





9th - 12th September 2013 Dipartimento di Fisica Università "La Sapienza", Roma



Space-like region: definitions and data



Rosenbluth/Polarization conflict(?)



Time-like region: definitions, data and more



The ratio G_E^p/G_M^p



Proton form factor at threshold





Nucleon Form Factors definition Space-like region $(q^2 < 0)$



- Electromagnetic current (q = p' p) $J^{\mu} = \langle N(p') | j^{\mu} | N(p) \rangle = e\overline{u}(p') \Big[\gamma^{\mu} F_1(q^2) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2M} F_2(q^2) \Big] u(p)$
- Dirac and Pauli form factors F₁ and F₂ are real

In the Breit frame

 $\begin{cases} p = (E, -\vec{q}/2) \\ p' = (E, \vec{q}/2) \\ q = (0, \vec{q}) \end{cases} \begin{cases} \rho_q = J^0 = e \left[F_1 + \frac{q^2}{4M^2} F_2 \right] \\ \vec{J}_q = e \, \overline{u}(p') \vec{\gamma} u(p) \left[F_1 + F_2 \right] \end{cases}$

• Sachs form factors $G_E = F_1 + \frac{q^2}{4M^2}F_2$ $G_M = F_1 + F_2$ Normalizations $F_1(0) = Q_N \qquad G_E(0) = Q_N$ $F_2(0) = \kappa_N \qquad G_M(0) = \mu_N$



pQCD asymptotic behavior Space-like region

Lett. Nuovo Cim. 7 (1973) 719 Phys. Rev. Lett. 31 (1973) 1153 JETP Lett. 96 (2012) 6-12



pQCD: as q² → -∞, asymptotic behaviors of F₁ and F₂, and G_E and G_M must follow counting rules

Valence quarks exchange gluons to distribute the momentum transfer *q*

Non-helicity-flip current $J^{\lambda,\lambda}(q^2)$

• $J^{\lambda,\lambda}(q^2) \propto G_M(q^2)$ • Two gluon propagators • $G_M(q^2) \underset{q^2 \to -\infty}{\sim} (-q^2)^{-2}$



Helicity-flip $J^{\lambda,-\lambda}(q^2)$

•
$$J^{\lambda,-\lambda}(q^2) \propto G_E(q^2)/\sqrt{-q^2}$$

• Two gluon propagators
$$/\sqrt{-q^2}$$

• $G_E(q^2) \underset{q^2 \to -\infty}{\sim} (-q^2)^{-2}$

Dirac and Pauli form factors
•
$$F_1(q^2) \underset{q^2 \to -\infty}{\sim} (-q^2)^{-2}$$

• $F_2(q^2) \underset{q^2 \to -\infty}{\sim} (-q^2)^{-3}$





Space-like data

Roma - September 11th, 2013

ß

Nucleon Form Factors in Experiments and Theory

Rosenbluth separation



Rosenbluth formula

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \frac{1}{1-\tau} \left[G_{E}^{2} - \frac{\tau}{\epsilon} G_{M}^{2}\right] \qquad \tau = \frac{q^{2}}{4M_{N}^{2}}$$

Mott pointlike cross section
 $\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{4\alpha^{2}}{(-q^{2})^{2}} \frac{E_{2}^{3}}{E_{1}} \cos^{2}(\theta_{e}/2)$
Photon polarization
 $\epsilon = \left[1 + 2(1-\tau) \tan^{2}(\theta_{e}/2)\right]^{-1}$





64

Roma - September 11th, 2013

G_{F}^{p} and G_{M}^{p} with Rosenbluth separation



Roma - September 11th, 2013

Polarization observables

A.I. Akhiezer, M.P. Rekalo, Sov. Phys. Dokl. 13, 572 (1968)



• Elastic scattering of **longitudinally polarized electrons** ($h = \pm 1$) on nucleon target

• Hadronic tensor: $W_{\mu\nu} = \underbrace{W_{\mu\nu}(0)}_{\text{no pol.}} + \underbrace{W_{\mu\nu}(\vec{P}) + W_{\mu\nu}(\vec{P}')}_{\text{ini. or fin. pol. of }N} + \underbrace{W_{\mu\nu}(\vec{P},\vec{P}')}_{\text{ini. and fin. pol. of }N}$

• In case of polarized electrons $(h = \pm 1)$ on unpolarized nucleon target:

$$P'_{x} = -\frac{2\sqrt{\tau(\tau-1)}}{G_{E}^{2} - \frac{\tau}{\epsilon}G_{M}^{2}} G_{E}G_{M} \tan\left(\frac{\theta_{e}}{2}\right) \qquad P'_{z} = \frac{(E_{e} + E'_{e})\sqrt{\tau(\tau-1)}}{M\left(G_{E}^{2} - \frac{\tau}{\epsilon}G_{M}^{2}\right)} G_{M}^{2} \tan^{2}\left(\frac{\theta_{e}}{2}\right)$$
$$\frac{P'_{x}}{P'_{z}} = -\frac{2M\cot(\theta_{e}/2)}{E_{e} + E'_{e}} \frac{G_{E}}{G_{M}}$$

