_{\ell\ell-ss;V}$

$$\left[\Delta\Pi(Q^2)\right]^{OPE}_{D=2} = \frac{3}{2\pi^2}\frac{\bar{m}_s^2}{Q^2}\sum_{k=0} c_k^{(2)}\,\bar{a}^k$$

$$\left[\Delta\Pi(Q^2)\right]^{OPE}_{D=4} = \frac{\left[\langle m_\ell\bar{\ell}\ell\rangle - \langle m_s\bar{s}s\rangle\right]}{Q^4}\sum_{k=0} c_k^{(4)}\,\bar{a}^k$$

$$\left(\bar{a} = \frac{\alpha_s(Q^2)}{\pi},\ \bar{m}_s = m_s(Q^2)\right)$$

$\Delta\Pi$	D	$c_0^{(D)}$	$c_1^{(D)}$	$c_2^{(D)}$	$c_3^{(D)}$
$\Delta\Pi_\tau$	2	1	7/3	19.93	208.75
$\Pi_{\ell\ell-ss;V}$	2	1	8/3	24.32	253.69
$\Delta\Pi_{\tau-EM}$	2	0	−1/3	−4.384	−44.943
$\Delta\Pi_\tau$	4	2	−2	−26/3	
$\Pi_{\ell\ell-ss;V}$	4	2	2/3	11	
$\Delta\Pi_{\tau-EM}$	4	0	−8/3	−59/3	

- Implications/issues for the $\Delta\Pi_\tau$ determination
  - $a(m_\tau^2) \sim 0.1 \Rightarrow D = 2$ $\Delta\Pi_\tau$ series asymptotic behavior apparently already set in at spacelike point for *all* kinematically accessible s_0
  - $D = 2$ scheme not unique (e.g., CIPT: "local scale" $\mu^2 = Q^2$ point-by-point on contour, c.f. FOPT: "fixed scale" $\mu^2 = s_0$)
  - "Conventional" analysis CIPT choice: *nominally* improved convergence misleading (4-, 5-loop contributions suppressed by cancellations on contour)
  - Slow true convergence reflected in ~ 0.0020 difference in $|V_{us}|$ from FOPT or CIPT for $D = 2$ at 4-loop and (estimated) 5-loop truncation orders

- Implications for the $\Delta\Pi_{\tau-EM}$ determination

  * Vanishing LO $D = 2, 4$ $\Delta\Pi_{\tau-EM}$ coefficients, suppressed HO $D = 2$ c.f. $\Delta\Pi_\tau \Rightarrow$ *expect* much reduced $|V_{us}|$ theoretical uncertainty from $[\delta R_\tau]^{OPE}$ analogue in $\Delta\Pi_{\tau-EM}$ FESRs

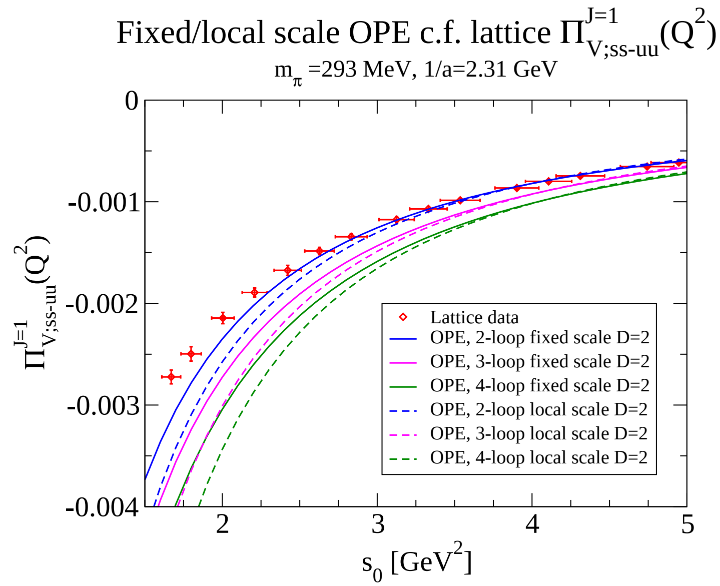
  * Some price to be paid in enhanced experimental errors (no cancellation of τ, EM normalization uncertainties; impact of ud V, EM cancellation)

