

Study of the rare decay $K^{\pm} \rightarrow \pi^{\pm} \gamma \gamma$
and high precision measurement
of the form factors of the
semileptonic decays $K \rightarrow \pi^0 \ell \nu$

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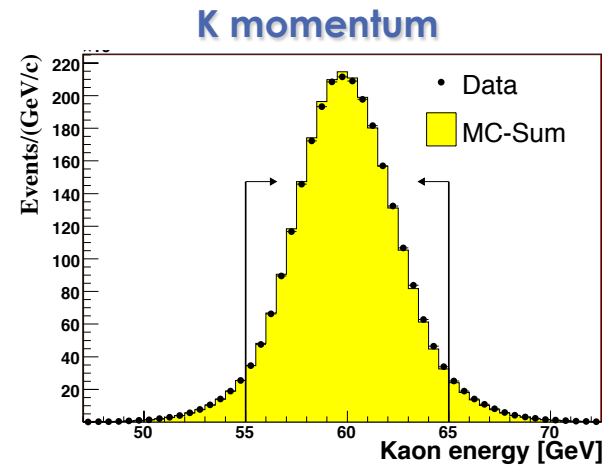
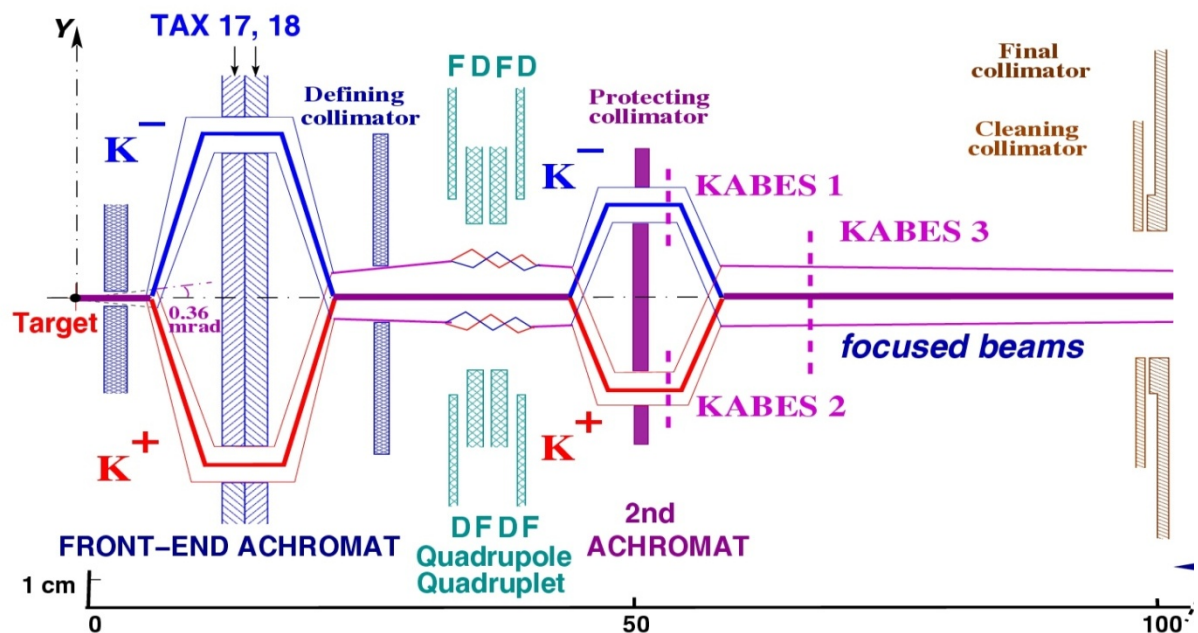
on behalf of the NA48/2 and NA62 collaborations

Outline

- ▣ The NA48/2 experiment
 - ▣ Beam line, detector, and data taking periods
- ▣ Semileptonic Kaon decays
- ▣ Recent results on $K_{\mu 3}$ and $K_{e 3}$ Form Factors slopes
 - ▣ $K_{\ell 3}$ form factor slopes fits status
- ▣ Recent result of NA48/2 and NA62 on $K^{\pm} \rightarrow \pi^{\pm} \gamma \gamma$
- ▣ Conclusions

The NA48/2 and NA62 beam line

Simultaneous K^+ and K^- beam with $N_{K^+}/N_{K^-} \sim 1.8$



NA48/2 = $60 \pm 2.2 \text{ GeV/c}$

NA62 = $74 \pm 1.4 \text{ GeV/c}$

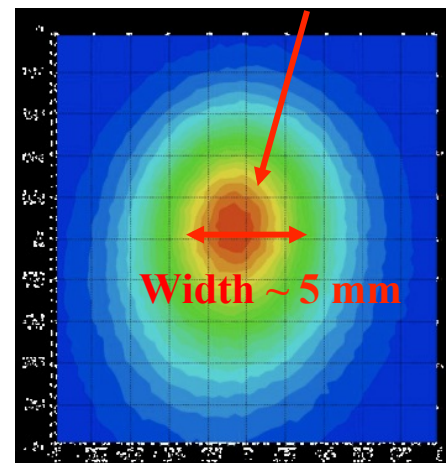
Beams within 1 mm

NA48/2 Data taking:

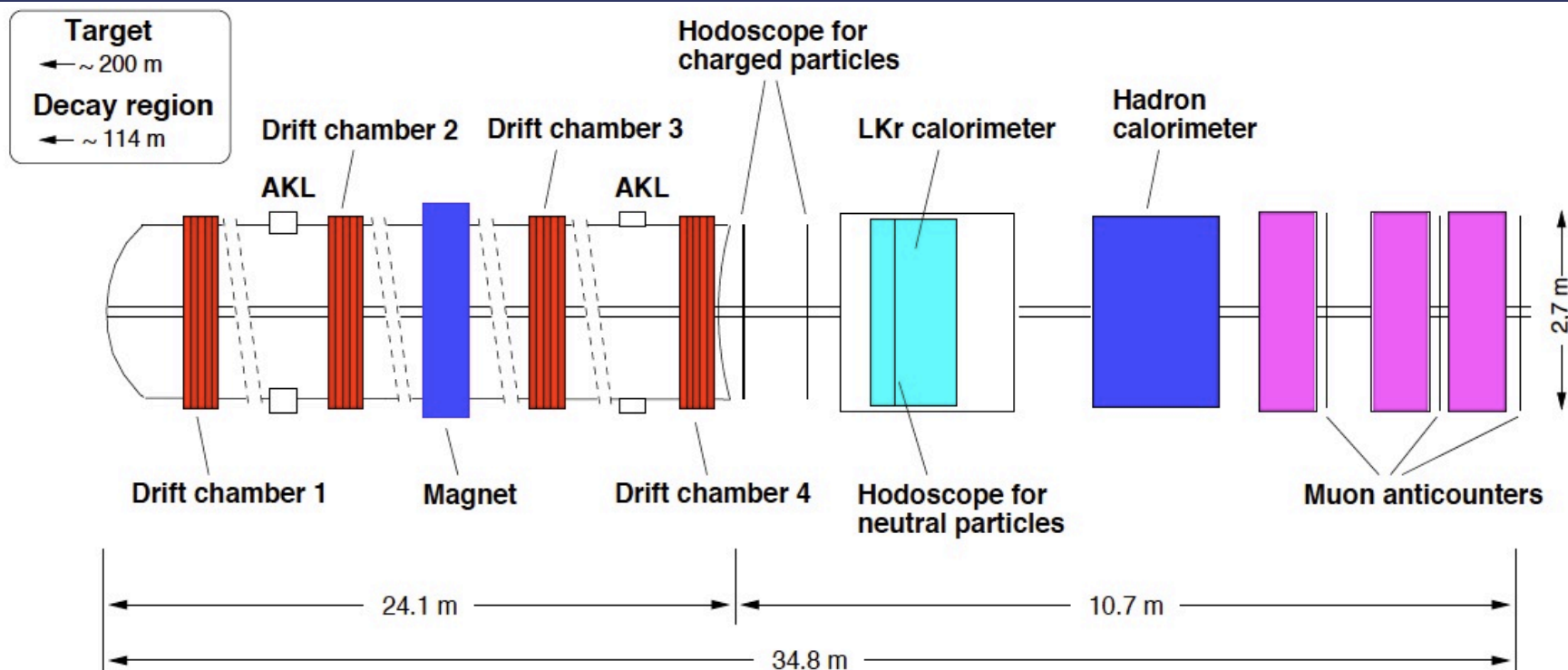
- 4 months in 2003 (K^\pm) + 4 months in 2004 (K^\pm)

NA62-RK Data taking:

- 2007 mostly K^+



The NA48/2 and NA62 detector



Magnetic Spectrometer

- 4 drift chambers and a dipole magnet

$$\frac{\sigma(p)}{p}_{NA48} = (1.02 \oplus 0.044 p)\%$$

p in GeV

$$\frac{\sigma(p)}{p}_{NA62} = (0.48 \oplus 0.009 p)\%$$

p

Liquid Krypton EM calorimeter (LKr)

- High granularity (13248 cells of 2x2 cm²)
- Quasi-homogeneous, 7m³ liquid Kr (27X₀)

$$\frac{\sigma(E)}{E} = \frac{3.2\%}{\sqrt{E}} \oplus \frac{9\%}{E} \oplus 0.4\% \quad E \text{ in GeV}$$

