e^+e^- hadron production cross sections at Belle

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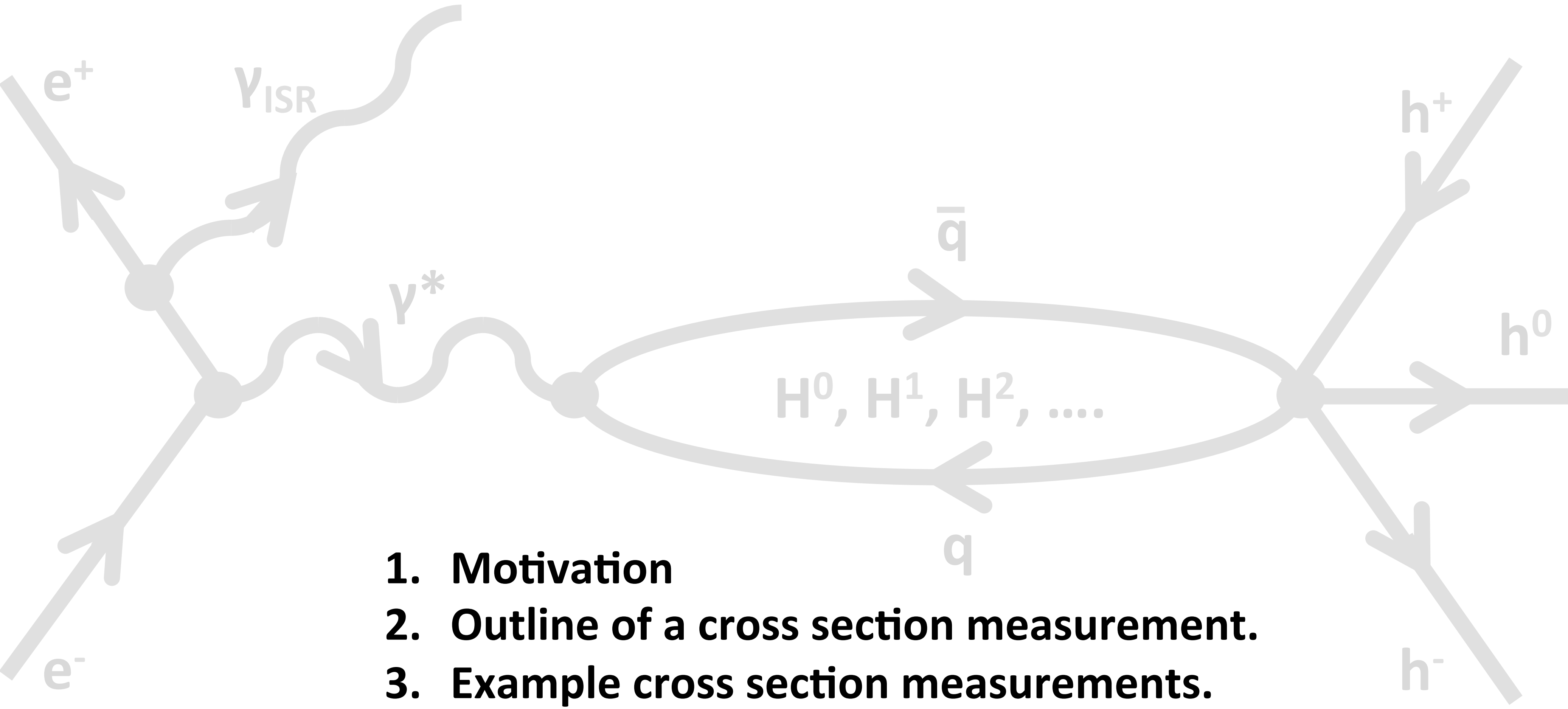
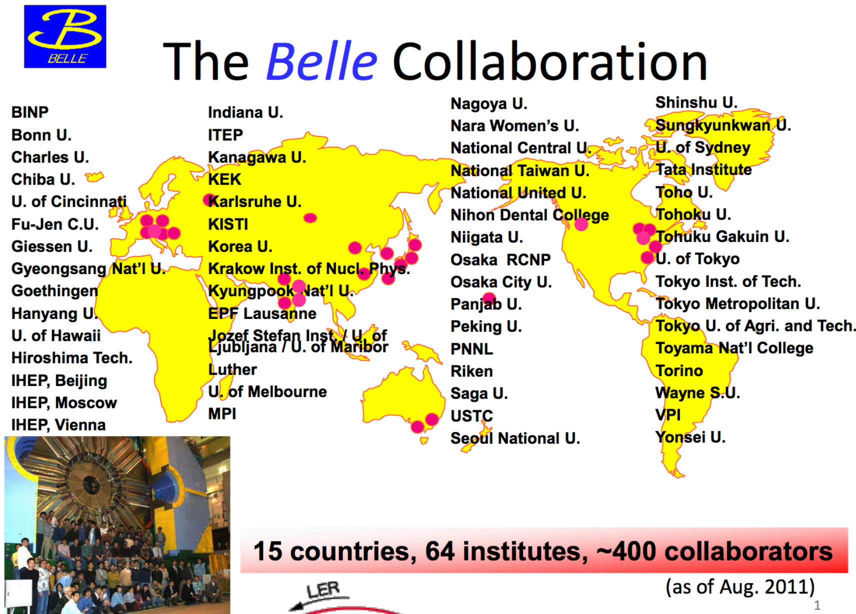
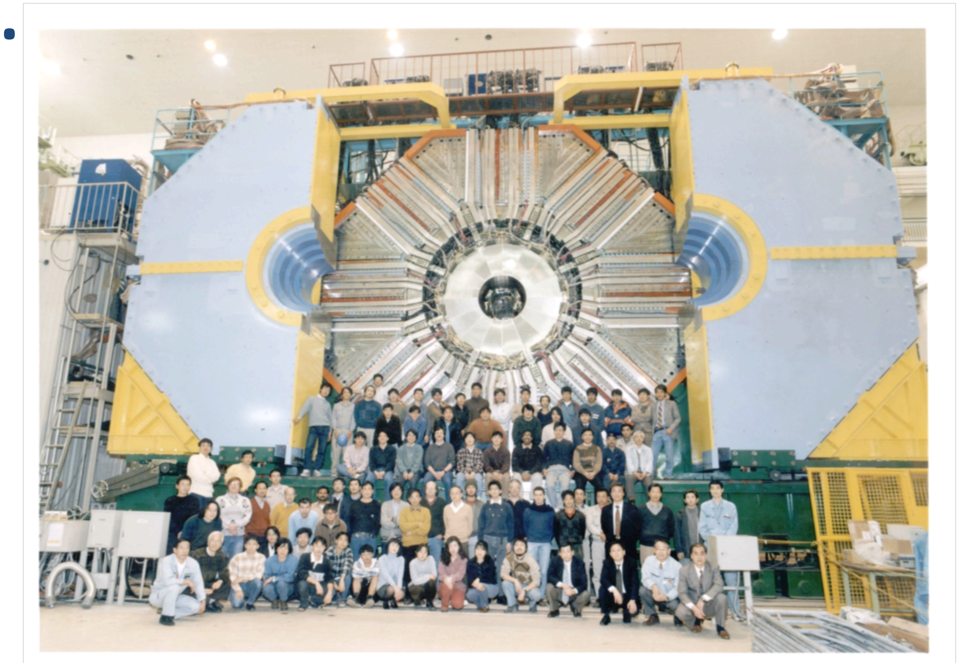
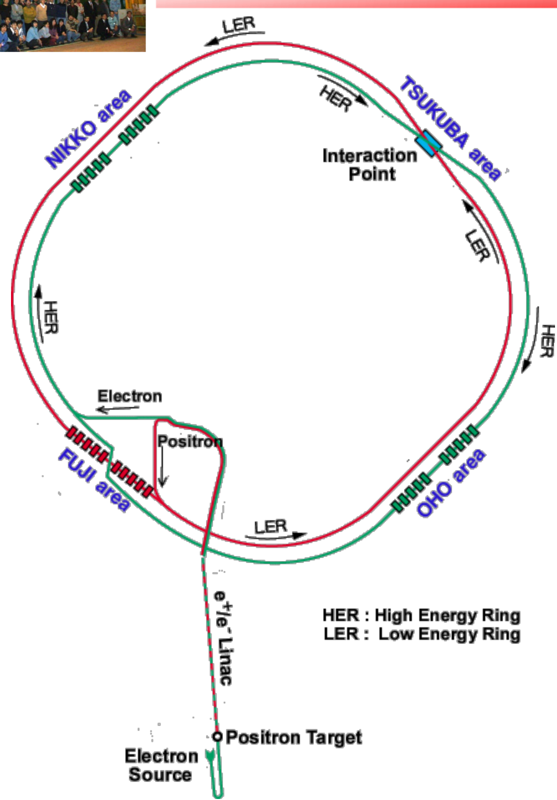


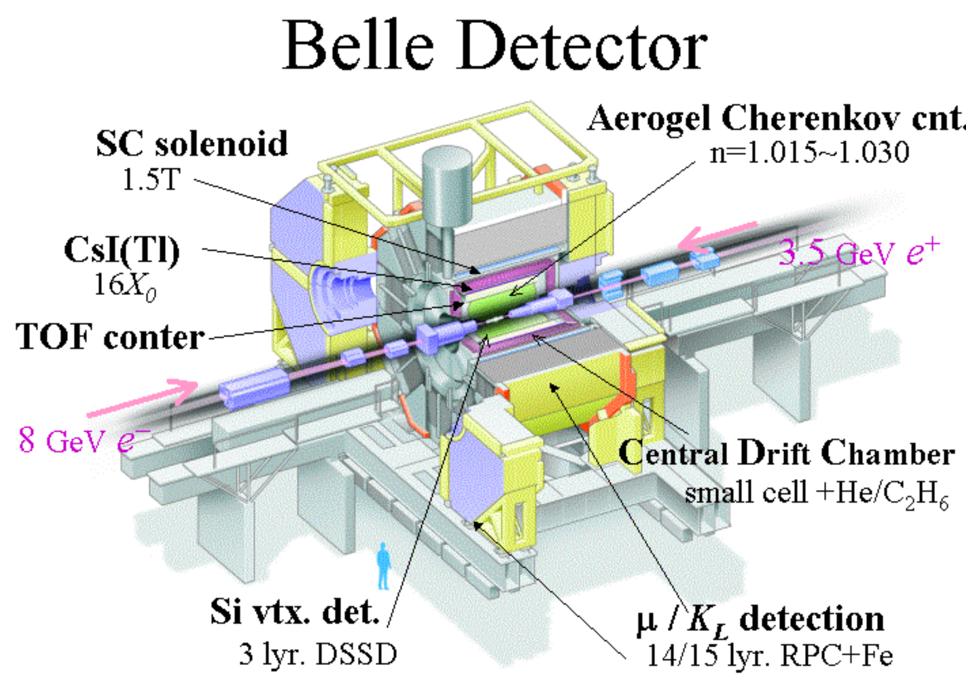
1. **Motivation**
2. **Outline of a cross section measurement.**
3. **Example cross section measurements.**

The international Belle experiment operated at a B-Factory, and uses a general purpose detector.





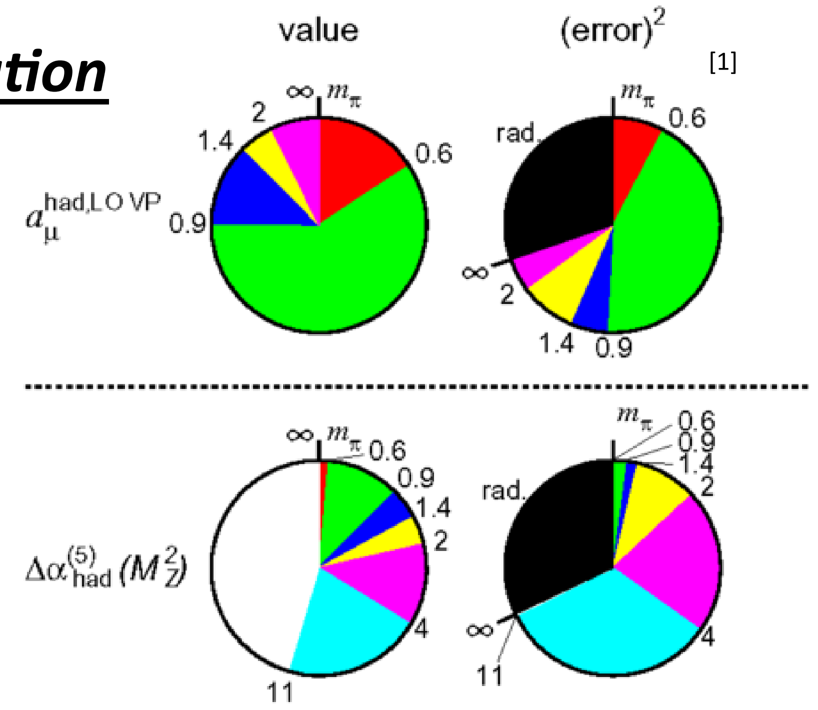




Why measure exclusive e^+e^- hadronic final state cross sections?

A. *Calculation of LO Hadronic Vacuum Polarization*

- a. **Muon anomalous magnetic moment.**
- b. **Running of alpha.**
- c. **Compare isovector e^+e^- cross sections to corresponding τ–spectral functions**



[1] T. Teubner, K. Hagiwara, R. Liao, A. D. Martin and D. Nomura, Nucl. Phys. Proc. Suppl. **225-227**, 282 (2012).

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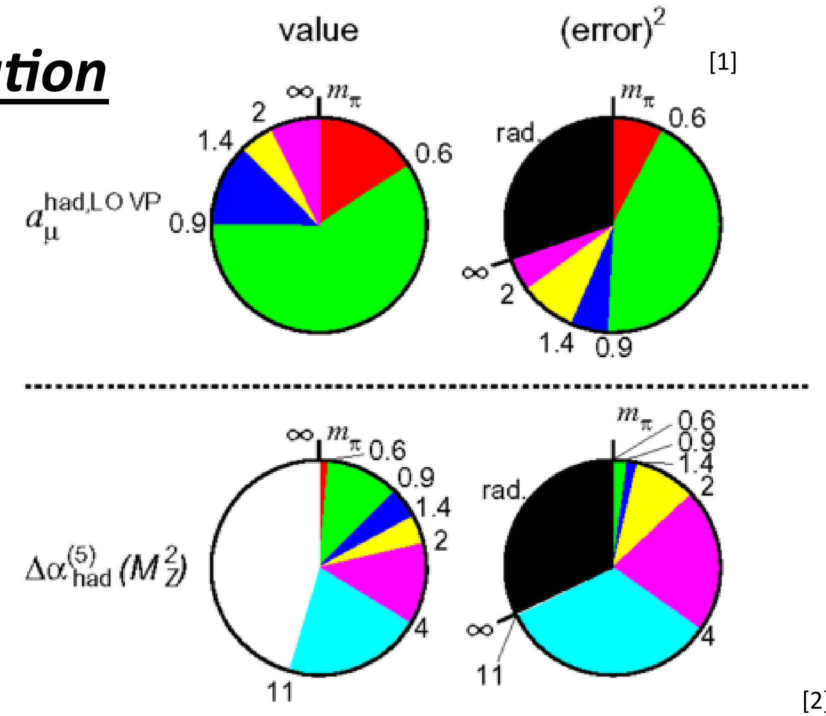
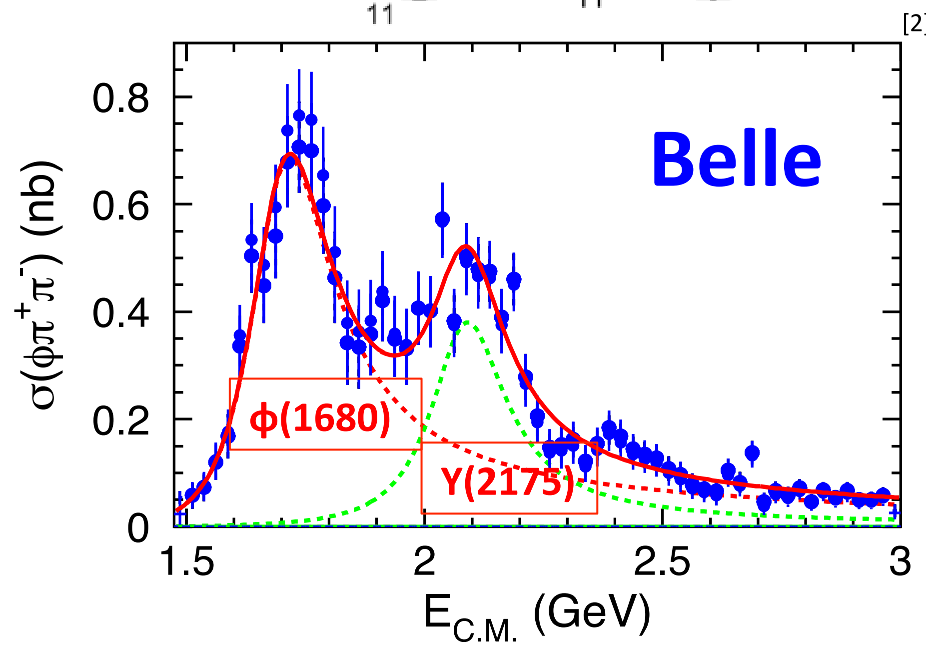
B. Hadron spectroscopy

a. New resonances (X, Y, Z, …)
b. Measure intermediate states
c. Measure resonance parameters
d. Check QCD models

