International Workshop on e+e- collisions from Phi to Psi 2013

PHI PSI 13


# ON THE SPIN CORRELATIONS OF MUONS AND TAU LEPTONS GENERATED IN THE ANNIHILATION PROCESSES $e^+e^- \to \mu^+\mu^-$, $e^+e^- \to \tau^+\tau^-$

**Valery V. Lyuboshitz** *) and **V.L. Lyuboshitz**

*Joint Institute for Nuclear Research, Dubna, Russia*

*) *E-mail: Valery.Lyuboshitz@jinr.ru*

Using the technique of helicity amplitudes, the electromagnetic process $e^+e^- \to \mu^+\mu^-$ is theoretically investigated in the one-photon approximation. The structure of the triplet states of the final $(\mu^+\mu^-)$ system is analyzed. It is shown that in the case of unpolarized electron and positron the final muons are also unpolarized, but their spins are strongly correlated. Explicit expressions for the components of the correlation tensor of the final $(\mu^+\mu^-)$ system are derived. The formula for the angular correlation at the decays of final muons $\mu^+$ and $\mu^-$, produced in the process $e^+e^- \to \mu^+\mu^-$, is obtained. It is demonstrated that spin correlations of muons in the process of electron-positron pair annihilation have the purely quantum character, since one of the incoherence inequalities for the correlation tensor components is always violated. Additional contribution of the weak interaction of lepton neutral currents through the virtual $Z^0$ boson is considered; it is established that, taking into account the weak interaction, the qualitative character of the muon spin correlations does not change. Analogous consideration can be wholly applied as well to the final $\tau$ leptons in the process $e^+e^- \to \tau^+\tau^-$ .

# 1 Helicity amplitudes for the annihilation process $e^+e^- \to \mu^+\mu^-$

In the first non-vanishing approximation over the electromagnetic constant $e^2/\hbar c$, the process of conversion of the electron-positron pair into the muon pair is described by the one-photon diagram (see Fig. 1).

Due to the electromagnetic current conservation, the virtual photon with a time-like momentum transfers the angular momentum $J = 1$ and negative parity. Taking into account that the internal parities of muons $\mu^+$ and $\mu^-$ are opposite, the $(\mu^+\mu^-)$ pair is generated in the triplet states (the total spin $S = 1$) with the total angular momentum $J = 1$ and with the orbital angular momenta $L = 0$ and $L = 2$, being the superpositions of the states ${}^3S_1$ and ${}^3D_1$ with the negative space parity.

The respective helicity amplitudes have the following structure:

$$f_{\Lambda'\Lambda}(\theta, \phi) = R_{\Lambda'\Lambda}(E)\; d^{(1)}_{\Lambda'\Lambda}(\theta)\; \exp(i\Lambda\phi), \tag{1}$$

where $\theta$ and $\phi$ are the polar and azimuthal angles of the flight direction of the positively charged muon $(\mu^+)$ in the center-of-mass (c.m.) frame of the considered reaction with respect to the initial positron momentum;

$d^{(1)}_{\Lambda'\Lambda}(\theta)$ are the Wigner functions (elements of the finite rotation matrix) for the angular momentum $J = 1$;

$\Lambda$ is the difference of helicities of the positron and electron, coinciding with the projection of total spin and with the projection of total angular momentum of the $(e^+e^-)$ pair onto the direction of positron momentum in the c.m. frame (the projection of orbital angular momentum onto the momentum direction equals zero);

$\Lambda'$ is the difference of helicities of the muons $\mu^+$ and $\mu^-$, coinciding with the projection of total angular momentum of the $(\mu^+\mu^-)$ pair onto the direction of momentum of the positively charged muon $\mu^+$ in the c.m. frame (see, for example, [1,2]).

Due to the factorizability of the Born amplitude, we can write:

$$R_{\Lambda'\Lambda}(E) = r^{(\mu)}_{\Lambda'}(E)\, r^{(e)}_{\Lambda}(E). \tag{2}$$

Here $\Lambda'$ and $\Lambda$ take the values $+1, 0, -1$; in doing so, the parameters $r^{(\mu)}_{\Lambda'}$, $r^{(e)}_{\Lambda}$ depend upon the initial energy $E$ of the positron (electron) in the c.m. frame of the pair $e^+e^-$, but do not depend upon the angles $\theta$ and $\phi$.

On account of the space parity conservation in the electromagnetic interactions, we have:

$$r^{(\mu)}_{+1} = r^{(\mu)}_{-1} = r^{(\mu)}_{1}, \quad r^{(e)}_{+1} = r^{(e)}_{-1} = r^{(e)}_{1}. \tag{3}$$

In accordance with the structure of electromagnetic current for the pairs $e^+e^-$ and $\mu^+\mu^-$ in the c.m. frame [1], the following relations are valid:

$$r^{(\mu)}_{0} = \frac{m_\mu}{E} r^{(\mu)}_{1} = \sqrt{1-\beta^2_\mu}\, r^{(\mu)}_{1}, \quad r^{(e)}_{0} = \frac{m_e}{E} r^{(e)}_{1}, \tag{4}$$

where $m_\mu$ and $m_e$ are the masses of the muon and electron, respectively, $\beta_\mu$ is the muon velocity in the c.m. frame. Since for the process $e^+e^- \to \mu^+\mu^-$ the inequality $E \geq m_\mu \gg m_e$ is always satisfied, the contribution of electron-positron states with antiparallel spins (equal helicities) can be neglected. In doing so, $R_{\Lambda 0}(E) \approx 0$.