The K^+ semileptonic decays

- $K^\pm \rightarrow \pi^0 \ell^\pm \nu$ decays provide the **most accurate** and **theoretically cleanest** way to **access** $|V_{US}|$:

$$\Gamma(K_{\ell 3(\gamma)}) = \frac{C_K^2 G_F^2 m_K^5}{192 \pi^3} S_{EW} |V_{US}|^2 |f_+(0)|^2 I_K^\ell(\lambda_{+0}) (1 + \delta_{SU(2)} + \delta_{EM}^\ell)^2$$

Experimental Inputs:

- $\Gamma(K\ell 3)$ Branching ratios and Kaon lifetimes
- $I_K^\ell(\lambda_{+0})$ Phase space integral depends on the form factors

Theory Inputs:

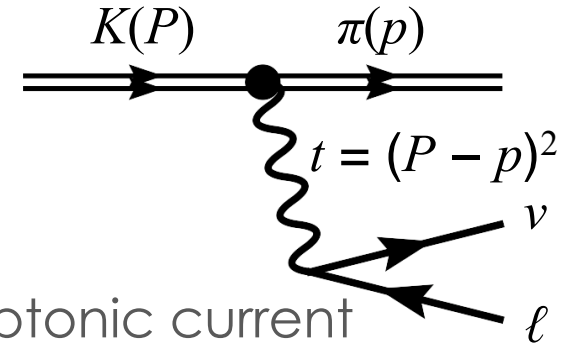
- S_{EW} Universal short distance EW corrections (1.0232 ± 0.0003)
- $f_+(0)$ Form factor at zero momentum transfer ($0.959(5)$) (EPJC 69 2010, 399-424)
- $\delta_{SU(2)}$ Correction for isospin breaking (ch. mode only $\sim (2.9 \pm 0.4)\%$) (EPJC 69 2010, 399-424)
- δ_{EM}^ℓ Long distance EM effects

Kℓ3 Form Factors

Hadronic matrix element:

$$\langle \pi | J_\alpha | K \rangle = f(0) \times [\tilde{f}_+(t)(P + p)_\alpha + \tilde{f}_-(t)(P - p)_\alpha]$$

f_- term multiplied by m_ℓ when contracted with leptonic current



Ke3 decays: Only **vector form factor**: $\tilde{f}_+(t)$

Kμ3 decays: Also need **scalar form factor**: $\tilde{f}_0(t) = \tilde{f}_+ + \tilde{f}_- \frac{t}{m_K^2 - m_\pi^2}$

$f_+(0)$ cannot be directly measured, therefore the form factors are normalised to $f_+(0)$:

$$\tilde{f}_+(t) = \frac{f_+(t)}{f_+(0)}$$

$$\tilde{f}_0(t) = \frac{f_0(t)}{f_+(0)}$$

For V_{US} , need integral over phase space of squared matrix element
Parameterize form factors and fit distributions in t (or related variables)

Form Factor parameterizations

Linear and quadratic parameterization (Taylor expansion):

$$\tilde{f}_{+,0}(t) = 1 + \lambda_{+,0} \left(\frac{t}{m_{\pi^+}^2} \right)$$

$$\tilde{f}_{+,0}(t) = 1 + \lambda'_{+,0} \left(\frac{t}{m_{\pi^+}^2} \right) + \lambda''_{+,0} \left(\frac{t}{m_{\pi^+}^2} \right)^2$$

Notes:

Many parameters: $\lambda'_+, \lambda''_+, \lambda'_0, \lambda''_0$
Large correlations, unstable fits
Limited sensitivity to $\lambda''_{+,0}$

Pole parameterization:

Assumes the exchange of vector and scalar resonances K^* with spin-parity $1^-/0^+$ and masses m_V/m_S ,

$f_+(t)$ described by $K^*(892)$, for $f_0(t)$ no obvious dominance is seen:

$$\tilde{f}_+(t) = \frac{m_V^2}{m_V^2 - t}$$

$$\tilde{f}_0(t) = \frac{m_S^2}{m_S^2 - t}$$

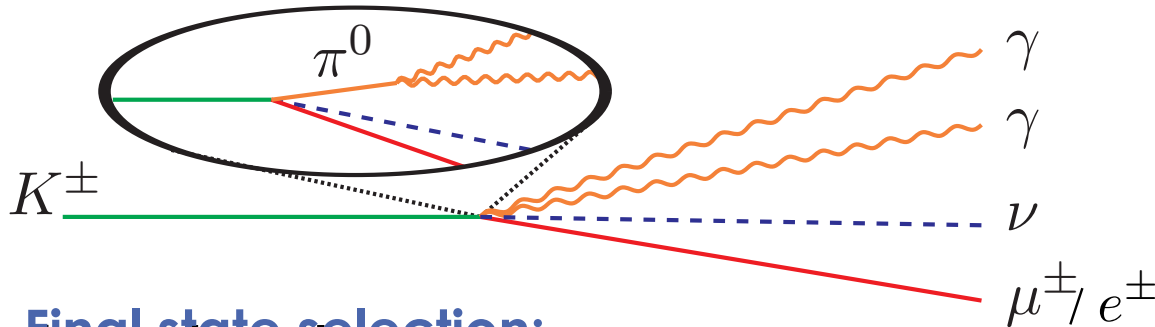
Dispersion relations:

$$\tilde{f}_+(t) = \exp \left[\frac{t}{m_{\pi^+}^2} (\Lambda_+ - H(t)) \right]$$

$$\tilde{f}_0(t) = \exp \left[\frac{t}{m_K^2 - m_{\pi^+}^2} (\ln C - G(t)) \right]$$

Not yet used in NA48/2 analysis. (PLB 638(2006) 480, PRD 80(2009) 034034)

Kℓ3 event selection

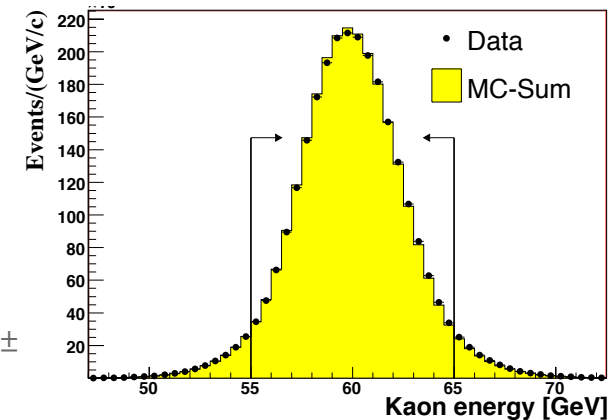
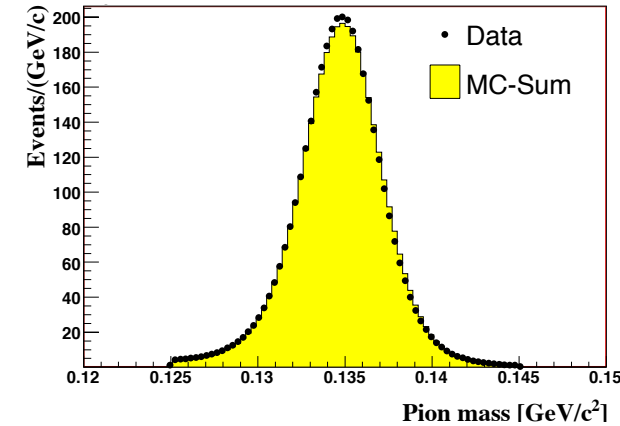


Final state selection:

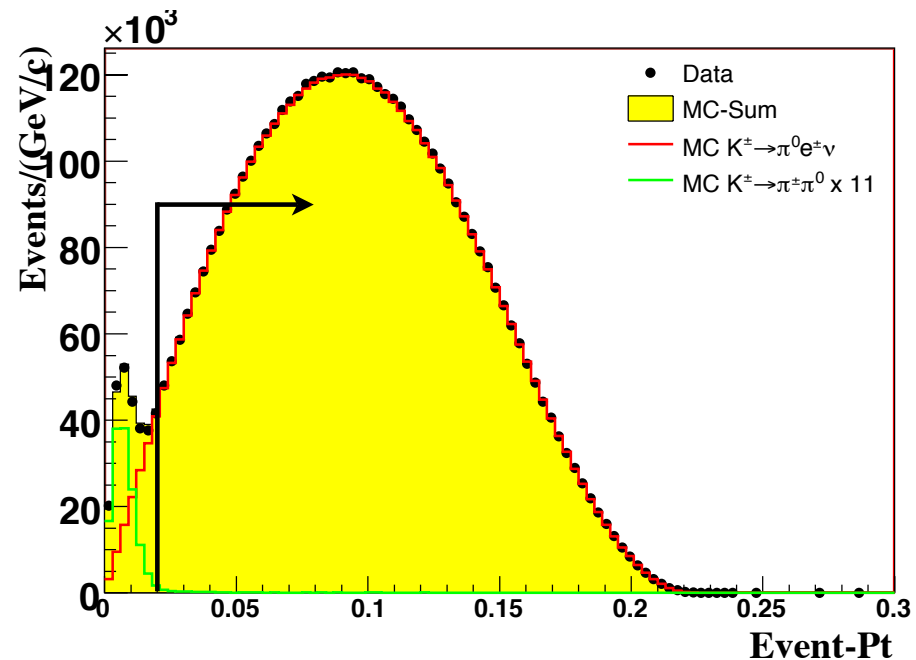
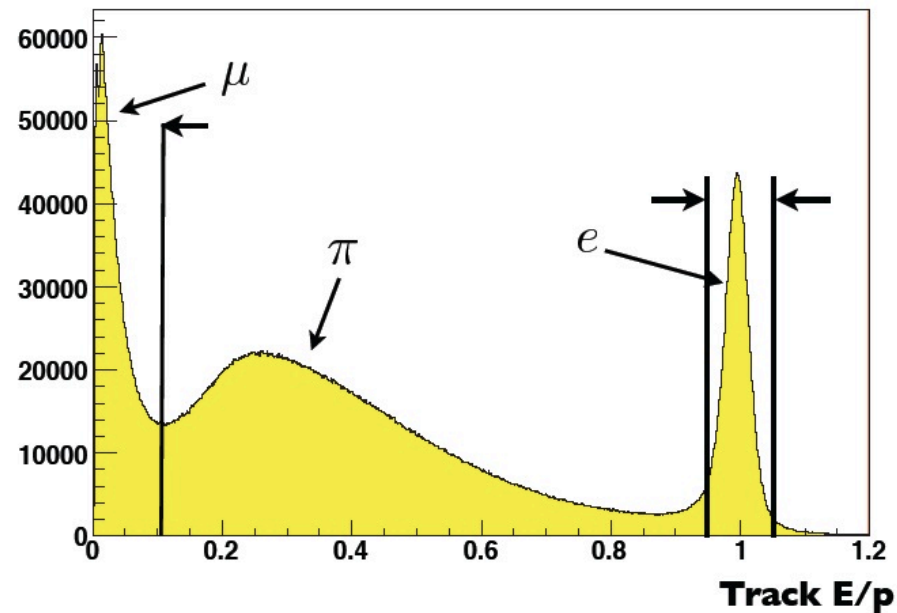
- ▣ 1 good track with particle ID cuts
 - ▣ Muon identified by muon veto and $E/p < 0.1$
 - ▣ Electron identified by $0.95 < E/p < 1.05$
- ▣ 1 good $\pi^0 \rightarrow \gamma\gamma$: $|m_{\gamma\gamma} - m_{\text{PDG}}(\pi^0)| < 10 \text{ MeV}$