$G_{F}^{\rho}/G_{M}^{\rho}$ in polarization transfer experiments





Radiative corrections in Rosenbluth separation Phys. Rev. D50 (1994) 5491

Sachs form factors G_E and G_M are extracted from Born cross sections (one- γ exchange)

The Born term is obtained from experimental cross sections correcting for radiative effects



G_E^n and G_M^n with different techniques







Nucleon form factors Time-like region $(q^2 > 0)$



Crossing symmetry:

$$\langle N(p')|j^{\mu}|N(p)
angle
ightarrow \langle \overline{N}(p')N(p)|j^{\mu}|0
angle$$

Form factors are complex functions of q^2

Optical theorem

 $\ln \langle \overline{N}(p')N(p)|j^{\mu}|0\rangle \sim \sum_{n} \langle \overline{N}(p')N(p)|j^{\mu}|n\rangle \langle n|j^{\mu}|0\rangle \implies \begin{cases} \ln F_{1,2} \neq 0\\ \text{for } q^{2} > 4M_{\pi}^{2} \end{cases}$ |*n*\arrow are on-shell intermediate states: 2\pi, 3\pi, 4\pi, ...

Time-like asymptotic behavior



$$\bigcup_{\substack{q^2 \to -\infty \\ \text{space-like}}} G_{E,M}(q^2) = \lim_{\substack{q^2 \to +\infty \\ \text{time-like}}} G_{E,M}(q^2)$$

$$\bigcup_{\substack{q^2 \to +\infty \\ q^2 \to +\infty}} (q^2)^{-2} \text{ real}$$

Cross section and analyticity



Annihilation cross section formula

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta C}{4q^2} \left[(1 + \cos^2 \theta) |G_M|^2 + \frac{1}{\tau} \sin^2 \theta |G_E|^2 \right]$$

• Outgoing nucleon velocity:
$$\beta = \sqrt{1 - 1/\tau}$$

• Coulomb correction:
$$C = \frac{\pi \alpha / \beta}{1 - \exp(-\pi \alpha / \beta)}$$



Roma - September 11th, 2013

Nucleon Form Factors in Experiments and Theory

Time-like magnetic proton form factor



Data obtained with $|{\it G}^{\rho}_{M}|=|{\it G}^{\rho}_{E}|\equiv |{\it G}^{\rho}_{eff}|$ (assumed true only at threshold)

$$|\mathbf{G}_{\text{eff}}^{p}|^{2} = \frac{\sigma_{p\bar{p}}(q^{2})}{\frac{16\pi\alpha^{2}\mathcal{C}}{3}\frac{\sqrt{1-1/\tau}}{4q^{2}}\left(1+\frac{1}{2\tau}\right)}$$



Time-like $|G_{F}^{p}/G_{M}^{p}|$ measurements

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2\beta\mathcal{C}}{2q^2} |G_M^p|^2 \left[(1+\cos^2\theta) + \frac{4M_p^2}{q^2\mu_p^2} \sin^2\theta |\mathbf{R}|^2 \right] \qquad \mathbf{R}(q^2) = \mu_p \frac{G_E^p(q^2)}{G_M^p(q^2)}$$



$\gamma\gamma$ exchange from $e^+e^- \rightarrow p\overline{p}\gamma$ **BABAR** 2013 data

Phys. Lett. B659 (2008) 197 arXiv:1302.0055









The ratio $R = \mu_p G_E^p / G_M^p$

- Dispersion relation for the imaginary part
 - Model-independent approach
 - First time-like $|G_E| |G_M|$ separation
 - \Rightarrow Ratio in the whole q^2 complex plane



Analytic $R(q^2)$



Analytic $R(q^2)$



Analytic **R(q**²)



Analytic **R(q**²)



Asymptotic $G_E^P(q^2)/G_M^p(q^2)$ and phase



$|G^{p}_{E}(q^{2})|$ and $|G^{p}_{M}(q^{2})|$ from $\sigma_{p\overline{p}}$ and DR



$$|G^p_{
m eff}(q^2)|^2 = rac{\sigma_{p\overline{p}}(q^2)}{rac{4\pilpha^2eta \mathcal{C}}{3s}}\left(1+rac{1}{2 au}
ight)^{-1}$$

Usually what is extracted from the cross section $\sigma(e^+e^- \rightarrow p\overline{p})$ is the effective time-like form factor $|G^{p}_{eff}|$ obtained assuming $|G^{p}_{E}| = |G^{p}_{M}|$ i.e. $|R| = \mu_{p}$

Using our parametrization for *R* and the *BABAR* data on $\sigma(e^+e^- \rightarrow p\overline{p})$, $|G_E^p|$ and $|G_M^p|$ may be disentangled



$|G^{p}_{E}(q^{2})|$ and $|G^{p}_{M}(q^{2})|$ from $\sigma_{p\overline{p}}$ and DR



$$|G^p_M(q^2)|^2 = rac{\sigma_{p\overline{p}}(q^2)}{rac{4\pilpha^2eta\mathcal{C}}{3s}}\left(1+rac{|R(q^2)|}{2\mu_p au}
ight)^{-1}$$

- Usually what is extracted from the cross section $\sigma(e^+e^- \rightarrow p\overline{p})$ is the effective time-like form factor $|G^{p}_{\text{eff}}|$ obtained assuming $|G^{p}_{E}| = |G^{p}_{M}|$ i.e. $|R| = \mu_{p}$
- Using our parametrization for *R* and the *BABAR* data on $\sigma(e^+e^- \rightarrow p\overline{p})$, $|G_{E}^{p}|$ and $|G_{M}^{p}|$ may be disentangled



Sommerfeld resummation factor needed?