## 2 Effective cross section and angular distribution

The cross section of the process $e^+e^- \to \mu^+\mu^-$ for the unpolarized electron and positron, integrated over the solid angle and summed over muon polarizations, is expressed through the helicity amplitudes $R_{\Lambda'\Lambda}(E)$:

$$\sigma = \frac{1}{4} 2\pi \left( \sum_{\Lambda'} \sum_{\Lambda} (R_{\Lambda'\Lambda}(E))^2 \int_{-1}^{1} (d^{(1)}_{\Lambda'\Lambda}(\theta))^2 d(\cos\theta) \right) \frac{p_\mu}{p_e} =$$
$$= \frac{\pi}{3} \left( \sum_{\Lambda'} \sum_{\Lambda} (R_{\Lambda'\Lambda}(E))^2 \right) \frac{p_\mu}{p_e}. \tag{5}$$

Here $p_\mu = \sqrt{E^2 - m^2_\mu}$ is the muon momentum in the c.m. frame, $p_e = \sqrt{E^2 - m^2_e} \approx E$ is the positron (electron) momentum in the c.m. frame. Since, in accordance with Eq.(4), $r^{(e)}_0 \approx 0$, it follows from Eqs. (2)-(5) that:

$$\sigma = \frac{4\pi}{3}\left(r_1^{(e)}(E)\, r_1^{(\mu)}(E)\right)^2 \left(1 + \frac{m_\mu^2}{2E^2}\right)\sqrt{1 - \frac{m_\mu^2}{E^2}}. \tag{6}$$

The term $m_\mu^2/2E^2$ in brackets in formula (6) is connected with the contribution of the amplitude $r_0^{(\mu)}(E)$. The calculation of the one-photon diagram gives:

$$r_1^{(\mu)}(E) = r_1^{(e)}(E) = \frac{|e|}{\sqrt{2E}}, \tag{7}$$

where $e$ is the electron charge. If the relativistic invariant

$$s = (p_{e^+} + p_{e^-})^2 = (p_{\mu^+} + p_{\mu^-})^2 = 4E^2$$

is introduced, the expression for the cross section of the process $e^+e^- \to \mu^+\mu^-$ takes the following form [1]

$$\sigma = \frac{4\pi}{3}\frac{e^2}{s}\left(1 + \frac{2m_\mu^2}{s}\right)\sqrt{1 - \frac{4m_\mu^2}{s}}. \tag{8}$$

Using the explicit formulas for $d$-functions corresponding to the angular momentum $J = 1$ [1,2]:

$$d^{(1)}_{+1+1}(\theta) = d^{(1)}_{-1-1}(\theta) = \frac{1+\cos\theta}{2}, \quad d^{(1)}_{+1-1}(\theta) = d^{(1)}_{-1+1}(\theta) = \frac{1-\cos\theta}{2},$$
$$d^{(1)}_{0+1}(\theta) = -d^{(1)}_{0-1}(\theta) = \frac{\sin\theta}{\sqrt{2}},$$

we find the angular distribution of muon emission, normalized by unity, in the c.m. frame for the considered annihilation process:

$$dW_{\mu^+\mu^-} = \frac{3}{16\pi}\,\frac{1 + \cos^2\theta + (m_\mu^2/E^2)\sin^2\theta}{1 + (m_\mu^2/2E^2)}d\,\Omega = \frac{3}{8\pi}\,\frac{2 - \beta_\mu^2\sin^2\theta}{3 - \beta_\mu^2}d\,\Omega, \tag{9}$$

where $d\,\Omega$ is the element of solid angle.

## 3 Structure of the triplet states of the $(\mu^+\mu^-)$ system formed in the process $e^+e^- \to \mu^+\mu^-$

Taking into account the relations (1) -(4) for the helicity amplitudes, it is clear that if the positron and electron are totally polarized along the positron momentum in the c.m. frame, then the $(\mu^+\mu^-)$ system is produced in the triplet state of the following form:

$$|\Psi\rangle^{(+1)} = \frac{\sqrt{2}}{\sqrt{2-\beta_\mu^2\sin^2\theta}}\times$$
$$\times\left(\frac{1+\cos\theta}{2}|+1\,\rangle - \sqrt{1-\beta_\mu^2}\,\frac{\sin\theta}{\sqrt{2}}|\,0\,\rangle + \frac{1-\cos\theta}{2}|-1\,\rangle\right). \tag{10}$$

Here $\beta_\mu = \sqrt{1-(m_\mu^2/E^2)}$ is the velocity of each of the muons, as before;

$$|+1\,\rangle = |+1/2\rangle^{(\mu^+)}\otimes|+1/2\rangle^{(\mu^-)}, \quad |-1\,\rangle = |-1/2\rangle^{(\mu^+)}\otimes|-1/2\rangle^{(\mu^-)},$$
$$|\,0\,\rangle = \frac{1}{\sqrt{2}}\left(|+1/2\rangle^{(\mu^+)}\otimes|-1/2\rangle^{(\mu^-)} + |-1/2\rangle^{(\mu^+)}\otimes|+1/2\rangle^{(\mu^-)}\right)$$

are the states with the projection of total spin of the $(\mu^+\mu^-)$ pair onto the direction of momentum of the muon $\mu^+$ in the c.m. frame of the reaction $e^+e^- \to \mu^+\mu^-$, equaling $+1$, $-1$ and 0, respectively.