Event reconstruction:

- ▣ LKr clusters and lepton track consistent in time
- ▣ Missing mass compatible with $M_\nu = 0$ using 60 GeV K^\pm hypothesis:
 - ▣ $M_{K\ell 3}^2 = (P_K - P_\ell - P_{\pi^0})^2 < 10 \text{ MeV}^2$
- ▣ K energy reconstructed under the assumption of a missing undetected neutrino:
 - $55 \text{ GeV} < E_K^\pm < 65 \text{ GeV}$



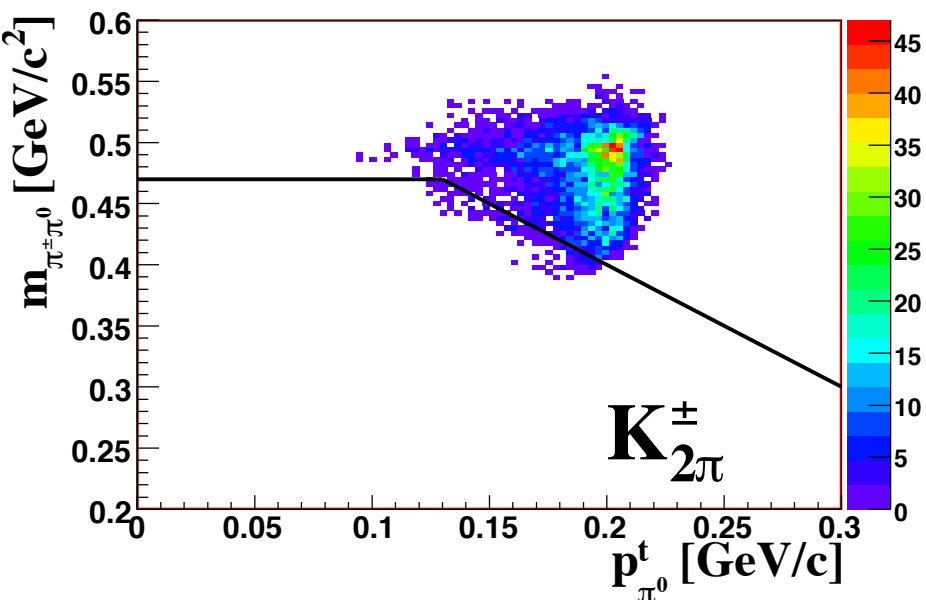
$\pi^+\pi^0$ background K_{e3}



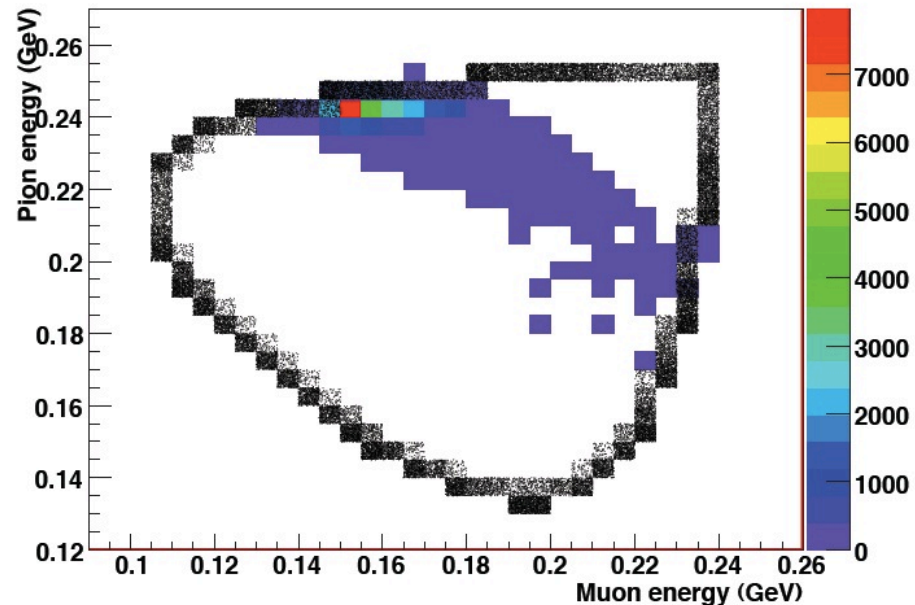
- Pion with $E/P > 0.95$ can fake a K_{e3}^\pm decay: $\pi^0 e_{\text{fake}} + \text{missing } E$
 - Missing energy coming from wrong assignment of electron mass to π^+
 - $\pi^+\pi^0$ will in any case have a small P_T due to no missing momentum
- Keep only events with: $p_{\text{event}}^T > 0.02 \text{ GeV}/c$
 - Background contamination reduced to $< 0.1\%$
 - Only about 3% of genuine K_{e3}^\pm events are lost

$\pi^+\pi^0$ background $K\mu 3$

Pion-Pt vs inv PiPi-Mass



Dalitz Plot PiPi0 background



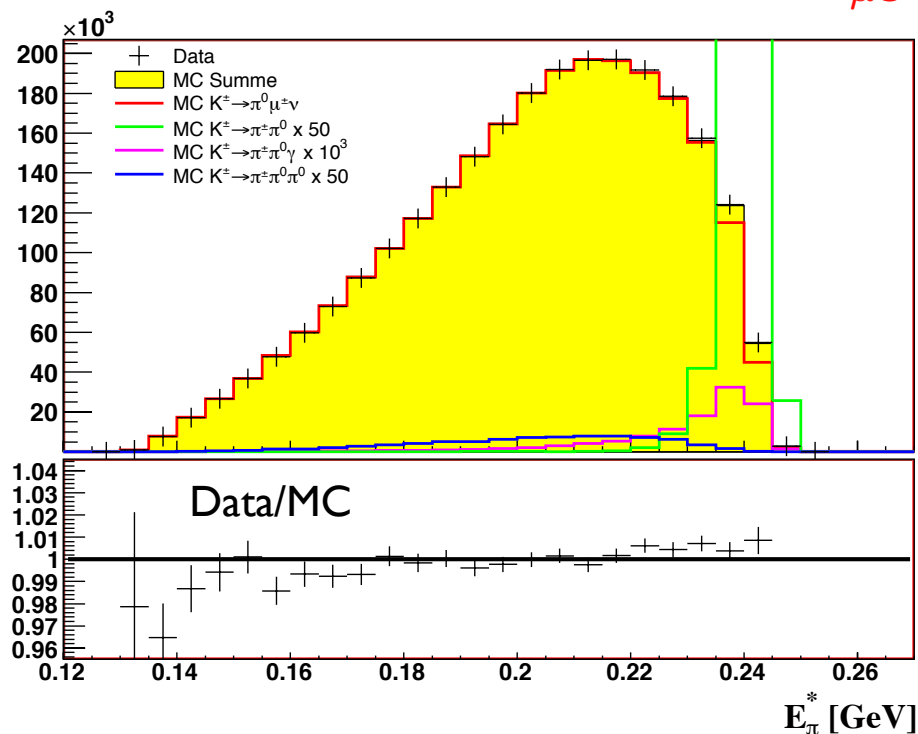
- $K^\pm \rightarrow \pi^\pm \pi^0$ with $\pi^\pm \rightarrow \mu^\pm \nu$ has same final state of the signal 2γ and 1μ
 - Without suppression, $K^\pm \rightarrow \pi^\pm \pi^0$ background at the level of 20%
- Cut in the invariant $\pi^\pm \pi^0$ mass and the transverse momentum of the pion:
 - Background contamination reduced to 0.5%
 - About 24% of $K^\pm \mu 3$ events are lost
 - Background is well localized in the Dalitz plot