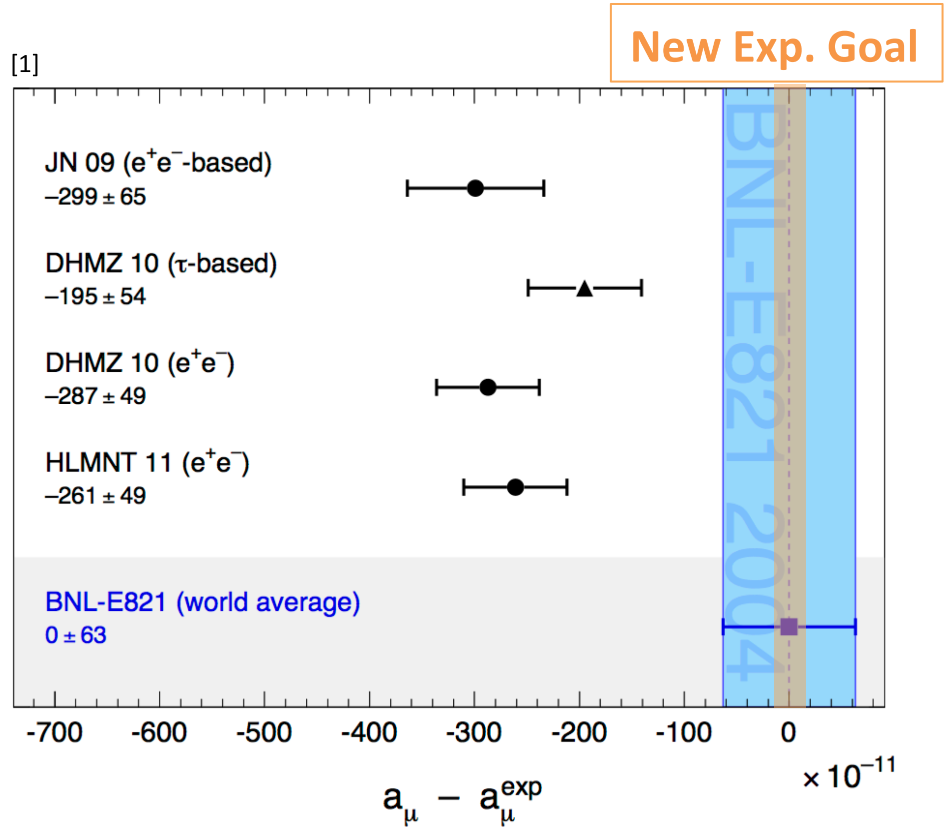


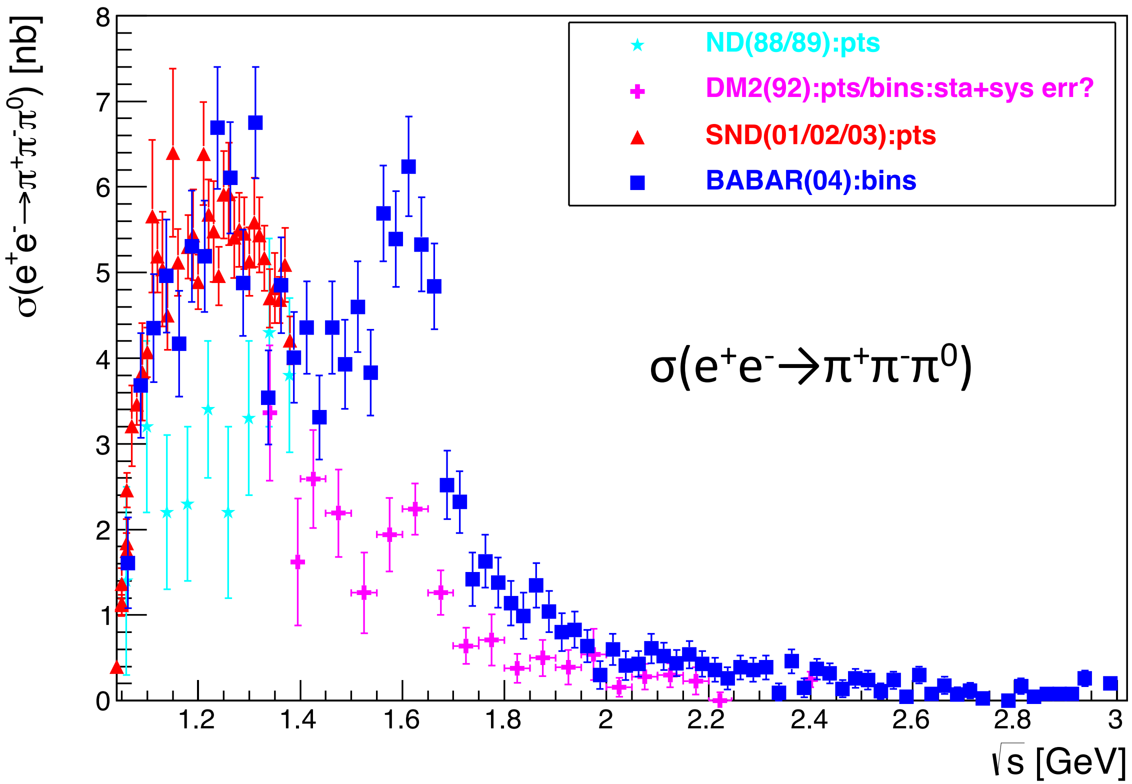


[1] T. Teubner, K. Hagiwara, R. Liao, A. D. Martin and D. Nomura, Nucl. Phys. Proc. Suppl. **225-227**, 282 (2012).
[2] C. P. Shen *et al.* [Belle Collaboration], Phys. Rev. D **80**, 031101 (2009) [arXiv:0808.0006 [hep-ex]].

Multi-hadron cross sections determine Hadronic Vacuum Polarization contributions for muon (g-2) and running of α_{QED}

- **Muon (g-2) hints at new physics beyond the Standard Model**
  - **New experiment aims at 4-fold increased precision**
  - **Improved cross sections required**
- **α_{QED} at *Z* mass important in global EW fits**
  - **Requires improved cross sections as well**





Measurements with increased precision is not enough; Cross checks between experiments is needed as well.

[1] J. Beringer *et al.* (Particle Data Group), Phys. Rev. D**86**, 010001 (2012).

Low energy Hadronic Vacuum Polarization contribution is from the measured cross sections

$$a_\mu^{LOHVP} = \frac{1}{4\pi^3(\hbar c)^2}\int_{4m_\pi^2}^{\infty} ds\,\sigma^{bare}\left(e^+e^- \rightarrow hadrons\right)K(s)$$

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1: Measure Exclusive Cross Sections.

Bare σ's include FSR and exclude VP & ISR:

$$\sigma^{bare}(e^+e^- \to hadrons) = \sigma^{bare}(e^+e^- \to \pi^+\pi^-) + \sigma^{bare}(e^+e^- \to \pi^+\pi^-\pi^0) + \sigma^{bare}(e^+e^- \to K^+K^-) + \cdots$$

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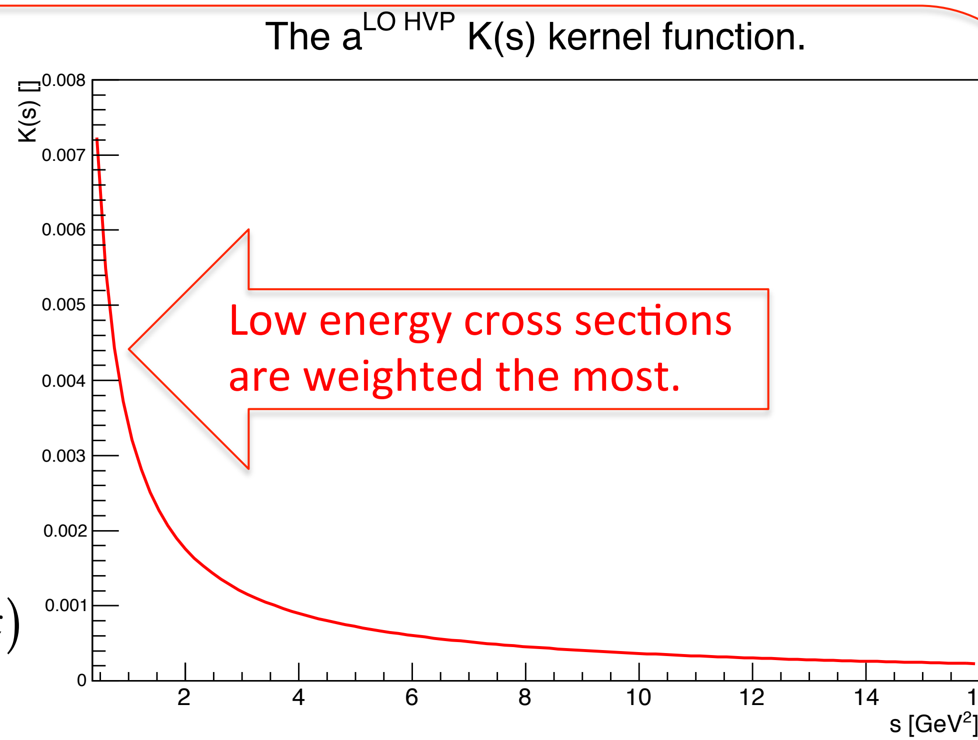
2: Integrate Cross Sections Over Known Kernel Function.

For $s > m_\mu^2$ & $0 \le x \le 1$:

$$x = \frac{1-\beta_\mu}{1+\beta_\mu} \qquad \beta_\mu = \sqrt{1-4m_\mu^2/s}$$

$$K(s) = \frac{x^2}{2}\left(2-x^2\right) + \frac{\left(1+x^2\right)(1+x)^2}{x^2}\left(\ln(1+x) - x + \frac{x^2}{2}\right) + \frac{(1+x)}{(1-x)}x^2\ln(x)$$



Relative importance of exclusive cross sections, ordered by impact on muon (g-2) … (α_{QED} slightly different)

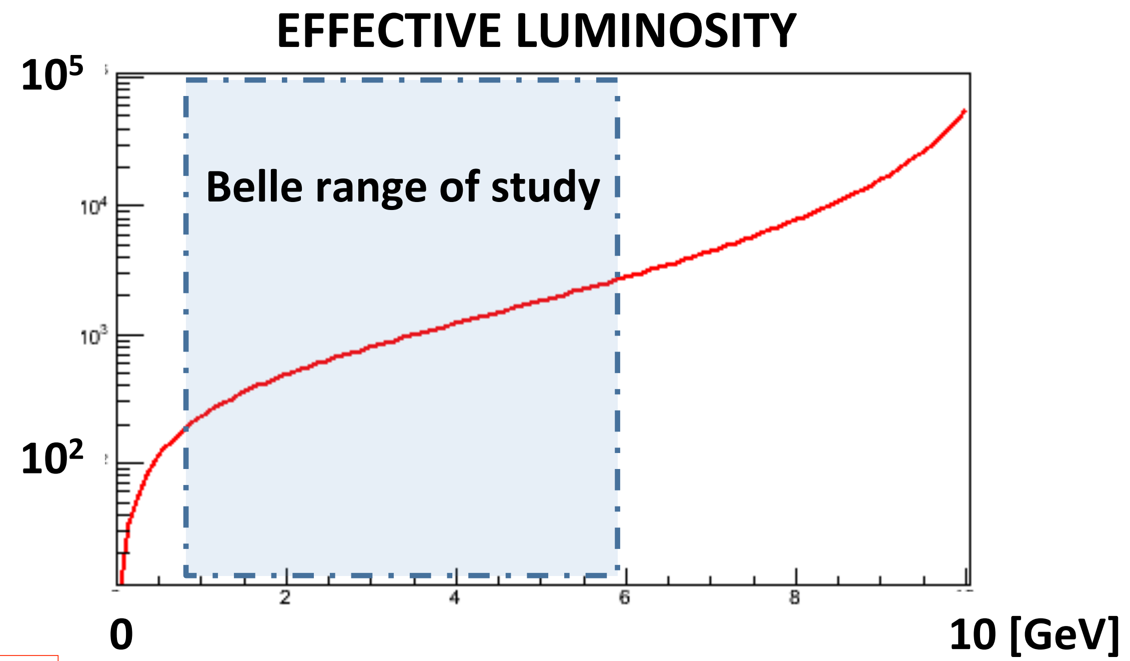
Err² is used as errors are added in quadrature.

10 Largest Exclusive Hadronic Final-State Contributions to $a_\mu^{HVP, LO}$ [1]

Final State	Contribution to $a_\mu^{had, LO}$ [x10⁻¹⁰] val ± sta. ± process sys. ± common sys.	Relative Val Contribution [%]	Relative Err² Contribution [%]
$\pi^+\pi^-$	507.80 ± 1.22 ± 2.50 ± 0.56	73.35	46.12
$\pi^+\pi^-\pi^0$	46.00 ± 0.42 ± 1.03 ± 0.98	6.64	12.59
K^+K^-	21.63 ± 0.27 ± 0.58 ± 0.36	3.12	3.09
$\pi^+\pi^-\pi^0\pi^0$	18.01 ± 0.14 ± 0.17 ± 0.40	2.60	1.19
$\pi^+\pi^-\pi^+\pi^-$	13.35 ± 0.10 ± 0.43 ± 0.29	1.93	1.60
$K_S^0K_L^0$	12.96 ± 0.18 ± 0.25 ± 0.24	1.87	0.87
$\pi^0\gamma$	4.42 ± 0.08 ± 0.13 ± 0.12	0.64	0.22
$K\bar{K}\pi$ (partly from isospin)	2.39 ± 0.07 ± 0.12 ± 0.08	0.35	0.15
$K\bar{K}\pi\pi$ (partly from isospin)	1.35 ± 0.09 ± 0.38 ± 0.03	0.20	0.88
$\pi^+\pi^-\eta$	1.15 ± 0.06 ± 0.08 ± 0.03	0.17	0.06
Total $a_\mu^{had, LO}$	692.3 ± 1.4 ± 3.1 ± 2.4 ± 0.2_ψ ± 0.3_{QCD}	-	-

[1] M. Davier, A. Hoecker, B. Malaescu, Z. Zhang, Eur. Phys. J. C (2011) 71: 1515 [DOI 10.1140/epjc/s10052-010-1515-z].

Fixed-energy experiments can measure a full range of center-of-mass energies

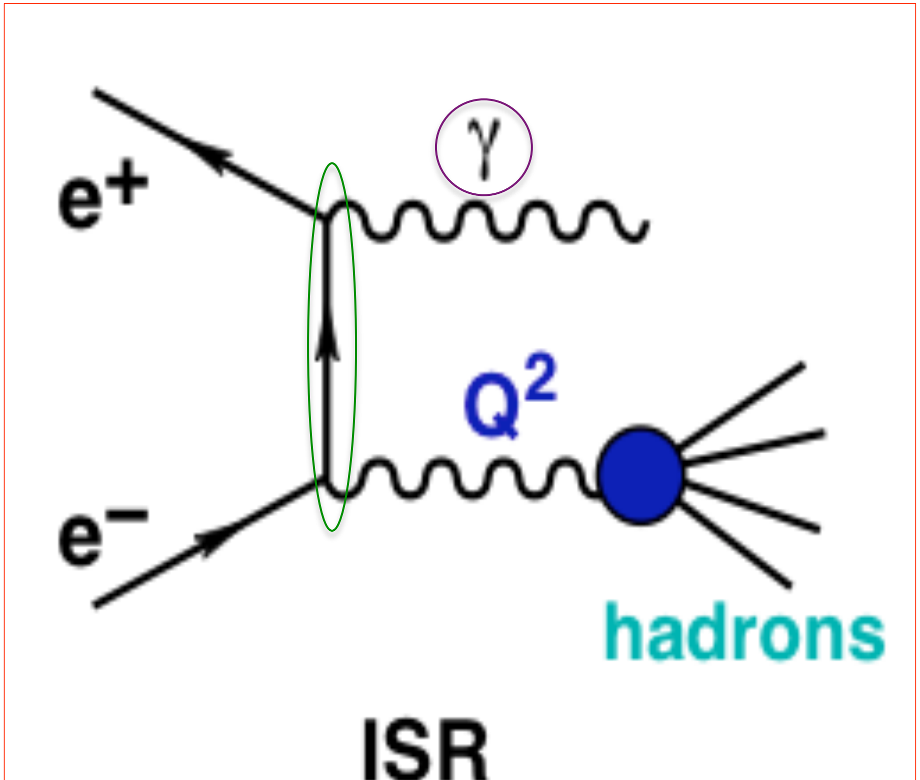


Initial State Radiation:

γ leaves e virtual.

γ ***lowers*** e^+e^- invariant mass.

[1]



[1] G. Rodrigo, Acta Phys. Polon. B 32, 3833 (2001) [arXiv:hep-ph/0111151].