Let us note that the real values of the coefficients of superposition of the triplet states $|+1\,\rangle$, $|\,0\,\rangle$ and $|-1\,\rangle$ in the state $|\Psi\rangle^{(+1)}$ (10) correspond to the choice of the quantization axes $z'$ and $z$ along the positron momentum and $\mu^+$ momentum, respectively, in the c.m. frame of the reaction $e^+e^- \to \mu^+\mu^-$, and the axis $y$ - along the normal to the plane of this reaction.

If the positron and electron are totally polarized in the direction being antiparallel to the positron momentum, then the $(\mu^+\mu^-)$ pair is generated in the following triplet state:

$$|\Psi\rangle^{(-1)} = \frac{\sqrt{2}}{\sqrt{2-\beta_\mu^2\sin^2\theta}}\times$$
$$\times\left(\frac{1-\cos\theta}{2}|+1\,\rangle + \sqrt{1-\beta_\mu^2}\,\frac{\sin\theta}{\sqrt{2}}|\,0\,\rangle + \frac{1+\cos\theta}{2}|-1\,\rangle\right). \tag{11}$$

## 4 Spin density matrix and correlation tensor of the $(\mu^+\mu^-)$ pair

If the positron and electron are not polarized, then, since $r^{(e)} \approx 0$, the final state of the $(\mu^+\mu^-)$ pair represents a noncoherent mixture of spin states $|\Psi\rangle^{(+1)}$ and $|\Psi\rangle^{-1}$, each of them being realized with the relative probability of 1/2. Taking into account Eqs. (10) and (11), the elements of the spin density matrix of the $(\mu^+\mu^-)$ system in the representation of triplet states $|+1\rangle$, $|0\rangle$ and $|-1\rangle$ have the form:

$$\rho_{+1,+1} = \rho_{-1,-1} = K\,\frac{1+\cos^2\theta}{2}, \quad \rho_{+1,-1} = \rho_{-1,+1} = K\,\frac{\sin^2\theta}{2},$$

$$\rho_{0,+1} = \rho_{+1,0} = -\rho_{0,-1} = -\rho_{-1,0} = -K\sqrt{\frac{1-\beta_\mu^2}{2}}\cos\theta\sin\theta,$$

$$\rho_{0,0} = K(1-\beta_\mu^2)\,\sin^2\theta, \quad K = \frac{1}{2-\beta_\mu^2\sin^2\theta}. \tag{12}$$

The spin states of two particles with spin 1/2 are characterized by the polarization vectors $\boldsymbol{\zeta}_1 = \langle\hat{\boldsymbol{\sigma}}^{(1)}\rangle$, $\boldsymbol{\zeta}_2 = \langle\hat{\boldsymbol{\sigma}}^{(2)}\rangle$ and the components of the correlation tensor $T_{ik} = \langle\hat{\sigma}_i^{(1)} \otimes \hat{\sigma}_k^{(2)}\rangle$. Here $\hat{\boldsymbol{\sigma}} = \{\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z\}$ is the vector Pauli operator, $\hat{\sigma}_i$, $\hat{\sigma}_k$ are the Pauli matrices, $i,k \to \{1,2,3\} \to \{x,y,z\}$; the axis $z$ is directed along the momentum of the positively charged muon $\mu^+$ in the c.m. frame of the considered reaction, and the axis $y$ is directed along the normal to the reaction plane; the symbol $\langle ... \rangle$ denotes the averaging over the quantum ensemble. If both the particles are not polarized and the correlations are absent, then $T_{ik} = 0$. For two independent particles with the polarization vectors $\boldsymbol{\zeta}_1$ and $\boldsymbol{\zeta}_2$ the correlation tensor is factorized : $T_{ik} = \zeta_i\zeta_k$; in so doing, the "trace" of the correlation tensor equals $T = T_{xx} + T_{yy} + T_{zz} = \boldsymbol{\zeta}_1\boldsymbol{\zeta}_2$.

In the case of triplet states of two spin-1/2 particles, the polarization parameters are expressed through the density matrix elements specified in the representation of triplet states $|+1\rangle$, $|0\rangle$ and $|-1\rangle$ with the definite total spin projections onto the axis $z$ [3,4] :

$$\boldsymbol{\zeta}_1 = \boldsymbol{\zeta}_2 = \boldsymbol{\zeta}; \quad \zeta_x = \sqrt{2}\,\mathrm{Re}\,(\rho_{+1,0} + \rho_{-1,0}), \quad \zeta_y = -\sqrt{2}\,\mathrm{Im}\,(\rho_{+1,0} - \rho_{-1,0}),$$

$$\zeta_z = \rho_{+1,+1} - \rho_{-1,-1}; \quad T_{xx} = \rho_{0,0} + 2\,\mathrm{Re}\,\rho_{+1,-1}, \quad T_{yy} = \rho_{0,0} - 2\,\mathrm{Re}\;\rho_{+1,-1},$$

$$T_{zz} = \rho_{+1,+1} + \rho_{-1,-1} - \rho_{0,0} = 1 - 2\rho_{0,0}; \quad T_{xy} = T_{yx} = -2\,\mathrm{Im}\,\rho_{+1,-1},$$

$$T_{zx} = T_{xz} = \sqrt{2}\,\mathrm{Re}\,(\rho_{+1,0} - \rho_{-1,0}), \quad T_{yz} = T_{zy} = -\sqrt{2}\,\mathrm{Im}\,(\rho_{+1,0} + \rho_{-1,0}). \tag{13}$$