Data – MC comparison $E_{\pi^0}^*$

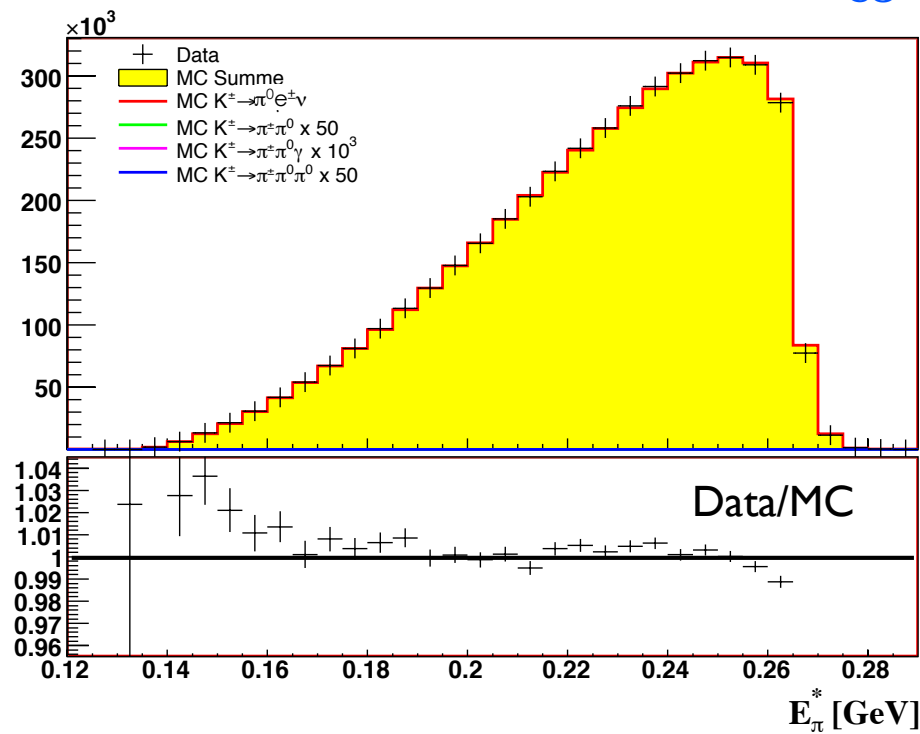
$K_{\mu 3}$ data sample
 2.5×10^6 $K_{\mu 3}$ events
 0.5% BG from $K^{\pm} \rightarrow \pi^{\pm} \pi^0$

Ke3 data sample
 4.0×10^6 Ke3 events
 BG < 0.1% from $K^{\pm} \rightarrow \pi^{\pm} \pi^0$

• Pion energy in the kaon rest frame: $K_{\mu 3}^{\pm}$



• Pion energy in the kaon rest frame: $K_{e 3}^{\pm}$



Radiative corrections

The $K\ell 3$ decay rate including first order radiative corrections can be written as:

$$\Gamma_{K\ell 3} = \Gamma_{K\ell 3}^0 + \Gamma_{K\ell 3}^1 = \Gamma_{K\ell 3}^0 (1 + 2\delta_{EM})$$

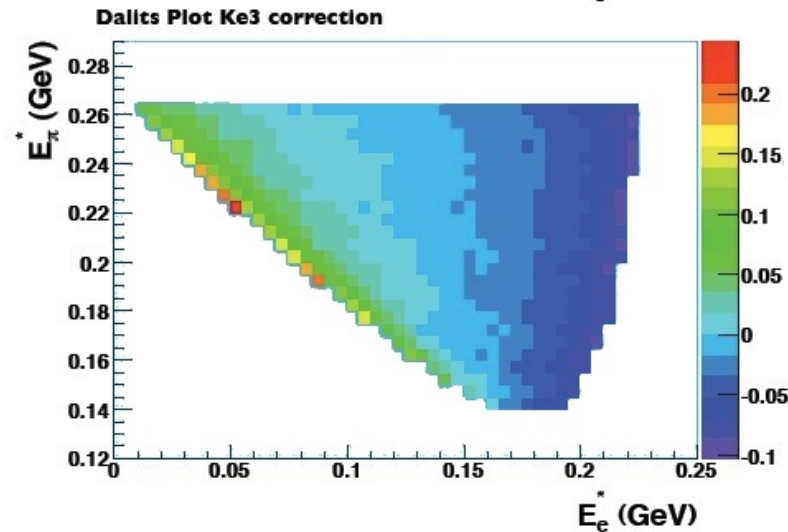
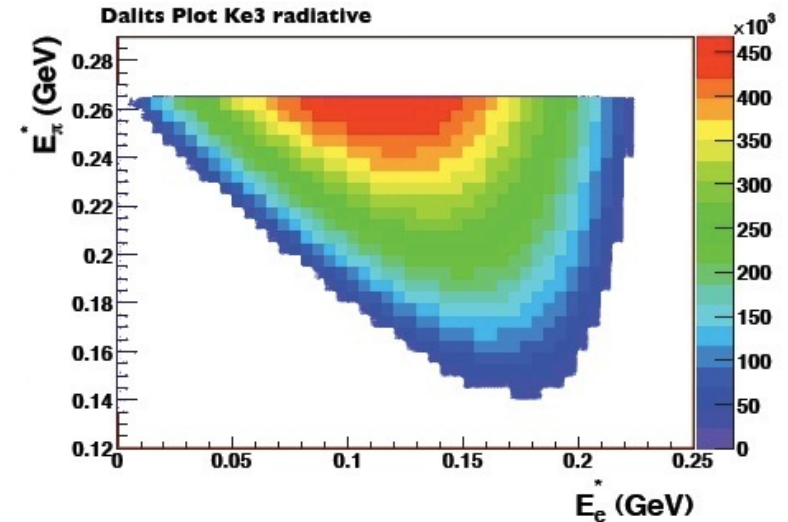
Simulation code provided by KLOE:
author C. Gatti, *EPJ C45 (2006) 417*

Parameters used for the normalization:

Mode	$\delta_{EM}(\%)$
$K^\pm e 3$ (<i>JHEP 11 (2008) 006</i>)	0.050 ± 0.125
$K^\pm \mu 3$ (<i>JHEP 11 (2008) 006</i>)	0.008 ± 0.125

Effects on the $Ke 3$ acceptance are bigger with respect to $K\mu 3$:

- ~10% effect on the Dalitz plot slope for $Ke 3$
- ~1% effect on slope for $K\mu 3$



Form factors fitting procedure

To extract the form factors a fit to the Dalitz plot density is performed:

$$\rho(E_l^*, E_\pi^*) = \frac{d^2 N(E_l^*, E_\pi^*)}{dE_\mu^* dE_\pi^*} \propto Af_+^2(t) + Bf_+(t)(f_0 - f_+) \frac{m_K^2 - m_\pi^2}{t} + C \left[(f_0 - f_+) \frac{m_K^2 - m_\pi^2}{t} \right]^2$$

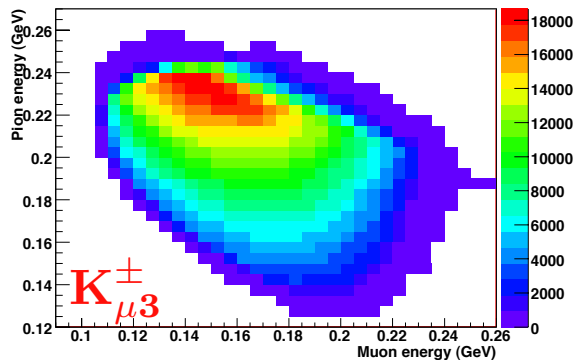
E_l^* and E_π^* are the energy of the lepton and of the pion in the kaon rest frame

A, B and C are kinematical terms

The fit is performed in cells of $5 \times 5 \text{ MeV}^2$

Cells which are outside or crossing the border of the physical region of the Dalitz plot are not used in the fit.

reconstructed data dalitz plot

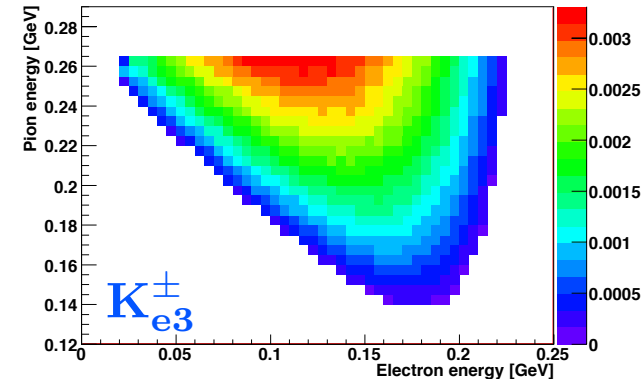
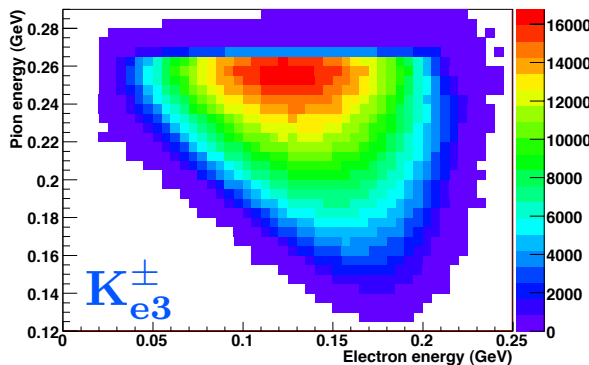
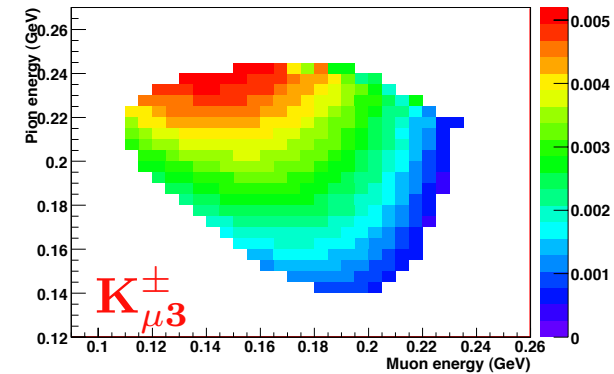