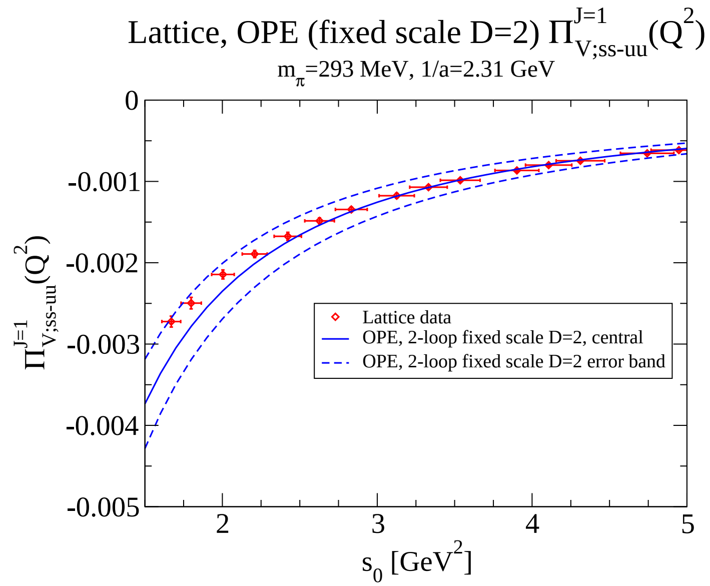
  * Key issue: Strong $\Delta\Pi_{\tau-EM}$ suppression c.f. $\Delta\Pi_\tau$ seen in OPE real, or an artifact of the limited number of remaining known $D = 2, 4$ series terms after deliberate LO coefficient suppression?

LATTICE c.f. OPE FOR THE VARIOUS FB $\Delta\Pi(Q^2)$

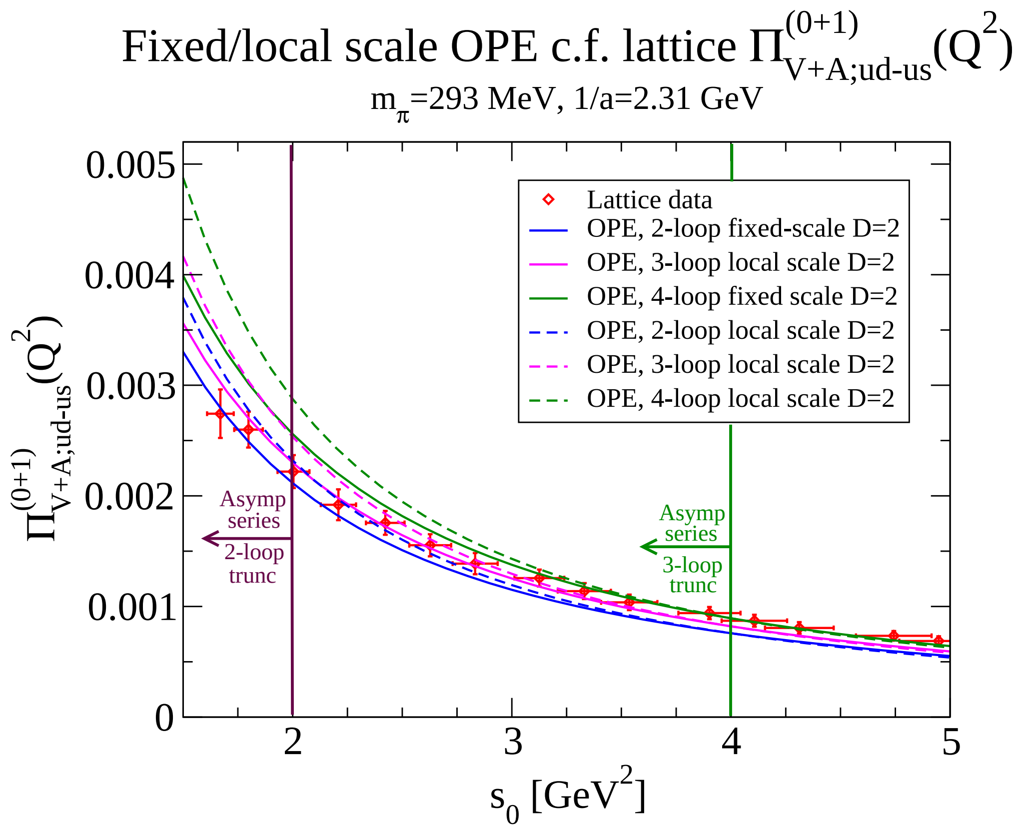
- RBC/UKQCD data on $\Delta\Pi(Q^2)$ from
  - Fine $1/a = 2.31$ GeV, $32^3 \times 64 \times 16_5$ Iwasaki DWF ensembles with m_π = 293, 349, 399 MeV, $m_\pi L$ = 4.1, 4.8, 5.5
  - New coarse $1/a = 1.37$ GeV, $32^3 \times 64^\times 32_5$ Iwasaki + DSDR DWF ensembles with m_π = 171, 248 MeV, $m_\pi L = 4.0,\ 5.5$
- *Simulation details: PRD83 (2011) 074508; PRD87 (2013) 094514*
- Compare lattice, OPE for lattice m_q, spacelike Q^2