Fixed-energy experiments can measure a full range of center-of-mass energies

Initial State Radiation:

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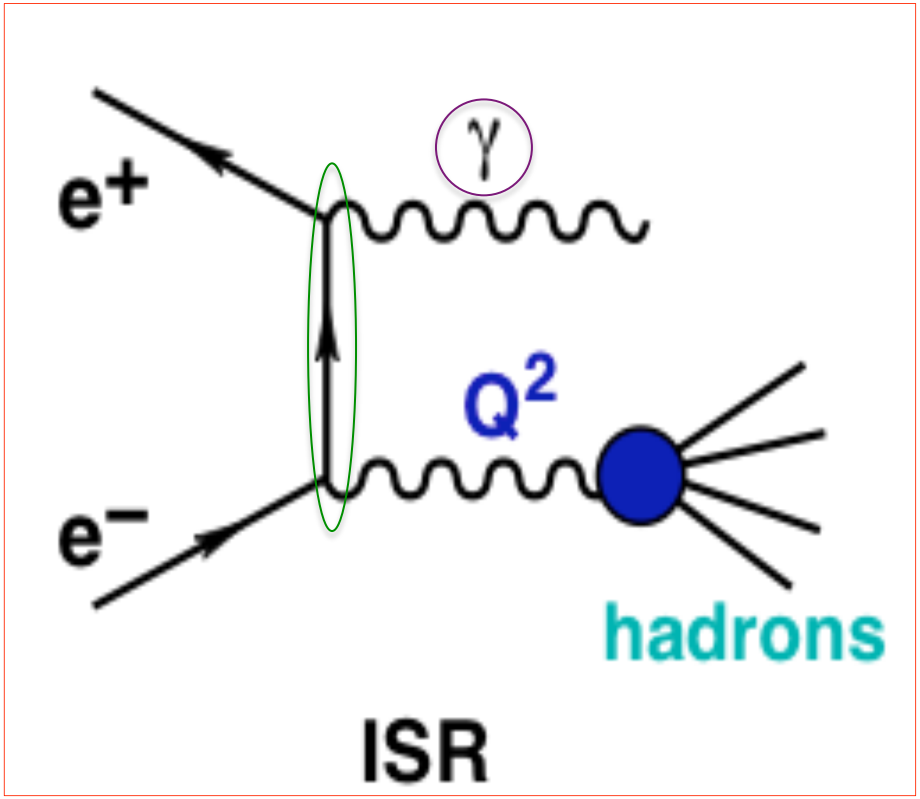
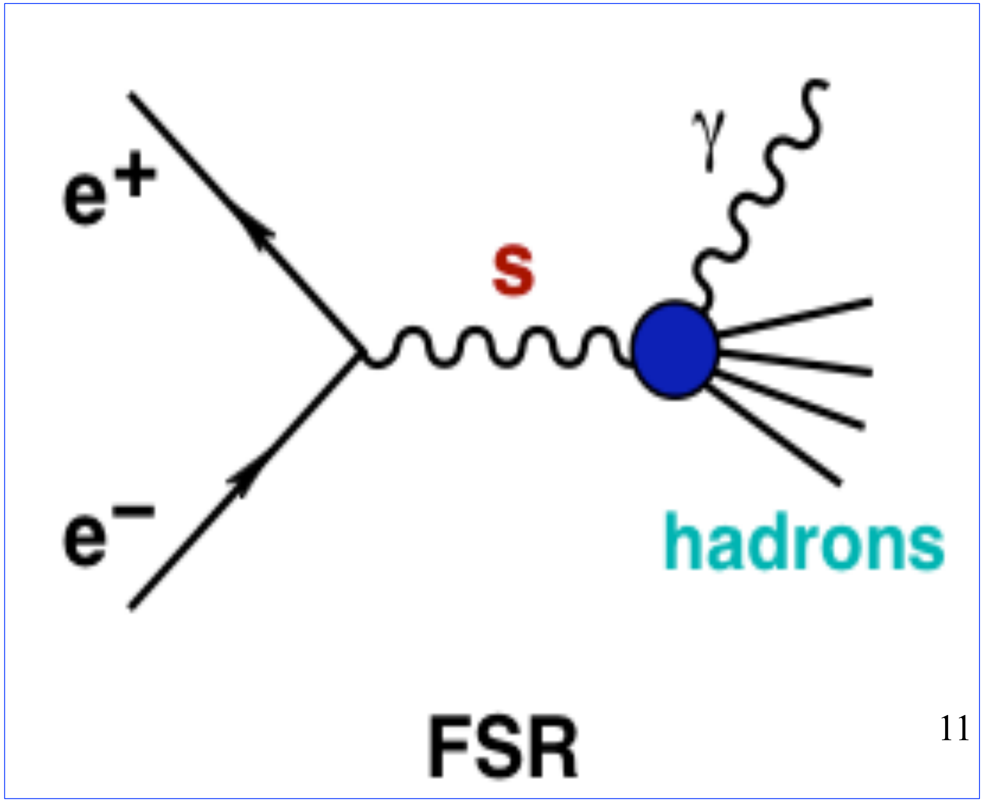
Final State Radiation:

NOT a problem at ***B***-factories:
Suppressed by kinematic cuts to < 1%.

[1]





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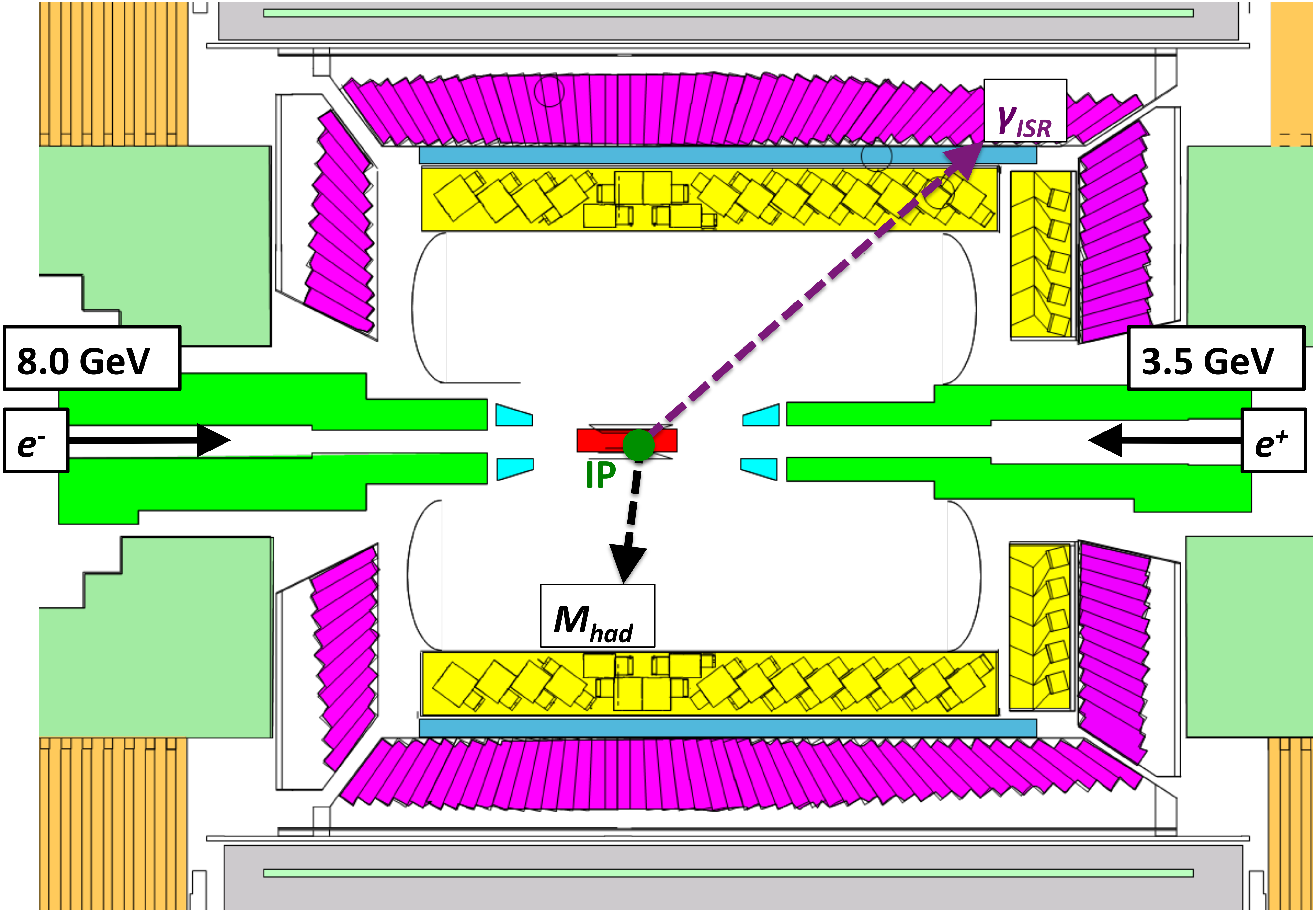
Fixed-energy colliders can measure a wide range of center-of-mass energies via ISR. Example: $e^+e^- \rightarrow \gamma_{ISR} M_{had}$



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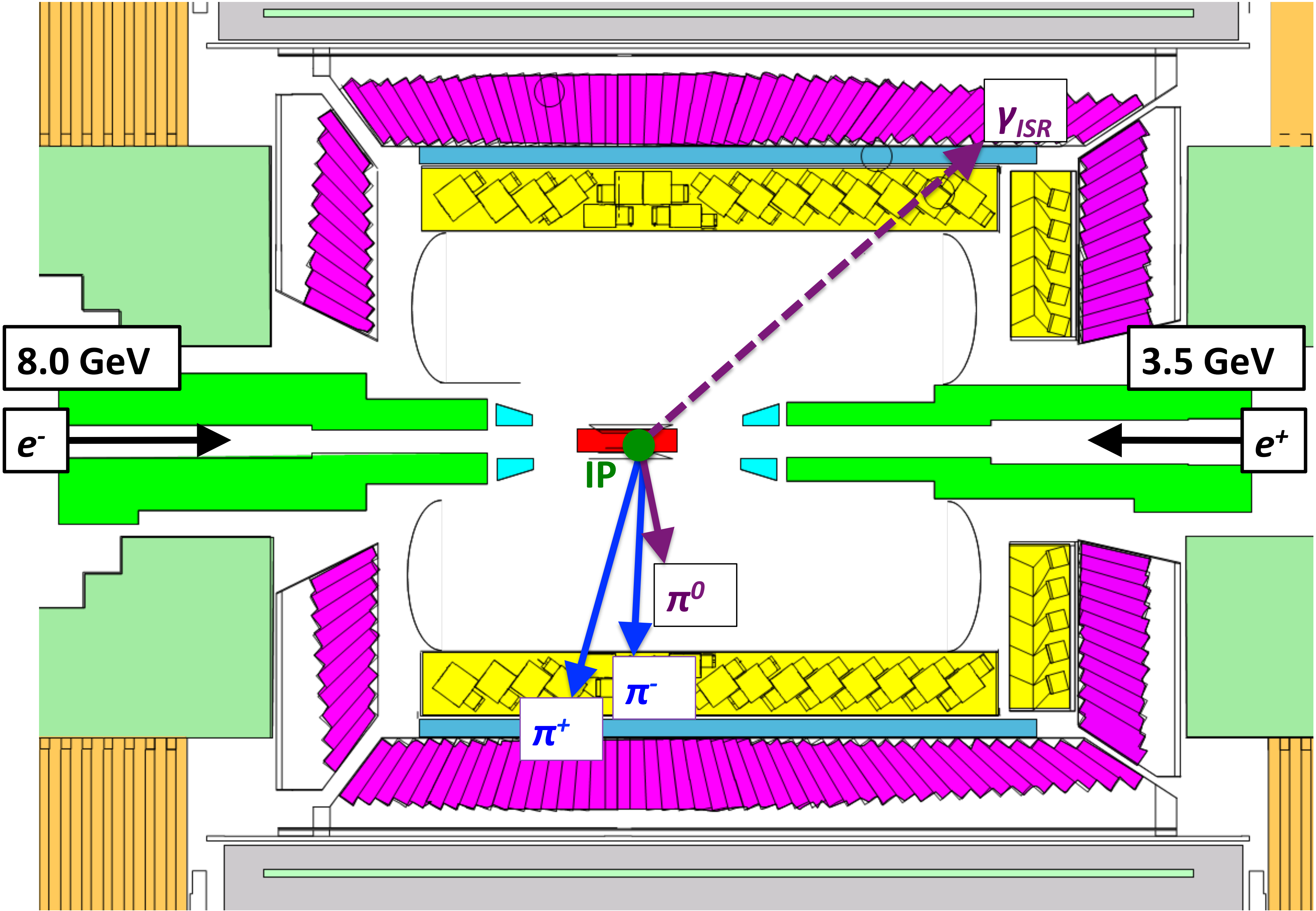

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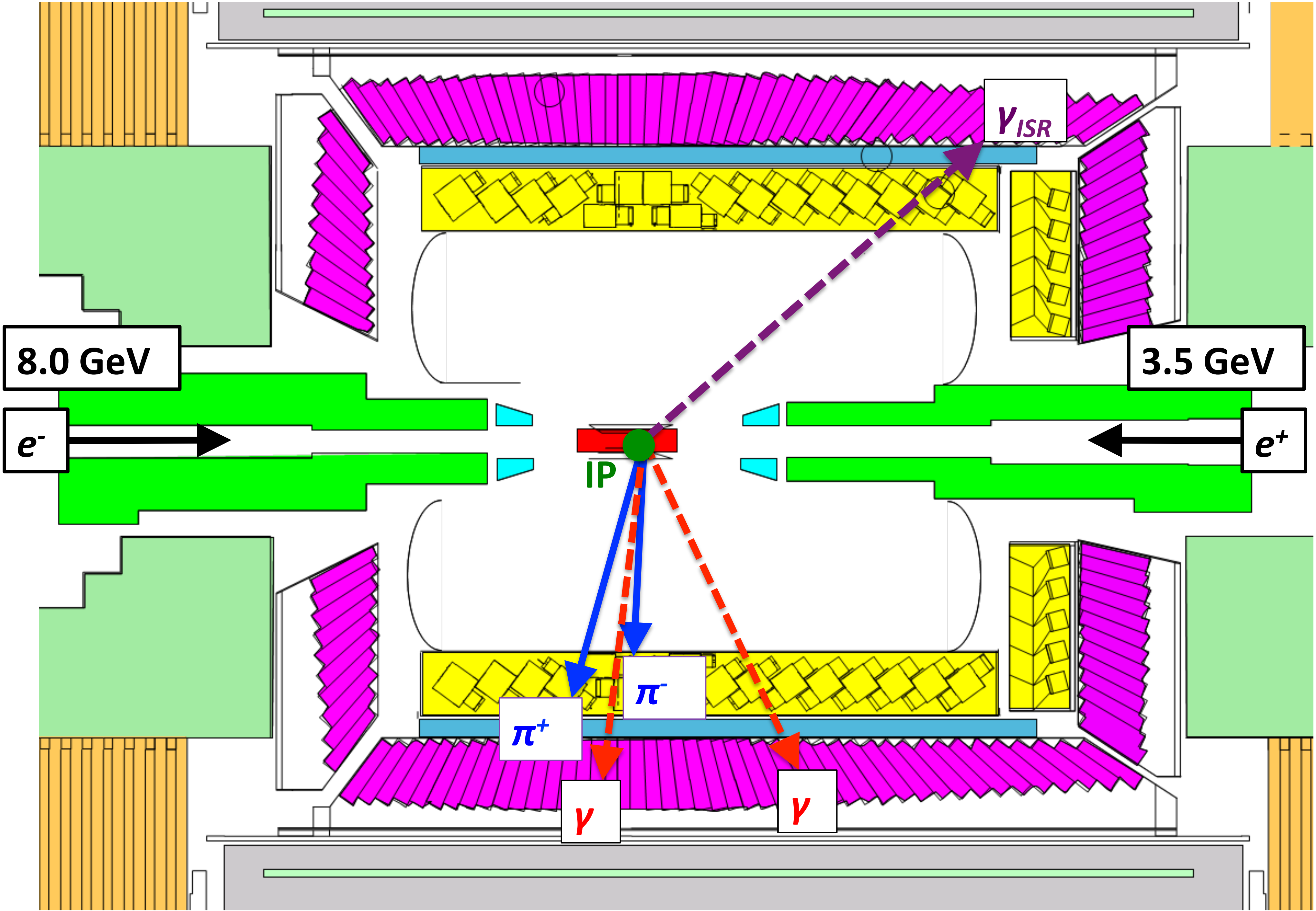

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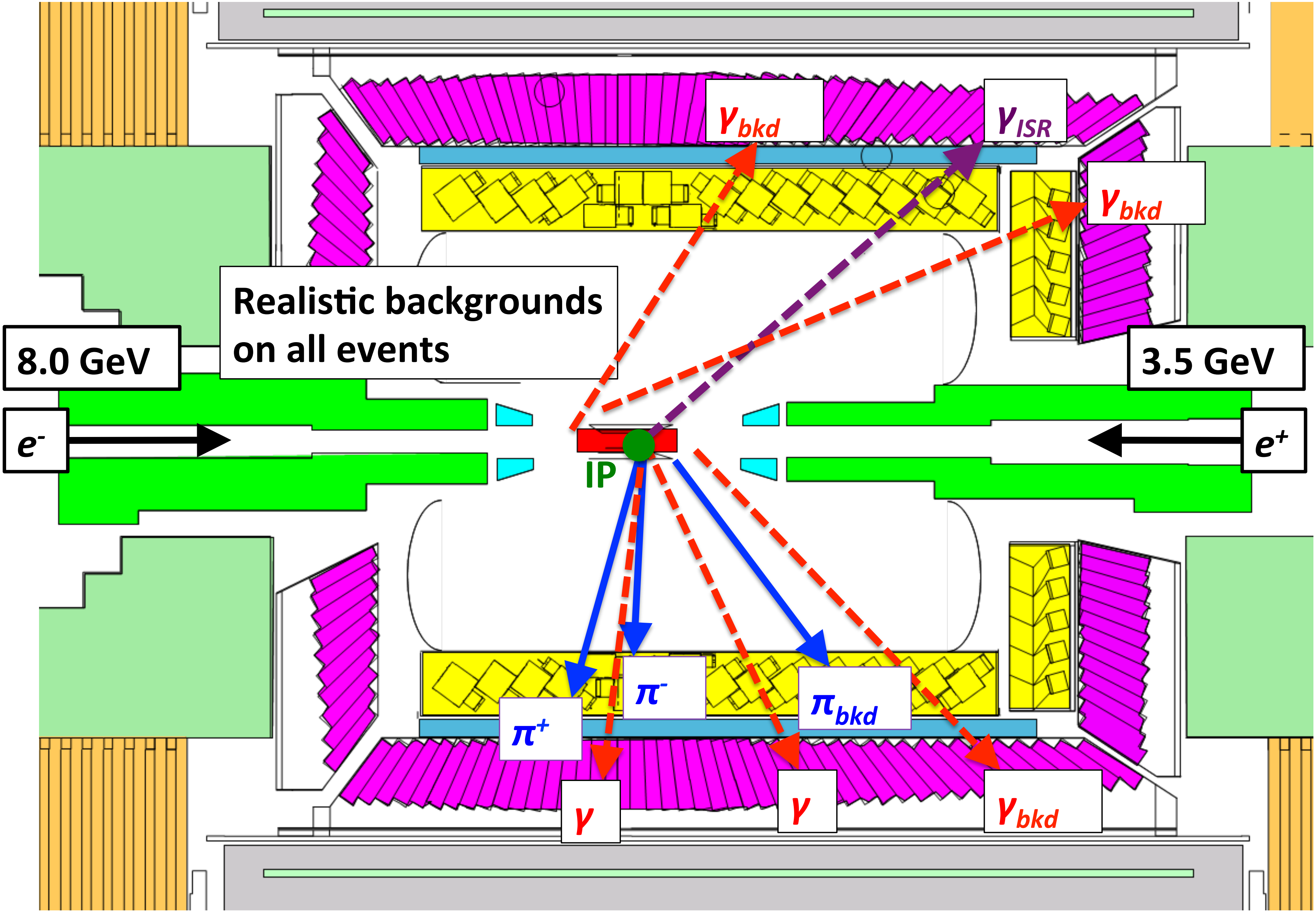

Four types of exclusive cross section measurements at Belle:

Direct Measurement

1. **Machine Operating Energy** ← Measured at a single energy.
2. **Scan Over Small Energy Range** ← Measured over an energy range.