Roma - September 11th, 2013

Nucleon Form Factors in Experiments and Theory

The Coulomb Factor



$$\sigma_{p\overline{p}} = \frac{4\pi\alpha^2\beta \,\mathcal{C}}{3q^2} \left[|G^p_M(q^2)|^2 + \frac{2M^2_p}{q^2} |G^p_E(q^2)|^2 \right]$$

C describes the pp Coulomb interaction as FSI [Sommerfeld, Sakharov, Schwinger, Fadin, Khoze]



Roma - September 11th, 2013

Nucleon Form Factors in Experiments and Theory

Enhancement and Resummation Factors





Roma - September 11th, 2013

Step and plateau in **BABAR** data ... another possibility (Vladimir F. Dmitriev)

Eur. Phys. J. A39 (2009) 315



Step and plateau in **BABAR** data ... another possibility (Vladimir F. Dmitriev)

Eur. Phys. J. A39 (2009) 315



BABAR: G_{eff}^{p} including threshold effects

$$\left|G_{\text{no corr}}^{p}(q^{2})\right|^{2} = \frac{\sigma_{p\bar{p}}(q^{2})}{\boldsymbol{\mathcal{ER}}\frac{16\pi\alpha^{2}}{3}\frac{\beta}{4q^{2}}\left(1+\frac{1}{2\tau}\right)}$$

$$ig|G^p_{\mathsf{eff}}(q^2)ig|^2 = rac{\sigma_{p\overline{p}}(q^2)}{\mathcal{E}\mathcal{R}_srac{16\pilpha^2}{3}rac{eta}{4q^2}ig(1+rac{1}{2 au}ig)}$$





BABAR: G_{eff}^{p} including threshold effects

$$\left|G_{\mathsf{no\,corr}}^{\rho}(q^2)\right|^2 = \frac{\sigma_{\rho\overline{\rho}}(q^2)}{\boldsymbol{\mathcal{ER}}\frac{16\pi\alpha^2}{3}\frac{\beta}{4q^2}\left(1+\frac{1}{2\tau}\right)}$$

$$\left|G_{\mathsf{eff}}^{p}(q^{2})
ight|^{2} = rac{\sigma_{p\overline{p}}(q^{2})}{\mathcal{E}\mathcal{R}_{s}rac{16\pilpha^{2}}{3}rac{eta}{4q^{2}}\left(1+rac{1}{2 au}
ight)}$$



Experiments: now and future





Nucleon Form Factors in Experiments and Theory

Nucleon form factors: theory and phenomenology

Radiative corrections

- Solution G_E^{ρ}/G_M^{ρ} from **Rosenbluth** and **polarization transfer** techniques, that have different radiative corrections, **do not agree**.
- Form factor values are obtained from cross section at **first order** (Born approx.). G_E and G_M appear in the Born amplitude with **different kinematical factors**.



Radiative corrections depending on relevant kinematical variables must be applied.

Models that reproduce proton and neutron, electric and magnetic form factors in space-like and time-like regions Light-front constituent quark model [G. Salmè et al.] Space-like and time-like nucleon form factors are determined in the framework of a relativistic quark model based on Bethe-Salpeter amplitudes and vector meson dominance. $P_{N'}$ Skyrme model with vector mesons [U.-G. Meissner et al.] Space-like form factors, obtained as Fourier transforms of charge and magnetic moment distributions, can be analytically continued in the time-like region. Desired time-like imaginary parts are obtained even though they appear quite "unstable". [See Simone Moretti poster]. VMD based models [Gari, Krümpelmann, Iachello, Jackson, Landé,...] Nucleon form factors are parametrized using a mixture of VMD and pQCD. At low energy the coupling to the photons is described through VM exchange. At high energy "hadron/guark" form factors drive the transition to pQCD.

24



Nucleon Form Factors in Experiments and Theory

"To do" list



Time-like $|G_E|$ - $|G_M|$ separation:

DR and data



Dispersive analyses: integral equation, sum rule,...

Experimental observation in $p\overline{p} \rightarrow \pi^0 I^+ I^-$ [PRC75,045205(07)]



Asymptotic behavior: DR and data for the phase



 $\textbf{Zeros} \leftrightarrow \textbf{phases: DR and data}$





"To do" list



Time-like $|G_E|$ - $|G_M|$ separation: DR and data BESIII Fanda







Dispersive analyses: integral equation, sum rule,...

Experimental observation in $p\overline{p} \rightarrow \pi^0 l^+ l^-$ [PRC75,045205(07)]







