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# SPIN CORRELATIONS OF THE FINAL LEPTONS IN THE TWO-PHOTON PROCESSES $\gamma\gamma \to e^+e^-,\ \mu^+\mu^-,\ \tau^+\tau^-$

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### Abstract

The spin structure of the process of electron-positron pair production by two photons $\gamma\gamma \to e^+e^-$ is theoretically investigated. It is shown that, if the primary photons are unpolarized, the final electron and positron are unpolarized as well but their spins are strongly correlated. Explicit expressions for the components of the correlation tensor of the final $(e^+e^-)$ system are derived, and the relative fractions of singlet and triplet states of the $(e^+e^-)$ pair are found. It is demonstrated that in the process $\gamma\gamma \to e^+e^-$ one of the incoherence inequalities of the Bell type for the correlation tensor components is always violated and, thus, spin correlations of the electron and positron in this process have the strongly pronounced quantum character. Analogous consideration can be wholly applied as well to the two-photon processes $\gamma\gamma \to \mu^+\mu^-$ and $\gamma\gamma \to \tau^+\tau^-$, which become possible at considerably higher energies.

## 1 Spin structure of the two-photon process $\gamma\gamma \to e^+e^-$ and correlation tensor of the $(e^+e^-)$ pair

Let us consider the process of electron-positron pair production by two photons, $\gamma\gamma \to e^+e^-$ .

The spin state of the electron-positron system is described, in the general case, by the two-particle spin density matrix :

$$\hat{\rho}^{(e^-\,e^+)} = \frac{1}{4}\,[\,\hat{I}^{(e^-)} \otimes \hat{I}^{(e^+)} + \hat{I}^{(e^-)} \otimes (\hat{\boldsymbol{\sigma}}^{(e^+)}\mathbf{P}^{(e^+)}) + (\hat{\boldsymbol{\sigma}}^{(e^-)}\mathbf{P}^{(e^-)}) \otimes \hat{I}^{(e^+)} +$$

$$+ \sum_{i=1}^{3}\sum_{k=1}^{3} T_{ik}\,\hat{\sigma}_i^{(e^-)} \otimes \hat{\sigma}_k^{(e^+)}\,], \tag{1}$$

where $\hat{I}$ is the two-row unit matrix, $\mathbf{P}^{(e^-)}$ and $\mathbf{P}^{(e^+)}$ are the polarization vectors of the electron and positron, respectively, $T_{ik}$ – components of the correlation tensor ( $T_{ik} = \langle \hat{\sigma}_i^{(e^-)} \otimes \hat{\sigma}_k^{(e^+)} \rangle$, $i, k = \{1, 2, 3\} = \{x, y, z\}$ ) .

In the absence of correlations, we have : $T_{ik} = P_i^{(e^-)}\, P_k^{(e^+)}$ .

The process $\gamma\gamma \to e^+e^-$ is described by two well-known Feynman diagrams [1]. Within the first nonvanishing approximation over the electromagnetic constant ( Born approximation ), in case of unpolarized primary photons the final electron and positron prove to be unpolarized as well, but their spins are correlated .

Thus, in the above formula for the spin density matrix $\hat{\rho}^{(e^+e^-)}$ (1)

$$\mathbf{P}^{(e^-)} = \mathbf{P}^{(e^+)} = 0.$$

The components of the correlation tensor of the electron-positron pair, generated in the interaction of unpolarized $\gamma$ quanta, may be calculated by applying the results of paper [2]. Finally we obtain the following expressions:

$$T_{zz} = 1 - \frac{2\,(1-\beta^2)\,[\beta^2\,(1+\sin^2\theta)+1]}{1 + 2\,\beta^2\,\sin^2\theta - \beta^4 - \beta^4\,\sin^4\theta}\,, \tag{2}$$

$$T_{yy} = \frac{(1-\beta^2)\,[\beta^2\,(1+\sin^2\theta)-1] - \beta^2\sin^4\theta}{1 + 2\,\beta^2\,\sin^2\theta - \beta^4 - \beta^4\,\sin^4\theta}\,, \tag{3}$$

$$T_{xx} = \frac{(1-\beta^2)\,[\beta^2\,(1+\sin^2\theta)-1]+\beta^2\sin^4\theta}{1+2\,\beta^2\,\sin^2\theta-\beta^4-\beta^4\,\sin^4\theta}\,. \tag{4}$$