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Direct Measurement

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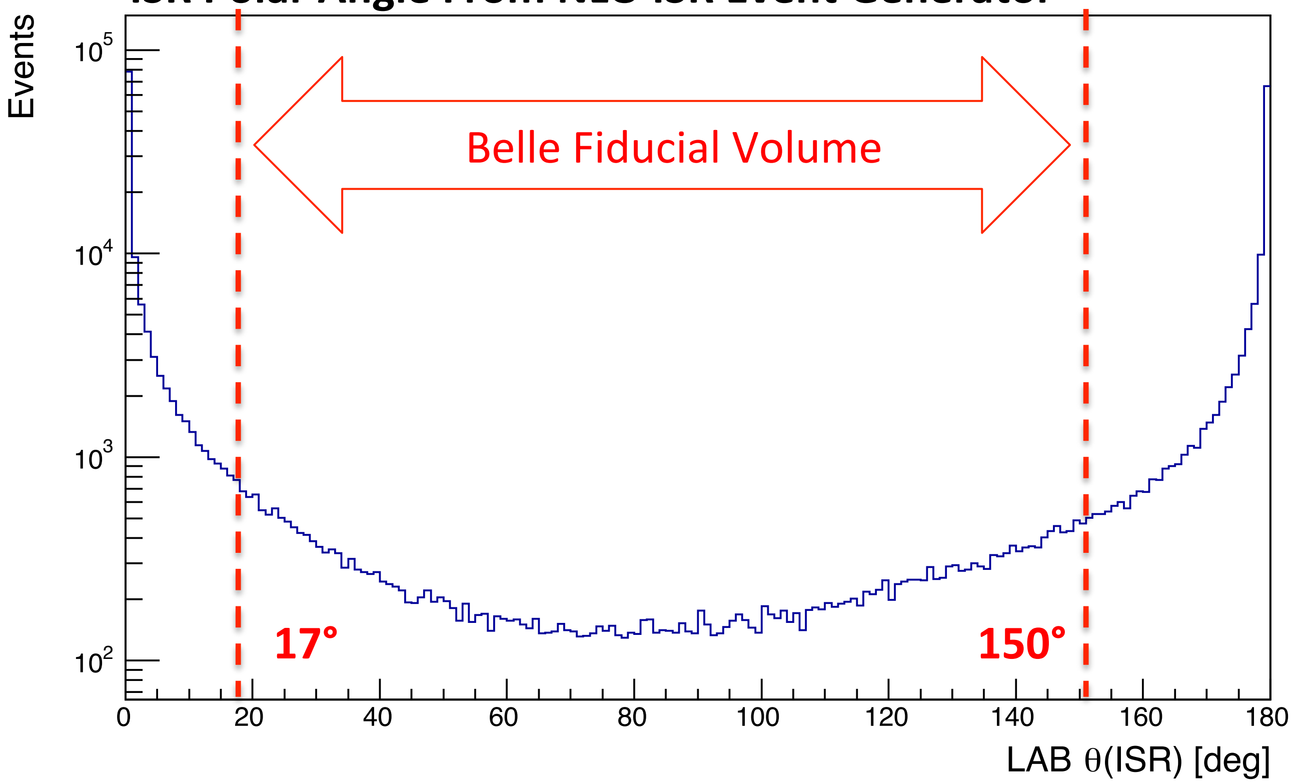
Measured at a single energy.

ISR Measurement

3. No Tag ISR (missing mass method)
4. Tag ISR (measure γ_{ISR})

Measured over an energy range.

ISR Polar Angle From NLO ISR Event Generator



No Tag ISR => Increased Yield
Tag ISR => Suppress Background

Goal: Determine absolute cross sections

$$\sigma_i = \frac{N_i^{corr}}{L_i \varepsilon_i^{corr} B(1+\delta_i^{rad})}$$

- σ_i = ***visible*** cross section for ith bin
- B = product of any branching ratios (PDG)
- N_i^{corr} = corrected signal yield (background subtractions and mass unfolding)

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- δ_i^{rad} = Next-to-leading order ISR radiative corrections are ~5.5% (realistic event generators)

Belle has directly measured a number of e^+e^- hadron production cross sections at the machine operating energy

Final State	Year	Int. Lum. [fb^{-1}]	σ [nb]	E Value [GeV]	Reff. (Slide 36)
$\Upsilon(1S)\pi^0\pi^0$	2013	121.4	$(1.16 \pm 0.06 \pm 0.10) \times 10^{-3}$	10.86	[S]
$\Upsilon(2S)\pi^0\pi^0$	2013	121.4	$(1.87 \pm 0.11 \pm 0.23) \times 10^{-3}$	10.86	[S]
$\Upsilon(3S)\pi^0\pi^0$	2013	121.4	$(0.98 \pm 0.24 \pm 0.15) \times 10^{-3}$	10.86	[S]
$\phi\eta$	2009	516	$(1.4 \pm 0.4 \pm 0.1) \times 10^{-6}$	10.58	[F]
$\phi\eta'$	2009	516	$(5.3 \pm 1.1 \pm 0.4) \times 10^{-6}$	10.58	[F]
$\rho\eta$	2009	516	$(3.1 \pm 0.5 \pm 0.1) \times 10^{-6}$	10.58	[F]
$\rho\eta'$	2009	516	$(3.3 \pm 0.6 \pm 0.2) \times 10^{-6}$	10.58	[F]
$\Upsilon(1S)\pi^+\pi^-$	2009	21.7	$(1.61 \pm 0.10 \pm 0.12) \times 10^{-3}$	≈10.87	[T]
$\Upsilon(2S)\pi^+\pi^-$	2009	21.7	$(2.35 \pm 0.19 \pm 0.32) \times 10^{-3}$	≈10.87	[T]
$\Upsilon(3S)\pi^+\pi^-$	2009	21.7	$(1.44^{+0.55}_{-0.45} \pm 0.19) \times 10^{-3}$	≈10.87	[T]
$\Upsilon(1S)K^+K^-$	2009	21.7	$(0.185^{+0048}_{-0.041} \pm 0.028) \times 10^{-3}$	≈10.87	[T]
$D_T^{*+}D_T^{*-}$	2004	88.9	$< 0.02 \times 10^{-3}$ @ 90% CL	10.58	[P]
$D_T^{*+}D_L^{*-}$	2004	88.9	$(0.55 \pm 0.03 \pm 0.05) \times 10^{-3}$	10.58	[P]
$D_L^{*+}D_L^{*-}$	2004	88.9	$< 0.02 \times 10^{-3}$ @ 90% CL	10.58	[P]
$D^+D_L^{*-}$	2004	88.9	$< 0.006 \times 10^{-3}$ @ 90% CL	10.58	[P]
$D^+D_T^{*-}$	2004	88.9	$(0.62 \pm 0.03 \pm 0.06) \times 10^{-3}$	10.58	[P]
D^+D^-	2004	88.9	$< 0.04 \times 10^{-3}$ @ 90% CL	10.58	[P]

Polarization:
T = Transverse
L = Longitudinal

Belle has also directly measured e^+e^- hadron production cross sections via energy scan

Final State	Year	Int. Lum. [fb^{-1}]	σ^{max} [nb]	E range [GeV]	Reff. (Slide 36)
$\omega\pi^0$	2013	913	6.01×10^{-6}	10.52 to 10.876	[U]
$K^*(892)^0\overline{K^0}$	2013	913	10.77×10^{-6}	10.52 to 10.876	[U]
$K^*(892)^-K^+$	2013	913	1.14×10^{-6}	10.52 to 10.876	[U]
$K_2^*(1430)^0\overline{K^0}$	2013	913	1.65×10^{-6}	10.52 to 10.876	[U]
$K_2^*(1430)^-K^+$	2013	913	8.36×10^{-6}	10.52 to 10.876	[U]
$\Upsilon(1S)\pi^+\pi^-$	2010	8.1	2.14×10^{-3}	10.83 to 11.02	[D]
$\Upsilon(2S)\pi^+\pi^-$	2010	8.1	5.31×10^{-3}	10.83 to 11.02	[D]
$\Upsilon(3S)\pi^+\pi^-$	2010	8.1	1.47×10^{-3}	10.83 to 11.02	[D]

Belle has measured a number of high energy (> 3 GeV) e^+e^- hadron production cross sections via ISR

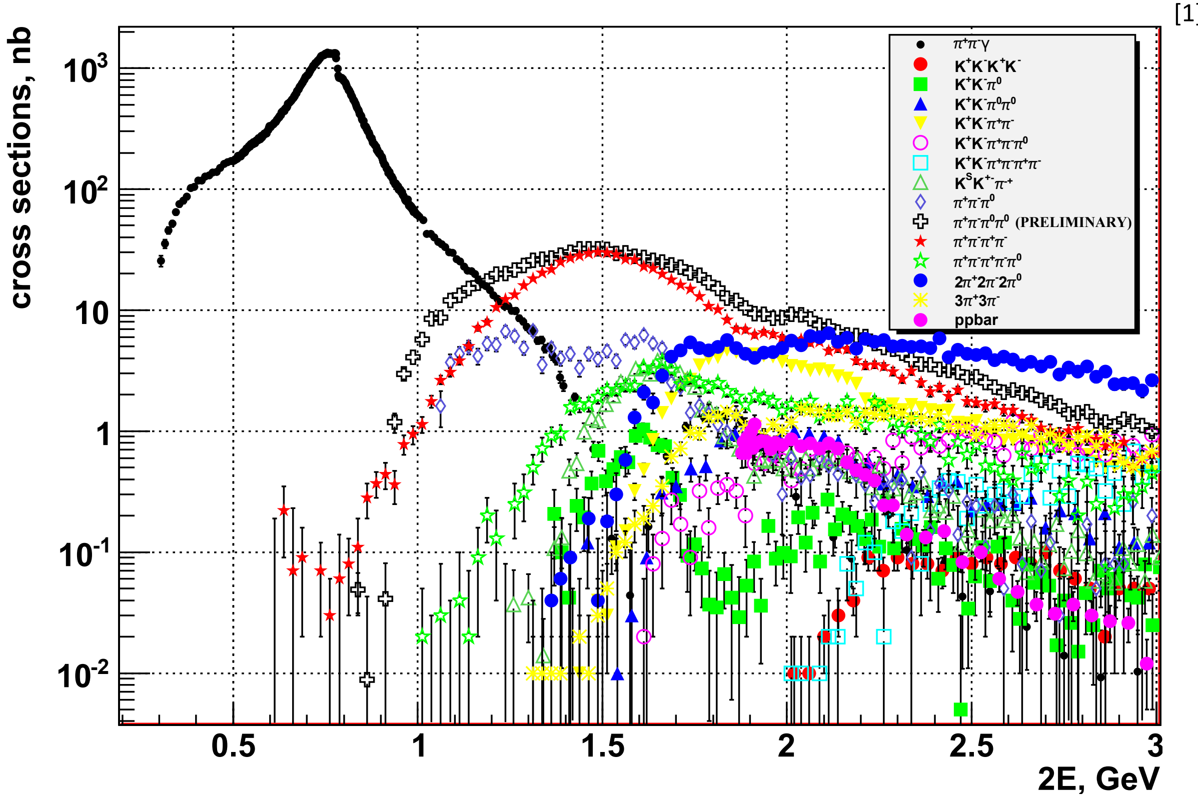
Final State	Year	Int. Lum. [fb^{-1}]	E range [GeV]	σ_{max} [nb]	Reff. (Slide 36)
$\pi^+\pi^- J/\psi$	2013	967	3.8 to 5.5	72×10^{-3}	[A]
$\eta J/\psi$	2012	980	3.8 to 5.3	80×10^{-3}	[B]
$D_s^+ D_s^-$	2011	967	3.8 to 5	0.45	[C]
$D_s^+ D_s^{*-}$	2011	967	4 to 5	0.9	[C]
$D_s^{*+} D_s^{*-}$	2011	967	4.2 to 5	0.5	[C]
$D^0 D^{*-} \pi^+$	2009	695	4 to 5.2	0.65	[E]
$\Lambda_c^+ \Lambda_c^-$	2008	695	4.56 to 5.4	0.55	[I]
$D^0 D^- \pi^+$	2008	673	4 to 5	0.6	[J]
$D\overline{D}$	2008	673	3.7 to 5	9	[K]
$D^+\overline{D}^-$	2008	673	3.7 to 5	4	[K]
$D^0\overline{D}^0$	2008	673	3.7 to 5	5.5	[K]
$K^+K^- J/\psi$	2007	673	4.1 to 6	10×10^{-3}	[L]
$\pi^+\pi^- \psi(2S)$	2007	673	4.1 to 5.5	80×10^{-3}	[M]
$\pi^+\pi^- J/\psi$	2007	548	3.8 to 5.5	70×10^{-3}	[N]
$D^{*+} D^{*-}$	2007	547.8	4 to 5	3.4	[O]
$D^+ D^{*-}$	2007	547.8	3.88 to 5	4.6	[O]

Belle still has the opportunity to measure a number of low energy (< 3 GeV) e^+e^- hadron production cross sections via ISR

Final State	Year	Int. Lum. [fb^{-1}]	E range [GeV]	σ_{max} [nb]	Reff. (Slide 36)
$\pi^+\pi^-\eta$	In prog.	562	0.9 to 3.5	≈4.3	-
$\pi^+\pi^-\pi^0$	In prog.	562	0.73 to 3.5	$\approx 1.5 \times 10^3$	-
$\phi\pi^+\pi^-$	2009	673	1.5 to 3	0.7	[G]

Example BaBar low energy cross section measurements via ISR

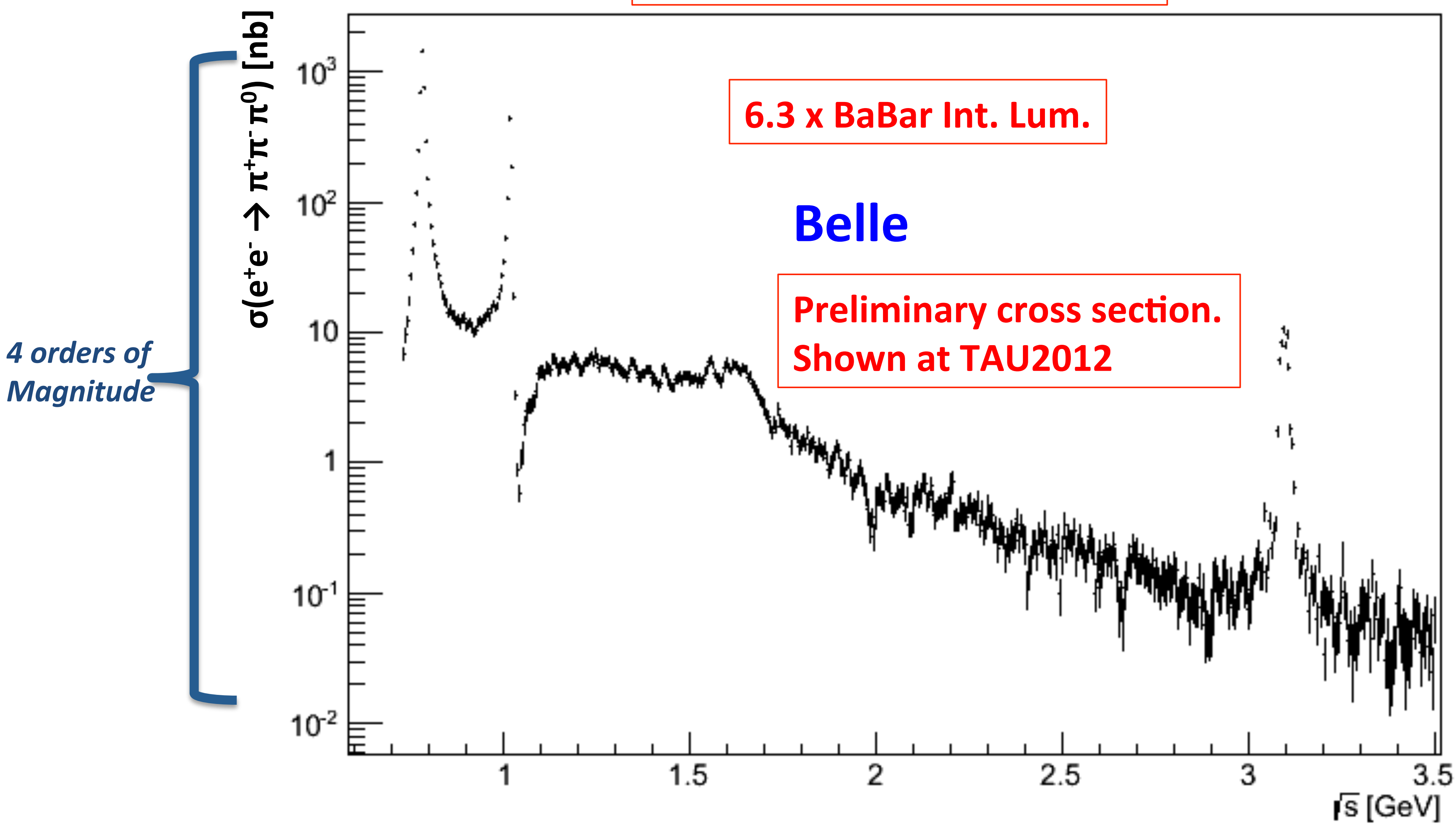
[1]



[1] A. Hafner [BaBar Collaboration], Nucl. Phys. Proc. Suppl. **207-208**, 133 (2010).