Lattice, OPE for $\Pi_{ss-\ell\ell}(Q^2)$, various $D = 2$ treatments

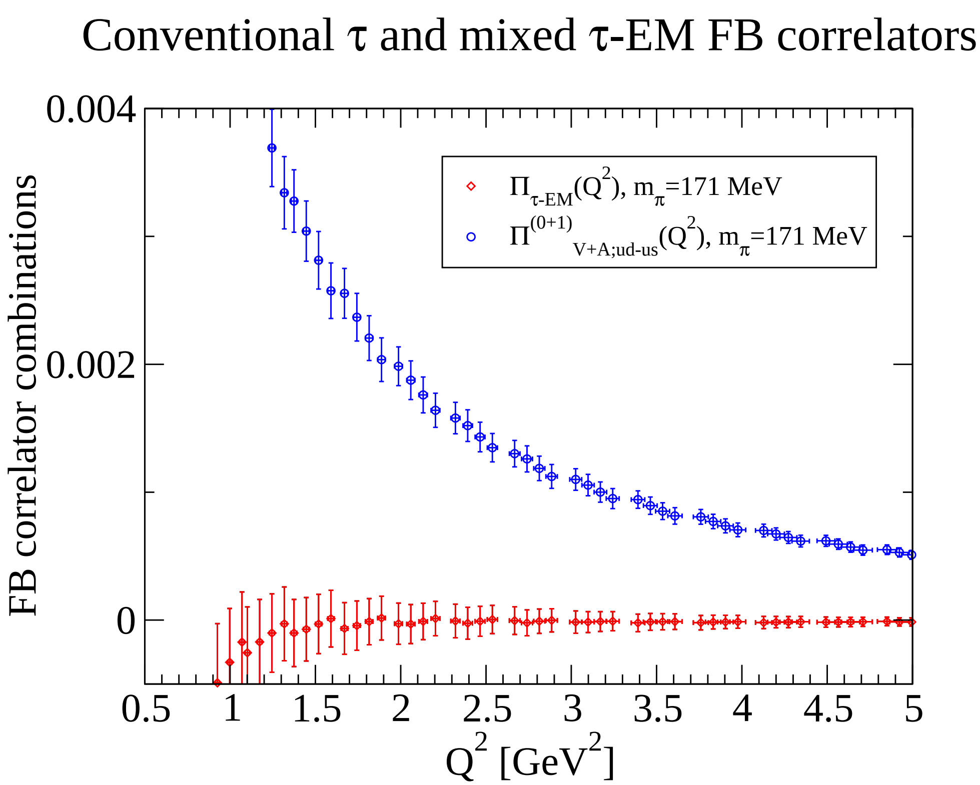




Lattice and OPE for $\Delta\Pi_\tau(Q^2)$, various truncations



Lattice input on the $\Delta\Pi_{\tau-EM}$ vs. $\Delta\Pi_{\tau}$ suppression



TENTATIVE LESSONS FROM THE LATTICE DATA

- $\Delta\Pi_\tau$ $D = 2$ series behaving "asymptotically": minimum in series at 3-loops, 4-loop, estimated 5-loop $D = 2$ terms *worsen* agreement with lattice $\Delta\Pi_\tau(Q^2)$

- 3-loop fixed-scale $D = 2$ provides good $\Delta\Pi_\tau(Q^2)$ OPE representation for $Q^2 \sim 2.3\ GeV^2 \rightarrow m_\tau^2$

- $D = 2$ scheme: fixed-scale favored over local-scale (suggests FOPT over CIPT for FESR integrals)

- Strong suppression of $\Delta\Pi_{\tau-EM}$ c.f. $\Delta\Pi_\tau$ confirmed (stronger than central OPE, but within errors)