Here the axis $z$ is aligned along the positron momentum in the c.m. frame, the axis $x$ lies in the reaction plane and the axis $y$ is directed along the normal to the reaction plane ; $\beta = \frac{v}{c}$ , $\mathbf{v}$ is the positron velocity in the c.m. frame ; $1-\beta^2 = \frac{m_e\,c^2}{E_+}$ , where $E_+$ is the positron ( or electron ) energy in the c.m. frame ; $\theta$ is the angle between the positron momentum and the momentum of one of the photons in the c.m. frame .

Meantime, the differential cross section of the process $\gamma\gamma \to e^+e^-$ in the c.m. frame has the following form [1,2] :

$$\frac{d\sigma}{d\Omega} = r_0^2\,\frac{1-\beta^2}{4}\,\beta\,\left[\frac{1+2\,\beta^2\,\sin^2\theta-\beta^4-\beta^4\,\sin^4\theta}{(1-\beta^2\cos^2\theta)^2}\right], \tag{5}$$

where $r_0 = \frac{e^2}{m_e\,c^2}$ .

The "trace" of the correlation tensor of the final $(e^+e^-)$ pair is determined by the formula :

$$T = T_{xx}+T_{yy}+T_{zz} = 1-\frac{4\,(1-\beta^2)}{1+2\,\beta^2\,\sin^2\theta-\beta^4-\beta^4\,\sin^4\theta}. \tag{6}$$

In doing so, the relative fraction of the triplet states is as follows [3] :

$$W_t = \frac{T+3}{4} = 1-\frac{1-\beta^2}{1+2\,\beta^2\,\sin^2\theta-\beta^4-\beta^4\,\sin^4\theta}, \tag{7}$$

and the relative fraction of the singlet state ( total spin $S=0$ ) equals

$$W_s = \frac{1-T}{4} = 1-W_t = \frac{1-\beta^2}{1+2\,\beta^2\,\sin^2\theta-\beta^4-\beta^4\,\sin^4\theta}. \tag{8}$$

At $\beta << 1$ we have: $W_t \approx 0$, $W_s \approx 1$ , i.e. the $(e^+e^-)$ pair is generated in the singlet state only .

On the contrary, under the conditions $\beta \approx 1$ and $2\sin^2\theta - \sin^4\theta > 1 - \beta^2$ we obtain : $W_t \approx 1$ , $W_s \approx 0$ ( almost pure triplet state of the pair ).

## 2 Generation of singlet and triplet $(e^+e^-)$ pairs

It should be noted that the charge parity $C$ of the system of two photons is positive and it is conserved in the process $\gamma\gamma \to e^+e^-$ . This means that electron-positron pairs generated in this process have the positive charge parity ( $C = 1$ ). According to the well-known formula for charge parity of a fermion-antifermion system, $C = (-1)^{L+S}$, where $L$ is the orbital momentum and $S$ is the total spin. Thus, in the process $\gamma\gamma \to e^+e^-$, singlet electron-positron pairs ( $S = 0$ ) are produced in the states with even orbital and total angular momenta ( $L = J$ ) and negative space parity $P = (-1)^{L+1}$. Due to the conservation of total angular momentum and space parity in the process $\gamma\gamma \to e^+e^-$, the contribution into generation of singlet $(e^+e^-)$ pairs is provided only by states of two photons with even total angular momenta and negative space parity, which, according to Bose symmetry, are antisymmetric over the polarizations of two photons (they correspond to the total spin of the two-photon system equaling 1). If both the photons are unpolarized, then the relative fraction of such photon pairs amounts to 1/4 .