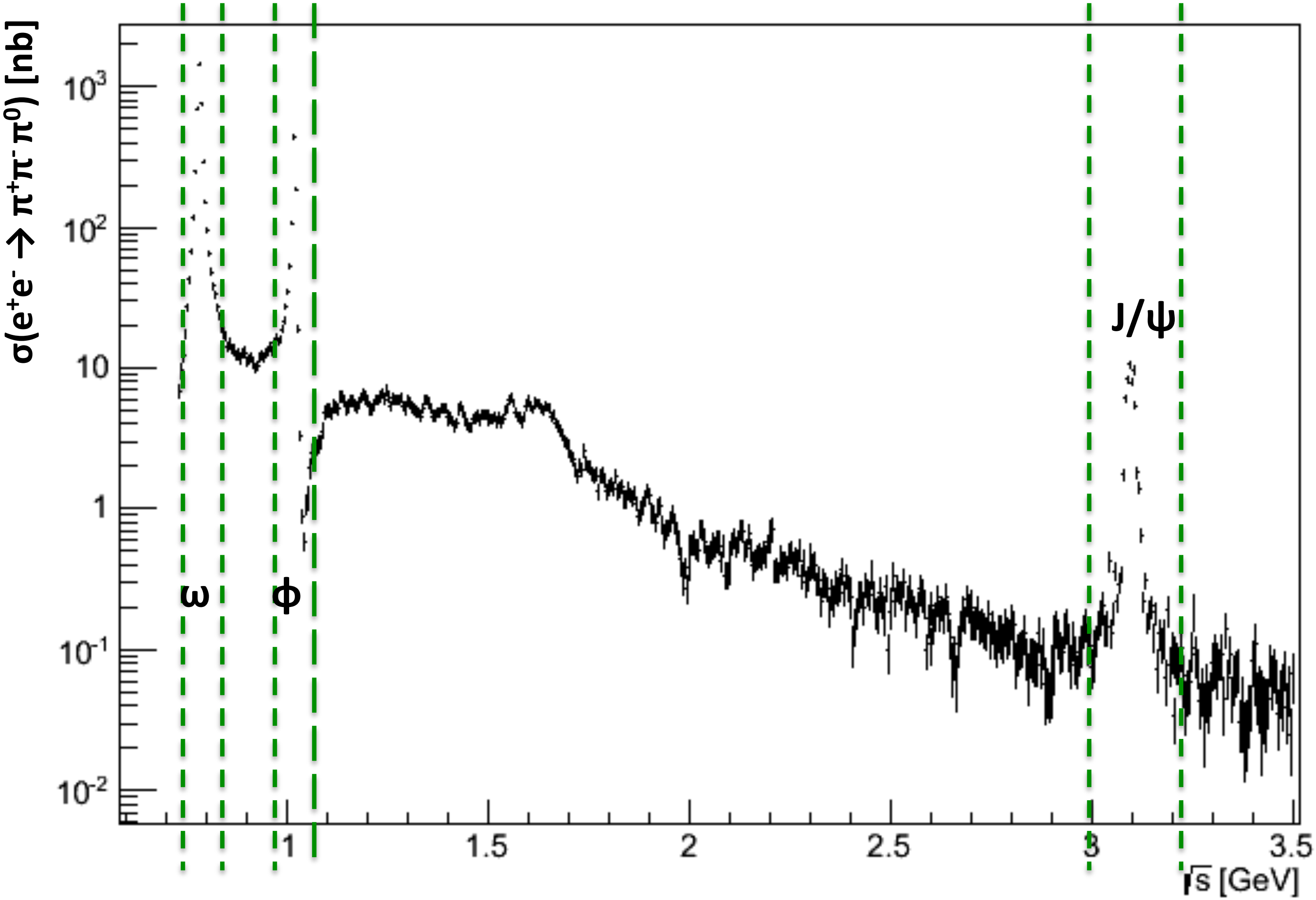
Low energy (> 3 GeV) cross section measurements are focused precision measurements.

Belle systematic error goal is 5%.



Systematic errors, background leakage, and small radiative correction checks to be completed in near future

Narrow resonance vector meson ISR cross sections can provide a cross check on the preliminary Belle $e^+e^- \rightarrow \pi^-\pi^+\pi^0$ cross section.



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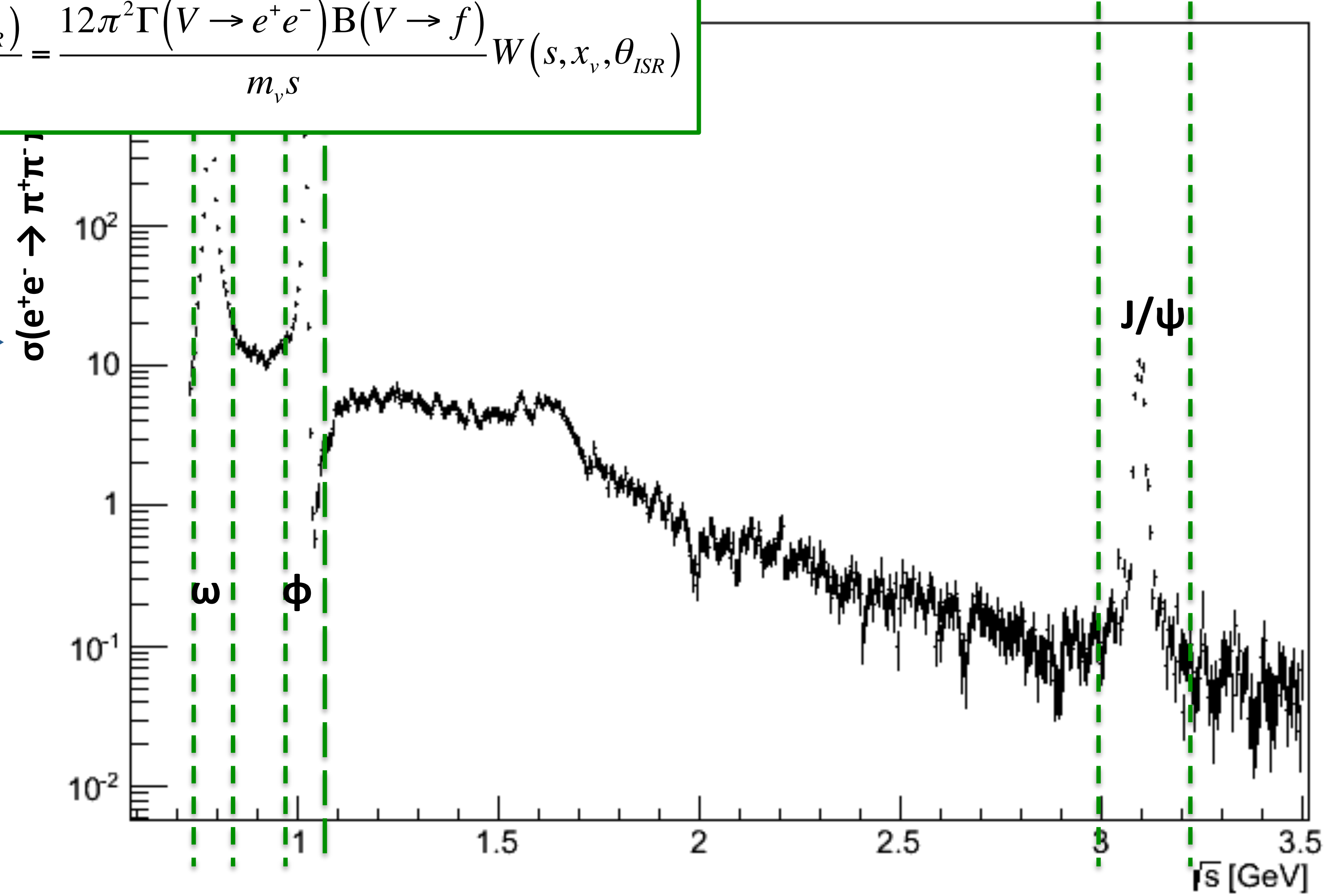
1st Step:

Integrate differential cross section over ISR fiducial volume

$$\frac{d\sigma_{ISR}(s,\theta_{ISR})}{d\cos\theta_{ISR}} = \frac{12\pi^2\Gamma(V \rightarrow e^+e^-)\mathrm{B}(V \rightarrow f)}{m_v s} W(s, x_v, \theta_{ISR})$$

NOT **the same cross sections**



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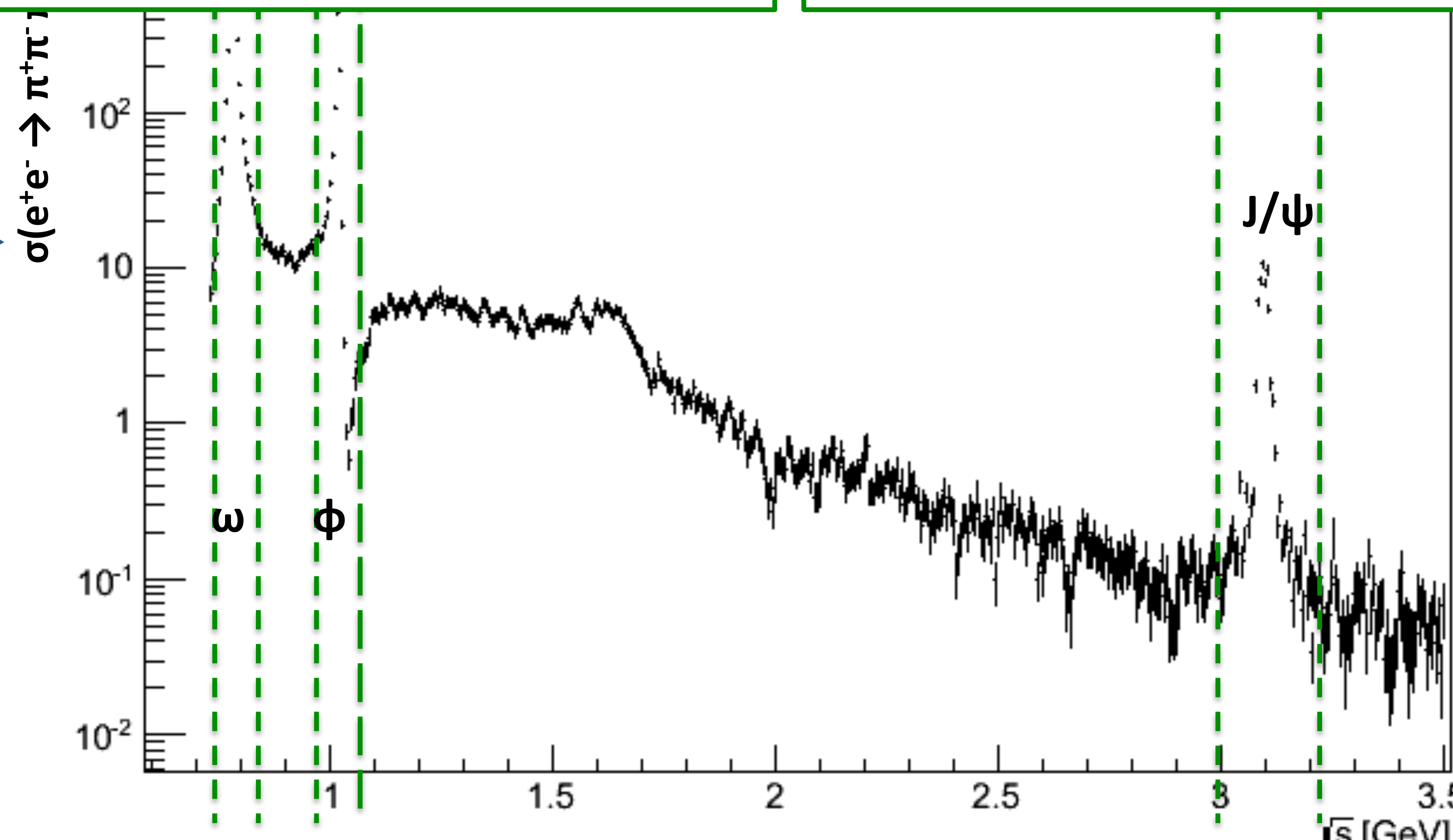
2^{nd} Step:

Integrate cross section over √s resonance-range; solve for *Γ(V → e⁺e⁻)B(V → f)*

$$\sigma_{ISR}\left(\theta_{ISR}^{\min} < \theta_{ISR} < \theta_{ISR}^{\max}\right) = \frac{N_{signal}}{\varepsilon RL\mathrm{B}}$$

***NOT* the same cross sections**



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Integrate differential cross section over ISR fiducial volume

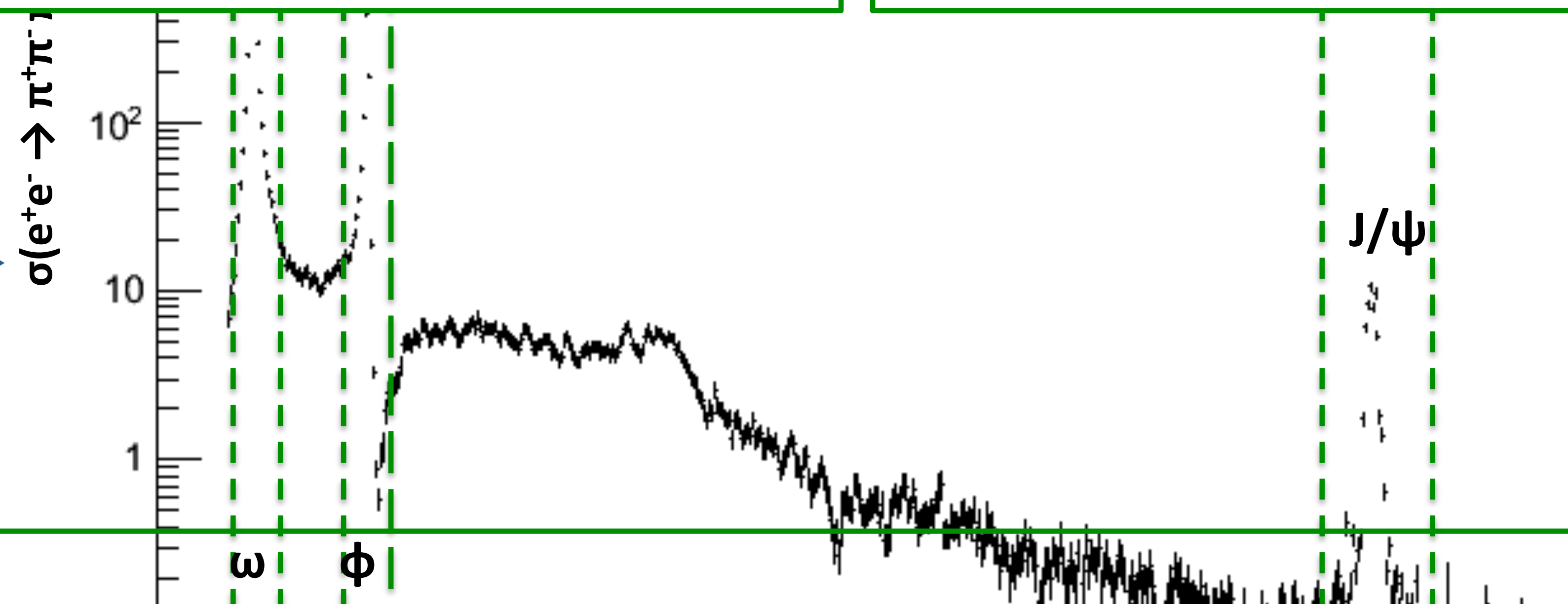
$$\frac{d\sigma_{ISR}(s,\theta_{ISR})}{d\cos\theta_{ISR}} = \frac{12\pi^2\Gamma(V \rightarrow e^+e^-)\mathrm{B}(V \rightarrow f)}{m_v s} W(s,x_v,\theta_{ISR})$$

2nd Step:

Integrate cross section over √s resonance-range; solve for *Γ(V → e⁺e⁻)B(V → f)*

$$\sigma_{ISR}\left(\theta_{ISR}^{\min} < \theta_{ISR} < \theta_{ISR}^{\max}\right) = \frac{N_{signal}}{\varepsilon R L B}$$

***NOT* the same cross sections**



Results:

Source	Γ(ω → e⁺e⁻)B(ω → π⁺π⁻π⁰) [eV]	Γ(ϕ → e⁺e⁻)B(ϕ → π⁺π⁻π⁰) [eV]	Γ(J/ψ→ e⁺e⁻)B(J/ψ → π⁺π⁻π⁰) [eV]
Prelim. Belle	563.0 ± 3.2 ± Sys. Err.	217.1 ± 2.1 ± Sys. Err.	117.3 ± 3.8 ± Sys. Err.
PDG [1]	551 ± 13	192.8 ± 4.8	116.4 ± 5.4