In doing so, the following equalities hold:

$$T = T_{xx} + T_{yy} + T_{zz} = 1, \quad T_{xx} + T_{yy} = 1 - 2w_0, \tag{14}$$

where $w_0 = \rho_{0,0}$ is the occupancy of the triplet state with the zero projection of total spin onto the axis $z$.

It is easy to see from Eqs. (12) and (13) that, at the annihilation $e^+e^- \to \mu^+\mu^-$ of the unpolarized positron and electron, the produced muons $\mu^+$ and $\mu^-$ are unpolarized ($\boldsymbol{\zeta}_{\mu^+} = \boldsymbol{\zeta}_{\mu^-} = 0$), but their spins are correlated: the correlation tensor components have the following form (see also [4]):

$$T_{xx}^{(\mu^+\mu^-)} = \frac{(2-\beta_\mu^2)\sin^2\theta}{2-\beta_\mu^2\sin^2\theta}, \quad T_{yy}^{(\mu^+\mu^-)} = -\frac{\beta_\mu^2\sin^2\theta}{2-\beta_\mu^2\sin^2\theta},$$

$$T_{zz}^{(\mu^+\mu^-)} = \frac{2\cos^2\theta + \beta_\mu^2\sin^2\theta}{2-\beta_\mu^2\sin^2\theta}, \quad T_{xz}^{(\mu^+\mu^-)} = -\frac{(1-\beta_\mu^2)^{1/2}\sin 2\theta}{2-\beta_\mu^2\sin^2\theta},$$

$$T_{xy}^{(\mu^+\mu^-)} = T_{yz}^{(\mu^+\mu^-)} = 0. \tag{15}$$

The "trace" of the correlation tensor of the $(\mu^+\mu^-)$ pair is:

$$T^{(\mu^+\mu^-)} = \langle \hat{\boldsymbol{\sigma}}_{\mu^+} \otimes \hat{\boldsymbol{\sigma}}_{\mu^-} \rangle = T_{xx}^{(\mu^+\mu^-)} + T_{yy}^{(\mu^+\mu^-)} + T_{zz}^{(\mu^+\mu^-)} = 1, \tag{16}$$

just as it should hold for any triplet state [1)].

[1)] For the singlet state $T = -3$ ; in the general case, $T = \rho_t - 3\rho_s$, where $\rho_t$ and $\rho_s$ are the fractions of the triplet and singlet state, respectively [3-5].

## 5 Angular correlations at the joint registration of decays of the muons $\mu^+$ and $\mu^-$

The "trace" of the correlation tensor $T$ determines the angular correlation between flight directions for the products of decay of two unstable particles with spin 1/2 in the case when space parity is not conserved [3-7].

Actually, the angular distribution at the decay of any polarized unstable particle with spin 1/2 under space parity nonconservation, normalized by unity, has the form (see, for example, [8]):

$$dW = \frac{1}{4\pi}(1 + \alpha\, \boldsymbol{\zeta}\, \mathbf{n})d\,\Omega_{\mathbf{n}},$$

where $\boldsymbol{\zeta}$ is the polarization vector of the unstable particle, $\alpha$ is the angular asymmetry coefficient, $\mathbf{n}$ is the unit vector along the momentum of the particle, formed in the decay, in the rest frame of the decaying unstable particle.

Then the double distribution for the flight directions of the decay products of two unstable particles under space parity nonconservation, normalized by unity, is as follows [3, 4]:

$$d^2W = \frac{1}{16\pi^2}\,(1 + \alpha_1\, \boldsymbol{\zeta_1}\, \mathbf{n_1} + \alpha_2\, \boldsymbol{\zeta_2}\, \mathbf{n_2} + \alpha_1\alpha_2 \sum_{i=1}^{3}\sum_{k=1}^{3} T_{ik}\, n_{1,i}\, n_{2,k})d\,\Omega_{\mathbf{n}_1}\, d\,\Omega_{\mathbf{n}_2}. \tag{17}$$

Here $\boldsymbol{\zeta_1}$ and $\boldsymbol{\zeta_2}$ are the polarization vectors of the first and second unstable particle, $\alpha_1$ and $\alpha_2$ are the coefficients of angular asymmetry for the decays of the first and second particle; $\mathbf{n}_1$ and $\mathbf{n}_2$ are unit vectors defined in the rest frames of the first and second unstable particle, respectively, and specified with respect to a unified system of spatial coordinate axes [6, 7]; just as before, $i, k \to \{1, 2, 3\} \to \{x, y, z\}$.