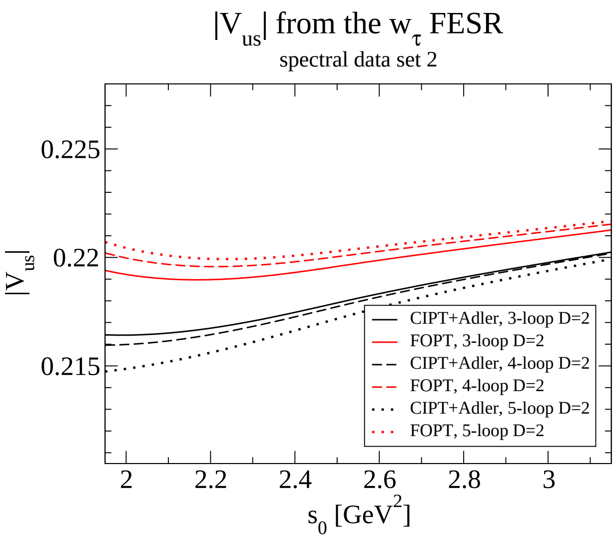
IMPACT OF LATTICE LESSONS ON $|V_{us}|$

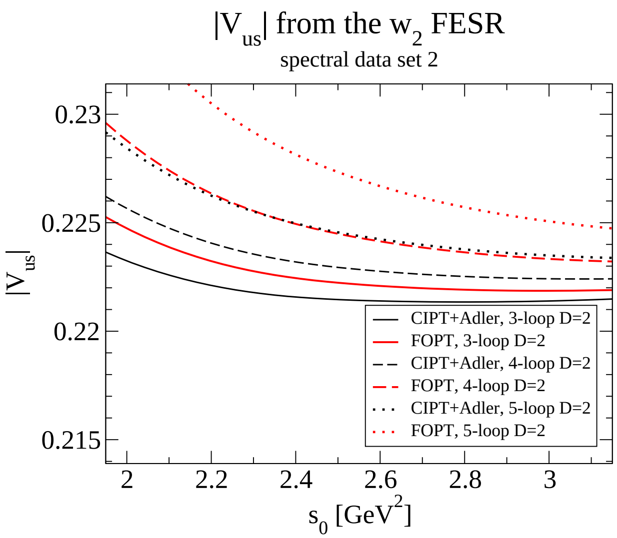
- Spectral function input for the updated analyses
  - B-factory $K\pi$, $K^-\pi^+\pi^-$, $\bar{K}^0\pi^-\pi^0$ distributions; all other us modes: ALEPH 1999 with current BFs
  - us BFs: HFAG fit with $\pi_{\mu 2}$, $K_{\mu 2}$ input, supplemented by
    - SET 1: Belle Tau 2012 $B[\tau^- \to K_s \pi^- \pi^0 \nu_\tau]$ only
    - SET 2: Additional preliminary BaBar [Adametz thesis] $B[\tau^- \to K^- n\pi^0 \nu_\tau]$ $(n = 1, 2, 3)$
  - Updated version OPAL ud distributions [PRD85 (2012) 093015] (+ global rescaling for small us BFs shifts)

- Results from the $\Delta\Pi_\tau$ FESRs (PRELIMINARY)
  - FESRs with $w = w_\tau$, w_N $(N = 2, 3, 4)$ (w_N has single numerically suppressed $D = 2N+2 > 4$ contribution, hence reduced OPE complexity)
  - For $w(s) = w_\tau(s)$
    * Significant s_0-instability for CIPT; reduced for FOPT (choice favored by lattice)
    * Above 3-loops, increase of FOPT-CIPT $|V_{us}|$ difference with increasing $D = 2$ truncation order
    * Improved s_0-stability for SET 2 c.f. SET 1

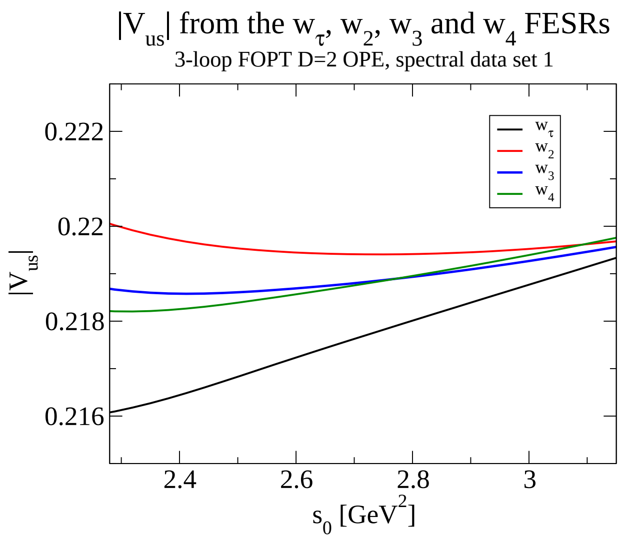
- For $w(s) = w_N(s)$
  - Reduced $|V_{us}|$ s_0-instability c.f. w_τ
  - SET 2 s_0-stability again better than SET 1
  - As for w_τ, increasing FOPT-CIPT $|V_{us}|$ difference with increasing $D = 2$ truncation order
  - Weak w-dependence for 3-loop FOPT $|V_{us}|$ in upper part of s_0 range
  - SET 2 $|V_{us}|$ compatible with 3-family unitarity (within errors)