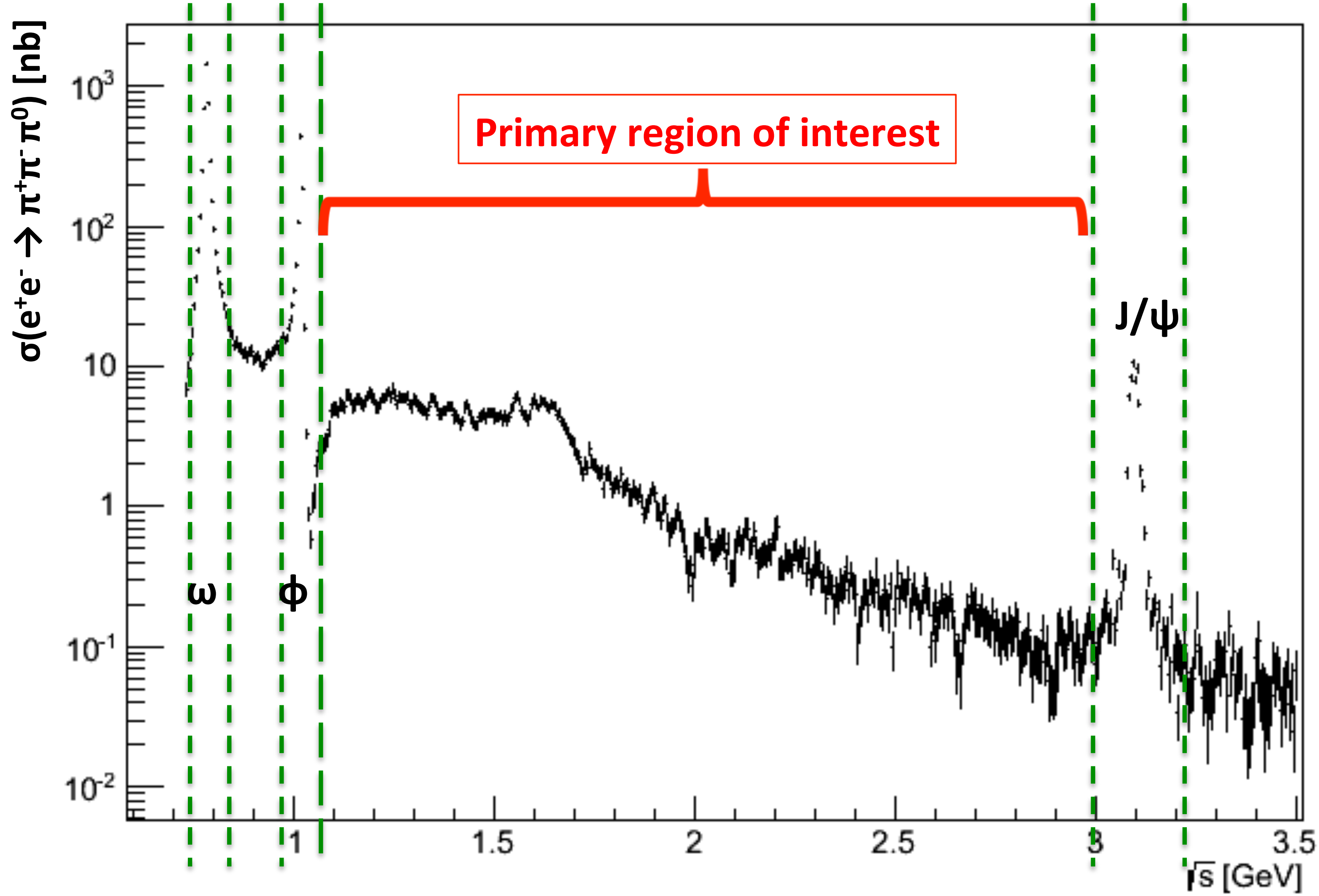
Belle: Sta. Err. only. (5% Sys. Err. goal)

PDG: ϕ → ρπ + π⁺π⁻π⁰

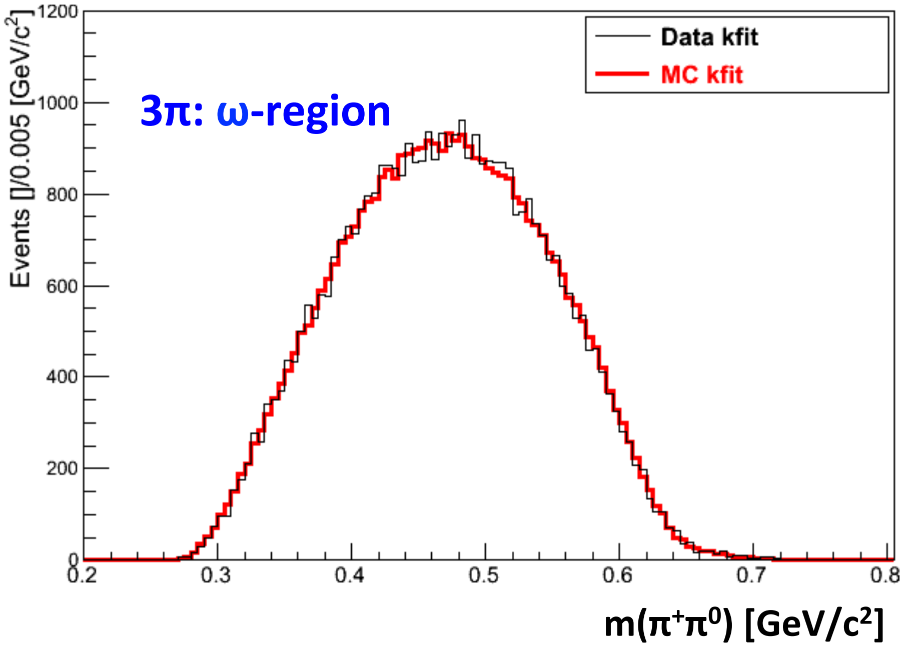
[1] J. Beringer et al. (Particle Data Group), Phys. Rev. D86, 010001 (2012) and 2013 partial update for the 2014 edition.

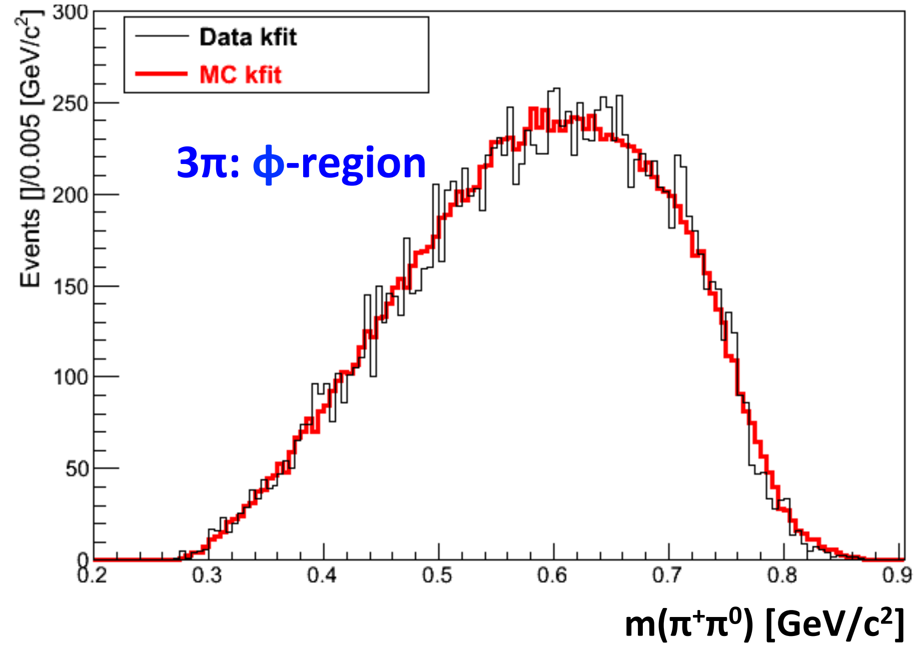
Narrow resonance vector meson ISR cross sections can provide a cross check on the preliminary Belle $e^+e^- \rightarrow \pi^-\pi^+\pi^0$ cross section.

- **This analysis may lock on to the well measured ω and ϕ resonances.**
- **Locking on will provide a more precise $\sigma(1.05 \leq \sqrt{s} \leq 3$ GeV).**

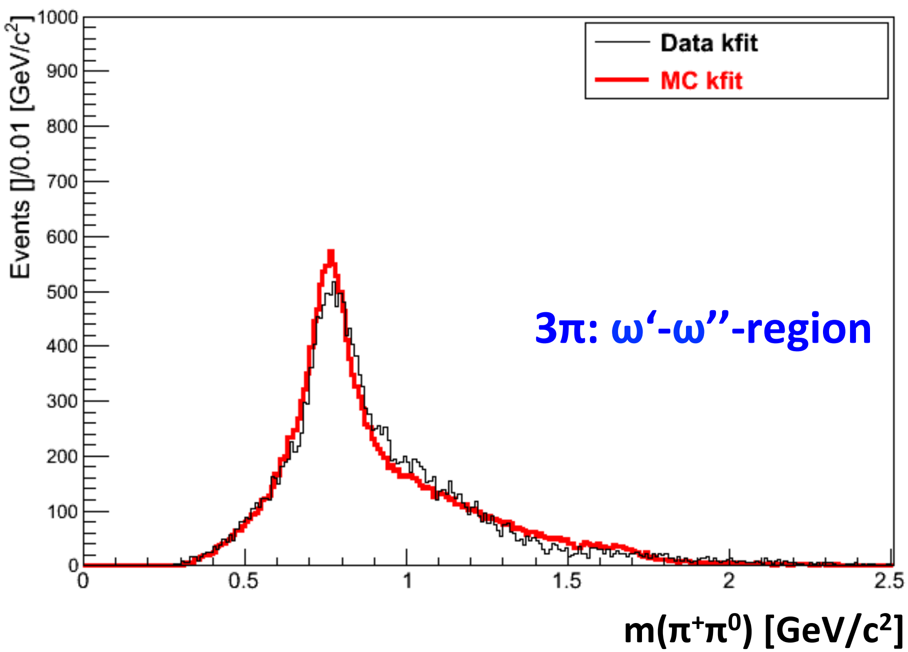


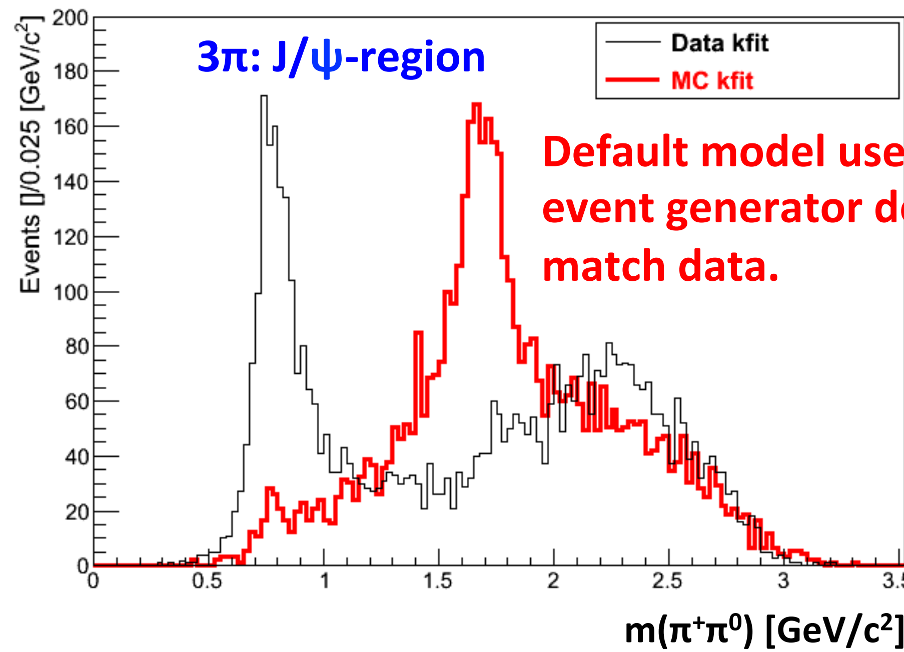
Belle cross section measurements are also interested in looking at intermediate states.





Belle $m(\pi^+\pi^0)$ distributions for $e^+e^- \rightarrow \pi^+\pi^-\pi^0\gamma_{ISR}$.

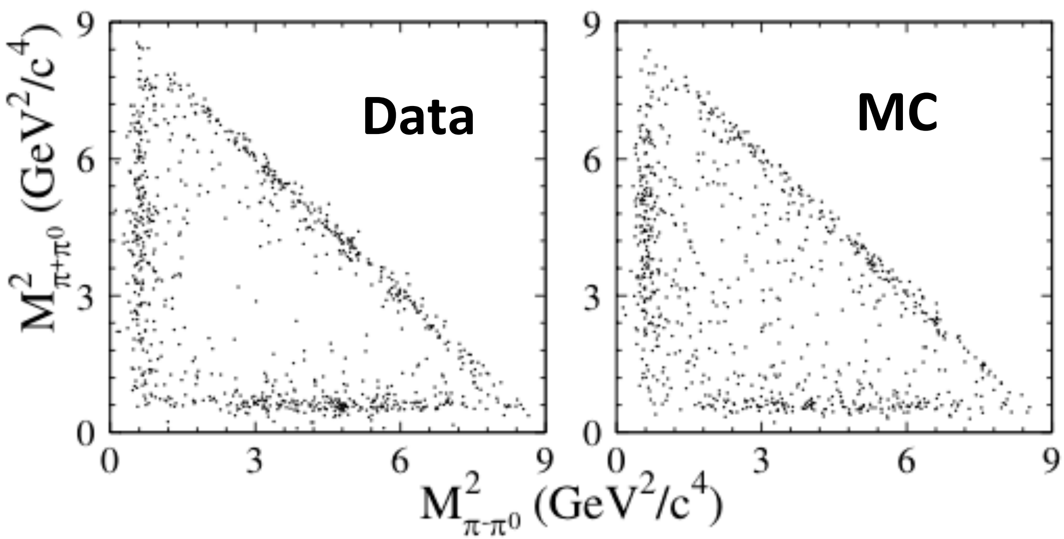


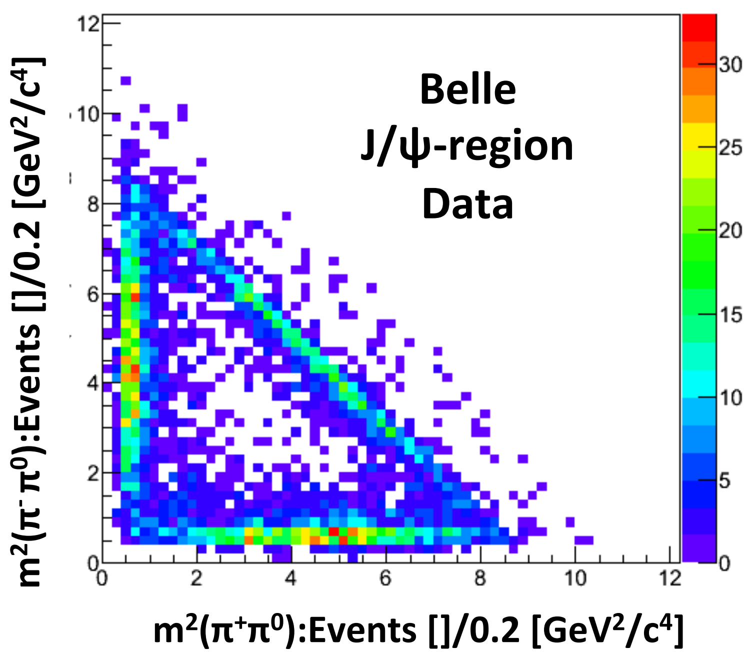


Default model used by event generator does not match data.

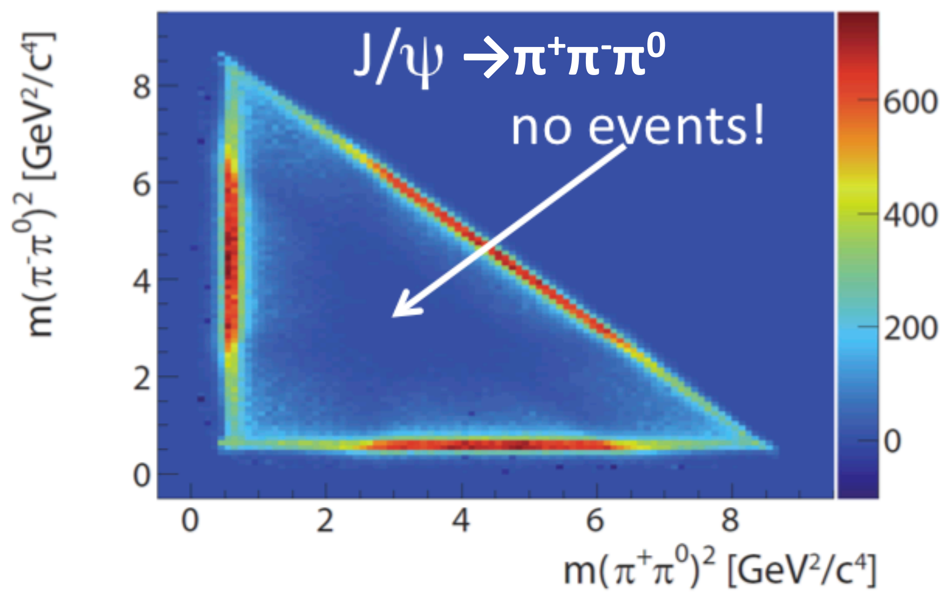
BaBar and BESIII obtain similar 2π Dalitz plots.