Using the method of moments, the components of the polarization vectors and correlation tensor can be found as a result of averaging the corresponding combinations of trigonometric functions of angles over the double distribution of decay directions [3]:

$$\zeta_{1,i} = \frac{3}{\alpha_1}\langle n_{1,i}\rangle, \quad \zeta_{2,k} = \frac{3}{\alpha_2}\langle n_{2,k}\rangle, \quad T_{ik} = \frac{9}{\alpha_1\alpha_2}\langle n_{1,i}n_{2,k}\rangle. \tag{18}$$

In doing so, the projections of the unit vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ onto the coordinate axes are:

$$n_{1,x} = \sin\theta_1 \cos\phi_1, \quad n_{1,y} = \sin\theta_1 \sin\phi_1, \quad n_{1,z} = \cos\theta_1;$$

$$n_{2,x} = \sin\theta_2 \cos\phi_2, \quad n_{2,y} = \sin\theta_2 \sin\phi_2, \quad n_{2,z} = \cos\theta_2, \tag{19}$$

where $\theta_1$, $\phi_1$ and $\theta_2$, $\phi_2$ are the polar and azimuthal angles in the rest frames of the first and second unstable particle, respectively, with respect to the unified system of axes $(x, y, z)$.

The integration of the double distribution of flight directions over all angles, except the angle $\delta$ between the vectors $\mathbf{n}_1$ and $\mathbf{n}_2$, leads to the following formula for the angular correlation [3, 4]:

$$dW = \frac{1}{2}\left(1 + \frac{\alpha_1\alpha_2 T}{3}\cos\delta\right) d(-\cos\delta); \quad \cos\delta = \mathbf{n}_1\mathbf{n}_2. \tag{20}$$

Let us apply Eq. (20) to the decays of the muons $\mu^+$ and $\mu^-$ produced in the process of electron–positron pair annihilation $e^+e^- \to \mu^+\mu^-$. According to Eq. (16), in this case the "trace" of the correlation tensor of the muon pair is equal to unity ($T = 1$). It is known that the asymmetry coefficient in the angular distribution of electrons at the decay of the polarized negatively charged muon $\mu^- \to e^-\nu_\mu\bar{\nu}_e$, integrated over the electron energy spectrum, equals $-1/3$ ($\alpha_1 = -1/3$) [8]. Due to the $CP$-invariance, the asymmetry coefficient in the angular distribution of positrons at the decay of the polarized positively charged muon $\mu^+ \to e^+\bar{\nu}_\mu\nu_e$, integrated over the positron energy spectrum, amounts to $+1/3$ ($\alpha_2 = +1/3$). As a result, we obtain the following formula for the angular correlation at the decays $\mu^- \to e^-\nu_\mu\bar{\nu}_e$ and $\mu^+ \to e^+\bar{\nu}_\mu\nu_e$ :

$$dW^{(\mu^+\mu^-)} = \frac{1}{2}\left(1 - \frac{1}{27}\cos\delta\right) d(-\cos\delta). \tag{21}$$

## 6 Coherent properties of the correlation tensor and violation of “classical” incoherence inequalities in the process $e^+e^- \to \mu^+\mu^-$

Previously it was shown in the papers [3,4] that in the case of incoherent mixtures of factorizable states of two particles with spin 1/2 the modulus of the sum of any two (and three) diagonal components of the correlation tensor cannot exceed unity, i.e. the following inequalities are satisfied:

$$|T_{xx}+T_{yy}| \leq 1, \quad |T_{xx}+T_{zz}| \leq 1, \quad |T_{yy}+T_{zz}| \leq 1,$$

$$|T| = |T_{xx}+T_{yy}+T_{zz}| \leq 1.$$

However, for nonfactorizable (entangled) states some of these inequalities may be violated. In particular, for the singlet state (zero total spin) we have:

$$T_{xx}+T_{yy} = T_{xx}+T_{zz} = T_{yy}+T_{zz} = -2, \quad T = T_{xx}+T_{yy}+T_{zz} = -3.$$

In the case of triplet state $|\,0\,\rangle$ with the zero projection of total spin onto the axis $z$, the diagonal components of the correlation tensor are: $T_{xx} = T_{yy} = 1, T_{zz} = -1$, and one of the incoherence inequalities is violated: $T_{xx}+T_{yy} = 2$.

In the process of annihilation of the unpolarized positron and electron $e^+e^- \to \mu^+\mu^-$, the muon pair is produced in the nonfactorizable two-particle quantum states $|\Psi\rangle^{(+1)}$ and $|\Psi\rangle^{(-1)}$ ( see Eqs. (10) and (11)). In so doing, one of the incoherence inequalities is violated: indeed, using Eqs. (15), we obtain at the angle $\theta \neq 0$:

$$T_{xx}^{(\mu^+\mu^-)} + T_{zz}^{(\mu^+\mu^-)} = 1 - T_{yy}^{(\mu^+\mu^-)} = \frac{2}{2-\beta_\mu^2 \sin^2\theta} > 1. \tag{22}$$

Our consideration relates, of course, also to the process $e^+e^- \to \tau^+\tau^-$, with the replacements $m_\mu \to m_\tau$, $\beta_\mu \to \beta_\tau$.

At very high energies $E \gg m_\mu$ $(m_\tau)$, when $\beta_\mu \approx \beta_\tau \to 1$, the nonzero components of the correlation tensor take the following values:

$$T_{xx} = \frac{\sin^2\theta}{1+\cos^2\theta}, \quad T_{yy} = -\frac{\sin^2\theta}{1+\cos^2\theta}, \quad T_{zz} = 1. \tag{23}$$

It is obvious that one of the incoherence inequalities is violated as before: $T_{xx} + T_{zz} \geq 1$.