Meantime, triplet electron-positron pairs ( $S = 1$ ) are produced, in the process $\gamma\gamma \to e^+e^-$, in the states with odd orbital momenta and positive space parity. In doing so, total angular momenta may take both even and odd values . The contribution into generation of triplet $(e^+e^-)$ pairs is provided only by states of two photons with positive space parity, being symmetric over polarizations, which correspond to the total spins of the two-photon system equaling 0 and 2 . If both the photons are unpolarized, then the relative fraction of such photon pairs amounts to 3/4 .

# 3 Violation of "classical" incoherence inequalities for correlation tensor components in the processes $\gamma\gamma \to e^+e^-$, $\mu^+\mu^-$, $\tau^+\tau^-$

Now let us consider the particular cases $\theta = 0$ and $\theta = \pi$ . In accordance with the formula for the differential cross section $\frac{d\sigma}{d\Omega}$ (5), here we obtain :

$$\frac{d\sigma}{d\Omega} = \frac{1}{4}\, r_0^2\, \beta\, (1+\beta^2). \tag{9}$$

In the ultrarelativistic limit formula (9) turns to $\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2}$ .

According to the general expressions (2)–(4) for the correlation tensor components, at $\theta = 0$ and $\theta = \pi$ we have :

$$T_{zz} = 1 - \frac{2\,(1+\beta^2)(1-\beta^2)}{1-\beta^4} = -1; \qquad T_{xx} = T_{yy} = -\frac{1-\beta^2}{1+\beta^2}. \tag{10}$$

In doing so, the "trace" of the correlation tensor (6) takes the value

$$T = 1 - \frac{4}{1+\beta^2} = -\frac{3-\beta^2}{1+\beta^2}, \tag{11}$$

and the relative fractions of the triplet states $W_t$ (7) and the singlet state $W_s$ (8) amount to

$$W_t = \frac{T+3}{4} = \frac{\beta^2}{1+\beta^2}, \qquad W_s = \frac{1-T}{4} = \frac{1}{1+\beta^2}. \tag{12}$$

At nonrelativistic velocities $W_t \approx 0$, $W_s \approx 1$, in accordance with the general case ; meantime, in the ultrarelativistic limit ( $\beta \to 1$ ) we have : $W_t = W_s = 1/2$ .

Let us remark that in the process $\gamma\gamma \to e^+e^-$ we observe the violation of the "incoherence" inequalities, established previously at the classical level , according to which for incoherent "classical" mixtures of factorizable two-particle spin states the sum of any two ( or three ) diagonal components of the correlation tensor cannot exceed unity [3]. Indeed, at $\theta = 0$ and $\theta = \pi$ , in particular, we obtain :

$$|T_{zz} + T_{xx}| = |T_{zz} + T_{yy}| = \frac{2}{1+\beta^2} > 1, \tag{13}$$

since $\beta < 1$ .

Thus, the spin correlations of the final electron and positron in the considered process have the strongly pronounced quantum character .

Finally, it should be noted that analogous results ( with the replacements $m_e \to m_\mu,\, m_\tau,\ \beta \to \beta_\mu,\, \beta_\tau$ ) hold as well for the processes of generation of a muon pair and a tau lepton pair by two photons : $\gamma\gamma \to \mu^+\mu^-$, $\gamma\gamma \to \tau^+\tau^-$, which become possible at considerably higher energies .

# 4 Summary

1. The theoretical investigation of spin structure of the processes $\gamma\gamma \to e^+e^-,\, \mu^+\mu^-,\, \tau^+\tau^-$ is performed. It is shown that, if the primary photons are unpolarized, the final leptons are not polarized as well but their spins are strongly correlated .

2. Explicit expressions for the components of correlation tensor of the final systems $(e^+e^-)$, $(\mu^+\mu^-)$, $(\tau^+\tau^-)$ are derived , and the relative fractions of singlet and triplet states for the pair of final leptons are found .

3. It is demonstrated that in the processes $\gamma\gamma \to e^+e^-,\, \mu^+\mu^-,\, \tau^+\tau^-$ the Bell-type "incoherence" inequalities for the correlation tensor components may be violated .

# References


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