BaBar $J/\psi \rightarrow \pi^+\pi^-\pi^0$ [2]





BESIII preliminary [1]

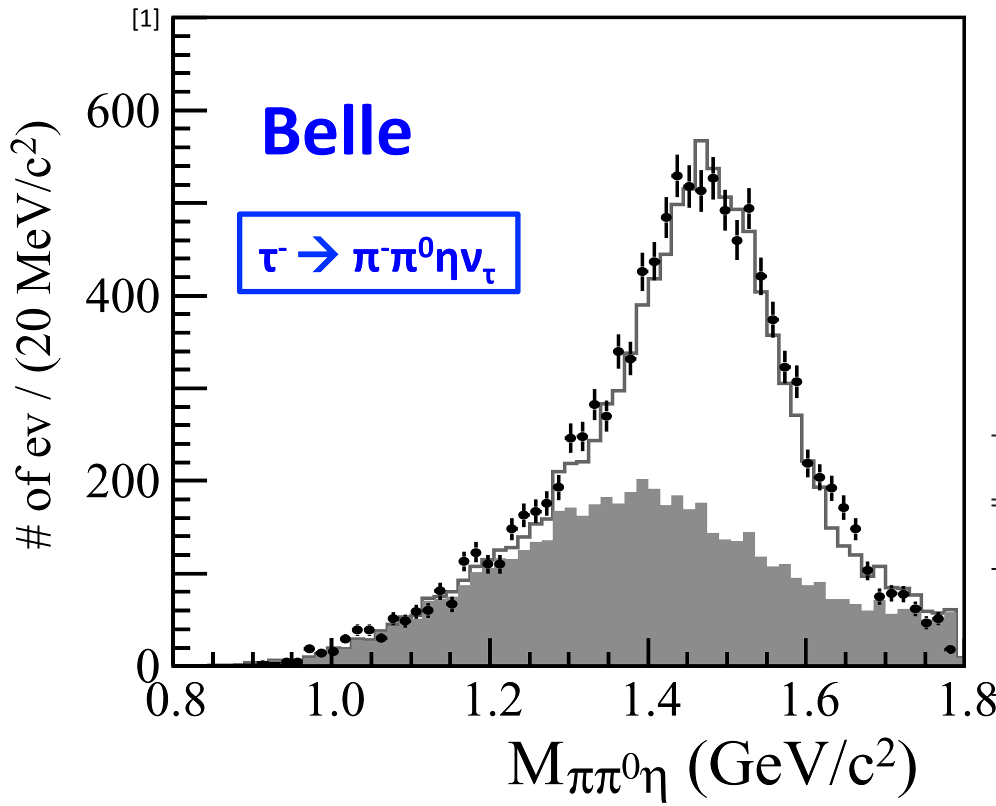


[1] S. L. Olsen [BESIII Collaboration], ``News from BESIII,'' arXiv:1203.4297 [nucl-ex]. http://docbes3.ihep.ac.cn/~talks/images/5/53/BORMIO2012-seteve.pdf

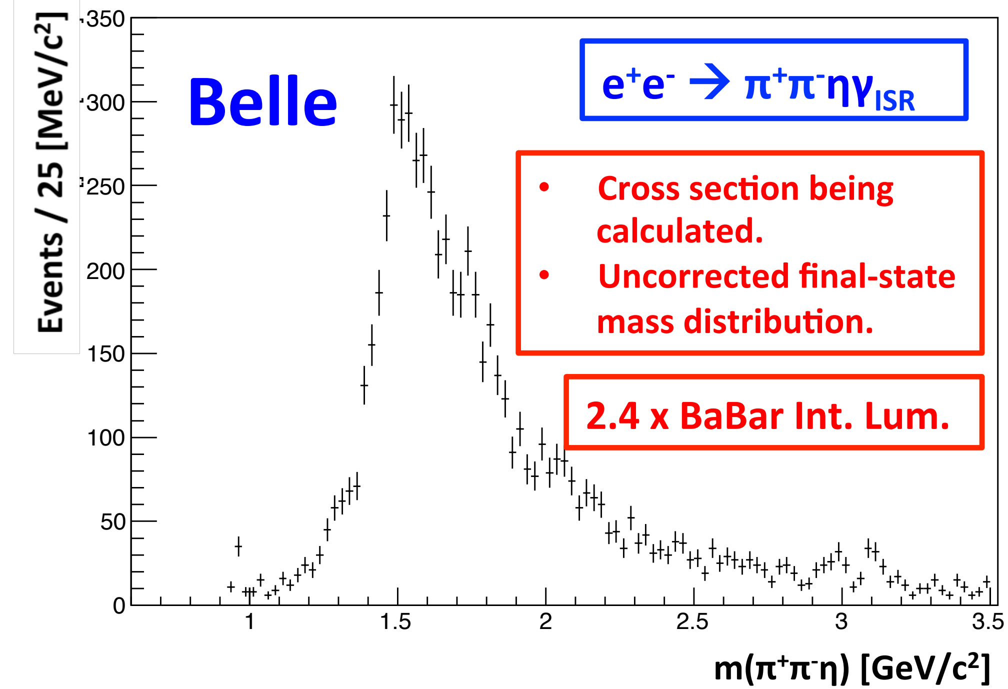
[2] B. Aubert *et al*. [BABAR Collaboration], Phys. Rev. D **70**, 072004 (2004) [arXiv:hep-ex/0408078].

The $e^+e^- \to \pi^+\pi^-\eta$ isovector final-state cross section can be compared to the $\tau^\pm \to \pi^\pm\pi^0\eta\nu_\tau$ spectral function.

Belle has already done a high precision measurement of the $\tau^- \to \pi^-\pi^0\eta\nu_\tau$ process.



- Points = Data
- Open Histogram = Signal MC
- Filled Histogram = $\tau^+\tau^-$ Bkg. MC
- $q\bar{q}$ Bkg. Strongly Suppressed & Negligible



[1]	Branching fraction $\mathcal{B}$ $(\times 10^{-4})$		
Mode	This work	Previous exp.	Reference
$\tau^- \to K^-\eta\nu_\tau$	$1.58 \pm 0.05 \pm 0.09$	$2.6 \pm 0.5 \pm 0.5$	CLEO [7]
		$2.9 \pm 1.3 \pm 0.7$	ALEPH [9]
$\tau^- \to \pi^-\pi^0\eta\nu_\tau$	$13.5 \pm 0.3 \pm 0.7$	$17 \pm 2 \pm 2$	CLEO [6]
		$18 \pm 4 \pm 2$	ALEPH [9]
$\tau^- \to K^-\pi^0\eta\nu_\tau$	$0.46 \pm 0.11 \pm 0.04$	$1.77 \pm 0.56 \pm 0.71$	CLEO [8]
$\tau^- \to \pi^-K^0_S\eta\nu_\tau$	$0.44 \pm 0.07 \pm 0.02$	$1.10 \pm 0.35 \pm 0.11$	CLEO [8]
$\tau^- \to K^{*-}\eta\nu_\tau$	$1.34 \pm 0.12 \pm 0.09$	$2.90 \pm 0.80 \pm 0.42$	CLEO [8]

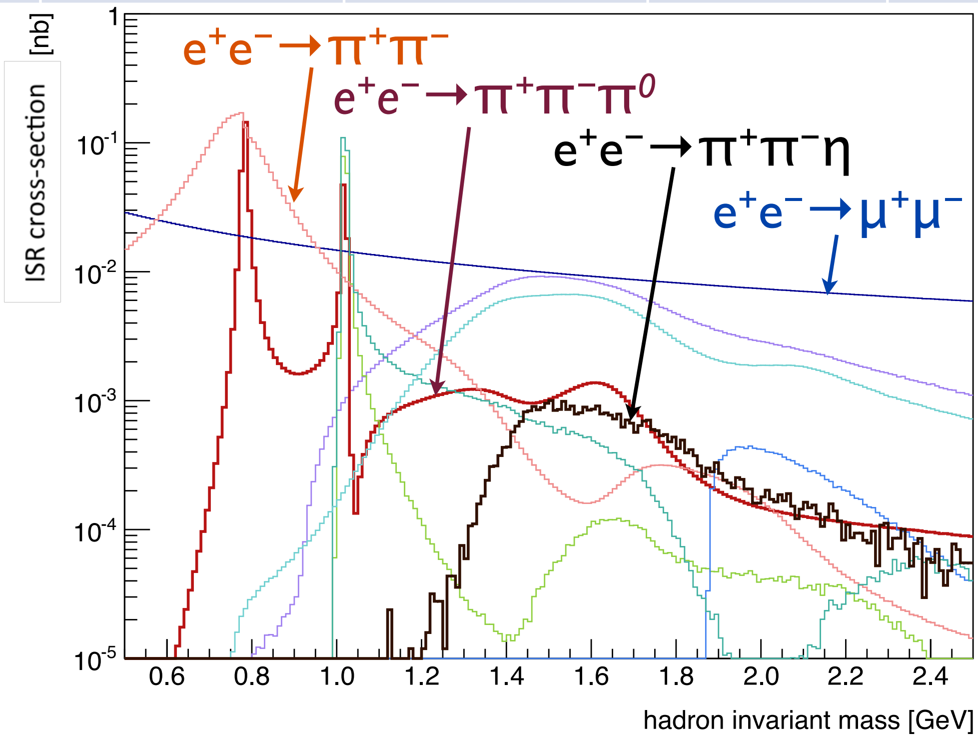
[1] K. Inami *et al.* [Belle Collaboration], Phys. Lett. B **672**, 209 (2009) [arXiv:0811.0088 [hep-ex]].

Conclusion

- A number of high energy (> 3 GeV) cross sections have been measured using Belle data
  - At machine operating energy
  - Energy scan
  - Via ISR
- Few low energy cross sections have been measured via ISR using Belle data
  - ***Belle data can still contribute a lot to high precession low energy cross section measurements***

Final State	Year	Int. Lum. [fb^{-1}]	E range [GeV]	σ_{max} [nb]	Reff. (Slide 36)
$\pi^+\pi^-\eta$	In prog.	562	0.9 to 3.5	≈4.3	-
$\pi^+\pi^-\pi^0$	In prog.	562	0.73 to 3.5	$\approx 1.5 \times 10^3$	-
$\phi\pi^+\pi^-$	2009	673	1.5 to 3	0.7	[G]

Belle σ's via ISR



Produced with PHOKHARA

List of Belle e^+e^- hadron production cross section references:

[A] Z. Q. Liu *et al.* [Belle Collaboration], Phys. Rev. Lett. **110**, 252002 (2013) [arXiv:1304.0121 [hep-ex]].
[B] X. L. Wang *et al.* [Belle Collaboration], Phys. Rev. D **87**, 051101 (2013) [arXiv:1210.7550 [hep-ex]].
[C] G. Pakhlova *et al.* [Belle Collaboration], Phys. Rev. D **83**, 011101 (2011) [arXiv:1011.4397 [hep-ex]].
[D] K. –F. Chen *et al.* [Belle Collaboration], Phys. Rev. D **82**, 091106 (2010) [arXiv:0810.3829 [hep-ex]].
[E] G. Pakhlova *et al.* [Belle Collaboration], Phys. Rev. D **80**, 091101 (2009) [arXiv:0908.0231 [hep-ex]].
[F] K. Belous *et al.* [Belle Collaboration], Phys. Lett. B **681**, 400 (2009) [arXiv:0906.4214 [hep-ex]].
[G] C. P. Shen *et al.* [Belle Collaboration], Phys. Rev. D **80**, 031101 (2009) [arXiv:0808.0006 [hep-ex]].
[H] P. Pakhlov *et al.* [Belle Collaboration], Phys. Rev. D **79**, 071101 (2009) [arXiv:0901.2775 [hep-ex]].
[I] G. Pakhlova *et al.* [Belle Collaboration], Phys. Rev. Lett. **101**, 172001 (2008) [arXiv:0807.4458 [hep-ex]].
[J] G. Pakhlova *et al.* [Belle Collaboration], Phys. Rev. Lett. **100**, 062001 (2008) [arXiv:0708.3313 [hep-ex]].
[K] G. Pakhlova *et al.* [Belle Collaboration], Phys. Rev. D **77**, 011103 (2008) [arXiv:0708.0082 [hep-ex]].
[L] C. Z. Yuan *et al.* [Belle Collaboration], Phys. Rev. D **77**, 011105 (2008) [arXiv:0709.2565 [hep-ex]].
[M] X. L. Wang *et al.* [Belle Collaboration], Phys. Rev. Lett. **99**, 142002 (2007) [arXiv:0707.3699 [hep-ex]].
[N] C. Z. Yuan *et al.* [Belle Collaboration], Phys. Rev. Lett. **99**, 182004 (2007) [arXiv:0707.2541 [hep-ex]].
[O] K. Abe *et al.* [Belle Collaboration], Phys. Rev. Lett. **98**, 092001 (2007) [hep-ex/0608018].
[P] T. Uglov *et al.* [BELLE Collaboration], Phys. Rev. D **70**, 071101 (2004) [hep-ex/0401038].
[Q] K. Abe *et al.* [Belle Collaboration], Phys. Rev. Lett. **89**, 142001 (2002) [hep-ex/0205104].
[R] K. Abe *et al.* [BELLE Collaboration], Phys. Rev. Lett. **88**, 052001 (2002) [hep-ex/0110012].
[S] P. Krokovny *et al.* [Belle Collaboration], arXiv:1308.2646 [hep-ex].
[T] K. F. Chen *et al.* [Belle Collaboration], Phys. Rev. Lett. **100**, 112001 (2008) [arXiv:0710.2577 [hep-ex]].
[U] C. P. Shen *et al.* [Belle Collaboration], arXiv:1309.0575 [hep-ex].

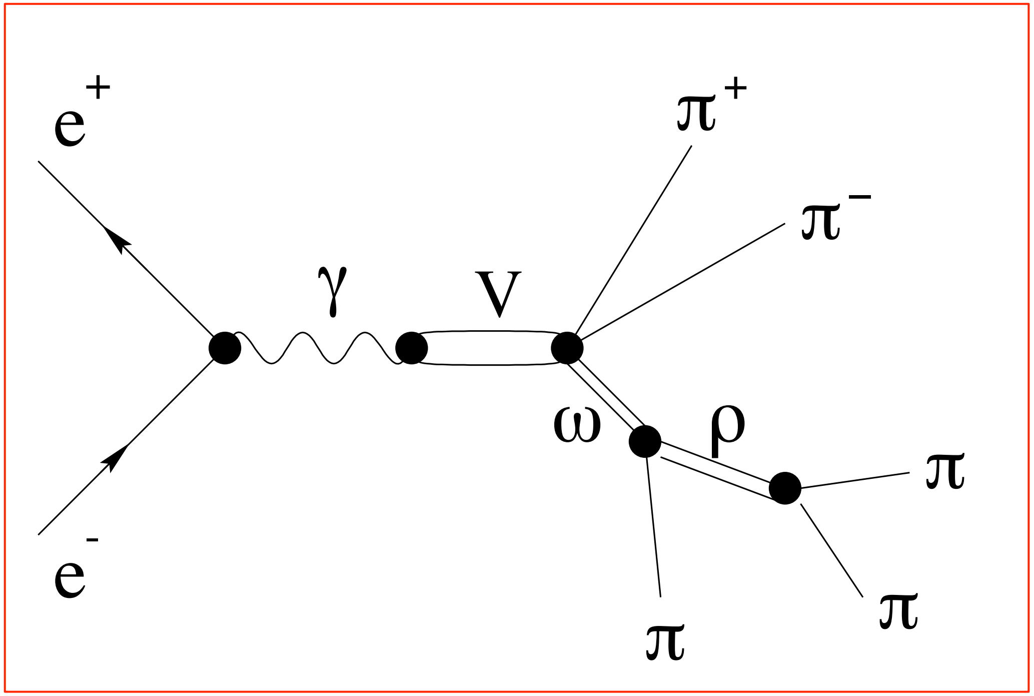
Backup Slides

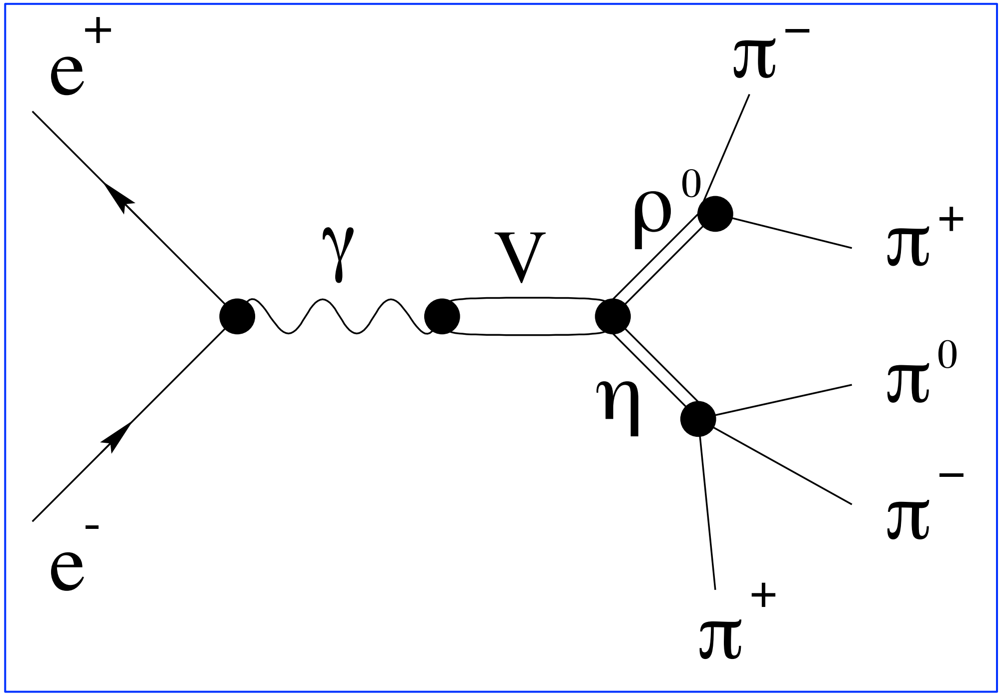
Hadronic final states can be divided into isoscalar and isovector processes

Isoscalar (I = 0) process

Isovector (I = 1) process

[1]





e^+e^- cross sections with I = 1 final states can be related to τ-spectral functions via CVC & unitarily.