## 7 Incorporation of the weak interaction of neutral currents through $Z^0$ boson

At very high energies the annihilation processes $e^+e^- \to \mu^+\mu^-$, $e^+e^- \to \tau^+\tau^-$ are conditioned not only by the electromagnetic interaction through the virtual photon, but also by the weak interaction of neutral currents through the $Z^0$ boson [9] ( see Fig. 2 ).

The interference of amplitudes of the purely electromagnetic and weak interaction leads to the charge asymmetry in lepton emission and to the effects of space parity violation. In the framework of the standard model of electroweak interaction, at the electron–positron pair annihilation the pairs $\mu^+\mu^-$, $\tau^+\tau^-$ are produced in the states $^3S_1$, $^3D_1$ with the negative space parity and, due to the weak interaction, also in the state $^3P_1$ with the positive space parity. In doing so, the total angular momentum is $J = 1$ and $CP$ parity of the pairs is positive.

Owing to the structure of "left" and "right" components of neutral currents, representing the combinations of the vector and axial currents $(V-A)$ and $(V+A)$, respectively, at high energies $E \gg m_\tau \gg m_\mu$ the nonzero helicity amplitudes of the processes $e^+e^- \to \mu^+\mu^-$, $e^+e^- \to \tau^+\tau^-$ in the c.m. frame have the following form [8]:

$$R_{11}(E) = \frac{e^2}{2E}\left[1 + x\left(\xi - \frac{1}{2}\right)^2\right], \quad R_{-1-1}(E) = \frac{e^2}{2E}[1 + x\xi^2],$$

$$R_{1-1}(E) = R_{-11}(E) = \frac{e^2}{2E}\left[1 + x\xi\left(\xi - \frac{1}{2}\right)\right]; \tag{24}$$

the helicity amplitudes $R_{0\Lambda}$, $R_{\Lambda'0}$ are practically equal to zero at high energies $E \gg m_\tau \gg m_\mu$. Here $\xi = \sin^2\theta_W < 1/4$ , $\theta_W$ is the Weinberg angle (angle of gauge boson mixing), the parameter $x$ determines the contribution of weak interaction. In doing so,

$$x = \frac{1}{\sin^2\theta_W \cos^2\theta_W} \; \frac{s}{s - (M_{Z^0} - i(\Gamma_{Z^0}/2))^2}, \tag{25}$$

where $s = (2E)^2$, $M_{Z^0} \approx 91.2$ GeV is the mass and $\Gamma_{Z^0} \approx 2.5$ GeV is the width of the $Z^0$ boson. According to the standard model of electroweak interaction (see, for example, [8]),

$$\frac{1}{\sin^2\theta_W \; \cos^2\theta_W} = \frac{1}{\xi(1-\xi)} = \frac{\sqrt{2} G_F M_{Z^0}^2}{\pi\alpha} \approx 6;$$

here $G_F = 1.166 \cdot 10^{-5}\,(\mathrm{GeV})^{-2}$ is the universal Fermi constant of weak interaction, $\alpha = 1/137$ is the electromagnetic constant. At energies $2E < M_{Z^0}$ (below the resonance energy) the parameter $x < 0$, and at energies $2E > M_{Z^0}$ (above the resonance energy)— $x > 0$; at the resonance energy $2E = M_{Z^0}$ the magnitude $x \approx -i\,208$ is the purely imaginary number.

Taking into account Eqs.(24), the differential cross section of the annihilation process at the collision of an unpolarized positron with an unpolarized electron is as follows:

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{1}{4}\{(|R_{11}(E)|^2 + |R_{-1-1}(E)|^2)\,\frac{(1+\cos\theta)^2}{4} +$$

$$+ (|R_{1-1}(E)^2 + |R_{-11}(E)|^2)\,\frac{(1-\cos\theta)^2}{4}\} =$$

$$= \frac{1}{4}\,\frac{e^4}{(2E)^2}\,[a_+(E)\,(1+\cos^2\theta) + 2a_-(E)\,\cos\theta]. \tag{26}$$

Outside the resonance region,when $\mathrm{Im}\, x \approx 0$, we have

$$a_+(E) = 1 + \frac{1}{2}x\left(\frac{1}{2} - 2\xi\right)^2 + \frac{1}{4}x^2\left[\left(\frac{1}{2} - \xi\right)^2 + \xi^2\right]^2,$$

$$a_-(E) = \frac{1}{8}x + \frac{1}{4}\left(\frac{1}{4} - \xi\right)^2 x^2. \tag{27}$$

In so doing, the factor of charge asymmetry is $A = a_-/a_+ = (x/8) + O(x^2)$. At the transition through the resonance region, the asymmetry factor sharply changes the magnitude and sign: when $E < M_{Z^0}/2$, $x < 0$, $A < 0$, then the positively charged muon flies predominantly into the backward hemisphere of the c.m. frame of the reaction with respect to the positron momentum; when $E > M_{Z^0}/2$, $x > 0$, then always $A > 0$, and the positively charged muon flies predominantly into the forward hemisphere with respect to the positron momentum. Meantime, at the absence of weak interaction, the angular distribution of leptons in the c.m. frame is symmetric in the first order over the electromagnetic constant $e^2/\hbar c$.