Applied corrections:

- Background subtraction
- Acceptance
- Radiative corrections



corrected dalitz plot



FF fit preliminary results

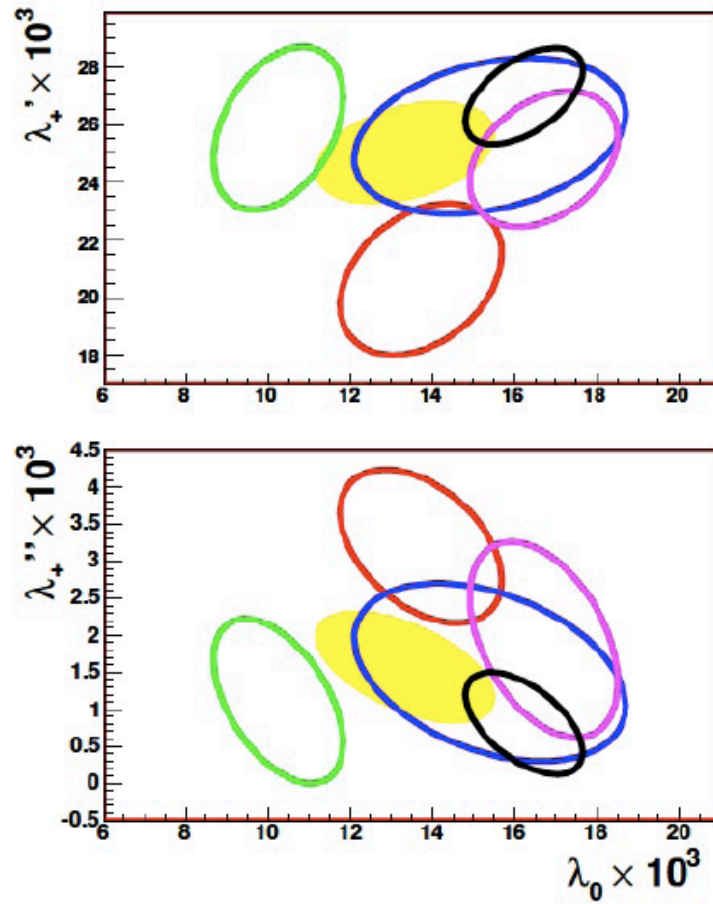
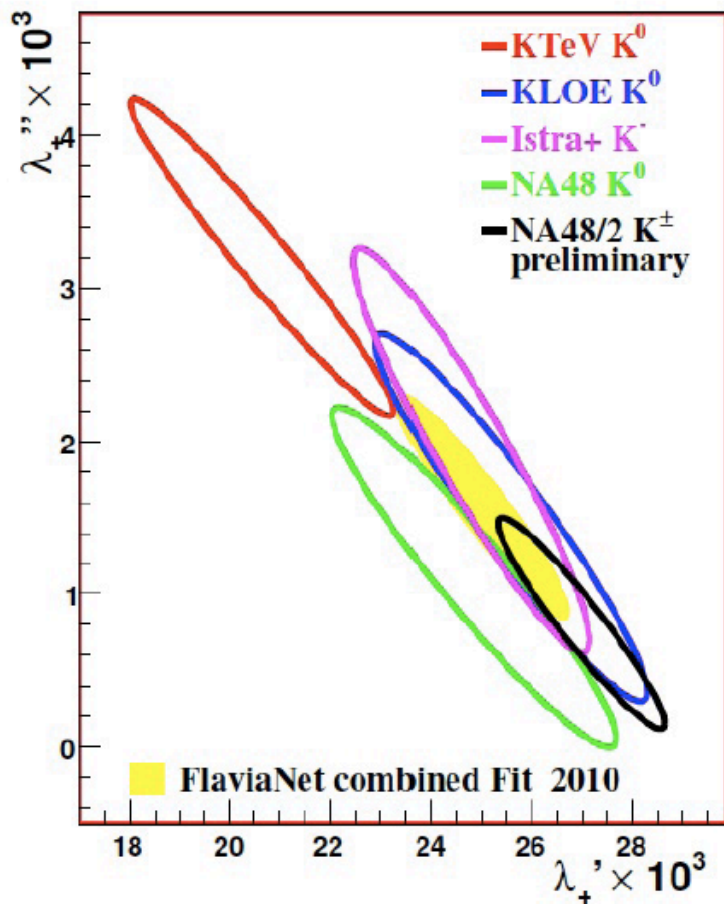
NA48/2 Form Factors fits preliminary results

Quadratic ($\times 10^{-3}$)	λ'_+	λ''_+	λ_0
$K_{\mu 3}^\pm$	$26.3 \pm 3.0_{\text{stat}} \pm 2.2_{\text{syst}}$	$1.2 \pm 1.1_{\text{stat}} \pm 1.1_{\text{syst}}$	$15.7 \pm 1.4_{\text{stat}} \pm 1.0_{\text{syst}}$
K_{e3}^\pm	$27.2 \pm 0.7_{\text{stat}} \pm 1.1_{\text{syst}}$	$0.7 \pm 0.3_{\text{stat}} \pm 0.4_{\text{syst}}$	
Pole (MeV/c ²)	m_V		m_S
$K_{\mu 3}^\pm$	$873 \pm 8_{\text{stat}} \pm 9_{\text{syst}}$		$1183 \pm 31_{\text{stat}} \pm 16_{\text{syst}}$
K_{e3}^\pm	$879 \pm 3_{\text{stat}} \pm 7_{\text{syst}}$		

Systematic errors

$K_{\mu 3}^\pm$	$\Delta\lambda'_+$	$\Delta\lambda''_+$	$\Delta\lambda_0$	Δm_V	Δm_S	K_{e3}^\pm	$\Delta\lambda'_+$	$\Delta\lambda''_+$	Δm_V
		$\times 10^{-3}$		MeV/c ²				$\times 10^{-3}$	
Kaon Energy	± 0.1	± 0.0	± 0.3	± 1	± 8	Kaon Energy	± 0.3	± 0.1	± 6
Vertex	± 1.0	± 0.5	± 0.1	± 2	± 7	Vertex	± 0.2	± 0.1	± 0
Bin size	± 0.8	± 0.4	± 0.7	± 3	± 10	Bin size	± 0.0	± 0.1	± 2
Energy scale	± 0.3	± 0.1	± 0.1	± 0	± 1	Energy scale	± 0.1	± 0.0	± 0
Acceptance	± 0.2	± 0.1	± 0.3	± 2	± 5	Acceptance	± 0.2	± 0.0	± 3
$K_{2\pi}$ background	± 1.7	± 0.5	± 0.6	± 3	± 0	2nd Ana	± 0.9	± 0.4	± 1
2nd Analysis	± 0.1	± 0.1	± 0.2	± 2	± 5	FF input	± 0.4	± 0.0	± 1
FF input	± 0.3	± 0.8	± 0.1	± 7	± 3	Systematic	± 1.1	± 0.4	± 7
Systematic	± 2.2	± 1.1	± 1.0	± 9	± 16	Statistical	± 0.7	± 0.3	± 3
Statistical	± 3.0	± 1.1	± 1.4	± 8	± 31				

Fit results comparison



Combined quadratic fit results for $K\ell 3$ decays.

The ellipses are 68% CL contours.

For comparison the combined fit from the FlaviaNet kaon working group is shown in yellow. [Eur. Phys. J. C 69, 399 \(2010\)](#)

FF fit preliminary results: combined

Quadratic ($\times 10^{-3}$)	λ'_+	λ''_+	λ_0
$K_{\mu 3}^\pm K_{e 3}^\pm$ combined	26.98 ± 1.11	0.81 ± 0.46	16.23 ± 0.95
Pole (MeV/c ²)	m_V		m_S
$K_{\mu 3}^\pm K_{e 3}^\pm$ combined	877 ± 6		1176 ± 31

Status of experimental measurements

- ▣ $K_L^0 \rightarrow \ell 3$: KLOE, KTeV and NA48
- ▣ $K^- \rightarrow \ell 3$: from ISTRA+
- ▣ $K^\pm \rightarrow \ell 3$: NA48/2 has the first measurement with $K_{e 3}$ and $K_{\mu 3}$
 - ▣ NA48/2 results for $K_{e 3}$ and $K_{\mu 3}$ are in good agreement
 - ▣ NA48/2 combined result has the smallest error.