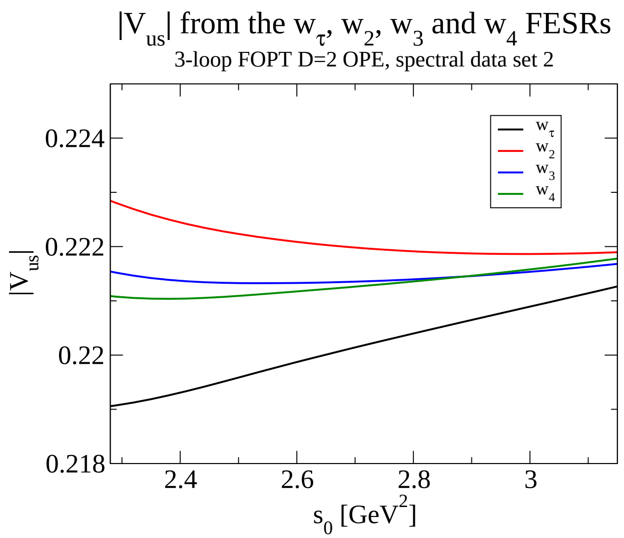
$|V_{us}|$ vs $D = 2$ truncation order, $\Delta\Pi_\tau$ FESRs





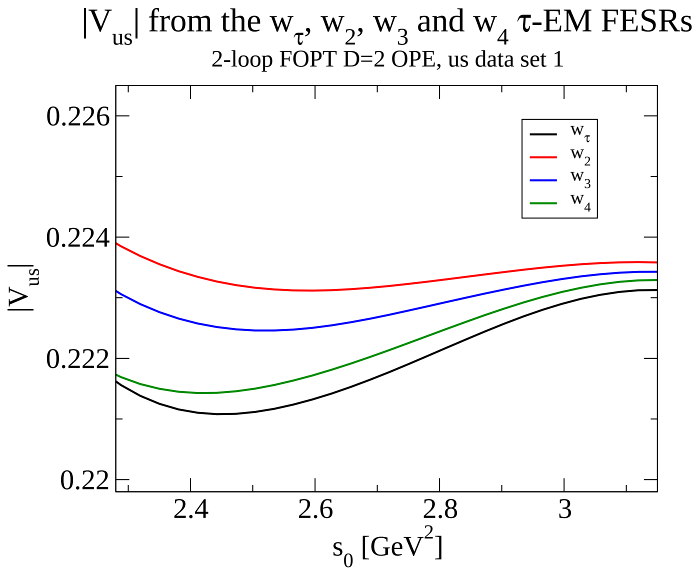
$|V_{us}|$ results, 3-loop FOPT $D = 2$ $\Delta\Pi_\tau$ FESRs