If the weak interaction contribution is neglected, then the lepton pairs, generated at the annihilation of the unpolarized positron and electron, are correlated but unpolarized. Due to the weak interaction through the exchange by the virtual $Z^0$ boson with the nonconservation of space parity, the final leptons acquire the longitudinal polarization. Since the lepton pairs are produced in the triplet states, the polarization vectors of the positively and negatively charged leptons are the same, and their average helicities $\lambda_+ = -\lambda_-$ have different signs in consequence of the opposite directions of momenta in the c.m. frame.

Taking into account Eqs. (24), the average helicities of the positively and negatively charged muons have the form

$$\lambda_+ = -\lambda_- = \frac{C(E)}{a_+(E)(1+\cos^2\theta) + a_-(E)\cos\theta}, \tag{28}$$

where the quantities $a_+(E)$ and $a_-(E)$ are described by Eqs. (27),

$$C(E) = \frac{1}{4}\left\{\left[1 + x\left(\frac{1}{2} - \xi\right)^2\right]^2 - (1 + x\xi^2)^2\right\} =$$
$$= \frac{1}{2}\left(\frac{1}{4} - \xi\right) x \left[1 + \frac{1}{2}\left(2\xi^2 - \xi + \frac{1}{4}\right)\right]. \tag{29}$$

At the energies below and above the resonance energy, the average helicities of each of the final leptons have different signs: when $E < M_{Z^0}/2$, then $x < 0$ and $\lambda_+ < 0$, $\lambda_- > 0$; when $E > M_{Z^0}$, then $x > 0$ and $\lambda_+ > 0$, $\lambda_- < 0$.

The structure of the correlation tensor of the final leptons is, on the whole, similar to that for the case of purely electromagnetic annihilation at high energy. In doing so, the nonzero components of the correlation tensor are: $T_{zz} = 1$, $T_{xx} = -T_{yy}$, as before. According to Eqs. (12) and (24),

$$T_{xx} = \sin^2\theta \frac{[1 + x\,(\xi^2 - \frac{1}{2}\,\xi + \frac{1}{8})]\,[1 + x\,\xi\,(\xi - \frac{1}{2})]}{a_+(E)(1 + \cos^2\theta) + 2a_-(E)\cos\theta}. \tag{30}$$

Again one of the incoherence inequalities for the correlation tensor components is violated: $T_{xx} + T_{zz} > 1$.

Thus, the consequences of the quantum-mechanical coherence for two-particle quantum systems with nonfactorizable internal states manifest themselves distinctly in spin correlations of lepton pairs produced in the annihilation processes $e^+e^- \to \mu^+\mu^-$, $e^+e^- \to \tau^+\tau^-$, and they can be verified experimentally ( see also [ 9 ] ) .

## 8 Summary

1. Using the technique of helicity amplitudes, the electromagnetic process $e^+e^- \to \mu^+\mu^-$ is theoretically investigated in the one-photon approximation. The structure of triplet states of the final $(\mu^+\mu^-)$ system is found.

2. It is shown that, if the primary electron and positron are unpolarized, the final muons $\mu^+$ and $\mu^-$ are also unpolarized but their spins are correlated. Explicit expressions for the correlation tensor of the final $(\mu^+\mu^-)$ system are derived.

3. Expression for the angular correlation at the decays of muons $\mu^+$ and $\mu^-$, produced in the process $e^+e^- \to \mu^+\mu^-$, into the channels $\mu^+ \to e^+\nu_e\bar{\nu}_\mu$ and $\mu^- \to e^-\bar{\nu}_e\nu_\mu$ is obtained .

4. It is shown that in the annihilation processes $e^+e^- \to \mu^+\mu^-$, $e^+e^- \to \tau^+\tau^-$ one of the incoherence inequalities for the correlation tensor components is always violated.

5. Additional contribution of weak interaction of neutral currents through the exchange by the virtual $Z^0$ boson into the annihilation processes $e^+e^- \to \mu^+\mu^-$, $e^+e^- \to \tau^+\tau^-$ at high energies is analyzed.