$K^\pm \rightarrow \pi^\pm \gamma\gamma$ decay

$K^\pm \rightarrow \pi^\pm \gamma \gamma$ decay theory

In the ChPT framework the differential rate of the decay $K^\pm(p) \rightarrow \pi^\pm(p_3) \gamma(q_1) \gamma(q_2)$ process (no $O(p^2)$ contribution) is:

$$\frac{\partial^2 \Gamma}{\partial y \partial z} = \frac{m_{K^\pm}}{(8\pi)^3} \cdot \left[z^2 \cdot (|A+B|^2 + |C|^2) + \underbrace{\left(y^2 - \frac{1}{4} \lambda(1, z, r_\pi^2) \right)^2}_{\text{relevant only @ low } m_{\gamma\gamma}} \cdot (|B|^2 + |D|^2) \right] \quad \begin{aligned} y &= \frac{p \cdot (q_1 - q_2)}{m_{K^\pm}^2} \\ z &= \frac{m_{\gamma\gamma}^2}{m_{K^\pm}^2} \end{aligned}$$

- ▣ The leading $O(p^4)$ contribution is given by $A(z, \hat{c})$ (loops) which is responsible for a cusp at $m_{\gamma\gamma} = m_{2\pi}$

$$\Gamma_A = (2.80 + 0.87\hat{c} + 0.17\hat{c}^2) \cdot 10^{-20} \text{ MeV}$$

- ▣ C (WZW) corresponds to $\sim 10\%$ of A at $O(p^4)$

[Ecker, Pich, de Rafael, Nucl. Phys. B303 (1988), 665]

- ▣ B, D = 0 @ $O(p^4)$

- ▣ The \hat{c} value can be related to fundamental ChPT parameters:

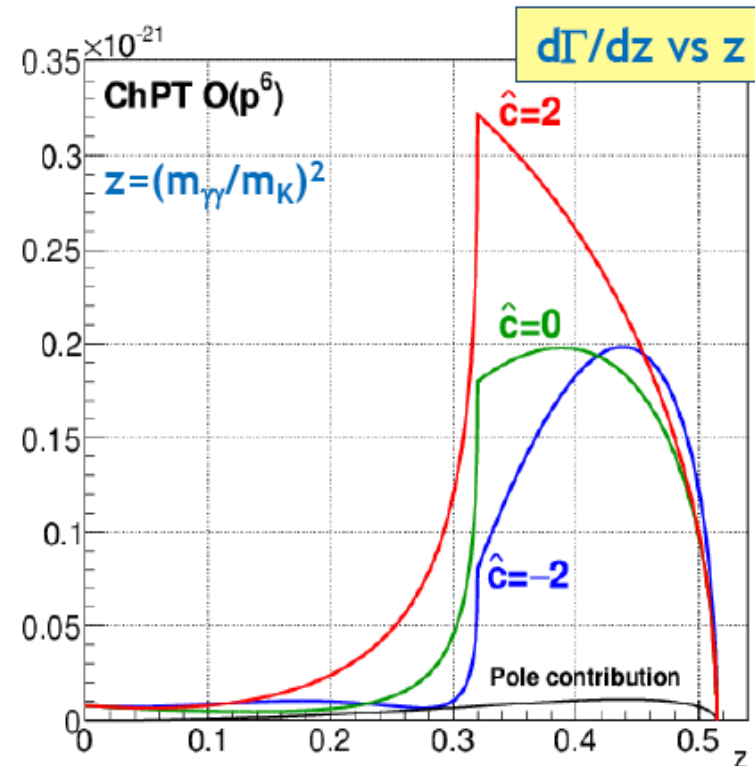
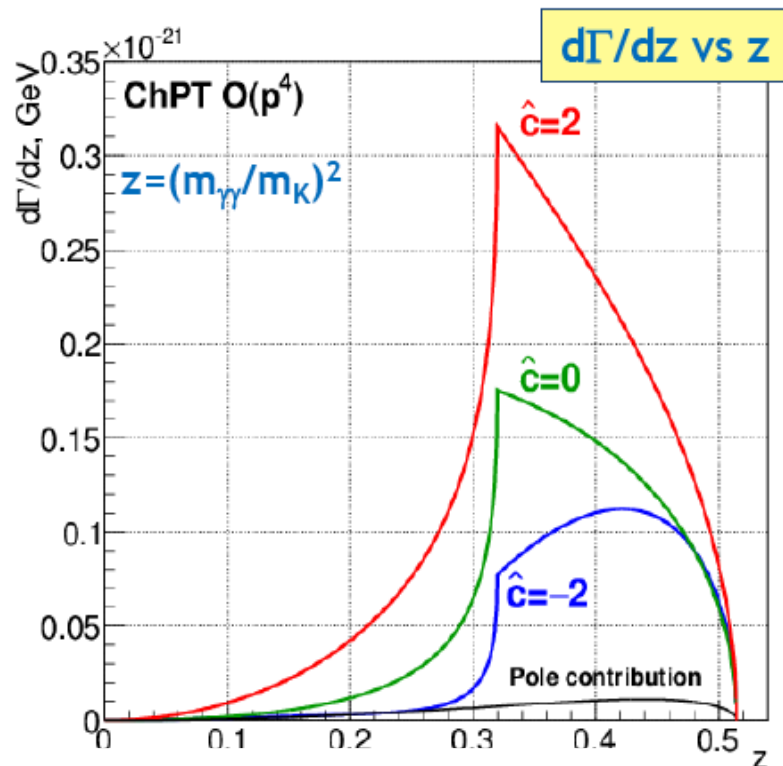
[D'Ambrosio, Portoles, PLB 386 (1996), 403]

$$\hat{c} = \frac{128\pi^2}{3} \left[3(L_9 + L_{10}) + (N_{14} - N_{15} - 2N_{18}) \right]$$

**Weak Chiral Lagrangian
QCD loop and counterterms**

$K^\pm \rightarrow \pi^\pm \gamma \gamma$ decay, ChPT $O(p^4)$ vs $O(p^6)$

- $O(p^6)$ loop amplitude correction, evaluated from $K \rightarrow 3\pi$ @ $O(p^4)$
- $O(p^6)$ A term gets y dependence, sizable correction to $d\Gamma/dz$ for $z < z^*$



$O(p^6)$ unitarity corrections may increase the BR by 30÷40%

[D'Ambrosio, Portolés, PLB386 (1996) 403]

$K^\pm \rightarrow \pi^\pm \gamma\gamma$ decay experimental status

Present PDG average based on:

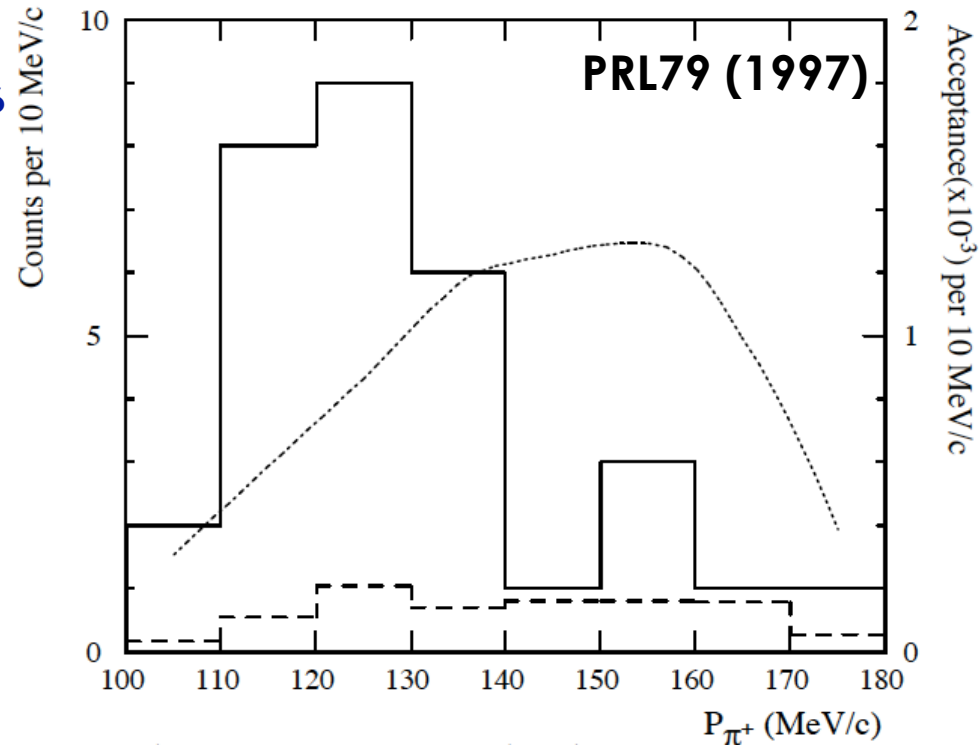
E787 (1997): BR = $(1.10 \pm 0.32) \times 10^{-6}$

31 candidates, 5 exp. Bkg

\hat{c} value = 1.6 ± 0.6 $O(p^4)$

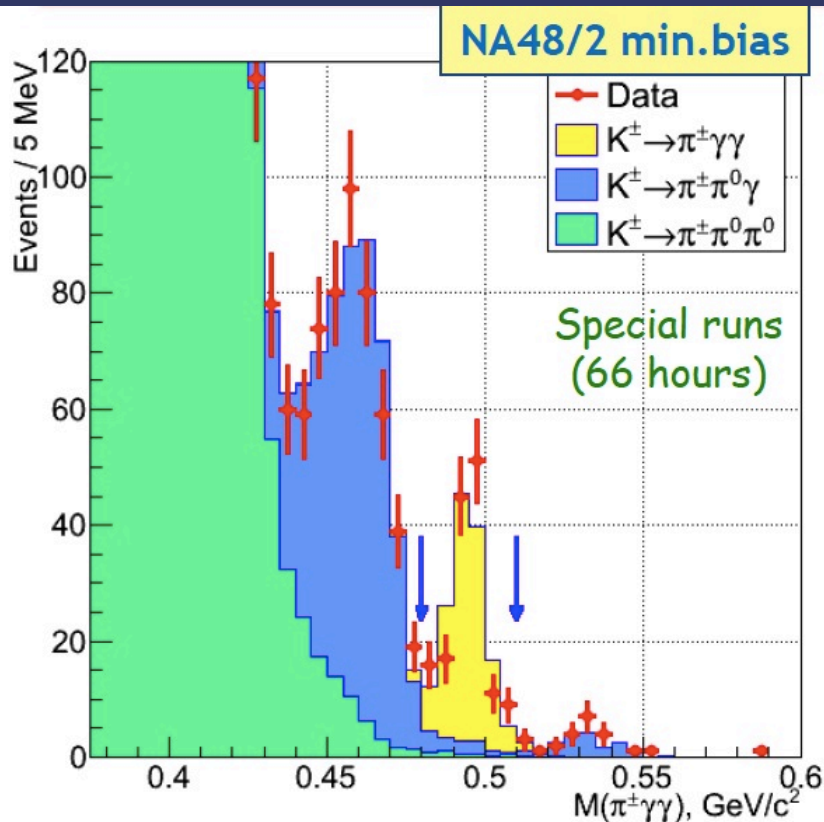
= 1.8 ± 0.6 $O(p^6)$

[PRL79 (1997) 4079]