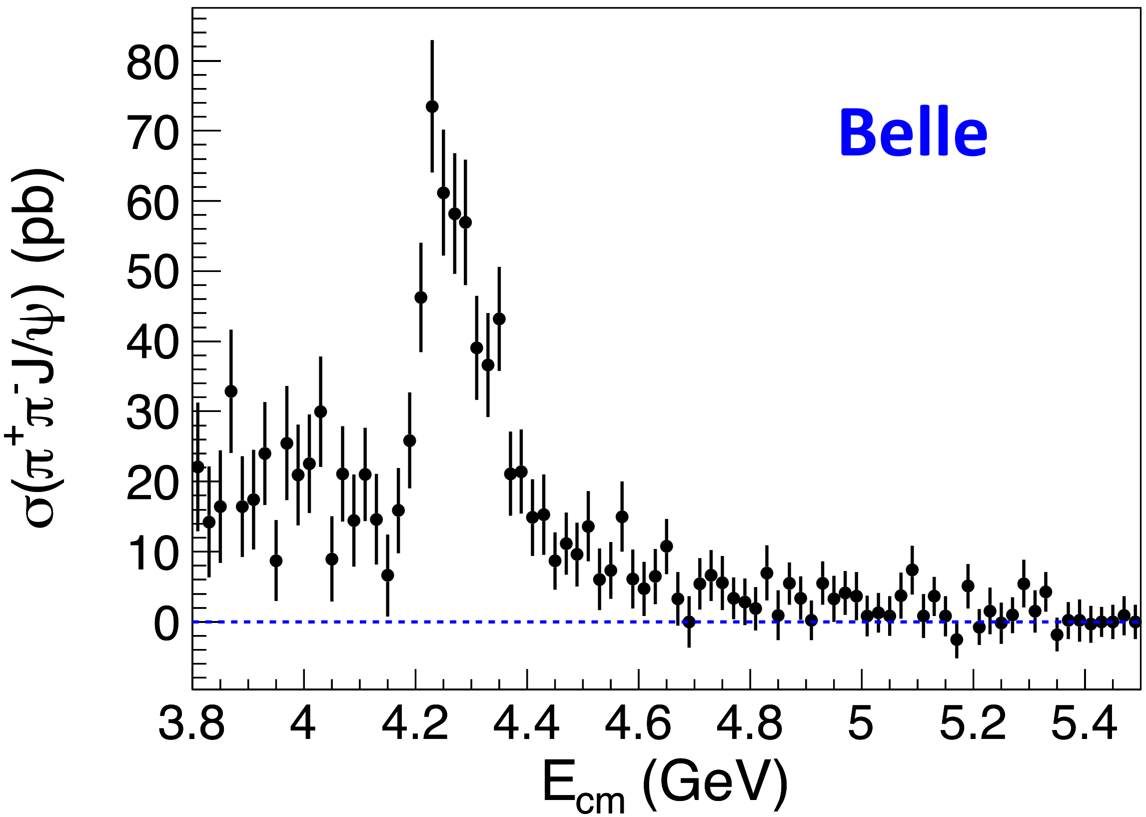
For Example:
$e^+e^- \rightarrow \pi^+\pi^-\eta \Leftrightarrow \tau^\pm \rightarrow W^\pm\nu_\tau \rightarrow \pi^\pm\pi^0\eta\nu_\tau$

[1] R. R. Akhmetshin *et al.* [CMD-2 Collaboration], Phys. Lett. B **489**, 125 (2000) [hep-ex/0009013].

Hadron spectroscopy involves measuring resonance parameters

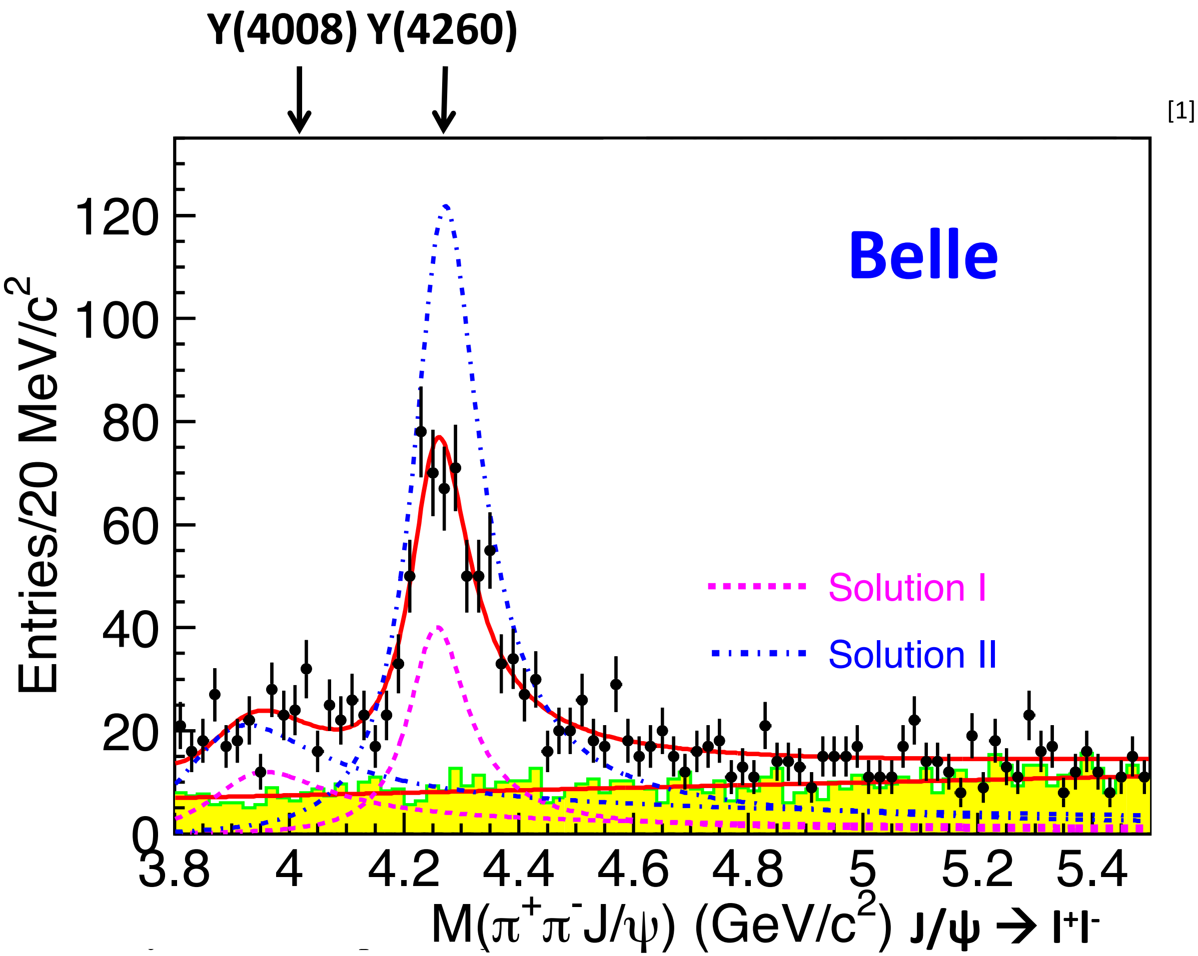
σ via ISR

[1]



[1] Z. Q. Liu *et al.* [Belle Collaboration], Phys. Rev. Lett. **110**, 252002 (2013) [arXiv:1304.0121 [hep-ex]].

Hadron spectroscopy involves measuring resonance parameters



Parameters	Solution I	Solution II
$M(R_1)$	$3890.8 \pm 40.5 \pm 11.5$	
$\Gamma_{\rm tot}(R_1)$	$254.5 \pm 39.5 \pm 13.6$	
$\Gamma_{ee}\mathcal{B}(R_1 \to \pi^+\pi^- J/\psi)$	$(3.8 \pm 0.6 \pm 0.4)$	$(8.4 \pm 1.2 \pm 1.1)$
$M(R_2)$	$4258.6 \pm 8.3 \pm 12.1$	
$\Gamma_{\rm tot}(R_2)$	$134.1 \pm 16.4 \pm 5.5$	
$\Gamma_{ee}\mathcal{B}(R_2 \to \pi^+\pi^- J/\psi)$	$(6.4 \pm 0.8 \pm 0.6)$	$(20.5 \pm 1.4 \pm 2.0)$
ϕ	$59 \pm 17 \pm 11$	$-116 \pm 6 \pm 11$

- **Fits with 2 coherent resonances.**
- **ϕ (degrees) is relative phase between resonances.**
- **Both fits are equal in quality.**

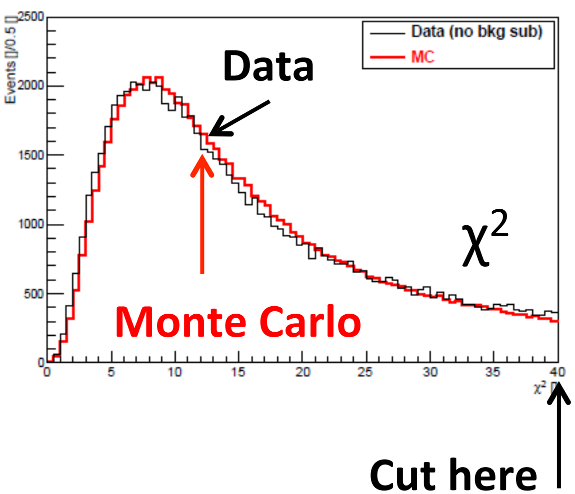
[1] Z. Q. Liu *et al.* [Belle Collaboration], Phys. Rev. Lett. **110**, 252002 (2013) [arXiv:1304.0121 [hep-ex]].

Data and Monte Carlo flow through the same analysis chain

Data → N_i^{corr}

526.6 fb^{-1}

Physics Skim



Simulation → ε_i^{corr}

Phokhara 6.2 — NLO ISR; FSR; Form Factors

GEANT — Physics Skim; Trigger Simulation; Goal: Detector eff

Pre Kinematic Fit Cuts — Mostly topological and quality of event

Post - Kinematic Fit Cuts — Good kinematic fit of $\pi^+\pi^-\gamma\gamma\gamma$; PID applied here

Background subtraction — m(γγ) distribution built to search for π^0.

Mass Unfolding

$$\sigma_i = \frac{N_i^{corr}}{L_i \varepsilon_i^{corr} B_{\pi^0 \to \gamma\gamma}}$$

Belle has directly measured several inclusive e^+e^- hadron production cross sections near the machine operating energy (≈10.6 GeV)

Final State	Year	Int. Lum. [fb^{-1}]	σ [nb]	Reff. (Slide 36)
$J/\psi c\bar{c}$	2009	673	$(0.74 \pm 0.08^{+0.09}_{-0.08}) \times 10^{-3}$	[H]
$J/\psi X_{non\text{-}c\bar{c}}$	2009	673	$(0.43 \pm 0.09 \pm 0.09) \times 10^{-3}$	[H]
$J/\psi D^{*+}X$	2002	46.2	$(0.53^{+0.19}_{-0.15} \pm 0.14) \times 10^{-3}$	[Q]
$J/\psi X$	2001	32.4	$(1.47 \pm 0.10 \pm 0.13) \times 10^{-3}$	[R]

Narrow resonance vector meson ISR cross sections can provide a cross check on the preliminary Belle $e^+e^- \rightarrow \pi^-\pi^+\pi^0$ cross section.

1^{st} Step:

$$\sigma_{ISR}\left(\theta_{ISR}^{\min} < \theta_{ISR} < \theta_{ISR}^{\max}\right) = \frac{N_{signal} - N_{side}}{\varepsilon R L B}$$

σ_{ISR} = ISR cross section
θ_{ISR} = γ_{ISR} polar angle
θ_{ISR}^{min} = minimum γ_{ISR} polar angle = 25°
θ_{ISR}^{max} = maximum γ_{ISR} polar angle = 155°
N_{signal} = total number of signal events over resonance
N_{side} = total number of events in side bands
ε = detector efficiency over resonance
R = radiative corrections
L = total integrated luminosity
B = branching fraction

1. Integrate signal yield over resonance.
2. Ignore N_{side} as it is a small correction.
3. Ignore R as it is a very small correction.

2^{nd} Step:

For narrow resonance vector mesons:

$$\frac{d\sigma_{ISR}\left(s,\theta_{ISR}\right)}{d\cos\theta_{ISR}} = \frac{12\pi^2\Gamma\left(V \rightarrow e^+e^-\right)\mathrm{B}\left(V \rightarrow f\right)}{m_v s} W\left(s, x_v, \theta_{ISR}\right)$$

σ_{ISR} = ISR cross section
$\sqrt{s}$ = CMS machine operating energy = 10.58 GeV
θ_{ISR} = γ_{ISR} polar angle = [25°, 155°]
Γ = resonance width
B = branching fraction
V = vector meson = ω, ϕ, J/ψ
f = hadronic final-state = $\pi^+\pi^-\pi^0$
m_V = vector meson mass
W = radiator function = describes γ_{ISR} emission probability.
$x_V = 1 - m_V^2/s$

1. Integrate differential equation w.r.t. $\cos\theta_{ISR}$, and over the analysis ISR fiducial volume.
2. Solve for $\Gamma(V \rightarrow e^+e^-)B(V \rightarrow f)$.

Results:

Source	$\Gamma(\omega \rightarrow e^+e^-)B(\omega \rightarrow \pi^+\pi^-\pi^0)$ [eV]	$\Gamma(\phi \rightarrow e^+e^-)B(\phi \rightarrow \pi^+\pi^-\pi^0)$ [eV]	$\Gamma(J/\psi \rightarrow e^+e^-)B(J/\psi \rightarrow \pi^+\pi^-\pi^0)$ [eV]
Prelim. Belle	563.0 ± 3.2	217.1 ± 2.1	117.3 ± 3.8
PDG [1]	551 ± 13	192.8 ± 4.8	116.4 ± 5.4

Belle: Sta. Err. only. (5% Sys. Err. goal)

PDG: $\phi \rightarrow \rho\pi + \pi^+\pi^-\pi^0$

[1] J. Beringer et al. (Particle Data Group), Phys. Rev. D86, 010001 (2012) and 2013 partial update for the 2014 edition.

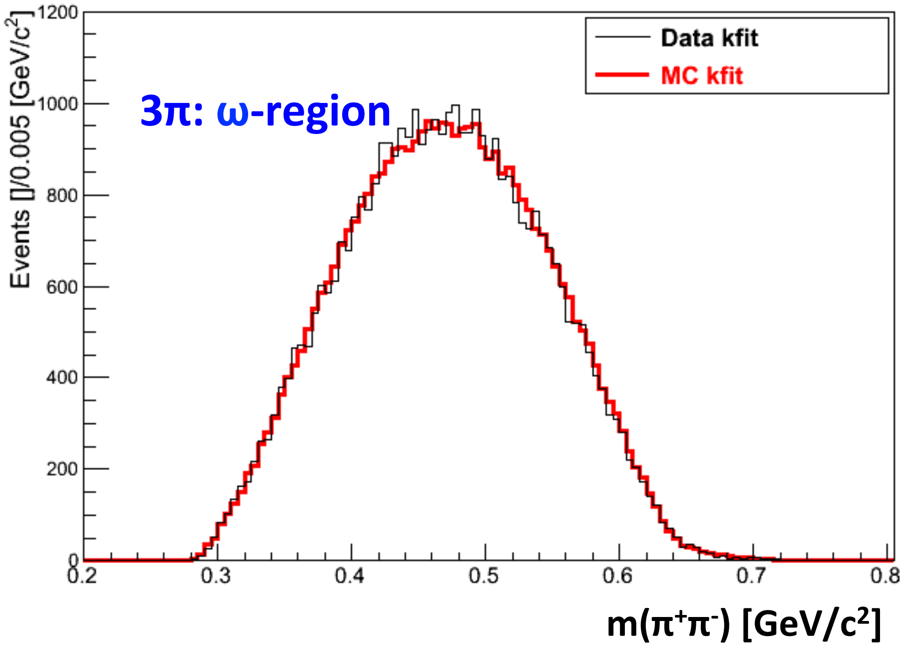
Preliminary Systematic Uncertainties

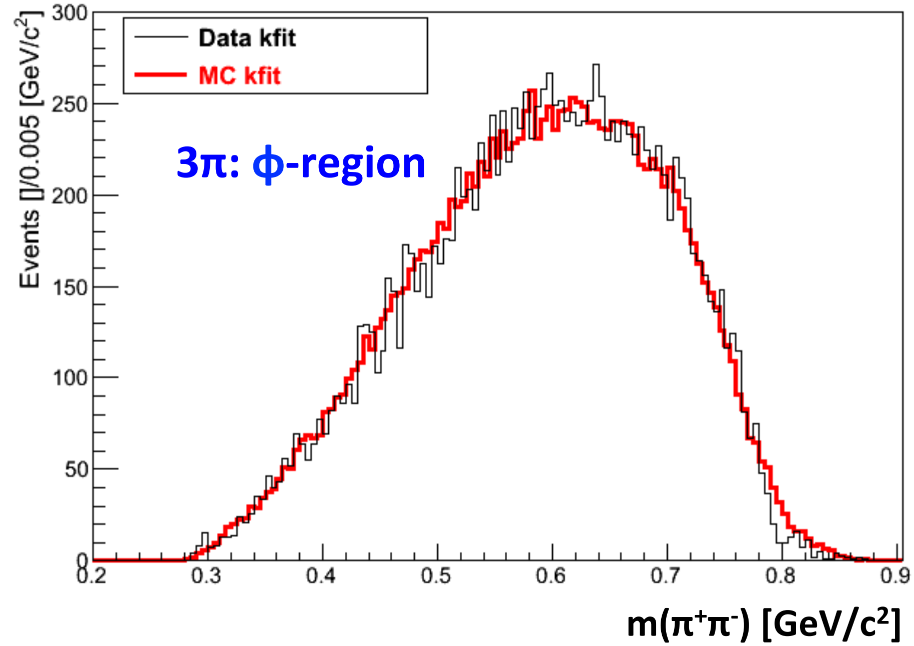
Systematic Uncertainty	**$\sigma(e^+e^- \rightarrow \pi^+\pi^-\pi^0)$ relative error (%)**	**Basis**
PID Efficiency	2.8*	Efficiency Correction From Data
Track Finding Efficiency	0.7	From Data
Total Integrated Luminosity	1.4	MC Bhabha Generator Accuracy
Trigger Efficiency	Work In Progress	Variation Of Trigger Masks
$E(\gamma)$ Cuts	1.2	Variation Of Cuts
$\theta(\gamma_{ISR})$ Cut	3.8*	Variation Of Cuts
Backgrounds	2.8*	Examine Largest Background
PID Cuts	0.94	Variation Of Cuts
Pre-Kfit p^μ Cuts	0.58	Variation Of Cuts
$m(\pi^0)$ Signal-Range	0.76	Variation Of Cuts
Total	**> 5.9**	**Add In Quadrature**

Goal: Total systematic at the ~5% level

* Conservative estimate: Error can be lowered with more work

Belle cross section measurements are also interested in looking at intermediate states.





Belle $m(\pi^+\pi^-)$ distributions for $e^+e^- \rightarrow \pi^+\pi^-\pi^0\gamma_{ISR}$.