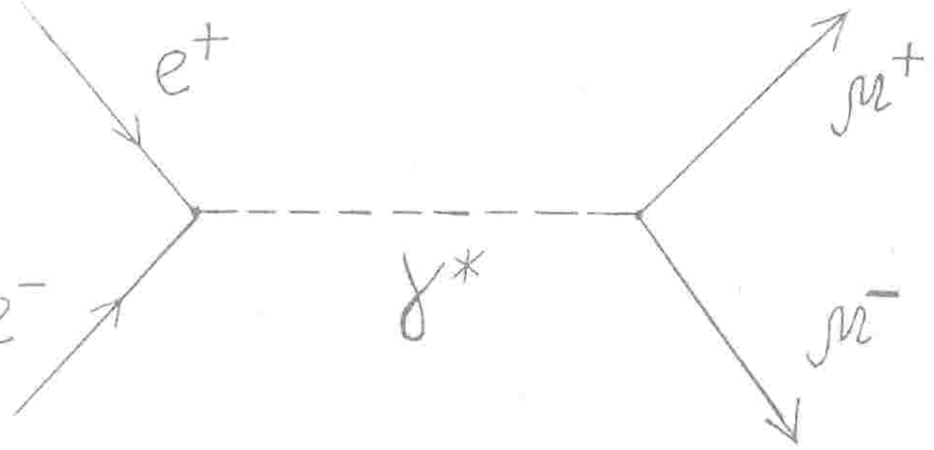


**Fig.1.** One-photon Feynman diagram for the electromagnetic process $e^+e^- \to \mu^+\mu^-$ .

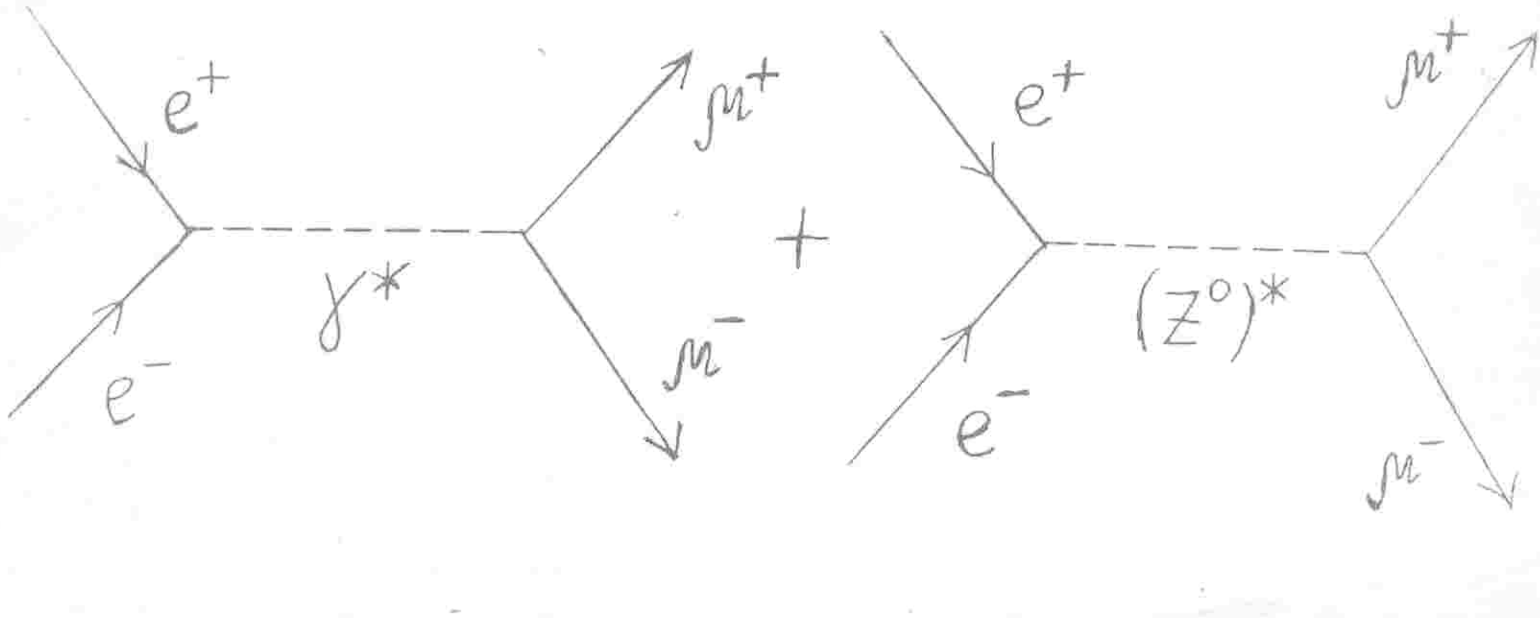


**Fig.2.** Process $e^+e^- \to \mu^+\mu^-$ with the additional contribution of weak interaction of neutral currents .

## References


[1] V.B. Berestetsky, E.M. Lifshitz and L.P. Pitaevsky, *Kvantovaya Elektrodynamika (Quantum Electrodynamics)* (Moscow, Nauka, 1989), §§ 68, 69, 81.

[2] A.M. Baldin, V.I. Gol'dansky, V.M. Maksimenko and I.L. Rosental', *Kinematika Yadernykh Reakcij (Kinematics of Nuclear Reactions)* (Moscow, Atomizdat, 1968), part II, § 11.

[3] V.L. Lyuboshitz, Proc. XXXIV Winter School of PNPI "Physics of Atomic Nuclei and Elementary Particles" ( St. Petersburg, 2000), p.402.

[4] R. Lednicky and V.L. Lyuboshitz, Phys. Lett. **B 508**, 146 (2001).

[5] G. Alexander and H.J. Lipkin, Phys. Lett. **B 352**, 162 (1995).

[6] R. Lednicky, V.V. Lyuboshitz and V.L. Lyuboshitz, Yad. Fiz. **66**, 1007 (2003) [ Phys. At. Nucl. **66**, 975 (2003) ] .

[7] R. Lednicky, V.L. Lyuboshitz and V.V. Lyuboshitz, Czech. J. Phys. (Suppl. B) **54**, B43 (2004).

[8] L.B. Okun, *Leptony i kwarki (Leptons and Quarks)* (Moscow, Nauka, 1990), §§ 3, 8, 22.

[9] V.L. Lyuboshitz and V.V. Lyuboshitz, Yad. Fiz. **72**, 340 (2009) [ Phys. At. Nucl. **72**, 311 (2009) ] .