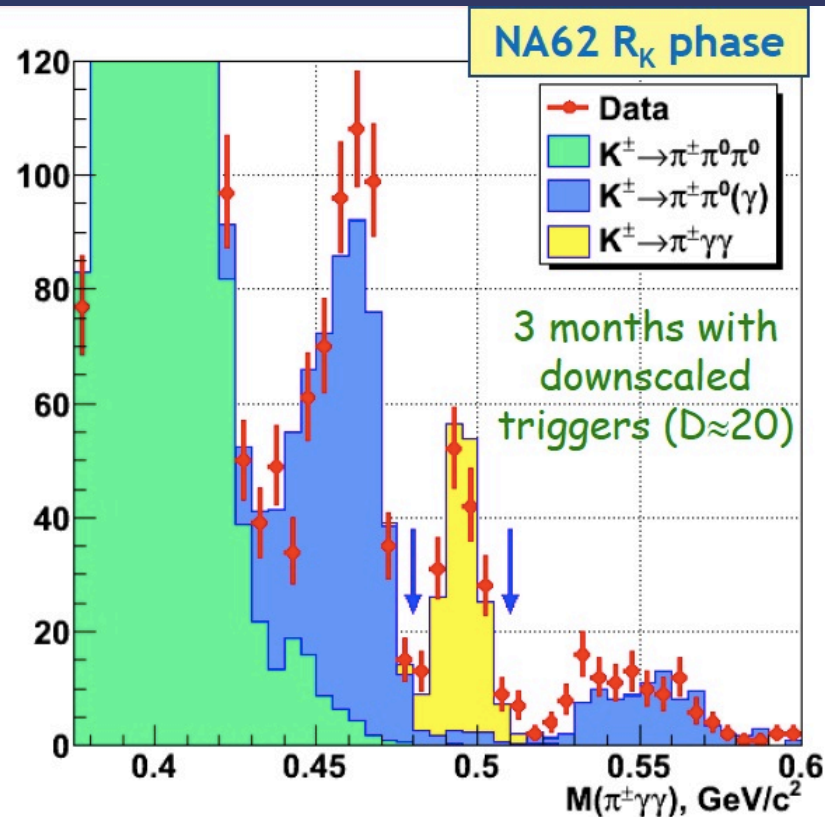


- NA48/2 main data set preliminary 2003/2004: measurement hindered by low trigger efficiency. Abandoned!
- **New strategy: minimum bias trigger** samples from NA48/2 and NA62.
 - 2004 data taking (66 hours special minimum bias run)
 - 2007 data taking (3 month control trigger downscaled by 20)

Minimum bias data samples



$\pi^\pm\gamma\gamma$ candidates	149
BG($\pi^\pm\pi^0\gamma$)	11.4 ± 0.6
BG($\pi^\pm\pi^0\pi^0$)	4.1 ± 0.4
Signal ($\pi^\pm\gamma\gamma$)	134 ± 12



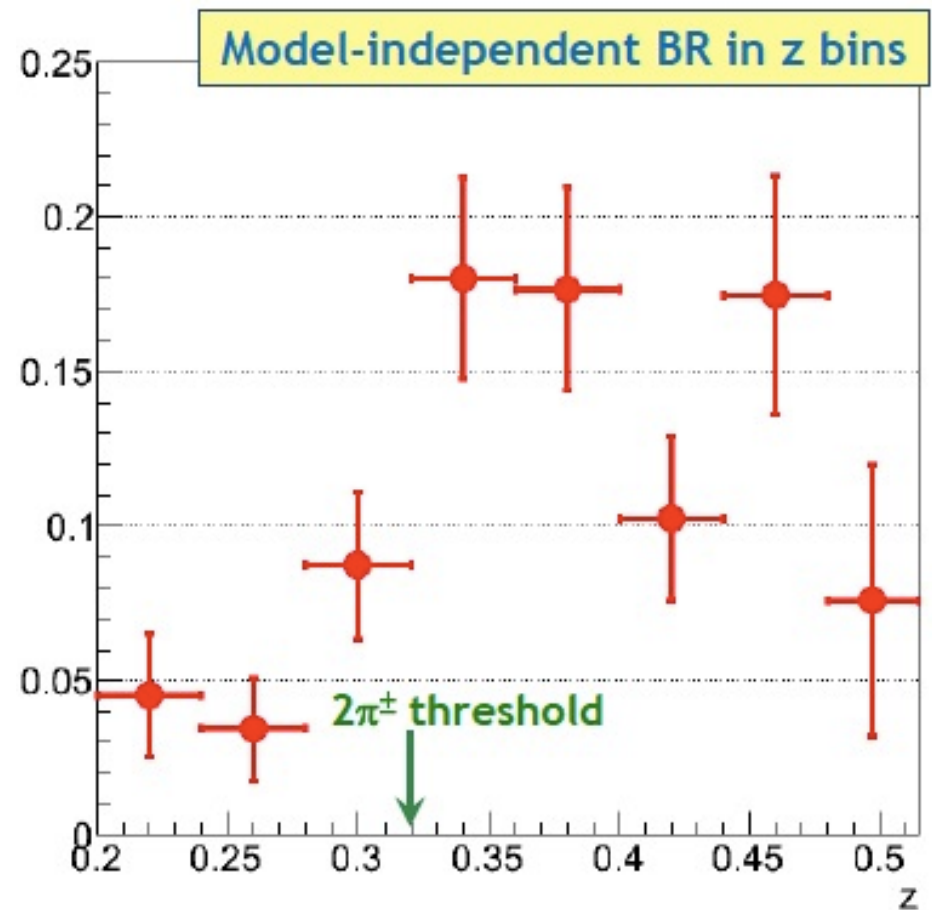
$\pi^\pm\gamma\gamma$ candidates	175
BG($\pi^\pm\pi^0\gamma$)	11.1 ± 1.0
BG($\pi^\pm\pi^0\pi^0$)	1.3 ± 0.3
Signal ($\pi^\pm\gamma\gamma$)	163 ± 13

~300 candidates 10 times the present world sample

Model independent BR NA48/2

z range	N_j	N_j^B	A_j	$B_j \times 10^6$
0.20–0.24	13	4.89	0.194	0.045 ± 0.020
0.24–0.28	9	2.73	0.198	0.034 ± 0.016
0.28–0.32	18	2.33	0.194	0.087 ± 0.024
0.32–0.36	33	1.30	0.190	0.180 ± 0.033
0.36–0.40	31	0.98	0.184	0.177 ± 0.033
0.40–0.44	18	1.61	0.173	0.103 ± 0.027
0.44–0.48	23	1.21	0.135	0.175 ± 0.038
$z > 0.48$	4	0.52	0.049	0.076 ± 0.044

Sufficiently small z bins: acceptance almost independent of kinematical distribution



FINAL NA48/2 Model Independent BR:

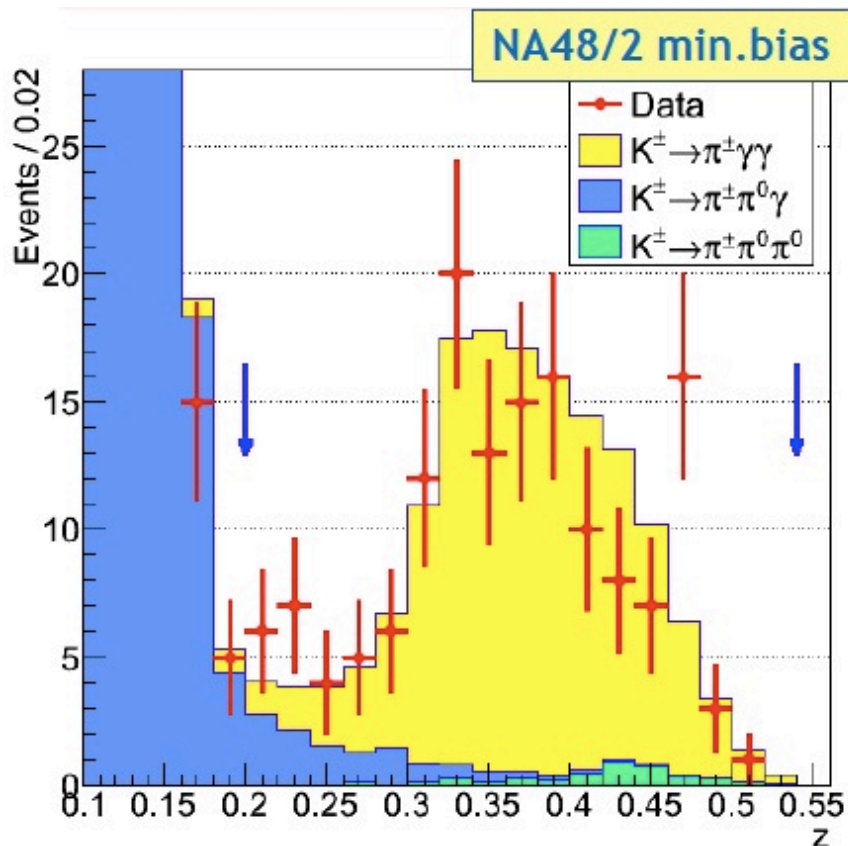
$$BR_{MI}(z > 0.2) = (0.877 \pm 0.087_{\text{stat}} \pm 0.017_{\text{syst}}) \times 10^{-6}$$