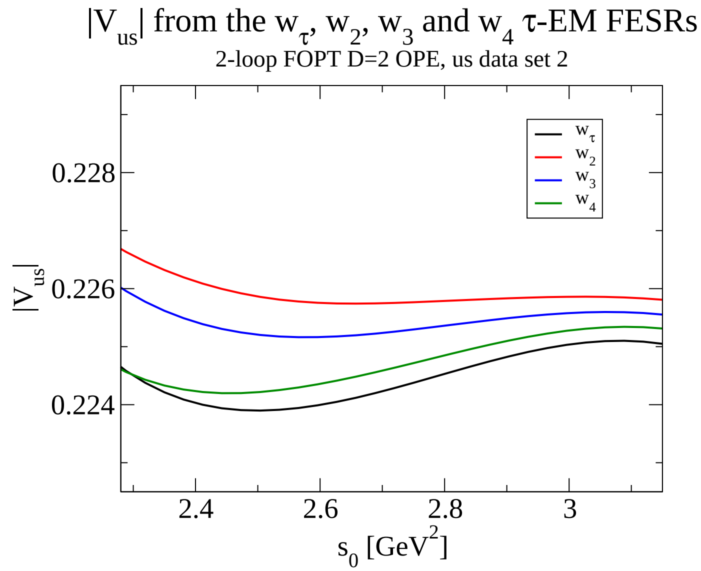




- Results for the $\Delta\Pi_{\tau-EM}$ FESRs (PRELIMINARY)
  - Same SET 1, SET 2 us, ud V, A spectral input choices; exclusive mode σ_{EM} to end of 2012
  - 2-loop FOPT D=2 OPE (best match to lattice)
  - As for $\Delta\Pi_\tau$ case, $w = w_\tau$, w_2, w_3, w_4
  - Improved s_0-stability, especially for w_N, SET 2
  - Weight dependence small in upper part of s_0 range
  - $|V_{us}|$ compatible with 3-family unitarity expectations

$|V_{us}|$ vs s_0 from the $\Delta\Pi_{\tau-EM}(Q^2)$ FESRs





SUMMARY

- $\Delta\Pi_\tau$-based $|V_{us}|$ determinations

  - $\Delta\Pi_\tau$ $D=2$ convergence problem confirmed; 3-loop FOPT truncation favored, $Q^2 \sim 2.3\ GeV^2 \to m_\tau^2$

  - $w = w_\tau$ case especially problematic (NO s_0-stability below $s_0 = m_\tau^2$) though reduced (~ 0.0010) FOPT-CIPT difference for 3-loop $D=2$

  - Conclusion: $D = 2$ OPE problems for "conventional" w_τ, $s_0 = m_\tau^2$, CIPT 4- or 5-loop $D=2$ analysis, systematic error $>>$ than previous estimates

  - **$\Rightarrow$ "Conventional" analysis (typically used by experimentalists) MUST be replaced**

- $\Delta\Pi_{\tau-EM}$-based $|V_{us}|$ determinations
  - Strong suppression of $\Delta\Pi_{\tau-EM}$ c.f. $\Delta\Pi_{\tau}$ suggested by OPE confirmed by lattice
  - Improved s_0-, $w(s)$-choice-stability for $|V_{us}|$
  - Central $s_0 \sim m_\tau^2$ $|V_{us}|$ results in agreement with expectations from 3-family unitarity, K physics
  - Ongoing work on us distributions relevant to clarifying extent of any residual $w(s)$-, s_0-instability
- In both cases, need completed Belle, BaBar analyses for us exclusive distributions to finalize errors

BACKUP SLIDES

- The scalar correlators in Minkowski and Euclidean space

- Experimental sources for the EM and $ij = ud, us$, $J = 0,1$ V, A spectral functions

- How well can the lattice do in generating the 2-point functions at large Q^2: the flavor ud V-A example

The scalar correlators

- Minkowski (continuum):

$$\Pi^{\mu\nu}_{V/A}(q^2) \equiv i \int d^4x\, e^{iq\cdot x} \langle 0| T \left(J^{\mu}_{V/A}(x) J^{\dagger\,\nu}_{V/A}(0) \right) |0\rangle$$
$$= \left(q^\mu q^\nu - q^2 g^{\mu\nu} \right) \Pi^{(1)}_{V/A}(q^2) + q^\mu q^\nu\, \Pi^{(0)}_{V/A}(q^2)$$

- Euclidean (lattice):

$$\Pi^{\mu\nu}_{V/A}(Q^2) = \left(Q^2 \delta^{\mu\nu} - Q^\mu Q^\nu \right) \Pi^{(1)}_{V/A}(Q^2) - Q^\mu Q^\nu\, \Pi^{(0)}_{V/A}(Q^2)$$

Experimental sources of ρ_{EM}, $\rho_{ud,us;V/A}$

$$\sigma_{bare}(s) = \frac{16\pi^3\alpha_{EM}(0)^2}{s}\rho_{EM}(s)$$

$$\frac{dR_{V/A;ij}}{ds} = \frac{12\pi^2\,|V_{ij}|^2 S_{EW}}{m_\tau^2}\left[w_\tau\left(\frac{s}{m_\tau^2}\right)\rho^{(0+1)}_{ij;V/A}(s) + w_L\left(\frac{s}{m_\tau^2}\right)\rho^{(0)}_{ij;V/A}(s)\right]$$

Lattice results for $\Pi^{0+1}_{ud;V-A}(Q^2)$ at $Q^2 \sim$ a few GeV^2