$K^\pm \rightarrow \pi^\pm \gamma \gamma$ ChPT fit $O(p^6)$ (preliminary)

External parameters of the $O(p^6)$ fit:

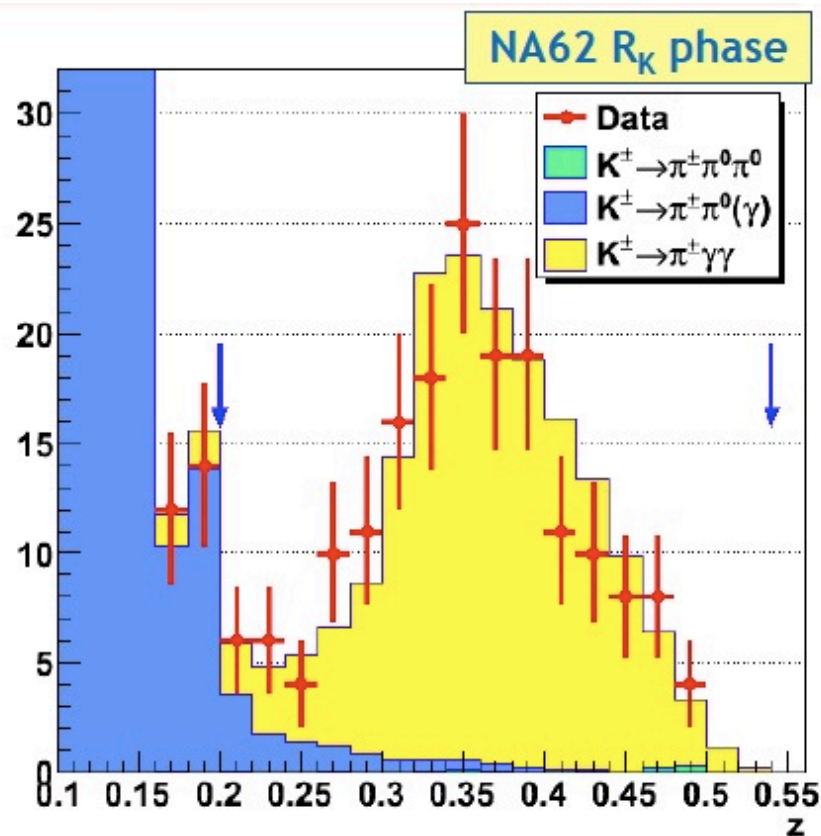
$K3\pi$ amplitude parameters: from fit to experimental data [NPB648 (2003) 317];

Polynomial contributions: $\eta_1=2.06$, $\eta_2=0.24$, $\eta_3=-0.26$ [PLB386 (1996) 403].



ChPT $O(p^4)$: $\hat{c} = 1.36 \pm 0.34$

ChPT $O(p^6)$: $\hat{c} = 1.67 \pm 0.40$



ChPT $O(p^4)$: $\hat{c} = 1.71 \pm 0.30$

ChPT $O(p^6)$: $\hat{c} = 2.21 \pm 0.32$

Combined fit results (preliminary)

NA48/2 and NA62-RK combined ChPT $O(p^4)$ fit:

$$\hat{c} = 1.56 \pm 0.22_{\text{stat}} \pm 0.07_{\text{syst}} = 1.56 \pm 0.23$$

NA48/2 and NA62-RK combined ChPT $O(p^6)$ fit:

$$\hat{c} = 2.00 \pm 0.24_{\text{stat}} \pm 0.09_{\text{syst}} = 2.00 \pm 0.26$$

Using ChPT formulation: D'Ambrosio, Portolés, PLB386 (1996) 403

Combined Model Dependent BR in full phase space:

$$\text{BR}_{O(p^6)} = (1.01 \pm 0.06) \times 10^{-6}$$

To be compared with PDG(BNL E787): $\text{BR}_6 = (1.10 \pm 0.32) \times 10^{-6}$

Summary and outlook

- NA48/2 has released a preliminary results on the $K^{\pm}\ell^3$ form factors:
 - Quadratic and Pole parameterizations used
 - First results studying both K^+ and K^- decays
 - Competitive result for $K_{\mu 3}$ and smallest error for $K_{e 3}$ and combined fits
 - Possibility to remeasure $K^{\pm}\ell^3$ BR and form factors in NA62-RK data set
 - A K^0_L data set has been also collected allowing measurement in neutral kaons as well
- $K^{\pm}\rightarrow\pi^{\pm}\gamma\gamma$ decay has been studied in NA48/2 and NA62-RK data sets
 - NA48/2 final result on model independent branching ratio:
$$BR_{MI}(z>0.2)=(0.877\pm 0.087_{\text{stat}}\pm 0.017_{\text{syst}})\times 10^{-6}$$
 - NA48/2 and NA62-RK combined Model Dependent BR in full phase space:
$$BR_{Op6}=(1.01\pm 0.06)\times 10^{-6} \quad \text{preliminary}$$
 - Measurement of \hat{c} NA48/2 and NA62-RK combined ChPT $O(p^6)$ fit:
$$\hat{c} = 2.00\pm 0.24_{\text{stat}}\pm 0.09_{\text{syst}} = 2.00\pm 0.26 \quad \text{preliminary}$$

Back up slides

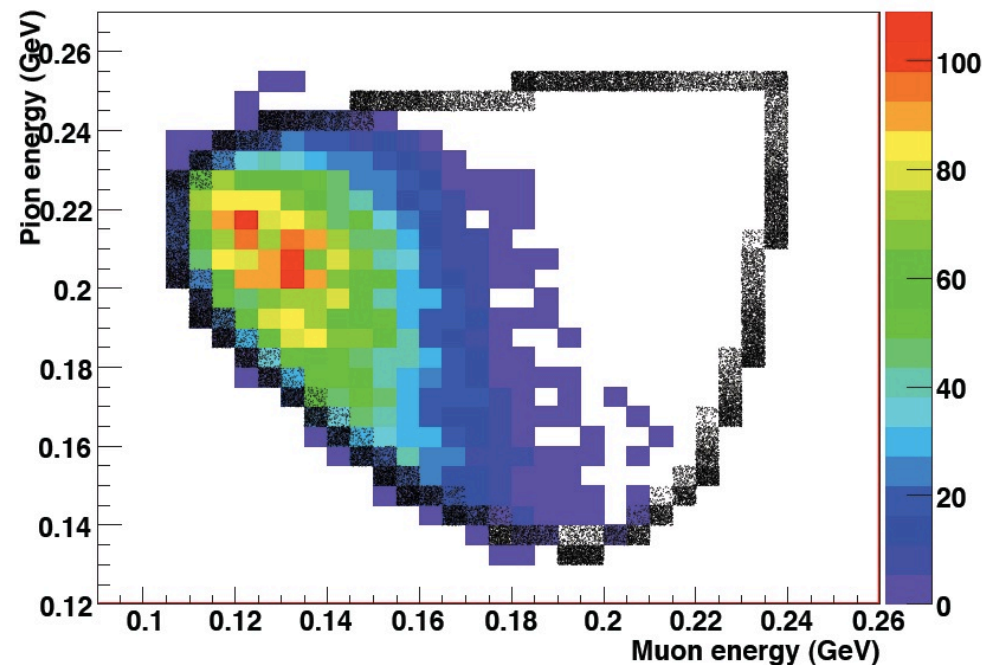
11/09/2013

$\pi^\pm\pi^0\pi^0$ background

- Background to $K\mu 3$
 - $\pi \rightarrow \mu$ decay + 2 lost photons from π^0 -decays.
 - **Small** but introduces slope in the Dalitz plot.
 - **No dedicated cut** to reduce the background is applied.
 - A correction is applied to take the background into account.
 - Without the correction the result shifts by $\sim 0.5\sigma_{\text{stat}}$

- Background to $Ke3$
 - Negligible

Dalitz Plot $\pi^+\pi^0\pi^0$ background



Dominant $K_{\pm}3$ BR in the V_{US} fit

Absolute BR of $Ke3$ and $K_{\mu}3$ from PDGlive

$Ke3(\text{PDG}) = (5.07 \pm 0.04) \% \quad \mathbf{0.8\%}$ error

$K_{\mu}3(\text{PDG}) = (3.353 \pm 0.034) \% \quad \mathbf{1.0\%}$ error

Ratio of $Ke3$ and $K_{\mu}3$ in NA48/2 2003 data

$$\mathcal{R}_{Ke3/K2\pi} = 0.2470 \pm 0.0009 (\text{stat}) \pm 0.0004 (\text{syst})$$

$$\mathcal{R}_{K\mu3/K2\pi} = 0.1637 \pm 0.0006 (\text{stat}) \pm 0.0003 (\text{syst})$$

$$\mathcal{R}_{K\mu3/Ke3} = 0.663 \pm 0.003 (\text{stat}) \pm 0.001 (\text{syst})$$

$BR(Ke3)/BR(K2\pi) = 0.4\%$ error dominated by statistic with 87×10^3 $Ke3$

$BR(K_{\mu}3)/BR(K2\pi) = 0.4\%$ error dominated by statistic with 77×10^3 $K_{\mu}3$